A Quantum Cosmic Censorship

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With: A. Orlandi, A. Giugno, F. Kühnel, **O. Micu,** F. Scardigli, **D. Stojkovic…**

Plan of the talk

1. Horizon Quantum Mechanics: gravitational radius of quantum state

2. Charged black holes I: inner horizon and mass inflation

3. Charged black holes II: naked singularity and cosmic censorship

4. Outlook

1) Horizon quantum mechanics

The Planck scale:

$$\sqrt{\hbar\, G_{\rm N}} = \ell_{\rm p} \simeq 10^{-35}\ {\rm m}$$

$$\sqrt{\hbar/G_{\rm N}} = M_{\rm p} \simeq 10^{19}\ {\rm GeV}$$

$$\frac{\hbar}{M_{\rm p}} \simeq G_{\rm N}\, M_{\rm p}$$

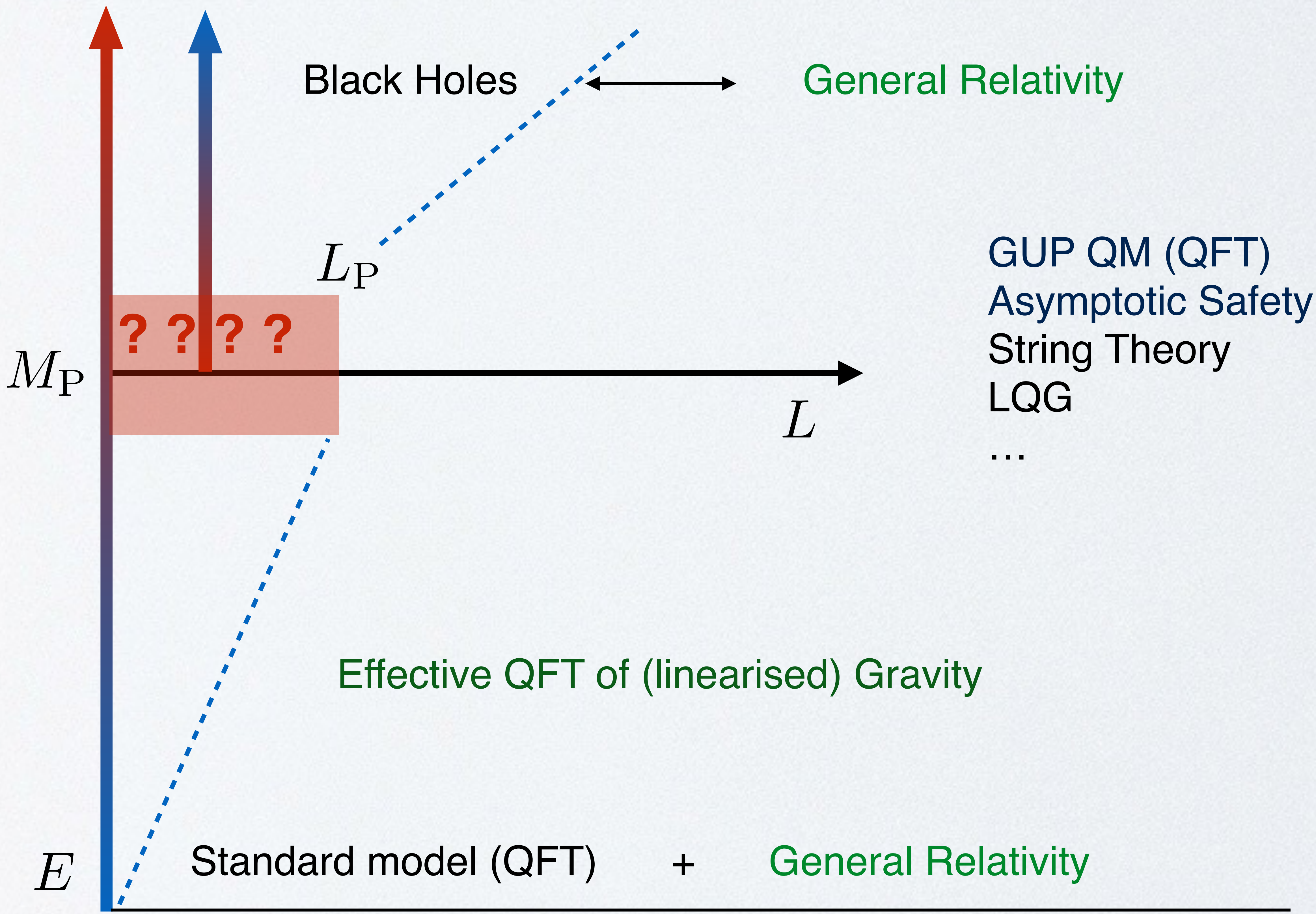
Q(FT) scale

(Strong) Gravity scale

Quantum Gravity
(black holes)

