Einstein-Charged Scalar Field Theory: Black Hole Solutions and Their Stability

Supakchai Ponglertsakul

in collaboration with: Sam Dolan, Elizabeth Winstanley

University of Sheffield

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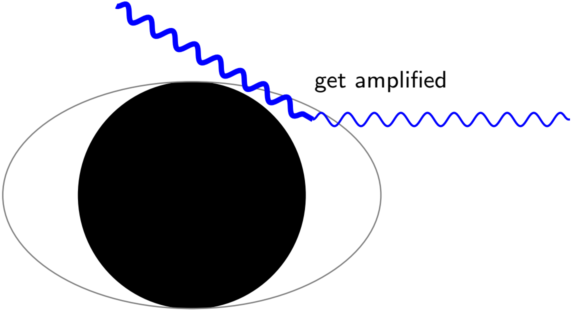
Plan

1. Motivation
2. The model
    - Einstein-charged scalar theory
    - Hairy black hole
3. Stability
    - Numerical results

Motivation

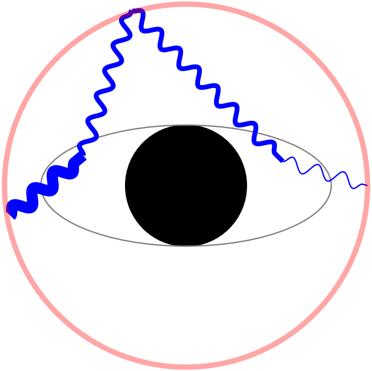
Kerr black hole



- Superradiant scattering (Zeldovich' 71, Misner' 72)
- Charge analogue (Bekenstein' 73)
- The amplification happens if

$$\omega < m\Omega_H + q\Phi_H$$

Kerr black hole surrounded by a reflecting 'mirror'



- A scattered wave is reflected back to a BH...
- Non-negligible back-reaction on the background
  $\Rightarrow$ superradiant instability
- Black hole bomb (Press & Teukolsky' 72)

What triggers instability?

Rotating:

- Massive fields (Damour et al.' 76, Detweiler' 80)
- AdS (Cardoso et al.' 04)
- Mirror (Cardoso et al.' 04)

Charge:

- AdS (Uchikata et al.' 11)
- Mirror (Herdeiro et al.' 13)
  - Massive charged scalar field in the RN background with a mirror.

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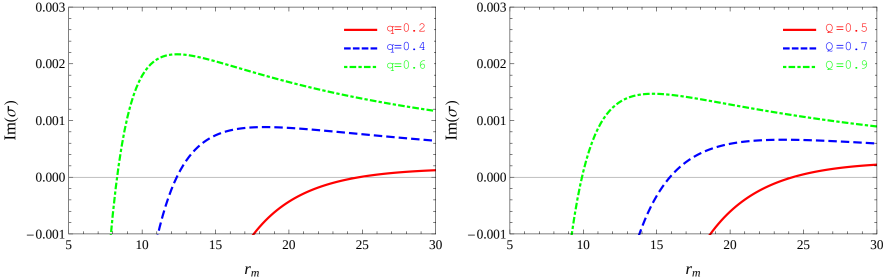
Charge case

Massless charged scalar field in RN background with a mirror

- Ansatz $\Phi \sim e^{-i\sigma t} R(r)$
- Boundary conditions

  $\Rightarrow R(r) \sim$ ingoing wave $r \to r_h$

  $\Rightarrow R(r_m) = 0$; r_m is the position of the mirror
- Numerical results $Q = 0.9$ or $q = 0.5$



$Im(\sigma) > 0 \Rightarrow$ exponentially growing modes

End-point?

Open question: what could be the **end-point** of these superradiant instabilities?

$\Rightarrow$ What sort of BH is the end-point of the instability?

Strategies

- Study fully coupled system of charged scalar field and gravity (Einstein-charged scalar).
- Black hole solutions???
- Stable???

The model

Einstein-Maxwell-Klein-Gordon

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}g^{ab}D^*_{(a}\Phi^* D_{b)}\Phi\right],$$

where $D_a = \nabla_a - iqA_a$ and $X_{(ab)} = \frac{1}{2}(X_{ab} + X_{ba})$.

- Einstein field equations $G_{ab} = 8\pi G\left(T^F_{ab} + T^\Phi_{ab}\right)$
- Maxwell equations $\nabla_a F^{ab} = \frac{iq}{2}\left(\Phi^* D^b\Phi - \Phi(D^b\Phi)^*\right)$
- Klein-Gordon equation $D_a D^a\Phi = 0$

The metric ansatz is

$$ds^2 = -f(r)h(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2,$$

and the vector potential $A_a = [A_0(r), 0, 0, 0]$.

