What if nature is bandlimited by a Planck-scale cutoff?

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Some philosophy

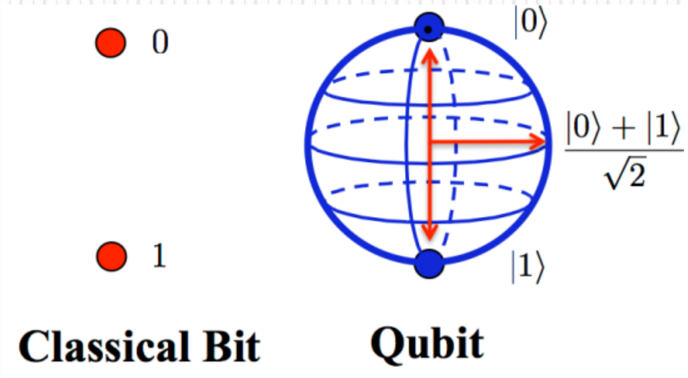
- Concepts can lose operational meaning:

  e.g., temperature, pressure, force, …

- In quantum gravity: space, time, matter, etc ?

- Most robust: (quantum) information ?

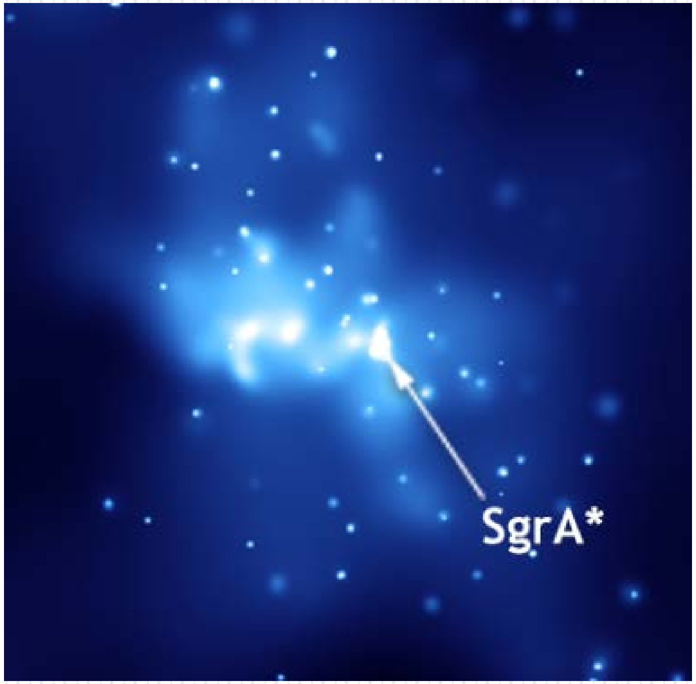
Is information a stronger concept?



Even when the meaning of the units of meters, seconds and kilograms fail, the meaning of bits and qubits may persist.

Information-theoretic foundation for physics ?

Concrete example:



- **BH entropy may be entanglement entropy.**

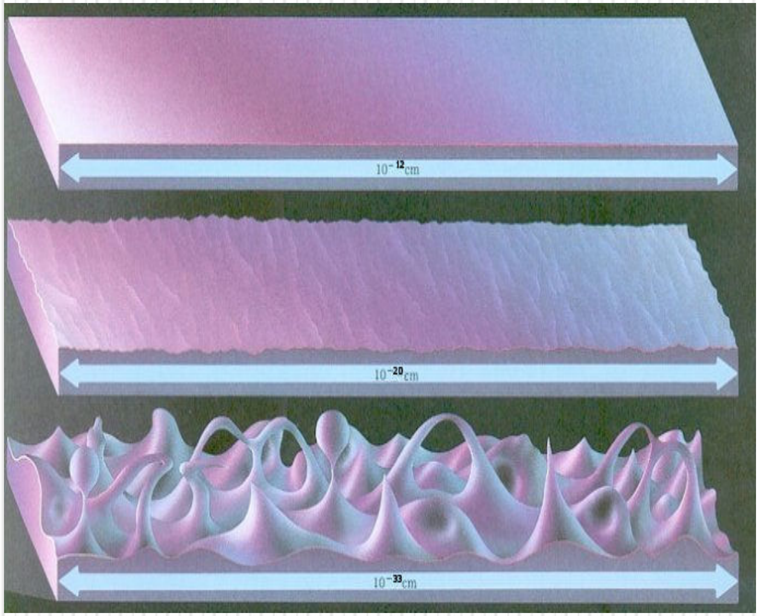
  (they scale the same way)

