CONFORMAL REGGE THEORY FROM STRING/GAUGE DUALITY AND INCLUSIVE CENTRAL PRODUCTION

Chung-I Tan, Brown University
Low-x Workshop, Sandomierz, Poland
Sept. 1-5, 2015

R. C. Brower, M. Djuric, I. Sarcevic and C-I Tan, "*String-Gauge Dual Description of Deep Inelastic Scattering at Small-x*", JHEP **1011**, 051 (2010), arXiv:1007.2259.

R. C. Brower, M. S. Costa, M. Djuric, T. Raben and C-I Tan, *"Strong Coupling Expansion for the Conformal Pomeron/Odderon Trajectories,"* JHEP **1502**, 104 (2015), arXiv:1312.1419.

R. Nally, T. Raben and C-I Tan, *"Central Inclusive Production from AdS/CFT,"* (to appear).

Outline

- **String/Gauge Duality:**
  - Unification and Universality
- Inclusive Central Production:
  - Universal features -Witten Diagram
  - Exclusive vs Inclusive: fixed-angle scattering/dim. counting rule
  - p_T-spectra for central production
- Conformal Regge Theory
  - Pomeron Spectral Curve in Strong Coupling
  - Pomeron and Odderon Intercepts in strong coupling

II. Unification and Universality:

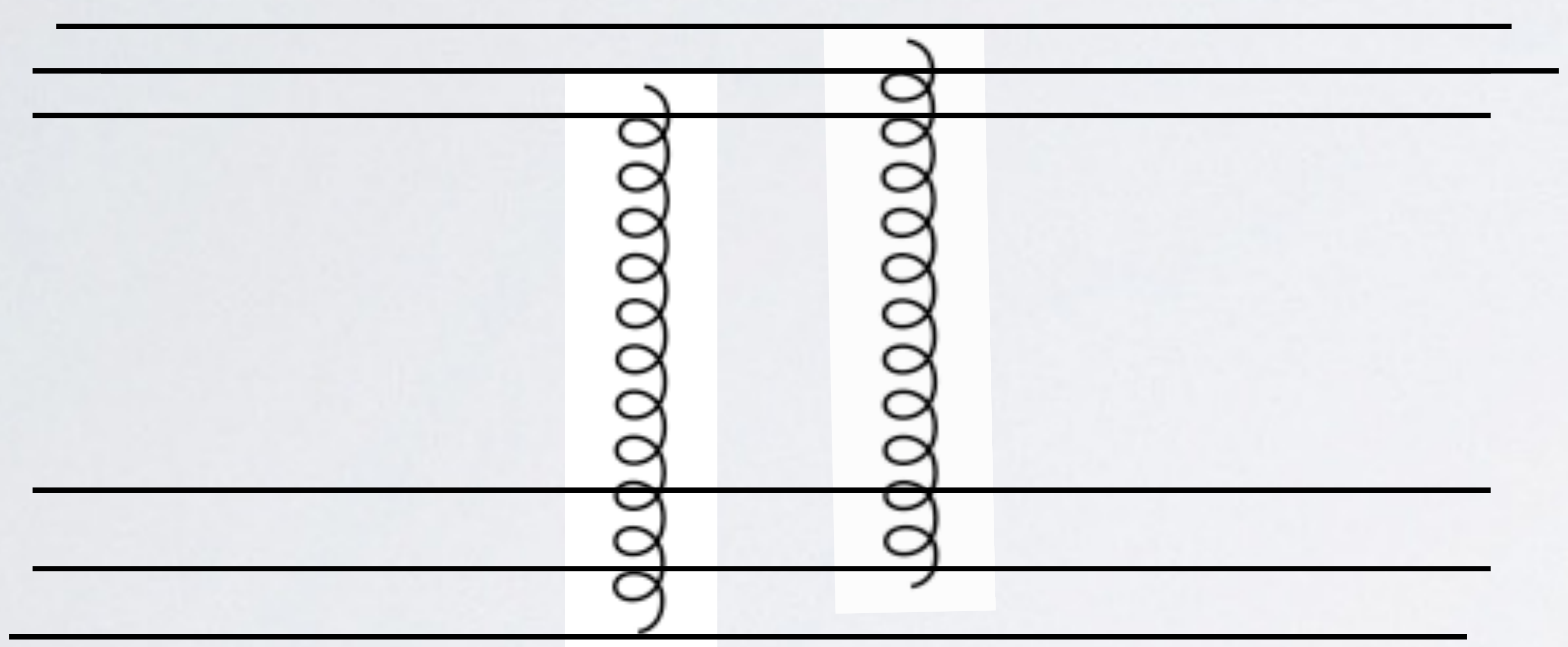
Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- "Pomeron" in QCD non-perturbatively,
- Unification of Soft and Hard Physics in High Energy Collision,
- Confinement Important,
- Looking for Generic Features following from Conformal Invariance!!

HIGH ENERGY SCATTERING <=> POMERON

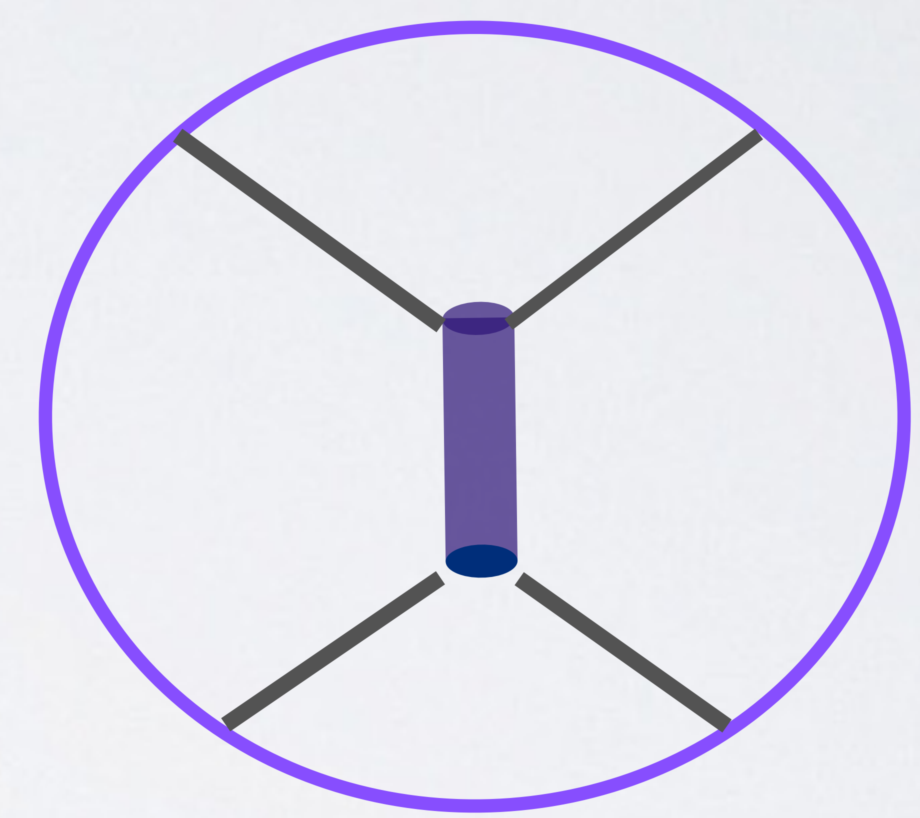
WHAT IS THE POMERON ?

WEAK: TWO-GLUON <=> STRONG: ADS GRAVITON



$$J_{cut} = 1 + 1 - 1 = 1$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.



$$J = 2$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253

Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar\psi(x)\psi(x), \;\; \bar\psi(x)D_\mu\psi(x)$

$S(x) = TrF_{\mu\nu}^2(x), \;\; O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \;\; etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar\psi \not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x), \;\; a(x), \;\; etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

$\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x) \mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string} \left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x) \right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

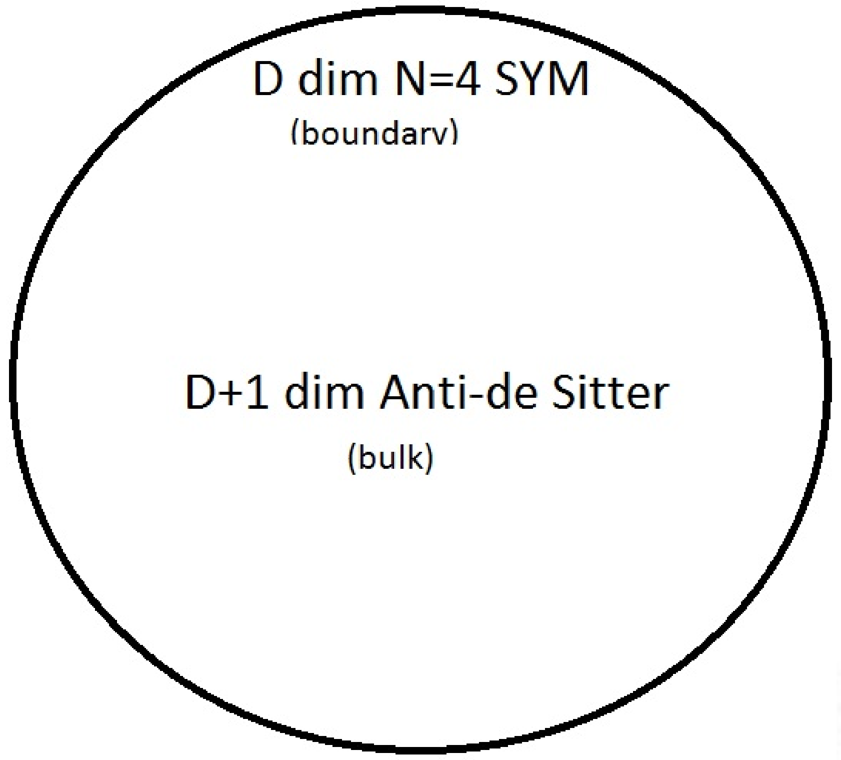
- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

Supergravity limit

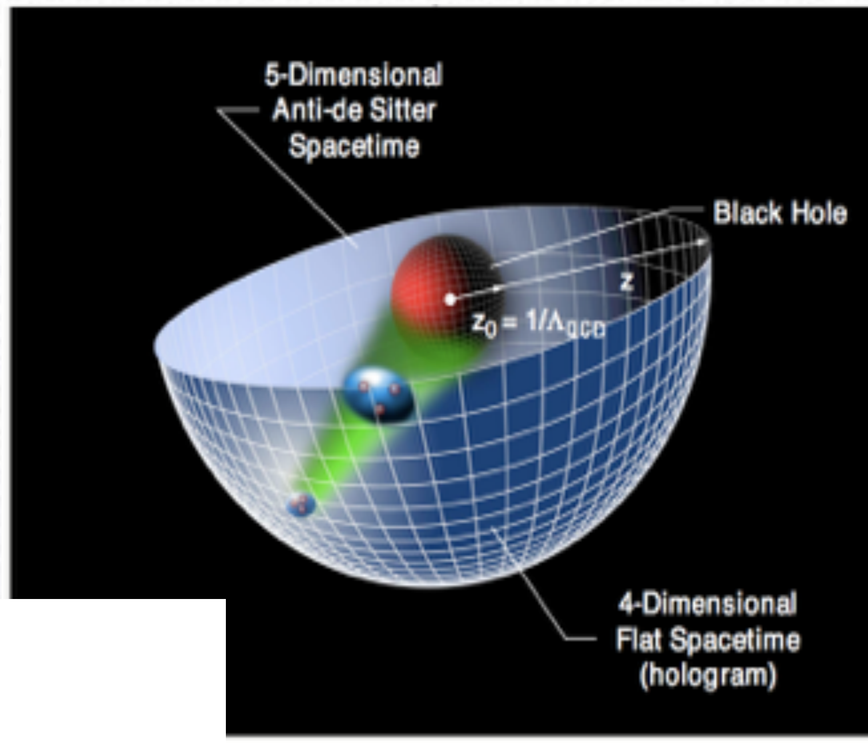
- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

Background and Motivation



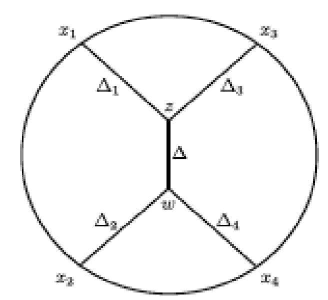
The AdS/CFT is a holographic duality
that equates a string theory (gravity) in high dimension
with a conformal field theory (gauge) in 4 dimensions.
Specifically, compactified 10 dimensional super string theory
is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory
in 4 dimensions in the limit of large 't Hooft coupling:

$$\lambda = g_s N = g_{ym}^2 N_c = R^4/\alpha'^2 >> 1.$$



$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5$$

Conformal Invariance and Pomeron Interaction from AdS/CFT



\+ ....

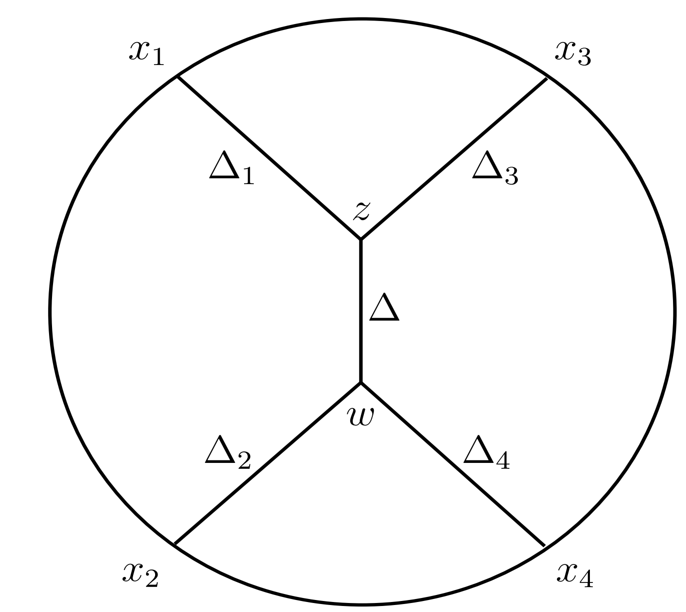
Technique: Summing generalized Witten Diagrams

Freedman et al., hep-th/9903196
Brower, Polchinski, Strassler, and Tan, hep-th/0003115

- Draw all "Witten-Feynman" Diagrams in AdS5,
- High Energy Dominated by Spin-2 Exchanges:

One Graviton Exchange at High Energy

$$p_1 + p_2 \to p_3 + p_4$$



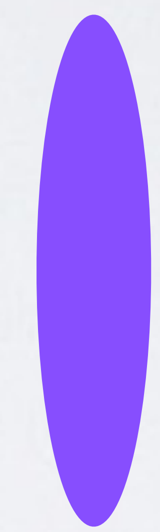
$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde\Phi_\Delta(p_1^2,z)\tilde\Phi_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde\Phi_\Delta(p_2^2,z')\tilde\Phi_\Delta(p_4^2,z')$$

$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

BASIC BUILDING BLOCK

- Elastic Vertex:

- Pomeron/Graviton Propagator:



$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\widehat{s}^{\,j}\; G_j(z,x^\perp,z',x'^\perp;j)$$

conformal:

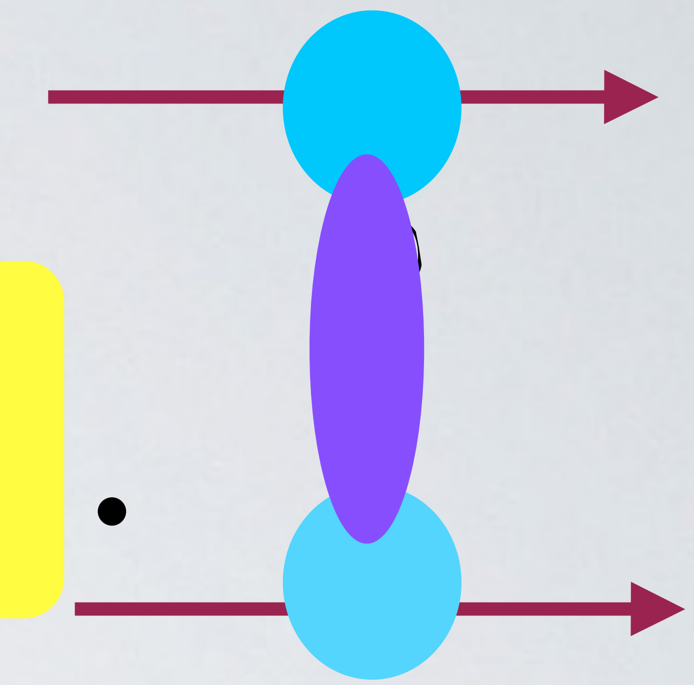
$$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi z z'}\,\frac{e^{(2-\Delta(j))\xi}}{\sinh \xi}\;,$$

$$\Delta(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

ADS BUILDING BLOCKS BLOCKS

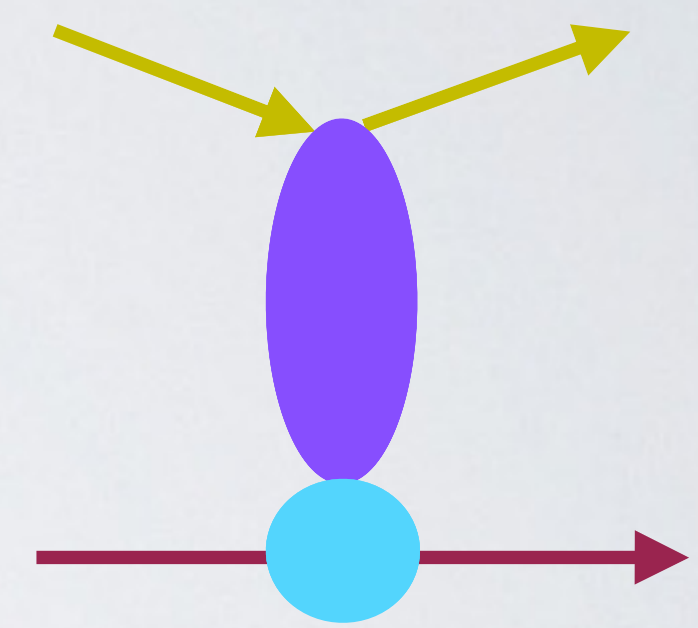
For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\,.$$



