Large contribution

of the Delbrück scattering

into process

of a photon emission

in collisions of relativistic nuclei

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1. Introduction

1.1. Motivation

Recently the electromagnetic (EM) processes in ultra-relativistic nuclear collisions were discussed in numerous papers
(see review *Baur et al. Phys. Rep.* **364**, *359 (2002)* and references therein).

For the RHIC and LHC colliders the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given in Table:

Collider	Z	γ
RHIC, Au-Au	79	108
LHC, Pb-Pb	82	3000

Only a few EM processes are related to Fundamental Physics, but

some of EM processes are of great importance mainly for two reasons:

1) They are **dangerous** or

2) They are **useful**

Two examples:

1) The e^+e^- pair production. The number of the produced electrons is so huge that some of them can be captured by nuclei, that immediately leads to loss of these nuclei from the beam. Thus, this very process is determined mainly **the life time of the beam and a possible luminosity of a machine**.

2) Coherent bremsstrahlung, not ordinary bremsstrahlung

$$Z_1\, Z_2 \to Z_1\, Z_2\, \gamma$$

but coherent one! The number of the produced photons at the RHIC is so huge in the region of the infrared light, that this process can be used for **monitoring beam collisions**:

R. Engel, A. Schiller, V.G. Serbo. A new possibility to monitor collisions of relativistic heavy ions at LHC and RHIC, Particle Accelerators **56***, 1 (1996)*

D. Trbojevic, D. Gasner, W. MacKay, G. McIntyre, S. Peggs, V. Serbo, G. Kotkin. Experimental set-up to measure coherent bremssrahlung and beam profiles in RHIC. 8th European Particle Accelerator Conference (EPAC 2002, 3–7 June, 2002, Paris) p. 1986

It means that various EM processes have to be estimated (their cross sections and distributions) **not to miss** something interesting or dangerous.

The ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by Feynman diagrams of Fig. 1

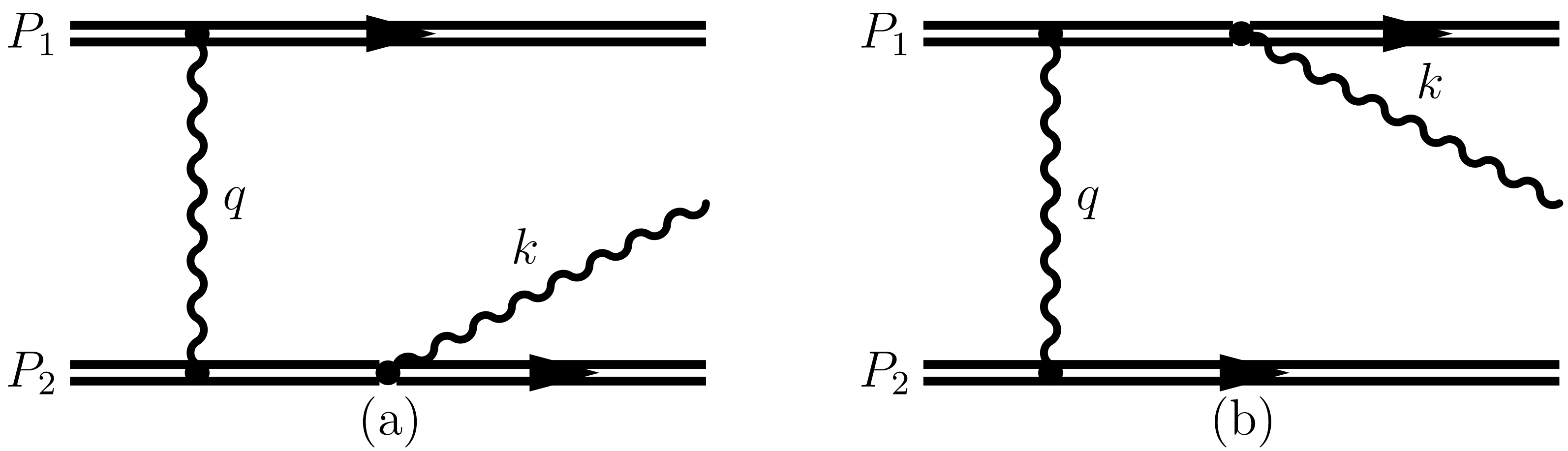

Fig. 1

and was known in detail many years ago

Bertulany, Baur Phys. Rep. **163**, *299 (1988)*

It can be described as **the Compton scattering** of the equivalent photon off opposite nucleus.

