Sense and nonsense on photon structure

Jiří Chýla

Institute of Physics, Prague

- **The nature of PDF of the photon**
- **Why semantics matters**
- **Factorization scale and scheme independence**
- **QCD analysis of photon structure function**
- **What is wrong with DIS_γ factorization scheme?**
- **Conclusions**

J. Chýla: JHEP04 (2000)007
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Basic facts

Colour coupling:

$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} \equiv \beta(\alpha_s(\mu)) = -\frac{\beta_0}{4\pi}\alpha_s^2(\mu) - \frac{\beta_1}{16\pi^2}\alpha_s^3(\mu) + \cdots$$

$\beta_0 = 11 - 2n_f/3$, $\beta_1 = 102 - 38n_f/3$

is actually a function of **renormalization scale μ** as well as **renormalization scheme RS**:

$$\alpha_s(\mu) = \alpha_s(\mu/\Lambda_{RS}, RS)$$

At NLO renormalization scheme can be labeled by Λ_{RS}

Parton distribution functions:

quark singlet and nonsinglet densities

$$\Sigma(x,M) \equiv \sum_{i=1}^{n_f} q_i^+(x,M) \equiv \sum_{i=1}^{n_f} [q_i(x,M)+\overline{q}_i(x,M)]$$

$$q_{\rm NS}(x,M) \equiv \sum_{i=1}^{n_f} \left(e_i^2 - \langle e^2\rangle\right)\left(q_i(x,M)+\overline{q}_i(x,M)\right),$$

as well as gluon density depend on **factorization scale M and factorization scheme (FS),** i.e.

$$PDF = PDF(x, M, FS)$$

Renormalization scale **μ** and factorization scale ***M*** are independent parameters that **should not be identified!**

Evolution equations:

$$\frac{\mathrm{d}\Sigma(x,M)}{\mathrm{d}\ln M^2} = \delta_\Sigma k_q + P_{qq}\otimes\Sigma + P_{qG}\otimes G$$

$$\frac{\mathrm{d}G(x,M)}{\mathrm{d}\ln M^2} = k_G + P_{Gq}\otimes\Sigma + P_{GG}\otimes G\,,$$

$$\frac{\mathrm{d}q_{\mathrm{NS}}(x,M)}{\mathrm{d}\ln M^2} = \delta_{\mathrm{NS}} k_q + P_{\mathrm{NS}}\otimes q_{\mathrm{NS}}\,,$$

inhomogeneous terms specific to photon

$$\delta_{\mathrm{NS}} = 6n_f\left(\langle e^4\rangle - \langle e^2\rangle^2\right)$$

$$\delta_\Sigma = 6n_f\langle e^2\rangle$$

Splitting functions:

<u>homogeneous:</u>

$$P_{ij}(x,M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x) + \cdots$$

<u>inhomogeneous:</u>

$$k_q(x,M) = \frac{\alpha}{2\pi}\left[k_q^{(0)}(x) + \frac{\alpha_s(M)}{2\pi} k_q^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_q^{(2)}(x) + \cdots\right]$$

$$k_G(x,M) = \frac{\alpha}{2\pi}\left[\qquad \frac{\alpha_s(M)}{2\pi} k_G^{(1)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 k_G^{(2)}(x) + \cdots\right]$$

where $k_q^{(0)}(x) = (x^2 + (1-x)^2)$ comes from **pure QED!**

$k_q^{(0)}(x),\ P_{ij}^{(0)}(x)$ are **unique**, whereas $k_q^{(i)}(x),\ P_{ij}^{(i)}(x), i \geq 1$

are **nonuniversal**, defining the **factorization scheme**.

