

# Minijets, soft gluon resummation and photon cross-sections



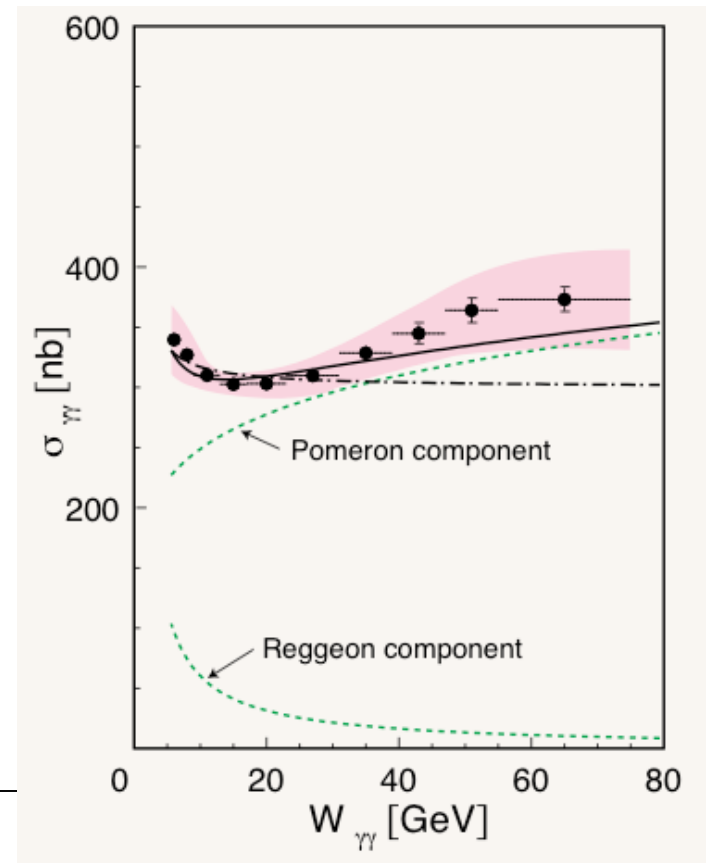
Giulia Pancheri - INFN-Frascati



AdA : Where  $e^+e^-$  collisions were born

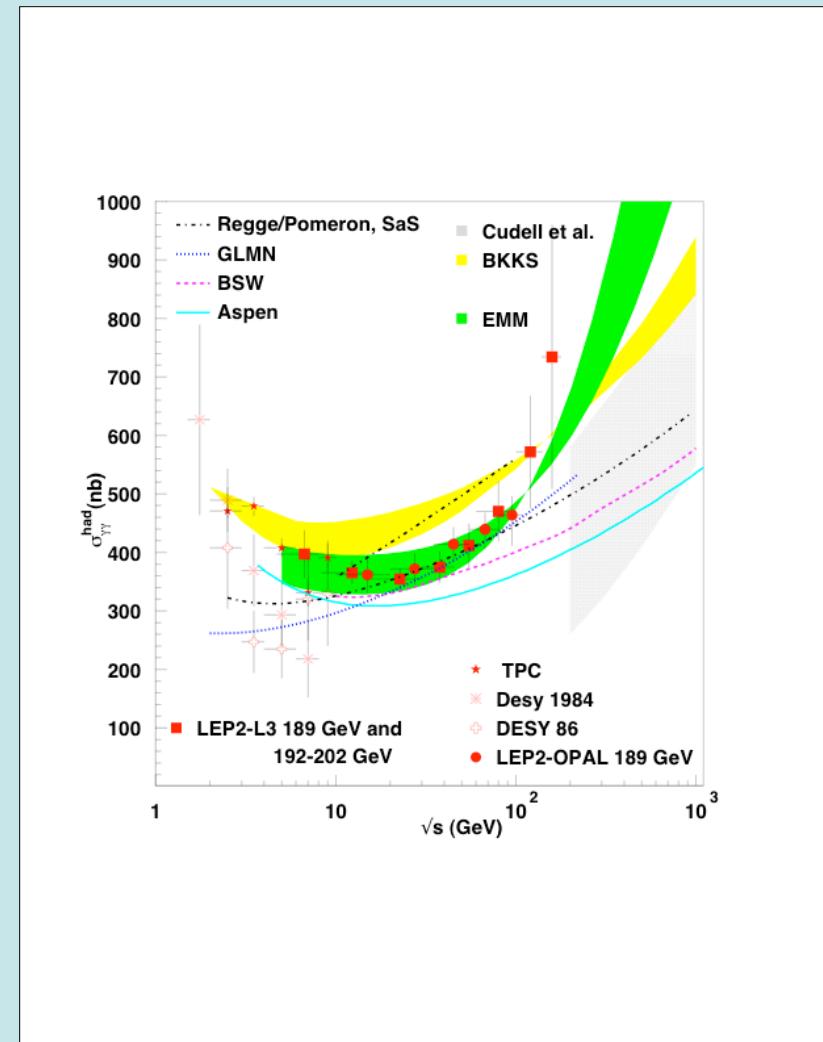
# Total cross-sections are a testing ground of our understanding of QCD beyond perturbative regime

