Four fermion two pair production from $\gamma\gamma$ collisions : from PLC to LHC

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Outline

1. Introduction
   - Four fermions two pairs production
   - Computation and analytic results
2. Monte-Carlo Generator
   - Cross section computation
   - LHC
3. Deal with Mixed QED and QCD
   - $\gamma g \longrightarrow q\bar{q}Q\bar{Q}$ case
   - $gg \longrightarrow q\bar{q}Q\bar{Q}$ case

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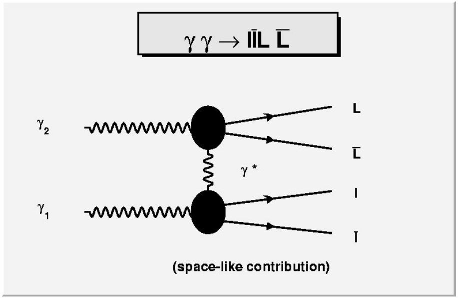
- Total cross section computation
  (cf L.N. Lipatov et al (1969), H. Chen and al. (1970)).
  two identical pair production - infinite energy in $\gamma\gamma$,...
- Total and differential cross section.
  (cf V. G. Serbo et al. (1970 - 1985-1998...)).
  different pair produced - main logarithmic approximation- polarisation of $\gamma\gamma$,....
- Factorisation Formulae.
  (cf. C. Carimalo thesis (1974) and Kessler).
  Helicity amplitude,....
- Motivation
  -Need for a reference process for luminosity measurement at a PLC
  - QED and QCD background source to rare processes
  - Only a realistic Monte-Carlo can give a correct result



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Factorisation Formulae

$$\sigma = \int_{u_{min}}^{u_{max}} \int_{u'_{min}}^{u'_{max}} \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dtdudu'} dudu'dt$$

$$\frac{d\sigma}{dtdudu'} = \frac{uu'}{8\pi^3 s^2 t^2}((1 + ch^2\theta)\sigma_T \sigma'_T$$
$$+sh^2\theta(\sigma_T \sigma'_L + \sigma_L \sigma'_T) + ch^2\theta \sigma_L \sigma'_L)$$

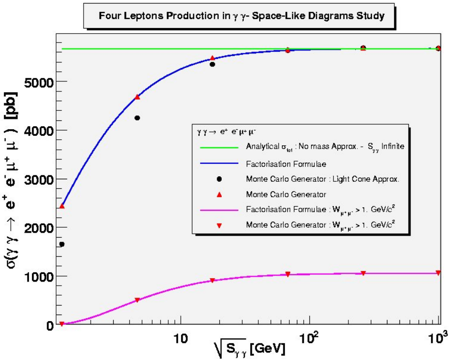
with

$$sh^2\theta = \frac{4st(t - t_{min})(t_{max} - t)}{(u+t)^2(u'+t)^2}$$

$$\sigma_T = \frac{4\pi\alpha^2\beta u}{(u+t)^2}(\beta^2 - 2 + 2\frac{2t}{u} - \frac{t^2}{u^2} + \frac{3 - \beta^4 + 2t^2/u^2}{2\beta}L)$$

$$\sigma_L = \frac{16\pi\alpha^2\beta t}{(u+t)^2}(1 - \frac{1-\beta^2}{2\beta}L)$$

$$L = \ln(\frac{1+\beta}{1-\beta}), \beta = \sqrt{1 - \frac{4m^2}{u}}$$



- Blue line : Factorisation Formula without cuts
- Pink line : Factorisation Formula with cuts on muons
- Other results explain later in talk

Analytic formulae for two pairs $\gamma\gamma$ production at Infinite Energy

In the case of infinite energy, ie when the $\gamma\gamma$ invariant mass goes to infinity we have :

$$\sigma = \int_{u_{min}}^{\infty} \int_{u'_{min}}^{\infty} \int_{t_{min}}^{\infty} \frac{d\sigma}{dtdudu'} dudu'dt$$

$$\frac{d\sigma}{dtdudu'} = \frac{uu'}{4\pi^3} \frac{(\sigma_T + \sigma_L)(\sigma'_T + \sigma'_L)}{(u+t)^2(u'+t)^2}$$