- **But for that there must be a natural UV cutoff.**
- **How does spacetime look at the Planck scale ?**

Concretely:

When we zoom in,

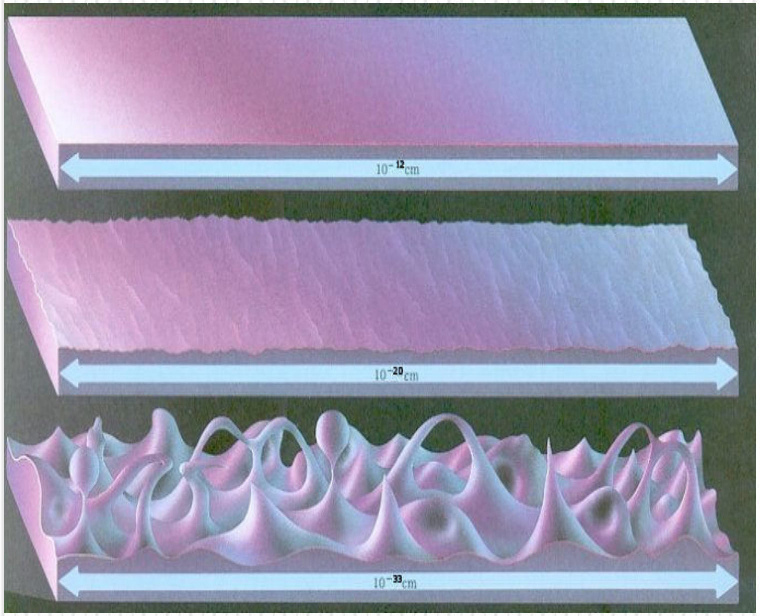
does space look like this ?



Concretely:

When we zoom in,

does space look like this ?



And does time look like that ?

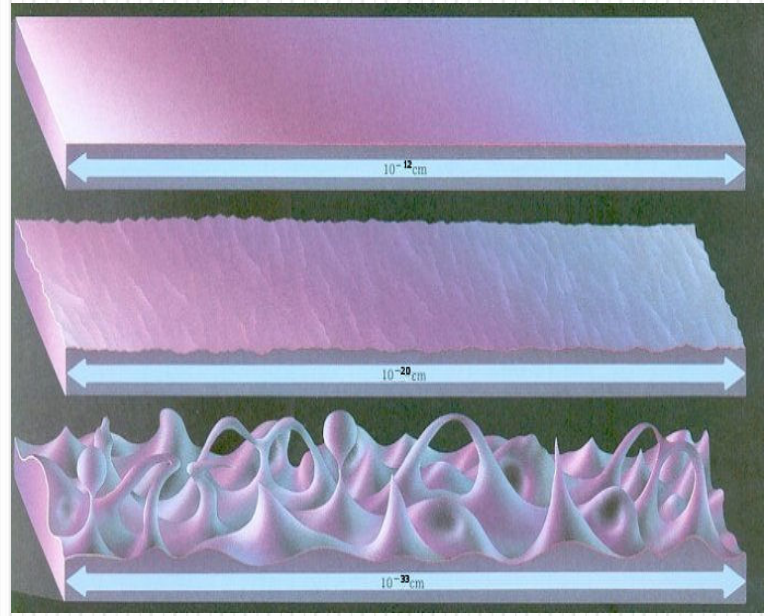
Concretely:

When we zoom in,

does space look like this ?