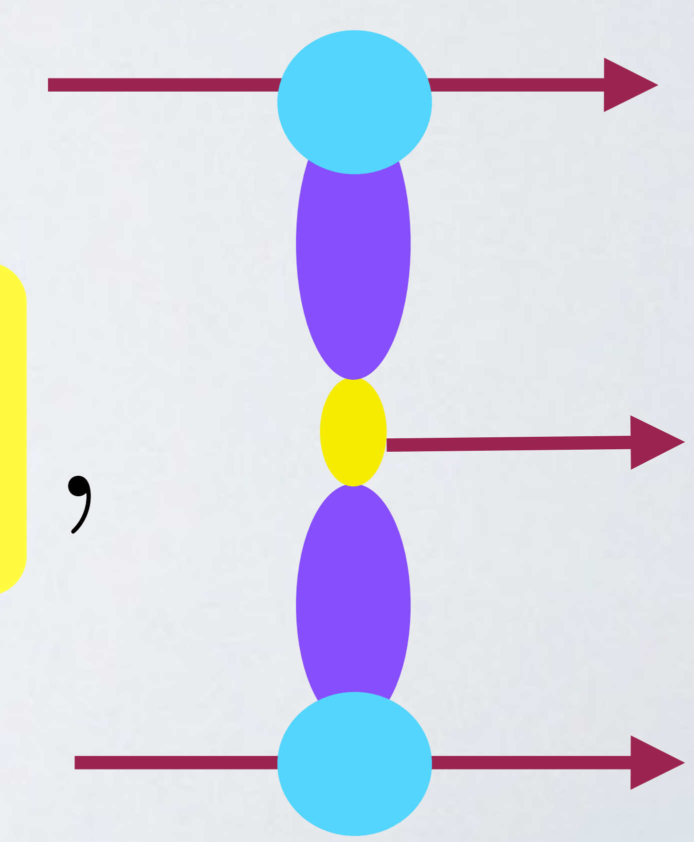
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}'\; e^{i\mathbf{q}_\perp \cdot (\mathbf{x}-\mathbf{x}')}\; \Phi_{13}(z)\; \mathcal{K}(s, \mathbf{x}-\mathbf{x}', z, z')\; \Phi_{24}(z')$$

$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

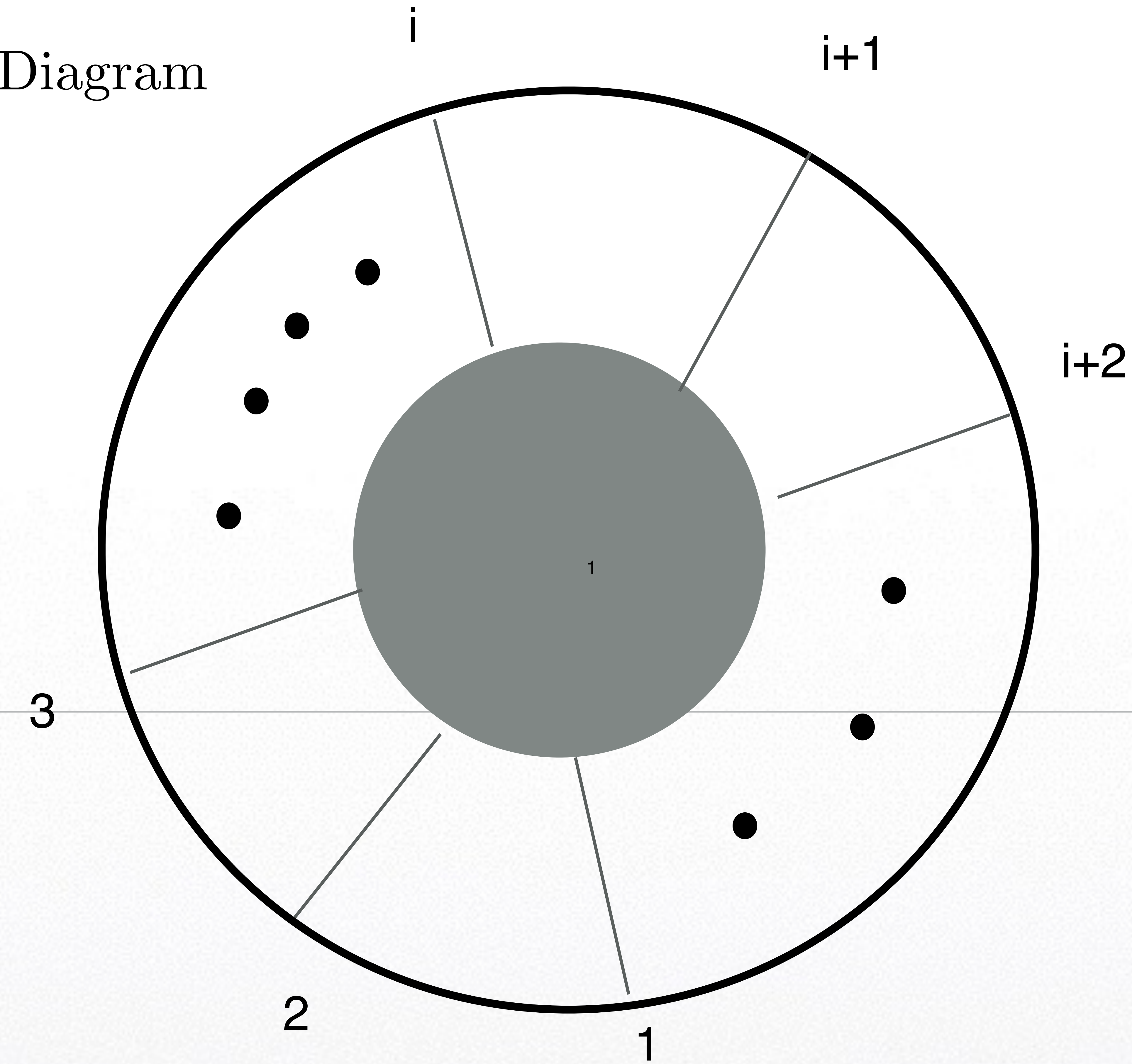


For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24}\,,$$



General Witten Diagram



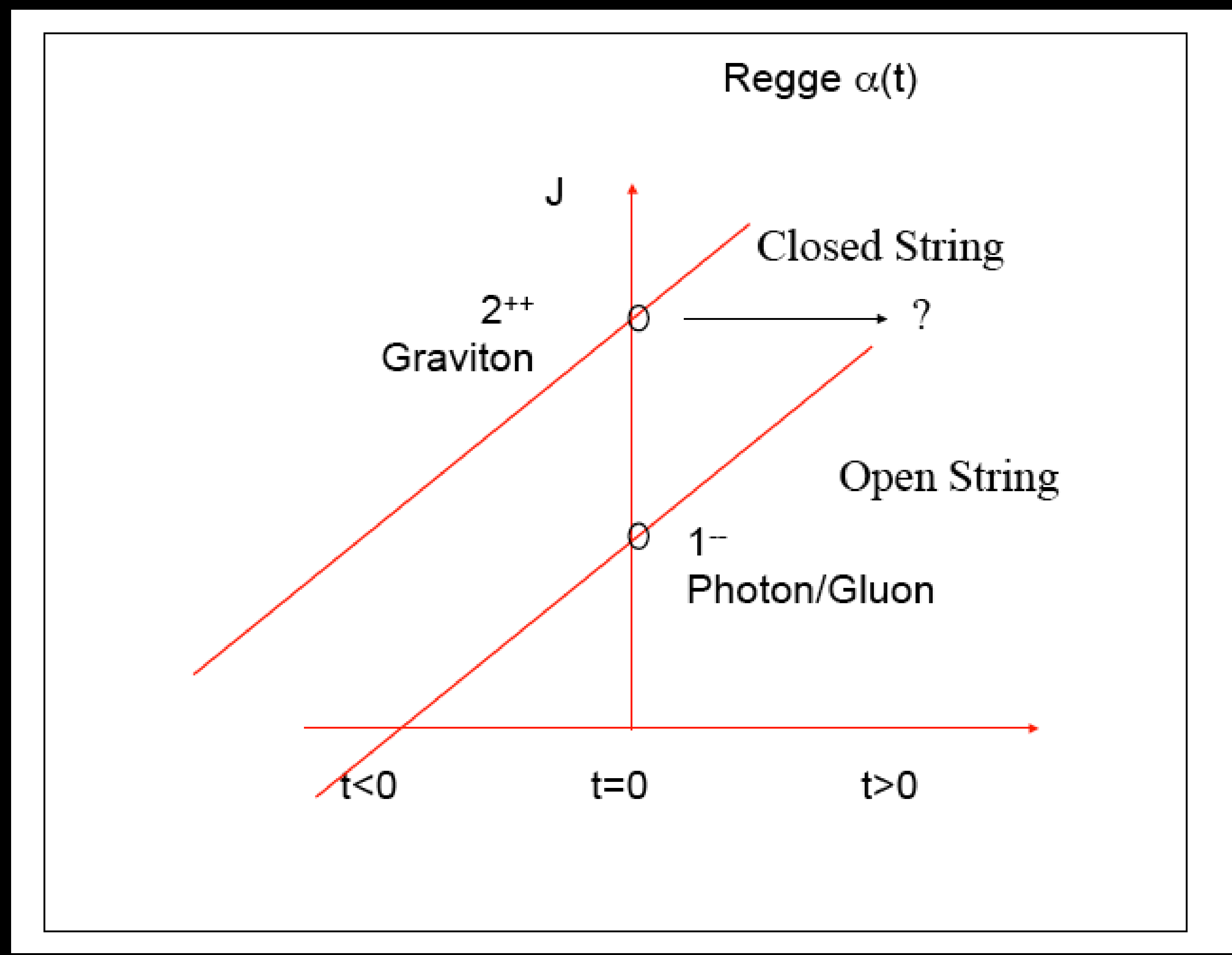
$$T_n(p_1, p_2, \cdots) = \int dz_1 dz_2 \cdots \Phi_1(p_1)\Phi_2(p_2)\Phi_3(p_3)\cdots$$
$$\mathcal{T}_n(p_1, p_2, \cdots, z_1, z_2, z_3, \cdots)$$

Additional Steps for QCD:

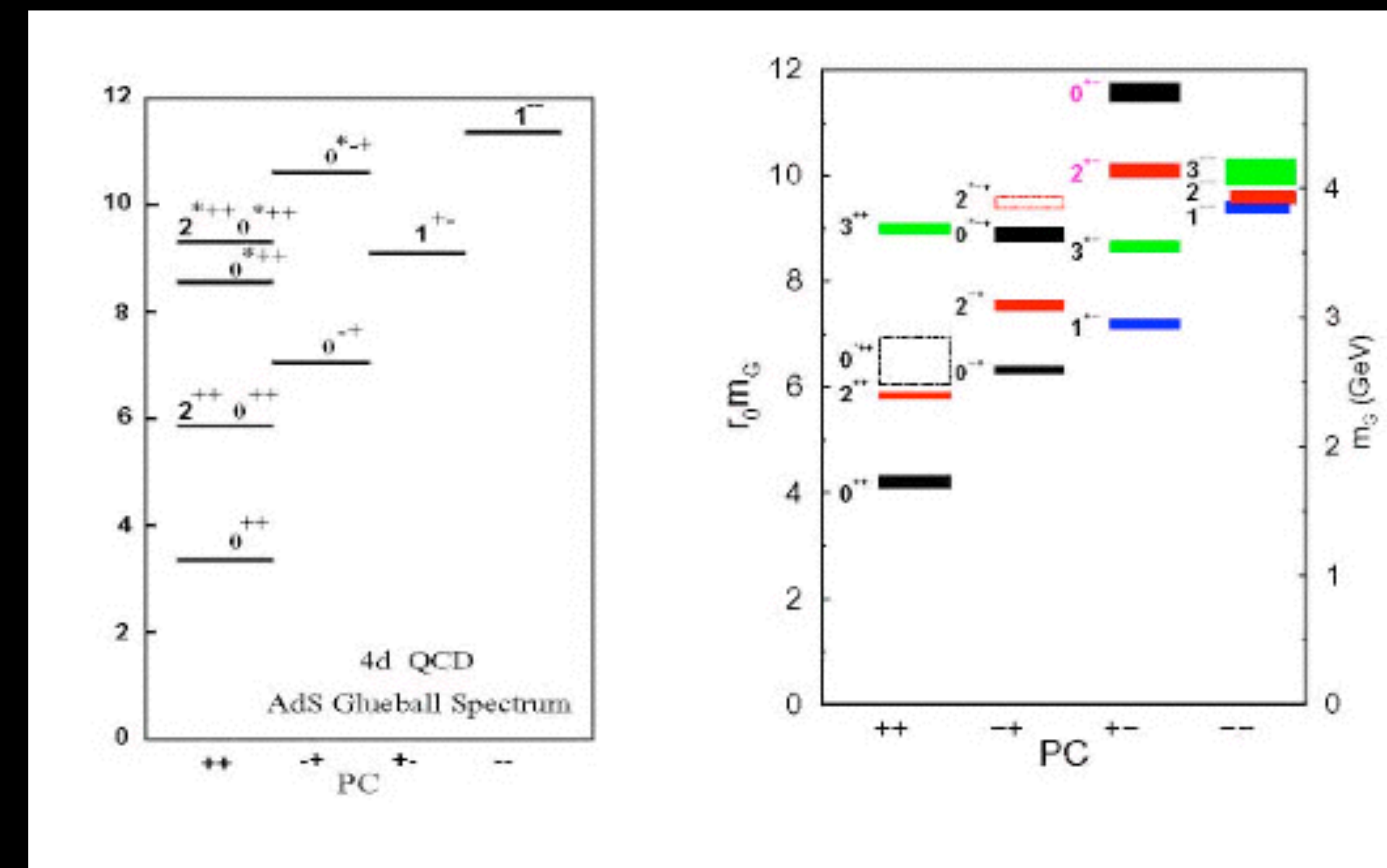
- Conformal, therefore no scale and no particles, etc.
  - Confinement
  - Need to consider running coupling in weak coupling
- Spin-2 leads to too rapid an increase for cross sections.
  - Need to consider $\lambda = g^2 N$ finite.

