Analysis of elastic pp and p$\bar{p}$ scattering

from a unitary extension of Bialas-Bzdak model

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p+p @ ISR and @ 7 TeV LHC
Real extension of Bialas-Bzdak
New: focusing ReBB on low t region of dσ/dt of pp and p$\bar{p}$
New: excitation functions

arXiv:1204.5617
arXiv:1306.4217
arXiv:1311.2308
arxiv:1505.01415
+ manuscript in preparation

S-matrix Unitarity, Optical Theorem

$$SS^\dagger = I\,,$$

$$S = I + iT$$

$$T - T^\dagger = iTT^\dagger$$

$$2\,\mathrm{Im}\,t_{el}(s,b) = |t_{el}(s,b)|^2 + \sigma(s,b)$$

Note: diffraction also measures |Fourier-transform|^2 images of sources of elastic scattering
- ideal for femtoscopic studies
- several similarities e.g. non-Gaussian sources etc

Black (grey) disc limit (important)
$\rightarrow \sigma(b) \sim \theta(R-b)$

Diffraction in quark-diquark models

$$\frac{d\sigma}{dt} = \frac{1}{4\pi}\left|T\left(\Delta\right)\right|^2 .$$

Bialas and Bzdak,
Acta Phys. Polon. B 38 (2007) 159
p= (q, d) or p = (q, (q, q))

$$T(\vec{\Delta}) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} t_{el}(\vec{b})e^{i\vec{\Delta}\cdot\vec{b}}\mathrm{d}^2b = 2\pi\int\limits_{0}^{+\infty} t_{el}\left(b\right)J_0\left(\Delta b\right)b\mathrm{d}b,$$

$$t_{el}(\vec{b}) = 1-\sqrt{1-\sigma(\vec{b})}.$$

σ(b) = b dependent prob. of interaction
→ connection to scattering centers

$$\sigma(\vec{b}) = \int\limits_{-\infty}^{+\infty}\ldots\int\limits_{-\infty}^{+\infty}\mathrm{d}^2s_q\mathrm{d}^2s_q'\mathrm{d}^2s_d\mathrm{d}^2s_d' D(\vec{s_q},\vec{s_d})D(\vec{s_q}',\vec{s_d}')\sigma(\vec{s_q},\vec{s_d};\vec{s_q}',\vec{s_d}';\vec{b}),$$

Structure of protons = ?
→ Diffractive pp at ISR (23.5 – 62.5 GeV) and LHC (7 - 8 TeV).

Diffraction a la Bialas and Bzdak

$$D\left(\vec{s_q},\vec{s_d}\right)=\frac{1+\lambda^2}{\pi R_{qd}^2}e^{-(s_q^2+s_d^2)/R_{qd}^2}\delta^2(\vec{s_d}+\lambda\vec{s_q}),\ \lambda=m_q/m_d,$$

$$\sigma(\vec{s_q},\vec{s_d};\vec{s_q}',\vec{s_d}';\vec{b})=1-\prod_{a,b\in\{q,d\}}\left[1-\sigma_{ab}(\vec{b}+\vec{s_a}'-\vec{s_b}')\right]$$

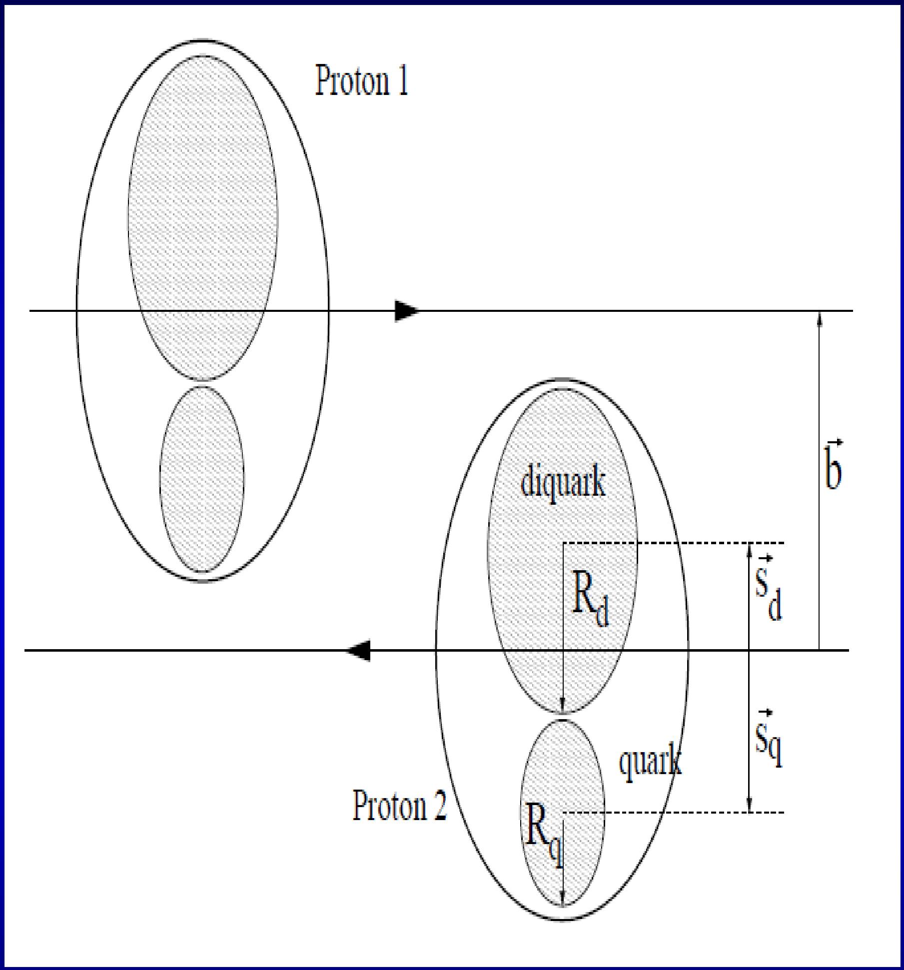
$$\sigma_{ab}\left(\vec{s}\right)=A_{ab}e^{-s^2/R_{ab}^2},\ R_{ab}^2=R_a^2+R_b^2,$$

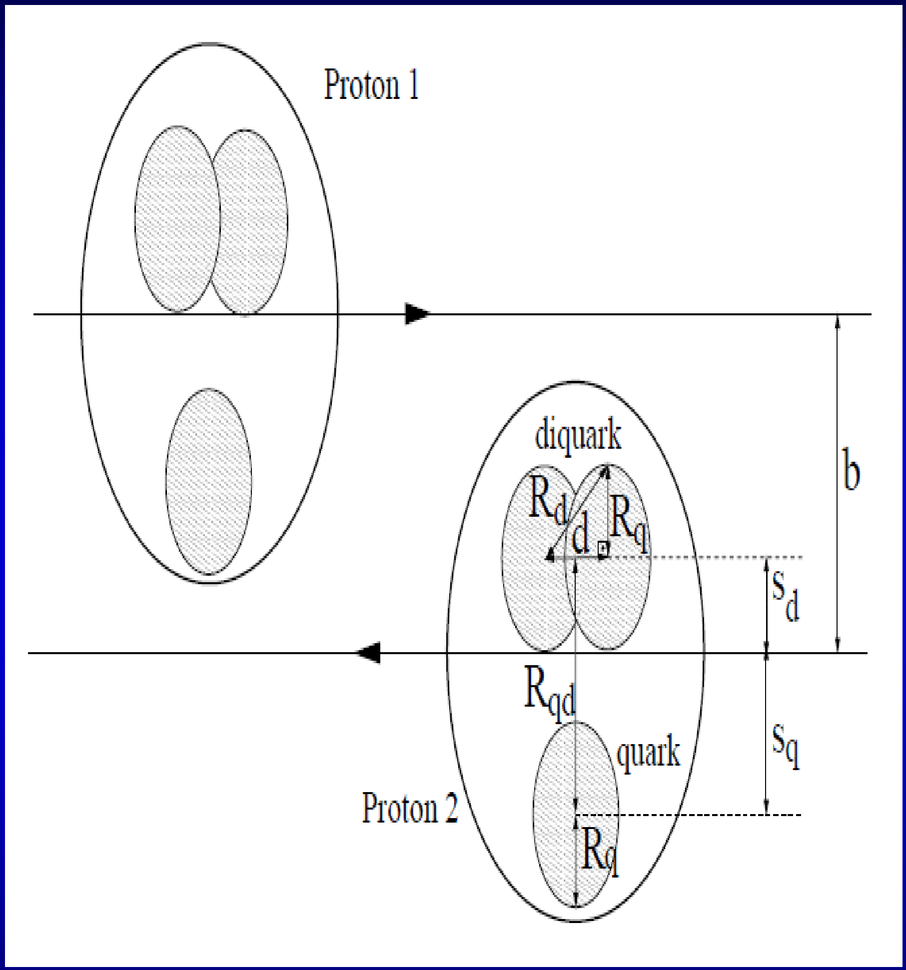
The quark-diquark model of Bialas and Bzdak has been analytically integrated in a Gaussian approximation, ***assuming*** that the real part of forward scattering is negligible.

Two different pictures: p = (q, d) or p = (q, (q,q))

Note: p= (q,q,q) model fails, quarks are correlated
W. Czyz and L. C. Maximon, Annals. Phys. 52 (1969) 59

Diffractive pp scattering





p = (q, d)

p = (q, (q,q))

Real extended BB model for the dip

$$\frac{d\sigma}{dt} = \frac{1}{4\pi}\left|T\left(\Delta\right)\right|^2.$$

$$T(\vec{\Delta}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} t_{el}(\vec{b}) e^{i\vec{\Delta}\cdot\vec{b}} \mathrm{d}^2 b = 2\pi \int_{0}^{+\infty} t_{el}\left(b\right) J_0\left(\Delta b\right) b \mathrm{d}b,$$

$$t_{el}(s,b) = i\left(1 - e^{-i\,\mathrm{Im}\,\Omega(s,b)}\sqrt{1-\sigma(s,b)}\right)$$

Bialas-Bzdak obtained if Re (t_{el}) = 0

$$t_{el}(s,b) = i\left(1 - e^{-\mathrm{Re}\,\Omega(s,b)}\right) = i\left(1 - \sqrt{1-\sigma(s,b)}\right)$$

$$\sigma(\vec{b}) = \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} \mathrm{d}^2 s_q \mathrm{d}^2 s_q' \mathrm{d}^2 s_d \mathrm{d}^2 s_d' D(\vec{s_q}, \vec{s_d}) D(\vec{s_q}', \vec{s_d}') \sigma(\vec{s_q}, \vec{s_d}; \vec{s_q}', \vec{s_d}'; \vec{b}),$$

Real extension of an imaginary t_{el}
New parameter Im Ω added

ReBB model for the dip (2)

$$\sigma(b) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \mathrm{d}^2 s_q \mathrm{d}^2 s'_q \mathrm{d}^2 s_d \mathrm{d}^2 s'_d D(\mathbf{s}_q, \mathbf{s}_d) D(\mathbf{s}'_q, \mathbf{s}'_d) \, , \sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b}) \, .$$

$$D\left(\mathbf{s}_q, \mathbf{s}_d\right) = \frac{1+\lambda^2}{R_{qd}^2 \pi} e^{-(s_q^2 + s_d^2)/R_{qd}^2} \delta^2(\mathbf{s}_d + \lambda\, \mathbf{s}_q), \;\; \lambda = \frac{m_q}{m_d} \, ,$$

$$\sigma(\mathbf{s}_q, \mathbf{s}_d; \mathbf{s}'_q, \mathbf{s}'_d; \mathbf{b}) = 1 - \prod_{a,b \in \{q,d\}} \left[1 - \sigma_{ab}(\mathbf{b} + \mathbf{s}'_a - \mathbf{s}_b)\right]$$

$$\sigma_{ab}\left(\mathbf{s}\right) = A_{ab} e^{-s^2/R_{ab}^2}, \;\; R_{ab}^2 = R_a^2 + R_b^2, \;\; a, b \in \{q, d\}$$

$$\sigma_{qq} : \sigma_{qd} : \sigma_{dd} = 1 : 2 : 4$$

Bialas-Bzdak model is „realized”:
p = (q,d)
p= (q, (q,q))