In the present report we consider not the Compton subprocess, but another one – **the Delbrück scattering subprocess** — which can given an essential contribution to emission of photons at the nuclear collisions without excitation of the final nuclei (see Fig. 2).

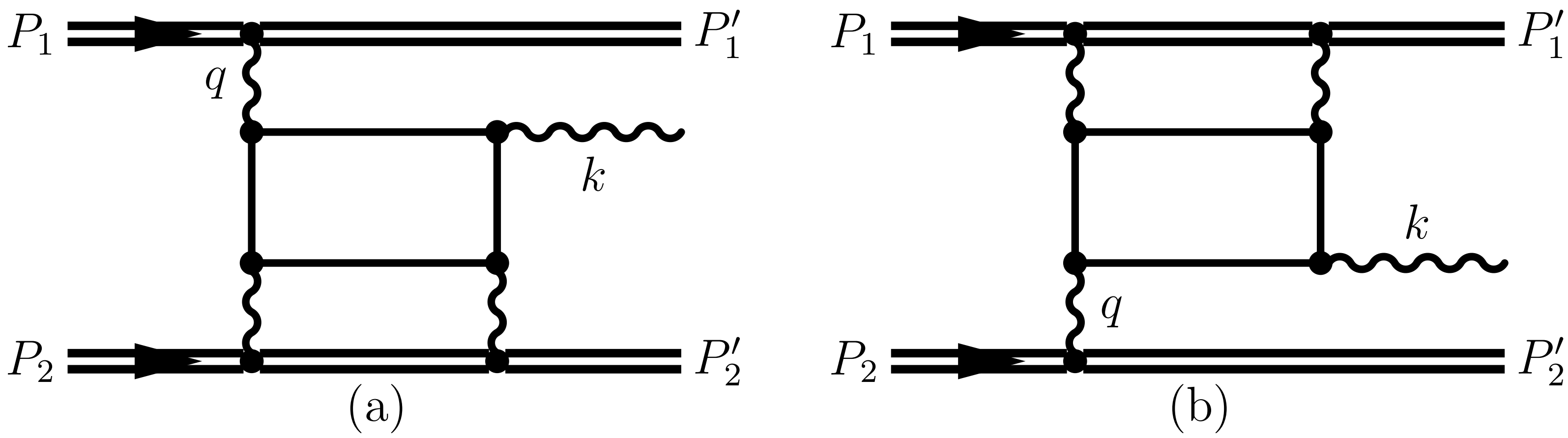

Fig. 2

First note: *Baur, Bertulany, Baur Z. f. Phys.* **A 330**, *77 (1988)*

1.2. Main result

At first sight, this is a process of **a very small cross section** since

$$\sigma \propto \alpha^7.$$

But at second sight, we should **add a very large factor**

$$Z^6 \sim 10^{11}$$

and take into account that the cross section **scale** is

$$1/m_e^2\,.$$

And the last, but not the least, we will show that this cross section has **an additional logarithmic enhancement** of the order of

$$L^2 \gtrsim 100\,, \quad L = \ln\left(\gamma^2\right).$$

As a result, the discussed cross section for the LHC collider is

$$\sigma \sim \frac{(Z\alpha)^6 \alpha}{m_e^2} L^2 \sim \mathbf{50\ \ barn}\,.$$

That is quite serious number!

