Phenomenology of microjets

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based on arXiv:1411.5182 and work in preparation

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Outline

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   - Jet algorithms
   - Perturbative properties of jets
2. Method
3. Observables
   - Filtering
   - Trimming
   - Inclusive jet spectrum
4. Conclusion

INTRODUCTION

Jet algorithms and choice of jet radius

A jet algorithm maps final state particle momenta to jet momenta.

$$\underbrace{\{p_i\}}_{\text{particles}} \Longrightarrow \underbrace{\{j_k\}}_{\text{jets}}$$

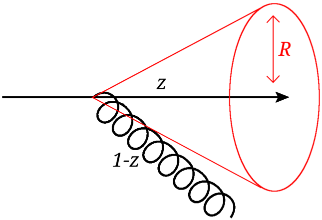
This requires an external parameter, the jet radius R, specifying up to which point separate partons are recombined into a single jet.

What are usual values for the jet radius R ?

- Most common choice is $R = 0.4 - 0.5$.
- In some environments (eg. heavy ions), values down to $R = 0.2$ are used to mitigate high pileup and underlying event contamination.
- Many modern jet tools (eg. trimming and filtering) resolve small subjets (typically with $R_{\text{sub}} = 0.2 - 0.3$) within moderate & large R jets.

Perturbative properties of jets

Jet properties will be affected by gluon radiation and $g \to q\bar{q}$ splitting.



Emissions outside of the jet reduce the jet energy.

We will try to investigate the effects of perturbative radiation on a jet analytically, in the small radius limit.

Perturbative properties: parton energy ≠ jet energy

Average energy difference between hardest final state jet and initial quark, considering emissions beyond the reach of the jet

$$\left\langle \frac{\text{quark } E - \text{jet } E}{\text{quark } E} \right\rangle = \int^{\mathcal{O}(1)} \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) \times \frac{\alpha_s}{2\pi} p_{qq}(z) \Theta(\theta - R)$$

$$= \frac{C_F}{\pi}\left(2\ln 2 - \frac{3}{8}\right) \alpha_s \ln R + \dots$$

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$$= \frac{C_F}{\pi}\left(2\ln 2 - \frac{3}{8}\right) \alpha_s \ln R + \dots$$

$\alpha_s \ln R$ implies large corrections for small R.

How relevant are small-R effects?

Energy loss has big effect on jet spectrum.

R	correction to jet spectrum
0.4	$\mathcal{O}(-25\%)$
0.2	$\mathcal{O}(-50\%)$

"In the small R limit, new clustering logarithms [of R ...] arise at each order and cannot currently be resummed."

— Tackmann, Walsh & Zuberi (arXiv:1206.4312)

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We aim to resum all leading logarithmic $(\alpha_s \ln R)^n$ terms in the limit of small R for a wide variety of observables.

METHOD

Evolution variable t

Use an evolution variable t corresponding to the integral over the angular emission probability weighted with α_s

$$t = \int_{R^2}^{R_0^2} \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t\theta)}{2\pi} \sim \frac{\alpha_s}{2\pi} \ln \frac{R_0^2}{R^2}, \quad R_0 \sim 1$$

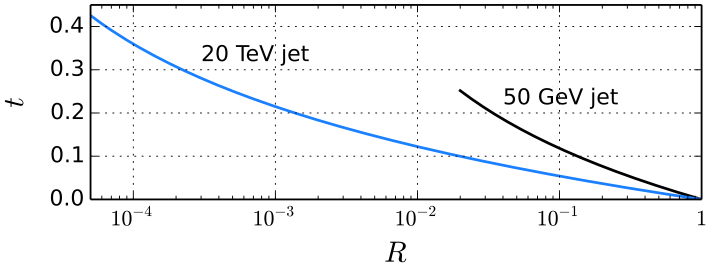


Figure – Plot of t as a function of R down to $Rp_t = 1\,\text{GeV}$.

Generating functionals for microjets

Definition

Quark generating functional $Q(x,t)$ encodes parton content observed when resolving an initial quark with momentum fraction x on an angular scale $t > 0$. (ie. $R < 1$).

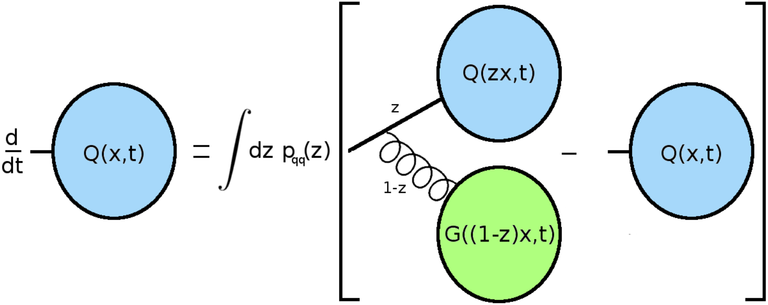
We can formulate the evolution equation for the quark generating functional from

$$Q(x,t) = Q(x,t-\delta_t)\left(1 - \delta t \int dz\, p_{qq}(z)\right) + \delta_t \int dz\, p_{qq}(z)\Big[Q(zx,t-\delta_t)G((1-z)x,t-\delta_t)\Big].$$

Gluon generating functional $G(x,t)$ defined the same way.

Quark evolution equation

Evolution equation for the quark generating functional can be rewritten as a differential equation.