QCD Pomeron <===> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

<u>Fixed cut in J-plane:</u>

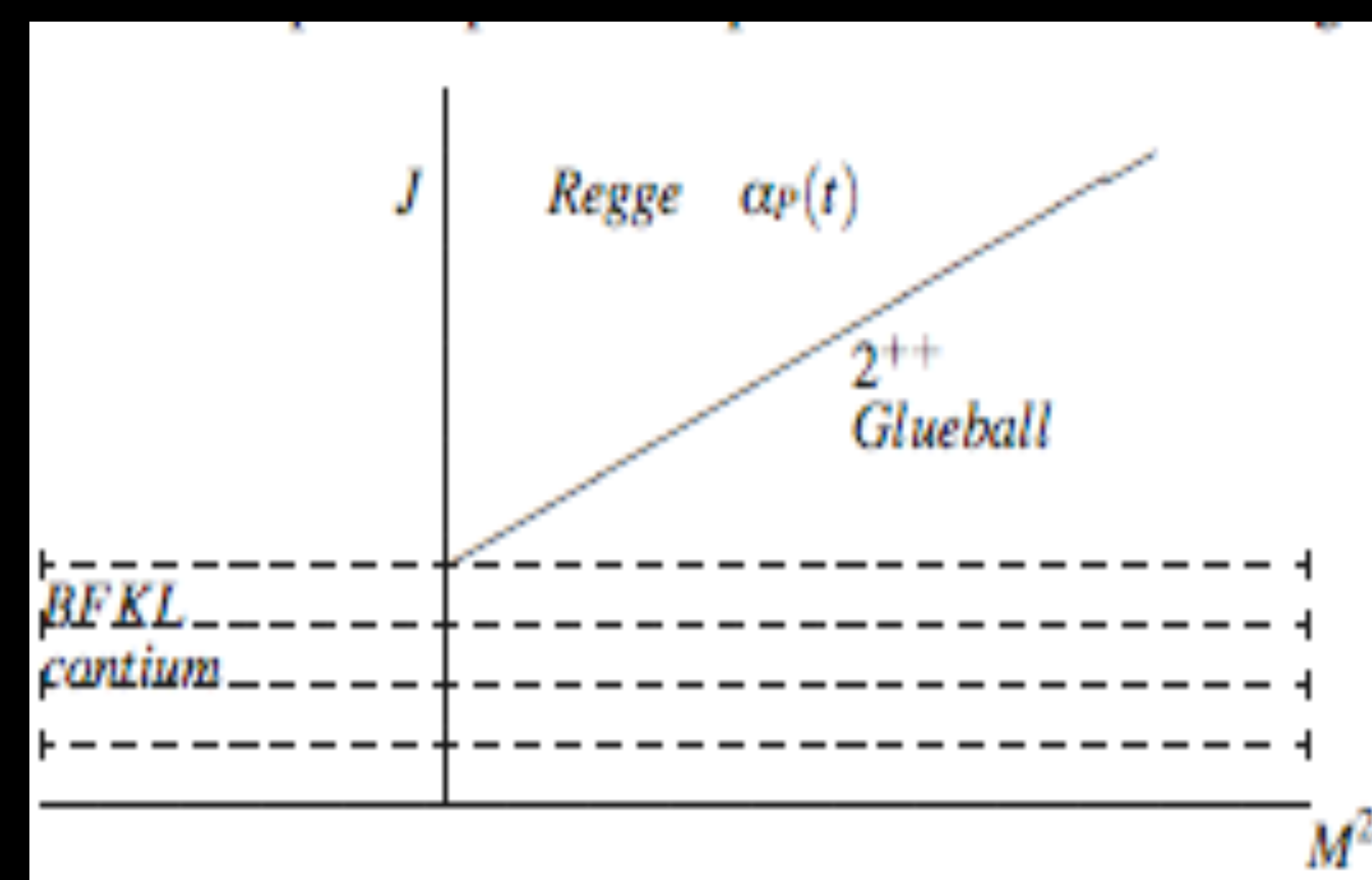
<u>Weak coupling:</u>
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

<u>Strong coupling:</u>

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

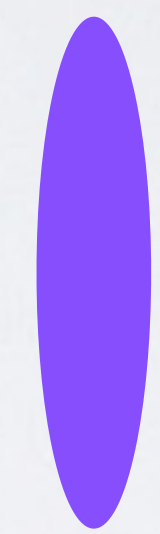
Pomeron in AdS Geometry



BASIC BUILDING BLOCK

- Elastic Vertex:

- Pomeron/Graviton Propagator:



$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right) \int \frac{dj}{2\pi i} \left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right) \widehat{s}^j \, G_j(z, x^\perp, z', x'^\perp; j)$$

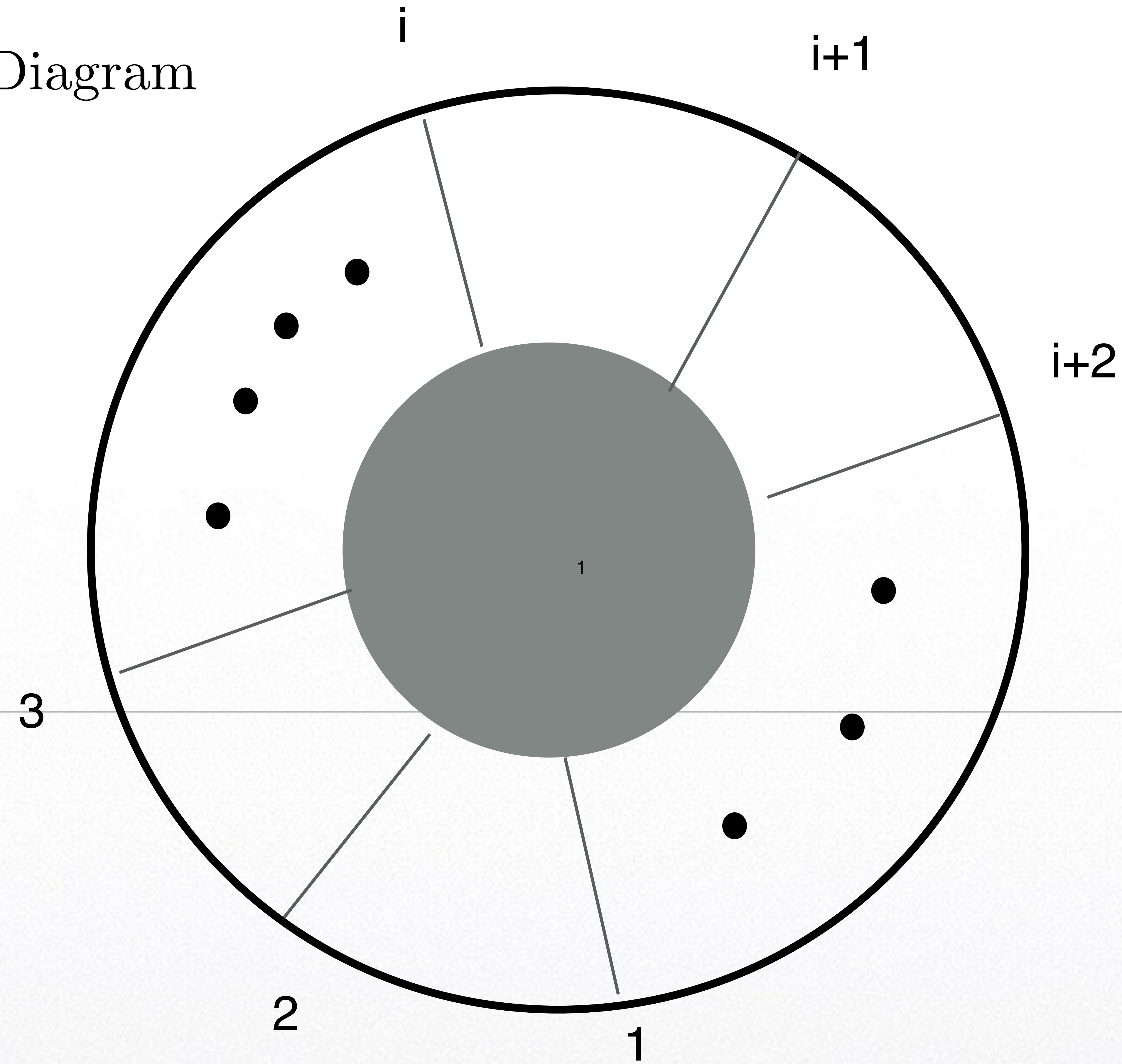
conformal:

$$G_j(z, x^\perp, z', x'^\perp) = \frac{1}{4\pi z z'} \frac{e^{(2-\Delta(j))\xi}}{\sinh \xi} \,,$$

$$\Delta(j) = 2 + \sqrt{2}\, \lambda^{1/4} \sqrt{(j - j_0)}$$

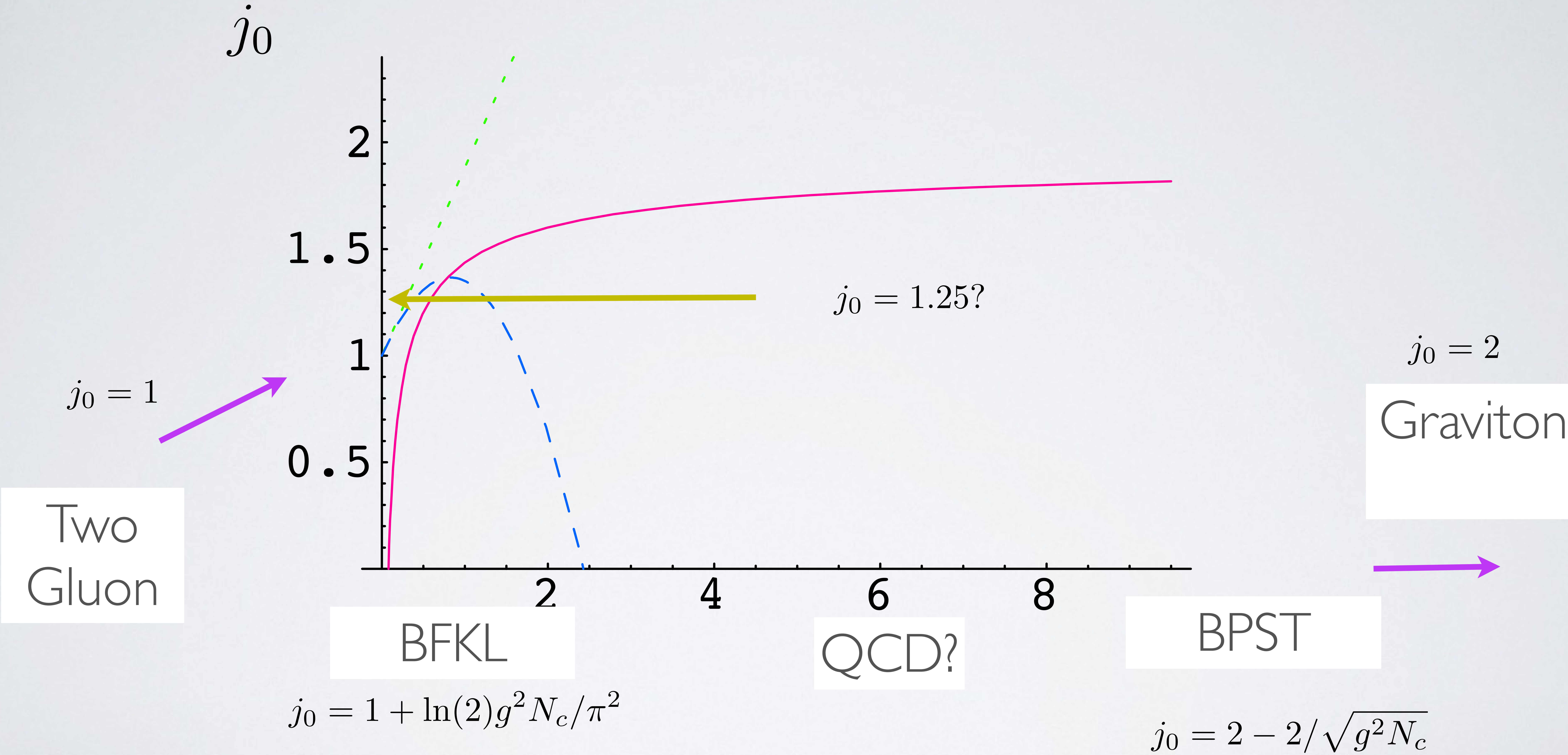
confinement: $G_j(z, x^\perp, z', x'^\perp; j)$ ⟶ discrete sum

General Witten Diagram



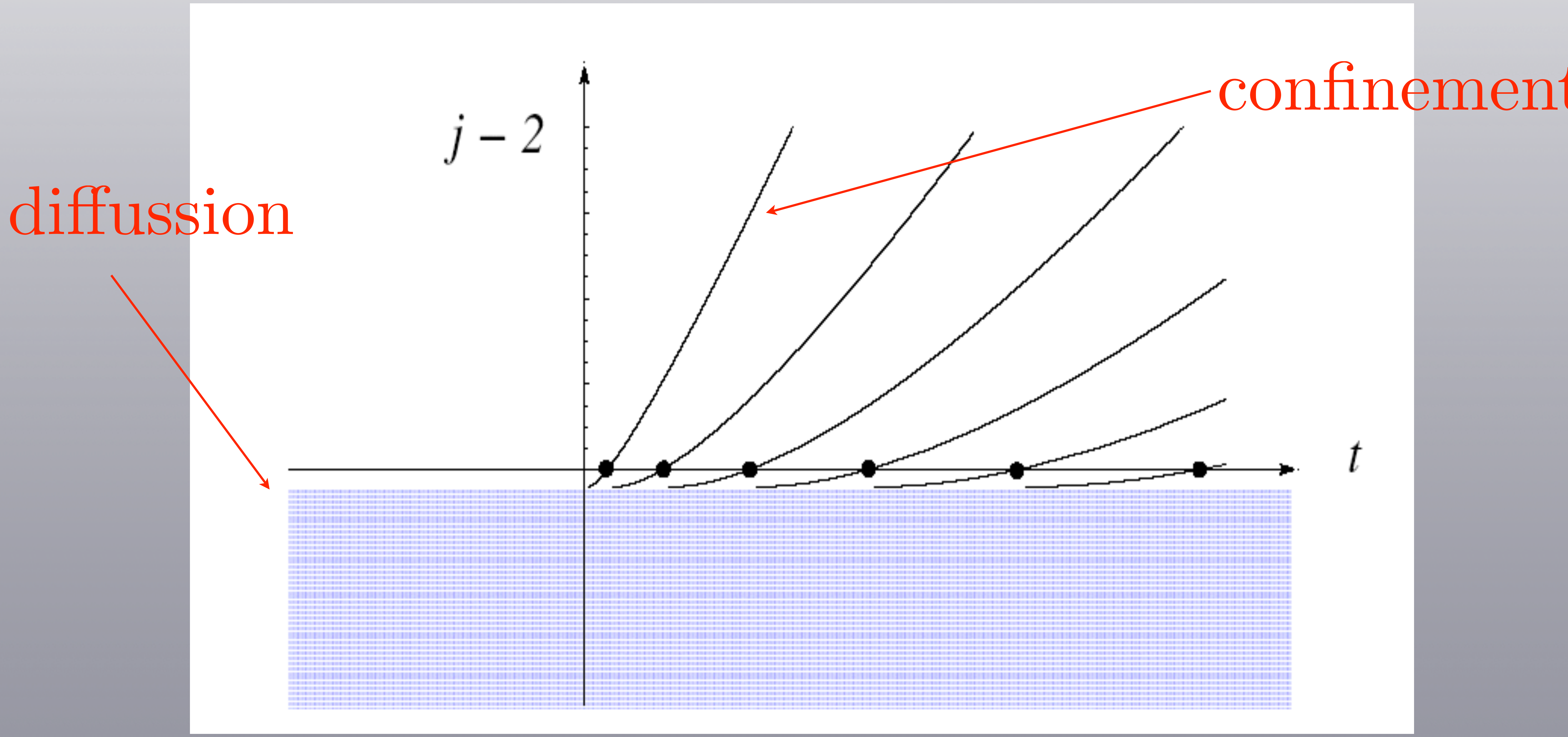
$$T_n(p_1, p_2, \cdots) = \int dz_1 dz_2 \cdots \Phi_1(p_1)\Phi_2(p_2)\Phi_3(p_3)\cdots$$
$$\mathcal{T}_n(p_1, p_2, \cdots, z_1, z_2, z_3, \cdots)$$

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$

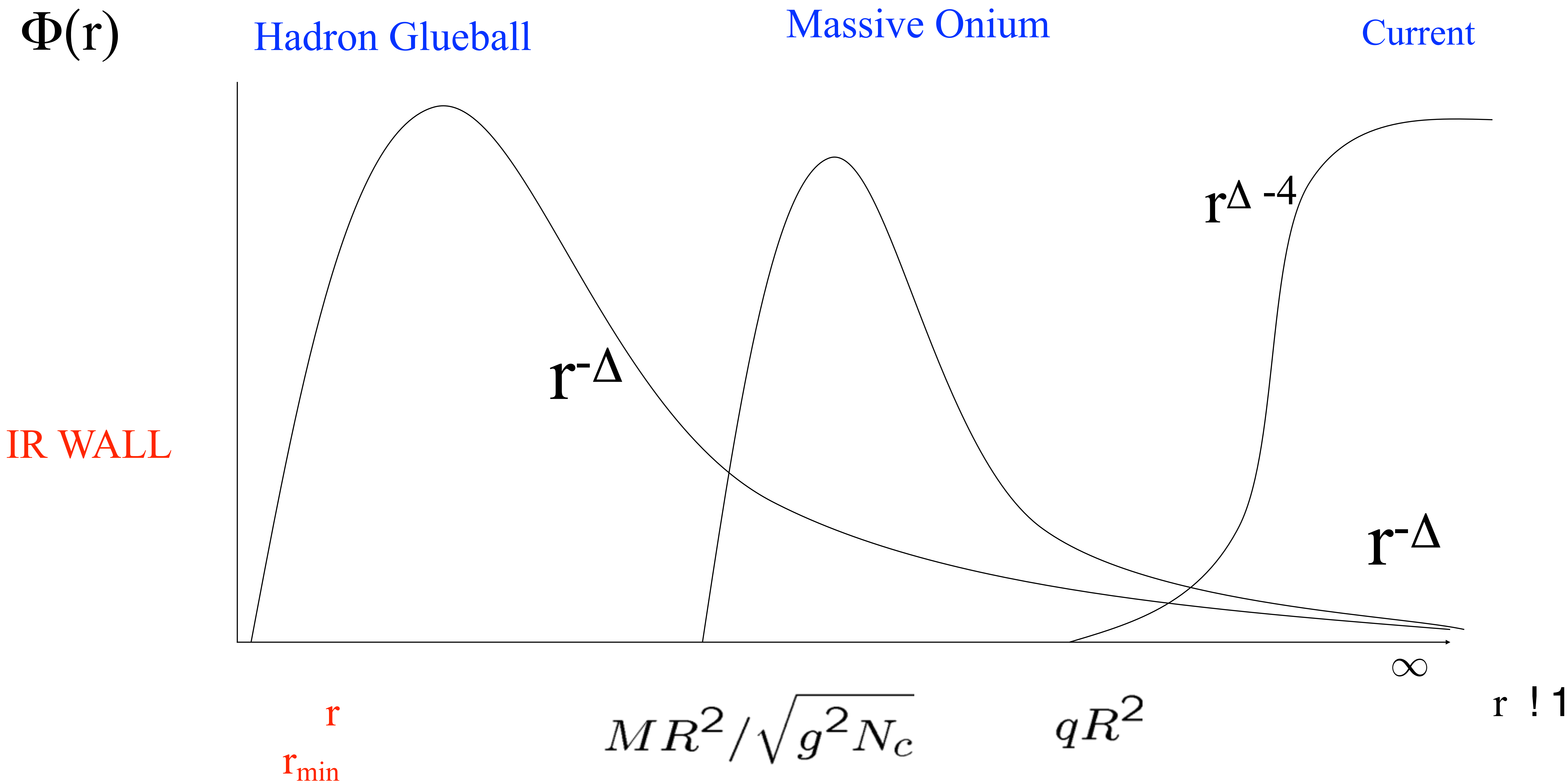


Unified Hard (conformal) and Soft (confining) Pomeron

At finite λ, due to Confinement in AdS, *at* $t > 0$ aymptotical linear Regge trajectories