Structure function:

$$\frac{1}{x}F_2^\gamma(x,Q^2) = q_{\rm NS}(M)\otimes C_q(Q/M)+\delta_{\rm NS}C_\gamma + \langle e^2\rangle\Sigma(M)\otimes C_q(Q/M)+\langle e^2\rangle\delta_\Sigma C_\gamma + \langle e^2\rangle G(M)\otimes C_G(Q/M)$$

where the coefficient functions

$$C_q(x,Q/M) = \delta(1-x) + \frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(x,Q/M)+\cdots$$

$$C_G(x,Q/M) = \frac{\alpha_s(\mu)}{2\pi}C_G^{(1)}(x,Q/M)+\cdots,$$

$$C_\gamma(x,Q/M) = \underbrace{\frac{\alpha}{2\pi}\left[C_\gamma^{(0)}(x,Q/M)\right.}_{\text{unique and pure QED}} + \underbrace{\left.\frac{\alpha_s(\mu)}{2\pi}C_\gamma^{(1)}(x,Q/M)+\cdots\right]}_{\text{FS dependent}}$$

unique and pure QED — FS dependent

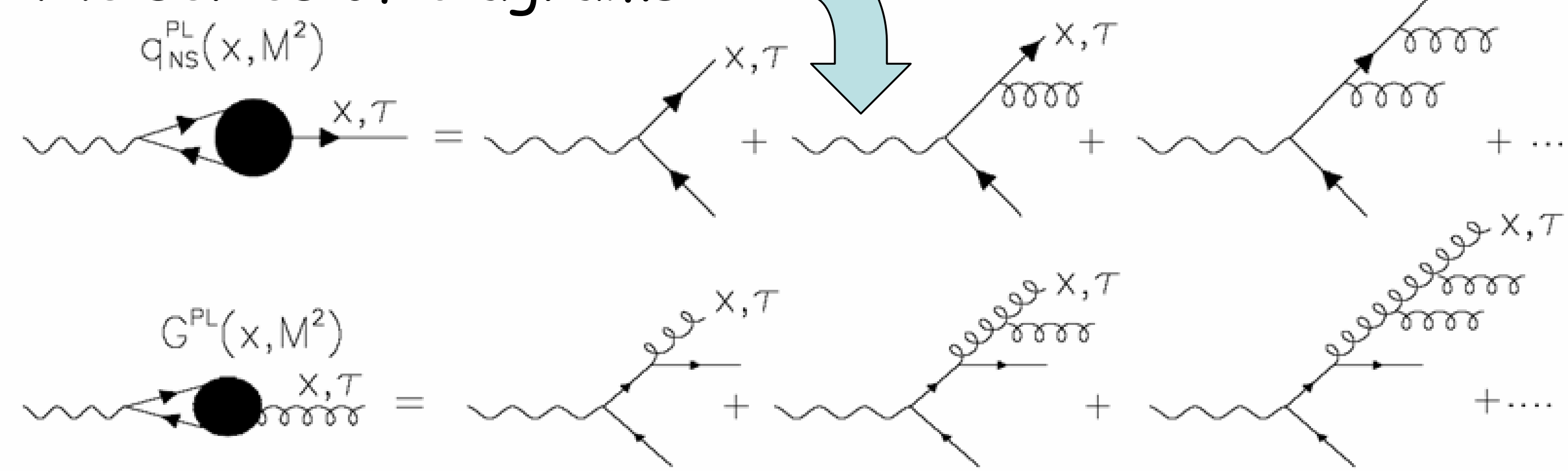
$$C_\gamma^{(0)}(x,Q/M) = \left(x^2+(1-x)^2\right)\ln\frac{Q^2(1-x)}{M^2x}+8x(1-x)-1$$

General solution of the evolution equations:

$$D(x,M) = D^{PL}(x,M,M_0) + D^{HAD}(x,M,M_0)$$

Point-like part (PL):
particular solution of the full inhomogeneous equation resulting from resummation of the series of diagrams:

Hadronic part (HAD):
general solution of the corresponding homogeneous one



This separation is **not unique, but depends on M_0!**

All the difference between hadron and photon structure functions comes from **the pointlike part** of PDF of the photon. **Only the pointlike solutions** of the evolution equations will be considered in the following.

$$q_{\rm NS}^{\rm PL}(n,M_0,M)=\frac{4\pi}{\alpha_s(M)}\left[1-\left(\frac{\alpha_s(M)}{\alpha_s(M_0)}\right)^{1-2P_{qq}^{(0)}(n)/\beta_0}\right]a_{\rm NS}(n)$$

$$\frac{\alpha}{2\pi\beta_0}\frac{k_{\rm NS}^{(0)}(n)}{1-2P_{qq}^{(0)}(n)/\beta_0}$$