After integrating on the invariant mass of each pair we find :

$$\sigma = \frac{8\alpha^4}{\pi} \int_0^{\infty} f(t,m) f(t,m') dt$$

where

$$f(t,m) = \frac{1}{3t}\left(1 + \frac{1}{2} v \left(5 - \frac{1}{v^2}\right) \ln\left(\frac{1+v}{1-v}\right)\right)$$

$$v = \sqrt{\frac{t}{t+4m^2}}$$

If we make the following variable change :

$$t = \frac{(1-z)^2 u_a y^2}{(1-zy^2)^2 - y^2(1-z)^2}, z = \frac{m-m'}{m+m'}$$

we obtain an easier integrable expression :

$$\sigma = \frac{8\alpha^4}{\pi} \int_0^1 c(y,z) g(y,z) g(y,-z) dy$$

with

$$c(y,z) = \frac{1}{18mm'(1-z^2)y^3}$$

$$g(y,z) = a(y,z) + b(y,z) \ln\left(\frac{(1+y)(1-zy)}{(1-y)(1+zy)}\right)$$

$$a(y,z) = 1 - zy^2$$

$$b(y,z) = \frac{y^2\left((5-y^2)z^2 - 8z + 5\right) - 1}{2(1-z)y}$$

We obtain finally :

$$\sigma = \frac{4\alpha^4}{9\pi m m'} \left\{ \frac{19}{16} \left[2 \left(\frac{1}{u} - u \right) \ln(u) \left(\frac{1}{u} + u \right) \left(2 + \ln^2(u) \right) \right] + \left[\frac{25}{4} + \frac{19}{32} \left(\frac{1}{u} - u \right)^2 \right] Poly(u) \right\}$$

with

$$Poly(u) = \ln^2(u) \ln\left(\frac{1+u}{1-u} \right) - 2\ln(u) \left(Li_2(u) - Li_2(-u) \right) + 2 \left(Li_3(u) - Li_3(-u) \right), u = \frac{m'}{m}$$

where $Poly(u) = Poly(1/u)$ can be shown explicitely. When the two masses are very different (ie $m >> m'$), we obtain :

$$\sigma \simeq \frac{28\alpha^4}{27 m^2 \pi} \left(\ln^2(u^2) - \frac{103}{21} \ln(u^2) + \frac{485}{63} \right)$$

in agreement with Serbo et al. computation.

When masses are equal $m = m'$ we get :

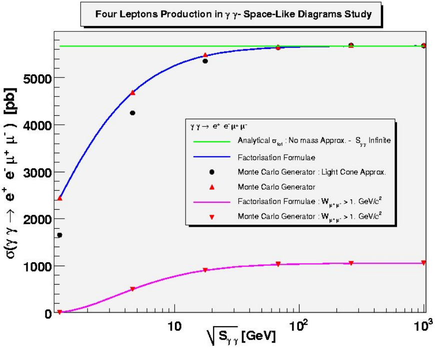
$$\sigma = \frac{2\alpha^4}{m^2\pi} (2z+1) \left(\frac{175}{36} \zeta(3) - \frac{19}{18} \right)$$

For identical masses we put z egal to zero, and we have to divide by a factor two in order to take in account the effect of identical particules. This result coincides with the well-known formula for identical pair production.