- A popular models for total cross-section



## Models for total cross-section

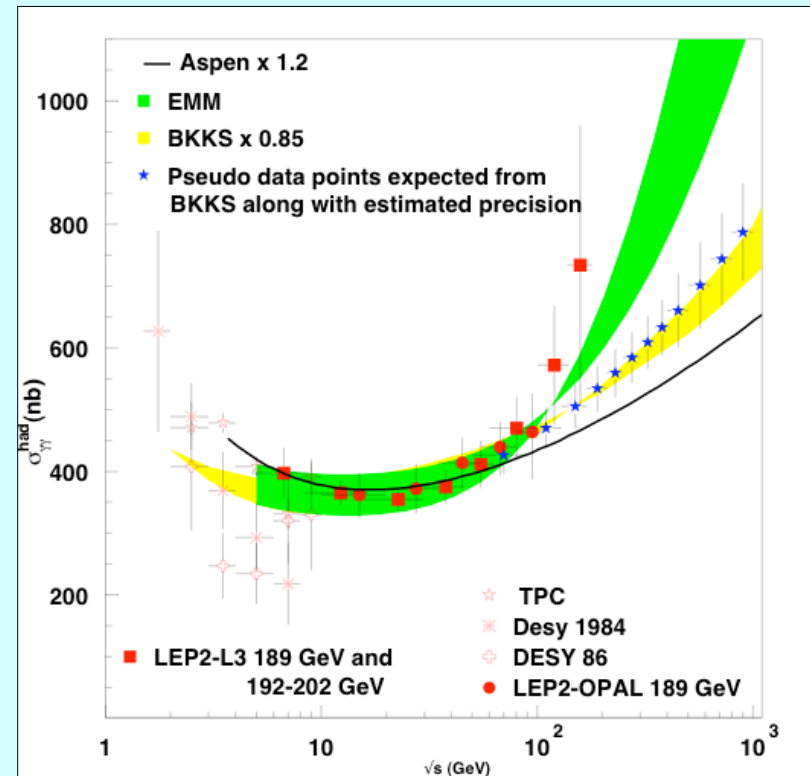
- The interest lies in QCD role
- What is the Pomeron?  
The Reggeon?
- Are these concepts universal?
- Or do they just phenomenologically describe our ignorance?
- How can ILC/LPC help ?



A.de Roeck, R. Godbole, A. Grau, G.Pancheri,  
JHEP 2003

$S^\varepsilon$  : Should  $\varepsilon$  be the same for all hadronic cross-sections?

- Yes if the model
  - is based on Regge poles and a universal Pomeron pole exchange
$$\sigma = Bs^{-\eta} + As^\varepsilon$$
- Not necessarily if
  - The model has some connection with QCD and parton densities play a role



A fit to LEP data shows that  $\varepsilon$  is not the same for proton and photon cross-sections

$$\sigma = B s^{-\eta} + A s^{\varepsilon} + C s^{\varepsilon_1}$$

- Fit3

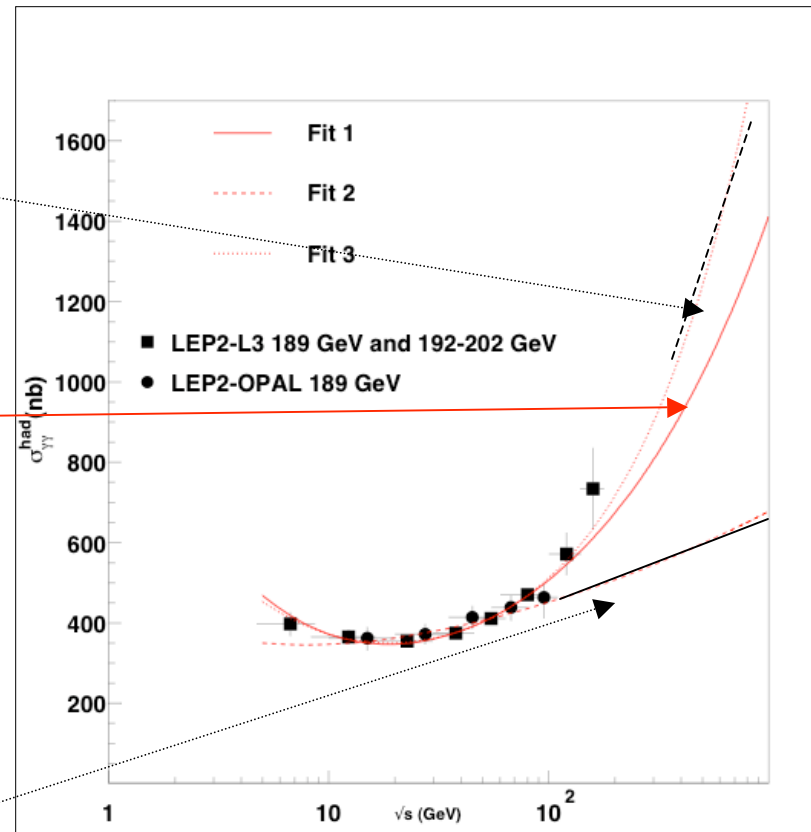
$C \neq 0$     $\varepsilon = 0.093$   
 $\varepsilon_1 = 0.418$

- Fit 1

$C = 0$     $\varepsilon = 0.250$

- Fit2

$C = 0$     $\varepsilon = 0.093$  as in pp



A.de Roeck, R. Godbole, A. Grau, G.Pancheri, JHEP 2003

A realistic QCD model should relate the fit to QCD phenomenological inputs quantities like densities etc.

# The Bloch-Nordsieck Eikonal Minijet model includes $k_+$ resummation


R.Godbole, A. Grau, G.Pancheri, Y.Srivastava PRD 2005  
A. Corsetti, A. Grau, G.Pancheri, Y. Srivastava PLB 1996

1. Multiple parton interactions : optical theorem and **eikonal** representation for  $T_{el}(s,t)$
2. Hard scattering to drive the **rise due to  $1/x$**
3. **Soft gluons** down to zero momentum to **tame** the rise