Standard, but **wrong** interpretation: $q_{NS}^{PL}=O(\alpha/\alpha_s)$

as switching off QCD by sending $\Lambda_{QCD}\rightarrow 0$

we get

$$q_{\rm NS}^{\rm PL}(x,M,M_0)\longrightarrow\frac{\alpha}{2\pi}k_{\rm NS}^{(0)}(x)\ln\frac{M^2}{M_0^2}$$

i.e. **as expected** the **pure QED contribution!**

Easy way to **wrong interpretation** of the correct result: start from **pure QED** (in units of α/2π and momentum space)

$$\frac{dq(n,\mu)}{d\ln\mu^2} = k^{(0)}(n) \quad \xrightarrow[\text{derivatives using}]{\text{replace}} \quad \frac{dq(n,\mu)}{d\alpha_s} = -\frac{4\pi}{\beta_0}\frac{k^{(0)}(n)}{\alpha_s^2}$$

$$\boxed{\frac{d\alpha_s(\mu)}{d\ln\mu^2} = -\frac{\beta_0}{4\pi}\alpha_s^2(\mu)}$$

integrating ↓   integrating ↓

$$q(n,\mu) = k^{(0)}(n)\ln\frac{\mu^2}{\mu_0^2} = \frac{4\pi k^{(0)}(n)}{\beta_0}\left(\frac{1}{\alpha_s(\mu)} - \frac{1}{\alpha_s(\mu_0)}\right)$$

define boundary condition

voilà: $q(n) \propto 1/\alpha_s$

but it is clear that this is just mirage as **q is of pure QED nature and has nothing to do with QCD!**

The fact that photon structure function behaves as $O(\alpha)$ follows directly from **factorization scale independence** of

$$F^{\gamma}(n,Q) = q(n,M)C_q(n,Q/M) + C_{\gamma}(n,Q/M)$$

As this expression is **independent of M**, we can take **any M** to evaluate it, for instance **M_0**. For $M=M_0$ the first term in vanishes and we get for the r.h.s.

$$\frac{\alpha}{2\pi}\left[C_{\gamma}^{(0)}(Q/M_0) + \frac{\alpha_s(\mu)}{2\pi}C_{\gamma}^{(1)}(Q/M_0) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 C_{\gamma}^{(2)}(Q/M_0, Q/\mu) + \cdots\right]$$

i.e. manifestly the expansion which **starts with O(α) pure QED contribution and** includes standard QCD corrections. These corrections **vanish** when QCD is switched off and **there is no trace** of the supposed α/α_s behaviour.

Why semantics matters

To avoid confusion, we should agree on the meaning of the terms **leading and next-to-leading order.**
Recall their meaning for

QED part

$$R_{e^+e^-}(Q) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \left(3\sum_{i=1}^{n_f} e_i^2\right)(1 + r(Q))$$

where
$$r(Q) = \frac{\alpha_s(M)}{\pi}\left[1 + \frac{\alpha_s(M)}{2\pi} r_1(Q/M) + \cdots\right]$$
contains **QCD effects**

For this quantity the QED contribution is subtracted and the terms **leading** and **next-to-leading** orders are applied **to QCD contribution r(Q) only**.

This procedure should be adopted for other physical quantities as well, including $F_2^\gamma(x,Q^2)$

Factorization scale and scheme independence

Recall that for the NS **proton** structure function and in momentum space

$$F^p(Q) = q(M)C_q(Q/M)$$

the scale and scheme independence means that

$$\frac{\mathrm{d}F(Q)}{\mathrm{d}\ln M^2} = q(M)\left[P(M)C_q(Q/M) + \dot{C}_q(Q/M)\right] = \frac{\mathrm{d}F(Q)}{\mathrm{d}C_q^{(j)}} = 0$$

which at the NLO implies

$$C_q^{(1)}(Q/M) = P^{(0)}\ln(Q^2/M^2) + C_q^{(1)}(1)$$

where

$$C_q^{(1)}(1) = \frac{2P^{(1)}}{\beta_0} + \kappa$$

FS invariant (κ) is related to the **nonuniversal NLO** splitting function **$P^{(1)}$**.

$P^{(1)}$ or **$C^{(1)}$**, but **not both**, can be chosen **to specify the FS**.