ReBB model: two choices

$$\mathrm{Im}\,\Omega(s,b) = -\alpha \cdot \mathrm{Re}\,\Omega(s,b)\,.$$

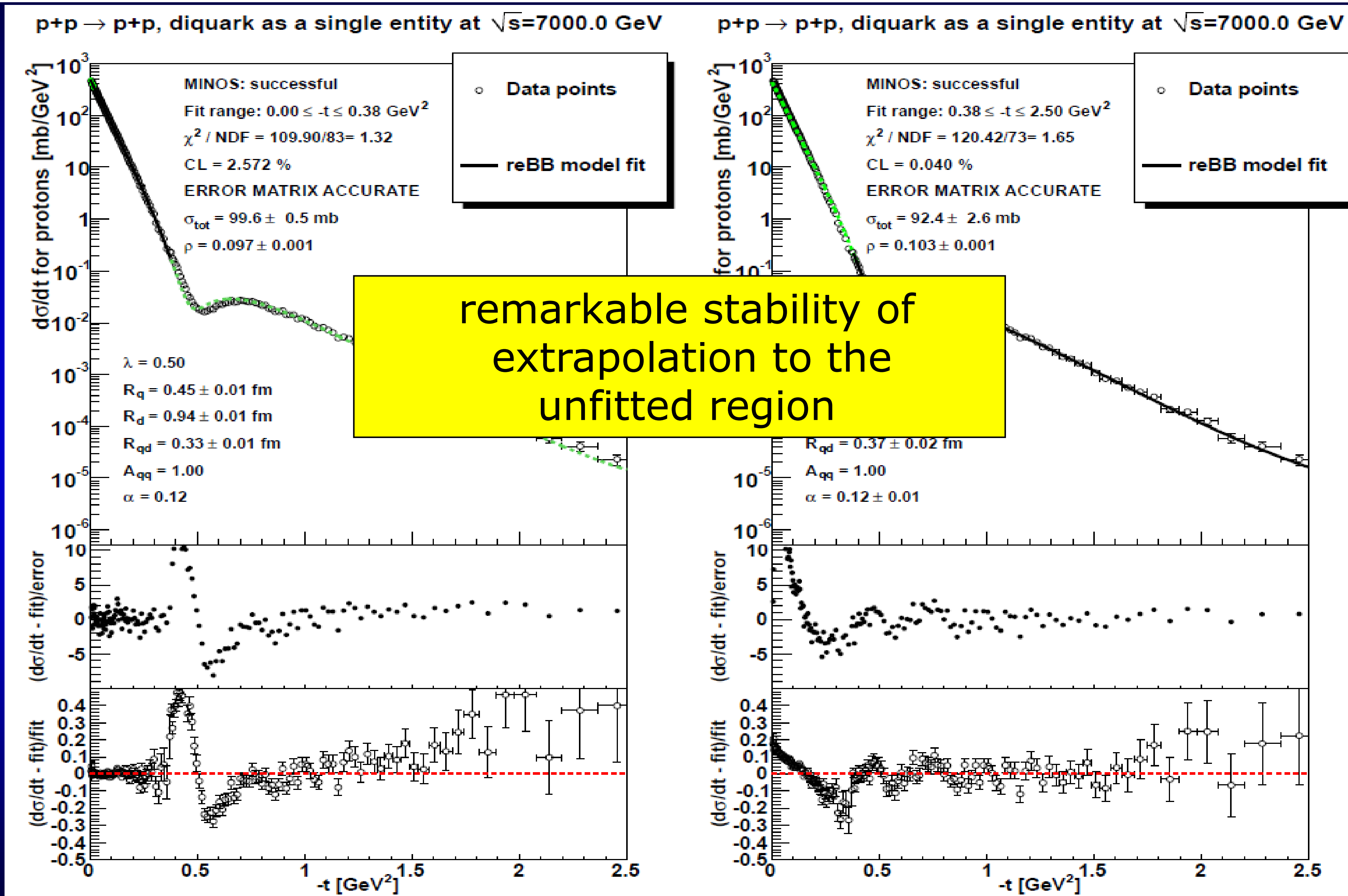
Similar to a constant ρ but not favored by data

$$\mathrm{Im}\,\Omega(s,b) = -\alpha \cdot \tilde{\sigma}_{inel}(s,b)\,,$$

This choice is also favoured by data
T. Cs., F. Nemes, arxiv:1306.4217

For small values of α we recover our first attempt, the αBB model

ReBB model, final fit range studies

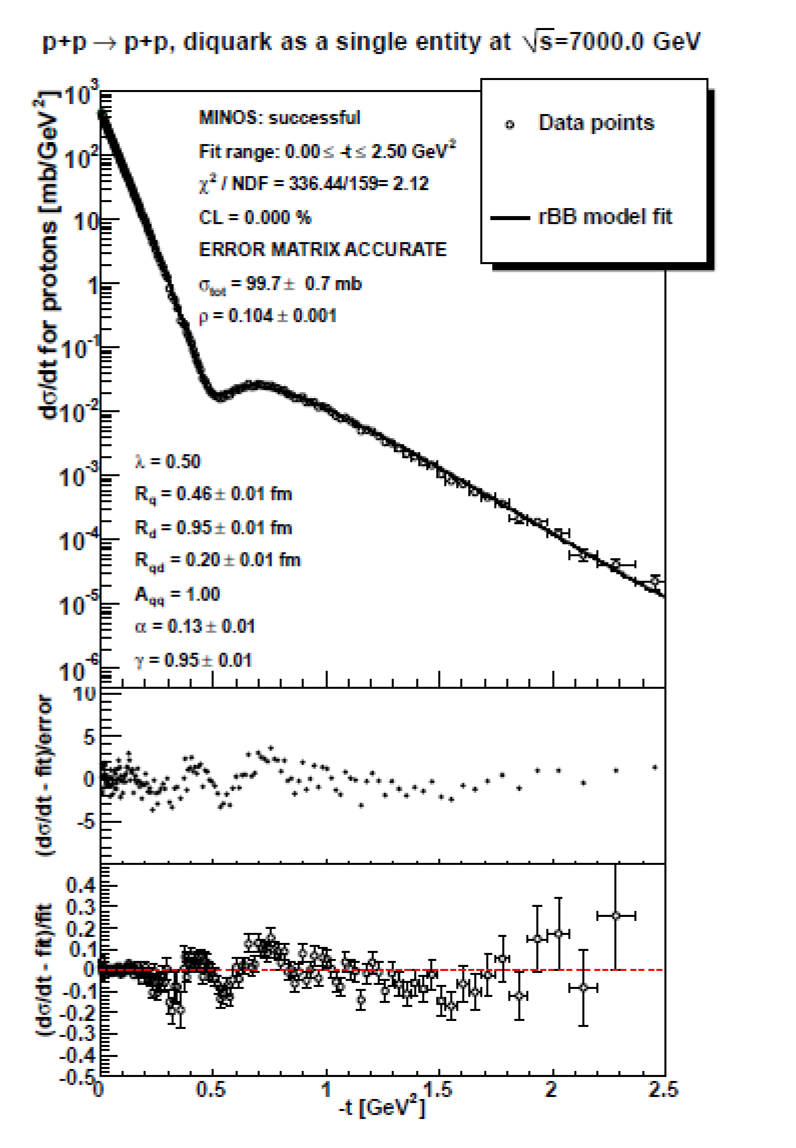


remarkable stability of extrapolation to the unfitted region

fit: $0.36 \leq -t \leq 2.5$ GeV^2, OK

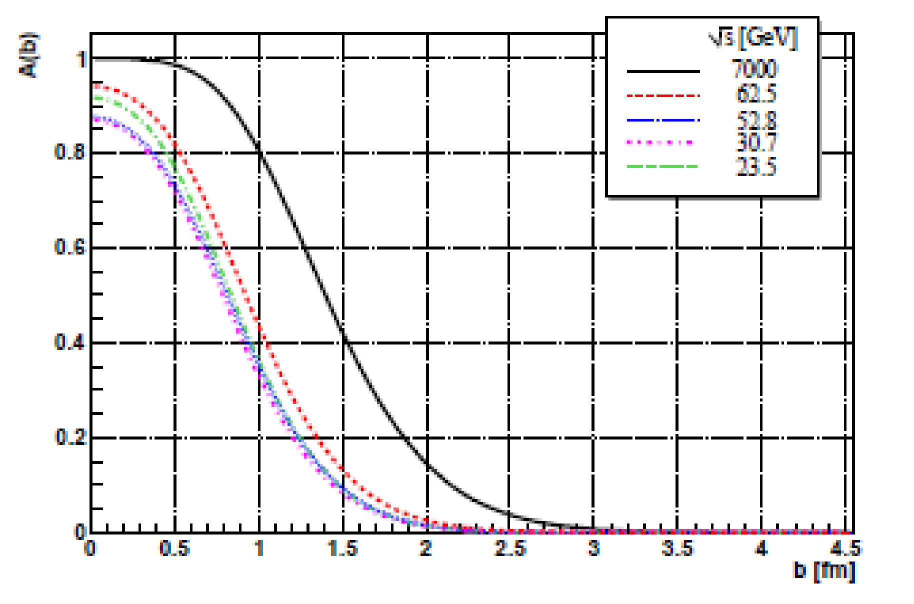
fit: $0 \leq -t \leq 2.5$ GeV^2, ~ OK

ReBB model, combined data sets



Shadow profile function

$$A(s,b) = 1 - \left|\exp\left[-\Omega(s,b)\right]\right|^2$$



$$\frac{d\sigma}{dt} \rightarrow \gamma \cdot \frac{d\sigma}{dt}$$

$$t_{sep} = -0.375\ \mathrm{GeV}^2$$

Fit range: $0 \le -t \le 2.5$ GeV², not quite OK → check @ 8 TeV

ReBB shadow profile functions

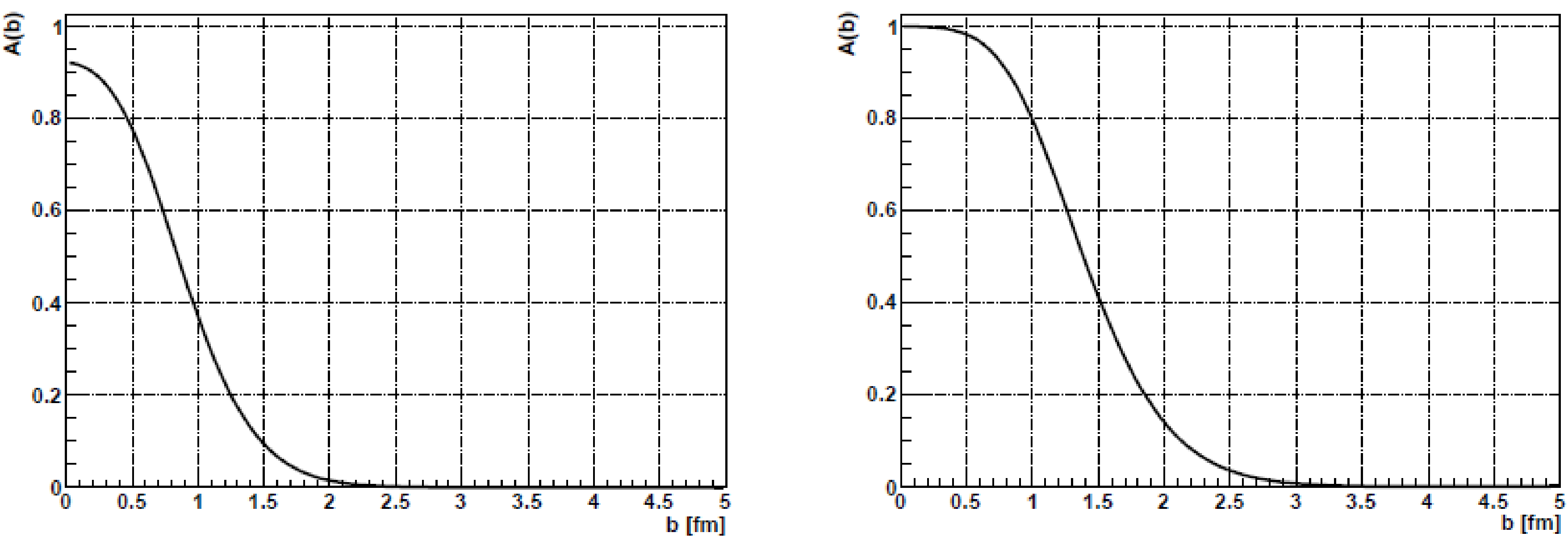
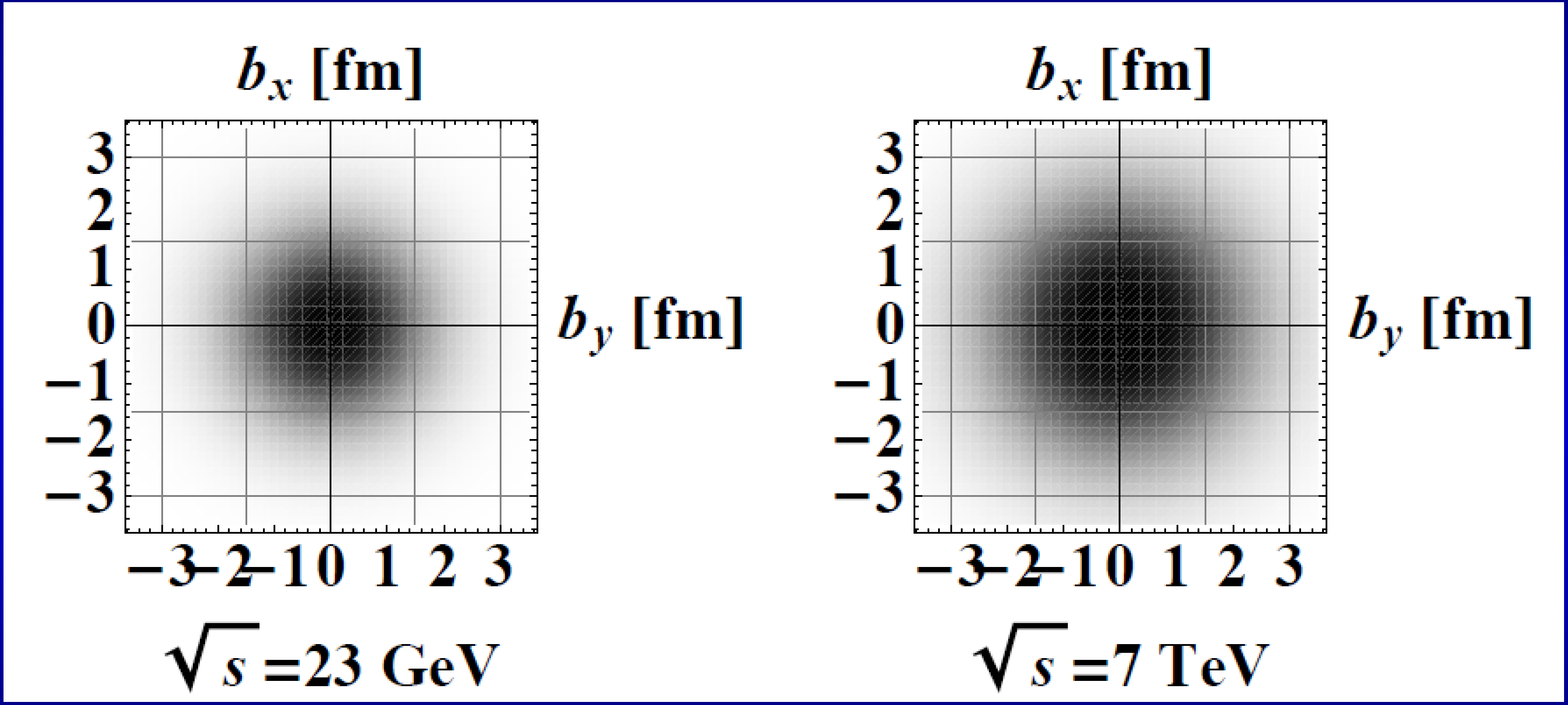


