# Extremal black holes in the near horizon limit Phase space and symmetry algebra

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# Black hole Thermodynamics

- Black holes are thermodynamical systems
- They have entropy, temperature, and chemical potentials
- They satisfy usual laws of thermodynamics
- Parametrized by mass and angular momenta $M, J_i$

**Extremal black holes**

- Black hole with vanishing temperature is called ***extremal***
- Extremality: Mass is a function of angular momenta $M = M(J_i)$
- Their entropy is finite $\quad T_H \longrightarrow 0 \quad \Longrightarrow \quad S \longrightarrow S_0$

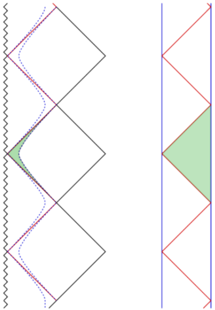
**Question.** Microscopic description of extremal black hole entropy with fixed angular momenta $J_i$?

# Motivation

## Near Horizon Geometry

- Black hole thermodynamics is associated to the horizon region

$$T_H = \frac{\kappa}{2\pi}, \qquad S = \frac{\text{Area of horizon}}{4G_N}$$



Near horizon limit

*picture from: Tom Hartman*

**Approach**

- In the extremal case, the near horizon region can be decoupled using the near horizon limit

- Enhancement of symmetries in NHEG $SL(2,\mathbb{R}) \times U(1)^{d-3}$

$$ds^2 = \Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sum_{i,j=1}^{d-3}\gamma_{ij}(\theta)(d\varphi^i + k^i r dt)(d\varphi^j + k^j r dt)\right]$$

# Kerr/CFT Correspondence

- A dynamical duality between asymptotic NHEK geometries and a two dimensional chiral CFT (`Strominger et.al.'09`)
- Microscopic counting of states in CFT matches with black hole entropy
- Extension to many other black holes in different theories and dimensions

**Challenges**

- No Dynamics in NHEK (`Reall et.al.'09 , Marolf,Horowitz, et.al '09`)
- Pathologies in construction of phase space

**Question**

How can this field theory description be meaningful?

## Lessons From $\mathbf{AdS_3/CFT_2}$ duality

- ***No dynamical degree of freedom*** in $AdS_3$, given the Brown Henneaux boundary conditions
- However, the duality still holds between ***nontrivial diffeomorphisms*** associated with nontrivial charges,
- The ***phase space*** is a set of geometries determined by two arbitrary functions $g_{\mu\nu}[L(z), \bar{L}(\bar{z})]$
- The symmetry of phase space is $Virasoro_L \times Virasoro_R$, coincide with the symmetries of a $CFT_2$

# Phase Space of Near Horizon Extremal Geometries

## Phase Space Field Configurations

- The phase space field configurations are metrics with a single arbitrary function $g_{\mu\nu}[F(\varphi^i)]$
- There is a well defined *symplectic structure* on this phase space
- Symmetries of phase space $x \to x + \xi\,[\epsilon(\varphi^i)]$

$$\xi\,[\epsilon(\vec{\varphi})] = \epsilon(\varphi^i)k^i\partial_{\varphi^i} - (k^i\partial_{\varphi^i}\epsilon)\,(\frac{1}{r}\partial_t + r\partial_r)$$

- Lie Algebra of vectors

$$\boxed{[\xi_{\vec{n}}, \xi_{\vec{m}}] = \vec{k}\cdot(\vec{n}-\vec{m})\;\xi_{\vec{n}+\vec{m}}}$$

## Charge Algebra

- Using the symplectic structure one can define conserved charges associated to each symmetry
- Quantized Algebra after $\{ \quad \} \to \frac{1}{i\hbar}[ \quad ]\,, H_{\vec{n}} \to \hbar\, L_{\vec{n}}$,

$$[L_{\vec{m}}, L_{\vec{n}}] = \vec{k}\cdot(\vec{m}-\vec{n})L_{\vec{m}+\vec{n}} + \frac{S}{2\pi}(\vec{k}\cdot\vec{m})^3\delta_{\vec{m}+\vec{n},0}\,.$$

- In $d = 4$ dimensions, it is a chiral Virasoro algebra
- In $d \geq 5$ we have an extended Virasoso algebra

## Charges

- Charge $H_{\vec{n}}[F]$ : Generator of $\xi_{\vec{n}}$ over the field configuration $g_{\mu\nu}[F(\varphi^i)]$

$$H_{\vec{n}} = \oint \boldsymbol{\epsilon}\; T[\Psi] e^{-i\vec{n}\cdot\vec{\varphi}},$$

where $e^{\Psi} = 1 + \vec{k} \cdot \vec{\partial} F$ and

$$T[\Psi] = \frac{1}{16\pi G}\Big((\Psi')^2 - 2\Psi'' + 2e^{2\Psi}\Big),$$

- Charges are Fourier modes of a Liouville-type stress tensor
- However $\Psi = \Psi(\varphi^i)$ , $\Psi' = \vec{k} \cdot \vec{\partial}\Psi$

# Summary and outlook

**Summary**

- We constructed the classical phase space of extermal black holes
- We obtrained the symmetry algebra (The NHEG algebra)
- We obtained the exact form of charges on the phase space

**Outlook**

- Look for a field theory with the same charges (it should be much similar to Liouville theory)
- Is there a notion of modular invariance here?
- Look for a Cardy like formula for counting of states? Is the black hole entropy reproduced?

**Thank you for you attention**

# Details

- Infinitesimal phase space transformation

$$\chi[\epsilon(\vec{\varphi})] = \epsilon(\varphi^i) k^i \partial_{\varphi^i} - (k^i \partial_{\varphi^i} \epsilon)\, (\frac{b}{r}\partial_t + r\partial_r) \tag{1}$$

- Phase space field configurations

$$ds^2 = \Gamma(\theta)\Big[ -(\boldsymbol{\sigma} - d\Psi)^2 + \Big(\frac{dr}{r} - d\Psi\Big)^2 + d\theta^2 + \gamma_{ij}(d\tilde{\varphi}^i + k^i\boldsymbol{\sigma})(d\tilde{\varphi}^j + k^j\boldsymbol{\sigma})\Big] \tag{2}$$

- Finite transformations

$$\bar{\varphi}^i = \varphi^i + k^i F(\vec{\varphi}), \qquad \bar{r} = r e^{-\Psi(\vec{\varphi})}, \qquad \bar{t} = t - \frac{b}{r}(e^{\Psi(\vec{\varphi})} - 1) \tag{3}$$

where $e^{\Psi} = 1 + \vec{k} \cdot \vec{\partial}_{\varphi} F$.