Inserting the previous relation into NLO approximation

$$F^p(Q) = A\left(\alpha_s(M)\right)^{\frac{-2P^{(0)}}{\beta_0}} \exp\left(-\frac{2P^{(1)}}{\beta_0}\frac{\alpha_s(M)}{2\pi}\right)\left[1+\frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(Q/M)\right]$$

F(Q) can be written **as a function of $C^{(1)}$** explicitly as

$$F^p(Q) = A\left(\alpha_s(M)\right)^{\frac{-2P^{(0)}}{\beta_0}} \exp\left[-\frac{\alpha_s(M)}{2\pi}\left(C_q^{(1)}-\kappa\right)\right]\left(1+\frac{\alpha_s(\mu)}{2\pi}C_q^{(1)}(Q/M)\right)$$

Mechanism guaranteeing FS invariance of F(Q):

Choice of $C^{(1)}$ here

is compensated by change of $C^{(1)}$ here

MS_{bar}:	$P^{(1)} \neq 0, C_q^{(1)} \neq 0$	used for **technical** reasons
DIS:	$P^{(1)} \neq 0, C_q^{(1)} = 0$	**F^p=q** to **all orders**
ZERO:	$P^{(1)} = 0, C_q^{(1)} \neq 0$	evolution equations **in LO form**

Factorization scale and scheme invariance of photon structure function

For the **pointlike part** of the nonsinglet quark distribution function of the photon the situation is more complicated as the expression involves **photonic coefficient function**

$$F^{\gamma}(Q) = q(M)C_q(Q/M) + C_{\gamma}(Q/M)$$

Factorization scale invariance

$$\begin{aligned} \dot{F}^{\gamma}(Q) &= \dot{q}(M)C_q(Q/M) + q(M)\dot{C}_q(Q/M) + \dot{C}_{\gamma}(Q/M) = 0 \\ &= \left[P(M)C_q(Q/M) + \dot{C}_q(Q/M)\right] q(M) + k(M)C_q(Q/M) + \dot{C}_{\gamma}(Q/M) \end{aligned}$$

then implies that the first non-universal inhomogeneous splitting function $k^{(1)}$ is, similarly as $P^{(1)}$, a function of $C_q^{(1)}$ (or the other way around). So $C_q^{(1)}$ **can again be used** to label the factorization scheme ambiguity.

QCD analysis of photon structure function

$$\underbrace{\frac{1}{x}F^{\gamma}_{2,\mathrm{NS}}(x,Q^2)}= \delta_{\mathrm{NS}}\left[q(M)\otimes C_q(Q/M)+\frac{\alpha}{2\pi}C_\gamma(Q/M)\right]$$

dropping charge factors we can separate contributions of individual **orders of QCD**

$$F(Q^2)=\underbrace{q_{\mathrm{QED}}+\frac{\alpha}{2\pi}C^{(0)}_\gamma}_{\mathrm{A_0;\ pure\ QED}}+$$

$$+\underbrace{q_{\mathrm{QCD}}+\frac{\alpha_s}{2\pi}C^{(1)}_q q_{\mathrm{QED}}+\frac{\alpha}{2\pi}\frac{\alpha_s}{2\pi}C^{(1)}_\gamma}_{\equiv A_1,\ \text{starting as } \mathcal{O}(\alpha\alpha_s)}+$$

$$+\underbrace{\frac{\alpha_s}{2\pi}C^{(1)}_q q_{\mathrm{QCD}}+\frac{\alpha}{2\pi}\left(\frac{\alpha_s}{2\pi}\right)^2C^{(2)}_\gamma+\left(\frac{\alpha_s}{2\pi}\right)^2C^{(2)}_q q_{\mathrm{QED}}}_{\equiv A_2,\ \text{starting as } \mathcal{O}(\alpha\alpha_s^2)}+\cdots$$

$$q^{\mathrm{PL}}(M)=q_{\mathrm{QED}}(M)+q_{\mathrm{QCD}}(M)$$

$$q_{\mathrm{QED}}(M) \to \frac{\alpha}{2\pi}k^{(0)}_q\ln\frac{M^2}{M_0^2}$$

Quantities taken into account (**QED**, lowest order **QCD**, second lowest order **QCD**)