Figure 4: The $A(b) = 1 - |e^{-\Omega(b)}|^2$ shadow profile function. 23.5 GeV (left) and 7 TeV (right).

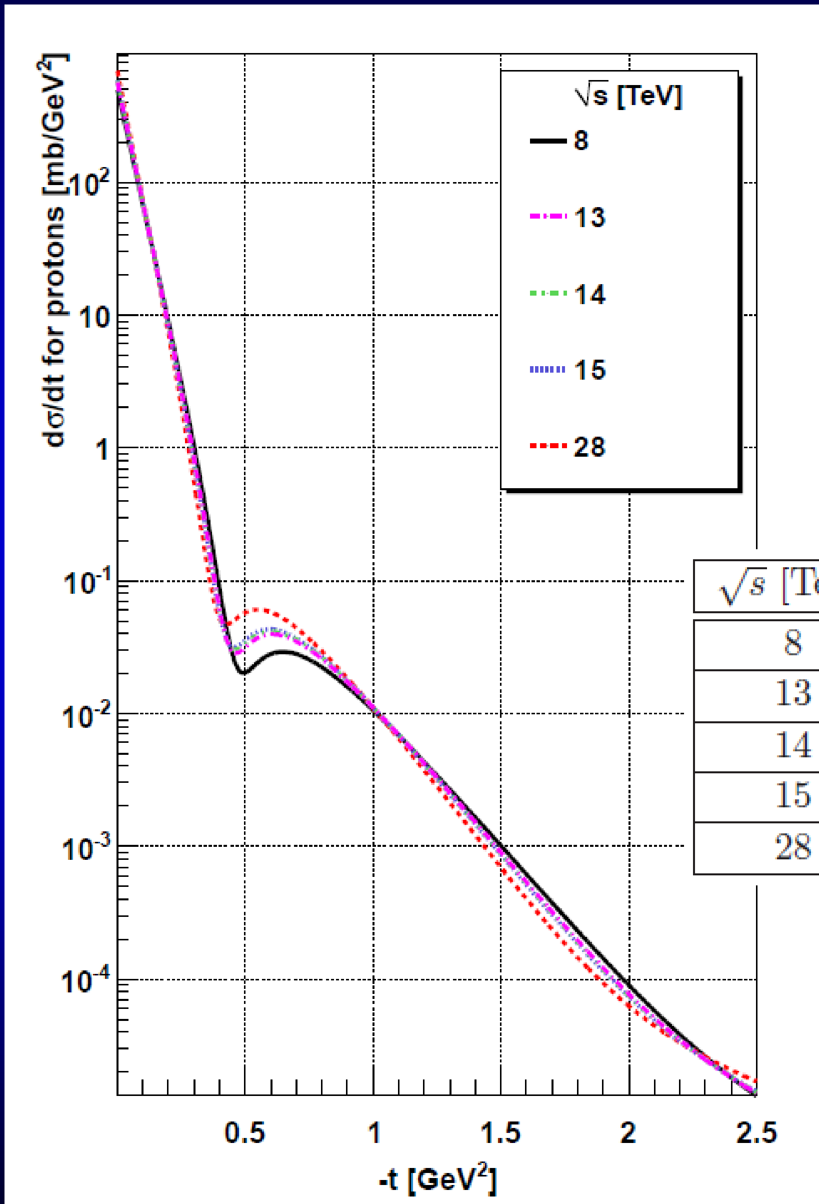
Indication of saturation at 7 TeV: A(b) ~ 1 at low b.
~ max probability of interaction at low b

Imaging on the sub-femtometer scale at 23 GeV ISR and 7 TeV LHC energy

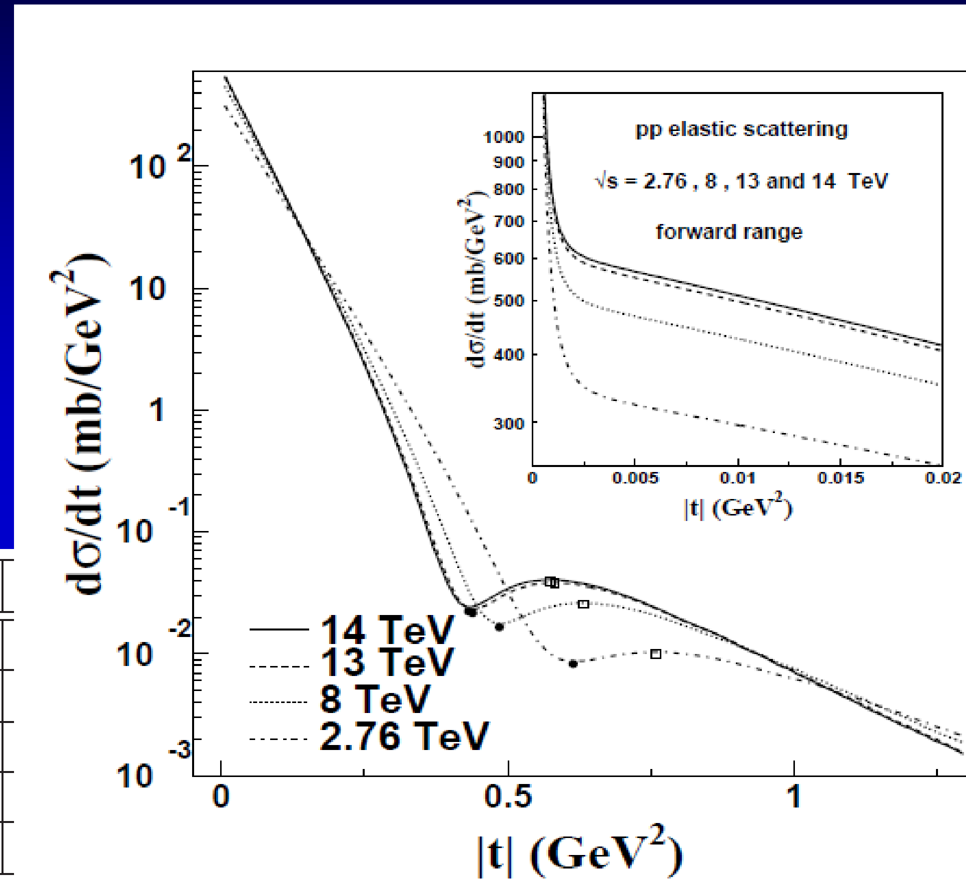


What about 8 TeV and future LHC energies?

Excitation function: dσ/dt



$\sqrt{s}$ [TeV]
8
13
14
15
28

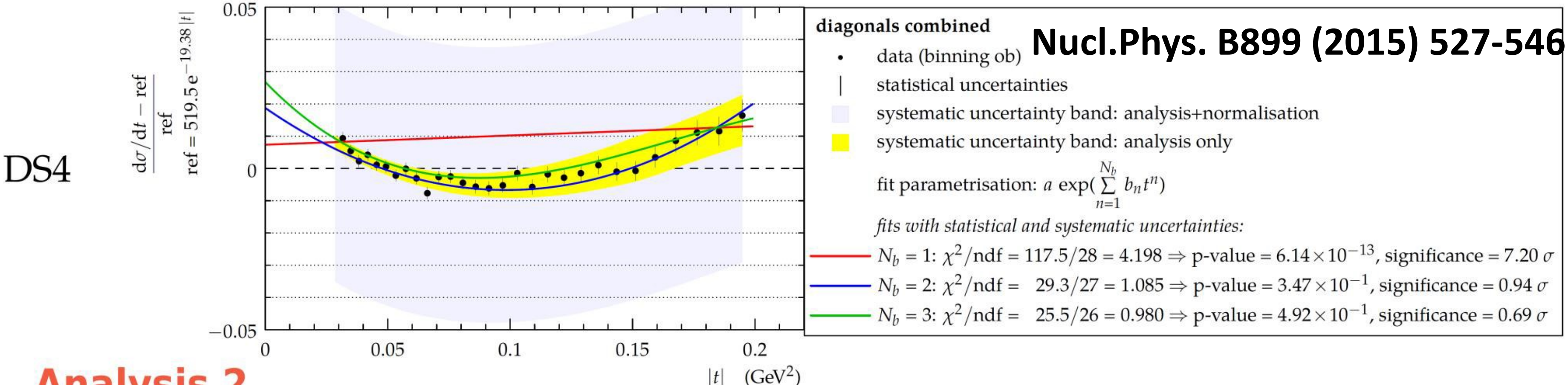


but $t_{dip}\ \sigma_{tot}$ ~ const (2 %)
Similar to:
K.A. Kohara, T. Kodama, E. Ferreira,
arXiv:1411.3518

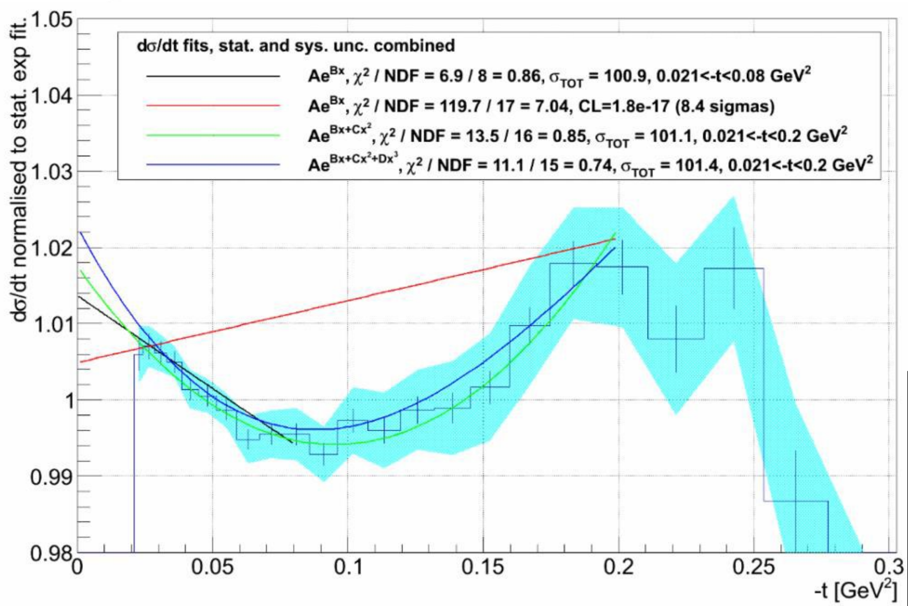
TOTEM 8 TeV pp data

Analysis 1: fits $A \exp(b_1 t + b_2 t^2 + \ldots)$, N_b parameters in exponent

Nucl.Phys. B899 (2015) 527-546



Analysis 2



TOTEM pp data at 8 TeV exponential shape excluded at 7+ σ

new determination
σ_{tot} = (101.4 ± 2.0) mb

Theoretical support, from ISR to LHC energies, unitarity
L. Jenkovszky and A. Lengyel,
arXiv:1410.4106

Non-exponential behaviour in ReBB

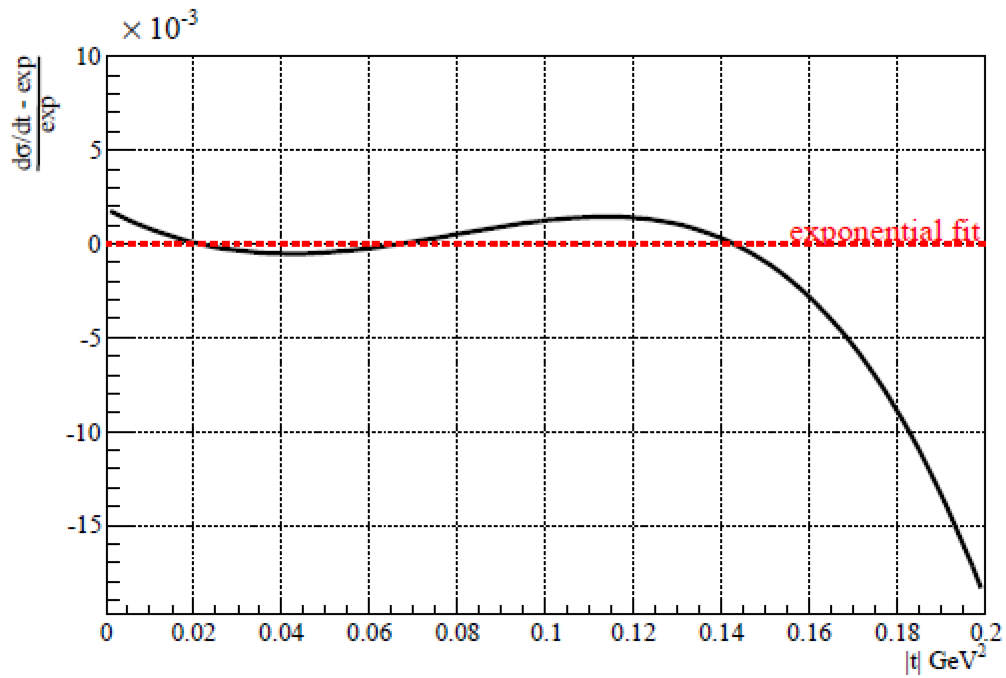
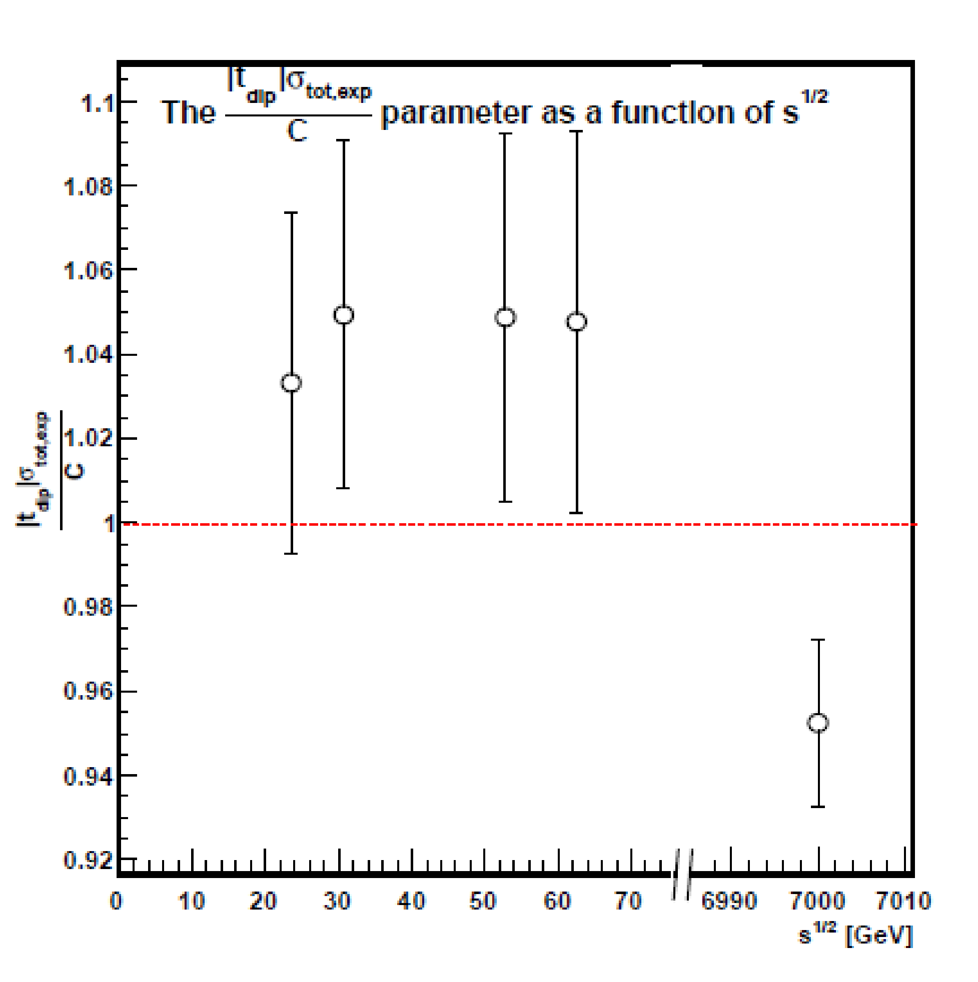


Fig. 5. The ReBB model, fitted in the $0.0 \leq |t| \leq 0.36$ GeV^2 range, with respect to the exponential fit of Eq. (33). In the plot only the $0.0 \leq |t| \leq 0.2$ GeV^2 range is shown. The curve indicates a significant deviation from the simple exponential at low $|t|$ values.

Similar non-exponential feature seen at 7 TeV as in 8 TeV TOTEM data

Black disc limit?



Geometric scaling,
but not the black disc limit:

T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

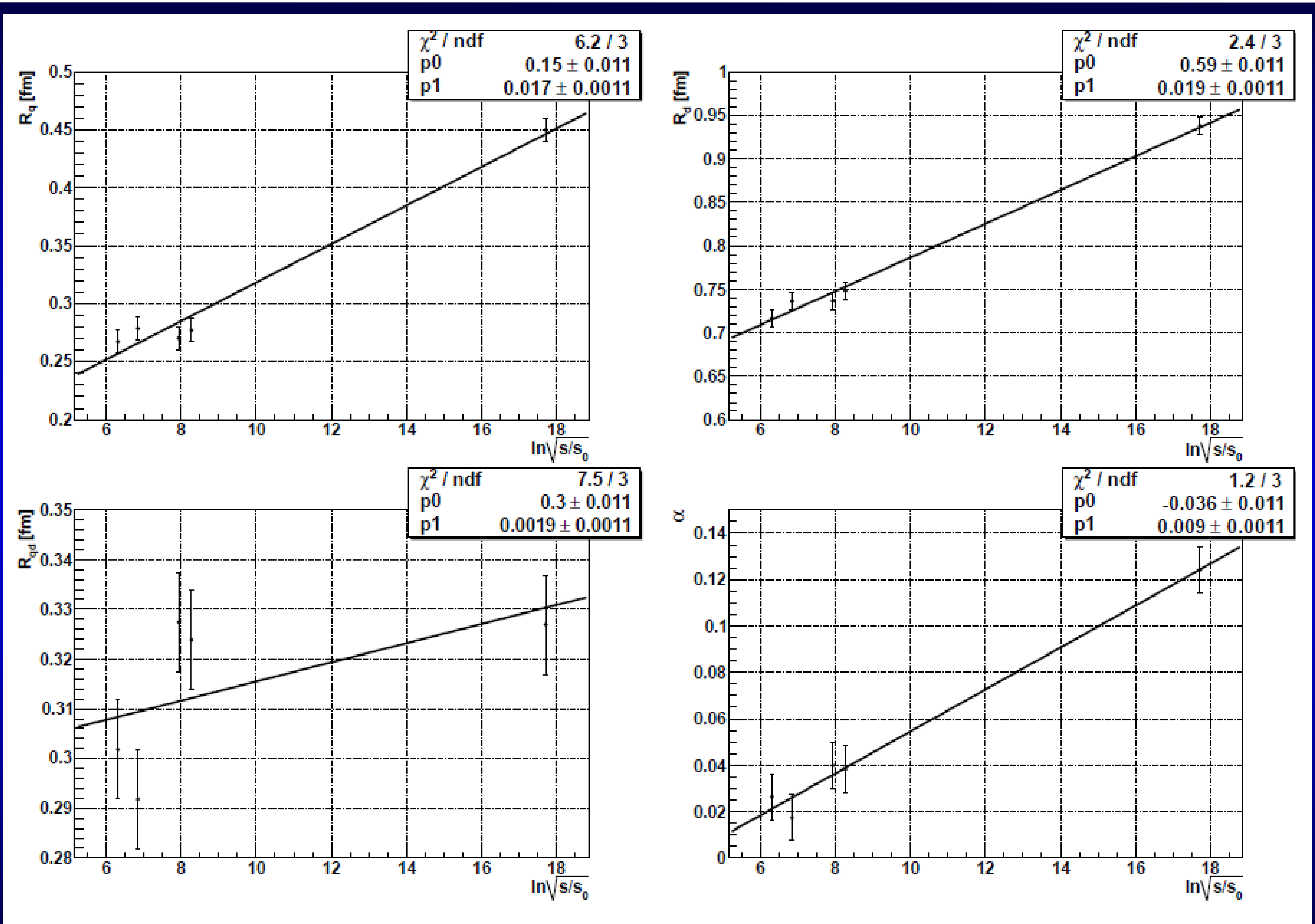
C(data) ~ 50 mb GeV^2
≠
C(black) ~ 36 mb GeV^2

$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR}\right]^2$$

$$\sigma_{tot,black} = 2\pi R^2 .$$

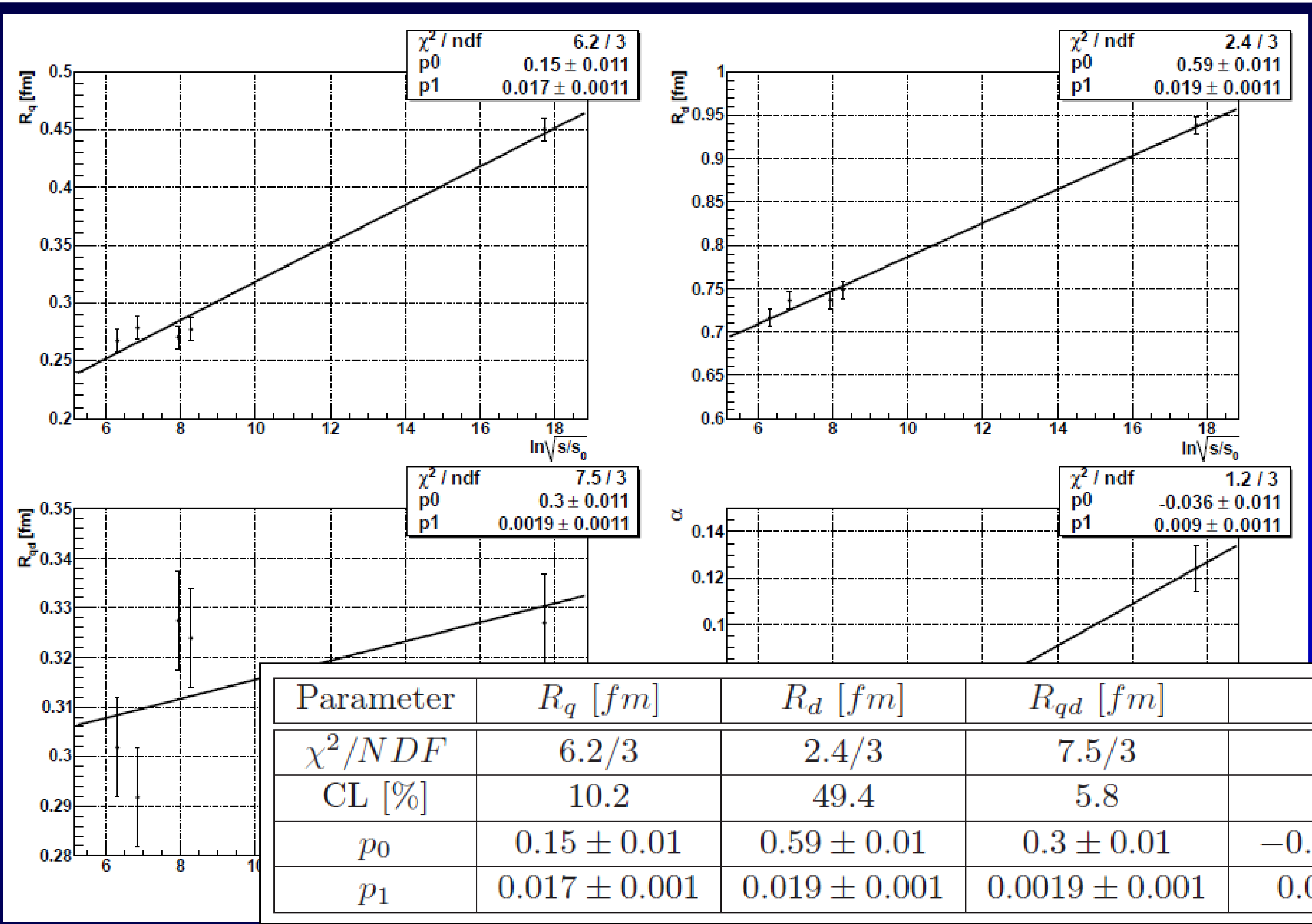
$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx\ 35.9\,\mathrm{mb\ GeV}^2$$

Excitation function: scaling in pp



Geometric scaling: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln(s/s_0)$

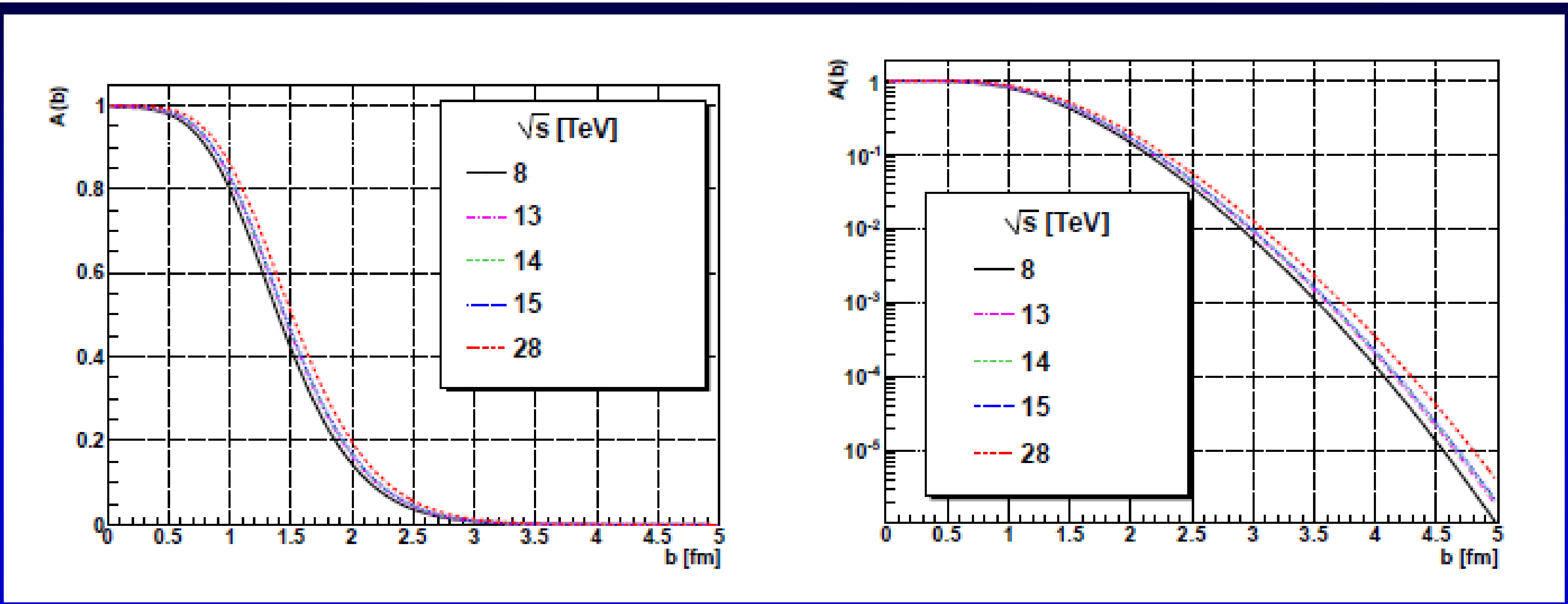
Geometric scaling in pp



Parameter	$R_q\ [fm]$	$R_d\ [fm]$	$R_{qd}\ [fm]$	α
χ^2/NDF	$6.2/3$	$2.4/3$	$7.5/3$	$1.2/3$
CL [%]	10.2	49.4	5.8	75.3
p_0	0.15 ± 0.01	0.59 ± 0.01	0.3 ± 0.01	-0.036 ± 0.01
p_1	0.017 ± 0.001	0.019 ± 0.001	0.0019 ± 0.001	0.009 ± 0.001

Geometric scaling: $\{R_q, R_d, R_{qd}, \alpha\} = p_0 + p_1 \ln(s/s_0)$

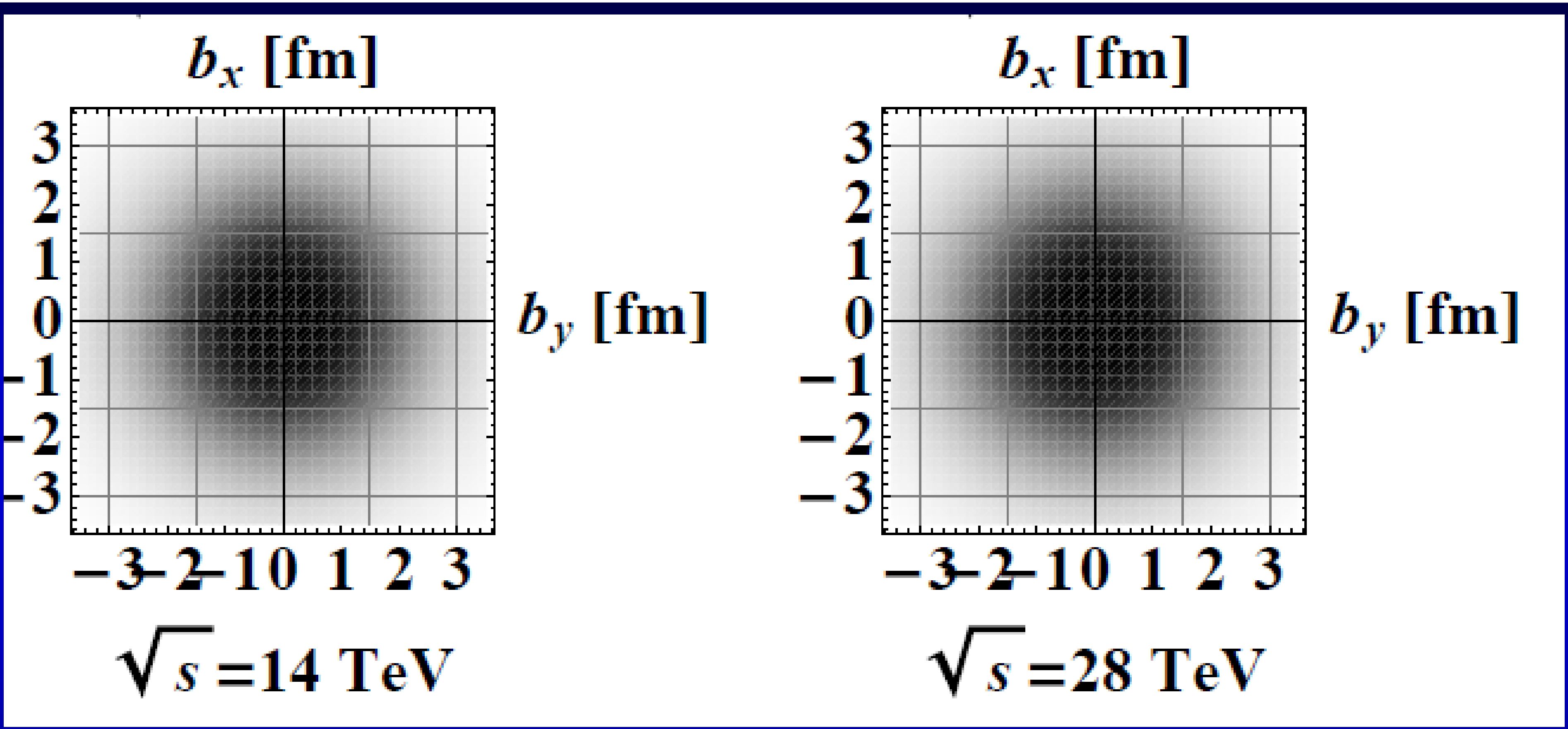
Predictions for the shadow profile



Blacker and Larger, but not Edgier: BnEL effect at LHC energies

Similar to: K.A. Kohara, T. Kodama, E. Ferreira, arXiv:1411.3518 but they also claim an asymptotic BEL effect