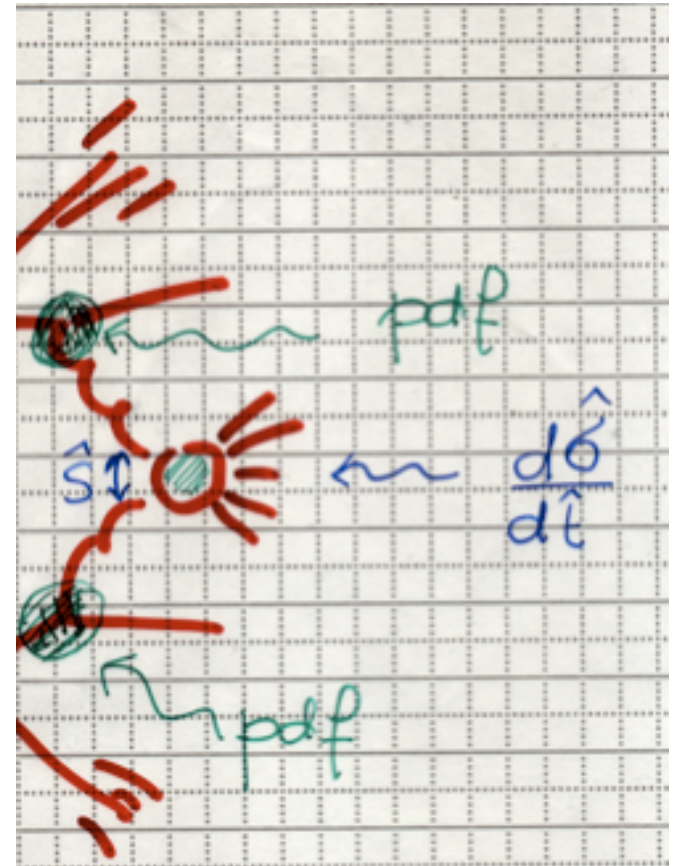


# The hard cross-section

- Mini-jet cross-section



$$\Sigma \int \text{densities} \int dp_t d\sigma/dp_t$$



In all mini-jet models densities make all the difference between photon and proton processes

Proton-proton and  
proton-antiproton

Most commonly used  
densities

- GRV
- CTEQ
- MRST

$\gamma$ -proton and  $\gamma\gamma$

Most commonly used  
densities

- GRV
- GRS
- CJKL

# Soft resummation

Probability of total  $K_T$  from infinite # of soft gluons

$$\int d^2b e^{iK_T b} \exp\{-\int d^3n(k)[1-e^{-ik_t b}]\}$$

depends upon single gluon energy

- maximum : use Kinematics
- **minimum : 0** from Bloch-Nordsieck theorem

# Role of resummation

An **infinite** number of soft quanta

- down to **zero momentum** but how?  
next slides
- Up to an energy dependent limit  $q_{\max}$ 
  - Higher hadron energy  $\Rightarrow$  possibility  
of more small  $x$  partons with “high energy”  
( $\approx 1-2$  GeV)  $\Rightarrow$  higher  $q_{\max}$

# Maximum soft gluon energy

- $q_1$  and  $q_2$  : any two partons
- $X$  : the 2-jet final state
- $Q^2 \geq 4 p_{tmin}^2$
- $q_{max}$  depends on  $X_1, X_2$
- We average over densities

$q_1 + q_2 \rightarrow g + X$

$q_{max} = \max\{R_t\} = \frac{\sqrt{\hat{S}}}{2} \left(1 - \frac{Q^2}{\hat{S}}\right)$

$\hat{S} = (q_1 + q_2)^2$

$Q^2 = X^2$

*Chiappetta, Greco 1981*

# Zero momentum quanta

- **Soft** gluons need to be resummed if they are indeed soft  $\approx 1/k$
- **Resummation** implies **integration** over  $dk_t$
- What matters will be  $\int \alpha_s(k_t) dk_t f(k_t)$  and not  $\alpha_s(0)$

Models for infrared behaviour

$$\alpha_s^{\text{FROZEN}}(R_t) = \frac{\text{constant}}{\ln\left(a^2 + \frac{R_t^2}{\Lambda^2}\right)} \xrightarrow{R_t \rightarrow 0} \frac{\text{const}}{\ln a^2}$$

$$\alpha_s^{\text{sing}}(R_t) = \frac{c}{\ln\left(1 + \left(\frac{R_t}{\Lambda}\right)^{2p}\right)} \sim \frac{1}{R_t^{2p}}$$

↑  
integrable

# Soft gluons give b-distributions

In eikonal representation

$$\sigma_{\text{tot}} \approx 2 \int d^2b [1 - e^{-n(b,s)/2}]$$

- $n(b,s)$  = average # of collisions at distance  $b$ , at energy  $\sqrt{s}$
- b-distribution is needed

Our ansatz:

b-distribution =  
Fourier transform of soft gluon  $K_t$  distribution

# Resummation of soft gluons down to $k_{\perp}=0$

- **Gluon emission in  $k_{\perp}$**  changes the collinearity of initial partons
- And for same energy and  $p_{\text{tmin}}$ , **acollinearity** of initial partons will bring loss of luminosity of the parton beams and parton-parton cross-sections will decrease
- As the energy available for soft gluon emission increases, so does the acollinearity of the parton-parton collision
- The rate of rise of total cross-sections due to **rising minijet cross-section** is **reduced** (softened by) by soft gluon emissions.
- Softening effect more important the more singular  $\alpha_s$

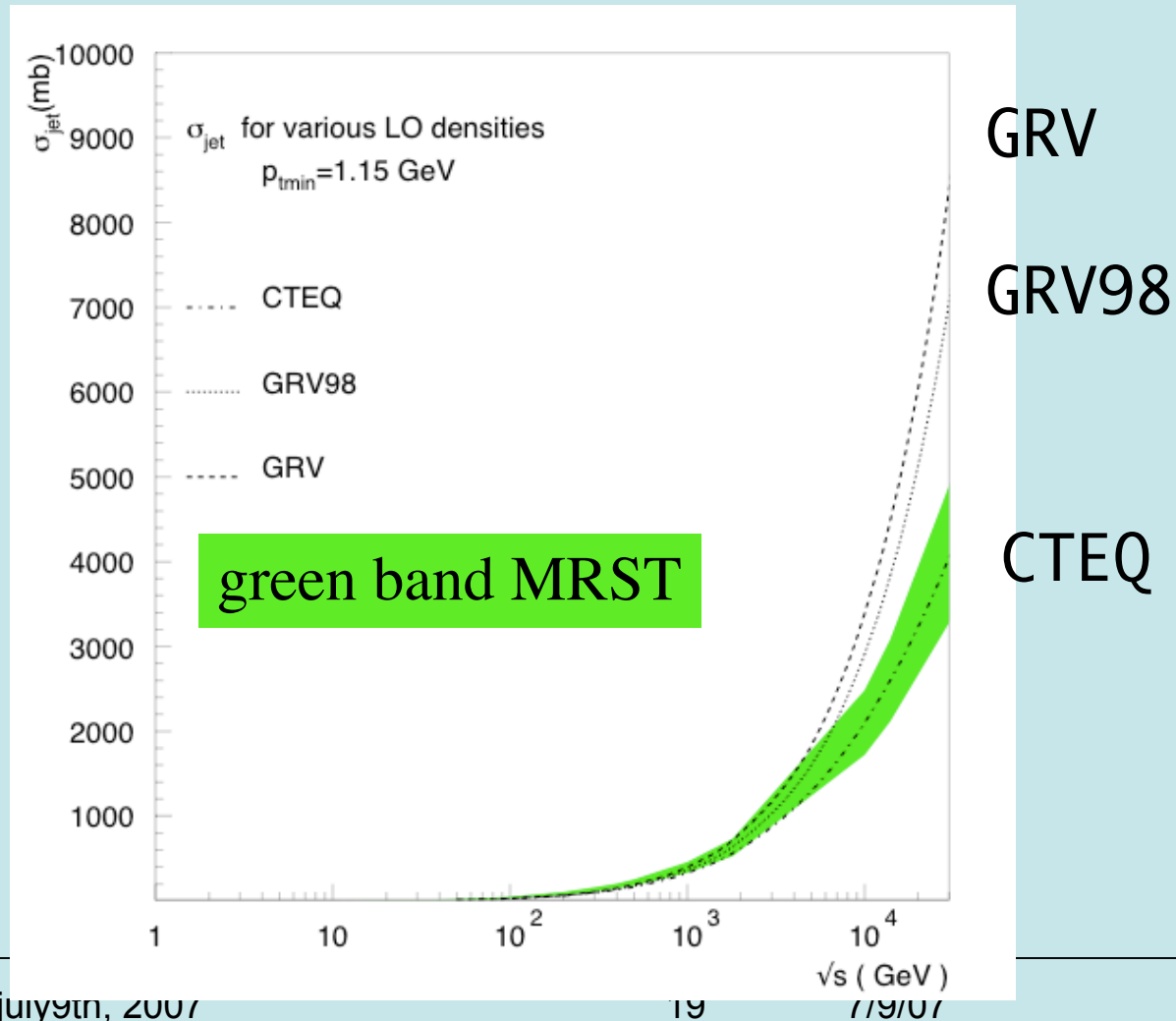


We shall illustrate how the model works  
for the proton-proton case and then  
show its application to  $\gamma\gamma$

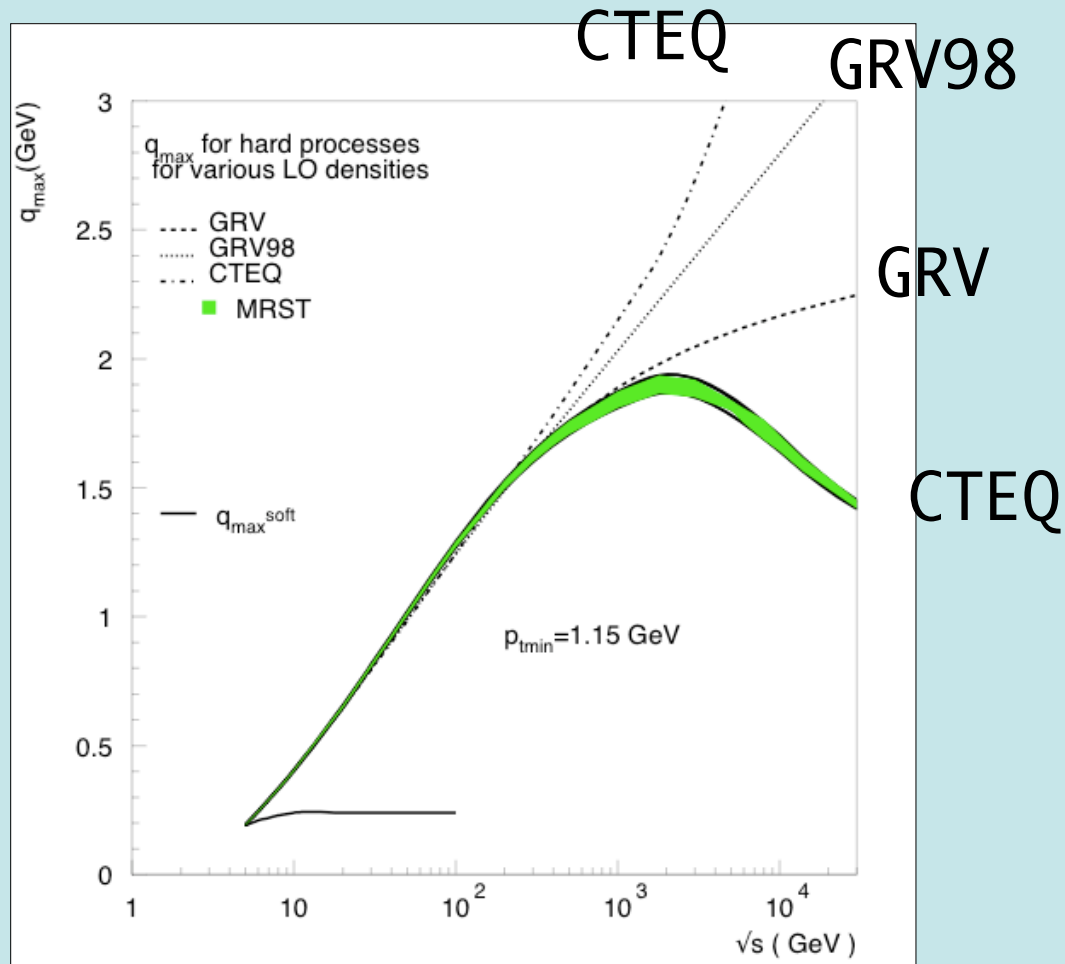
# How the model works

- Choose  $p_{\text{tmin}} = 1\div 2$  GeV for mini-jets
- Choose parton densities
- Calculate minijet x-section
- Calculate  $q_{\text{max}}$  for soft gluons
- Calculate  $A(b,s)$  for given  $q_{\text{max}}$
- Calculate  $n_{\text{hard}}(b,s) = A(b,s) \sigma_{\text{jet}}(p_{\text{tmin}}, s)$
- Parametrize  $n_{\text{soft}}$
- Evaluate  $n(b,s) = n_{\text{soft}} + n_{\text{hard}}$
- Eikonalize  $\sigma_{\text{tot}} \approx 2 \int d^2b [1 - e^{-n(b,s)/2}]$

# $\sigma_{\text{jet}}$ for $p_{t\text{min}}=1.15 \text{ GeV}$





# $q_{\max}$ for $p_{t\min}=1.15$ GeV

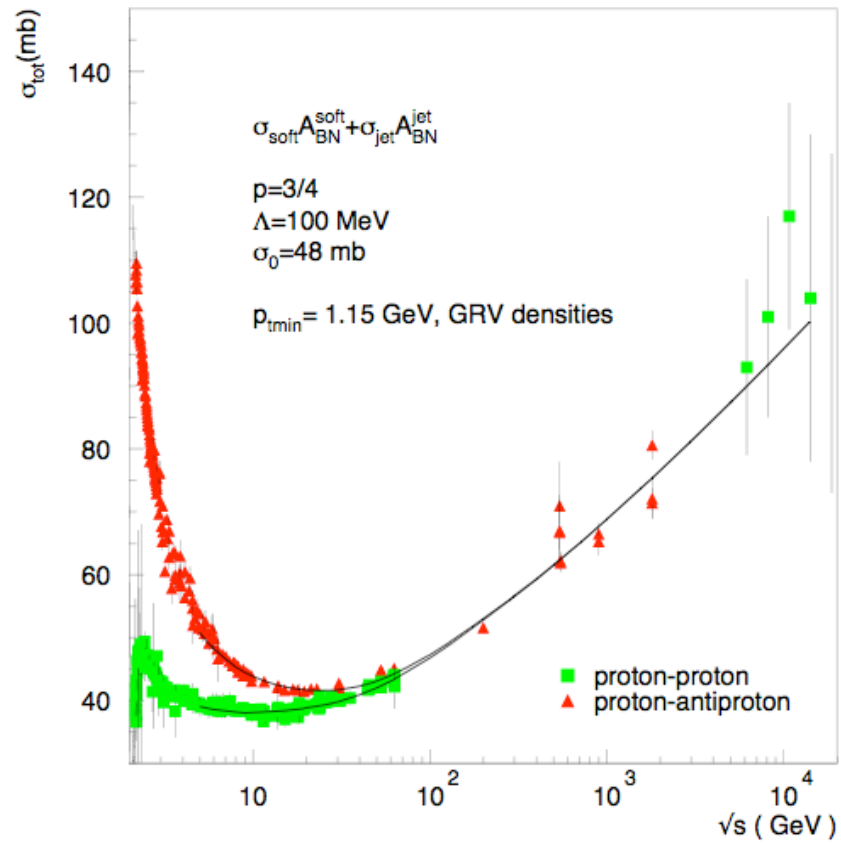


# Which parameters in soft emission?

$p_{tmin}$  and  $p$  regulate how large is the maximum energy, but PDF's also play a role

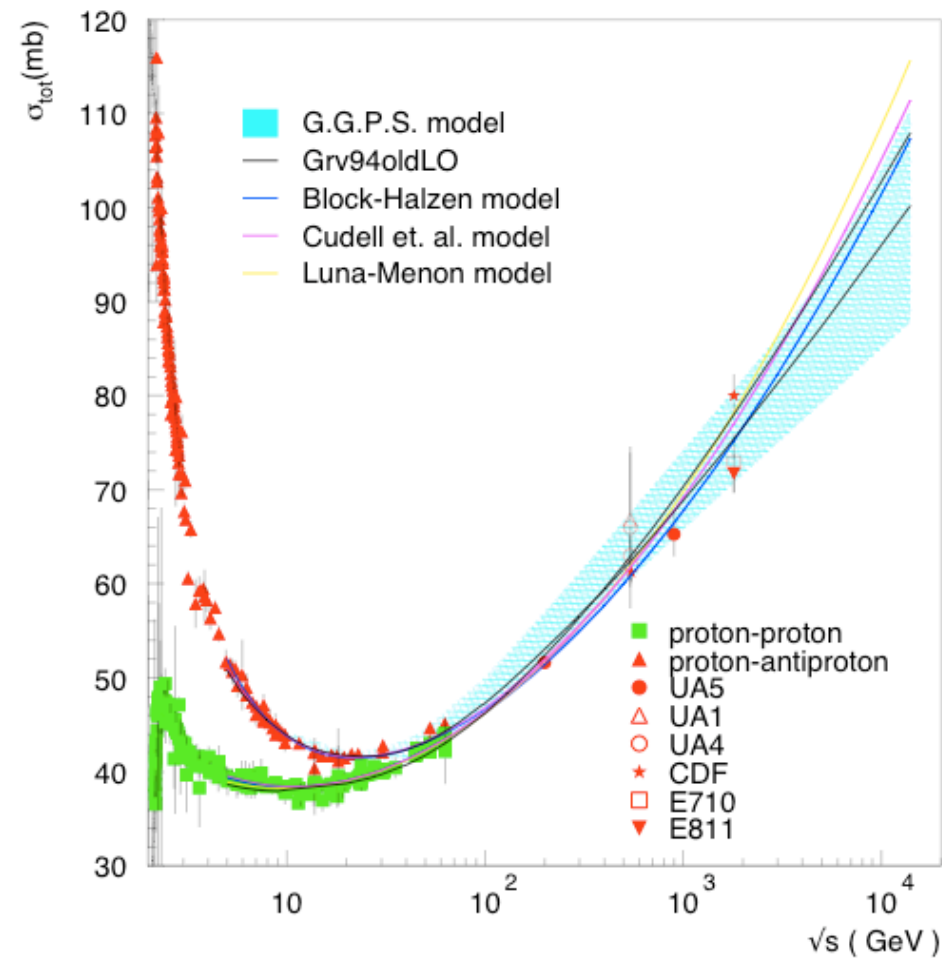
- $P_{tmin}$   hard scattering lowest scale
- $p$   infrared (integrable) behaviour

$\sigma_{\text{tot}} = 102 \text{ mb}$   
at LHC



GRV

# Comparison with proton data(I)



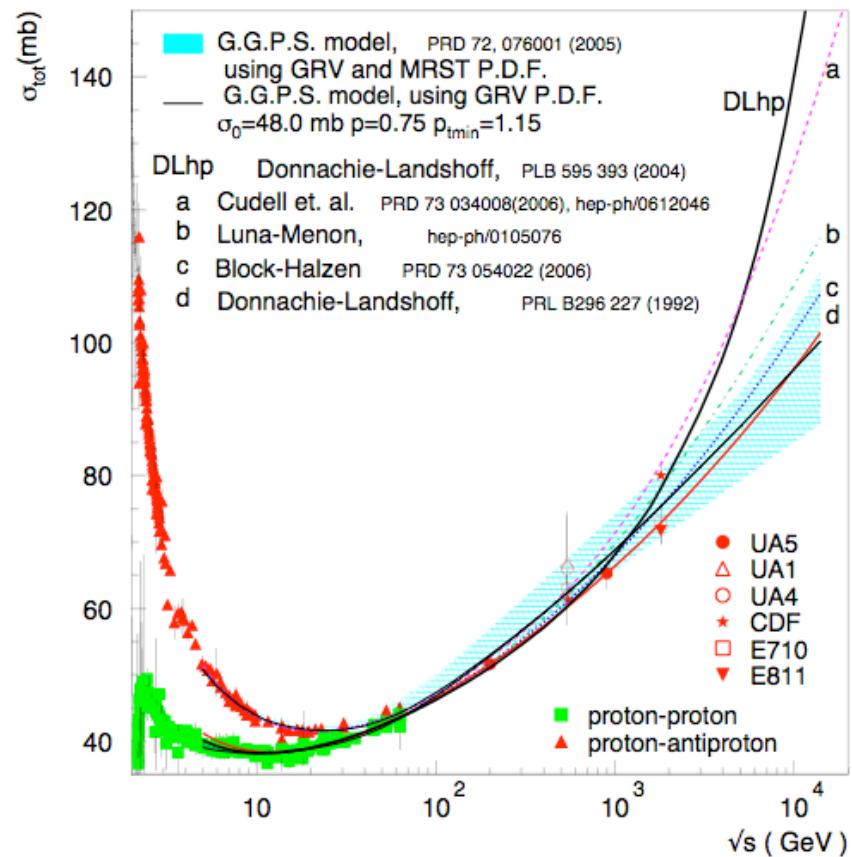
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Pramana **67** (2006)

GGPS PRD **2005**

# Comparison with proton data

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# Survival probability

Probability of not having an inelastic collision

$$P_{inel} = e^{-n(b,s)}$$

Can be used to calculate the survival probability of Large Rapidity Gaps for collisions at given b-value