name	standard approach	alternative approach
QED:	**does not** introduce	$k^{(0)}, C_\gamma^{(0)}$
LO:	$k^{(0)}, P^{(0)}$	$k^{(0)}, C_\gamma^{(0)}, k^{(1)}, C_\gamma^{(1)}, P^{(0)}, C_q^{(1)}$
NLO:	$k^{(0)}, C_\gamma^{(0)}, k^{(1)}, P^{(0)}, C_q^{(1)}$	$k^{(0)}, C_\gamma^{(0)}, k^{(1)}, C_\gamma^{(1)}, P^{(0)}, C_q^{(1)}$
	$P^{(1)}$	$k^{(2)}, P^{(1)}, C_q^{(2)}, C_\gamma^{(2)}$

Note, in particular, that in the standard approach the pure **QED** coefficient function $C_\gamma^{(0)}$ appears **first at the "NLO"**

Comparison of **LO** analyses in the standard and alternative approaches will be presented by **J. Hejbal**.

What is wrong with DIS$_\gamma$ FS?

In the standard approach the lowest order, **purely QED** photonic coefficient function $C_\gamma^{(0)}$, is treated **in the same way** as genuine **QCD** NLO coefficient function $C_q^{(1)}$, i.e. is absorbed in the definition of PDF of the photon in the

$$\overline{q}(M, M_0) \equiv q(M, M_0) + \frac{\alpha}{2\pi} C_\gamma^{(0)}(1)$$

in the so called **"DIS$_\gamma$"** factorization scheme,

with the same boundary condition

$$\overline{q}(M = M_0, M_0) = q(M = M_0, M_0) = 0$$

Note that such mechanism must hold also for **pure QED** contribution as it must operate at any fixed order and **cannot thus mix orders of QED and QCD.**

However, getting rid of the troubling $C_\gamma^{(0)}$ term via this redefinition **violates factorization scheme invariance**.

The QED box diagram regularized by quark mass m_q gives

$$F^\gamma_{\rm QED}(Q) = \frac{\alpha}{2\pi} C_\gamma^{(0)}(Q/m_q) = \frac{\alpha}{2\pi}\left[k^{(0)}\ln\frac{Q^2}{m_q^2} + C_\gamma^{(0)}(1)\right]$$

$$= q_{\rm QED}(M) + \frac{\alpha}{2\pi} C_\gamma^{(0)}(Q/M)$$

$$\equiv \frac{\alpha}{2\pi} k^{(0)} \ln\frac{M^2}{m_q^2}$$

defines the **QED part** of the quark distribution function of the photon

Note that redefining the quark distribution function as

$$q_{{\rm QED},f}(M) \equiv q_{\rm QED}(M) + \frac{\alpha}{2\pi} f$$

does not change the evolution equation in f-scheme:

$$\frac{\mathrm{d}q_{\mathrm{QED},f}(M)}{\mathrm{d}\ln M^2} = \frac{\alpha}{2\pi}k^{(0)}$$

To keep the sum $F^{\gamma}_{\mathrm{QED}}(Q) = q_{\mathrm{QED},f}(M) + \frac{\alpha}{2\pi}C^{(0)}_{\gamma,f}(Q/M)$

f-independent implies $C^{(0)}_{\gamma,f}(Q/M) \equiv C^{(0)}_{\gamma}(Q/M) - f$

But **we cannot** impose on the same boundary condition

$$q_{QED,f}(M = m_q) = 0$$ in all f-schemes!!

since we would get **manifestly f-dependent** expression

$$F^{\gamma}(Q) = \frac{\alpha}{2\pi}\left(C^{(0)}_{\gamma}(Q/m_q) - f\right)$$

Conclusions

The organization of finite order QCD approximations of the photon structure function which follows closely that of the e^+e^- annihilation to hadrons and **separates the pure QED contribution** has been discussed.