Predictions for the shadow profile

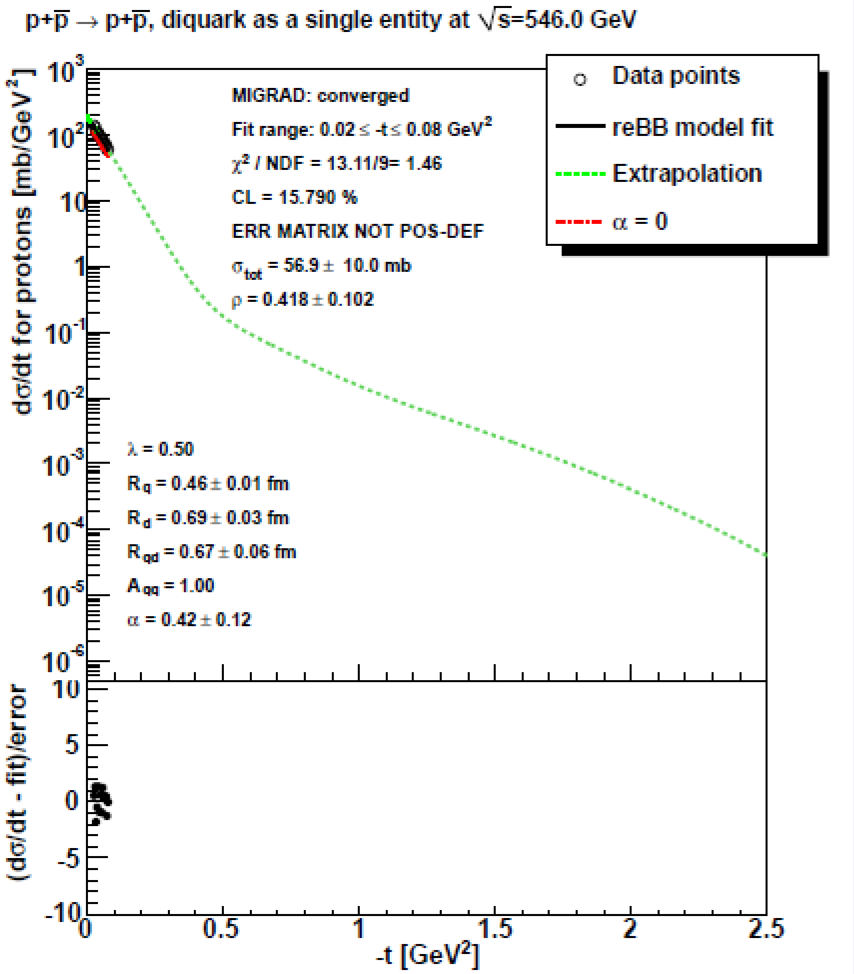


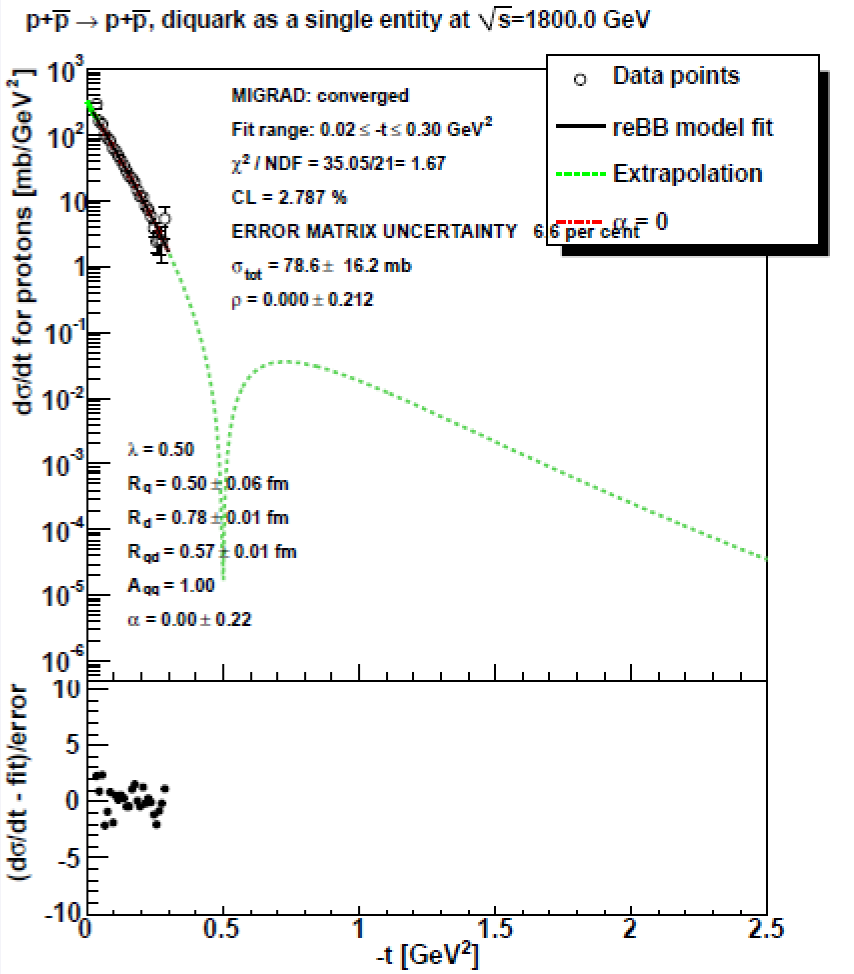
Blacker and Larger,
but not Edgier:
BnEL effect
at LHC energies

Results presented so far:
arxiv:1505.01415

New results: $p\bar{p}$ data with ReBB model

All the usual BB fit parameters are free ($\sqrt{s} = 546$ GeV, 1.8 TeV)

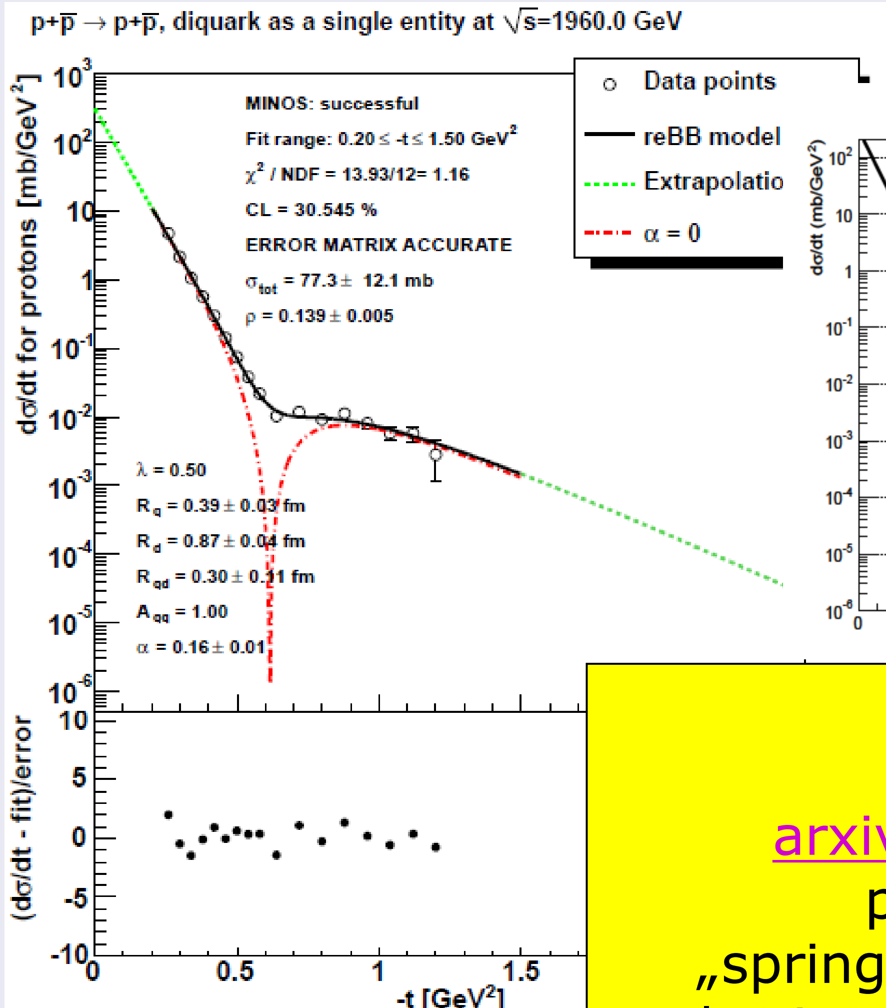


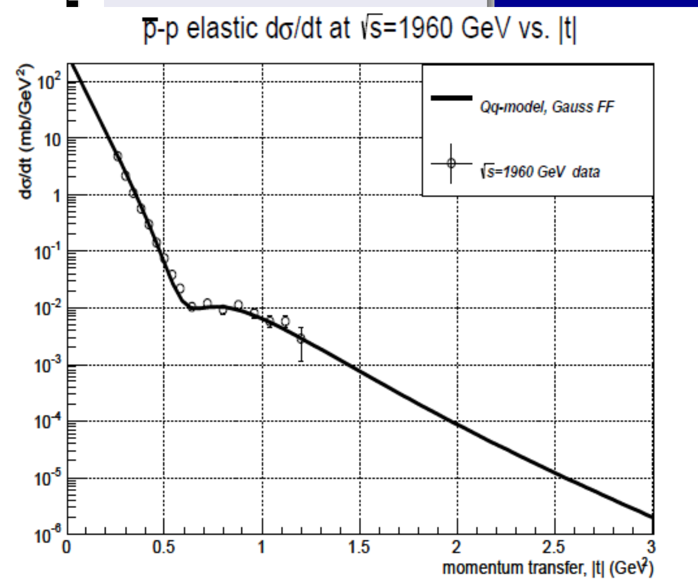


There are not enough points to pin down the shape.

Tevatron $p\bar{p}$ data with ReBB model

All the usual BB fit parameters are free (1.96 TeV)



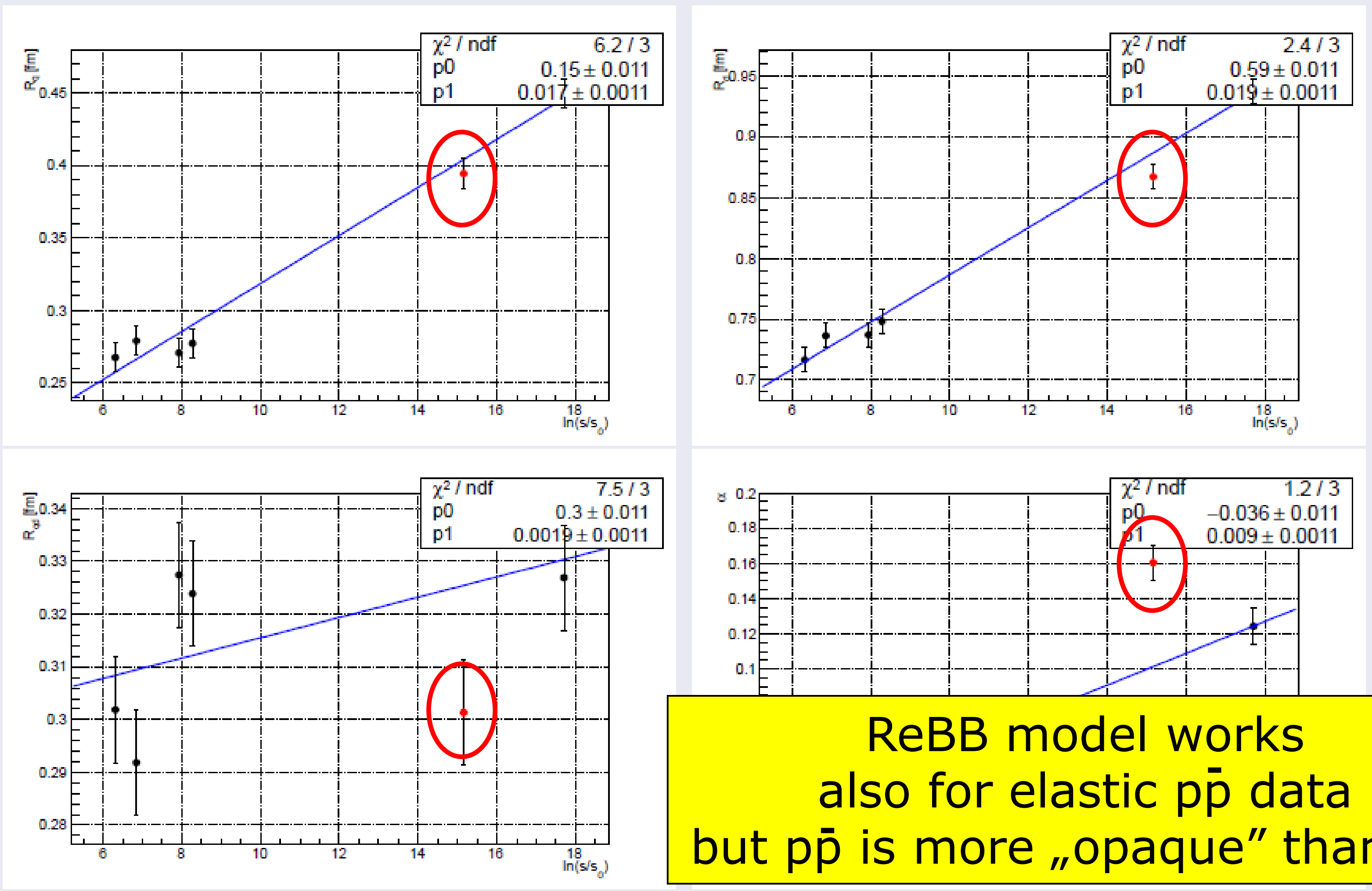


Similar to:,
V. Grichine,
arxiv.org:1404.5768
p = (q, d) in a
„springy” Pomeron picture
also increases real part of t_{el}

Ok.

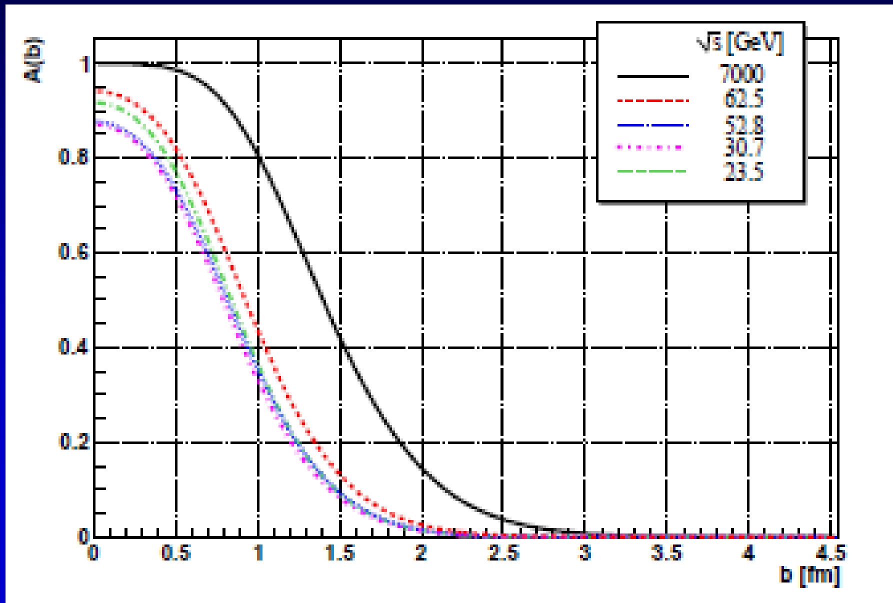
Tevatron $p\bar{p}$ data trends ReBB model

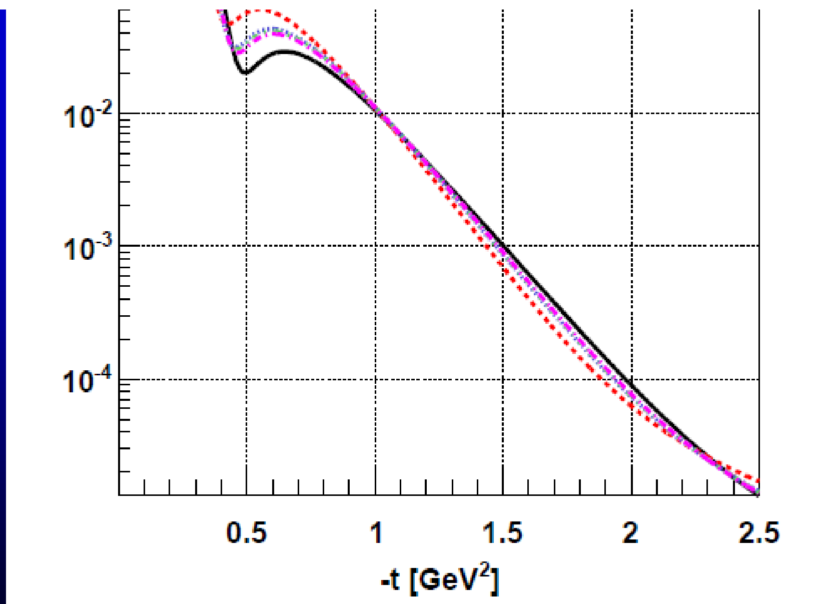
The good fit at $\sqrt{s} = 1.96$ TeV compared with the extrapolations based only on *pp* fits of our ReBB paper



ReBB model works
also for elastic $p\bar{p}$ data
but $p\bar{p}$ is more „opaque" than pp.

What have we learned?





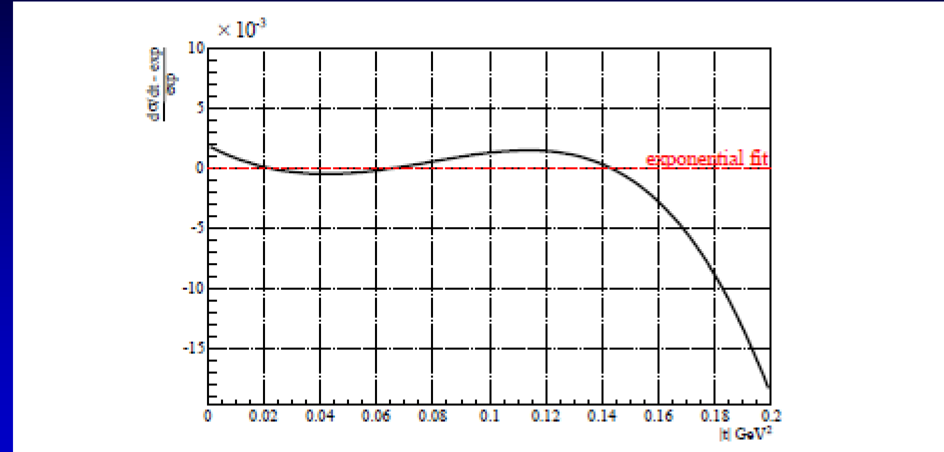


FIG. 5. The rBB model, fitted in the $0.0 \le |t| \le 0.36$ GeV^2 range, with respect to the exponential

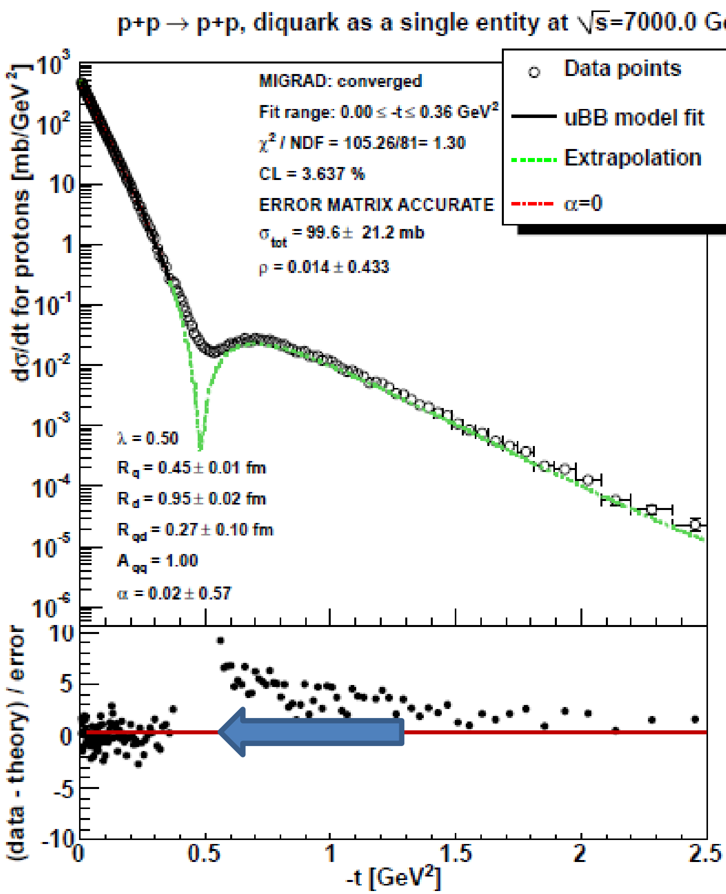
Really extended Bialas-Bzdak:
p = (q, d) at LHC
non-trivial structure at low-t
describes both pp and pp

BnEL: Blacker, ~~Edgier,~~ Larger

ReBB model works naturally
also for elastic p$\bar{\text{p}}$ data
but antiproton is more „opaque”.

Backup slides – Questions?

Focusing reBB on the low-t region



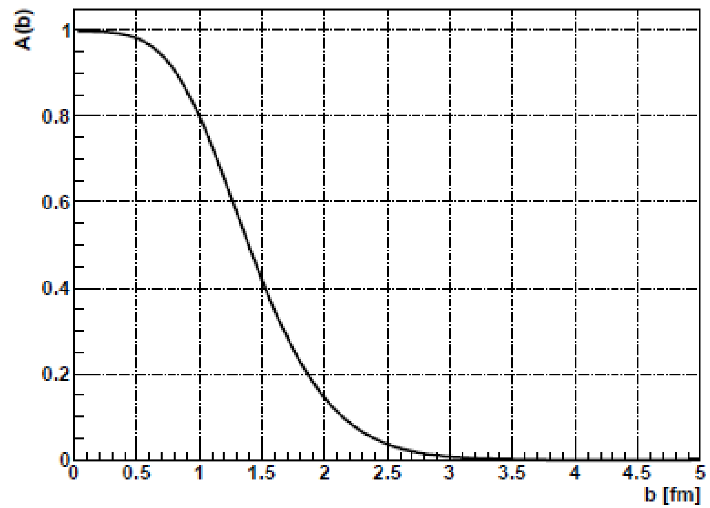


Figure 5: 0 - 0.36 GeV^2. ρ has large error, since the dip is not part of the fit.

Saturation is apparent if fit range is limited to |t| < 0.36 GeV²

Focusing reBB on even lower -t region

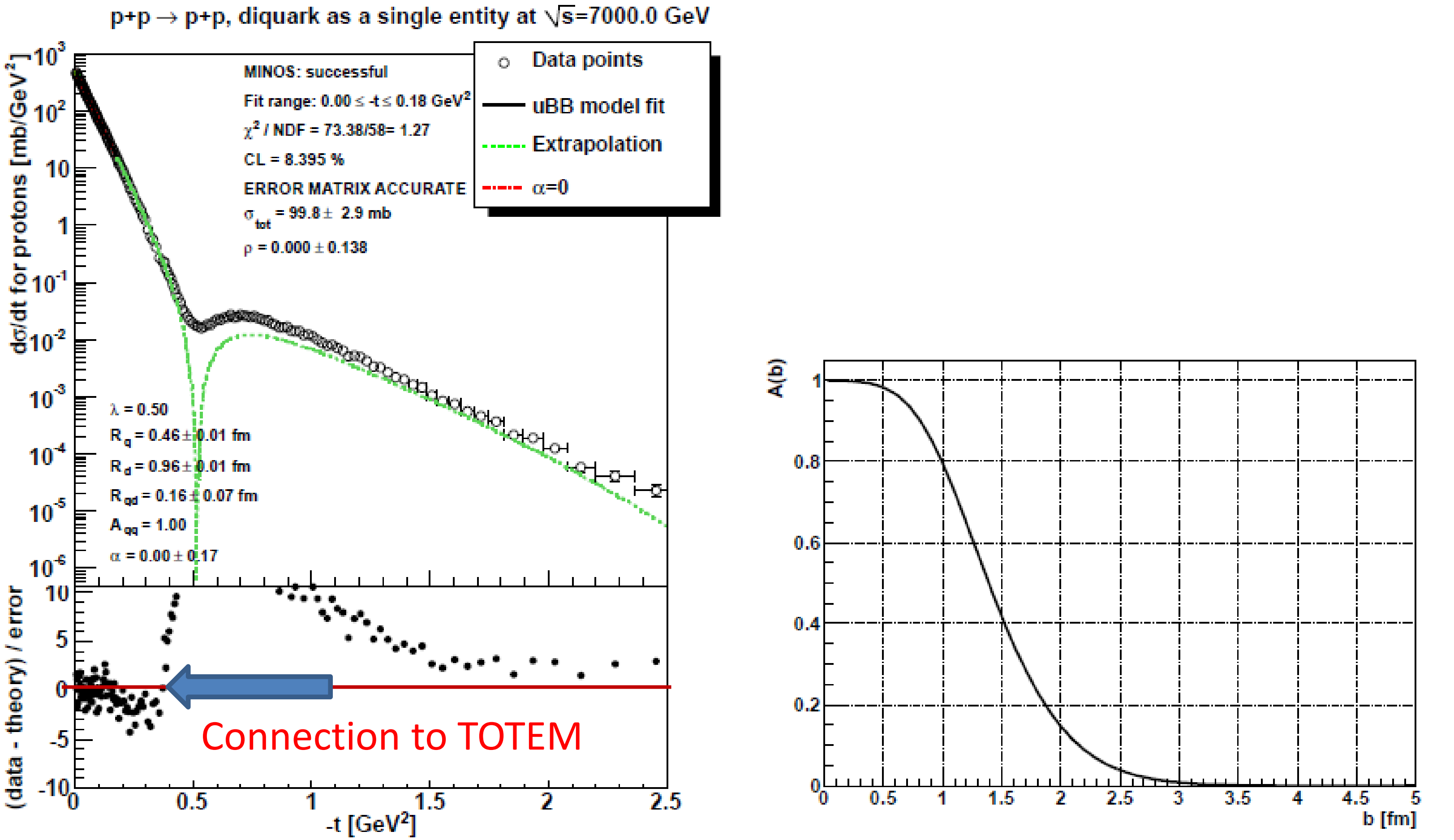
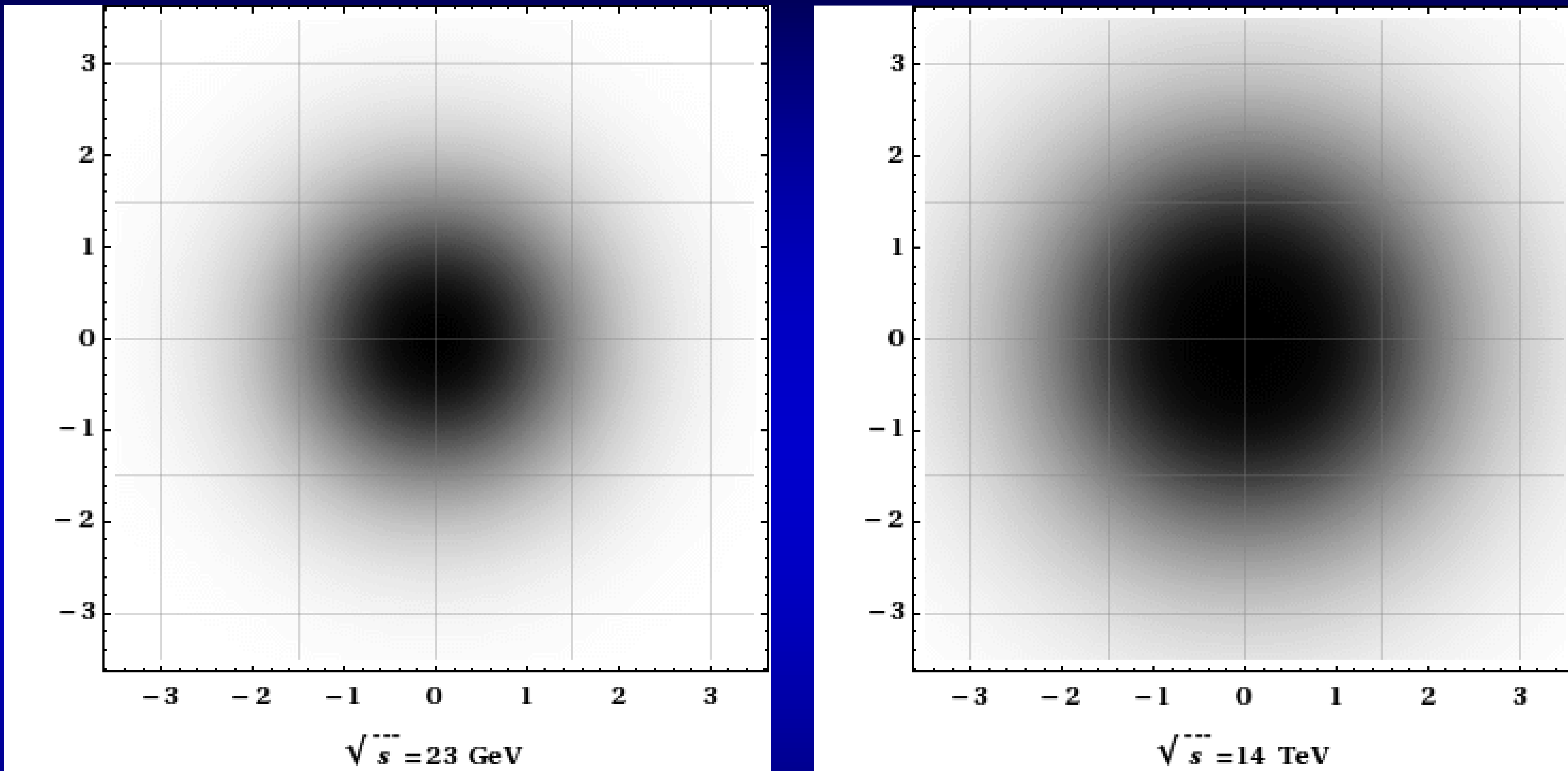


Figure 6: 0 - 0.18 GeV^2. ρ has large error, since the dip is not part of the fit.

Saturation still apparent, fit range $|t| < 0.18$ GeV^2

Backup slides – Discussion

Motivation: Is the proton a black disc?

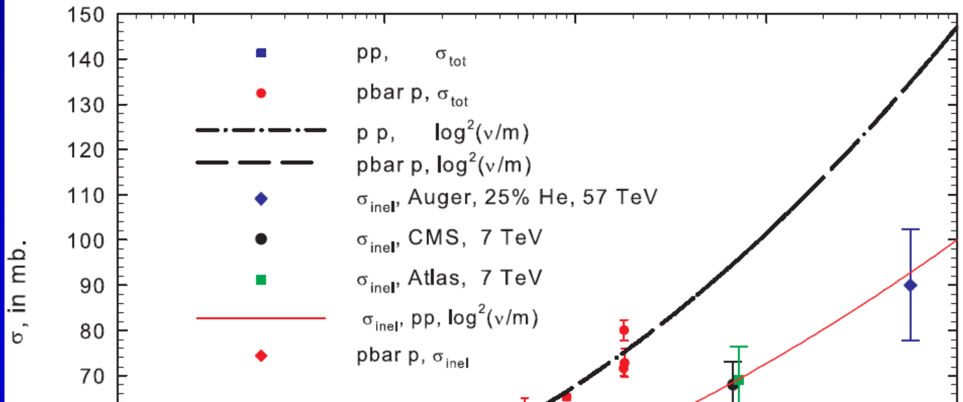


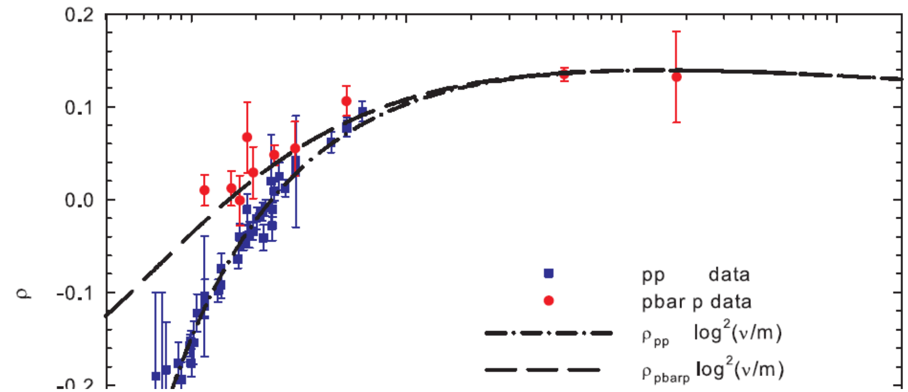
Recent papers by M. Block and F. Halsen address this topic :
Experimental confirmation: the proton is asymptotically a black disc,
arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002

Properties of a black disc

Properties of a black disk: In impact parameter space b, the elastic and total cross sections are given by

$$\sigma_{\rm el} = 4\int d^2b\, |a(b,s)|^2, \qquad \sigma_{\rm tot} = 4\int d^2b\, {\rm Im}\, a(b,s). \tag{7}$$





Conclusions: We find that the $\ln^2 s$ Froissart bound for the proton for $\sigma_{\rm tot}$ [7] and $\sigma_{\rm inel}$ [9] is saturated and that at infinite s, (1) the experimental ratio $\sigma_{\rm inel}/\sigma_{\rm tot} = 0.509 \pm 0.011$, compatible with the black disk ratio of 0.5 and (2) the forward scattering amplitude is purely imaginary. We thus conclude that the proton becomes an expanding black disk at sufficiently ultra-high energies that are probably never accessible to experiment. The theory for these bounds is predicated on the pillar stones of analyticity and unitarity, which have now been experimentally verified up to 57000 GeV. Further, since $\sigma_{\rm tot}$ has been extrapolated up from the Tevatron, we expect no new large cross section physics between 2000 and 57000 GeV.

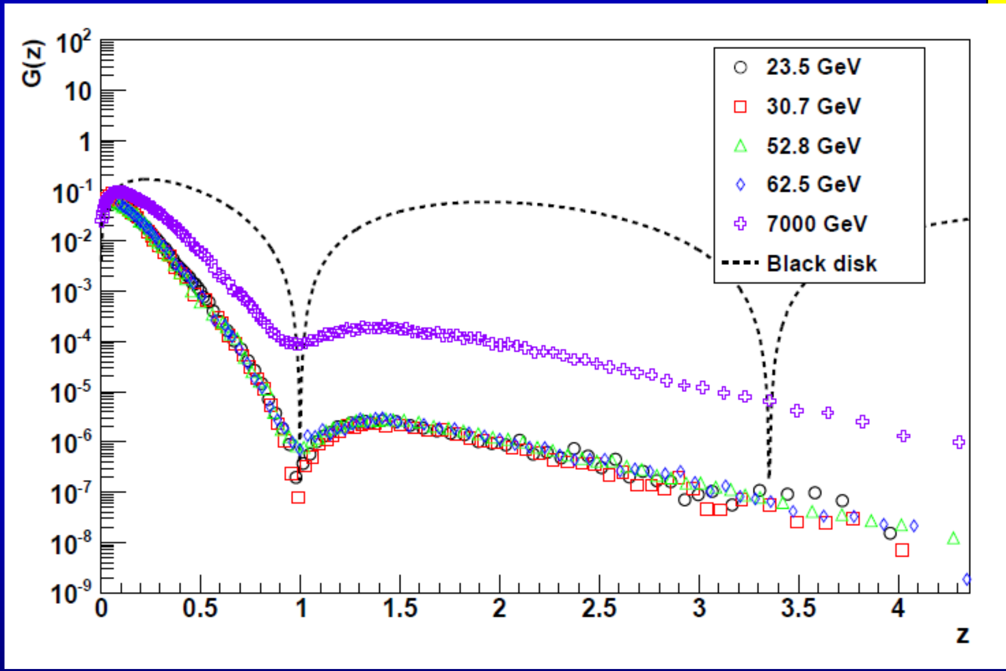
Finally, the lowest-lying glueball mass is measured to be $M_{\rm glueball} = 2.97 \pm 0.03$ GeV. Reproducing these experimental results will be a task of lattice QCD.

arXiv:1109.2041, Phys. Rev. Lett. 107 (2011) 212002
arXiv:1208.4086, Phys.Rev. D86 (2012) 051504
arXiv:1409.3196

Black Disc (BD) limit?

Scaling but
not in the black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
Int. J. Mod. Phys. A (2014)

C(data) ~ 50 mb GeV²
≠
C(black) ~ 36 mb GeV²

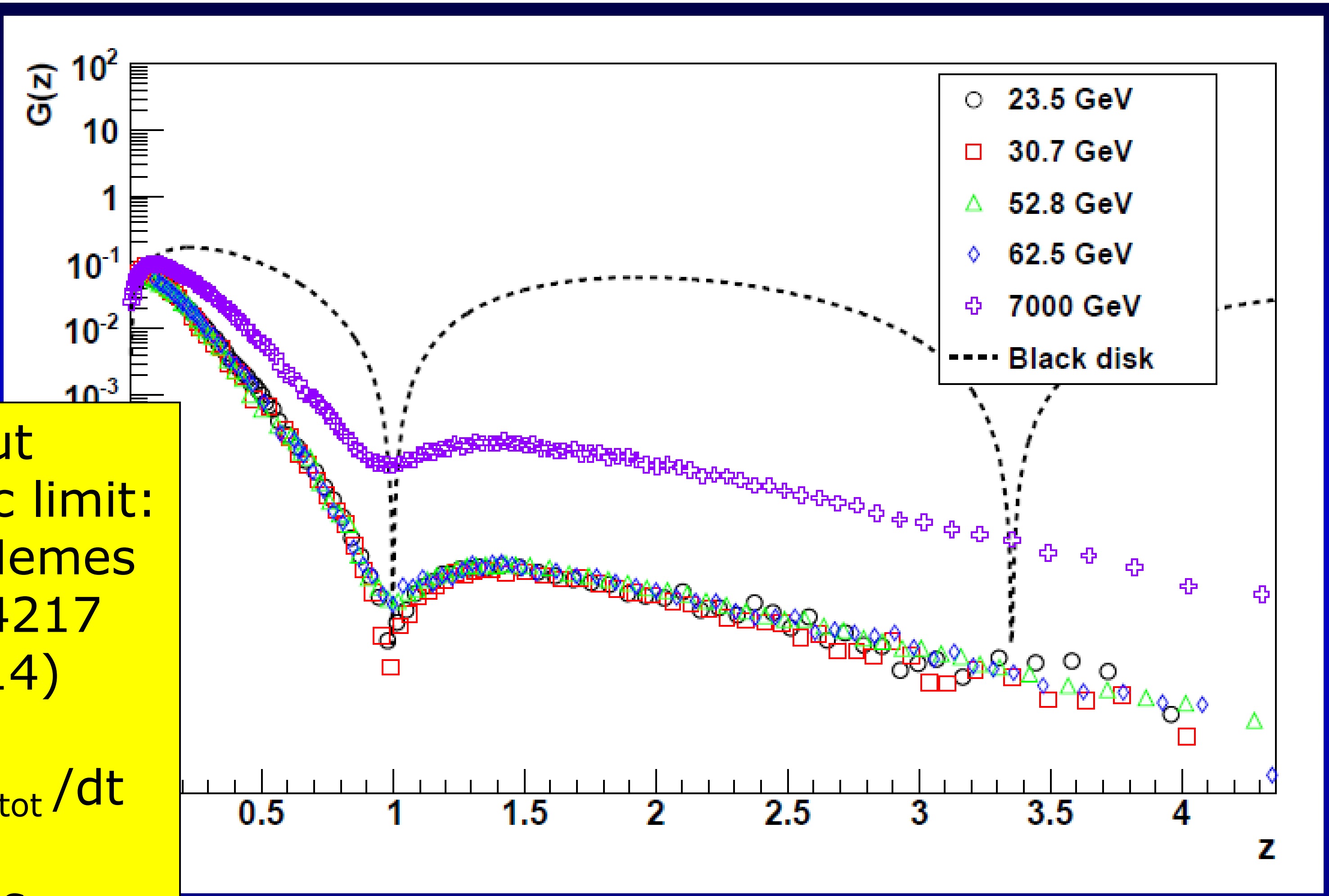


$$\frac{d\sigma_{black}}{dt} = \pi R^4 \left[\frac{J_1(qR)}{qR}\right]^2$$

$$\sigma_{tot,black} = 2\pi R^2 \,.$$

$$C_{black} = |t_{dip,black}| \cdot \sigma_{tot,black} = 2\pi j_{1,1}^2 (\hbar c)^2 \approx \; 35.9\,\mathrm{mb\;GeV}^2$$

Geometric scaling, but not BD limit?



Scaling but
not a black disc limit:
T. Cs. and F. Nemes
arXiv:1306.4217
IJMPA (2014)

$G(z) = t\, d\sigma/\sigma_{tot}/dt$

plotted vs

$z = t/t_{dip}$