# Survival probability

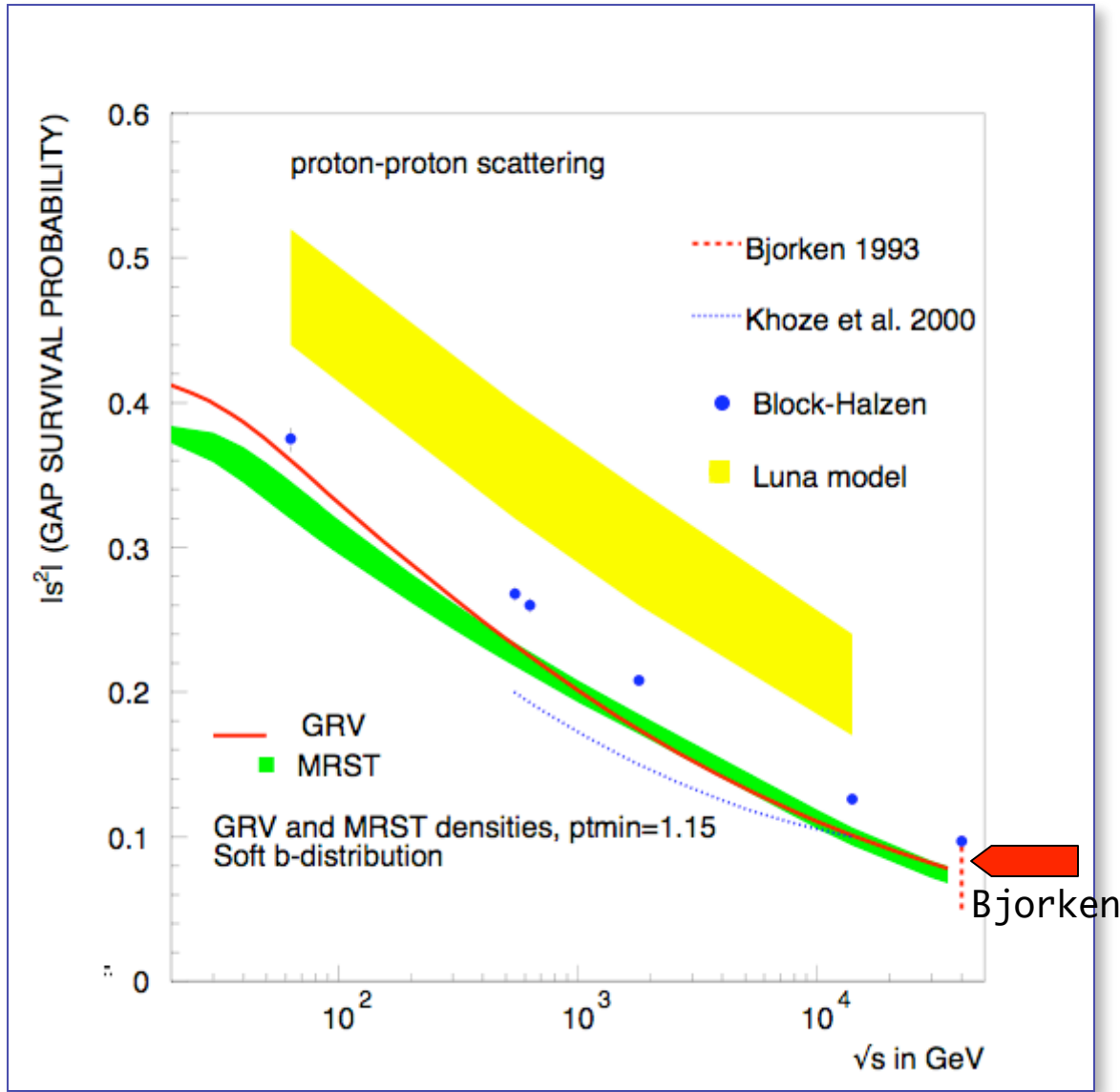
$$\langle |S|^2 \rangle = \int d^2\vec{b} A(\vec{b}, q_{max}^{soft}) |S(\vec{b})|^2$$

we use the soft b-distribution  $A(\vec{b}, q_{max}^{soft})$

$$\int d^2\vec{b} A(\vec{b}, q_{max}^{soft}) = 1$$

$$|S(\vec{b})|^2 = P_{inel}$$

# Comparison with other models



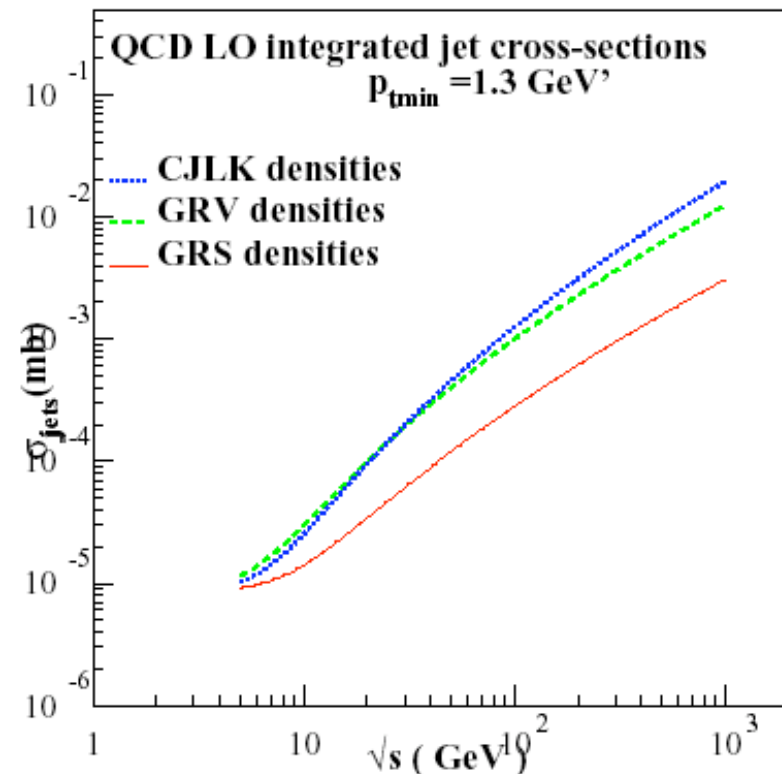
Now apply the model to  $\gamma\gamma$

Choose  $p_{t\min} = 1 \div 2 \text{ GeV}$  for mini-jets  
and parton densities

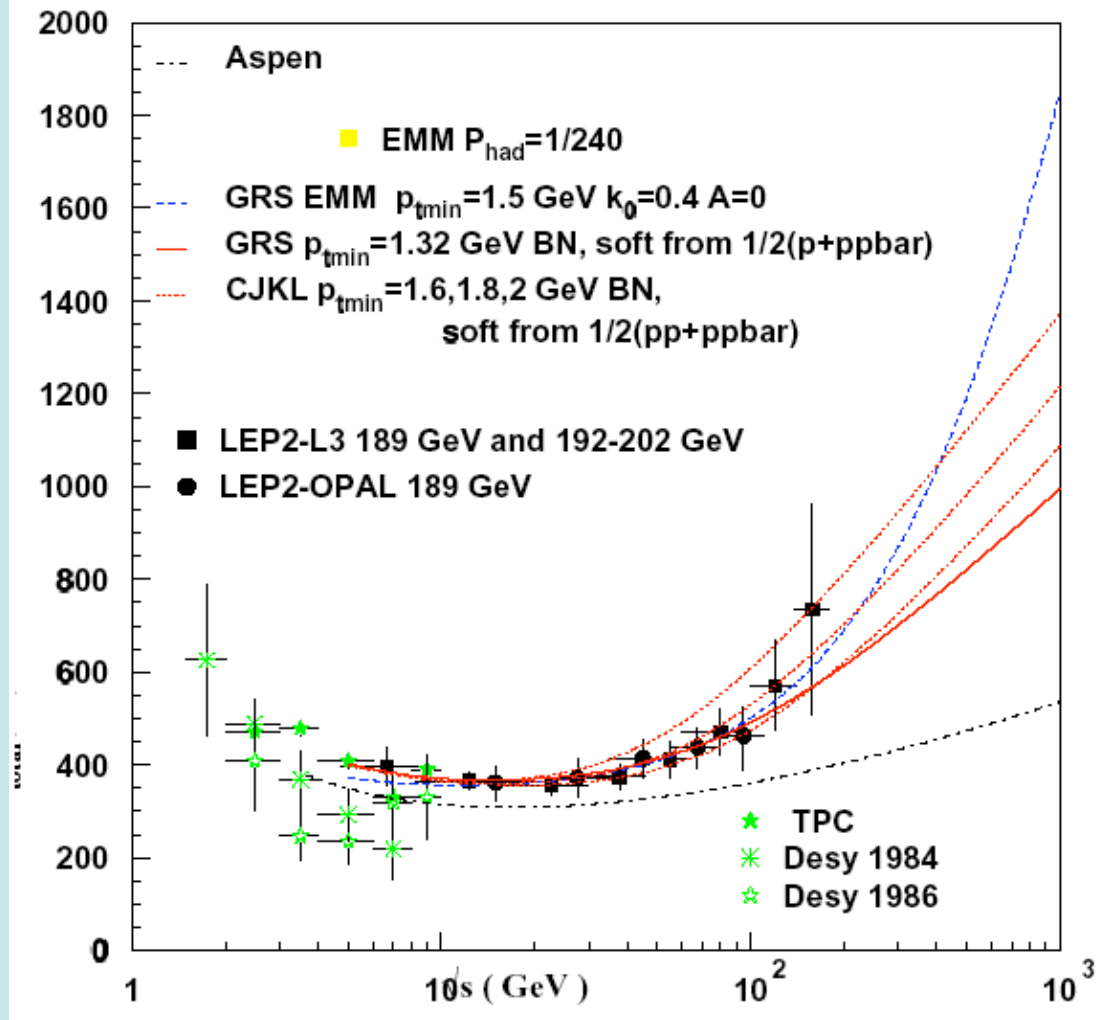
For photons, LEP data  
suggest

$p_{t\min} \sim 1.3 \div 1.8 \text{ GeV}$

- Gluck Reya Vogt
- Gluck Reya Shielbein
- Cornet Jankowski Lorca  
Krawczyk



Eikonalize  $\sigma_{\text{tot}} \approx 2P_{\text{had}} \int d^2b [1 - e^{-n(b,s)/2}]$   
 and compare with data



$P_{\text{had}}$  is a Phenomenological input describing the hadronic content of the photon in eikonal models

R.Fletcher, T.Gaisser, F.Halzen, 1993

# An aside on the Froisart bound and minijet cross-sections

# About the Froissart bound and QCD minijets

For all densities we find

$$\sigma_{jet}^{PDF}(s, p_{tmin}) \approx s^\epsilon$$

with

$\epsilon \approx 0.4$  for GRV and GRV98  $\rightarrow$  more singular

$\epsilon \approx 0.3$  for CTEQ and MRST  $\rightarrow$  less singular



# QCD Mini-jets violate the Froissart bound

- Consequence of **infinite** range of QCD
- One needs to introduce a **finite** distance of the interaction
- The **eikonal** does it through the hadron finite size

# Minijets : how to reduce their growth?

- Embedding the minijets into the eikonal

but

- It depends on how  $A(b,s) \longrightarrow 0$   
as  $b \longrightarrow \text{infinity}$

# Finite size of hadrons

- The finite size can be introduced through the Form Factor

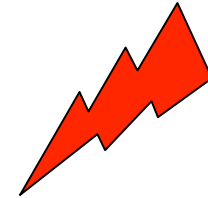
$$A(b) \sim e^{-b \text{ constant}} \text{ as } b \sim \text{very large} :$$

not enough to tame the rise because the growth of

$\sigma_{\text{jet}}^{\text{PDF}}$  is too strong!!

or

energy dependent soft gluon emission where satisfaction of the Froissart bound would imply that the finite range of the interaction is restored through soft gluon emission down into the infrared



# For all pdf's

- For different PDF , with soft gluon emission to give an energy dependent size and QCD hard gluon minijets to drive the rise
- All the Bloch-Nordsieck type curves

$$\sigma_{\text{tot}}^{pp/p\bar{p}} = a_0 + a_1 s^b + a_2 \ln(s) + a_3 \ln^2(s).$$

even though  $\sigma_{jet} \uparrow s^\epsilon$

# Conclusions (I)

- We have built a model for the total cross-section which
  - Incorporates **hard** and **gluon** effects
  - Satisfies the limits from the **Froissart** bound
  - Can be used to study other minimum bias effects
  - Easily extended to  $\gamma p$  and  $\gamma\gamma$

# Conclusions (II)

- Predictions at ILC vary according to which densities better describe the behaviour at low  $x$
- Total cross-sections measurements in Collider mode would allow clean information on  $\gamma\gamma$  cross-sections, reducing the errors due to modelling of diffractive components
- Even in regular mode, difference in the model predictions are measurable and can give insights into the soft or non perturbative region of QCD.