Outline

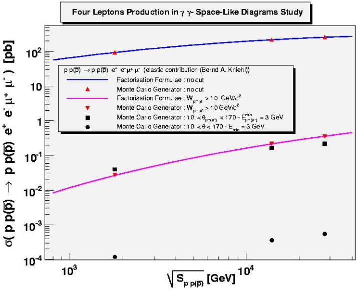
- **Monte-Carlo generator status** :
  Fully integrated in ROOT (cf R. Brun et al.)
  old version (cf old PHOTON Conference) $\Longrightarrow$ new version
  Language : Fortran $\Longrightarrow$ C++
  Integration : Vegas $\Longrightarrow$ FOAM (cf S. Jadach)
- **Computation** : helicity amplitude - only space-like diagrams - use impact factor method
  $\mathrm{M} = \frac{1}{\mathrm{t}} \sum^{\mathrm{helicity}} \mathrm{L}_\mu \mathrm{R}_\nu \mathrm{g}^{\mu\nu}, \mathrm{g}^{\mu\nu} = \frac{1}{2} \mathrm{T}^{+\mu} \mathrm{T}^{-\nu} + \ldots., \ \mathrm{T}^{+} = \frac{\mathrm{k}_{\gamma 1}}{\omega} \mathrm{T}^{-} = \frac{\mathrm{k}_{\gamma 2}}{\omega}$
  The dominant term at low angle is :
  $I = |M|^2 \sim L_{++} R_{--} = \frac{1}{4} L_\mu \mathrm{T}^{+\mu} L^*_\nu \mathrm{T}^{+\nu} R_\alpha R^*_\beta \mathrm{T}^{-\alpha} \mathrm{T}^{-\alpha}$
- **Computation of square amplitude** : dominant term only :
  $L_{++} = 4\omega \left(l_+ P_{\bar{l}\gamma}(\bar{x}) \right) + \bar{l}_+ P_{l\gamma}(x)) + 8 \vec{l_T}.\vec{\bar{l}_T} \, (\mathrm{x}(1 - \bar{\mathrm{x}}) + \bar{\mathrm{x}}(1 - \mathrm{x})) - 8 \frac{m^2}{l_- \bar{l}_-}$
  $(l_+, \bar{l}_-, \vec{\bar{l}_T}$: lepton light cone variable)

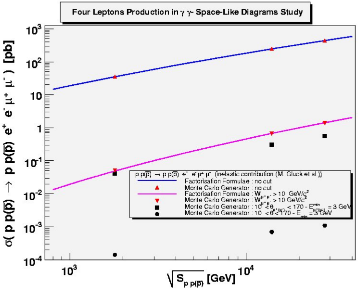
- $P_{l/\gamma}(x)$ is the lepton spectrum distribution :
  $P_{l/\gamma}(x) = \frac{2\ m^2\ x}{\omega^2\ l_-^2} + \frac{1}{\omega\ l_-} \ (x^2 + (1 - x)^2)$
  and $x = 1 - \frac{l_+}{\omega}$ is the fraction of the photon energy taken by the quasi-real lepton l.



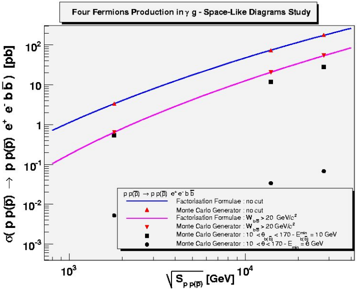
Outline

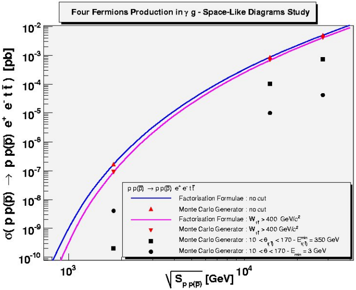
- Cross Section Computation :
  $\sigma = \int_{z_{min}}^{z_{max}} dz 2z \int_{\frac{z^2}{z_{max}}}^{z_{max}} \frac{dy}{y} f_{\gamma/p}(y) f_{\gamma/p}(\frac{z^2}{y}) \sigma^{\gamma\gamma \rightarrow l\bar{l}Q\bar{Q}}$
  $f_{\gamma/p}(y,\mu^2) = f_{\gamma(el)/p}(y) + f_{\gamma(inel)/p,Q^2}(y)$
  $y = \frac{E_\gamma}{E_{beam}}$ and Q^2 is the resolution scale at which the proton is probed.
- Photon content of proton used :
  -elastic contribution (Bernd A. Kniehl)
  -inelastic comtribution (M. Glück et al),
- Results : clear signature (pseudo pair)
  $\sigma_{CUT}^{LHC} = 100. - 0.1$ fb (depending on CUTS)
  if $L(LHC) = 100\ \mathrm{fb}^{-1} \rightarrow \mathrm{N_{evt}} \leq 10^4 - 10$
  (need realistic simulation for bgk rejection $\Rightarrow$ conclusion)
- Use Roman pot (silicon detector) in ATLAS/LHC to tag proton(s)