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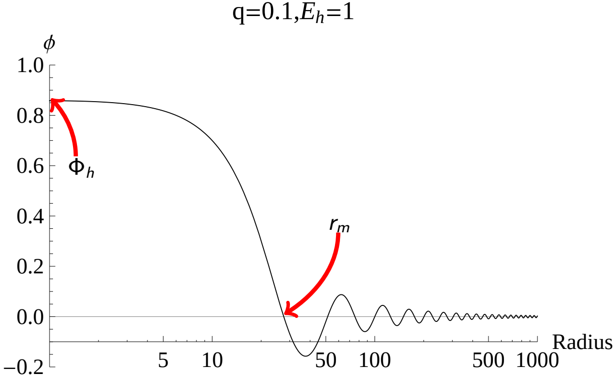
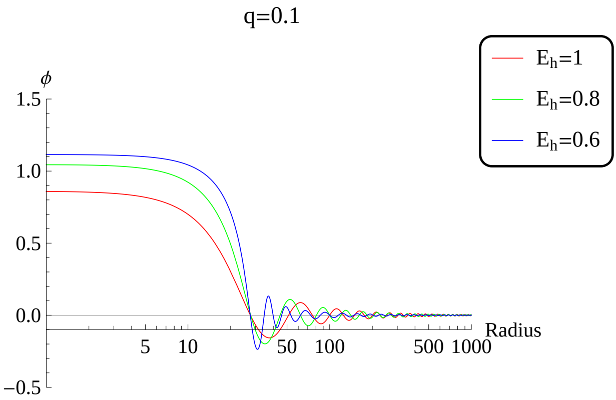
Hairy black hole



- $E_h \equiv A_0'(r_h)$
- Mirror at a node of Φ
- Focus on the first node $\Rightarrow$ more likely to be stable
- Each (Φ_h, r_m) represents one BH solution.

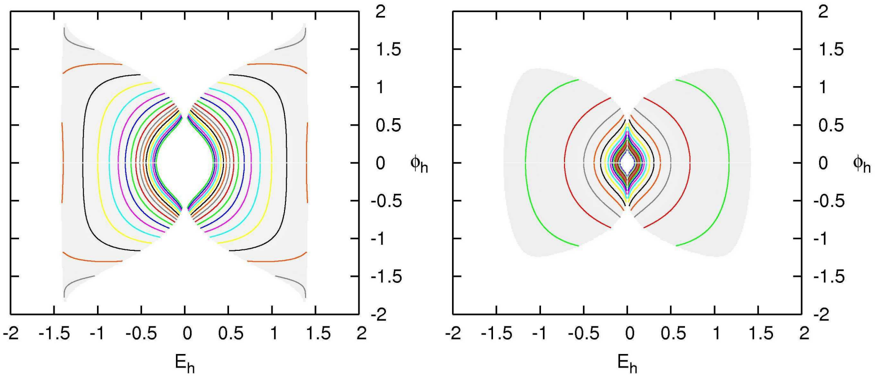
More solutions...



- Different BH solutions, but share the same r_m.

Solution space

$$q = 0.1 \textit{ and } q = 0.4$$



- Grey area: solution exists with a node (mirror).
- Contour line: the radius of the first node.

Stability

Perturbations

$$f = \bar{f}(r) + \delta f(t,r), \qquad h = \bar{h}(r) + \delta h(t,r),$$

$$A_0 = \bar{A}_0(r) + \delta A_0(t,r), \quad \Phi = \bar{\Phi}(r) + \delta\Phi(t,r).$$

- $\delta\Phi \sim e^{-i\sigma t}\tilde{\Phi}(r)$
- Boundary conditions
  $\Rightarrow \tilde{\Phi}(r) \sim$ ingoing wave $r \to r_h$
  $\Rightarrow \tilde{\Phi}(r_m) = 0$
- $Im(\sigma) > 0$ unstable, exponential growth with time
  $Im(\sigma) < 0$ stable.

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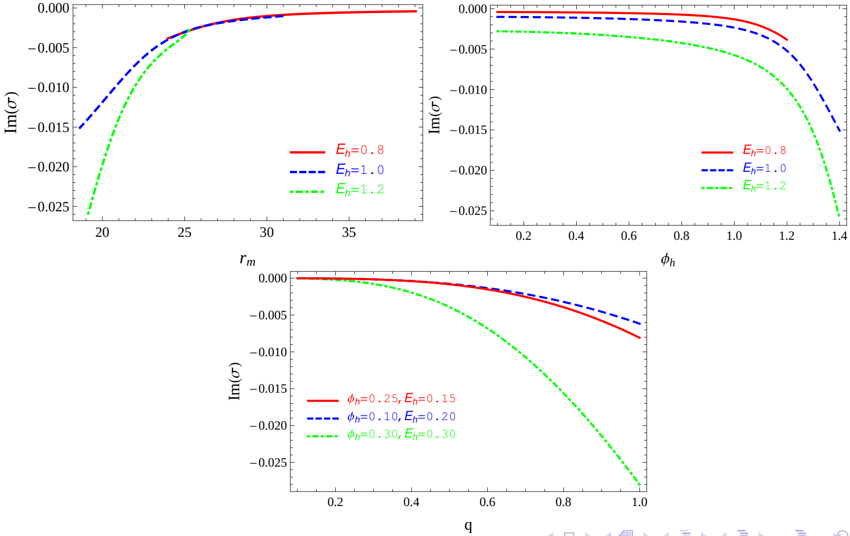
Example results

$q = 0.1, \Phi_h = 0.1 - 1.4$



Summary

- We are interested in a fully coupled Einstein-charged scalar theory with a reflecting mirror.
- Hairy black hole solutions are obtained numerically.
- Early results suggests that these BH are stable.
- (Possible?) end-point of superradiant instability of the charged scalar field in RN background.