And does time look like that ?

Check operational meaning!



What happens, operationally, as one approaches the Planck Scale?

Resolve a distance more and more precisely.

=> **increasing momentum uncertainty,**

=> **increasing curvature uncertainty,**

=> **increasing distance uncertainty.**

➔ **Cannot resolve distances below** 10^(-35)m.

What is the structure of spacetime ?

(and does information theory come up naturally?)

Paradox:

General relativity:

- Fields live on a differentiable spacetime manifold.

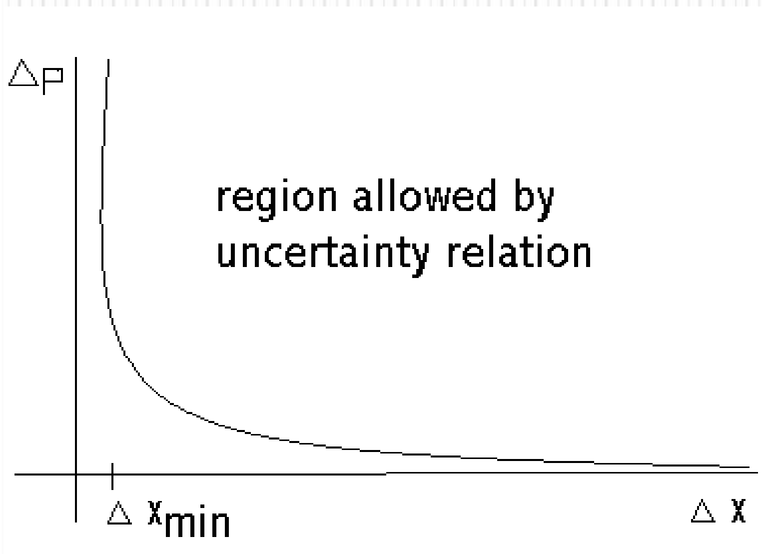
Quantum field theory:

- QFT generally only well defined if spacetime is discrete.

Possible resolution

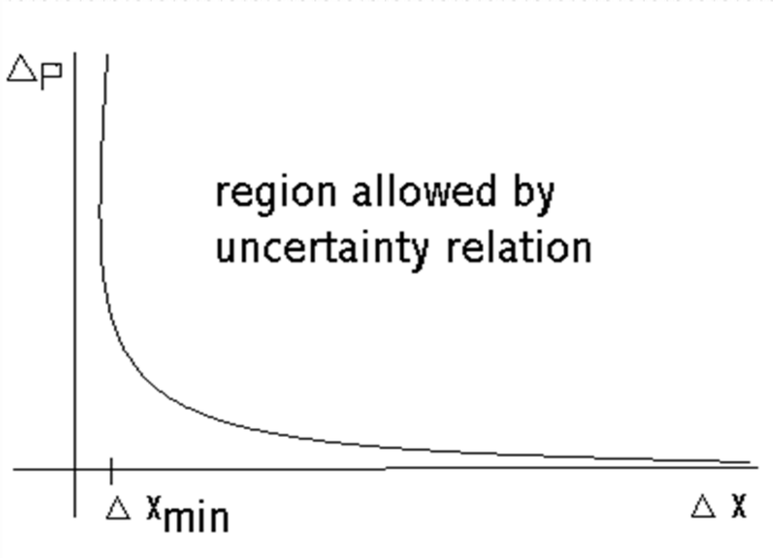
Studies in quantum gravity and string theory

=>



Canonical commutation relations and Hilbert space representation:

- A. Kempf, J. Math. Phys. **35**, 4483 (1994), hep-th/9311147
- A. Kempf, G. Mangano, and R.B. Mann, Phys. Rev. D 52, 1108 (1995), hep-th/9412167



➔ ***If so, fields must possess a finite bandwidth !***

➔ **Spacetime is both discrete *and* continuous,**

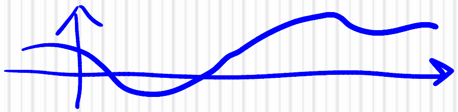
in the same mathematical way that information is.