$$G_{\rm N} = \frac{\ell_{\rm p}}{m_{\rm p}}$$

1) Horizon quantum mechanics



1) Horizon quantum mechanics

QFT on fixed (classical) background shows existence of bound states as resonances in scattering process

Hydrogen atom (= spatially extended wave-function) is a **non-perturbative** state (QFT can then describe the Lamb shift around the proper QM state…)

Black Holes ~ Hydrogen atom of gravity?

Non-perturbative description of BHs ~ GR

1) Horizon quantum mechanics

Horizon in a classical spherically symmetric system:

$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

Einstein equations:

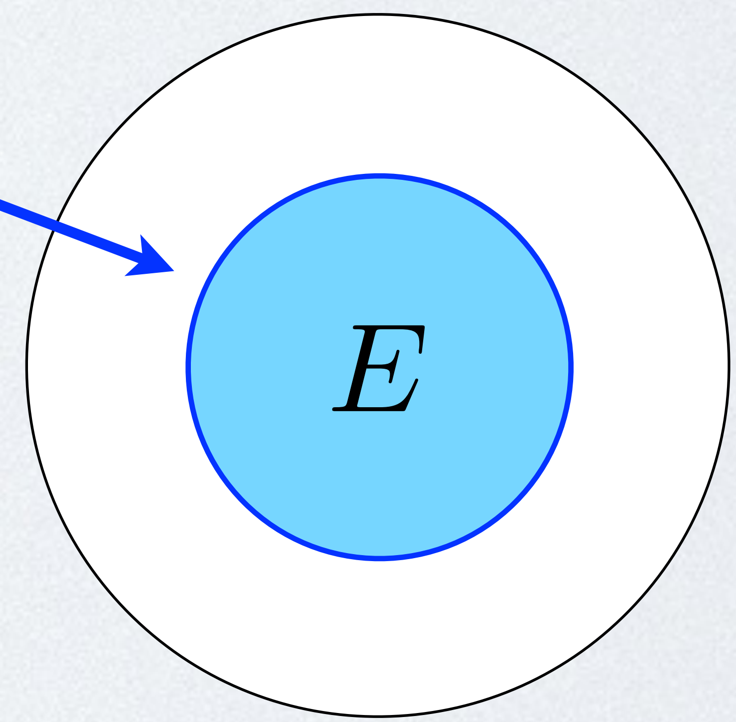
$$g^{rr} = 1 - \frac{2\,G_{\rm N}\,E(t,r)}{r}$$

Misner-Sharp mass

$$E = \frac{4\,\pi}{3}\int_0^r \rho(t,r')\,r^2\,dr'$$

$$G_{\rm N} = \frac{\ell_{\rm p}}{m_{\rm p}}$$

$$R_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$



Sphere is a **trapping surface** ("horizon") if

$$4\,\pi\,R_{\rm H}^2 = 4\,\pi\,r^2$$

Schwarzschild radius

Areal radius

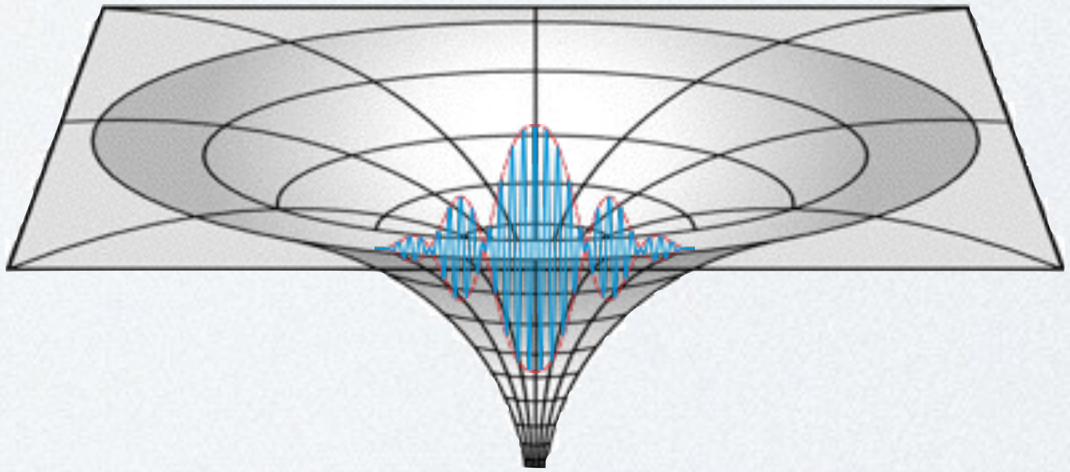
1) Horizon quantum mechanics

What is the Schwarzschild radius of QM states?

Many attempts at quantising "pure" black hole/horizon degrees of freedom
(independently of source)

"Background field approach"

But **GR is non-linear:**
(non-perturbatively) quantise black hole/horizon **and** matter source!



1) Horizon quantum mechanics

[ArXiv:1305.3195]

1) Localised source at rest: $\langle x|\psi_{\rm S}\rangle$ ~ packet

Energy (modes) of choice!

2) Spectral decomposition: $|\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$

N.B. equality between operators acting multiplicatively on kets

$$r_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$

3) Horizon wave-function: $\langle r_{\rm H}|\psi_{\rm H}\rangle \simeq C(r_{\rm H})$

"Probability amplitude for the size of gravitational radius"

1) Horizon quantum mechanics

[ArXiv:1305.3195]

Probability density particle is inside its own gravitational radius = horizon:

$$\mathcal{P}_<(r < r_{\mathrm{H}}) = P_{\mathrm{S}}(r < r_{\mathrm{H}})\,\mathcal{P}_{\mathrm{H}}(r_{\mathrm{H}})$$

$$P_{\mathrm{S}}(r < r_{\mathrm{H}}) = 4\,\pi \int_0^{r_{\mathrm{H}}} |\psi_{\mathrm{S}}(r)|^2\, r^2\, dr$$

$$\mathcal{P}_{\mathrm{H}}(r_{\mathrm{H}}) = 4\,\pi\, r_{\mathrm{H}}^2\, |\psi_{\mathrm{H}}(r_{\mathrm{H}})|^2$$

↓

Probability particle is a Black Hole:

$$P_{\mathrm{BH}} = \int_0^{\infty} \mathcal{P}_<(r < r_{\mathrm{H}})\, dr_{\mathrm{H}}$$

1) Horizon quantum mechanics

[ArXiv:1411.5848]

Localised particle at rest:

Gaussian wave-function:

$$\psi_{\rm S}(r) \simeq e^{-\frac{r^2}{2\,\ell^2}}$$

Energy spectrum: $|\psi_{\rm S}\rangle = \sum_E C(E)\,|E\rangle$

Fourier transform:

$$\psi_{\rm S}(p) \simeq e^{-\frac{p^2}{2\,\Delta^2}}$$

$\Delta = \frac{\hbar}{\ell} \sim m$

$E^2 = p^2 + m^2$ (flat space)

Horizon wave-function:

$$r_{\rm H} = 2\,\ell_{\rm p}\,\frac{E}{m_{\rm p}}$$

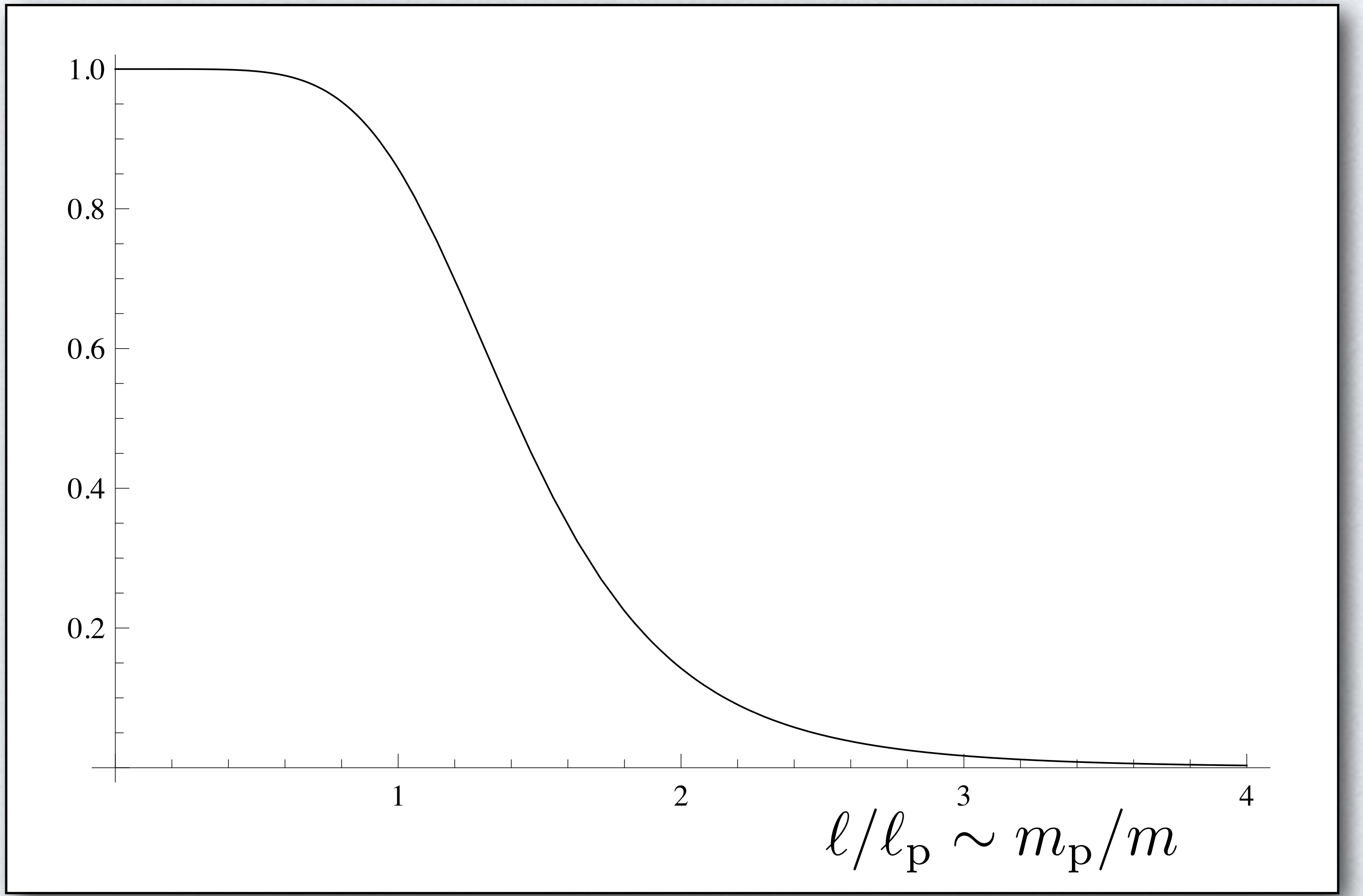
$$\psi_{\rm H}(r_{\rm H}) \simeq e^{-\frac{\ell^2\,r_{\rm H}^2}{8\,\ell_{\rm p}^4}}$$

1) Horizon quantum mechanics

[ArXiv:1411.5848]

$$\psi_{\rm S}(r) \simeq e^{-\frac{r^2}{2\,\ell^2}}$$

Probability particle is a Black Hole:



$$M_{\rm BH} \gtrsim m_{\rm p}$$

"Fuzzy" minimum mass

1) Horizon quantum mechanics

Applications:

1) GUP and quantum black hole decay: R.C., F. Scardigli, EPJC *74* (2014) 1, 2685

2) Quantum hoop conjecture: R.C., O. Micu, F. Scardigli, PLB *732* (2014), 105

3) BEC black holes: R.C., A. Giugno, O. Micu, A. Orlandi, PRD 90 (2014) 084040; PRD 91 (2015) 124069

4) Time evolution: R.C., EPJC *75* (2015) 4, 160

5) Corpuscular CMB: R.C., F. Kühnel, A. Orlandi, arXiv:1502.04703

6) Minimum mass black holes: X. Calmet, R.C., to appear

7) Charged sources: R.C., O. Micu, D. Stojkovic, JHEP 05 (2015) 096; PLB *747* (2015) 68

2) Charged black holes I

[RC, O Micu, D Stojkovic, arXiv:1503.01888]

Reissner-Nordstrøm metric:

$$ds^2 = -f\, dt^2 + f^{-1}\, dr^2 + r^2 d\Omega^2$$

$$0 < \alpha \equiv \frac{|Q|\, m_{\rm p}}{\ell_{\rm p}\, M} < 1$$

→

$$f = 1 - \frac{2\,\ell_{\rm p}\, M}{m_{\rm p}\, r} + \frac{Q^2}{r^2} = (1 - R_+)\,(1 - R_-)$$

$$R_\pm = \ell_{\rm p}\, \frac{M}{m_{\rm p}} \left(1 \pm \sqrt{1 - \alpha^2}\right)$$

$$\psi_{\rm S}(r) = \frac{e^{-\frac{r^2}{2\,\ell^2}}}{\ell^{3/2}\, \pi^{3/4}}$$

$$E^2 = p^2 + m^2$$

2 HWFs

$$\psi_{\rm H}(r_\pm) = \mathcal{N}_\pm\, \Theta\,(r_\pm - R_{{\rm min}\pm})\, \exp\left\{-\frac{m_{\rm p}^2\, r_\pm^2}{2\, m^2\, \ell_{\rm p}^2\, (1 \pm \sqrt{1-\alpha^2})^2}\right\}$$

$$R_{{\rm min}\pm} = R_\pm(m)$$

2) Charged black holes I

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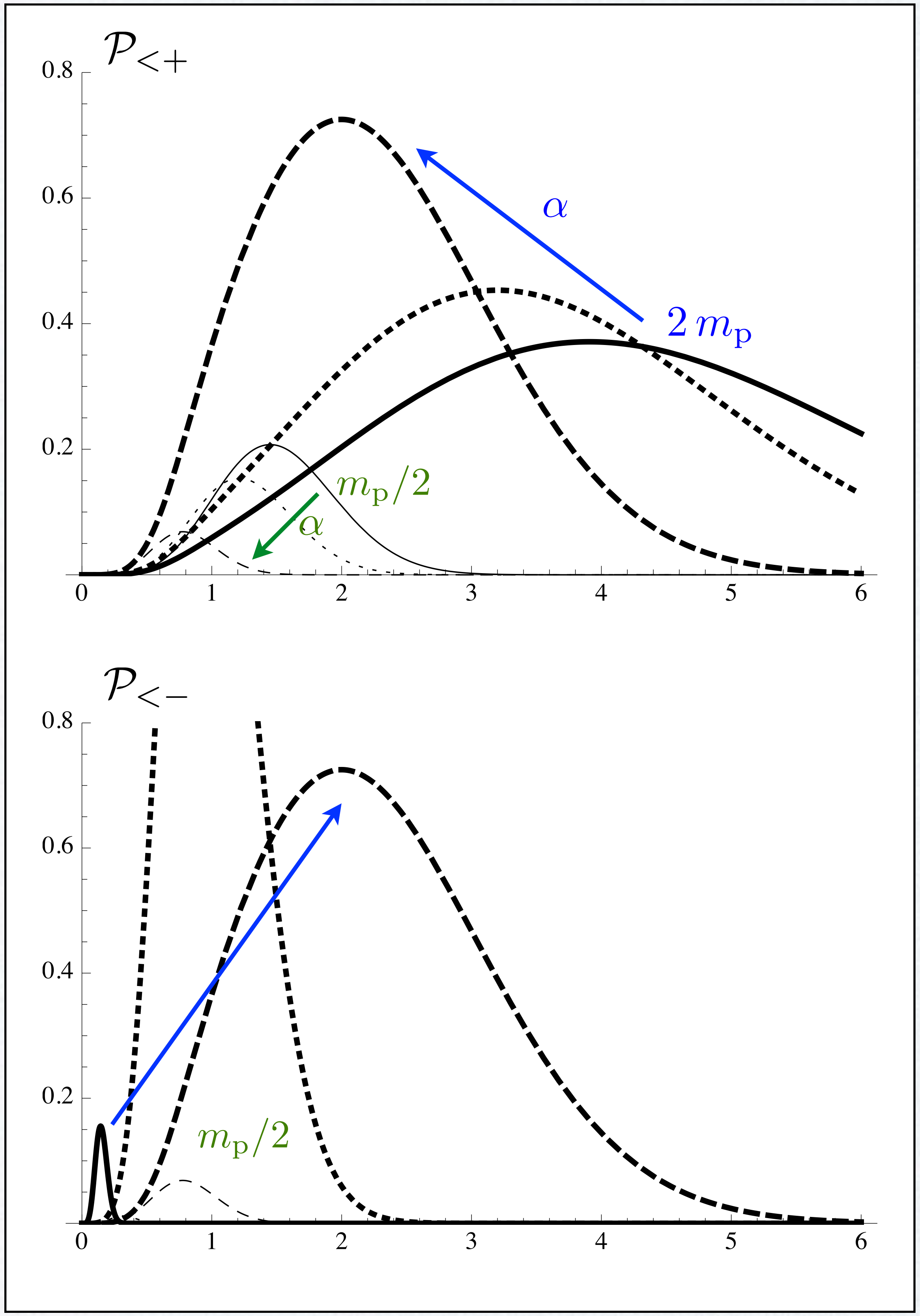
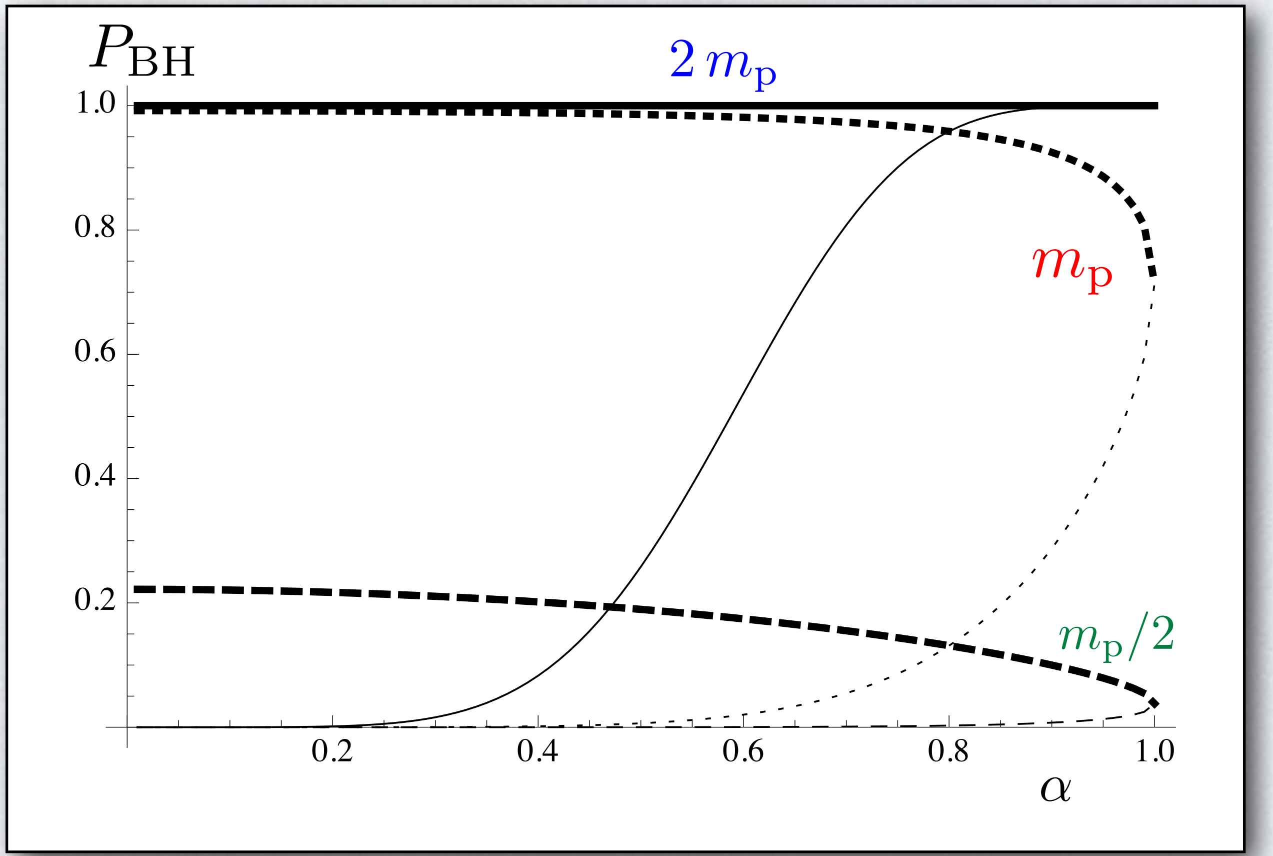




Inner horizon only realised for large mass and charge-to-mass ratio

↓

Mass-inflation overcome by QM?

3) Charged black holes II

[RC, O Micu, D Stojkovic, arXiv:1503.02858]

Reissner-Nordstrøm metric: $ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2 d\Omega^2$

$$\alpha \equiv \frac{|Q|\,m_{\rm p}}{\ell_{\rm p}\,M} > 1 \quad \longrightarrow \quad f > 0$$

Naked singularity

Assume continuity of HWF and its observables through $\alpha = 1$

↓

$$|\psi_{\rm H}(R)|^2 = \mathcal{N}^2 \exp\left\{-\frac{2-\alpha^2}{\alpha^4}\,\frac{m_{\rm p}^2 R^2}{m^2\,\ell_{\rm p}^2}\right\}$$

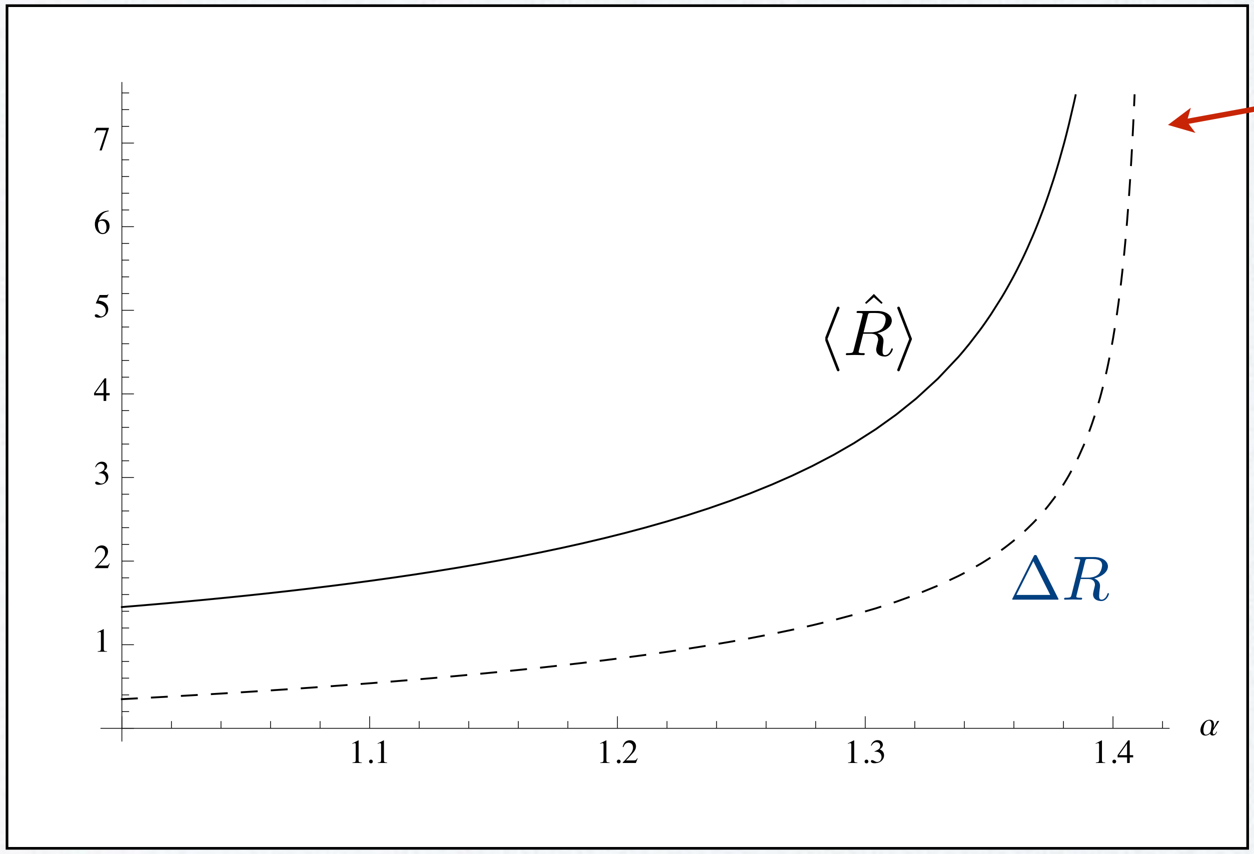
$$1 \le \alpha^2 < 2$$

↑

Normalisable

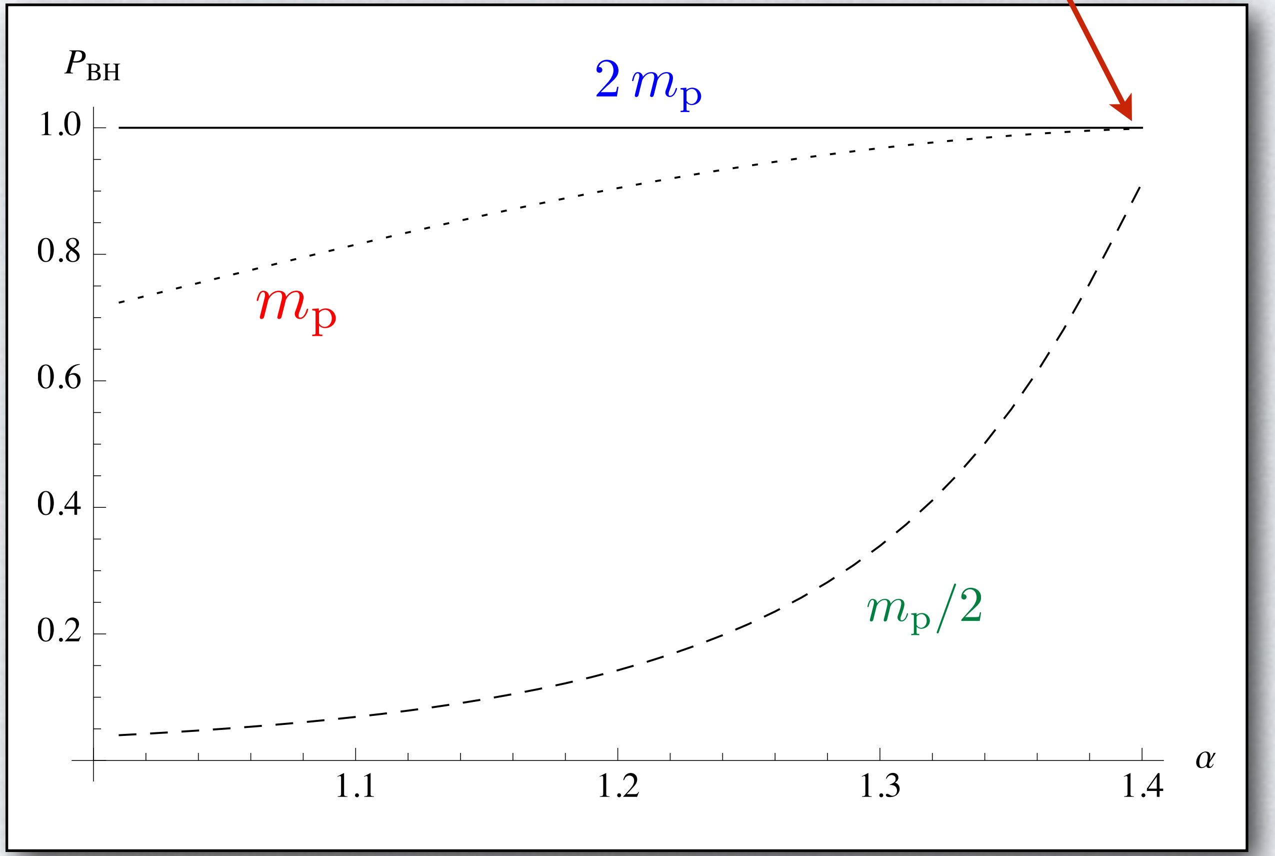
3) Charged black holes II

[RC, O Micu, D Stojkovic, arXiv:1503.02858]



Observables diverge!

Infinitely large BH



Quantum Cosmic Censorship

No charged source can exceed

$\alpha^2 \gtrsim 2$

4) Outlook

1. Analyse other spherical systems
2. Generalise HWF to non-spherical **spinning** systems
3. Analyse (2-)particle collisions with angular momentum+spin
4. (Hope for) fully quantum description of gravitational collapse

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Thank you!