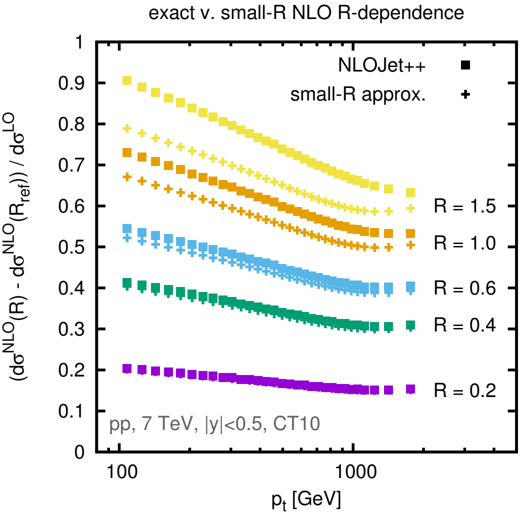
Blobs represent the generating functionals at a scale t.

A similar equation can be obtained for the gluon evolution.

These equations allow us to resum observables to all orders numerically. They effectively exploit angular ordering.

Validity of small-R approximation

Small-R is a valid for $R \leq 1$, but starts to break down around $R \sim 1$.



Compare inclusive spectrum from NLOJet++ with small-R approximation

We look at differences between R values.

Agreement of squares and crosses indicates that the small-R approximation is good.

OBSERVABLES

Fragmentation functions

Two broad classes of observables:

- Inclusive microjet observables obtained from the inclusive microjet fragmentation function $f_{j/i}^{\text{incl}}(z,t)$.
  In this case, momentum conservation translates to

$$\sum_j \int dz\, z\, f_{j/i}^{\text{incl}}(z,t) = 1.$$

- Hardest microjet observables obtained from the momentum distribution of the hardest microjet $f^{\text{hardest}}(z,t)$.
  Here we have a probability sum rule

$$\int dz\, f^{\text{hardest}}(z,t) = 1.$$

Observables

We computed small-R effects for a number of observables

- Filtering (= keep n_{filt} hardest subjets).
- Trimming (= keep subjets with $p_t^{\text{subjet}} > f_{\text{cut}}\, p_t^{\text{jet}}$)
- Inclusive jet spectrum (resummation is DGLAP-like).
- Jet vetoes in H & Z production.
- Dijet mass spectrum.
- Jet flavour (eg. hardest microjet flavour-change probability).

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Filtering

Definition

Reclustering of a jet on a smaller angular scale $R_{\text{filt}} < R_0$, discarding all but the n_{filt} hardest subjets.

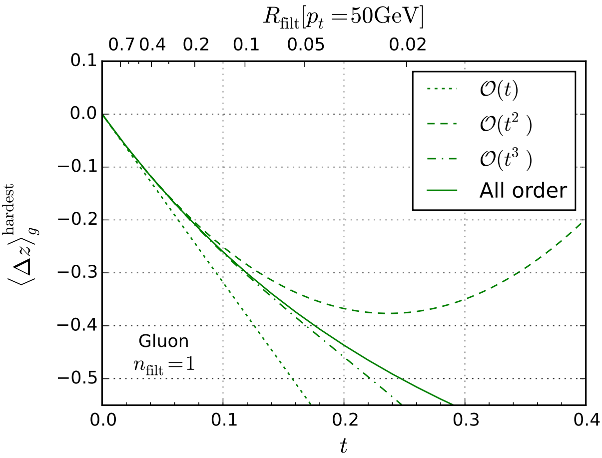
Define $f^{k\text{-hardest}}(z)$ the probability that the k-th hardest subjet carries a momentum fraction z of the initial parton.

We can express the energy loss between the filtered jet and the initial parton as

$$\langle \Delta z \rangle^{\text{filt},n} = \left[\sum_{k=1}^{n} \int dz\, z\, f^{k\text{-hardest}}(z) \right] - 1.$$

As n_{filt} increases, the filtered jet retains more of the original parton momentum.

$$t \sim \frac{\alpha_s}{2\pi} \ln \frac{1}{R^2}$$

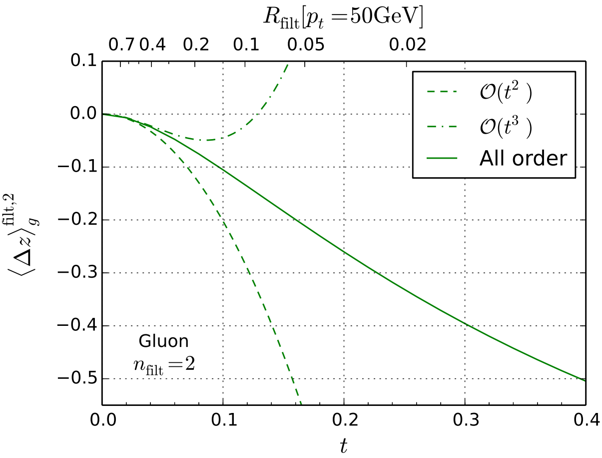


Average jet energy loss Δz after filtering with $n_{\text{filt}} = 1$.

As n_{filt} increases, the filtered jet retains more of the original parton momentum.

Convergence of the power series is extremely slow.

Resummation essential to get a stable answer.