See: A. Kempf, Phys. Rev. Lett. **85**, 2873 (2000)
A. Kempf, Phys. Rev. Lett. **92**, 221301 (2004)
A. Kempf, R. Martin, Phys. Rev. Lett. **100**, 021304 (2008)
A. Kempf, Phys. Rev. Lett. **103**, 231301 (2009)
D. Aasen, T. Bhamre, A. Kempf, Phys. Rev. Lett. **110**, 121301 (2013)

Information theory does come up naturally

Information can be:

- continuous (e.g., music):



- discrete (letters, digits, etc): R 7 2 5 B

Unified in 1949 by Shannon, through: <u>Sampling theory</u>

Applications ubiquitous:

- communication engineering & signal processing
- scientific data taking, e.g., in astronomy.

Shannon's sampling theorem

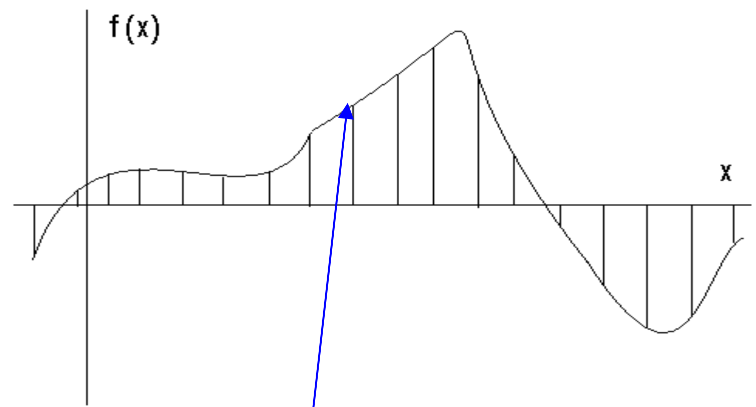
- Assume f is "bandlimited", i.e:

$$f(x) = \int_{-\omega_{\max}}^{\omega_{\max}} \tilde{f}(\omega)\, e^{-2\pi i \omega x} d\omega$$

- Take samples of f(x) at Nyquist rate:

$$x_{n+1} - x_n = (2\omega_{\max})^{-1}$$

- **Then, exact reconstruction is possible:**



samples

$$f(x) = \sum_n f(x_n) \frac{\sin[\,2\pi (x - x_n)\,\omega_{\max}\,]}{\pi (x - x_n)\,\omega_{\max}}$$

Properties of bandlimited functions

- **Differential operators are also finite difference operators.**
- **Differential equations are also finite difference equations.**
- **Integrals are also series:**

$$\int_{-\infty}^{\infty} f(x)^* g(x)\,dx = \frac{1}{2\omega_{\max}} \sum_{n=-\infty}^{\infty} f(x_n)^* g(x_n)$$

Remark:

Useful also as a summation tool for series

(traditionally used, e.g., in analytic number theory)

Covariant "bandlimitation" ?

Cut off of the spectrum of

the Laplacian or d'Alembertian.

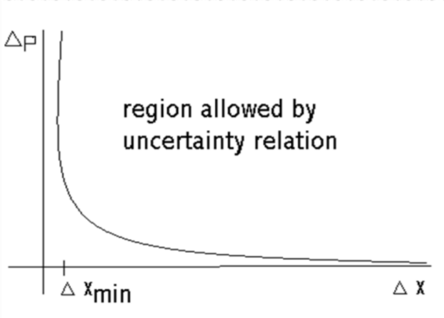
$$Z[J] = \int_F e^{iS[\phi]+i\int J\phi\ d^nx} D[\phi]$$

➔ The space of fields, F, in the QFT path integral

is spanned by the eigenfunctions w. eigenvalues:

$$\lambda_i < \Lambda$$

What if physical fields are "bandlimited"?