2. Delbrück scattering (DS)

The DS is an elastic scattering of a photon in the Coulomb field of a nucleus via a virtual electron-positron loop (Fig. 3)

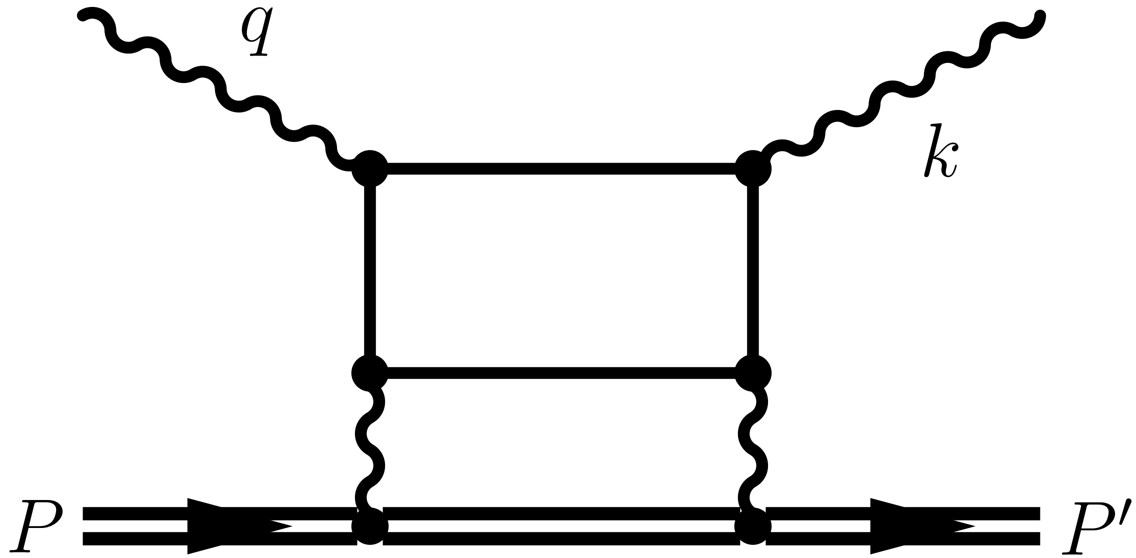

Fig. 3

Its properties are well known

see review *Milstein, Schumacher*, *Phys. Rep.* **243**, *183 (1994)*

The total cross section of this process **vanishes** at small energies

$$\sigma_{\mathrm{D}}(\omega_L, Z) \sim (Z\alpha)^4 \frac{\alpha^2}{m^2} \left(\frac{\omega_L}{m}\right)^4 \text{ at } \omega_L = qP/M \ll m\,, \quad m \equiv m_e\,, \qquad (1)$$

but **tends to constant** at $\omega_L \gg m$.
In the lowest order of the perturbative theory this constant is

$$\sigma_{\mathrm{D}}^{(0)}(Z) = 1.07\,(Z\alpha)^4\,\frac{\alpha^2}{m^2} \quad \text{at} \quad \omega_L \gg m\,. \tag{2}$$

The Coulomb corrections $\sim (Z\alpha)^{2n}$ **decrease** it significantly

$$\sigma_{\mathrm{D}}(\omega_L, Z)_{\omega_L \gg m} \to \sigma_{\mathrm{D}}(Z) = \frac{\sigma_{\mathrm{D}}^{(0)}(Z)}{r_Z}\,, \tag{3}$$

For example, for DS off the Au ($Z = 79$) and Pb ($Z = 82$) nuclei

$$\sigma_{\mathrm{D}}(Z = 79) = 5.5 \ \text{mb}\,, \ \ \sigma_{\mathrm{D}}(Z = 82) = 6.2 \ \text{mb}\,, \tag{4}$$

this corresponds to $r_{79} = \mathbf{1.7}$ and $r_{82} = \mathbf{1.8}$.

Comparison:

Cross section for **the nuclear Thomson scattering** is

$$\sigma_{\mathsf{T}}(Z) = \frac{8\pi}{3}\frac{Z^4\alpha^2}{M^2}\,, \tag{5}$$

where $M \approx Am_p$.