Average jet energy loss Δz after filtering with $n_{\text{filt}} = 2$.

Filtering of gluon jets

As n_{filt} increases, the filtered jet retains more of the original parton momentum.

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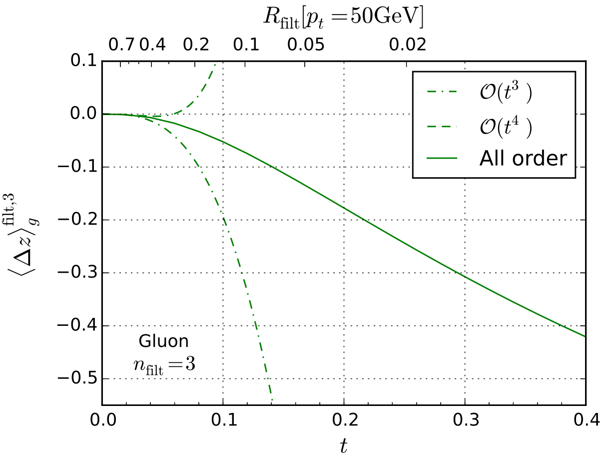


Average jet energy loss Δz after filtering with $n_{\text{filt}} = 3$.

Trimming

Definition

- Recluster all particles within a jet into subjets with $R_{\text{trim}} < R_0$.
- Resulting microjets with $p_t \geq f_{\text{cut}} p_t^{\text{parton}}$ are merged and form the trimmed jet, others are discarded.

Energy difference between the trimmed jet and the initial parton of flavour i can then be expressed as a function of f_{cut}

$$\langle \Delta z(f_{\text{cut}}) \rangle_i^{\text{trim}} = \left[\sum_j \int_{f_{\text{cut}}}^1 dz\, z\, f_{j/i}^{\text{incl}}(z,t) \right] - 1\,.$$

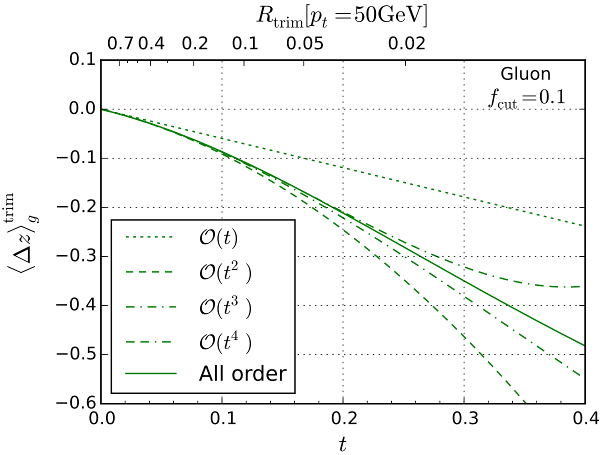
Caveat: this would need double resummation of $\ln R$ and $\ln f_{\text{cut}}$.

Trimming of gluon jets

As with filtering, energy loss from trimmed jets with a given R_{trim} is much reduced relative to that from a single microjet with that same radius.

Convergence of the power series is (maybe) better than for filtering.

Resummation of $\ln f_{\text{cut}}$ would be required as well.



Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.1$.

As with filtering, energy loss from trimmed jets with a given R_{trim} is much reduced relative to that from a single microjet with that same radius.

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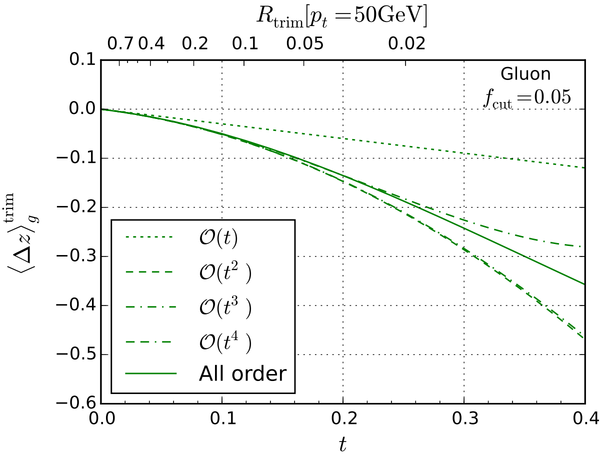
Resummation of $\ln f_{\text{cut}}$ would be required as well.



Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.05$.

Trimming of gluon jets

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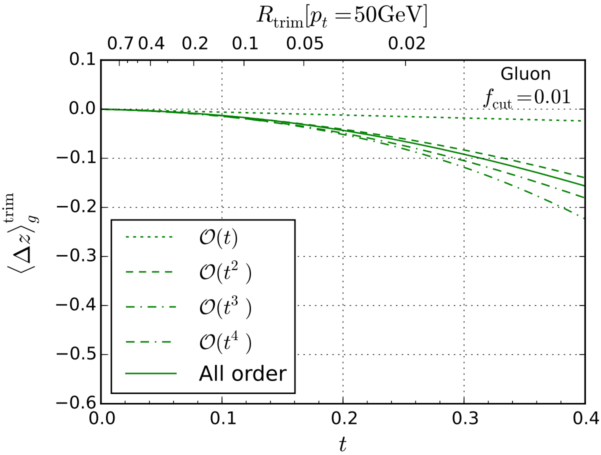


Average jet energy loss Δz after trimming with $f_{\text{cut}} = 0.01$.

Inclusive jet spectrum

The jet spectrum can be obtained from the convolution of the inclusive microjet fragmentation function with the inclusive partonic spectrum from hard $2 \to 2$ scattering

$$\frac{d\sigma_{\text{jet}}}{dp_t} = \sum_i \int_{p_t} \frac{dp'_t}{p'_t} \frac{d\sigma_i}{dp'_t} f^{\text{incl}}_{\text{jet}/i}(p_t/p'_t, t),$$

Small-R effects enhanced by $\ln n$ factors

$$\sim \alpha_s \ln \frac{1}{R^2} \ln n$$

To estimate corrections, assume that the partonic spectrum is dominated by a single flavour i and that its p_t dependence is $d\sigma_i/dp_t \sim p_t^{-n}$ then

$$\frac{d\sigma_{\text{jet}}}{dp_t} \simeq \frac{d\sigma_i}{dp_t} \int_0^1 dz\, z^{n-1} f^{\text{incl}}_{\text{jet}/i}(z, t) \equiv \frac{d\sigma_i}{dp_t} \langle z^{n-1} \rangle^{\text{incl}}_i .$$

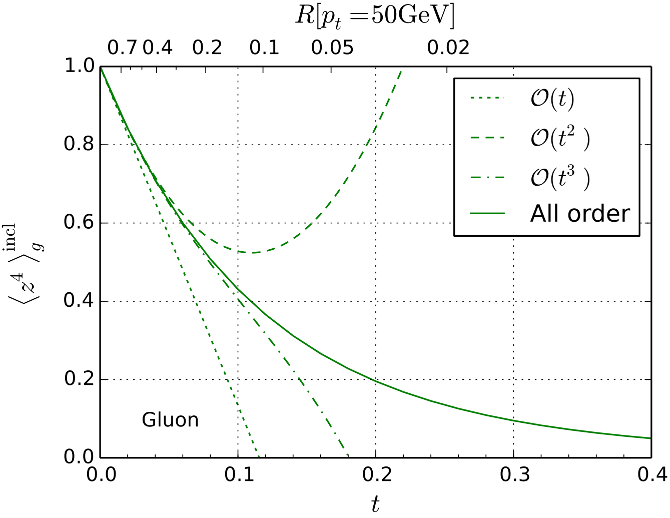
At the LHC typical n values for the partonic spectrum range from about 4 at low p_t to 7 or even higher at high p_t.

Moment of inclusive microjet spectrum $\langle z^{n-1} \rangle$

Convergence is slow, and small-R terms are important, amounting to a 30 – 50% effect (for $R = 0.4 - 0.2$) on gluonic inclusive spectrum.

Corresponds to correction to $1/p_t^5$ spectrum.

NNLO corrections (order t^2) deviate noticeably from all-orders result below $R = 0.3$.



Matching NLO and LL_R

Necessary condition that matching must satisfy

$$\frac{d\sigma^{\text{LL}_R+\text{NLO}}}{d\sigma^{\text{LO}}} \to 0 \qquad \text{for } R \to 0\,.$$

For this reason, we adopt multiplicative matching,

$$d\sigma^{\text{LL}_R+\text{NLO,mult.}} = \frac{d\sigma^{\text{LL}_R}}{d\sigma^{\text{LO}}} \times \left(d\sigma^{\text{LO}} + d\sigma^{\text{NLO}} - d\sigma^{\text{LL}_R@\text{NLO}}\right)\,.$$

This suggests the following alternative expression for the NLO cross section

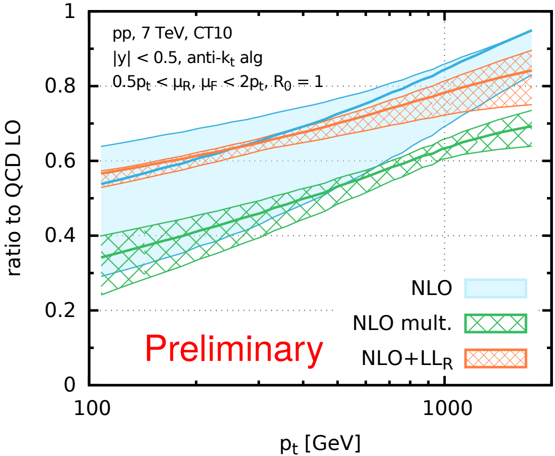
$$d\sigma^{\text{NLO,mult.}} = \left(1 + \frac{d\sigma^{\text{NLO}}(R) - d\sigma^{\text{NLO}}(R_0)}{d\sigma^{\text{LO}}}\right) \times \left(d\sigma^{\text{LO}} + d\sigma^{\text{NLO}}(R_0)\right)\,.$$

Scale-dependence of inclusive jet spectrum

Small-R resummation changes the scale dependence.

Large cancellations between scale dependence of partonic scattering & small-R fragmentation contributions.

correlated scale choice, R = 0.1



There are different ways of estimating uncertainties due to the matching scheme.

$$\max_{\mu_F,\mu_R}\left[\frac{\text{resum}}{\text{LO}}\times(\text{NLO}+\text{LO}-\text{NLO}_{\text{small-}R})\right]$$

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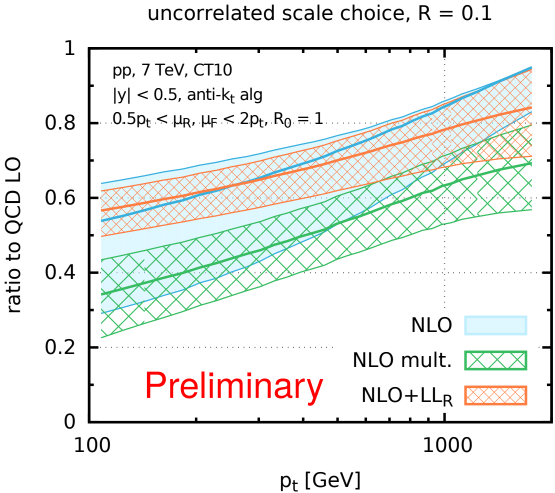
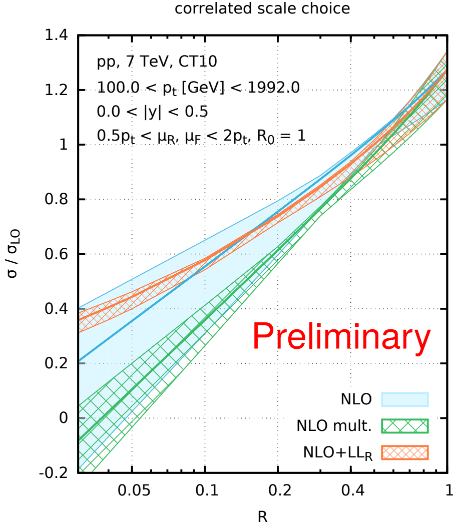
$\Rightarrow$ so add scale variation from those two components in quadrature.



There are different ways of estimating uncertainties due to the matching scheme.

$$\max_{\mu_F,\mu_R}\left(\frac{\text{resum}}{\text{LO}}\right) \otimes \max_{\mu_F,\mu_R}\left(\text{NLO} + \text{LO} - \text{NLO}_{\text{small-}R}\right)$$

R-dependence of the cross-sections



Unphysical vanishing of scale uncertainty for certain R values with correlated scale choice.

LL_R resummation cures negative cross-sections arrising at $R < 0.05$.

Transition point at $R \sim 0.5$ where LL_R resummation reduces scale dependence and improves stability.

Changes in prescription have significant impact in range $R \in [0.1, 0.5]$.

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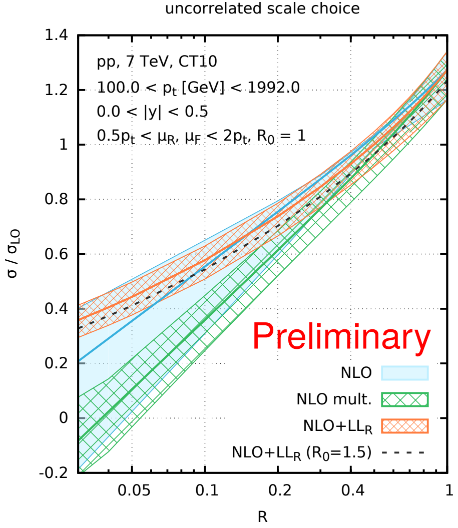

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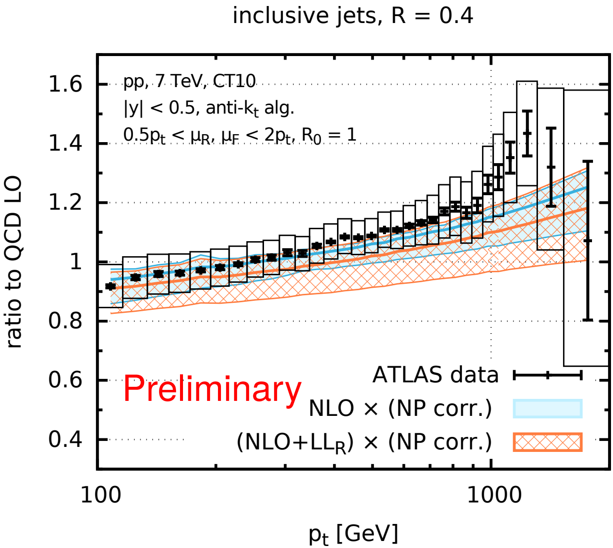
Changes in prescription have significant impact in range $R \in [0.1, 0.5]$.

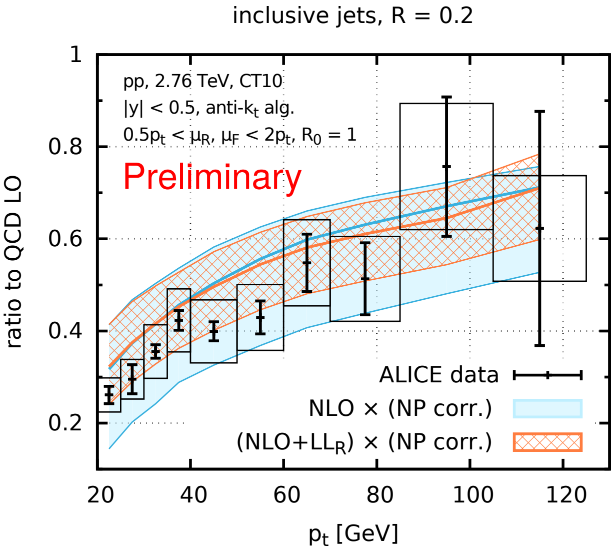
Comparison to data: ATLAS with $R = 0.4$

Small-R resummation shifts the spectrum by $5 - 10\%$, and changes the scale dependence of the NLO prediction.