Fields possess equivalent representations

- **on a differentiable spacetime manifold**

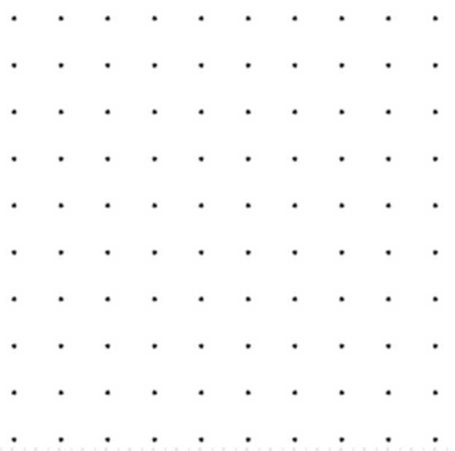
  (which shows preservation of external symmetries)

- **on any lattice of sufficiently dense spacing**

  (which shows UV finiteness of QFTs).

Entanglement entropy

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi$$

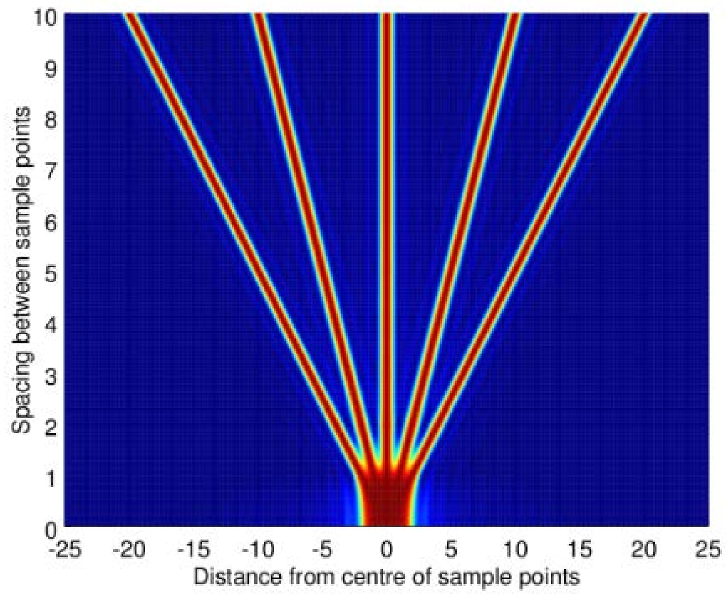


For lattice QFT:

- Coupled harmonic oscillators
- Their ground state is short-range entangled
- Entanglement entropy obeys volume and area laws.

Ent. entropy with bandwidth cutoff

With Jason Pye (UW) and William Donnelly (UCSB):



- Find smooth transition from volume to area law.

The physics of the Nyquist rate?

- Area law kicks in at Nyquist rate.
- On flat space, Nyquist rate is constant.
- How does curvature affect the Nyquist rate?

Density of degrees of freedom ?

We can use [Gilkey 1975]:

Consider any compact 4-dim Riemannian manifold. Then:

$$N = \frac{1}{16\pi^2}\int d^4x\sqrt{|g|}\ \left\{\frac{\Lambda^2}{2} + \frac{\Lambda}{6}R + O(R^2, \Lambda^{-1})\right\}$$

Can now read off:

- Cosmological constant is density of DOF: $\dfrac{N}{V} = \dfrac{\Lambda^2}{32\pi}$
- Curvature is local perturbation of density of DOF.

Re-expressing the Einstein action

=> Einstein action takes the simple form:

$$
\begin{aligned}
S &= \frac{6\pi}{\Lambda}\frac{1}{16\pi^2}\int d^4x\sqrt{|\,g\,|}\ \left\{\frac{\Lambda^2}{2} + \frac{\Lambda}{6}R + O(R^2,\Lambda^{-1})\right\} \\
&= \frac{6\pi}{\Lambda}N \\
&= \frac{6\pi}{\Lambda}\,Tr(1)
\end{aligned}
$$

Notice: The Einstein action is the integral over the density of degrees of freedom, where the cosmological constant sets the baseline, modulated by curvature.

