BFKL resummation effects in di-hadron production at LHC

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Outline

Outline

Di-hadron production

Process: $\mathrm{proton}(\mathrm{p}_1) + \mathrm{proton}(\mathrm{p}_2) \to \mathrm{h}_1(\mathrm{k}_1) + \mathrm{h}_2(\mathrm{k}_2) + \mathrm{X}$ *...LHC physics!*

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}(x_1 x_2 s,\mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$

- ◇ *large hadron transverse momenta:* $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\mathrm{QCD}} \Rightarrow$ pQCD allowed
- ◇ *large rapidity gap between hadrons (high energies)* $\Rightarrow \Delta y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$

  $\Rightarrow$ *BFKL resummation:* $\sum_n \left(a_n^{(0)} \; \alpha_s^n \ln^n s + a_n^{(1)} \; \alpha_s^n \ln^{n-1} s \right)$

- *Collinear fragmentation of the parton i into a hadron h*
  $\Rightarrow$ *convolution of* D_i^h *with a coefficient function* C_i^h

$$d\sigma_i = C_i^h(z) dz \to d\sigma^h = d\alpha_h \int_{\alpha_h}^1 \frac{dz}{z} D_i^h\left(\frac{\alpha_h}{z},\mu\right) C_i^h(z,\mu)$$

where α_h *is the momentum fraction carried by the hadron*

Outline

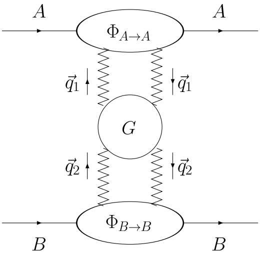
The BFKL resummation

pQCD, total cross section for $A+B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$ $\Leftarrow$ **optical theorem**

- ◇ Pomeron channel: $t = 0$ + singlet colour representation in the t-channel
- ◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

- **BFKL resummation**:

  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



- ▶ $\mathrm{Im}_s\left(\mathcal{A}_{AB}^{AB}\right)$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles.

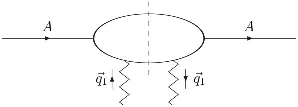
$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent**

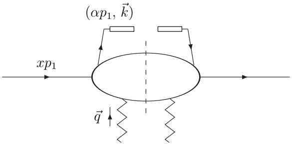
$\longrightarrow$ determined through the **BFKL equation**
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent**

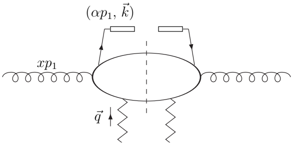
$\longrightarrow$ known in the NLA just for few processes



* forward identified hadron production



quark jet vertex



gluon jet vertex

[D.Yu. Ivanov, A. Papa (2012)]

Outline

The cross section

Azimuthal correlation momenta

$$\langle \cos\left[n\left(\phi_1-\phi_2-\pi\right)\right]\rangle \;\equiv\; \frac{\mathcal{C}_n}{\mathcal{C}_0}\,,$$

$$\mathcal{R}_{m,n}=\frac{\langle\cos\left(m\Delta\phi\right)\rangle}{\langle\cos\left(n\Delta\phi\right)\rangle}$$

$$\frac{d\sigma}{dx_1\,dx_2\;d|\vec{k}_1|\;d|\vec{k}_2|\,d\phi_1\,d\phi_2}=\frac{1}{(2\pi)^2}\left[\mathcal{C}_0+\sum_{n=1}^{\infty}2\cos(n\phi)\,\mathcal{C}_n\right]$$

...useful definitions:

$$Y \;=\; \ln\frac{x_1x_2s}{|\vec{k}_1||\vec{k}_2|}\,,\qquad Y_0 \;=\; \ln\frac{s_0}{|\vec{k}_1||\vec{k}_2|}$$

BFKL cross section

$$\mathcal{C}_n \equiv \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \cos[n(\phi_1 - \phi_2 - \pi)] \frac{d\sigma}{dy_1 dy_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\phi_1 d\phi_2}$$

$$= \frac{e^Y}{s} \int_{-\infty}^{+\infty} d\nu \left(\frac{\alpha_1 \alpha_2 s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\left[\chi(n,\nu)+\bar{\alpha}_s(\mu_R)\left(\bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu)+\frac{10}{3}+i\frac{d\ln\frac{c_1(n,\nu)}{c_2(n,\nu)}}{d\nu}+2\ln\mu_R^2\right)\right)\right]}$$

$$\times \alpha_s^2(\mu_R) c_1(n,\nu,|\vec{k}_1|,\alpha_1) c_2(n,\nu,|\vec{k}_2|,\alpha_2)$$

$$\times \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1)}{c_1(n,\nu,|\vec{k}_1|,\alpha_1)} + \frac{c_2^{(1)}(n,\nu,|\vec{k}_2|,\alpha_2)}{c_2(n,\nu,|\vec{k}_2|,\alpha_2)}\right)\right] .$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$

$$K^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + \imath\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right)\right)$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa, (2014)]

Outline

BLM method

NLO BFKL corrections to C_0 with opposite sign with respect to the leading order (LO) result and large in absolute value.

- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part
  - ✓ $\overline{\text{MS}} \to$ MOM $\quad + \quad$ choose BLM scale $\quad + \quad$ MOM $\to \overline{\text{MS}}$

* "Exact" BLM:

suppress NLO IFs + NLO Kernel β_0-dependent factors

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, (2015)]

Outline

Observables and kinematics

- **Observables:**

$$\langle \cos\left[n\left(\phi_1 - \phi_2 - \pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}, \quad \text{with} \quad n = 1, 2, 3$$

$$\frac{\langle \cos\left[2\left(\pi - \Delta\phi\right)\right]\rangle}{\langle \cos\left(\pi - \Delta\phi\right)\rangle} = \frac{\mathcal{C}_2}{\mathcal{C}_1}, \quad \frac{\langle \cos\left[3\left(\pi - \Delta\phi\right)\right]\rangle}{\langle \cos\left[2\left(\pi - \Delta\phi\right)\right]\rangle} = \frac{\mathcal{C}_3}{\mathcal{C}_2}, \quad \text{with} \quad \Delta\phi = \phi_2 - \phi_1 .$$

◇ *Integrated coefficients:*

$$C_n = \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \int_{k_{1,\min}}^{\infty} dk_1 \int_{k_{2,\min}}^{\infty} dk_2 \delta\left(y_1 - y_2 - Y\right) \mathcal{C}_n\left(y_1, y_2, k_1, k_2\right)$$

- **Kinematic settings:**
  - ◇ $R = 0.5$ and $\sqrt{s} = 7, \ 13$ TeV
  - ◇ $-2.4 \le y_i \le 2.4$, with $i = 1, 2$
  - ◇ 5 GeV $\le k_1 \le$ 21 GeV
    5 GeV $\le k_2 \le$ 21 GeV
- **BFKL expansion order:**
  - ◇ LLA (NS, BLM)
  - ◇ LLA + NLA kernel (BLM)

Numerical stuff

- **Numerical tools:**

  FORTRAN → weak time dependence on multidim. integration ranges

  \+ NLO MSTW 2008 PDFs

  [A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, (2009)]

  \+ **three** different FF parametrizations!

  - **AKK**

    [S. Albino, B.A. Kniehl, G. Kramer, (2008)]

  - **DSS**

    [D. de Florian, R. Sassot, M. Stratmann, (2007)]

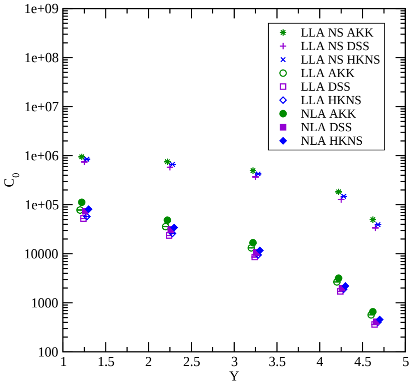
  - **HKNS**

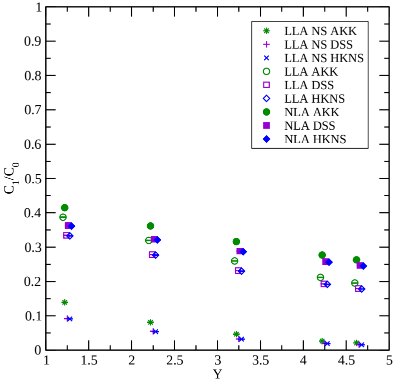
    [M. Hirai, S. Kumano, T.-H. Nagai, K. Sudoh, (2007)]

  \+ CERNLIB

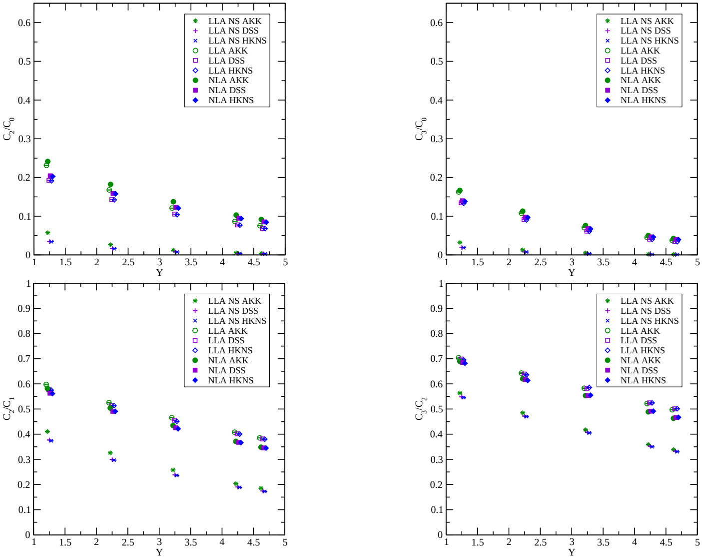
  http://cernlib.web.cern.ch/cernlib

C_0 and C_1/C_0 @ 13 TeV

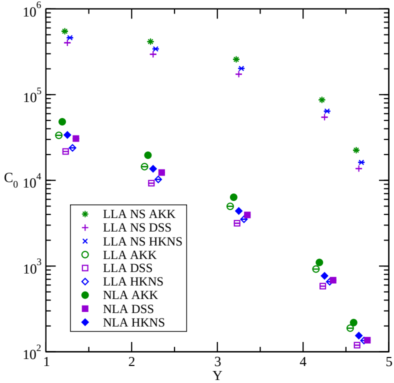


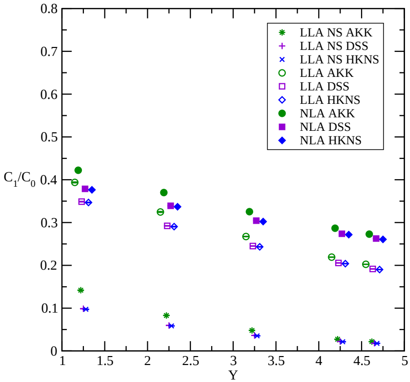


R_{nm} @ 13 TeV

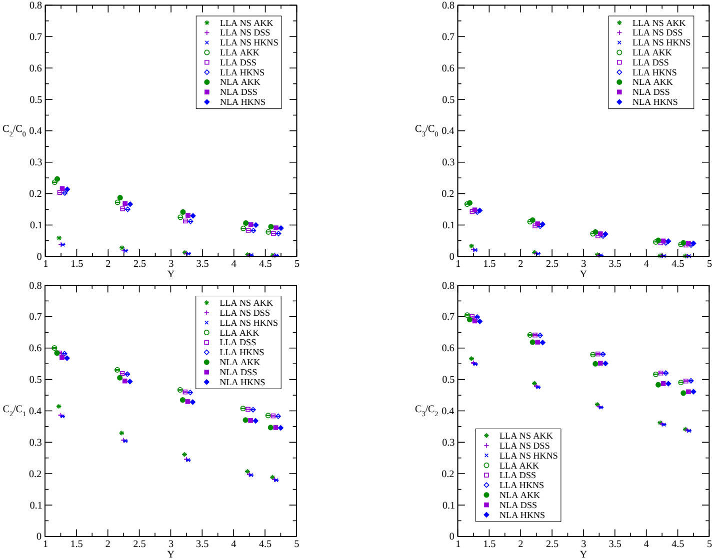

LLA NS AKK
LLA NS DSS
LLA NS HKNS
LLA AKK
LLA DSS
LLA HKNS
NLA AKK
NLA DSS
NLA HKNS
C2/C0
C3/C0
C2/C1
C3/C2
Y

C_0 and C_1/C_0 @ 7 TeV





R_{nm} @ 7 TeV


C2/C0
C3/C0
C2/C1
C3/C2
Y
LLA NS AKK
LLA NS DSS
LLA NS HKNS
LLA AKK
LLA DSS
LLA HKNS
NLA AKK
NLA DSS
NLA HKNS

Conclusions...

Comparison of predictions for C_0 and several R_{nm} in LLA and LLA + NLA kernel BFKL approach.

- Implementation of exact **BLM method**!
- @BLM scales $\Rightarrow$ good agreement between LLA and LLA + NLA kernel
- LLA @NS $\Rightarrow$ C_0 overestimated and ratios exhibit stronger decorrelation

$\Rightarrow$ **Reliability test of the BLM method!**

...Outlooks

...full NLA calculations $\Leftarrow$ NLO **impact factor**

Conclusions...

Comparison of predictions for C_0 and several R_{nm} in LLA and LLA + NLA kernel BFKL approach.

- Implementation of exact **BLM method**!
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...Outlooks

...full NLA calculations $\Leftarrow$ **NLO impact factor**

Thanks for your attention!!

BACKUP slides

The BFKL BLM cross section

$$C_n^{\rm BLM} = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \, \exp\left[(Y - Y_0)\, \bar\alpha_s^{\rm MOM}(\mu_R^*) \left\{ \chi(n,\nu) + \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(\bar\chi(n,\nu) + \frac{T^{\rm conf}}{C_A} \chi(n,\nu) \right) \right\} \right] 4 (\alpha_s^{\rm MOM}(\mu_R^*))^2 \frac{C_F}{C_A} \frac{1}{|\vec k_1||\vec k_2|} \left(\frac{\vec k_1^2}{\vec k_2^2} \right)^{i\nu}$$

$$\times \int_{\alpha_1}^1 \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1 + \bar\alpha_s^{\rm MOM}(\mu_R^*)\chi(n,\nu)} \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x}\right) + \sum_{a=q,\bar q} f_a(x) D_a^h\left(\frac{\alpha_1}{x}\right) \right]$$

$$\times \int_{\alpha_2}^1 \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1 + \bar\alpha_s^{\rm MOM}(\mu_R^*)\chi(n,\nu)} \left[\frac{C_A}{C_F} f_g(z) D_g^h\left(\frac{\alpha_2}{z}\right) + \sum_{a=q,\bar q} f_a(z) D_a^h\left(\frac{\alpha_2}{z}\right) \right]$$

$$\times \left[1 + \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(A(s_0) + 2 \frac{T^{\rm conf}}{C_A} \right) \right],$$

with the μ_R^* scale chosen as the solution of the following integral equation...

...choosing the μ_R^* scale

$$C_n^\beta = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \; \exp\left[(Y - Y_0)\, \bar{\alpha}_s^{\rm MOM}(\mu_R) \chi(n,\nu) \right]$$

$$\times 4 \left(\alpha_s^{\rm MOM} \left(\mu_R \right) \right)^3 \frac{C_F}{C_A} \frac{1}{|\vec{k}_1||\vec{k}_2|} \left(\frac{\vec{k}_1^2}{\vec{k}_2^2} \right)^{i\nu}$$

$$\times \int_{\alpha_1}^{1} \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1 + \bar{\alpha}_s^{\rm MOM}(\mu_R)\chi(n,\nu)} \left[\frac{C_A}{C_F} f_g(x) D_g^h \left(\frac{\alpha_1}{x} \right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h \left(\frac{\alpha_1}{x} \right) \right]$$

$$\times \int_{\alpha_2}^{1} \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1 + \bar{\alpha}_s^{\rm MOM}(\mu_R)\chi(n,\nu)} \left[\frac{C_A}{C_F} f_g(z) D_g^h \left(\frac{\alpha_2}{z} \right) + \sum_{a=q,\bar{q}} f_a(z) D_a^h \left(\frac{\alpha_2}{z} \right) \right]$$

$$\times \frac{\beta_0}{2 C_A} \left[\frac{5}{3} + \ln \frac{\mu_R^2}{k_1 k_2} + f\left(\nu\right) - 2\left(1 + \frac{2}{3} I\right) \right.$$

$$\left. + \, \bar{\alpha}_s^{\rm MOM}(\mu_R) \left(Y - Y_0 \right) \frac{\chi(n,\nu)}{2} \left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln \frac{\mu_R^2}{k_1 k_2} + f(\nu) - 2\left(1 + \frac{2}{3} I\right) \right) \right] = 0$$

...which represents the condition that terms proportional to β_0 in C_n disappear

$$\alpha^{\rm MOM} = -\frac{\pi}{2T}\left[1 - \sqrt{1 + 4\alpha_s\left(\mu_R\right)\frac{T}{\pi}}\right] ,$$

with $T = T^{\beta} + T^{\rm conf}$,

$$T^{\beta} = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right) ,$$

$$T^{\rm conf} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}\left(I - 1\right)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right] ,$$

where $I = -2\int_0^1 dx \frac{\ln(x)}{x^2 - x + 1} \simeq 2.3439$ and ξ is a gauge parameter.