The ratio

$$\frac{\sigma_{\mathsf{T}}(Z)}{\sigma_{\mathsf{D}}(Z)} = 7.83\, r_Z \left(\frac{m}{\alpha^2 Am_p}\right)^2 \approx \frac{1}{30} \quad \text{for} \quad {}^{208}\text{Pb} \tag{6}$$

is small for heavy nuclei.

The main contribution to DS is given by the region of transverse momenta of the final photon $\mathbf{k}_\perp \sim \mathbf{m}$.
For **larger** transverse momentum,

$$d\sigma_{\mathrm{D}} = (Z\alpha)^4 \, \alpha^2 \, f(k_\perp/m, Z) \, \frac{dk_\perp^2}{m_\perp^4} \quad \text{at } \; m \lesssim k_\perp \ll \omega_L \, , \tag{7}$$

where

$$m_\perp = \sqrt{m^2 + k_\perp^2}$$

and $f(k_\perp/m, Z)$ is **slowly varying function** of $k_\perp/m$.

For $Z = 82$ this function is:

$f(k_\perp/m, Z) \approx \mathbf{1.2}$ at $k_\perp \gg m$,

$f(k_\perp/m, Z) \approx \mathbf{0.48}$ at $k_\perp = m$.

It should be noted that such a distribution is valid for not very large transverse momentum:

at $k_\perp < 1/R$,

where $R = 1.2\, A^{1/3}$ fm

is the nucleus radius,

$R = 7$ fm, $1/R = 28$ MeV for Au and Pb.

We assume further that $m_\perp < 1/R$.

3. Contribution of the DS into emission of photons in nuclear collision

3.1. Total cross section

Below we assume that $Z_1 = Z_2$ and $\gamma_1 = \gamma_2$ for the sake of simplicity.

The cross section is given by (see Fig. 2)

$$d\sigma = d\sigma_a + d\sigma_b \tag{8}$$

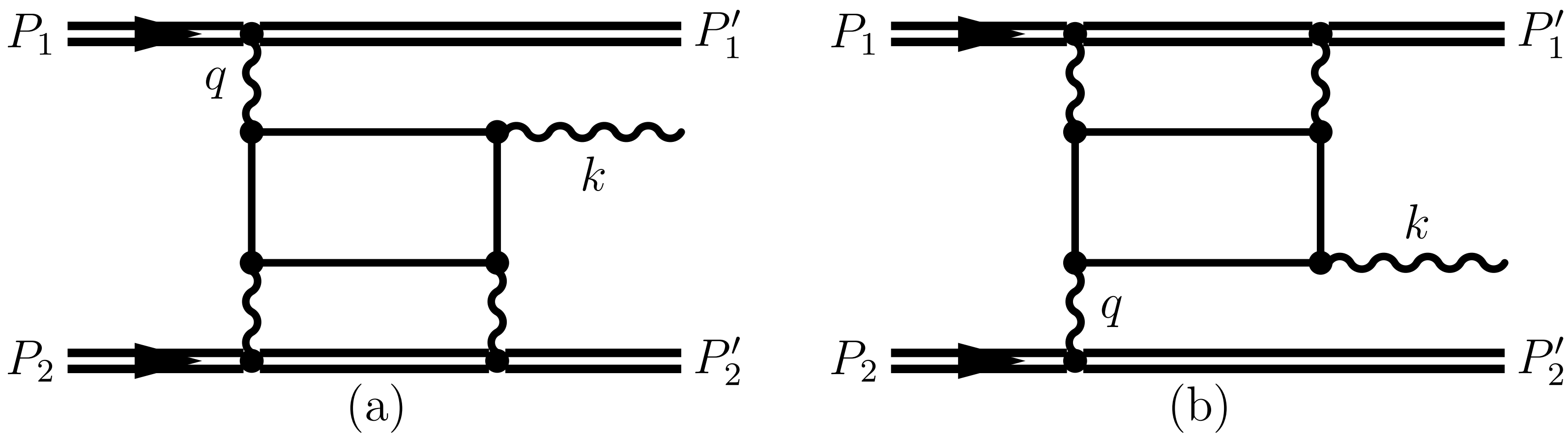

Fig. 2

The interference term is small and can be safely neglected.