- Cross section :
  -$\sigma^{\gamma g \to l\bar{l}Q\bar{Q}} = \frac{1}{8} 4e_Q^2 \frac{\alpha_s}{\alpha} \frac{1}{2} \sigma^{\gamma\gamma \to l\bar{l}Q\bar{Q}}$
  Initial gluon color average $\to \frac{1}{8}$, quark color $\to$ 4 and symetrisation $\to \frac{1}{2}$
  - Gluon content of the proton used : CTEQ6
  - Photon content of the proton used : elastic (Bernd A. Kniehl) and inelastic (M. Glück et al).
- Results
  $\sigma_{CUT}^{e^+e^-t\bar{t}} \leq 0.1$ fb (very low)
  $\sigma_{CUT}^{e^+e^-b\bar{b}} = 1$ pb $-$ 10 fb (depending on CUTS)
  if $L(LHC) = 100\ \mathrm{fb}^{-1} \Rightarrow \mathrm{N_{evt}} \leq 10^5 - 10^3$
  (also needed realistic simulation for bgk rejection $\Rightarrow$ conclusion)
  -Also use Roman pots (silicon detectors) in LHC to tag proton





Outline

What we want :

$$M = \sum_{u=1}^{24} C_u'^{(\mathrm{Color})} T_u = \sum_{u=1}^{8} C_u' T_u^{(Space)} + \sum_{u=9}^{24} C_u' T_u$$

$$M = F_1^{(\mathrm{Col.})} \sum_{u=1}^{8} T_u^{(Space)} + F_2^{(\mathrm{Col.})} \sum_{u=1}^{24} \alpha_u T_u + F_3^{(\mathrm{Col.})} \sum_{u=1}^{24} \alpha_u' T_u$$

$$\sigma = \frac{1}{8}|F_1|^2 \sigma_{QED} + \frac{1}{8}|F_2|^2 \sigma_{\text{“}QED''} + \frac{1}{8}|F_3|^2 \sigma_{3g}$$

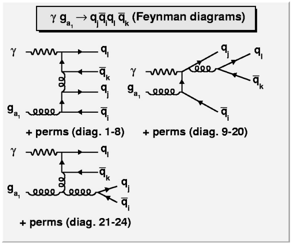
$$\sigma_{QED} \sim |\sum_{u=1}^{8} T_u^{(Space)}|^2, \sigma_{\text{“}QED''} \sim |\sum_{u=1}^{24} \alpha_u T_u|^2 \sigma_{3g} \sim |\sum_{u=1}^{24} \alpha_u' T_u|^2$$

How we proceed : (6 independent color structures) :

$$C_1 = \bar{u}^l \mathrm{T}^c u_k \bar{u}^j \mathrm{T}^c \mathrm{T}^{a_1} u_i \quad (\text{same way for } C_2, C_3, C_4, C_5 \text{ and } C_6)$$

$$= (\mathrm{T}^c \mathrm{T}^{a_1})_f^e (\mathrm{T}^c)_h^g \left((1_i^j)_e^f + (8_i^j)_e^f\right)\left((1_k^l)_g^h + (8_k^l)_g^h\right)$$

$$= \frac{1}{2}(\mathrm{T}^{a_1})_e^f \left((1 \otimes 8_{ik}^{jl})_{bf}^{eb} + (8s_{ik}^{jl})_{bf}^{eb} - (8a_{ik}^{jl})_{bf}^{eb}\right)$$



- γg color vector basis computation using tensorial representation of 8_g and $(1 \oplus 8)_q \otimes (1 \oplus 8)_Q = ... \oplus (1 \otimes 8) \oplus (8 \otimes 1) \oplus 8_s \oplus 8_a \oplus ..$

Mixed QED and QCD : dealing with color structure in $\gamma g \to q\bar{q}Q\bar{Q}$

4 color vectors basis

$$e_{1\ ik}^{a_1 jl} = \frac{\sqrt{3}}{2}(T^{a_1})_e^f (1 \otimes 8_{ik}^{jl})_{bf}^{eb} = \frac{1}{2\sqrt{3}}\delta_k^l (T^{a_1})_i^j,\ e_{2\ ik}^{a_1 jl} = ..,$$

$$e_{4\ ik}^{a_1 jl} = \frac{1}{\sqrt{6}}(T^{a_1})_e^f (8a_{ik}^{jl})_{bf}^{eb} = \frac{1}{2\sqrt{6}}\left(\delta_i^l (T^{a_1})_k^j - \frac{1}{2}\delta_k^j (T^{a_1})_i^l\right)$$

with $\sum_{a_1,i,j,k,l} e_{n\ ik}^{a_1 jl}\left(e_{p\ ik}^{a_1 jl}\right)^\star = \delta_n^p$ (orthonormal base)

$\Longrightarrow$ Rotation Color Components matrix

$$M = u_{2\ ik}^{a_1 jl}(M_1 + M_2 + ... + M_8) +$$

$$\frac{1}{\sqrt{6}} u_{3\ ik}^{a_1 jl}(M_1 - M_2 ... + M_{20}) + \sqrt{\frac{3}{2}} u_{4\ ik}^{a_1 jl}(M_1 + ... + M_{24})$$

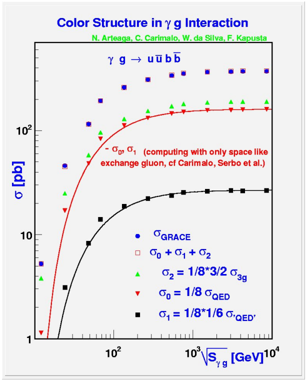
$$\sigma = \frac{1}{8}\sigma_{QED-Space} + \frac{1}{8}\frac{1}{6}\sigma_{\text{“}QED\text{''}} + \frac{1}{8}\frac{3}{2}\sigma_{3g}$$

($\frac{1}{8}$ = average gluon color, $|F_1|^2 = 1, |F_2|^2 = \frac{1}{6}, |F_3|^2 = \frac{3}{2}$)

Modification of GRACE color basis generator

- $\sigma_{QED-Space}$: space-like contibution of QED with α_s
- $\sigma_{\text{“}QED\text{''}}$: space-and-time like contribution of QED but sign -1 in front of some amplitude
- σ_{3g} : including the 3 gluons coupling contribution

	$u_{1\ ik}^{a_1 jl}$,	$u_{2\ ik}^{a_1 jl}$,	$u_{3\ ik}^{a_1 jl}$,	$u_{4\ ik}^{a_1 jl}$
C_1	0	1	$\frac{1}{\sqrt{6}}$	$-\sqrt{\frac{3}{2}}$
C_2	0	1	$\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}$
C_3	0	1	$-\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}$
C_4	0	1	$-\frac{1}{\sqrt{6}}$	$-\sqrt{\frac{3}{2}}$
C_5	0	0	0	$-\sqrt{\frac{3}{2}}$
C_6	0	0	0	$\sqrt{\frac{3}{2}}$