Compare: scalar field action

- In the eigenbasis of the Laplacian is not only the Einstein action diagonal but also the action of a scalar field:

$$S_{matter} = \int d^n x \sqrt{|g|}\ \tfrac{1}{2}\phi(x)(\Delta + m^2)\phi(x)$$

$$= \sum_{i=1}^{N} \tfrac{1}{2}\phi_i(\lambda_i + m^2)\phi_i$$

$$= Tr[\ \tfrac{1}{2}(\Delta + m^2)|\phi)(\phi|\ ]$$

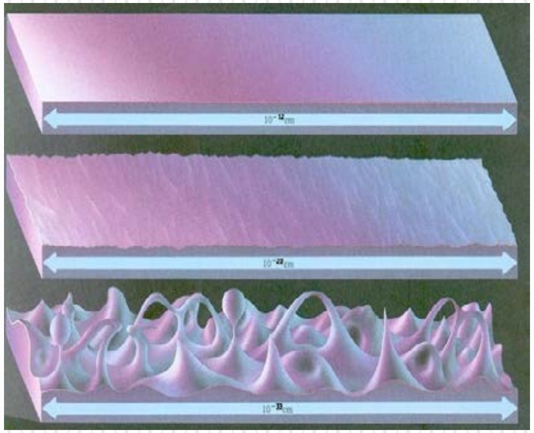
- **Actions are traces, and gravity could be a leading constant.**

But what is bandlimitation for spacetime itself ?

Is there a Shannon-like reconstruction of space from discrete sets of samples?

With bandwidth / min uncertainty cutoff,

what could supercede rulers and clocks ?



Idea:
Noise correlator as proxy for distance

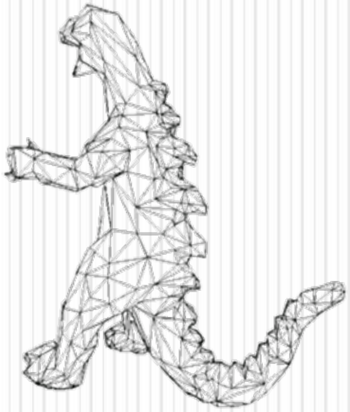
- Quantum field correlators indicate distance.

- Does the entanglement structure of the vacuum encode spacetime's curvature ?

Idea: correlators as proxy for distances

At N points x_i of a finite piece of the manifold,

sample the propagator's matrix elements:



$$\langle x_a \,|\, 1/\Delta \,|\, x_b \rangle$$

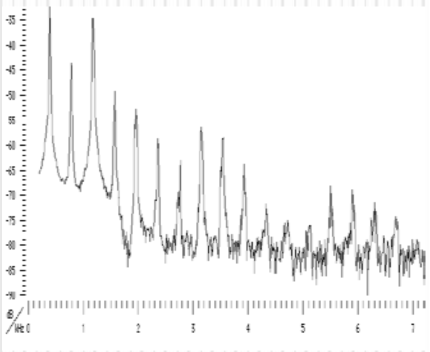
- Work w. Aslanbeigi and Saravani: One can reconstruct the metric.
- Basis independent information ➔ eigenvalues of Δ .
- **Does the spectrum tell the shape?**

Spectral Geometry:

- **“ How far is shape determined by sound ? ”**

$$-d^2\phi/dt^2 = \Delta_g \phi$$

⇔

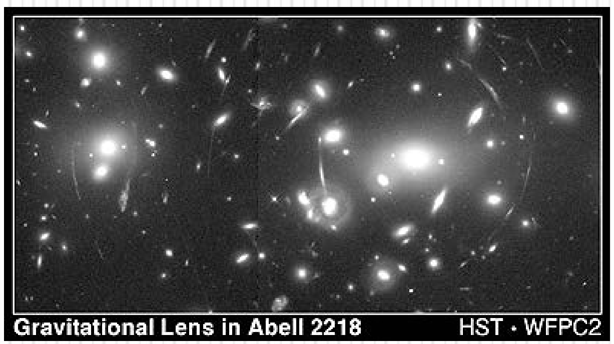


(M,g) spec(Δ)

There are some positive results, e.g., on shapes of revolution!

Prospect:

Can one hear a spacetime's curvature in its quantum noise?

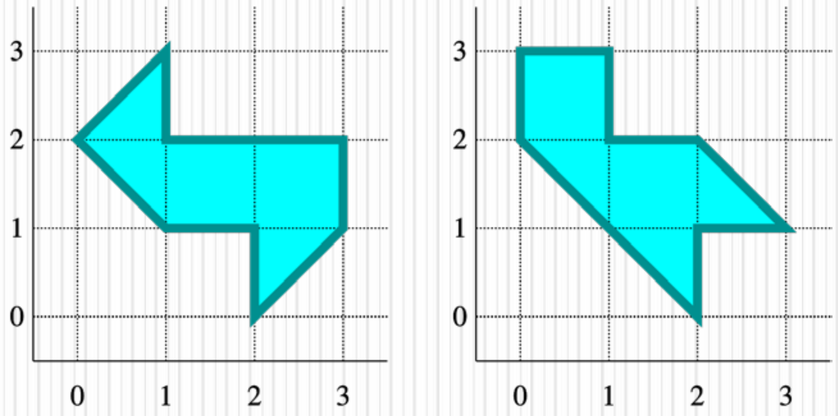


⇔

Deep link between gravity and quantum theory?

Problem !

- **Spectral geometry has counter examples !**
- Work by Milnor, Sunada, Gordon …



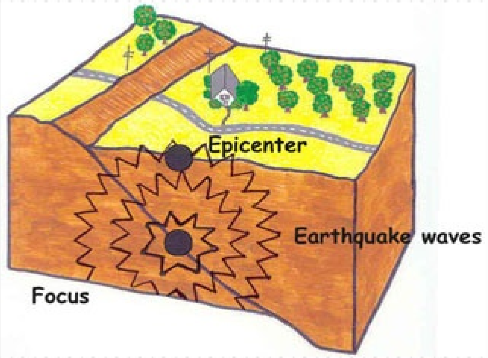
Solution:

Infinitesimal spectral geometry & tensor spectra

In dimensions d>2, not every perturbation of a Riemannian manifold can be described by a scalar function f.

$$g_{\mu\nu}(x) \rightarrow (1+f(x))\, g_{\mu\nu}(x)$$

Need to use scalar, vector and tensor perturbations:

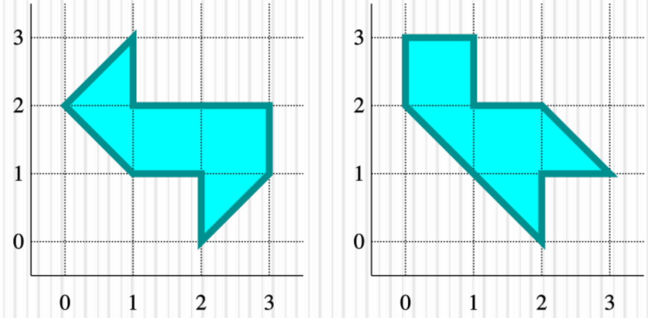


$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \delta s_{\mu\nu}(x) + \delta v_{\mu\nu}(x) + \delta h_{\mu\nu}(x)$$

(Seismic waves of different types carry independent information too)

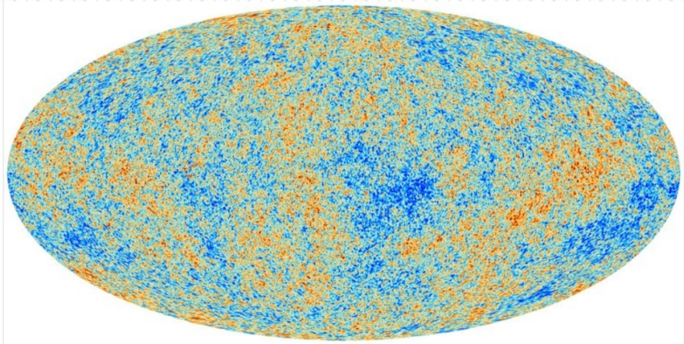
Work with Mikhail Panine (UW)

- We showed that even spectral geometry of planar domains works generically.



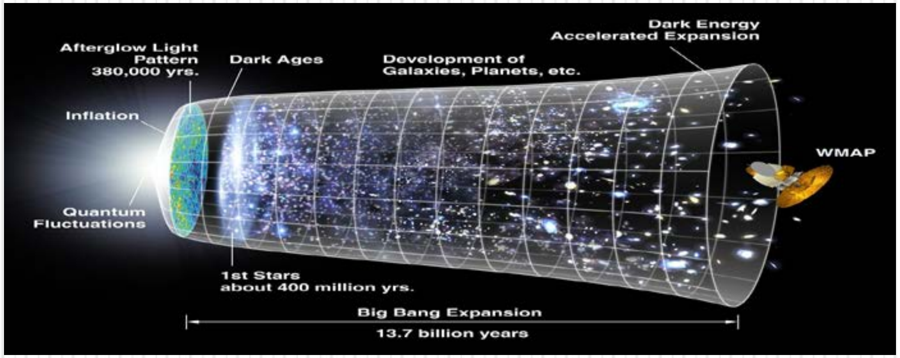
Experimental predictions ?

CMB is closest to Planck scale

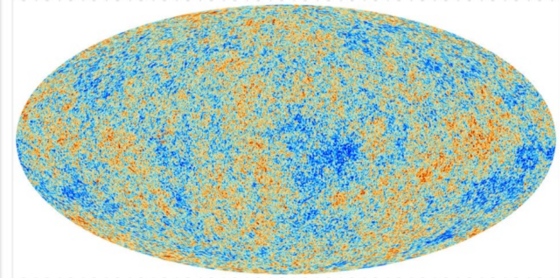


Why?

Hubble scale in inflation only about 5 orders from Planck scale.



Applied to cosmology



Multiple groups have non-covariant predictions for CMB.

- Characteristic, O(10^-5) or O(10^-10) modulations
- Characteristic deviation from scalar/tensor consistency relation in B-polarization data.

Big challenge:

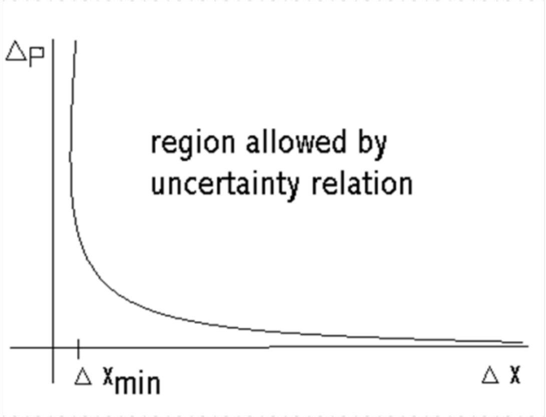
Predictions with local Lorentz covariant bandlimit cutoff!

Upcoming work with:

Aidan Chatwin-Davies (CalTech) and Robert Martin (U. Cape Town)

Summary

- Philosophy: Only information theoretic concepts may survive at Planck scale



- Found: Spacetime may be bandlimited

- Thus discrete = continuous

  for spacetime, as is the case for information.

- Notion of distance replaced by info-theoretic notion of correlation.

- Most also already works Lorentz-covariantly (pls ask).

Thank you