In **the equivalent photon approximation**

$$d\sigma_a = dn_1(\omega)\,\sigma_{\mathrm{D}}(\omega_L, Z)\,, \tag{9}$$

where the number of equivalent photons is

$$dn_1(\omega) = 2\,\frac{Z^2\alpha}{\pi}\,\frac{d\omega}{\omega}\,\ln\frac{m\gamma}{\omega}\,. \tag{10}$$

Then integrating the cross section (9) over ω in the region

$$\frac{m}{\gamma} \lesssim \omega \lesssim m\gamma\,, \tag{11}$$

we obtain **the total cross section** in the leading log approximation

$$\sigma = \sigma_a + \sigma_b = 2\frac{Z^2\alpha}{\pi}\,\sigma_{\mathrm{D}}(Z)\,L^2\,, \quad L = \ln\frac{P_1P_2}{2M_1M_2} = \ln(\gamma^2)\,. \tag{12}$$

In particular, for the Au-Au collisions at the RHIC collider the total cross section

$\sigma = 14$ **barn**,

for the Pb-Pb collisions at the LHC collider the total cross section

$\sigma =$ **50 barn**.

The size of these cross sections is **larger** that the total hadronic/nuclear cross section for the Pb-Pb collisions of **7.9 barn**.

3.2. Energy and angular distribution

In the same way, we can go from the energy of the equivalent photon ω to the energy of the final photon E_γ and to obtain

the inclusive cross section

$$d\sigma = \frac{2}{\pi^2}(Z\alpha)^6\alpha\,\frac{f(k_\perp/m, Z)}{\left(m^2+k_\perp^2\right)^2}\,L\,\frac{d^3k}{E_\gamma}\,, \quad m_\perp \ll E_\gamma \ll m_\perp\gamma \tag{13}$$

and **the spectrum of photons**

$$d\sigma = \frac{4}{\pi}Z^2\alpha\sigma_{\mathrm{D}}(Z)\,L\,\frac{dE_\gamma}{E_\gamma}\,, \quad m \ll E_\gamma \ll m\gamma\,. \tag{14}$$

The typical emission angle of the photon is not very small:

$$\frac{1}{\gamma} \ll \theta_\gamma = \frac{k_\perp}{E_\gamma} \ll 1\,.$$

Remark. The spectral distribution of photons (14) has the form

$$d\sigma \propto \frac{dE_\gamma}{E_\gamma}\,,$$

which is typical for the bremsstrahlung spectrum of soft photons and **usually leads to the infrared divergency**. In our case this type of distribution is valid for not soft photons in the region $m \ll E_\gamma \ll m\gamma$. When the photon energy tends to zero, we should take into account that the Delbrück cross section vanishes for soft photons. As a result, **the discussed cross section has no infrared divergency**.

4. Comparison

Comparison **the obtained** spectrum with that for **the ordinary bremsstrahlung** spectrum shows that for the same photon energy the ratio

$$\frac{d\sigma^a_{\mathsf{bremsstrahlung}}}{d\sigma_a} \sim \frac{\sigma_{\mathsf{T}}(Z)}{\sigma_{\mathsf{D}}(Z)} \approx \frac{1}{30} \quad \text{for} \quad {}^{208}\text{Pb} \tag{15}$$

is **small for heavy nuclei**.

5. Conclusions

1) The total cross section, the energy and angular distribution of photons, emitted due to the Delbrück subprocess are calculated

2) It was found out that the corresponding cross section is large enough

3) If this process can be detected, one can study the Delbrück scattering in the energy range up to

$$\omega_L \sim 2m\gamma^2 ,$$

which is

10 GeV for RHIC and

8 TeV for LHC.

Not bad!

It should be stressed that the discussed process is very sensitive to high order corrections $\sim (Z\alpha)^{2n}$, which reduce the lowest order result almost two times.