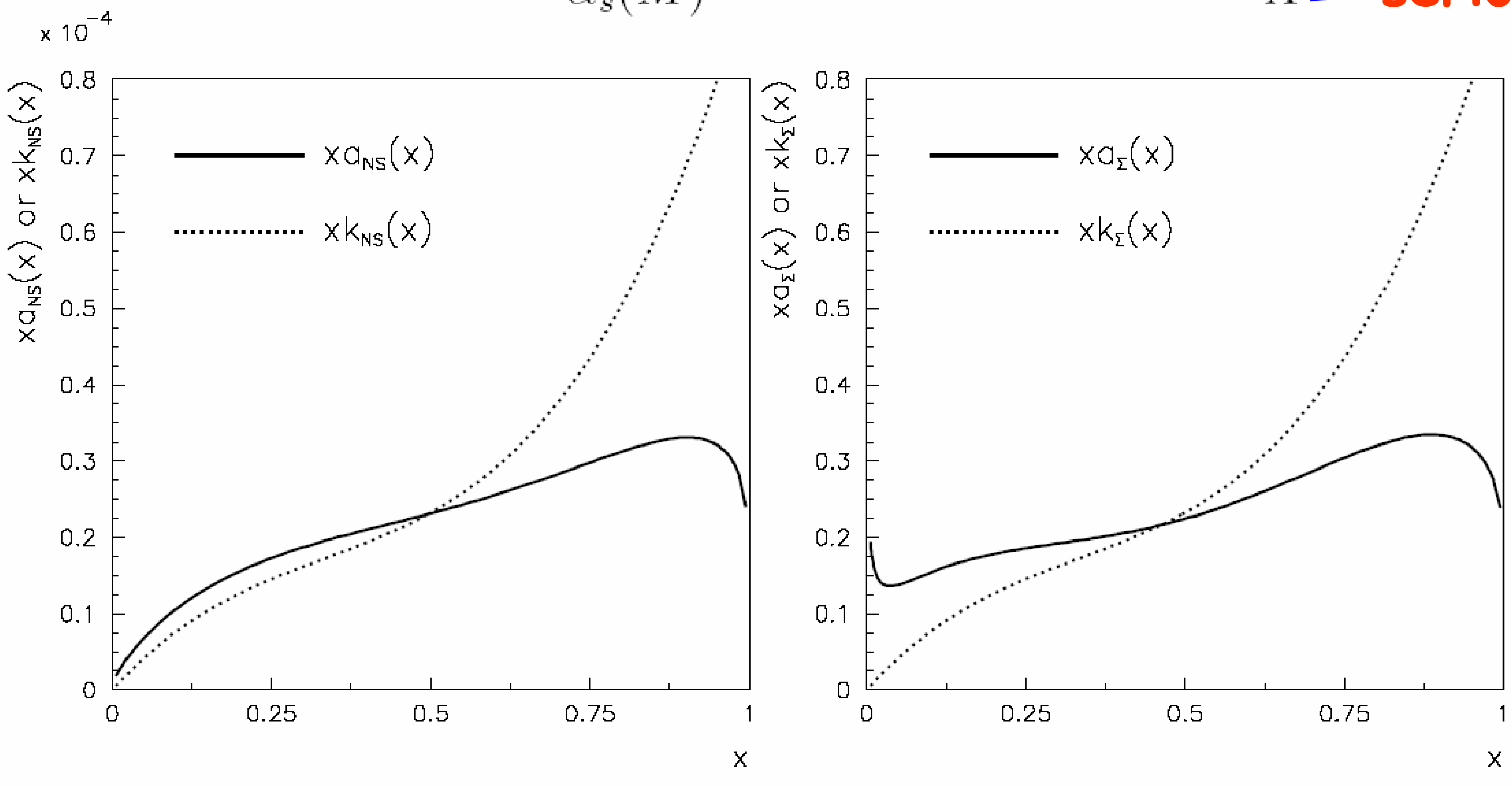
It differs significantly from the standard one by the set of terms included at each finite order. As shown in the talk of J. Hejbal, **this difference is sizable** and of **phenomenological relevance**.

Asymptotic pointlike solution

for asymptotic values of M

$$q_{\rm NS}^{\rm PL}(x, M_0, M) \longrightarrow \frac{4\pi}{\alpha_s(M)} a_{\rm NS}(x) \equiv q_{\rm NS}^{\rm AP}(x, M) \propto \ln\frac{M^2}{\Lambda^2}$$

not to be taken **seriously**



What is wrong with DIS FS?

Moch, Vermaseren, Vogt in Photon05: