Photon 2007, 9-13 July, Paris

Determination of parton distribution functions of the proton

Emmanuelle Perez (CERN-PH)

How to determine the non-perturbative inputs necessary to calculate a process in a collision involving hadron(s) :

$$\sigma(AB \to X) = \sum_{a,b} \int dx_a\, dx_b\, G_{a/A}(x_a, \mu_F^2)\, G_{b/B}(x_b\, \mu_F^2)\, \hat{\sigma}^{ab\to X}(x_a, x_b, \mu_F^2, \ldots)$$

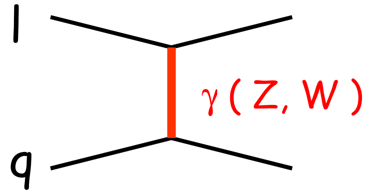
Using data from :

- Deep-inelastic scattering (inclusive & semi-inclusive)
- Drell-Yan measurements
- jet data (Tevatron & HERA)
- Electroweak processes at the Tevatron

Emmanuelle.Perez@cern.ch

Inclusive deep-inelastic scattering (ep, μp, μd, ν_μ N & $\bar{\nu}_\mu$ N)

$$\frac{d^2\sigma^{NC}_{e\mp p}}{dx dQ^2} = \frac{2\pi\alpha^2 Y_+}{xQ^4}\left(F_2 - \frac{y^2}{Y_+}F_L \pm \frac{Y_-}{Y_+}xF_3\right) \quad Y_\pm = 1 \pm (1-y)^2$$



Leading-order relations :

$F_2 = \sum e^2_{qi} [\, xq_i(x) + x\bar{q}_i(x) \,]$

In lepton-proton : ~ 4 $(u + \bar{u}) + (d + \bar{d})$

In lepton-deuterium : ~ $(u + d + \bar{u} + \bar{d})$

$u_n = d_p \equiv d$

$d_n = u_p \equiv u$

lp & ld allows to "separate" the flavors.

$xF_3 \sim \sum 2e_q a_q [\, xq_i(x) - x\bar{q}_i(x) \,]$ valence

$dF_2 / d\ln Q^2 \sim \alpha_S(Q^2)\, xg(x,Q^2)$ Scaling violations give access to xg(x)

F_L = 0 at leading order. Beyond LO, F_L ~ gluon density. So far, direct meas. only from fixed target experiments, i.e. at high x.

$\sigma_{CC}(e^+P) \sim (1-y)^2\ (xd + xs) + (x\bar{u} + x\bar{c})$ → mainly d

$\sigma_{CC}(e^-P) \sim (xu + xc) + (1-y)^2\ (x\bar{d} + x\bar{s})$ → mainly u

Flavor separation

$\sigma(\nu N) + \sigma(\bar{\nu} N)$ ~ $F_2^\nu = x\,(U + D + \bar{U} + \bar{D})$

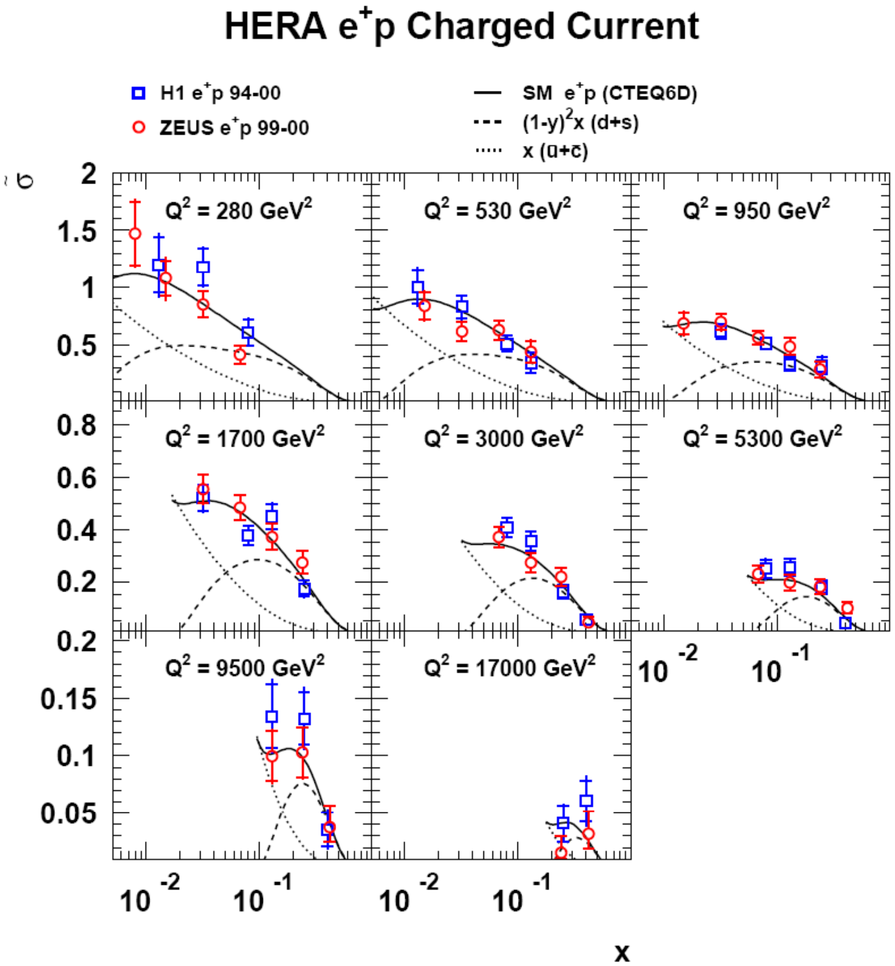
$\sigma(\nu N) - \sigma(\bar{\nu} N)$ ~ $xF_3^\nu = x\,(U - \bar{U} + D - \bar{D})$

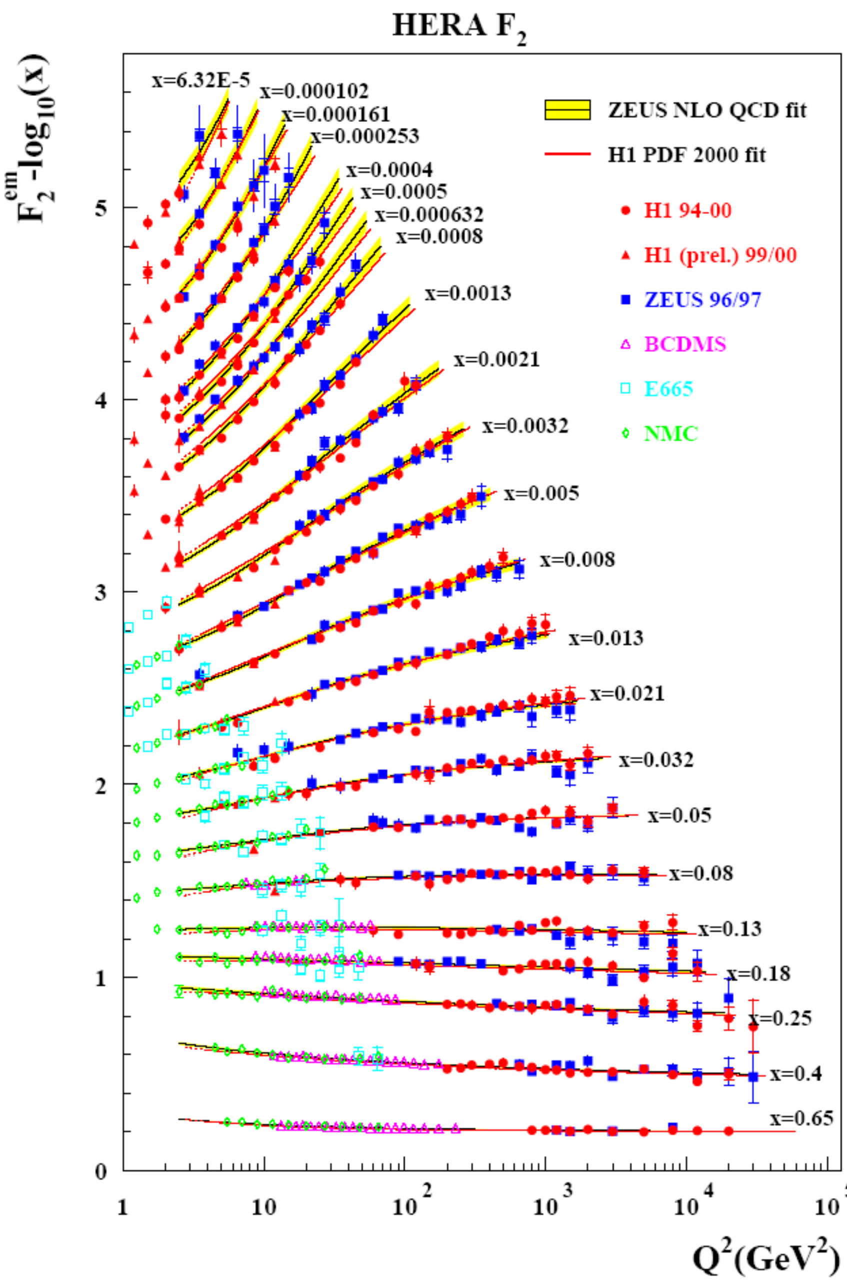
νN data require non-trivial nuclear corrections.

Example of DIS measurements

NC : strong scaling violations at low x

CC : flavor separation with ep data only.





QCD fits in a nutshell

- Parameterize a set of pdfs at a "starting scale" Q_0^2

  e.g. $xg(x) = A\, x^{\alpha}\, (1-x)^{\beta}\, P(x)$

  and a set of quark pdfs, e.g. u_{val}, d_{val}, TotalSea = $\Sigma\, \bar{q}$, $\bar{d} - \bar{u}$

  - quite some freedom in choosing what to parameterize
  - quite some freedom in choosing the form of the parameterization

- and do assumptions to supplement the lack of sensitivity of the fitted data.

  e.g. if only lp data are fitted, no information on $\bar{d} - \bar{u}$, set to zero or to smth consistent with other data.

- Usually impose number sum rules : $\int_0^1 [s(x) - \bar{s}(x)]dx = 0$ Id. for c, b

$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2 \qquad \int_0^1 [d(x) - \bar{d}(x)]dx = 1$$

And momentum sum rule : $\int_0^1 x[g(x) + \sum (q(x) + \vec{q}(x))]dx = 1$

- Helps fix the gluon normalisation
- "connects" the low x and high x behaviors of g(x)

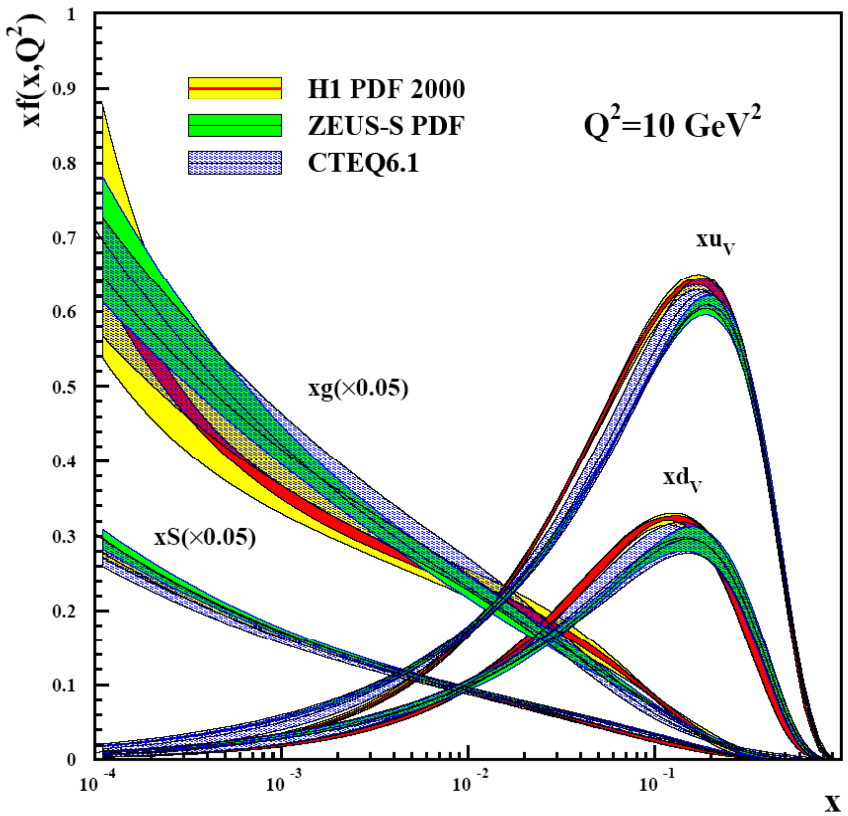
- DGLAP equations give $f(x,Q^2)$ at any Q^2, once $f(x, Q_0^2)$ is known.

  Allows to calculate σ_{theo} (DIS, DY, jet data,...) and fit theory to data.

QCD fits (NLO) from the H1 and ZEUS collaborations

Main differences:
- data included
- params. at Q_0^2
- treatment of heavy quarks

	H1 PDF 1997 Eur.Phys.J C21 (2001)	H1 PDF 2000 Eur.Phys.J C30 (2003)	ZEUS-S Phys.Rev.D67 (2003)	ZEUS-JET Eur.Phys.J C42 (2005)
other experiments used	BCDMS (μp)		BCDMS, NMC, E665, CCFR (μp, μd) (νFe)	– (but jets)
fitted distributions	ep valence and sea terms	$u+c,\ \bar{u}+\bar{c}$ $d+s,\ \bar{d}+\bar{s},\ g$	$u_v,\ d_v$ $S,\ \bar{d}-\bar{u},\ g$	$u_v,\ d_v$ $S,\ \bar{d}-\bar{u},\ g$
Q_0^2 Q^2_{min}	4 3.5	4 3.5	7 2.5	7 2.5
main aim	α_s $g(x)$	pdfs	pdfs α_s	pdfs α_s

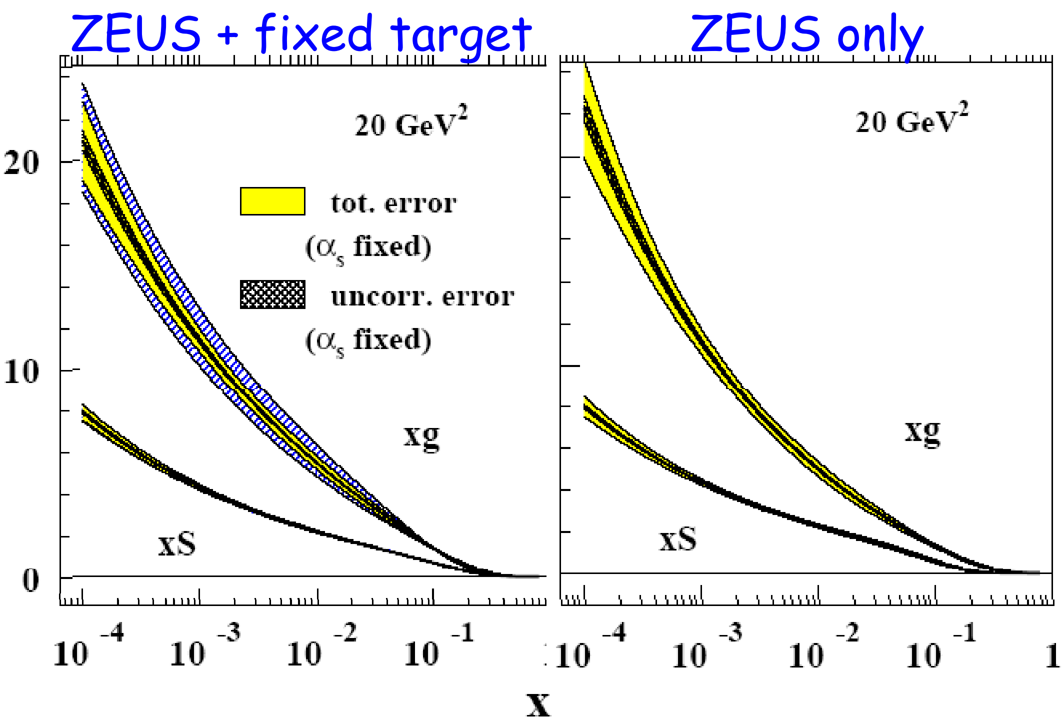


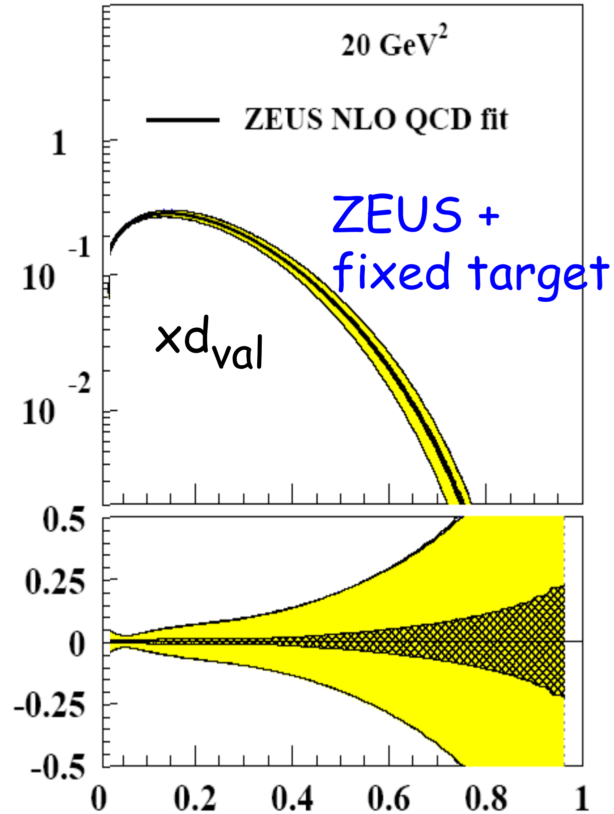
Reasonable agreement…

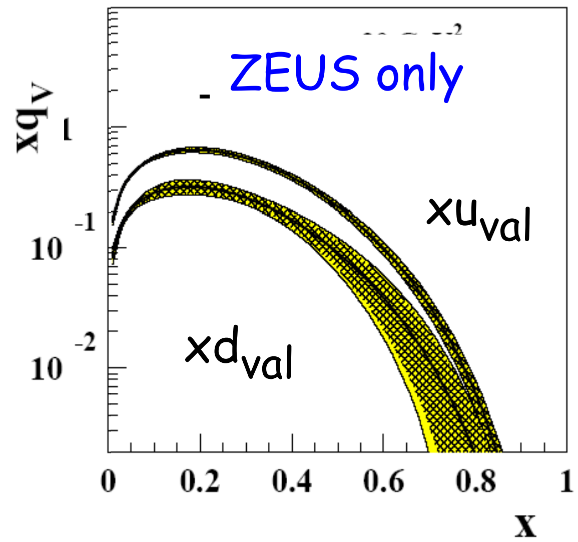
Differences however that are not embedded in the error bands, esp. for the valence distributions.

Sensitivity to those has a different origin in the H1 and ZEUS fits :
- H1 : uses mainly CC DIS to do the flavor separation. Note the pretty good determination with ep data only (free of corrections)
- ZEUS : this comes mainly from μp vs. μd and xF_3 measured in fixed target experiments.

Comparison of ZEUS alone / ZEUS with fixed target



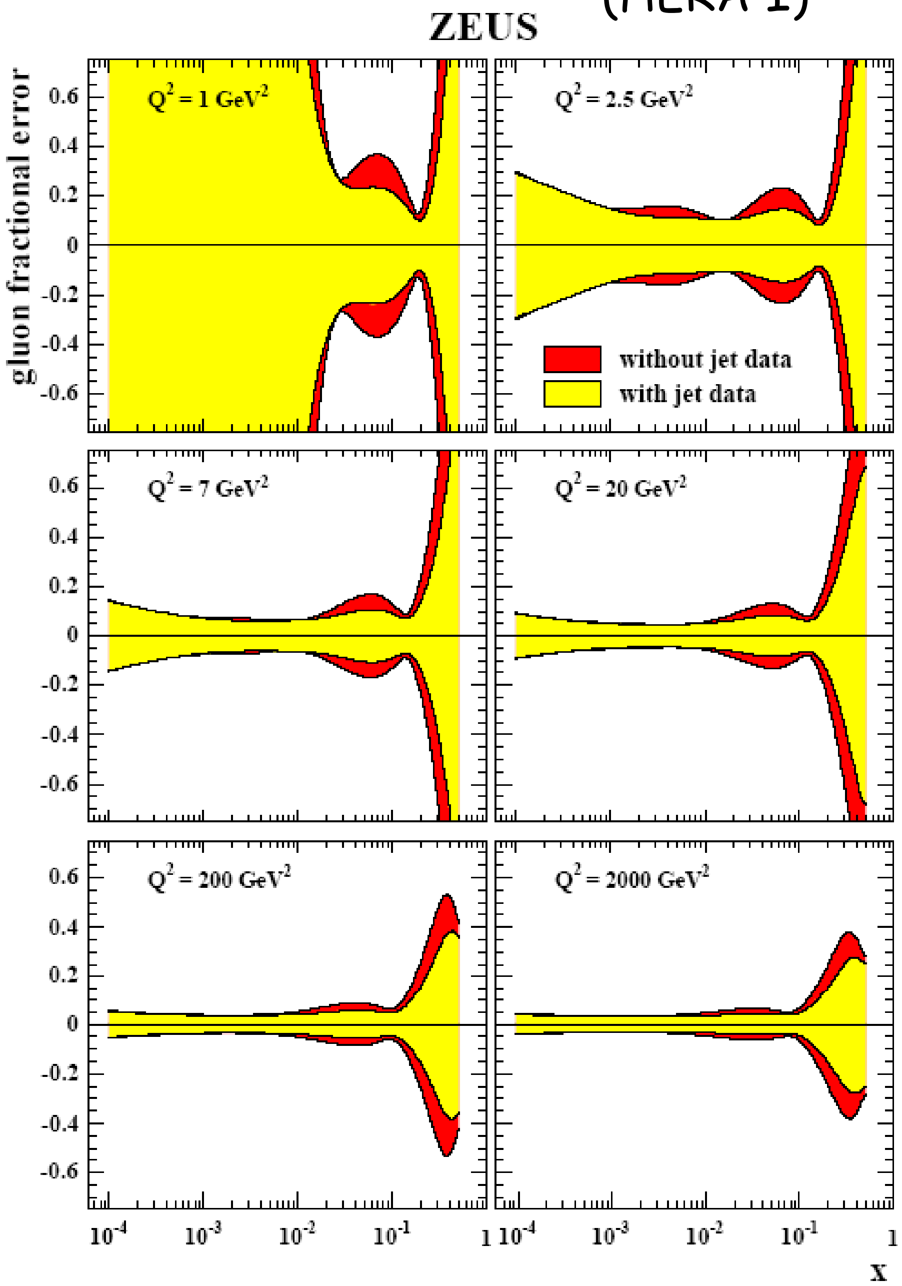


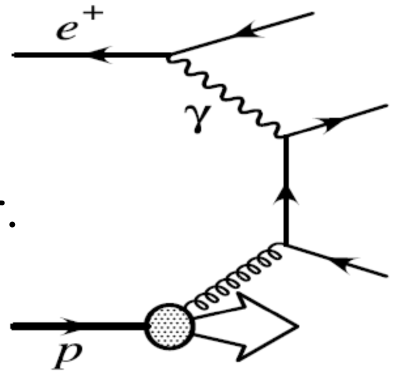


- the gluon and sea densities are mainly determined by the HERA data (for x below ~ 0.1)

- valence distribution : adding fixed target data reduces the uncertainties by a factor of ~ 2.
  u_{val} remains well determined from ZEUS alone.
  For d_{val} : deuterium data more constraining than HERA high Q^2 CC data.

(HERA I)





HERA jet data have small system. uncert. (typically ~ 5%)

comparison of gluon distribution from fits with and without jets :

no significant change in shape:
no tension between jet and inclusive data (QCD factorisation)

HERA jet cross sections bring constraints on the gluon density in the range x = 0.01 – 0.4

Impact in global fits will be more prominent with the full HERA statistics (increase x 5 with the HERA II data).

Adding more data : global fits

Global fits performed mainly by the MRST/MSTW and the CTEQ groups.

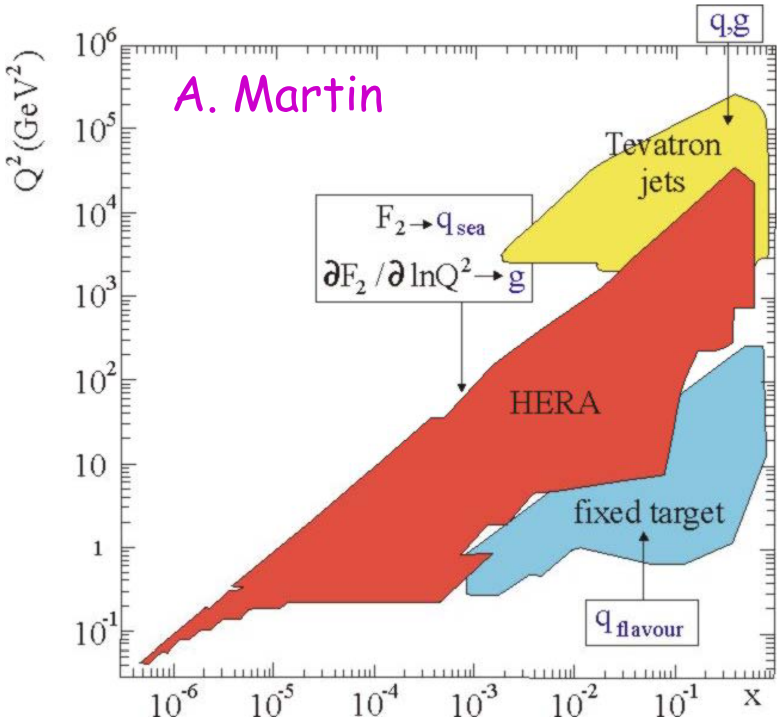
Non-inclusive DIS data that are usually included :

- Tevatron jet cross-sections → high x gluon
- Drell-Yan measurements $pN \to \mu\mu$ → large x quarks, $\bar{d} - \bar{u}$
- Dimuon production in νN and $\bar{\nu} N$ ($\nu_\mu s \to \mu c \to \mu\mu X$) → s and $\bar{s}$
- η asymmetry of W production at Tevatron → d/u at medium x

Recent fits also include HERA jet data and F_2^b & F_2^c measurements.

Some data used to be included in global fits, as prompt photon production which in principle brings constraints on the gluon density – but hampered by too large theoretical uncertainties, and disagreements within datasets.

Typically this leads to ~ 3000 points in the fits, with a large number of systematic error sources.



Recent updates of global fits

See e.g. presentations of R. Thorne and W.K. Tung at DIS'07 for details

- Detailed study of the strange content of the nucleon
  $\rightarrow$ by including dimuon data from CCFR and NuTeV

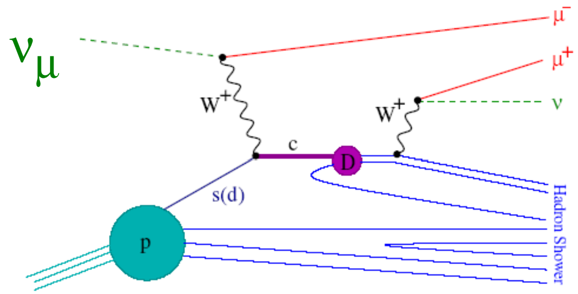
- Other new datasets included :
  - semi-inclusive measurements of F_2^b and F_2^c from H1
  - HERA and Tevatron Run II jet data included in MRST/MSTW07, as well as CDF RunII lepton asymmetry from W
    $\rightarrow$ things are nicely consistent

- First global analysis allowing an intrinsic charm component from CTEQ

- Treatment of heavy flavors in NLO fit :
  CTEQ6.1 $\rightarrow$ CTEQ6.5 : new implementation of the general mass Variable Flavor number scheme taking into account heavy quark mass effects. Large effect.

- Complete set of NNLO parton distribution functions with their uncertainties from MRST. W.r.t. the approximate MRST2004NNLO, full treatment of heavy flavors. Large effect.
  Also include NNLO for Drell-Yan.

The strange content of the nucleon

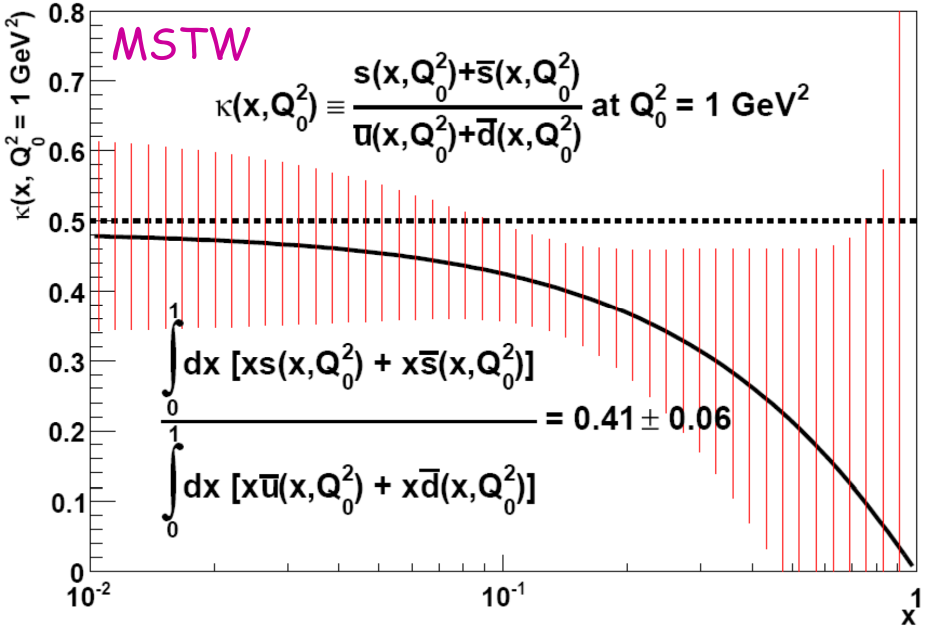


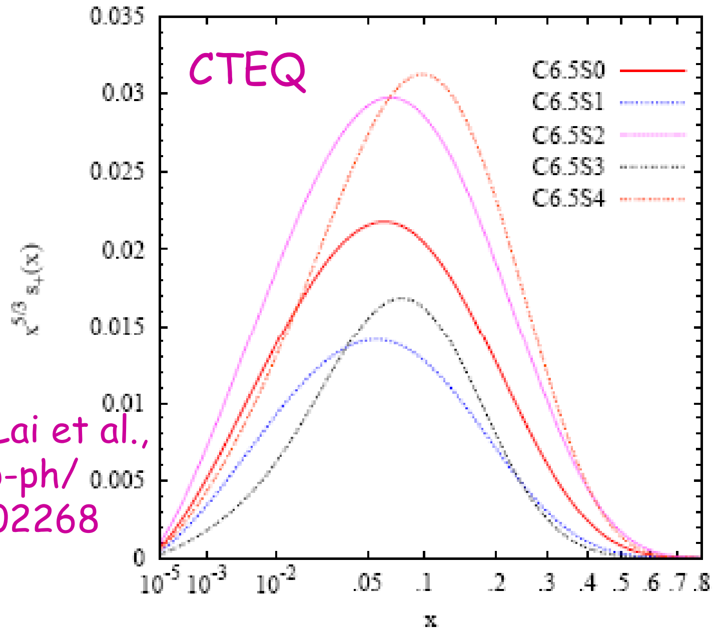
Experimentally : NuTev, CCFR : $W^+ \, s \to c \to \mu^+ X$
$W^- \, \bar{s} \to \bar{c} \to \mu^- X$

Previously, fits assumed that, at Q_0^2 :
$s = \bar{s} = r \, (\bar{u} + \bar{d}) \, / \, 2$ with $r \sim 0.5$

Inclusion of NuTeV & CCFR data in the fit allows to relax this assumption :
fit $s^+ = s + \bar{s}$. Low-x behavior = that of $\bar{u} + \bar{d}$ or of TotalSea (Regge inspired).
Additional new parameter for the high x behavior ($(1 - x)^\beta$).

Both groups observe a significant improvement of the χ^2 with this new param.,
i.e. data prefer an independent shape for $s + \bar{s}$.





H. Lai et al., hep-ph/ 0702268

Reduced ratio compared to r = 0.5.
Suppression at high x, i.e. low W^2.
Effect of ms ?

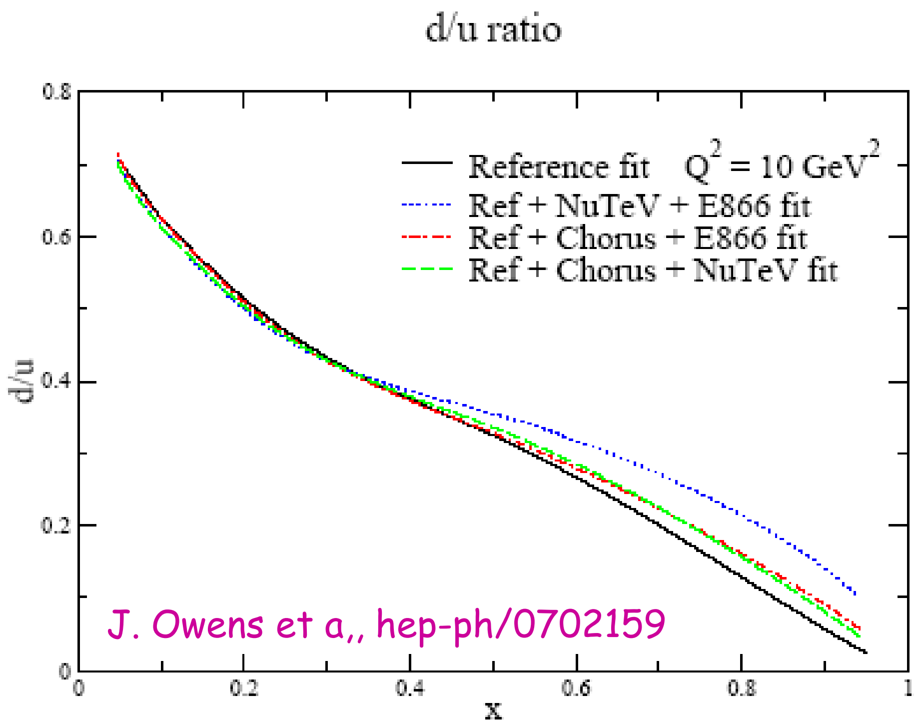
Neutrino DIS, Drell-Yan data and large x pdfs

- Latest ν DIS cross-sections from NuTeV (ν Fe) at high x : larger than older data from CCFR. Understood from calibration of magnetic field in CCFR spectrometer. NuTev also differs from ν DIS measured at Chorus (ν Pb).
  → discrepancies at high x.
    In the new MRST/MSTW fit, NuTev and Chorus data are included (replace CCFR) but cut out data at x > 0.5 (most relevant information is at x < 0.3 anyway).

- CTEQ analyzed these recent NuTeV & Chorus data together with latest DY measurements from E866. Tension seen at high x :
    - NuTeV data pull the valence distributions upward at high x. Also pull against the BCDMS and NMC data.
    - E866 data prefer lower valence distributions at high x.

Nuclear corrections to NuTeV data do not help…

Affects esp. the d/u at high x.
Specific fit performed without νDIS, giving more weight to DY, W asym, F_2^p/F_2^d.



Fit to RunII jet data

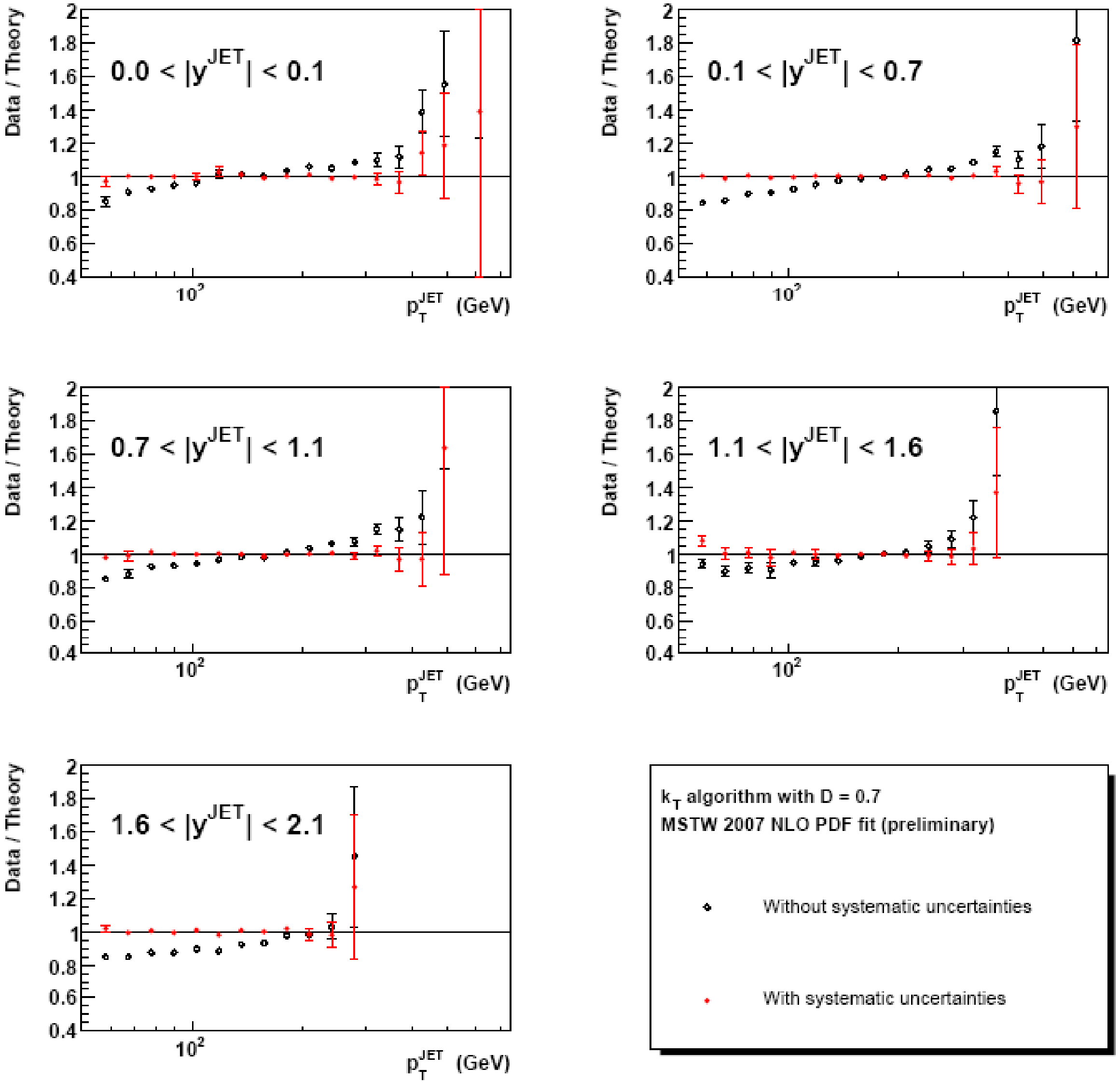
Very good χ^2.

Taking fully into account the (large) correlated systematic errors is mandatory to get a good global fit including Tevatron jet data.

New data prefer a slightly lower gluon at high x compared to Run I data, but consistent within 1σ.

Via sum rules, affect a bit the gluon at low x.

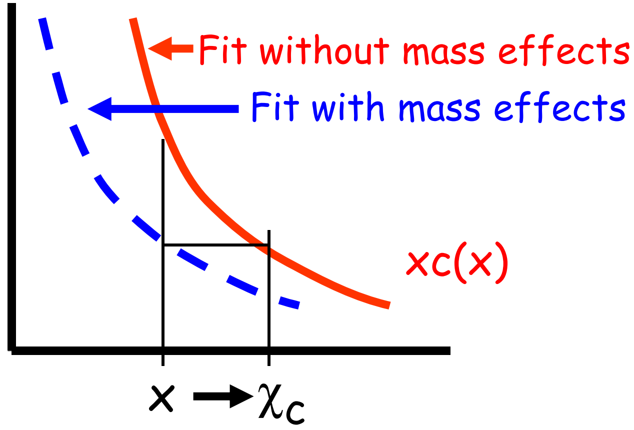
CDF Run II inclusive jet data, χ^2 = 56/76 pts.



Improved treatment of heavy flavors (NLO CTEQ fit)

Latest CTEQ fit (CTEQ6.5) : use the general VFNS with quark mass effects accounted for (kinematics + Wilson coeff.)

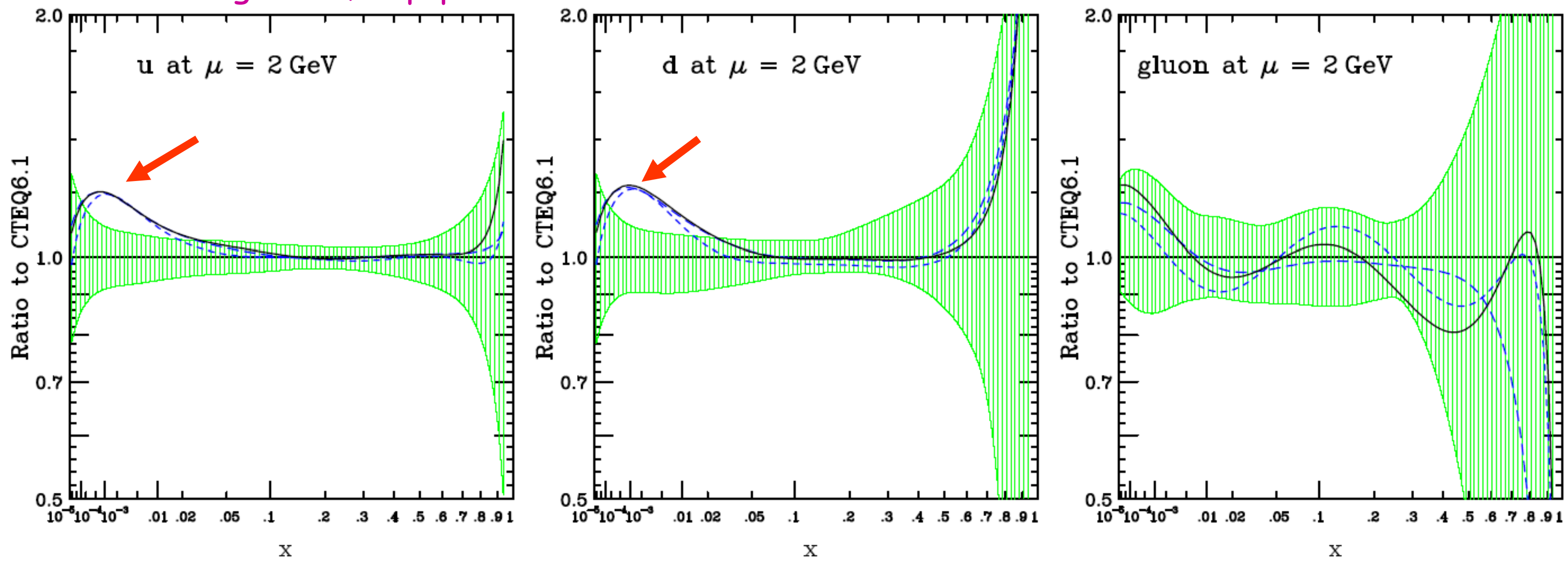
Main effect : rescaling of the momentum fraction carried by the incoming quark. E.g. in $\gamma c \to c$, $x \to \chi_c = x\,(1 + 4\,M^2_c\,/\,Q^2)$



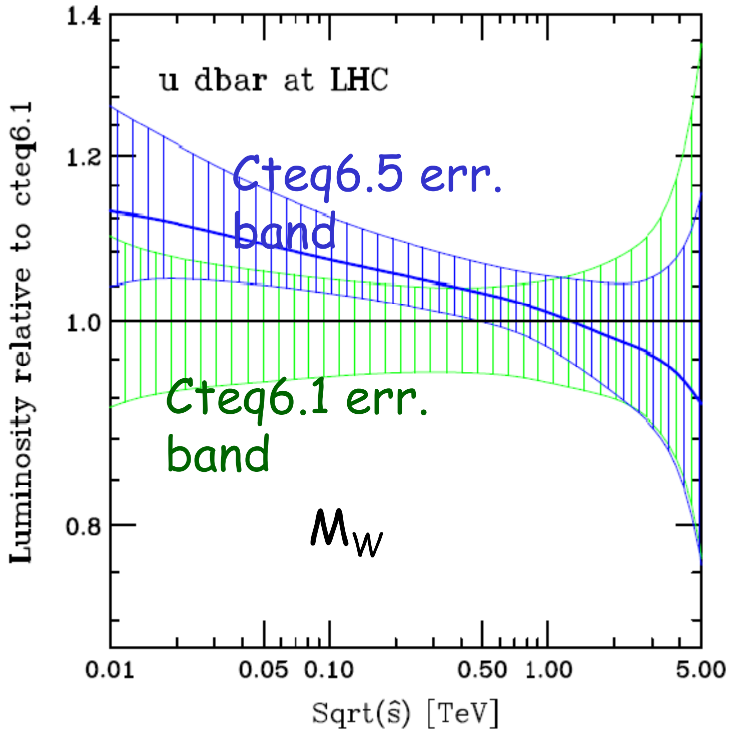
Largest effect when $xf(x,Q^2)$ varies quickly, i.e. at low x and Q^2. New formalism suppresses the HQ contributions relative to the zero-mass case.

Causes u and d to increase, with differences persisting at higher Q^2.

W.K. Tung et al., hep-ph/0611254

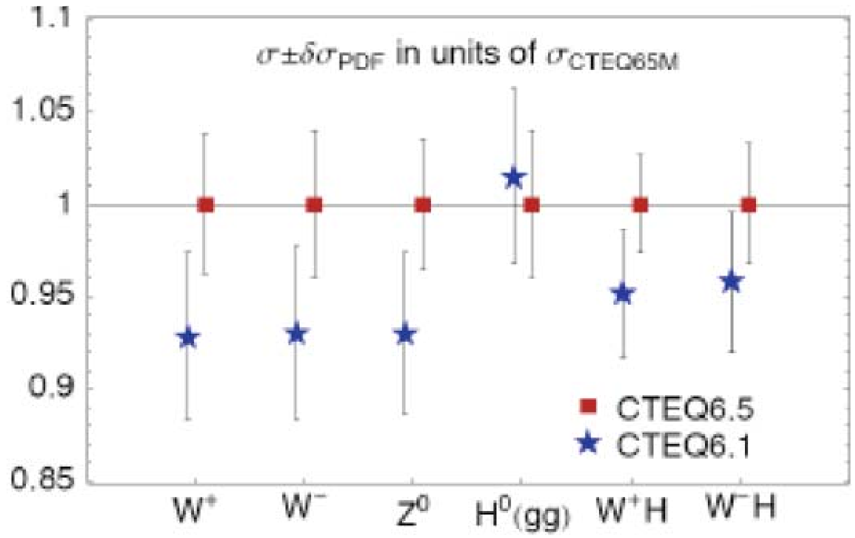


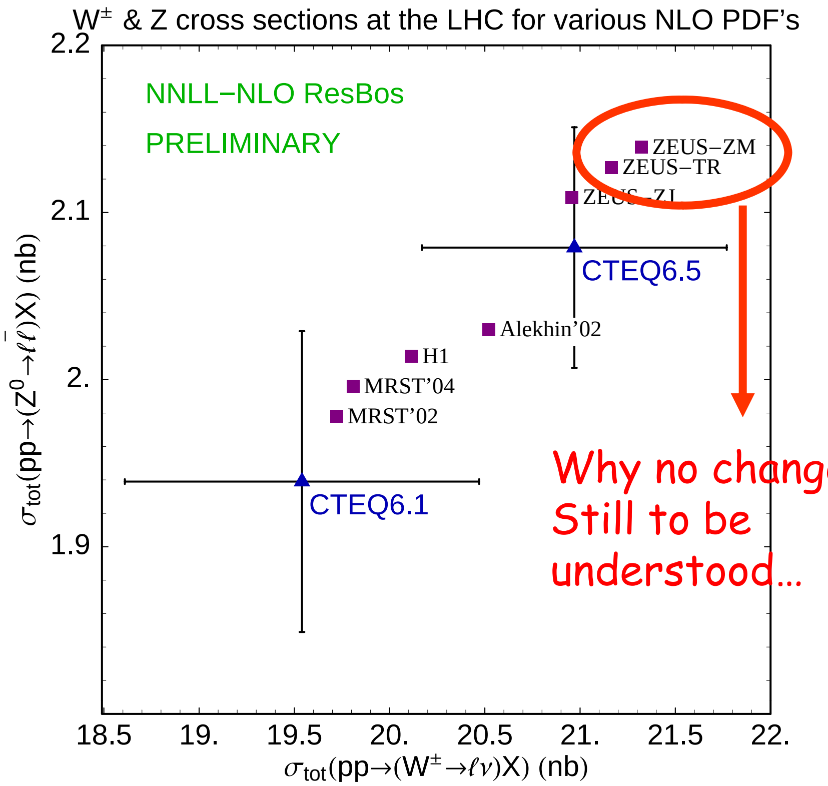
CTEQ6.1 → CTEQ6.5 and LHC predictions



The change affects mostly the q-q parton luminosities.

Hence the W and Z cross-sections at the LHC (<x> ~ 7 10^{-3} at central rapidity) :
x-sections larger by 8% with CTEQ6.5

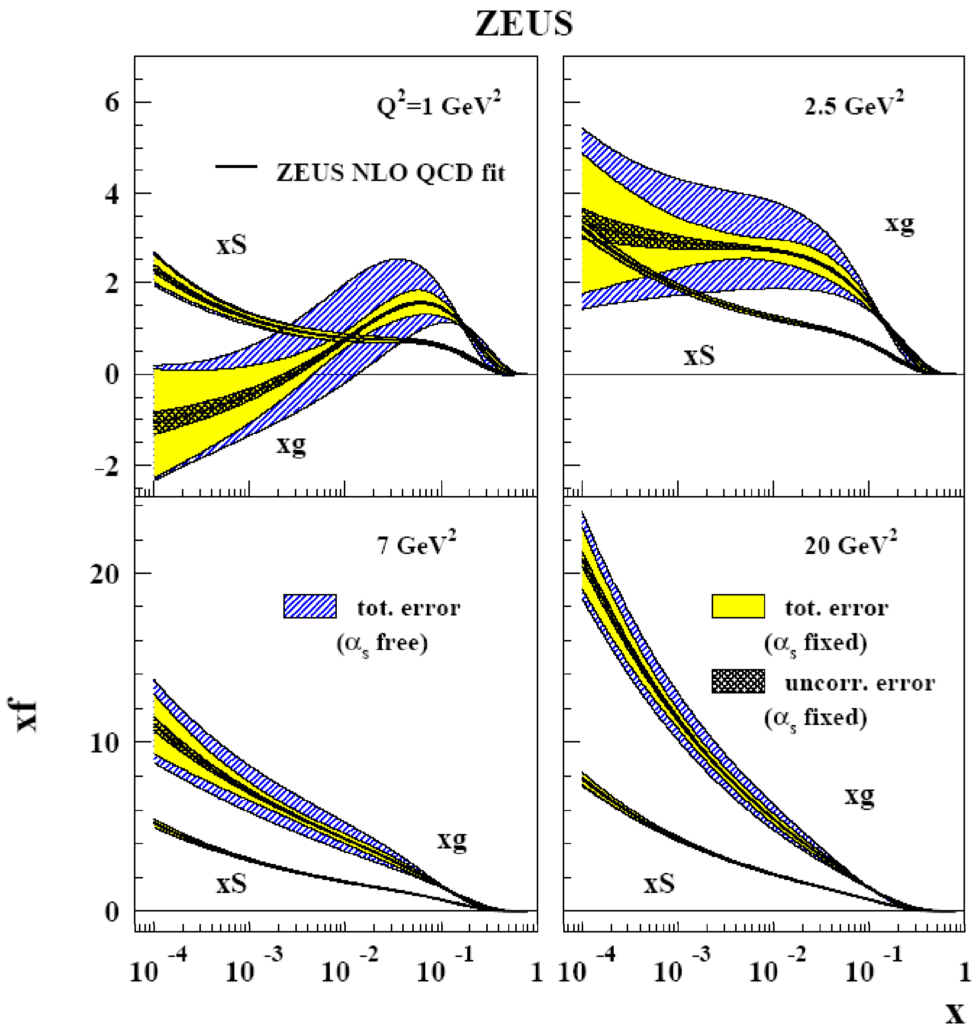




Why no change ? Still to be understood…

Photon, July 2007

A closer look to the low x and low Q^2 region



For low Q^2 : the sea continues to rise at low x, while the gluon density is suppressed !!

Gluon density even becomes negative at Q^2 = 1 GeV^2.

This gluon density results in a negative F_L at lowest Q^2.

MRST/MSTW also gets a negative xg and F_L at low x, for Q^2 ~ 2-3 GeV^2.

Sign that the approximations done in the QCD calculations are not valid in this regime.

Non-convergence could be due to important terms in α_S ln(1/x).
Cured by NNLO ? Or a full resummation of these ln(1/x) is needed ?

Drastic approach: cut out the lowest x data in the fits…

Was tried by the MRST group ("MRST03 conservative pdfs").
The cut $x > x_{min}$ was made more and more severe, until fits are stable.
Stability was obtained for $x > \sim 5.\,10^{-3}$!

These fits do not describe the HERA data at lowest x.
And give very different predictions from "standard" fits, for many observables at the Tevatron or the LHC.

Hence one needs to better understand the limitations of our calculations at low x…

NLO DGLAP predictions at low x and Q^2 could be wrong due to :

- Large terms in ln(1/x) $\rightarrow$ look at NNLO, or at a resummation of these logs
- unitarization (saturation) effects which tame the low x rise of F_2, e.g. due to gluon recombinations $\rightarrow$ make the evolution equations non-linear.