Comparison to data: ALICE with $R = 0.2$

Small-R resummation somewhat improves agreement with Alice $R = 0.2$ data, and reduces the scale dependence of the NLO prediction.



CONCLUSION

Conclusion

- Using generating-functional approach, carried out numerical leading logarithmic resummation of $\ln R$ enhanced-terms in small-R jets.
- Discussed small-R effects in filtering and trimming. Perturbative convergence is particularly bad for filtering with $n_{\text{filt}} > 1$.
- Small-R effects can be substantial, for example reducing the inclusive jet spectrum by $30 - 50\%$ for gluon jets for $R = 0.4 - 0.2$.
- Preliminary: introduced new matching scheme and studied scale dependence of inclusive jet spectrum.
- Preliminary: small-R terms for inclusive jet spectrum can have noticeable effect beyond NLO. Comparison to ATLAS and ALICE data.

We intend to make the code public.

BACKUP SLIDES

Generalised k_t algorithm with incoming hadrons

Basic idea is to invert QCD branching process, clustering pairs which are closest in metric defined by the divergence structure of the theory.

Definition

1. For any pair of particles i, j find the minimum of

$$d_{ij} = \min\{k_{ti}^{2p}, k_{tj}^{2p}\}\frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}, \quad d_{jB} = k_{tj}^{2p}$$

   where $\Delta R_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$.
2. If the minimum distance is d_{iB} or d_{jB}, then the corresponding particle is removed from the list and defined as a jet, otherwise i and j are merged.
3. Repeat until no particles are left.

The index p defines the specific algorithm, with $p = \pm 1, 0$.

Jet radius value

Jet radius values for different experiments, excluding substructure R choices

	ATLAS	CMS	ALICE	LHCb
R	$0.2^{*}, 0.4 - 0.6$	$0.3^{*}, 0.5, 0.7$	$0.2 - 0.4$	$0.5, 0.7$

* for PbPb only

Evolution equations

We can write the complete evolution equations as differential equations, for the quark the previous graph corresponds to

Quark

$$\frac{dQ(x,t)}{dt} = \int dz\, p_{qq}(z)\,[Q(zx,t)\,G((1-z)x,t) - Q(x,t)]\,.$$

In the gluon case we find,

Gluon

$$\begin{aligned}\frac{dG(x,t)}{dt} &= \int dz\, p_{gg}(z)\,[G(zx,t)G((1-z)x,t) - G(x,t)] \\ &\quad + \int dz\, n_f\, p_{qg}(z)\,[Q(zx,t)Q((1-z)x,t) - G(x,t)]\,.\end{aligned}$$

Jet flavour

Given a parton flavour, we look at the probability that the hardest resulting microjet has changed flavour.

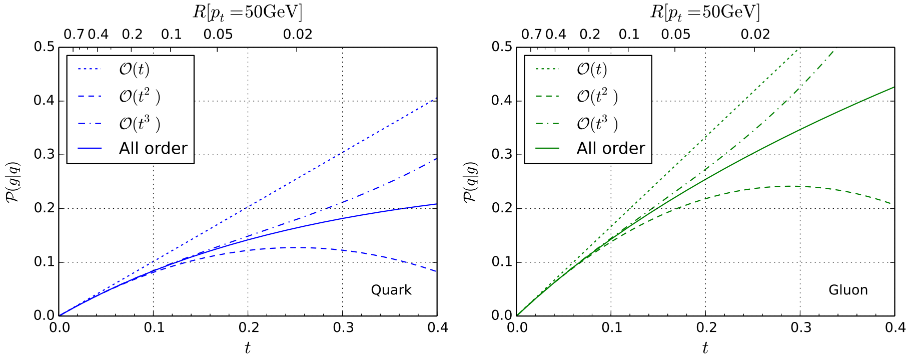


Figure – Flavour change probability.

Microjet vetoes

Jet veto resummation for Higgs production has terms

$$\alpha_s^m \ln^{2m} \frac{Q}{p_t} + \text{subleading}$$

Among the subleading terms there are small-R enhanced terms

$$\alpha_s^{m+n} \ln^m \frac{Q}{p_t} \ln^n \frac{1}{R^2} + \dots$$

Suspected of having important impact, and calculated by several groups

NNLL jet vetoes	$n = 1$	[Banfi, Monni, Salam, Zanderighi PRL 109 (2012) 202001]
		+ [Becher, Neubert, Rothen JHEP 1301 (2013) 125]
		+ [Stewart, Tackmann, Walsh, Zuberi PRD 89 (2014) 054001]
Alioli & Walsh	$n = 2$ (numerically)	[JHEP 1403 (2014) 119, corr. in arXiv-v3]
Our work	$n = 2$ (analytically)	[JHEP 1504 (2015) 039]
	+ $n \to \infty$ (numerically)	