Outline

Mixed QED and QCD : dealing with color structure in $gg \to q\bar{q}Q\bar{Q}$

- 36 amplitudes and 12 independent color structures
- Computation of color factor C_1,....C_{12} :
  $C_1 = \bar{u}^l \mathrm{T}^{a_2}\mathrm{T}^c u_k \bar{u}^j \mathrm{T}^c \mathrm{T}^{a_1} u_i$
  $= \left(\frac{1}{2}\delta^d_a\delta^f_g - \frac{1}{6}\delta^f_a\delta^d_g\right)\delta^b_e\delta^h_c$ (delta coefficient)
  $(\mathrm{T}^{a_1})^e_f\,(\mathrm{T}^{a_2})^g_h\left((1^j_i)^a_b + (8^j_i)^a_b\right)\left((1^l_k)^c_d + (8^l_k)^c_d\right)$
- Use tensorial representation of
  $8_g \otimes 8_g = 1 \oplus 8_a \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27$
  $\Longrightarrow$ example : color singlet gluons :
  $(1^{a_1a_2})^{ac}_{bd} = \frac{1}{24}(\mathrm{T}^{a_1})^u_v(\mathrm{T}^{a_2})^v_u(-\delta^a_b\delta^c_d + 3\delta^a_d\delta^c_b)$
  for the quark : $(1 \oplus 8)_q \otimes (1 \oplus 8)_Q =$
  $1 \oplus 1 \otimes 1 \oplus 1 \otimes 8 \oplus 8 \otimes 1 \oplus 8_a \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27$
  $\Longrightarrow$ exemple color singlet and 27 quark :
  $(1 \otimes 1^{jl}_{ik})^{ac}_{bd} = (1^j_i)^a_b(1^l_k)^c_d$
  $(1^{jl}_{ik})^{ac}_{bd} = \frac{1}{24}(8^j_i)^u_v(8^l_k)^v_u(-\delta^a_b\delta^c_d + 3\delta^a_d\delta^c_b)$
  $(27^{jl}_{ik})^{ac}_{bd} =$
  $\frac{1}{4}((8^j_i)^a_b(8^l_k)^c_d + (8^j_i)^c_b(8^l_k)^a_d + (8^j_i)^a_d(8^l_k)^c_b +$
  $(8^j_i)^c_d(8^l_k)^a_b - \frac{1}{5}(\delta^a_d(8s^{jl}_{ik})^c_b + \delta^c_d(8s^{jl}_{ik})^a_b + \delta^a_b(8s^{jl}_{ik})^c_d +$
  $\delta^c_b(8s^{jl}_{ik})^a_d) - \frac{1}{6}(\delta^c_d\delta^a_b + \delta^c_b\delta^a_d)(8^j_i)^u_v(8^l_k)^v_u))$

- Built program in Mathematica to contract all delta coefficients with the 13 irreductible tensors (tested in the γg case)
  $\Longrightarrow$obtention of 13 base's vectors orthonormal
  $e_1 = \frac{1}{6\sqrt{2}}\delta^{a_1a_2}\delta^j_i\delta^l_k$
  $e_2 = \frac{1}{24}\delta^{a_1a_2}(3\delta^l_i\delta^j_k - \delta^j_i\delta^l_k)$, $e_3 = ..$,
  $e_{13} = \frac{1}{360\sqrt{3}}\big(-60(\mathrm{T}^{a_1})^l_k(\mathrm{T}^{a_2})^j_i -$
  $60(\mathrm{T}^{a_1})^j_k(\mathrm{T}^{a_2})^l_i - 60(\mathrm{T}^{a_1})^l_i(\mathrm{T}^{a_2})^j_k -$
  $60(\mathrm{T}^{a_1})^j_i(\mathrm{T}^{a_2})^l_k +$
  $12d^{a_1a_2c}((\mathrm{T}^c)^l_k\delta^j_i + (\mathrm{T}^c)^j_k\delta^l_i +$
  $(\mathrm{T}^c)^l_i\delta^j_k + (\mathrm{T}^c)^j_i\delta^l_k) + 5\delta^{a_1a_2}(\delta^l_i\delta^j_k +$
  $\delta^j_i\delta^l_k))$
  $\Longrightarrow$ after rotation need only 12 vectors for writing the 12 C_i
- Next step : obtention of the QED-space component and modification of color GRACE

Summary

- Our Monte-carlo is ready (fully integrated in ROOT software).
- We provide a new formula for $\gamma\gamma \to 2\,\mathrm{pairs}$ (no mass approximation).
- We provide an explicit orthogonal color basis for $\gamma g \to q\bar{q}Q\bar{Q}$ and $gg \to q\bar{q}Q\bar{Q}$.

- Outlook
  - Include more realistic photon fluxes in our Monte-Carlo in p p case (Q^2 dependence (cf V. Serbo et al paper $pp \to ppe^+e^-e^+e^-$)
  - Include time-like photon exchange
  - Continue to investigate $gg \to q\bar{q}Q\bar{Q}$ case.