To study this experimentally, one needs more observables than just F_2. E.g :

- the longitudinal structure function F_L $\rightarrow$ talk of A. Petrukhin
- the slopes of F_2 (revealed no sign of a taming of the rise at low x)

(- exclusive final states)

Complete set of NNLO pdfs

- Without any major approximation
- With full uncertainties

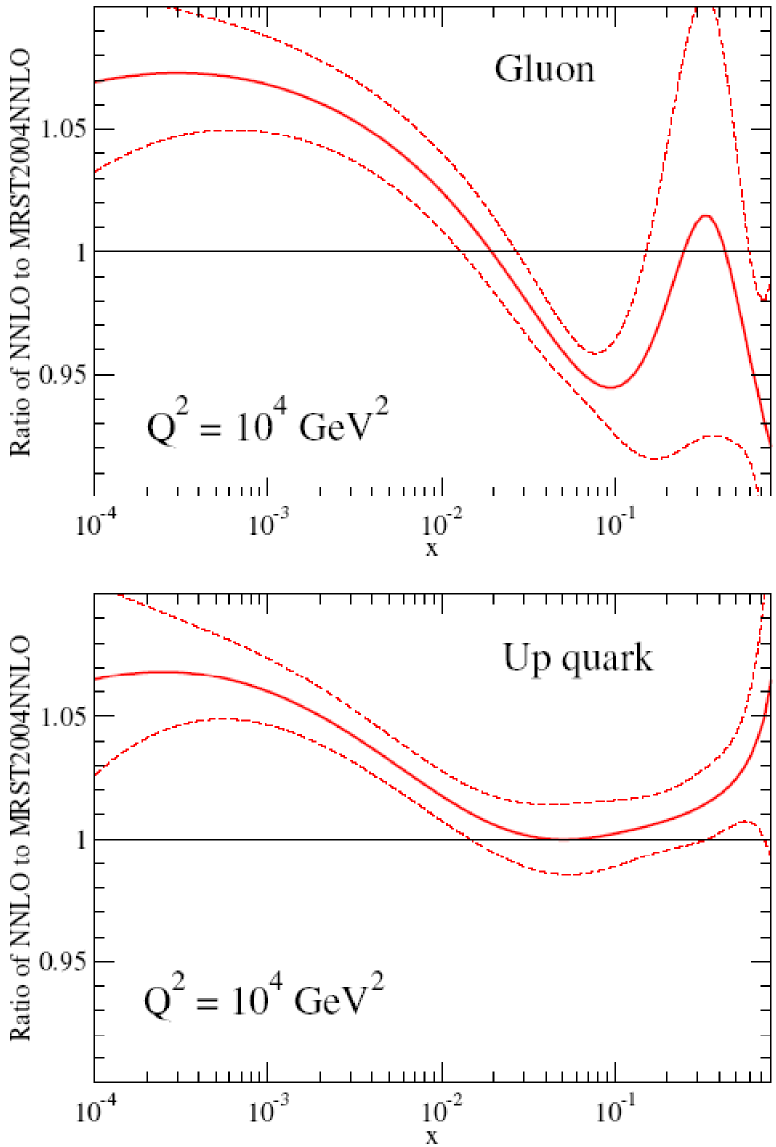
A. Martin et al., arXiv:0706.0459

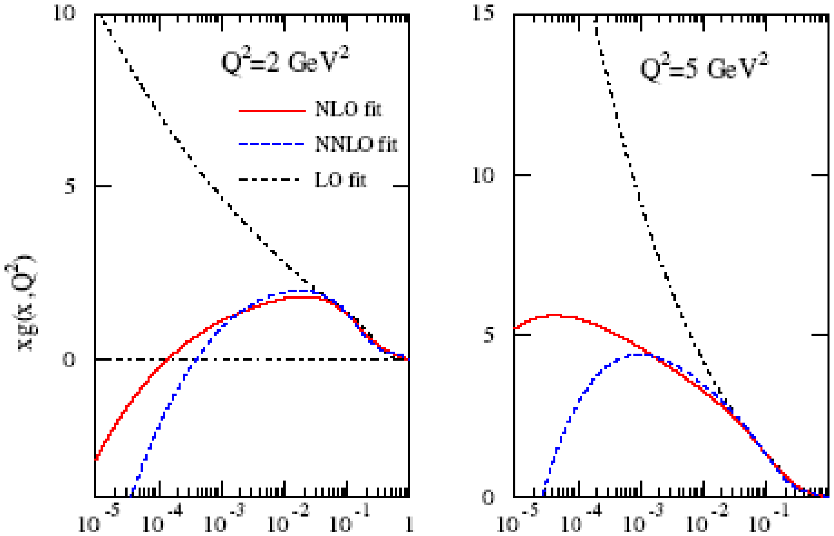
Main change w.r.t. previous, approximate NNLO : full VFNS (maintains the continuity of physical observables, by introducing disc. in coeff. functions which counter those in the pdfs).

Leads to a flatter evolution of charm.
Means that the light quarks have to evolve a bit more quickly.
Also affects the gluon pdf.

Big changes !
Previous approx. pdfs lie outside of the error bands.

Results in a 6% increase of $\sigma(W)$ and $\sigma(Z)$ at the LHC.

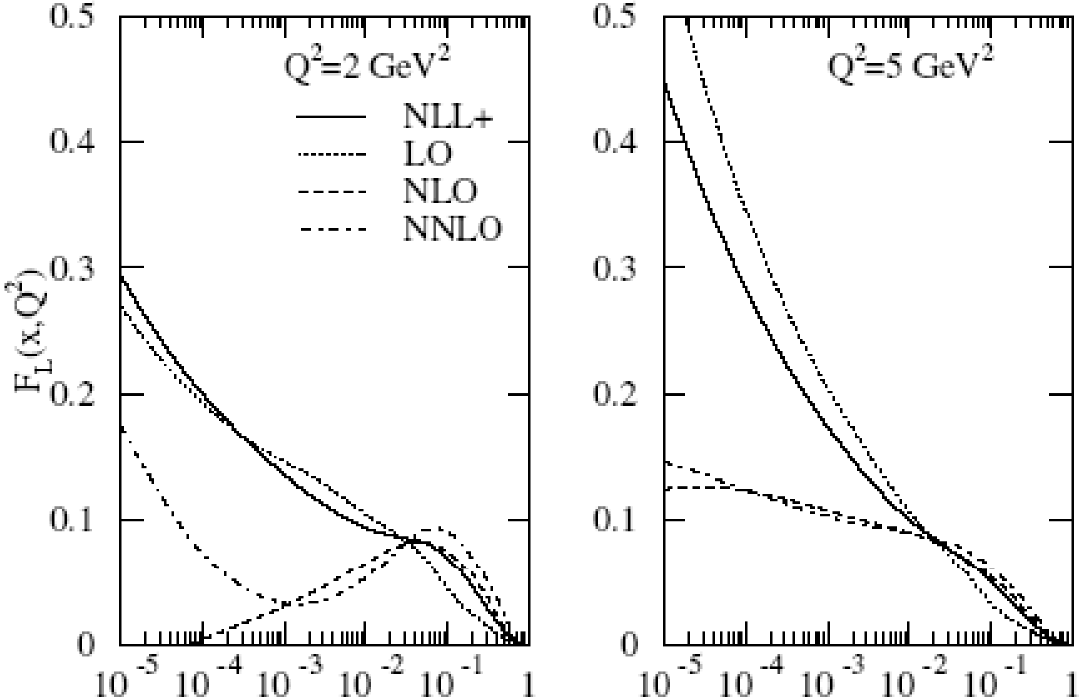




At NNLO, the gluon density at low x and low Q^2 becomes even more negative than at NLO.

Compensated by positive terms in the $O(\alpha_S{}^3)$ coefficient function for $F_L \rightarrow F_L$ at NNLO is positive.

R. Thorne, C. White, PRD 75 (2007) 034005



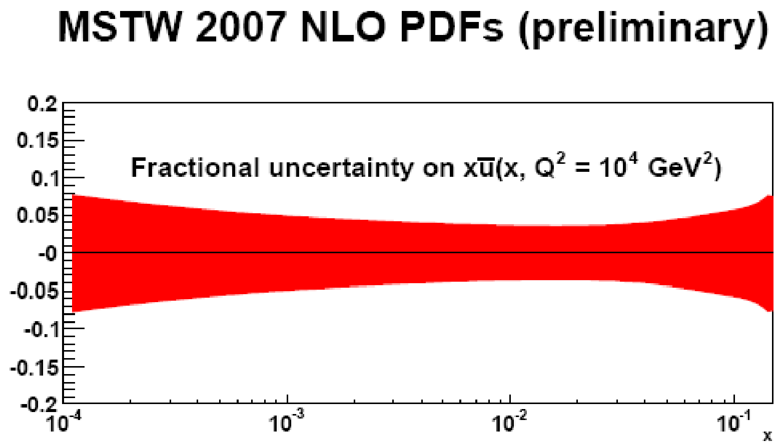
F_L measurements at HERA may tell us more about the correct approach at low x :

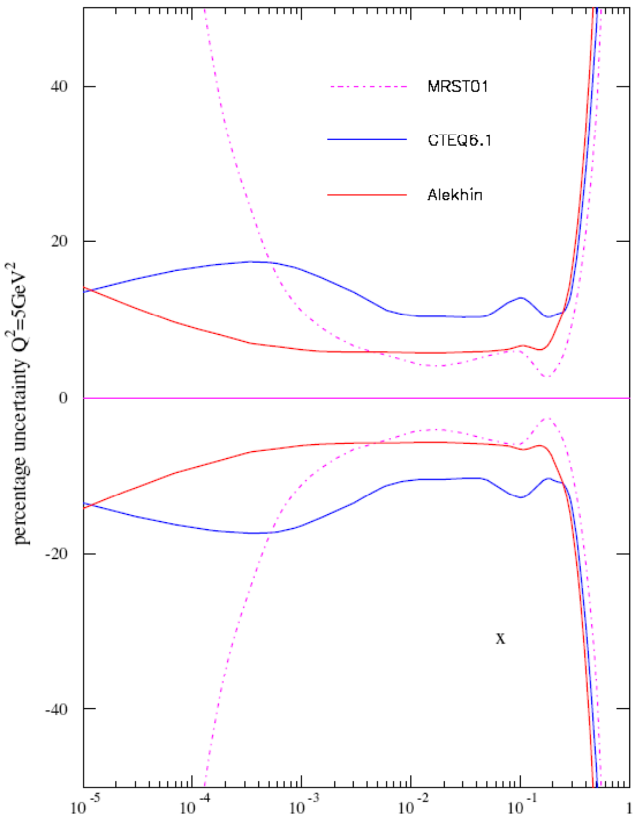
- NNLO enough ?
- or need a full resummation of ln(1/x) terms ?

Few remarks about the pdf uncertainties…

Keep in mind that the error bands shown do not include any "uncertainty" related to the parameterization choice / assumptions.

Example 1 : relaxing the hypothesis that $s = \bar{s} = 0.25\ (\bar{u} + \bar{d})$ results in a larger uncertainty on s, which feeds into that on $\bar{u}$ and $\bar{d}$. Uncertainty on low x antiquarks roughly doubles w.r.t. before.





Example 2 : uncertainty on the gluon at low x

Very different shapes for the error band on the gluon density in different global fits.

- MRST/MSTW parameterize at $Q_0^2 = 1\ GeV^2$ and allows the gluon to become negative.

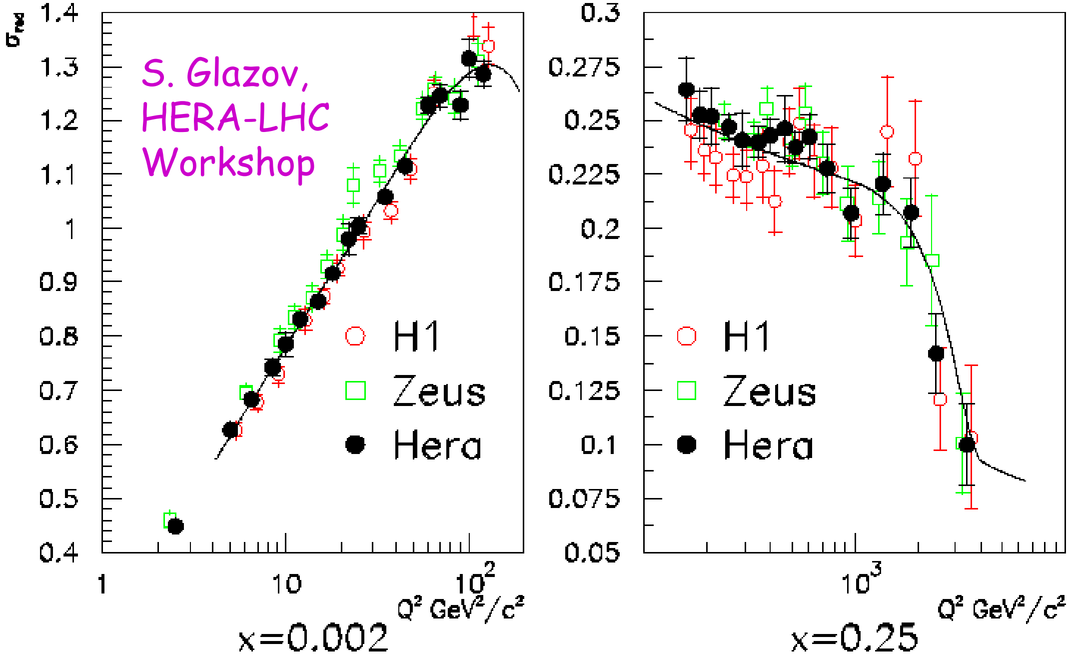
- CTEQ param. at $Q_0^2 = m_c^2 = 1.7\ GeV^2$. Input gluon is valence like and very small at low x, i.e. very small absolute error. At higher Q^2, all uncertainty is due to the evolution driven by the higher x gluon, which is well-determined.

Low x gluon w/o any theo. prejudice is very uncertain !

Conclusions : the near future… on the experimental side

- Analysis of the full HERA data
  - → high Q^2 measurements benefit from the large increase of luminosity with the inclusion of HERA II data (100 pb^{-1} → 500 pb^{-1}) i.e. constraints from xF_3, CC DIS, high Et jets will be much stronger.
  - → Final analysis of the low Q^2 HERA I data : Larger statistics (up to x2) compared to what is currently included in the fits, better understanding of systematics, expect a precision of 1 – 1.5 %.
  - → Direct F_L measurement with a precision of ~ 20%.

- Combined H1 + ZEUS dataset : averaging of the σ measurement in a model-independent way.
  "cross-calibration" of syst. uncertainties leads to an improvement which is better than √2 in regions where the measuremer are dominated by systematics.



- More precise jet data from Tevatron Run II (Jet energy scale unc. reduced)

- The LHC comes in operation.
  Constraints from EW processes (ATLAS, CMS and LHCb).

Backup

First global analysis allowing for an "intrinsic charm" component

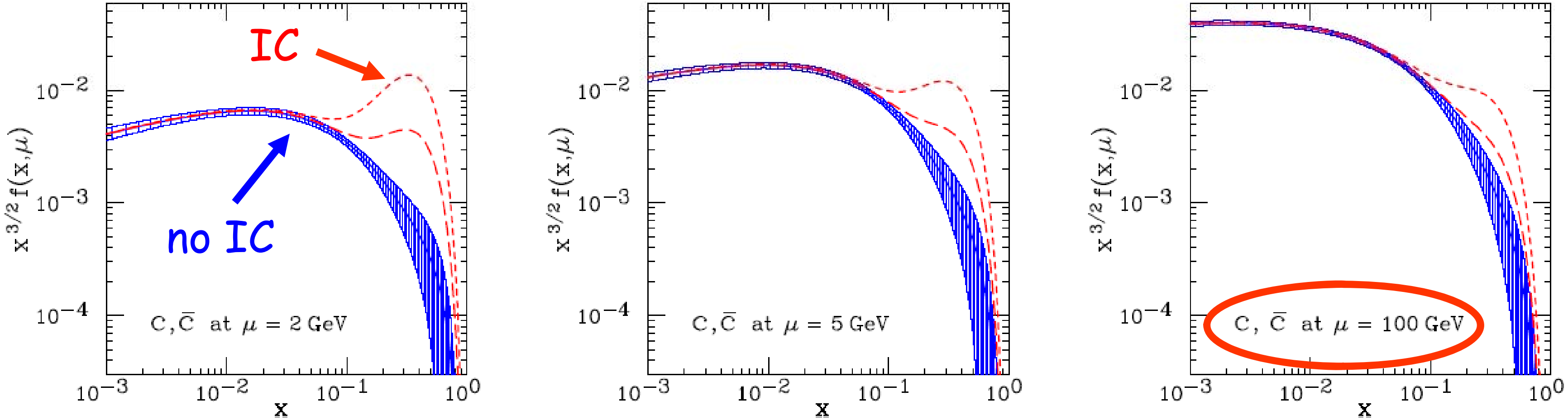
All pdf fits usually assume that charm is "radiatively" generated, i.e. originate only from QCD evolution, starting from a null distribution at $\mu \approx m_c$.
But an "intrinsic" charm component of non-perturbative origin could exist.

E.g. models of Brodsky et al, $\psi_p > = uud > + uud u\bar{u} > + uud c\bar{c} > + \ldots$

In a frame where the proton is moving, the $uudc\bar{c}$ state can exist only if all partons travel at the same rapidity, $y_i = \ln (k_i^+ / m_{T,i}) \sim \ln (x_i / m_i)$
i.e. the intrinsic charm quarks should be at high x.

The existing data have no sensitivity to such a component because it is at too large x. They allow that IC carries a few % of the proton momentum.

(Pumplin et al., hep-ph/070122)



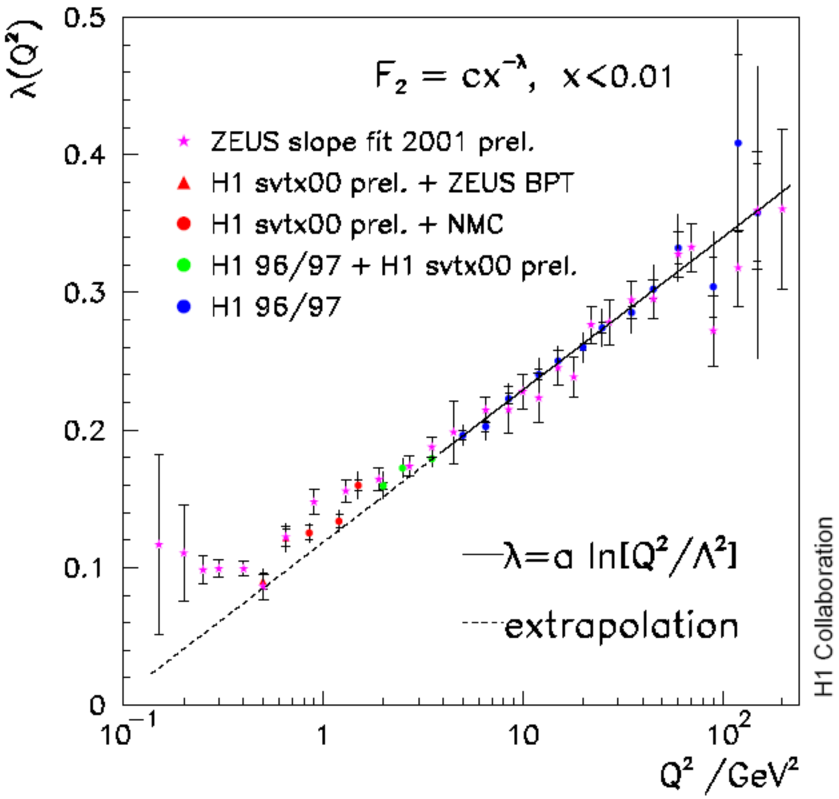
Could enhance drastically e.g. the production of H^+ via $c\,\bar{s} \to H^+$ at LHC

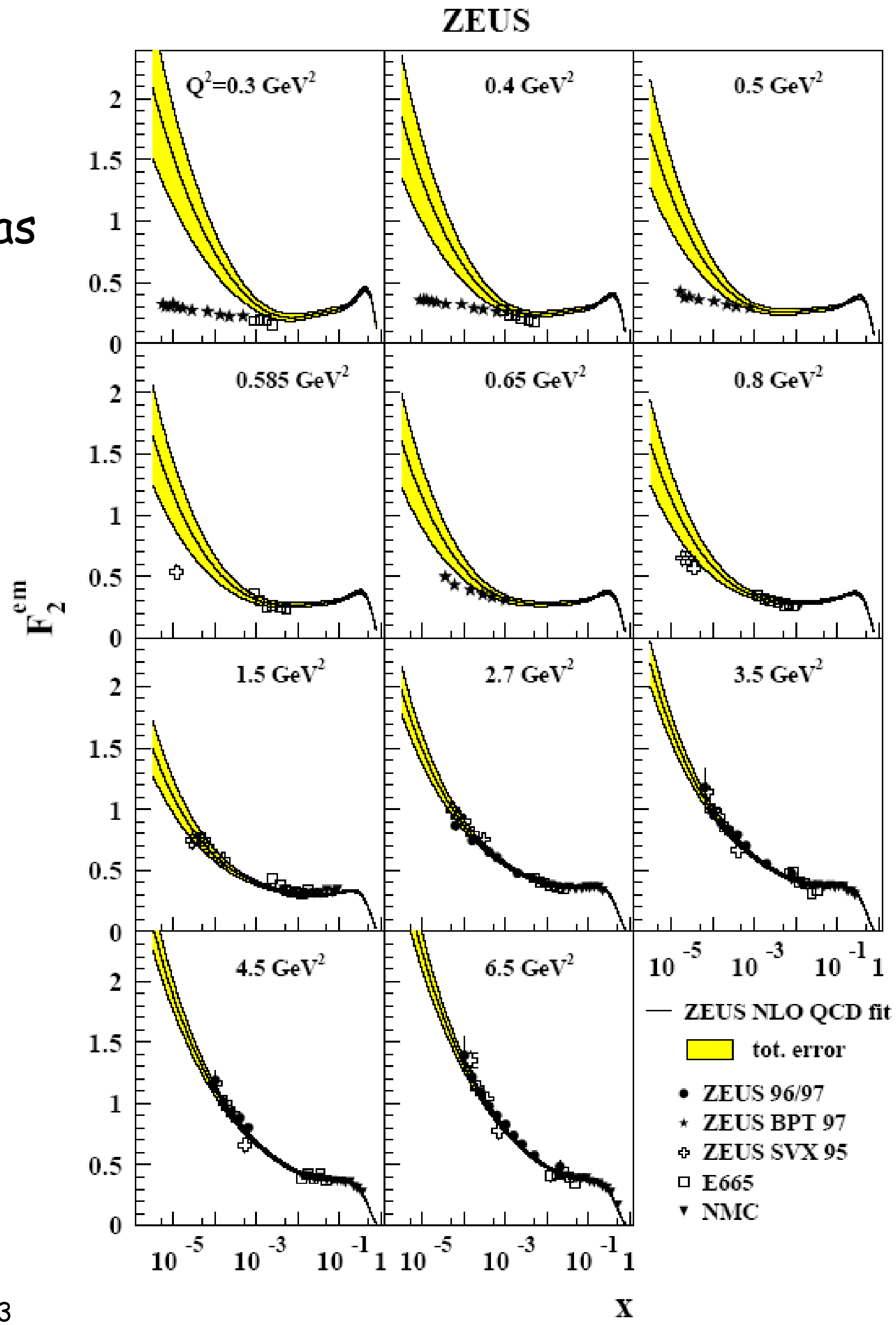
Strong rise of F_2 as $x \to 0$.
The rise increases with increasing Q^2.

F_2 at low x and low-medium Q^2 behaves as

$$F_2(x,Q^2) = c\ x^{-\lambda(Q^2)}$$

Deviation from $\lambda(Q^2) \sim \ln(Q^2)$ for Q^2 below ~ 1 GeV^2.

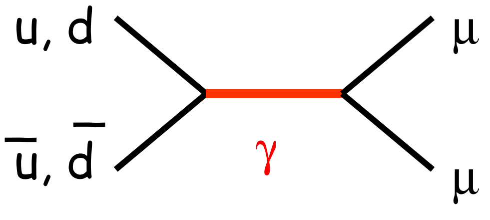




Drell-Yan measurements $\rightarrow$ constraints on $\bar{d} - \bar{u}$

$\bar{d} = \bar{u}$ was a "natural" assumption in global fits, until the NA51 experiment (CERN) reported that $\bar{d} > \bar{u}$ at $x = 0.18$ (some hints before from NMC…)

Follow-up by E866 (Fermilab) : fixed target, DY in pp and pd, E_{beam} = 800 GeV.



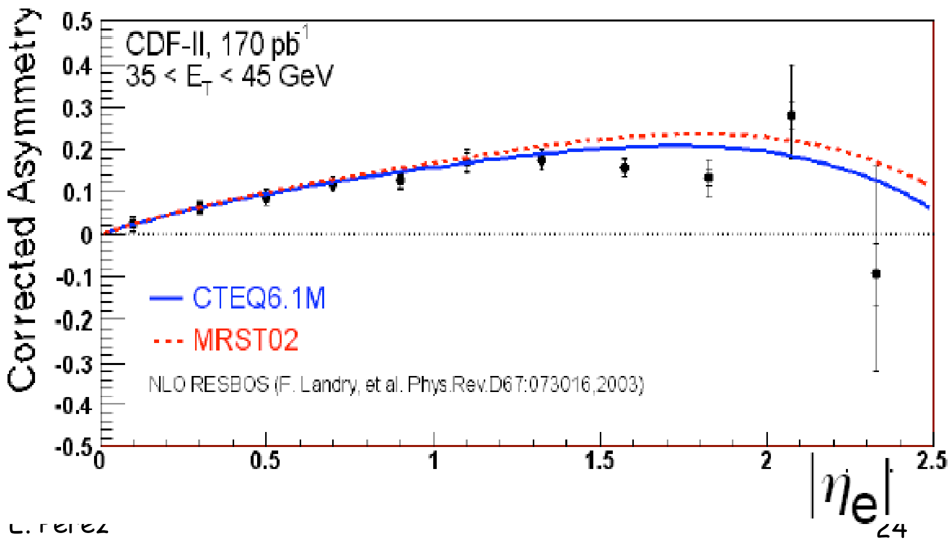
For $x_1 \gg x_2$ one has :

where $x_1 = x_{Beam}$, $x_2 = x_{Target}$

$$\frac{\sigma_{pd}}{2\sigma_{pp}} \sim \frac{1}{2} \frac{\left[1 + \frac{1}{4}\frac{d(x_1)}{u(x_1)}\right]}{\left[1 + \frac{1}{4}\frac{d(x_1)}{u(x_1)}\frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right]} \left[1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right]$$

E866 measures this ratio down to $\langle x_2 \rangle \sim 0.03$.

η asymmetry in W production at the Tevatron $\rightarrow$ d/u at medium x



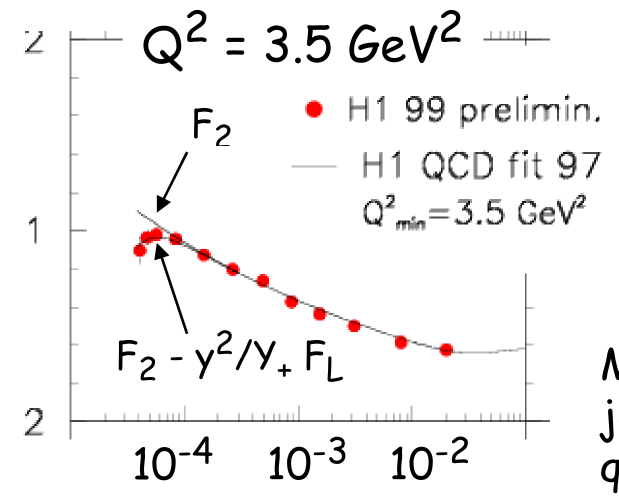
$x_{1,2} = (M^2_W / S) \exp(\pm \eta_W)$

At central rapidity, $x_1 = x_2 \sim 2\ 10^{-3}$

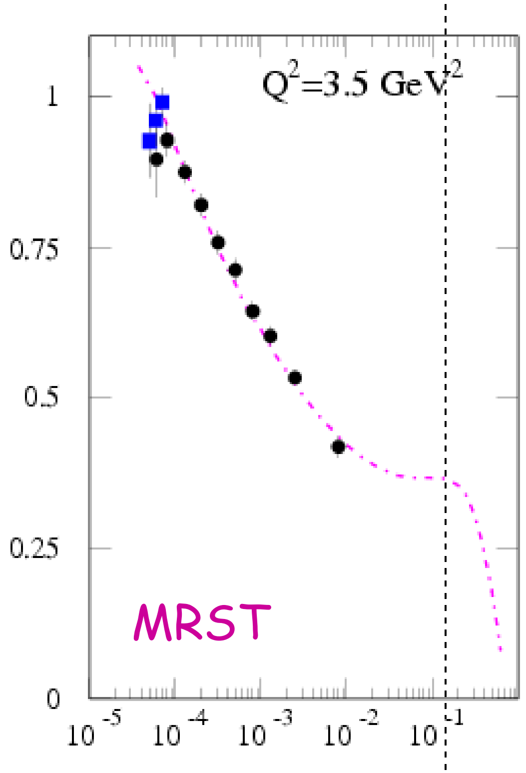
At $\eta \sim 2.5$: $x_1 = 2\ 10^{-2}$, $x_2 \sim 2\ 10^{-4}$

$\sigma(W+) - \sigma(W-) \sim$

$u(x_1)\, d(x_2)\, (1-\cos\theta)^2 - d(x_1) u(x_2) (1+\cos\theta)^2$

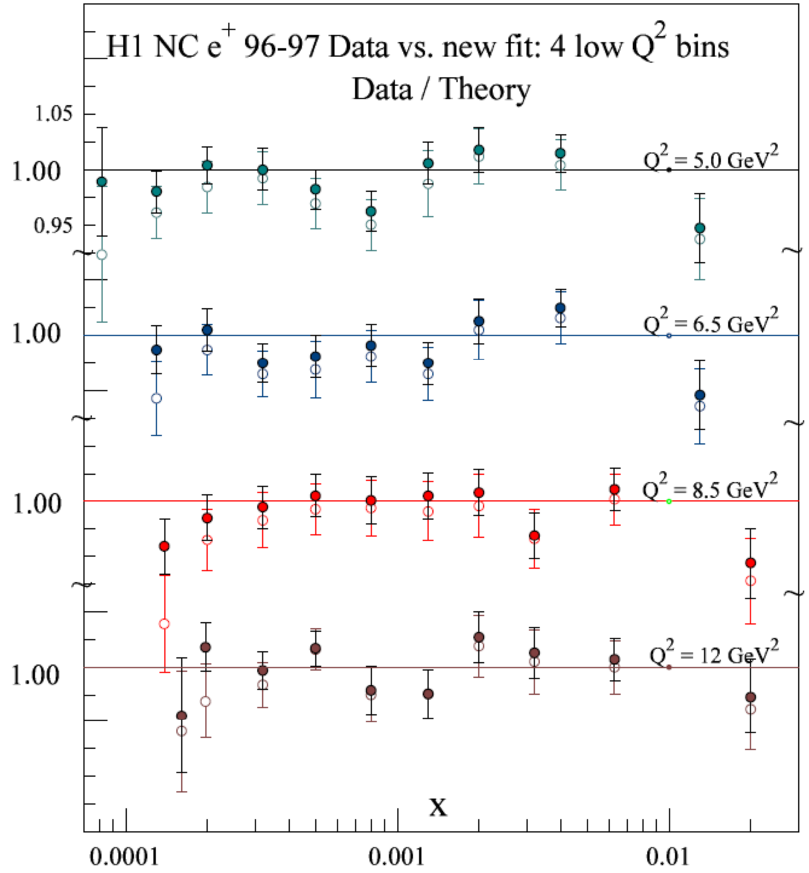


"HERA fits" reproduce well the turn-over observed in the NC cross-section at high y (low x), interpreted as the effect of FL ("loose" the contribution from longitudinal photons).



MRST : Tevatron jet data require a quite high g(x) at high x, resulting in a lower g(x) at low x, getting negative at Q^2 ~ 2-3 GeV^2. Leads to F_L < 0... Which does not reproduce the turn-over.

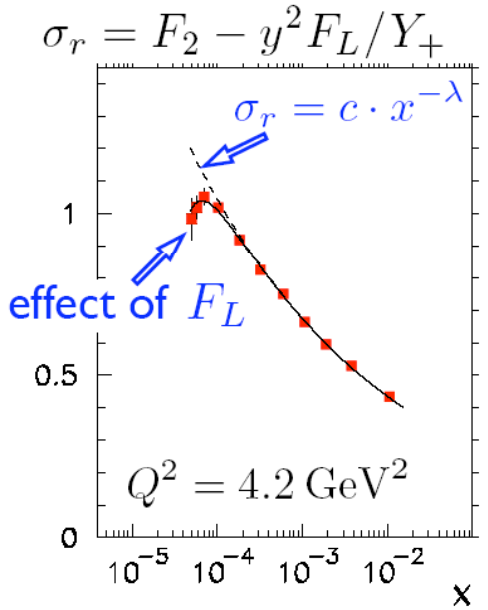
The CTEQ fit results in systematic shifts (within 2σ) which sweep away the turn-over.



Maybe a hint that HERA data at lowest x are less consistent with DGLAP within a global fit.

The longitudinal structure function F_L at low x

F_L is more directly related to the gluon density than is F_2.
Hence it is a good experimental observable to study the importance of the ln(1/x) terms.

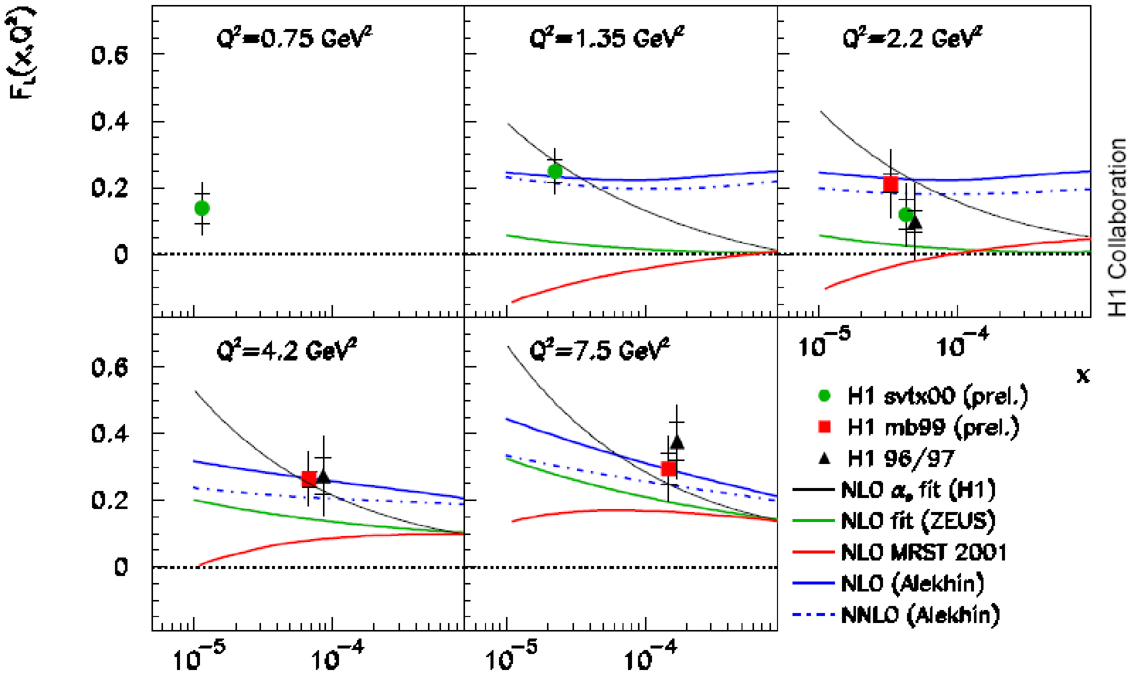


But no direct measure of F_L at low x has been done yet !

Only indirect determinations so far :

DGLAP fit to the data for y < ycut, i.e. cutting out the lowest x domain.

→ use the fit to predict F_2 at lower x. The difference between the measured cross-section and F_2 is ~ F_L.

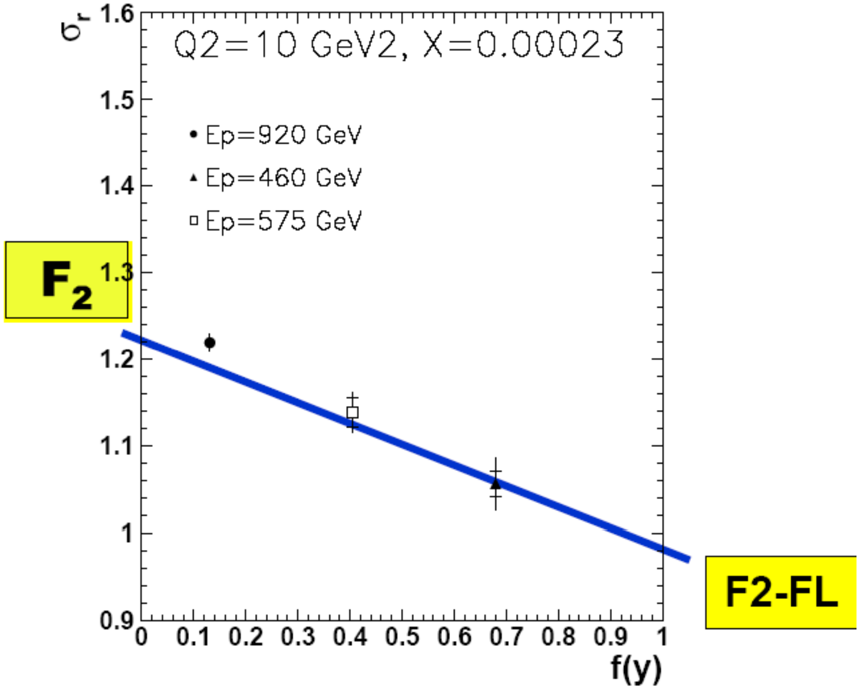


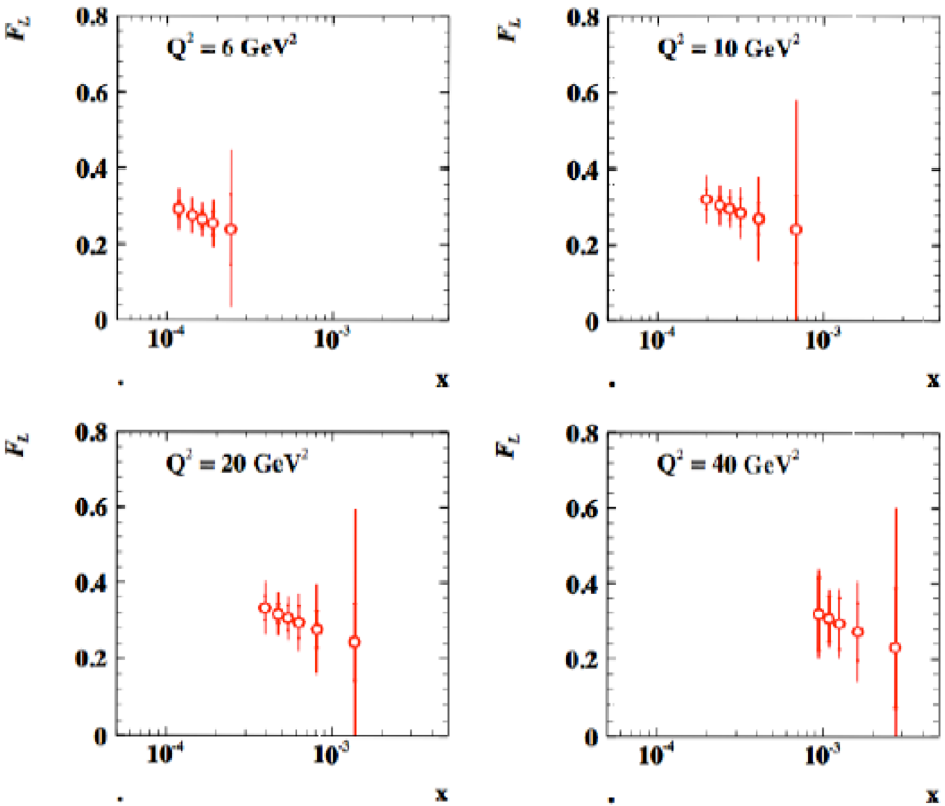
This is more a "consistency check" of the overall framework.

… which fails e.g. for the MRST gluon…

A direct measurement requires measuring $\sigma(x,Q^2) \sim F_2 - y^2/Y_+ F_L$ for at least two values of $y = Q^2/xS$, i.e. at two different values of the center of mass energy.

On March 21st, HERA started a "low energy run" with Ep = 460 GeV.
Was very successful ! More than 10 pb^{-1} collected in ~ 2 months.





On June 1st, moved to an intermediate energy (575 GeV) for the last month of data taking.

The plots show a simulation of what is expected with 10 pb^{-1} at 460 GeV and 7 pb^{-1} at 575 GeV.

The slope of F_2 and the low Q^2 - high Q^2 transition region

In the double asymptotic limit, DGLAP predicts that $F_2(x,Q^2)$ is close to $x^{-\lambda(Q2)}$. A power-behavior is also predicted by the BFKL evolution, with $\lambda \sim 0.3 - 0.5$.

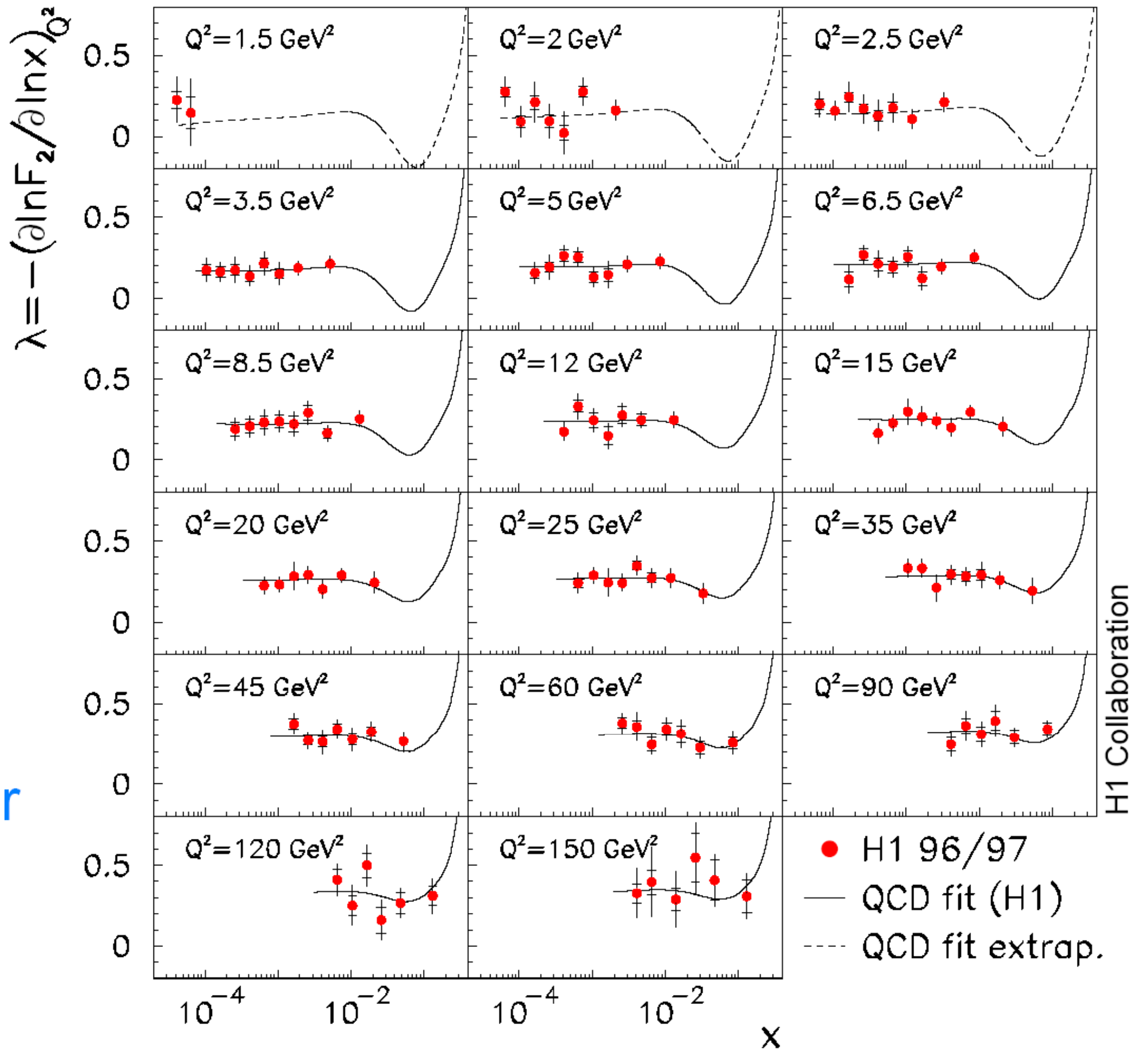
→ Extract

$$\lambda(x,Q^2) \equiv (\partial F_2 / \partial \ln x)_{Q^2}.$$

A decrease of λ with decreasing x may sign a breakdown of the theory due to saturation effects.

No evidence for such a decrease in the data.

r

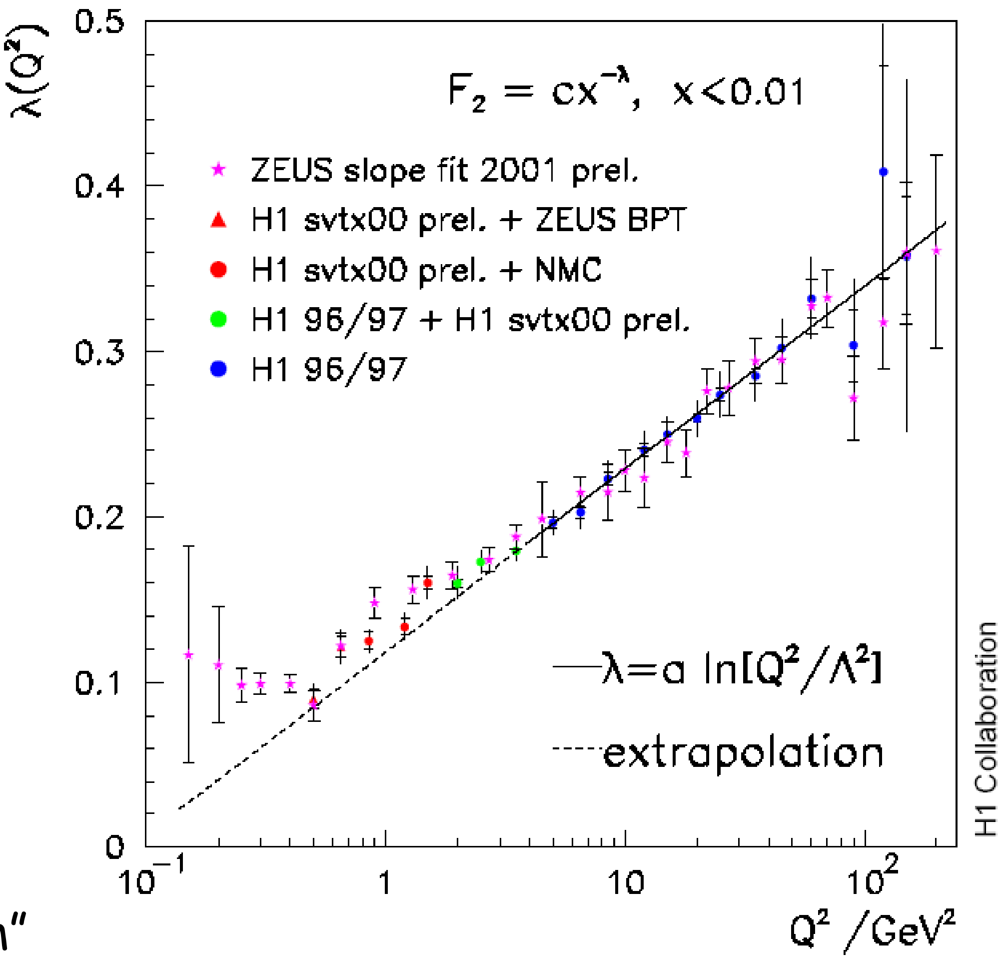


The previous plot shows that the data can be parameterized indeed by

$$F_2(x,Q^2) = c(Q^2)\ x^{-\lambda(Q^2)}$$

For Q^2 > 2-3 GeV^2 :

$\lambda(Q^2)$ depends logarithmically on Q^2 and c ~ constant - as ~ expected from the DGLAP equations.



For Q^2 < ~ 1 GeV^2 :

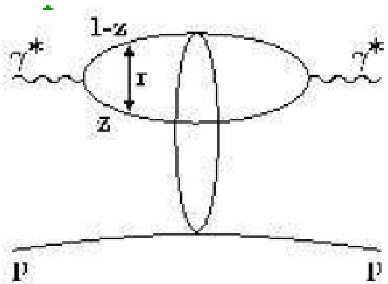
$\lambda(Q^2)$ deviates from a $\ln(Q^2)$ behavior and tends to a value close to $\alpha_{Pom}(0)$-1 ~ 0.08

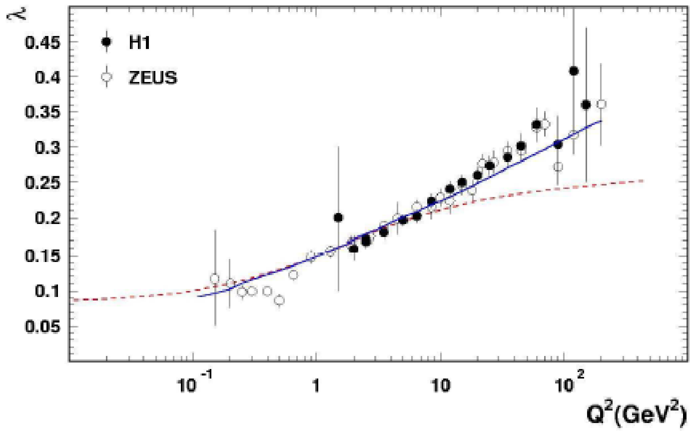
Observation of a "confinement transition" between "partonic degrees of freedom" to "hadronic degrees of freedom" at a scale of about 0.3 fm.

The low Q^2 - high Q^2 transition in dipole models

Dipole models provide a nice description of this transition :

At low x, $\gamma^* \to q\bar{q}$ and the long-lived dipole scatters from the proton



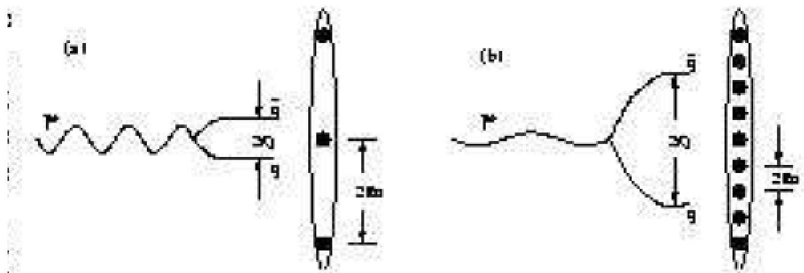


QCD improved dipole

GBW dipole

Regge region pQCD generated slope

The dipole-proton cross section depends on the relative size of the dipole $r \sim 1/Q$ to the separation of gluons in the target R_0



$$\sigma = \sigma_0(1 - \exp(-r^2/2R_0(x)^2)),\ R_0(x)^2 \sim (x/x_0)^{\lambda} \sim 1/xg(x)$$

r/R_0 small $\Rightarrow$ large Q^2, x
$\sigma \sim r^2 \sim 1/Q^2$

r/R_0 large $\Rightarrow$ small Q2, x
$\sigma \sim \sigma_0 \Rightarrow$ saturation of the dipole cross-section

GBW dipole model

Original model was improved by relating $\sigma(x,r)$ to $1/g(x,Q^2)$.

$\to$ Describes the λ slopes both at low and high Q^2.

Golec-Biernat, Wustoff

See lectures by F. Gelis.

GBW model predicts a "geometric scaling" property :

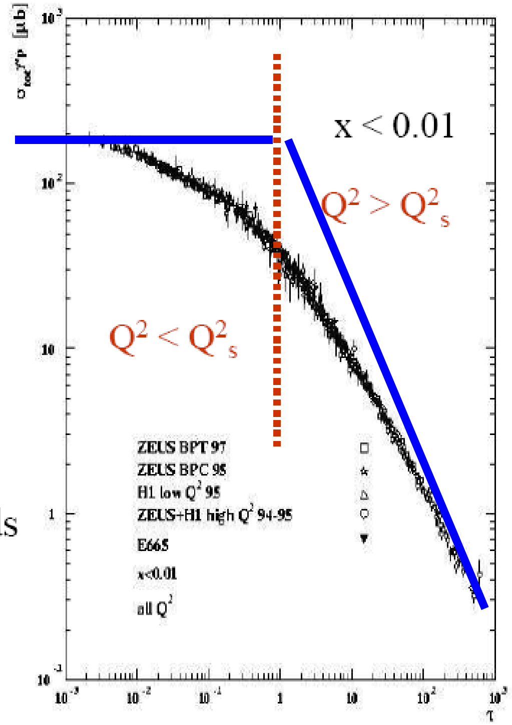
$\sigma = \sigma_0 \, (1 - \exp(-1/\tau))$

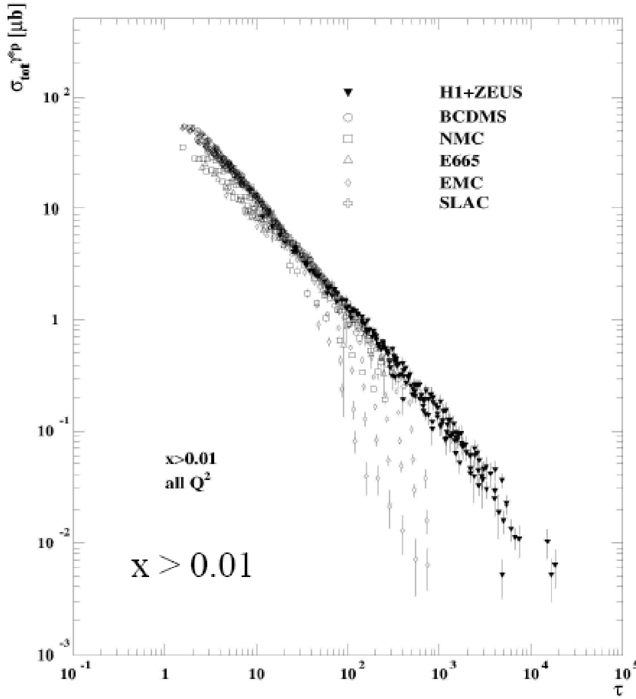
Involves only

$\tau = Q^2 R_0^2(x)$

$\tau = Q^2/Q_0^2 \, (x/x_0)^{\lambda}$

And INDEED, for $x<0.01$, $\sigma(\gamma^* p)$ depends only on τ, not on x, Q^2 separately





τ is a new scaling variable, applicable at small x

Slide from M. Cooper

It can be used to define a `saturation scale' , $Q^2_s = 1/R_0^2(x) \approx x^{-\lambda} \sim x\, g(x)$, gluon density

- such that saturation extends to higher Q^2 as x decreases

Which is borne out by the low x data indeed. Transition between $\sigma(\gamma^* p) \sim \sigma_0$ (τ small) to $\sigma(\gamma^* p) \sim \sigma_0 / \tau$ (τ large) observed for $\tau \sim 1$.

Not a proof of saturation… but shows that the low x HERA data have many of the attributes of a saturated system.