Microjet vetoes

Writing the probability of no gluon emissions above a scale p_t as

$$P(\text{no primary-parton veto}) = \exp\left[-\int_{p_t}^{Q}\frac{dk_t}{k_t}\bar{\alpha}_s(k_t)2\ln\frac{Q}{k_t}\right],$$

one can show that including small-R corrections and applying the veto on the hardest microjet, we have

$$\begin{aligned}\mathcal{U} &\equiv P(\text{no microjet veto})/P(\text{no primary-parton veto})\\ &= \exp\left[-2\bar{\alpha}_s(p_t)\ln\frac{Q}{p_t}\int_0^1 dz\, f^{\text{hardest}}(z,t(R,p_t))\ln z\right].\end{aligned}$$

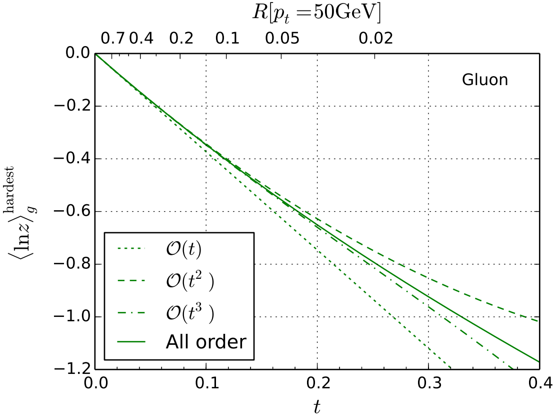
The R-dependent correction generates a series of terms

$$\alpha_s^{m+n}(Q)\ln^m(Q/p_t)\ln^n R\,.$$

The logarithmic moment of f^{hardest} is

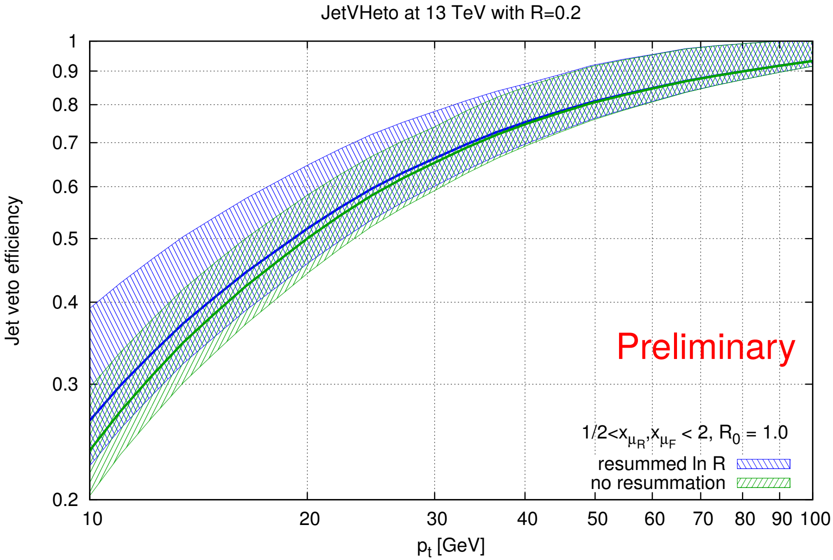
$$\langle \ln z \rangle^{\text{hardest}} \equiv \int_0^1 dz\, f^{\text{hardest}}(z) \ln z\,.$$

This seems to have a particularly stable perturbative expansion.



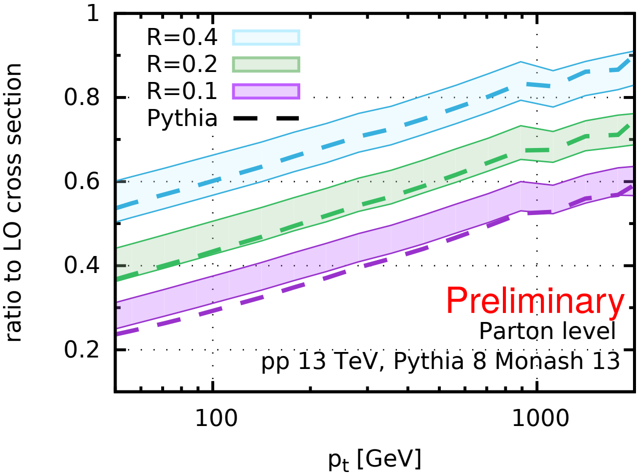
Small-R effects in jet veto efficiency

Inclusion of small-R terms leads to better handle on uncertainties.



Comparison with Pythia 8

Small-R resummation yields spectrum similar to parton shower.



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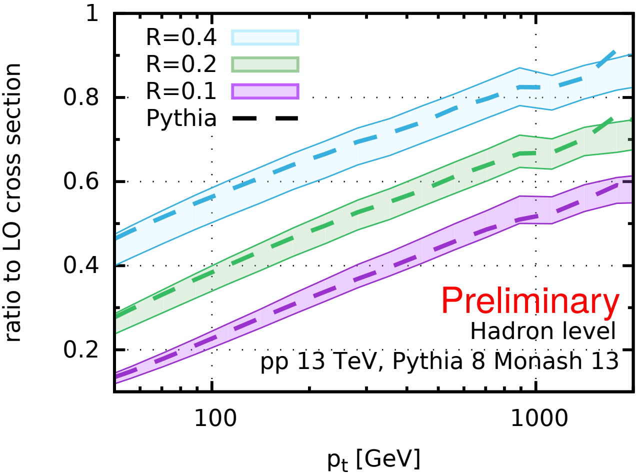

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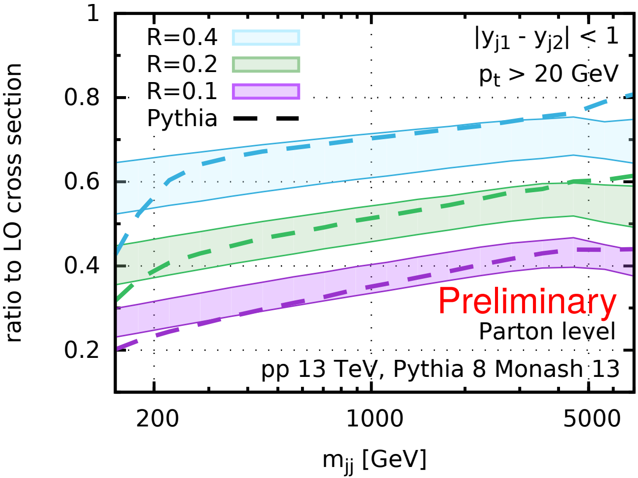

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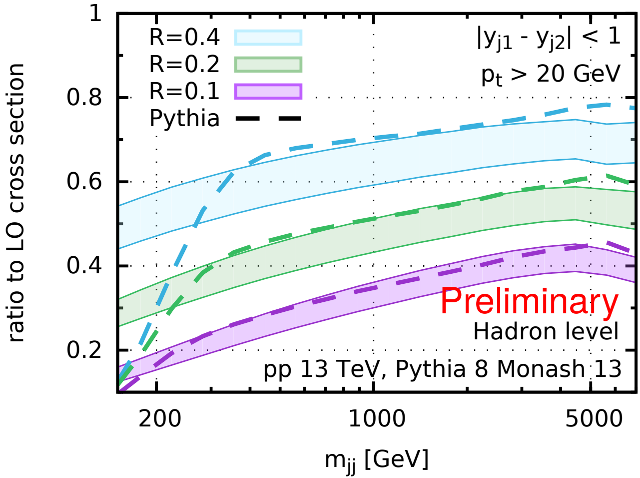
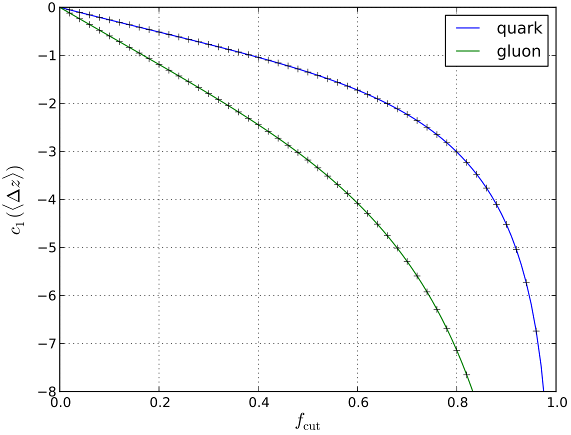


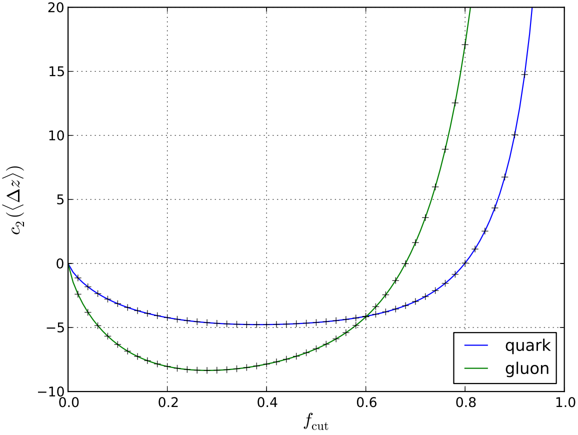
Trimming coefficients

First order coefficients $c_1(\Delta z)$ as a function of f_{cut}.



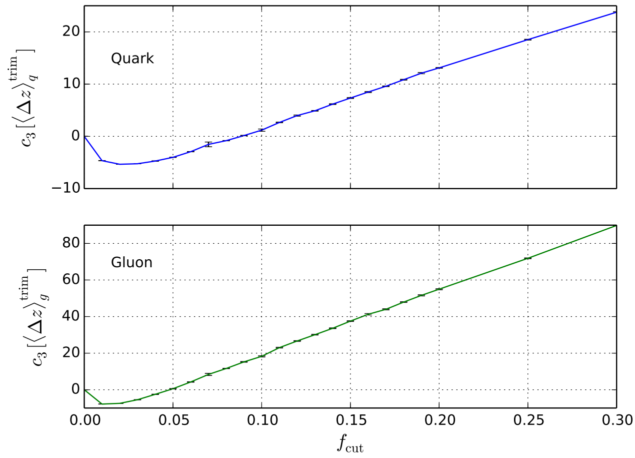
Trimming coefficients

Second order coefficients $c_2(\Delta z)$ as a function of f_{cut}.



Trimming coefficients

Third order coefficients $c_3(\Delta z)$ as a function of f_{cut}.



Trimming coefficients

Fourth order coefficients $c_4(\Delta z)$ as a function of f_{cut}.