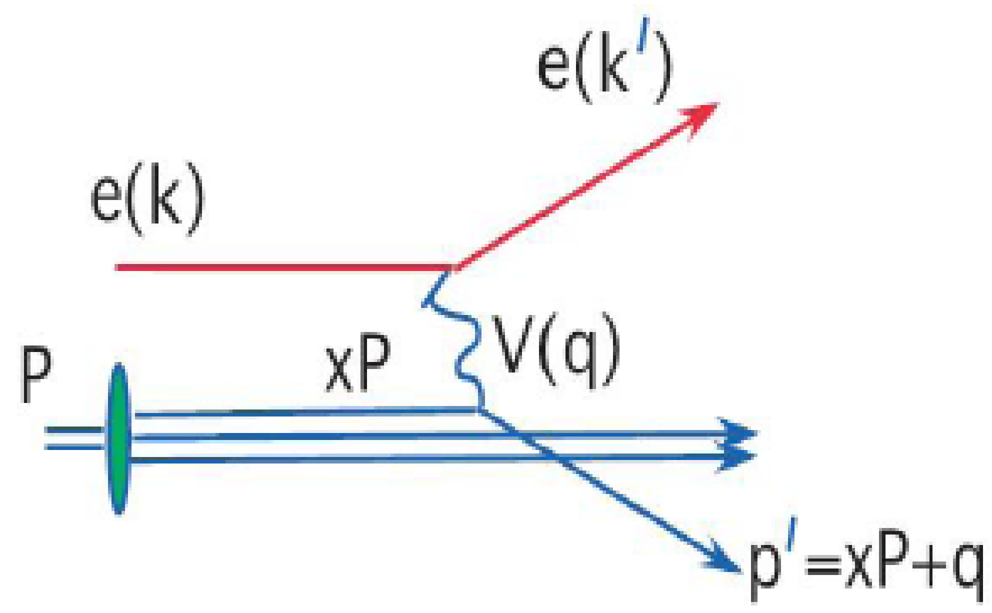
Approx. Scale Invariance and the 5th dimension



$$z = r^{-1}$$

III. Central Inclusive Spectrum:

Conformal Invariance?
Confinement?
Satuation?



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$x \equiv \frac{Q^2}{s}$$

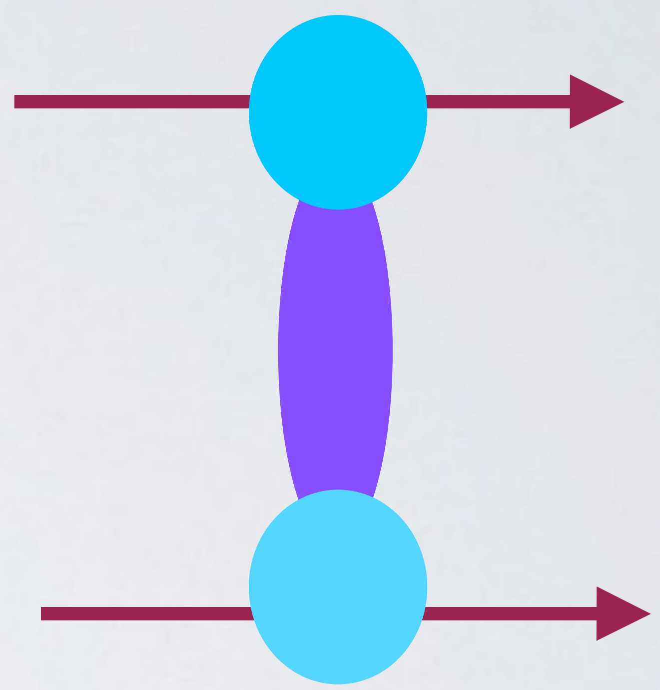
$$Small\ x : \ \frac{Q^2}{s} \to 0$$

$$Optical\ Theorem$$

$$\sigma_{total}(s, Q^2) = (1/s)\mathrm{Im}\ A(s, t = 0; Q^2)$$

ELASTIC VS DIS ADS BUILDING BLOCKS

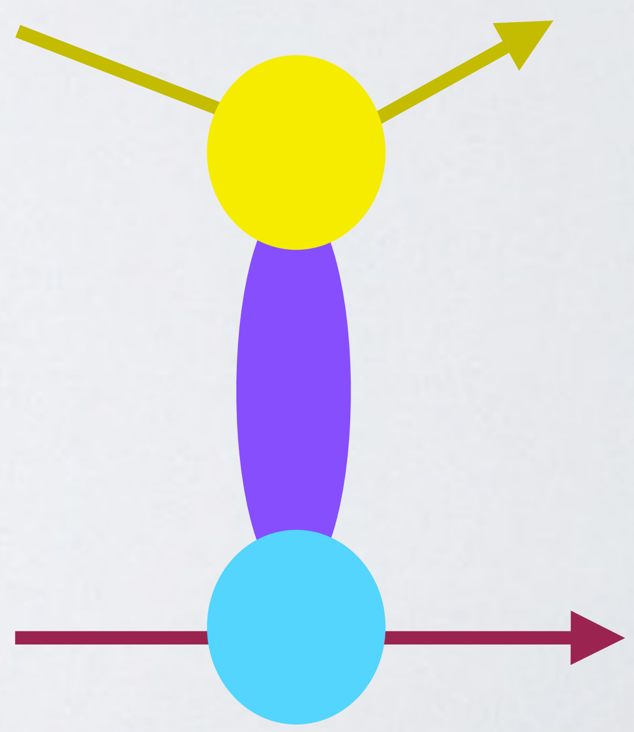
$$A(s, x_\perp - x'_\perp) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \Phi_{12}(z) G(s, x_\perp - x'_\perp, z, z') \Phi_{34}(z')$$



$$\sigma_T(s) = \frac{1}{s} Im A(s, 0)$$

for $F_2(x, Q)$

$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z, Q) = \frac{1}{z}[Qz)^4(K_0^2(Qz) + K_1^2(Qz)]$$

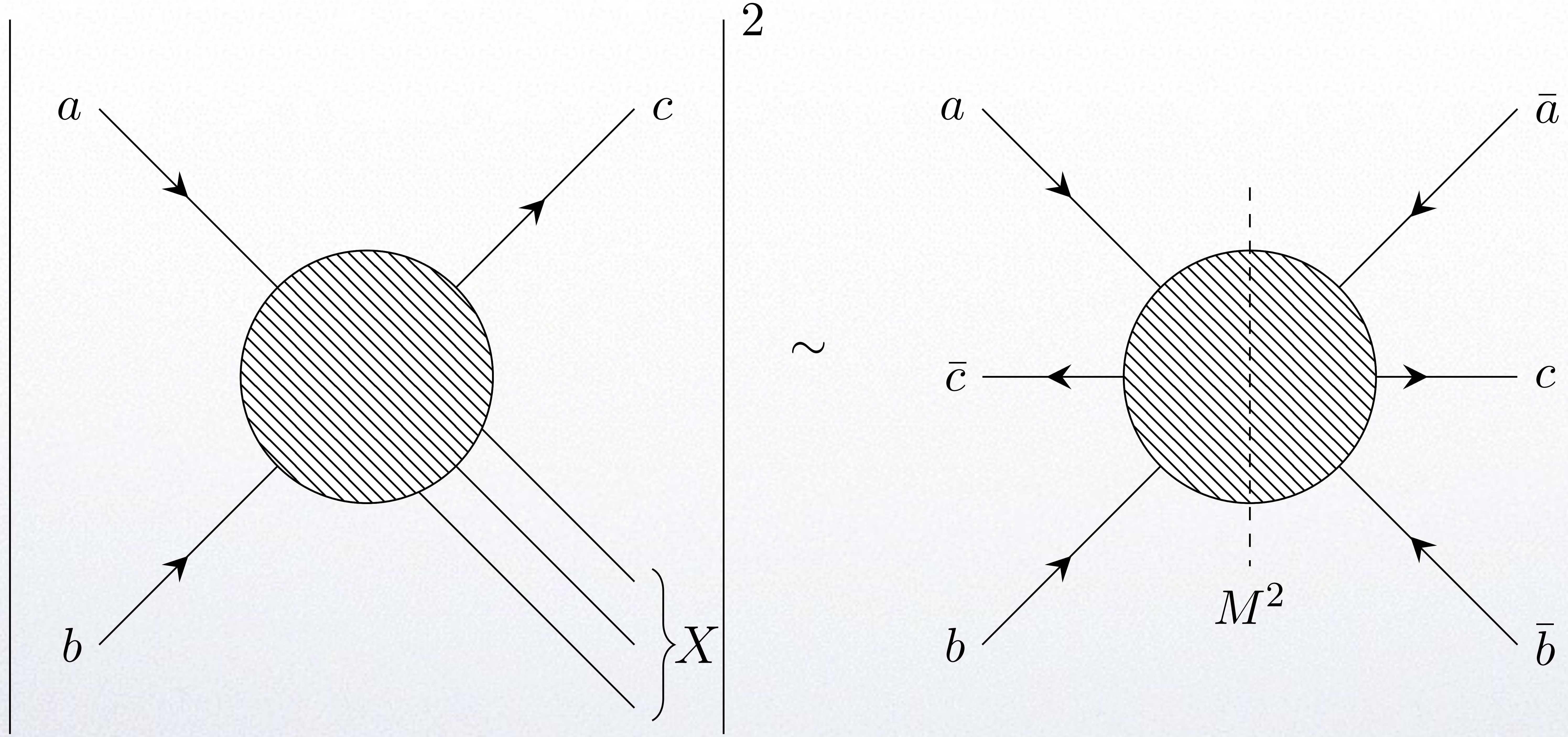


$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

Central Inclusive Single-Particle Production

$a + b \to c + X$

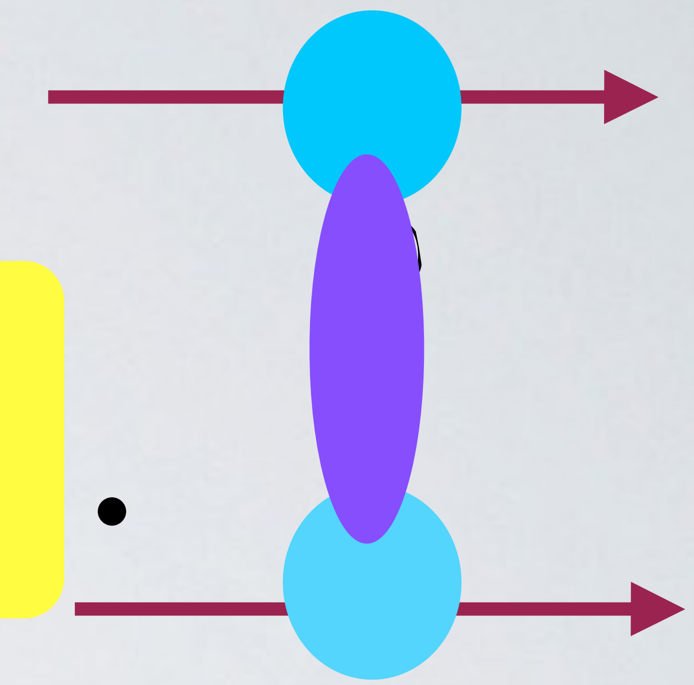
$$\frac{d\sigma}{(d^3k/E)} = (1/s)(1/2i) Disc_{M^2}\, T_{3\to 3}(k_a, k_b, k'_c; k'_a, k'_b, k_c)$$

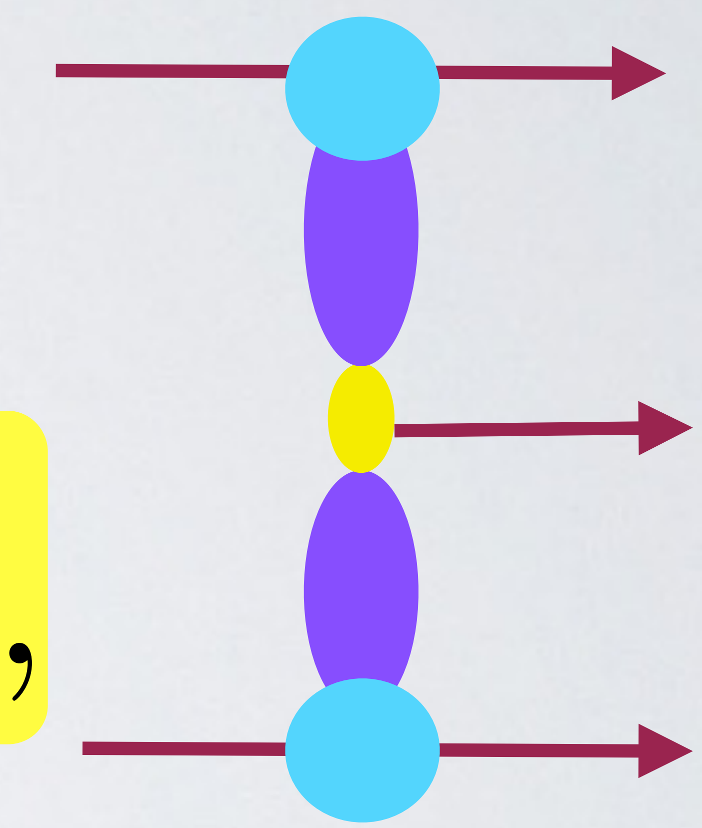


ADS BUILDING BLOCKS BLOCKS

For 2-to-2

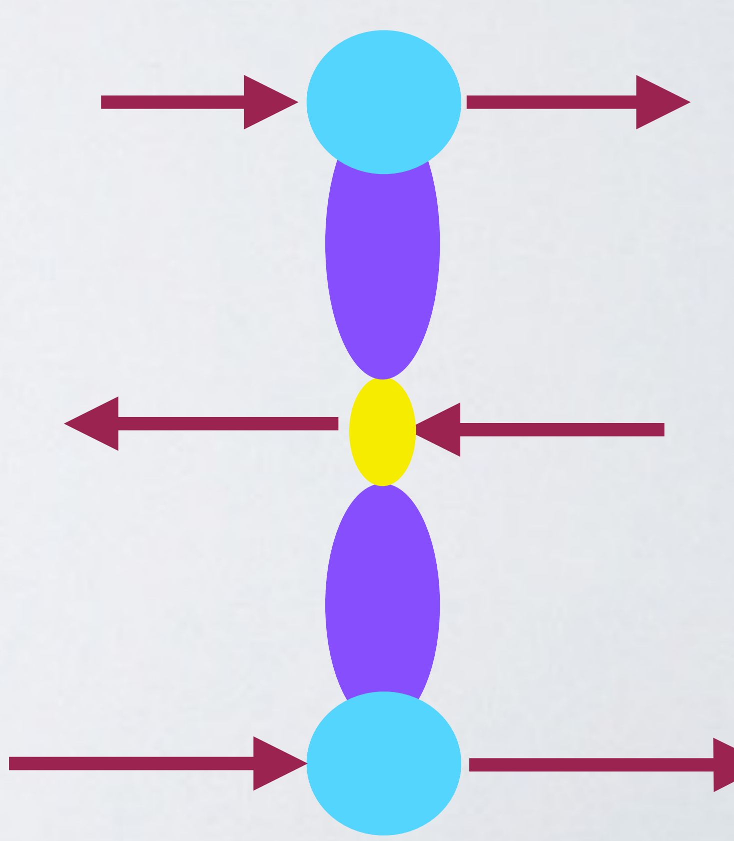
$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24} \; .$$





For 2-to-3

$$A(s, s_1, s_2, t_1, t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24} \; ,$$



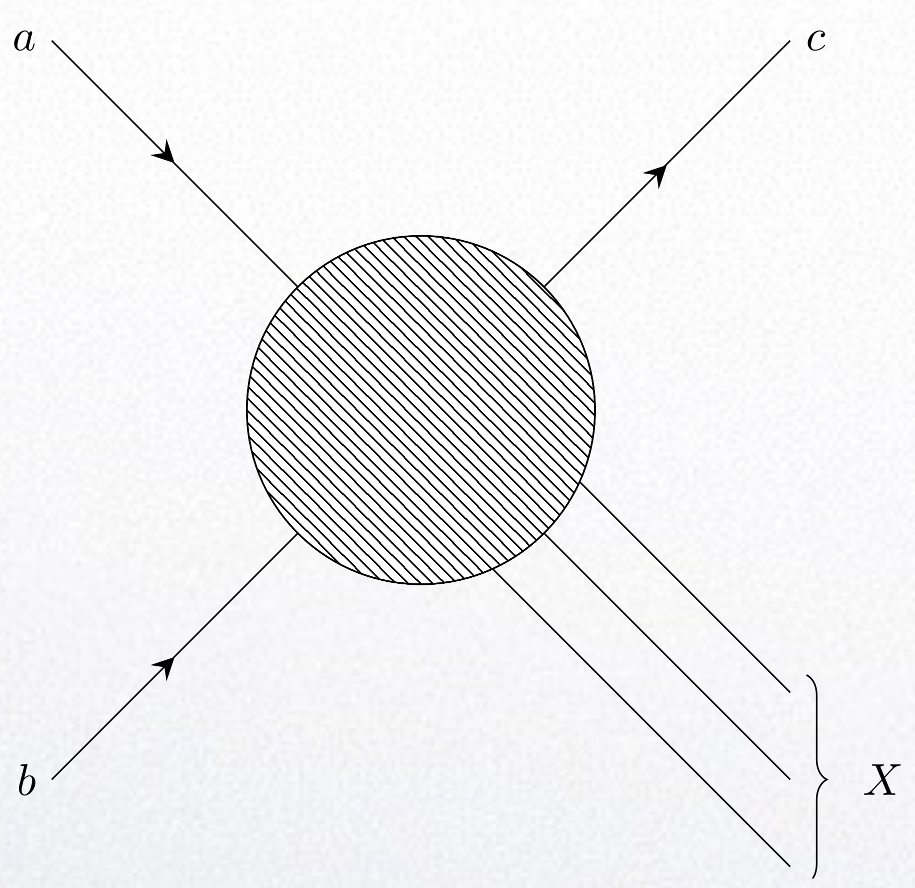
$$T_{3-3}(s, M^2, t_1, t_2, \kappa, ...) = \Phi_{a,\bar{a}}\,\tilde{}\!* \mathcal{K}_P * \mathcal{V}_{\bar{c},c} * \widetilde{\mathcal{K}}_P * \Phi_{b,\bar{b}}$$

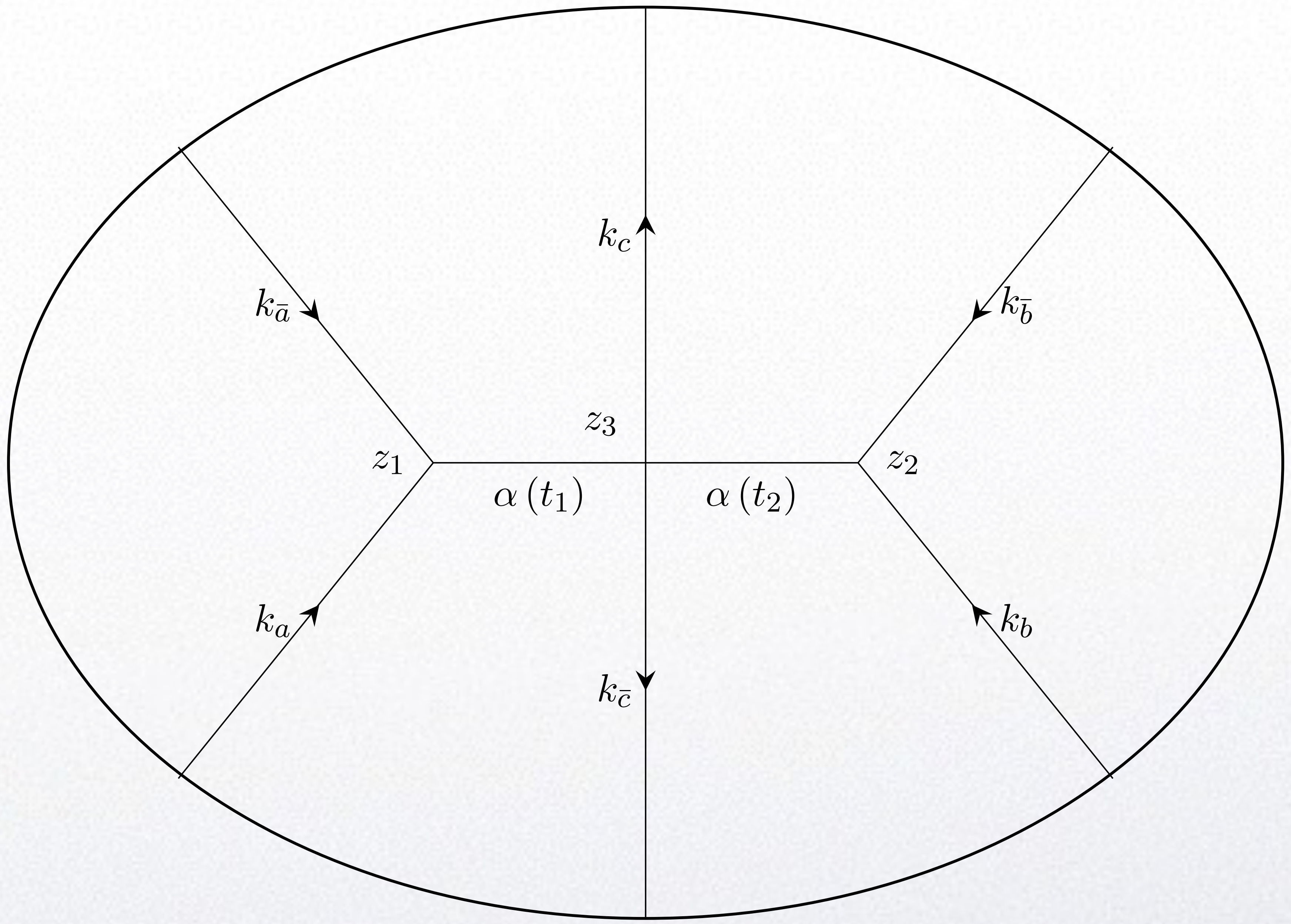
$$\mathcal{V}_{\bar{c},c} \;\sim\; \phi_c \, \phi_{\bar{c}} \; V_{\bar{c},c}(t_1, t_2, z_c, \kappa)$$

Central Inclusive Single-Particle Production

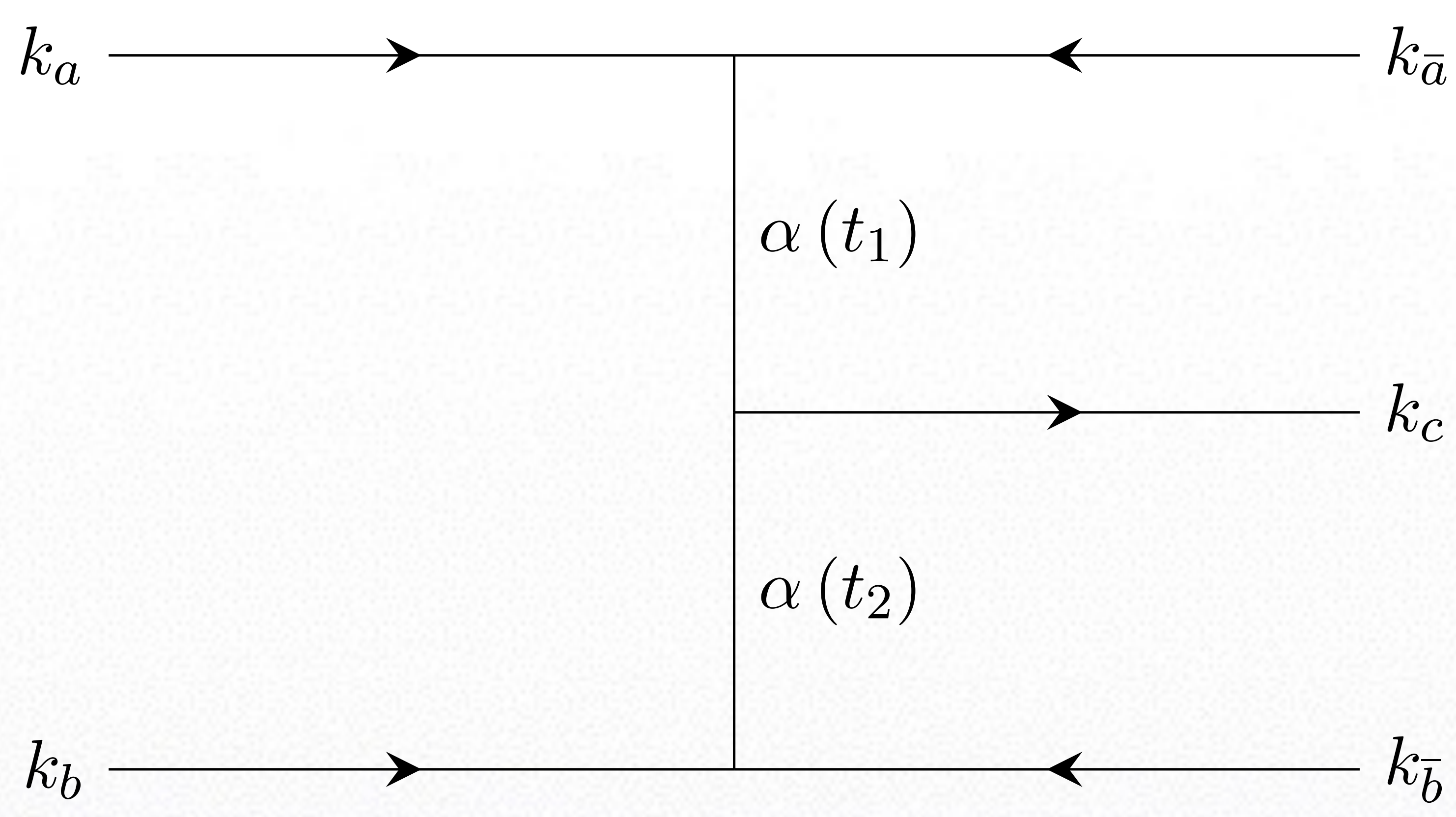
$$a + b \to c + X$$

$$\frac{d\sigma}{(d^3k/E)} = (1/s)(1/2i)Disc_{M^2}\, T_{3\to 3}(k_a, k_b, k'_c; k'_a, k'_b, k_c)$$





How to calculate this Mueller Discontinuity within AdS/CFT?



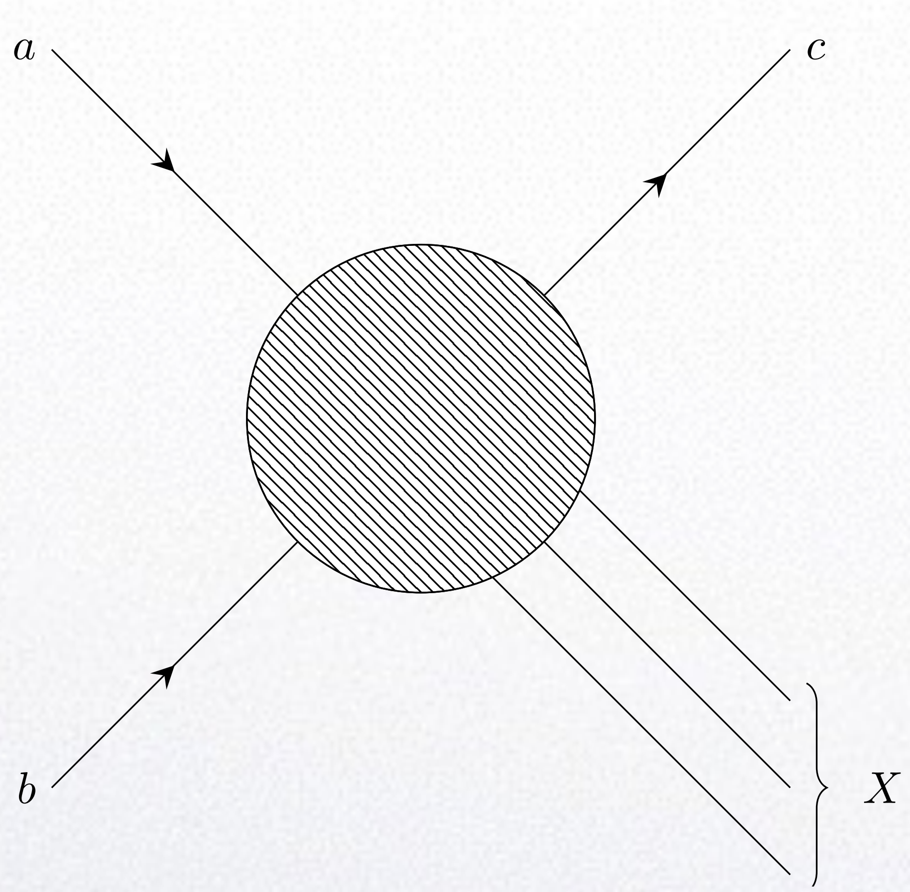
$$T_{2\to 3}(p_1, p_2, p_3, p_c, p_4) = \int (dz_1\sqrt{g(z_1)})(dz_2\sqrt{g(z_1)})\{\Psi_1(z_1)\Psi_3(z_1)\}\mathcal{T}_{2\to 3}(p_i; z_1, z_c, z_2)\{\Psi_2(z_2)\Psi_4(z_2)\}$$

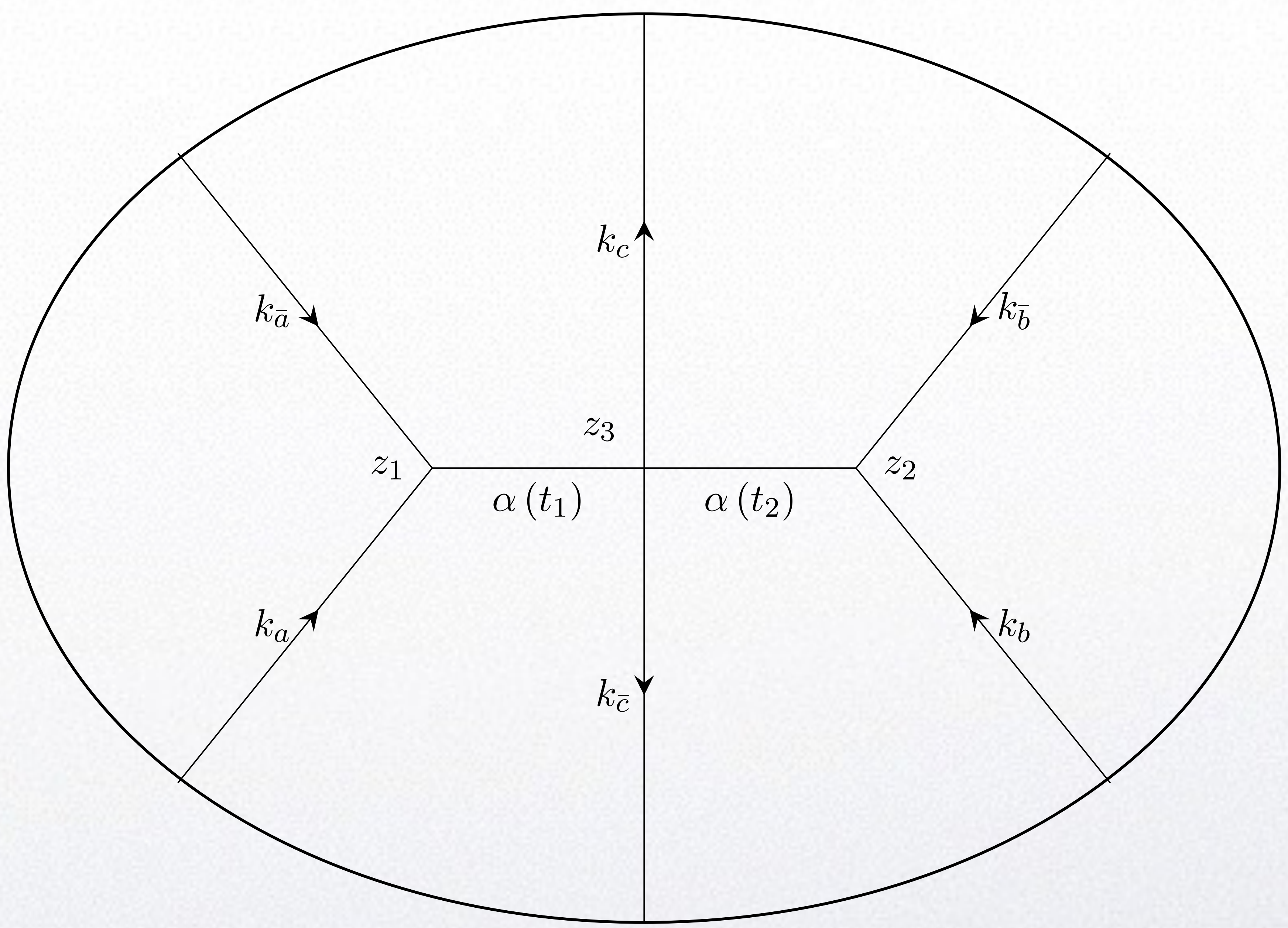
$$\mathcal{T}_{2\to 3}(p_i; z_1, z_2) = V(z_1)\int dz_c\sqrt{g(z_c)}\mathcal{K}(s_1, z_1, z_c)V(z_1)G(t_1, \kappa, t_2, z_c)\Psi_c(z_c)V(z_c)\mathcal{K}(s_2, z_c, z_2)V(z_2)$$

Central Inclusive Single-Particle Production

$a + b \to c + X$

$$\frac{d\sigma}{(d^3k/E)} = (1/s)(1/2i)Disc_{M^2}\, T_{3\to 3}(k_a, k_b, k_c'; k_a', k_b', k_c)$$





Fixed Angle Exclusive Scattering

$$T(p_1, p_2, \cdots) = \int \prod_{i=1,2,3,4\cdots} \{(dz_i\sqrt{g(z_i)})\Psi_i(z_i)\}\mathcal{T}(p_i; z_i)$$

$$\to \int (dz\sqrt{g(z)}) \prod_{i=1,2,3,4\cdots} \{\Psi_i(z)\}\bar{\mathcal{T}}(zp_i)$$

Assuming that $p_i \sim \sqrt{s}$, and $\bar{\mathcal{T}}(zp_i) \simeq e^{-z/z_{sc}(s)}$ where $z_{sc} \simeq \sqrt{s}$, it follows

$$T(p_1, p_2, \cdots) \sim (\sqrt{s})^{-(n-4)}$$

where $n = \sum \Delta_i$.

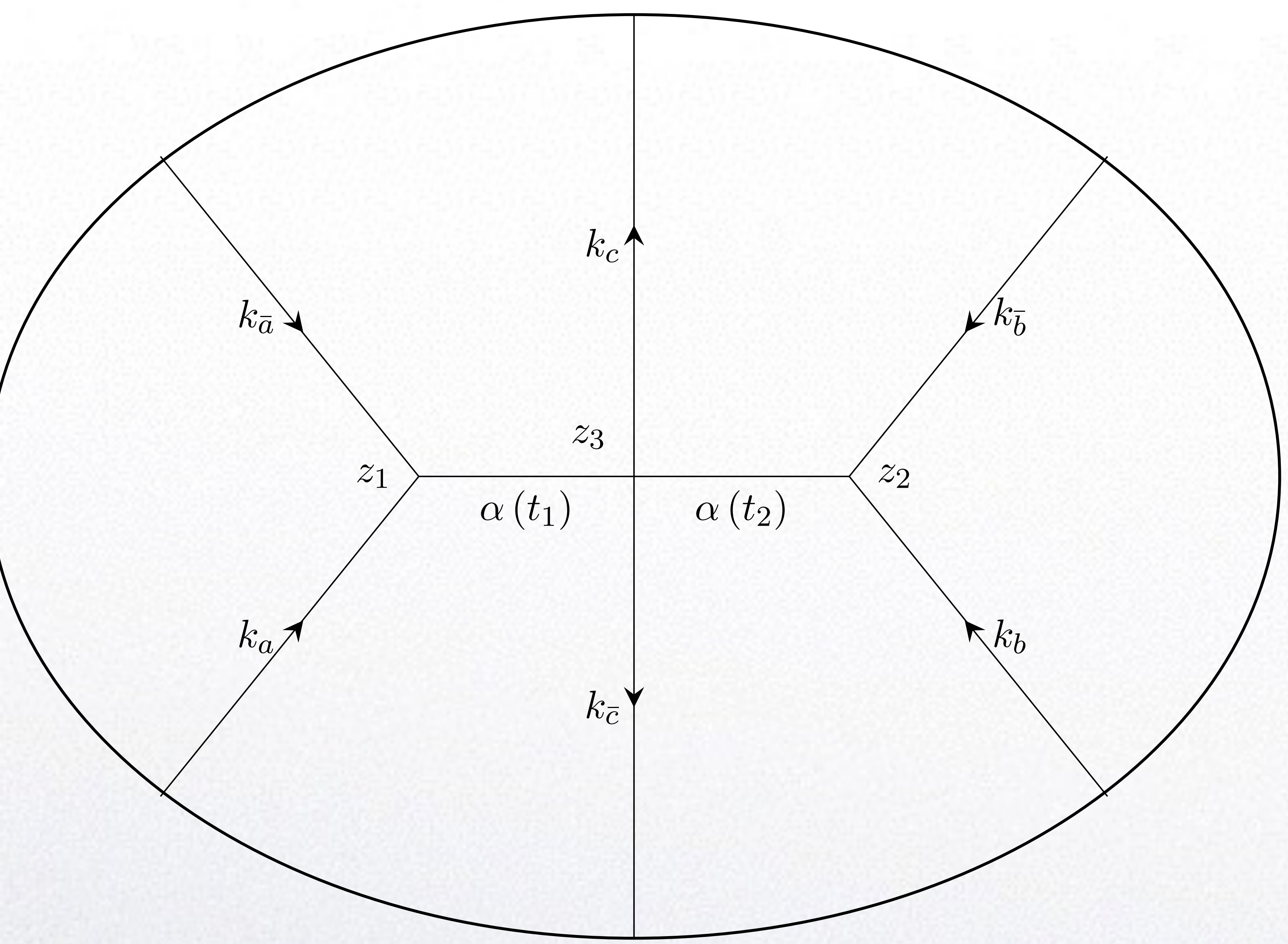
With spin, n_i becomes twist, where

$$n_i = \tau_i = \Delta_i - S_i.$$

Central Inclusive Single-Particle Production

$$a + b \to c + X$$

$$\frac{d\sigma}{(d^3k/E)} = (1/s)(1/2i)Disc_{M^2}\, T_{3\to 3}(k_a, k_b, k'_c; k'_a, k'_b, k_c)$$



$$\mathrm{Disc}_{M^2} T_6 \sim (g_0^2 s^{\alpha_0}\beta/R^{4+4\alpha_0}) \int_0^{z_{sc}} (dz_3/z_3)(z_3^2\kappa)^{\alpha_0}\phi_c(z_3)\phi_{\bar{c}}(z_3)$$

$$z_{sc}^2\kappa \simeq O(1)$$

$$\kappa = m^2 + p_T^2$$

$$\frac{d\sigma}{d^2p_t}|_{y=0} \sim s^{\alpha_0 - 1}\kappa^{-\Delta_c} \sim p_t^{-2\Delta_c}$$

Production via glueballs, $\frac{d\sigma}{d^2p_t}|_{y=0} \sim p_t^{-8}$

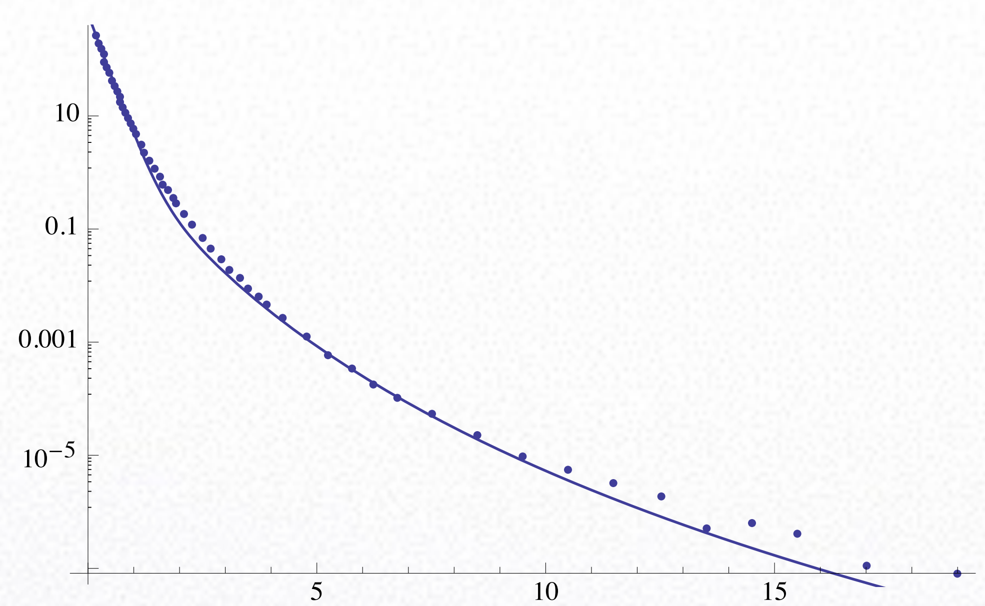
Direct meson production, $\frac{d\sigma}{d^2p_t}|_{y=0} \sim p_t^{-6}$

Inclusive Production

$a + b \to c + X$

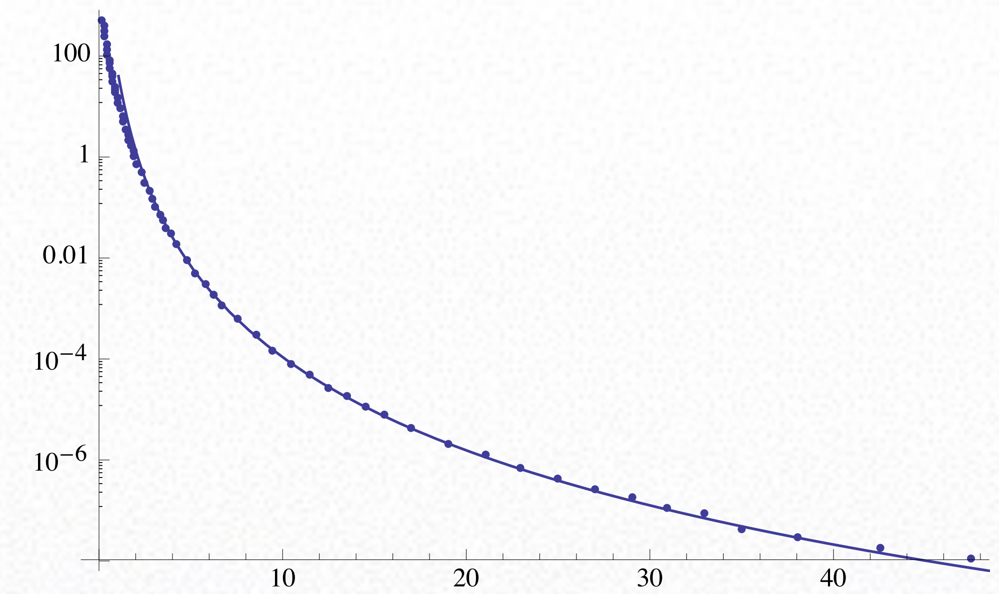
$$\frac{d\sigma}{d^2 p_t}\Big|_{y=0} \sim s^{\alpha_0 - 1} \kappa^{-\Delta_c} \sim p_t^{-2\Delta_c}$$

$Alice : arXiv : 1307.1093$



$\sqrt{s} = 900\text{GeV}$

$n_{eff} : 6 \sim 9$



$\sqrt{s} = 7\,\text{TeV}$

$n_{eff} : 6.5 \sim 8$

E. Nally, T. Raben and C-I Tan, *"Central Inclusive Production from AdS/CFT,"* (to appear).

Inclusive Production

$$a + b \to c + X \qquad \frac{d\sigma}{(d^3k/E)} = (1/s)(1/2i)Disc_{M^2}\, T_{3\to 3}(k_a, k_b, k_c'; k_a', k_b', k_c)$$

$$\frac{d\sigma}{d^2p_t}|_{y=0} \sim s^{\alpha_0 - 1}\kappa^{-\Delta_c} \sim p_t^{-2\Delta_c} \qquad \frac{d\sigma}{d^2p_t}|_{y=0} \sim p_t^{-8} \quad \text{(gluon dominance)}$$

- can also treat other regions, e.g., triple-Regge limit
- can be generalized to multi-particle inclusive production

$$\frac{d\sigma}{d^2p_{t,1}d^2p_{t,2}\cdots}|_{y=0} \sim p_t^{-2\sum \Delta_i}$$

- study both conformal behavior and effects of Confinement

IV: Pomeron in the conformal Limit, OPE, and Anomalous Dimensions

$$G_{mn} = g^0_{mn} + h_{mn}$$

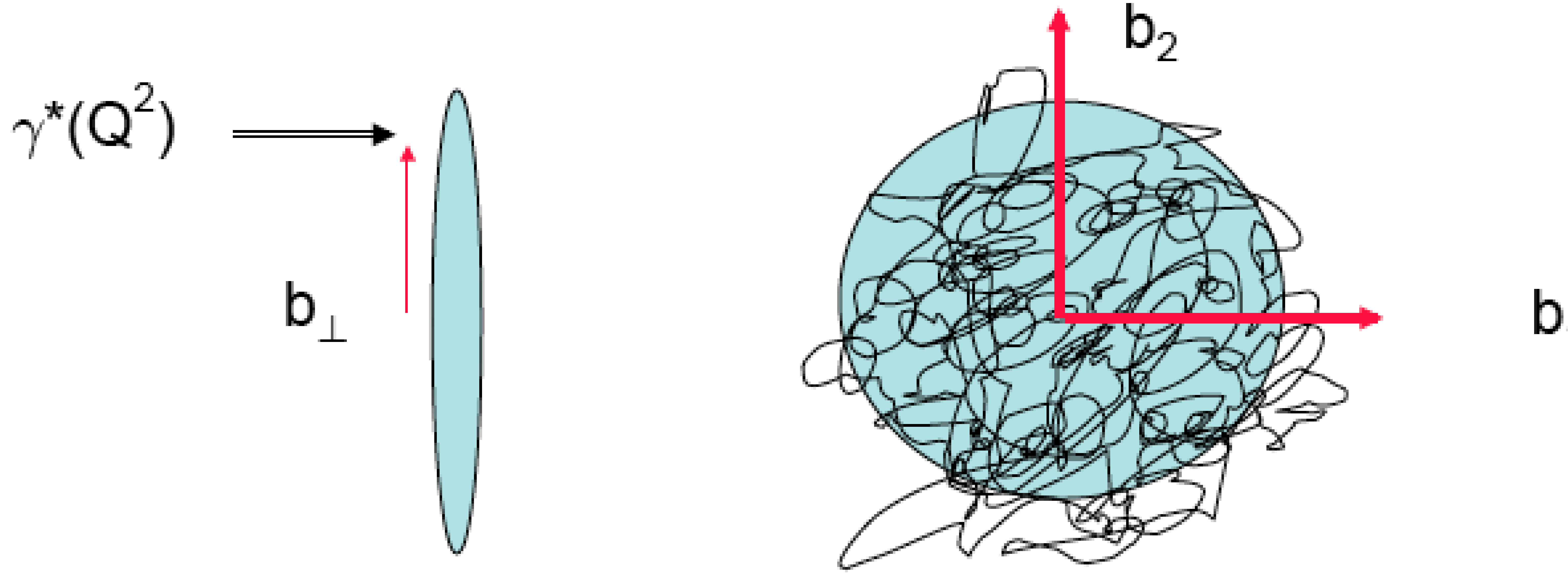
Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

QCD EMERGENCE OF 5-DIM ADS

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

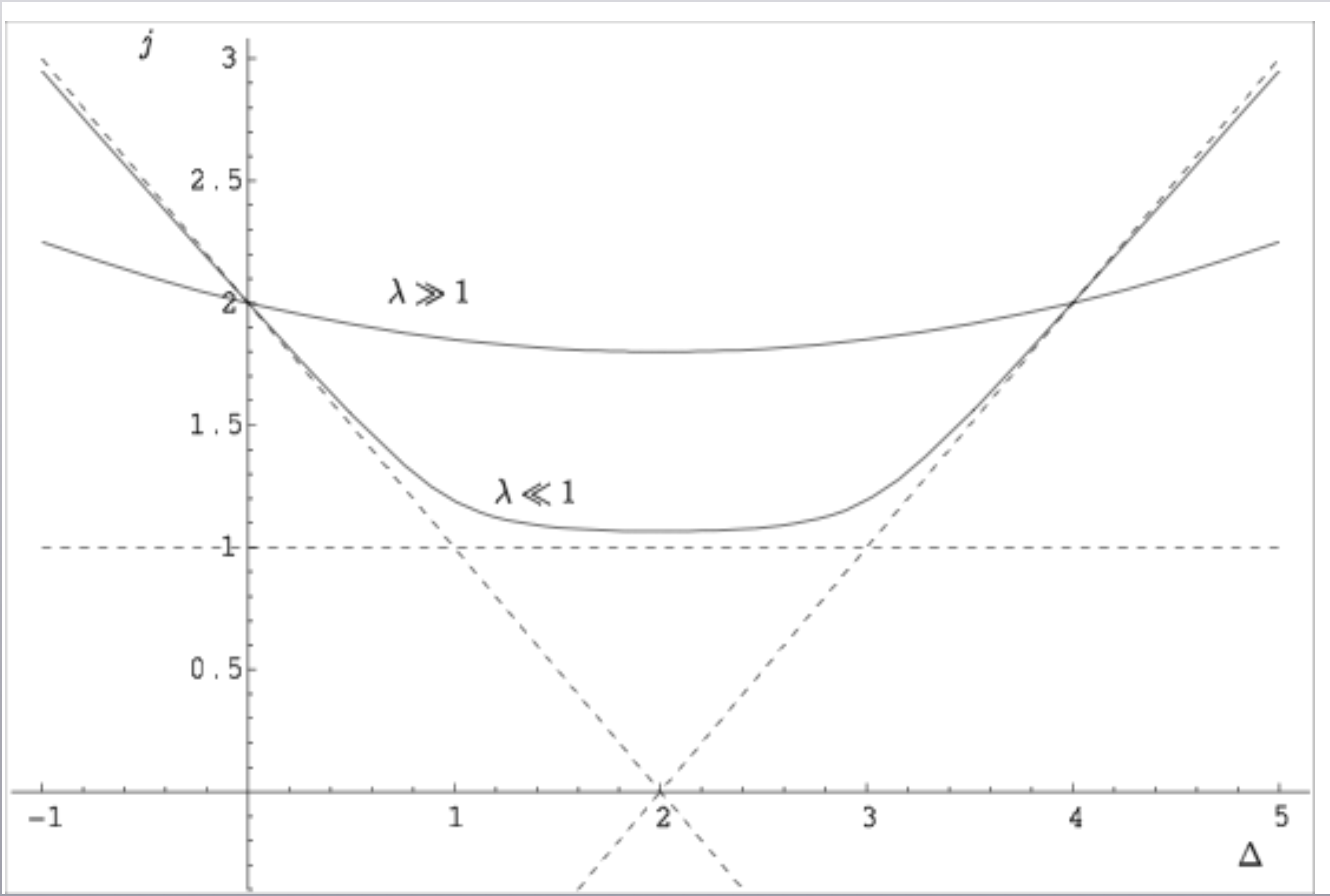
2-d Longitudinal $p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$

2-d Transverse space: $x'_\perp - x_\perp = b_\perp$

1-d Resolution: $z = 1/Q$ (or $z' = 1/Q'$)

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Operators that contribute are the twist 2 operators

$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$

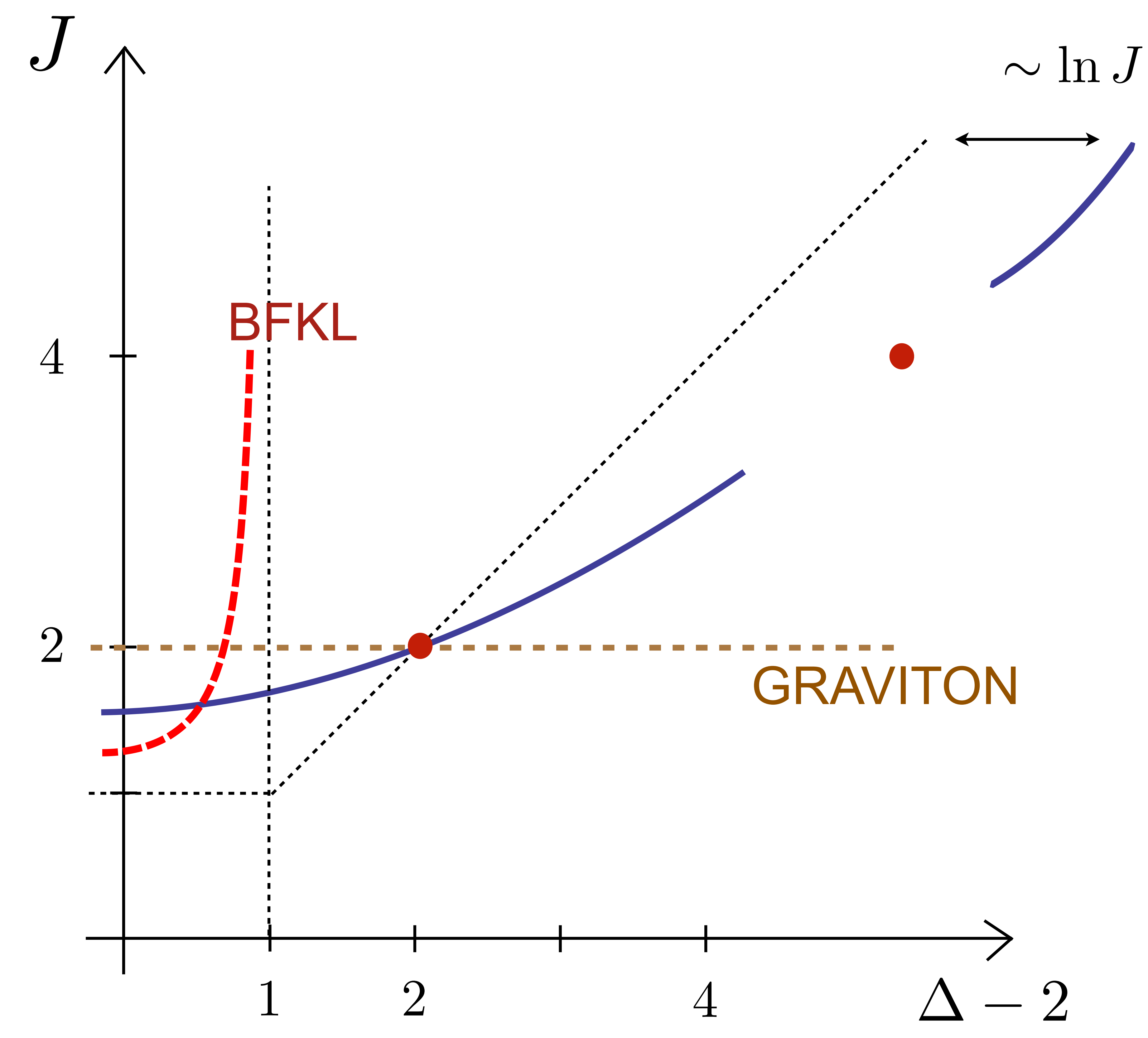
- Dual to string theory spin J field in leading Regge trajectory

$$\left(D^2 - m^2\right) h_{a_1 \dots a_J} = 0$$

$$m^2 = \Delta(\Delta - 4) - J\,, \qquad \Delta = \Delta(J)$$

- Diffusion limit

$$J(\Delta) = J_0 + \mathcal{D}\,(\Delta - 2)^2 \;\Longrightarrow\; m^2 = \frac{2}{\alpha'}(J-2) - \frac{J}{L^2}$$



POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}}$$

Brower, Polchinski, Strassler, Tan

Kotikov, Lipatov, et al.

ODDERON

Solution-a: $\alpha_O = 1 - \frac{8}{\lambda^{1/2}} -$

Solution-b: $\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

Kotikov, Lipatov, et al.

Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

Gromov et al.

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Costa, Goncalves, Penedones (1209.4355)

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Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1823}{16}}{\lambda^{5/2}} + \frac{720\zeta(5)+1344\zeta(3)-\frac{3585}{4}}{\lambda^3}$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

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Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} - \frac{0}{\lambda} + \frac{0}{\lambda^{3/2}} + \frac{0}{\lambda^2} + \frac{0}{\lambda^{5/2}} + \frac{0}{\lambda^3} + \cdots$$

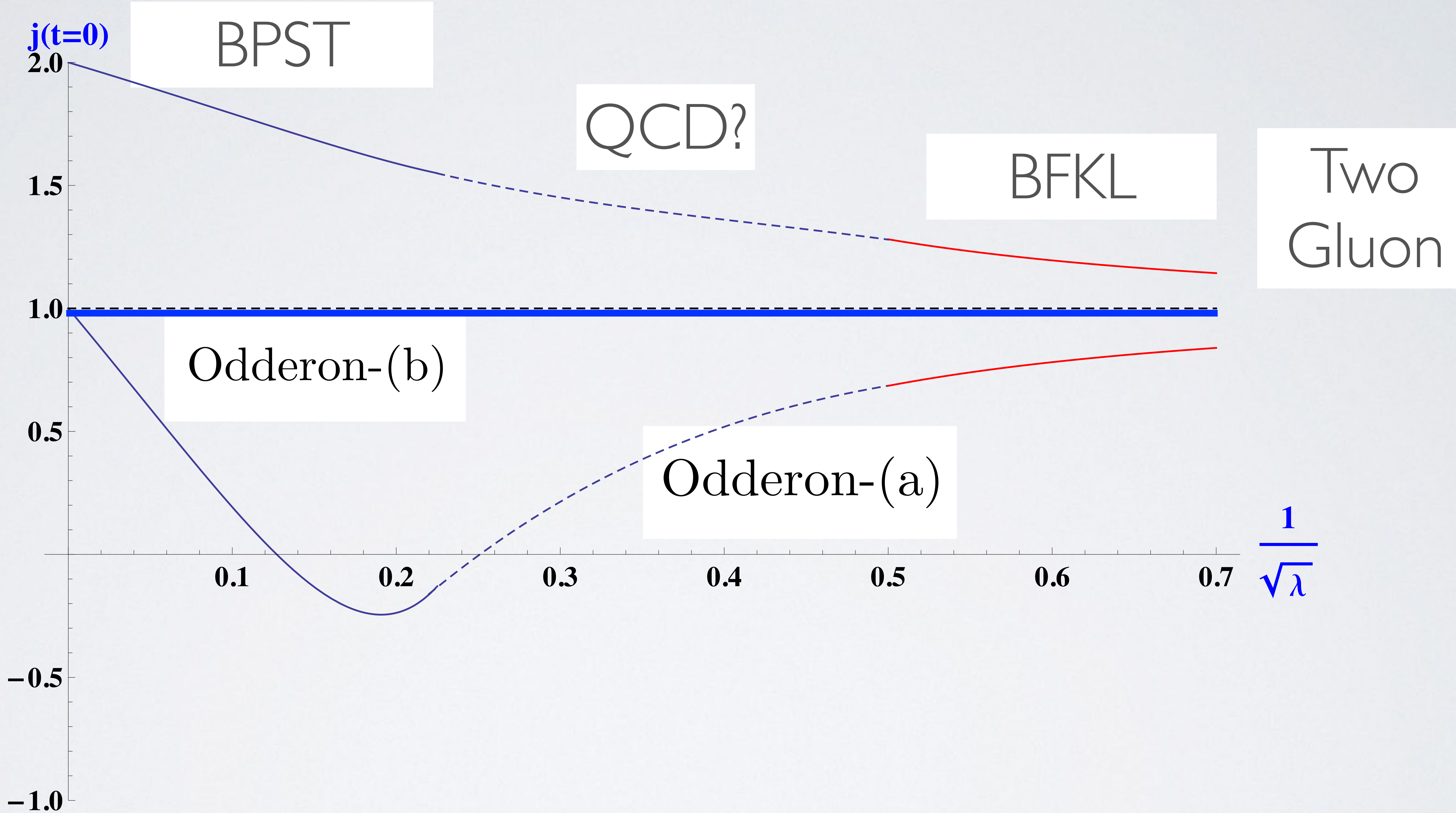
Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

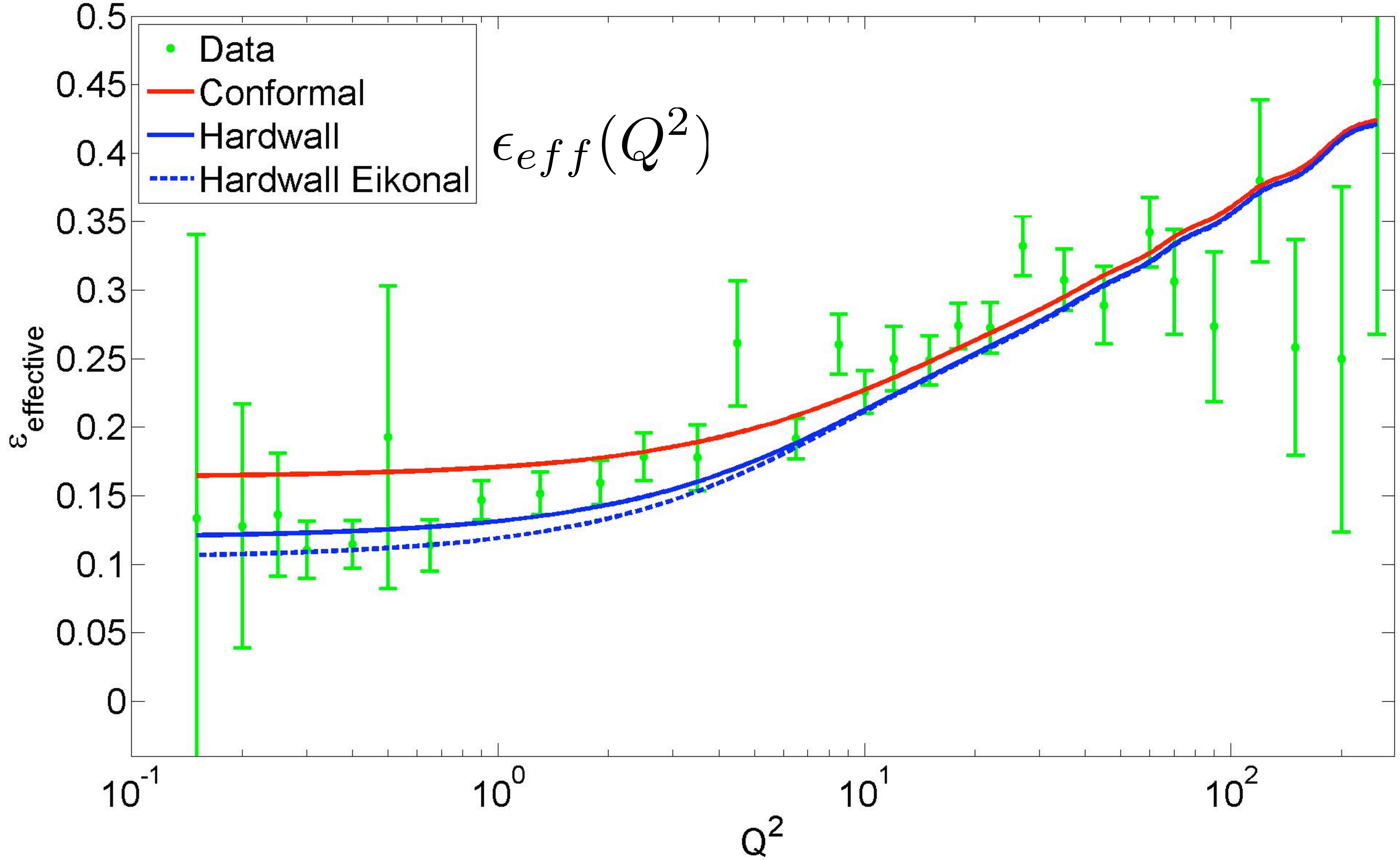
$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$

Graviton



$j_0 = 1$

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effecti}}$$



VII. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- First principle description of elastic/total cross sections, DIS at small-x, Central Diffractive Glueball production at LHC, etc.
- Inclusive Production and Dimensional Scalings.

Backup Slides

Conformal Invariance as Isometry of AdS

Longitudinal Boost: $\tau = \log(\rho z z' s/2)$ $\quad \mathcal{K}(s,\vec{b},z,z') = \int \frac{dj}{2\pi i}\Big(\frac{e^{-i\pi j}+1}{\sin \pi j}\Big) e^{j\tau} \mathcal{K}(j,\vec{b},z,z')$

Conformal Invariance in Transverse AdS_3: $\xi = \sinh^{-1}\Big(\frac{b^2+(z-z')^2}{2zz'}\Big)$

$$\mathcal{K}(j,\vec{b},z,z') = \int \frac{d\nu}{2\pi}\Big(\frac{e^{i\nu\xi}}{\sinh\xi}\Big) G(j,\nu)$$

Pomeron as a pole in AdS: $G(j,\nu) = \frac{1}{j - j_0 + \nu^2/2\sqrt{\lambda}}$

Full Conformal Invariance:

$$\mathrm{Im}\, \mathcal{K}(s,\vec{b},z,z') = \int \frac{dj}{2\pi i} \int \frac{d\nu}{2\pi}\Big(\frac{e^{j\tau} e^{i\nu\xi}}{\sinh\xi}\Big) G(j,\nu)$$

$$\Delta(j) = 2 + 2\sqrt{(j-j_0)/\rho}$$

$$\mathcal{K}(j,\vec{b},z,z') \sim \frac{e^{(2-\Delta(j))\xi}}{\sinh\xi} \qquad \mathcal{K}(s,b,z,z') \sim e^{j_0}\Big(\frac{\xi}{\sinh\xi}\,\frac{\exp(-\frac{\xi^2}{\rho\tau})}{\tau^{3/2}}\Big)$$

Propagators and Wave functions

In this framework the pomeron propagator obeys:

$$\Big[-\partial_z^2+\Lambda^4 z^2+(2\Lambda^2-t)+\frac{\alpha^2(j)-1/4}{z^2}\Big]\chi_P(j,z,z',t)=\delta(z-z')$$

$$\alpha(j)=\Delta(j)-2$$

Where as for a continuous t spectrum the solution becomes a combination of Whittaker's functions (generalized hyper geometric functions)

$$\chi_P\sim ...M_{\kappa,\mu}(z_<)W_{\kappa,\mu}(z_>) \qquad (2)$$

for $\kappa=\kappa(t)$ and $\mu=\mu(j)$

$$\kappa(t)=t/4\Lambda^2-1/2 \qquad \mu(j)=\alpha(j)/2$$

Special Limits, Behavior, and Symmetry

- Λ controls the strength of the soft wall and in the limit $\Lambda \to 0$ one recovers the conformal solution

$$\mathrm{Im}\chi_P^{conformal}(t=0) = \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}(zz')\frac{e^{(1-\rho)\tau}}{\tau^{1/2}}\,exp\left(\frac{-(\mathrm{Log}z - \mathrm{Log}z')^2}{\rho\tau}\right)$$

where $\tau = \mathrm{Log}(\rho zz's/2)$ and $\rho = 2 - j_0$. Note: this has a similar behavior to the weak coupling BFKL solution where

$$\mathrm{Im}\chi(p_\perp, p'_\perp, s) \sim \frac{s^{j_0-1}}{\sqrt{\pi\mathcal{D}\mathrm{Log}s}}\exp(-(\mathrm{Log}p'_\perp - \mathrm{Log}p_\perp)^2/\mathcal{D}\mathrm{Log}s)$$

- If we look at the energy dependence of the pomeron propagator, we can see a softened behavior in the forward regge limit.

$$\chi_{conformal} \sim \frac{s^{j_0-1}}{\sqrt{\log s}} \to \chi_{HW} \sim \frac{s^{j_0-1}}{(\log s)^{3/2}}$$

Analytically, this corresponded to the softening of a j-plane singularity from $1/\sqrt{j-j_0} \to \sqrt{j-j_0}$. Again, we see this same softened behavior in the soft wall model.

For two-to-two scattering involving on-shell hadrons, it is convenient to express the amplitude as

$$A_4(s,t) \simeq 2s \int d^2b e^{-i\mathbf{b}\mathbf{q}_\perp} \int dz dz' P_{13}(z) P_{24}(z')\, \chi(s,b,z,z'),$$

where, for scalar glueball states,

$$P_{ij}(z) = \sqrt{-g(z)}(z/R)^2 \phi_i(z)\phi_j(z)$$

involves a product of two external normalizable wave functions. We have introduced function $\chi(s,b,z,z')$, the "eikonal", where

$$\chi(s,b,z,z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s,b,z,z')$$

and $\mathcal{K}(s,b,z,z')$ is the BPST Pomeron kernel.

- We are interested in calculating the structure function $F_2(x, Q^2)$, which is simply the cross section for an off-shell photon. Using the optical theorem we obtain

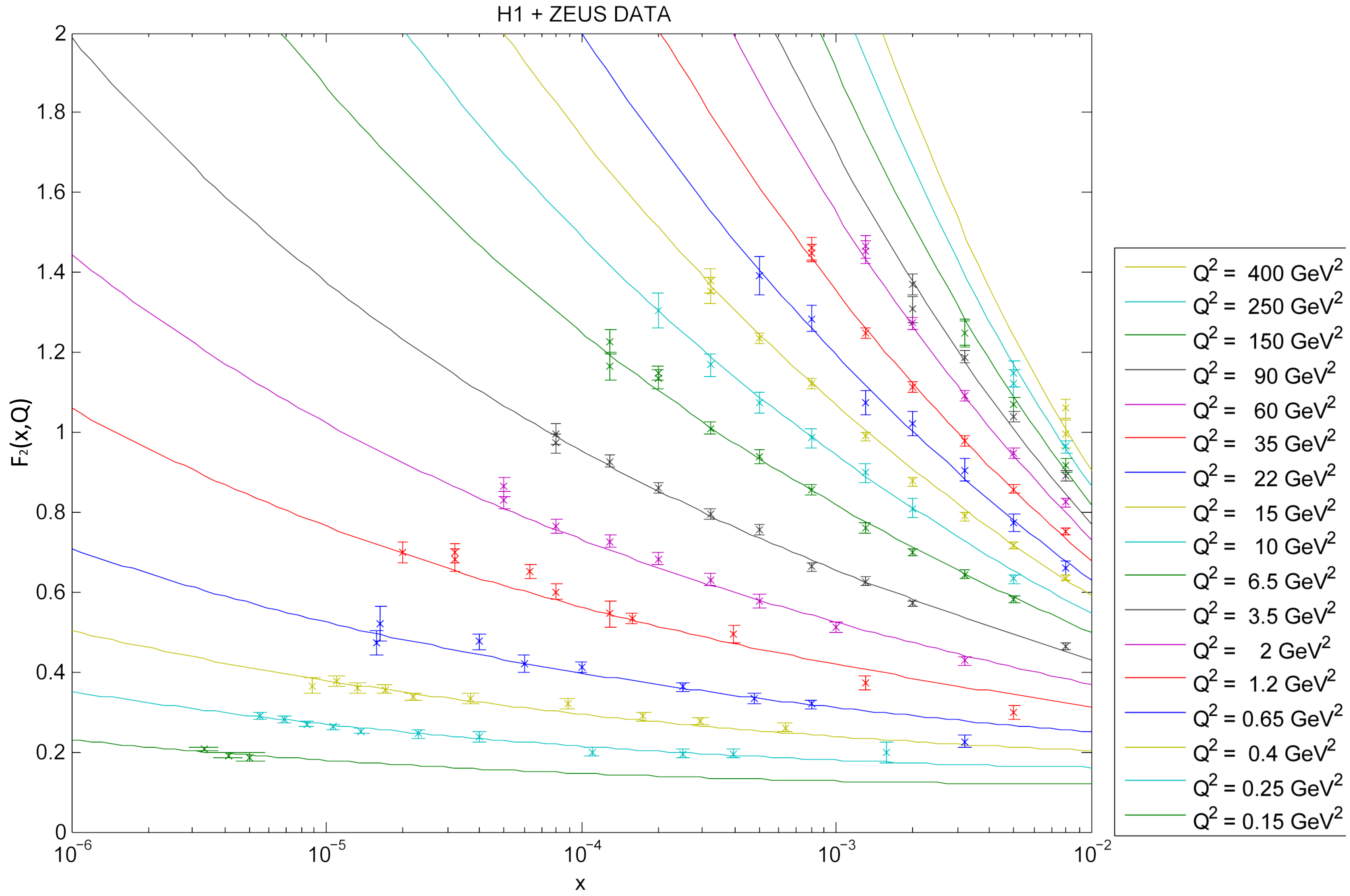
$$\sigma_{tot} \simeq 2 \int d^2b \int dz dz' P_{13}(z) P_{24}(z') \, Im \, \chi(s, b, z, z')$$

- For DIS, P_{13} should present a photon on the boundary that couples to a spin 1 current in the bulk. This current then propagates through the bulk, and scatters off the target.
- The wave function, in the conformal limit, is

$$P_{13}(z) \rightarrow P_{13}(z, Q) = \frac{1}{z}(Qz)^4 (K_0^2(Qz) + K_1^2(Qz))$$

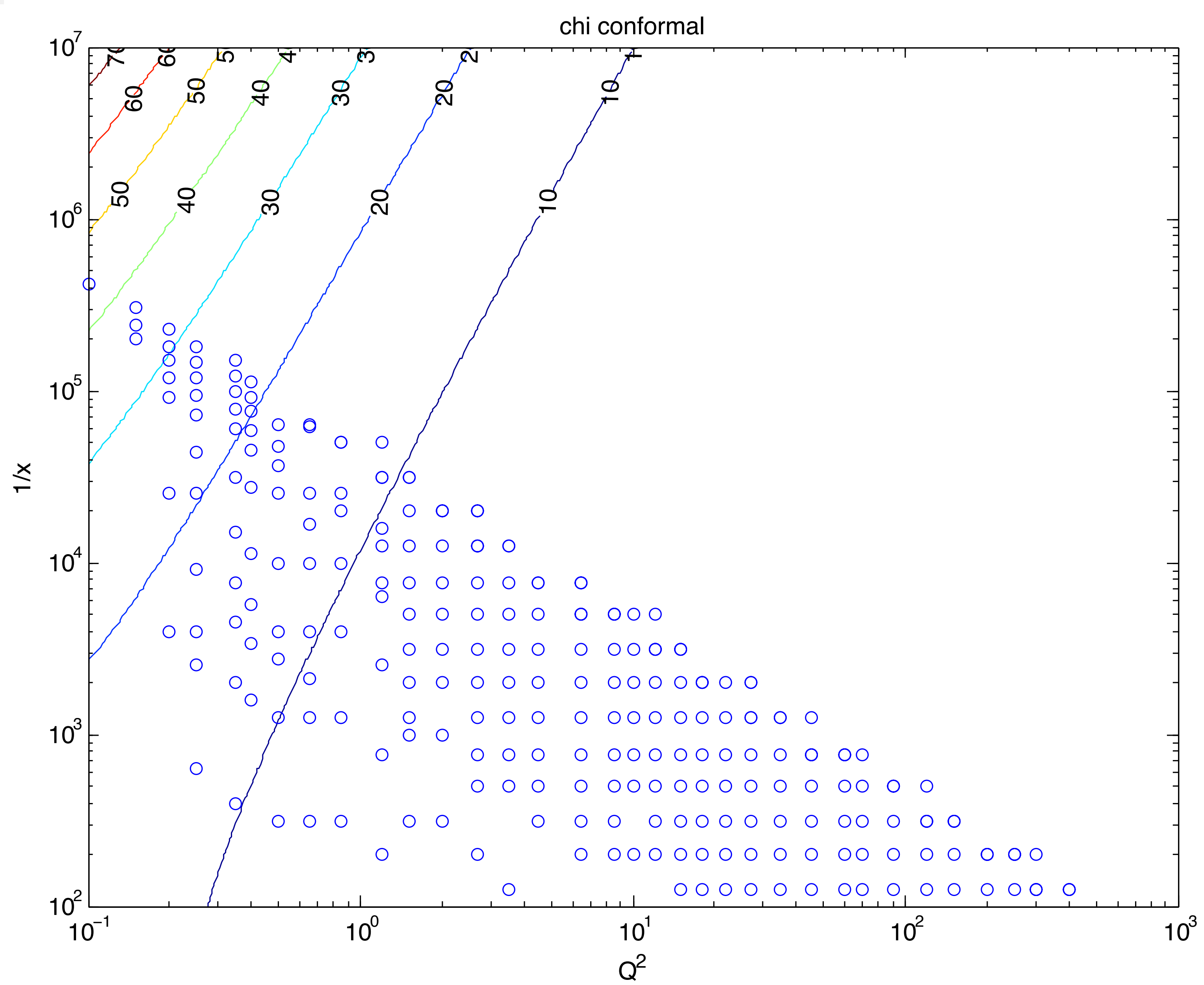
- For the proton, one for now treats it as a glueball of mass $\sim \Lambda = 1/Q'$.

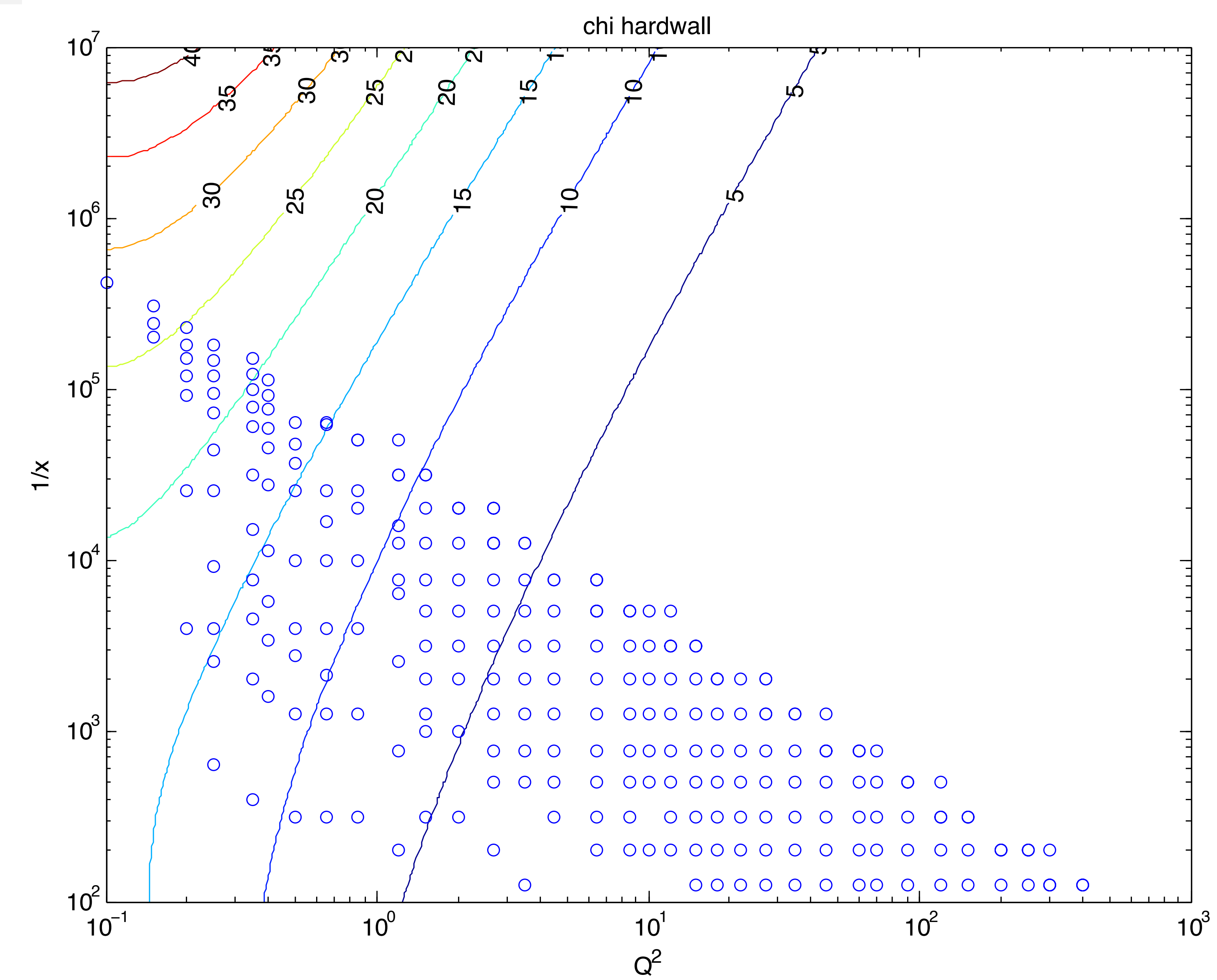
Plots

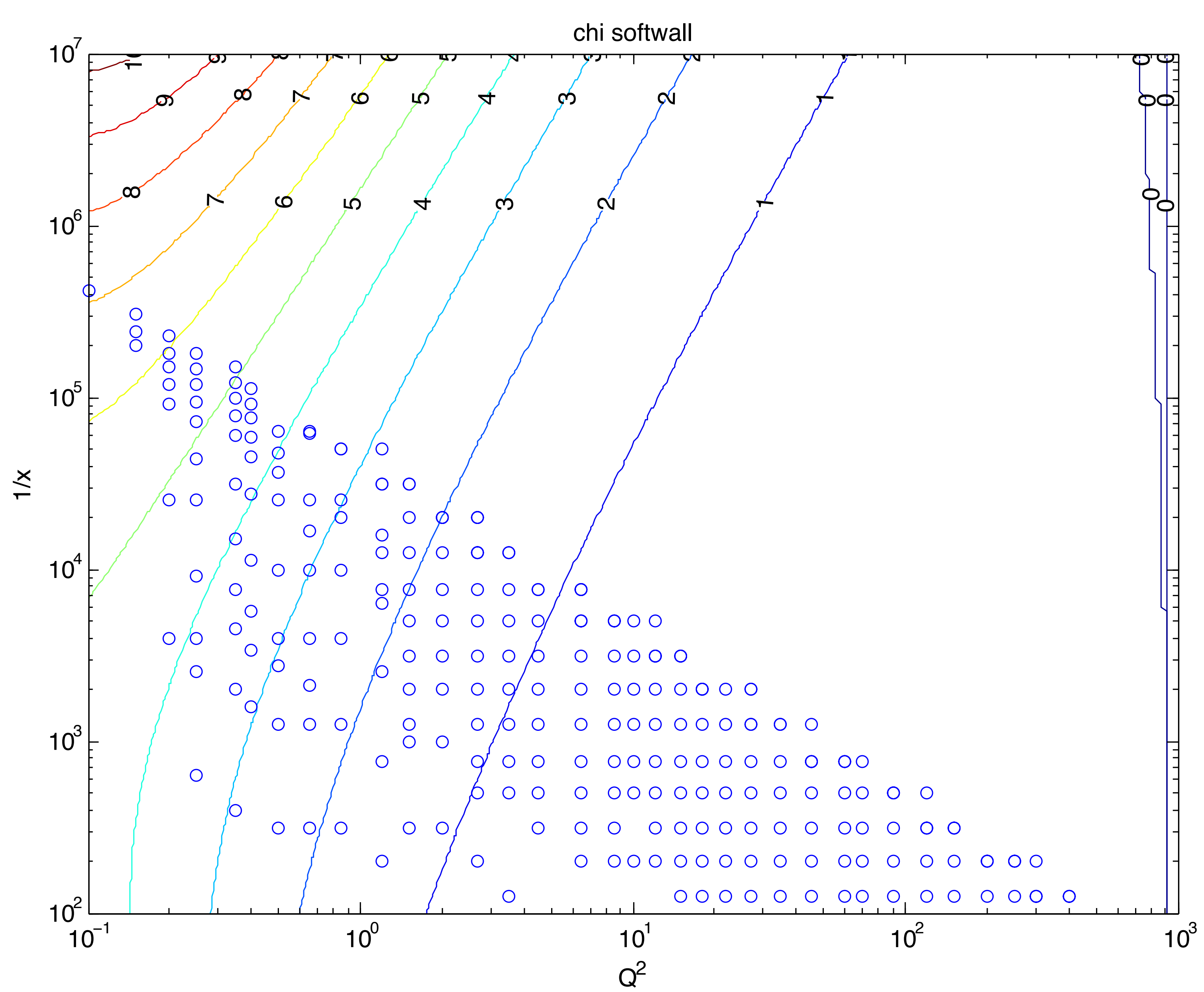


The structure function $F_2(x, Q^2)$ plotted for farious values of Q^2. The data points are from the H1-Zeus collaboration and the solid lines are the soft wall fit values.

Plots Cont.



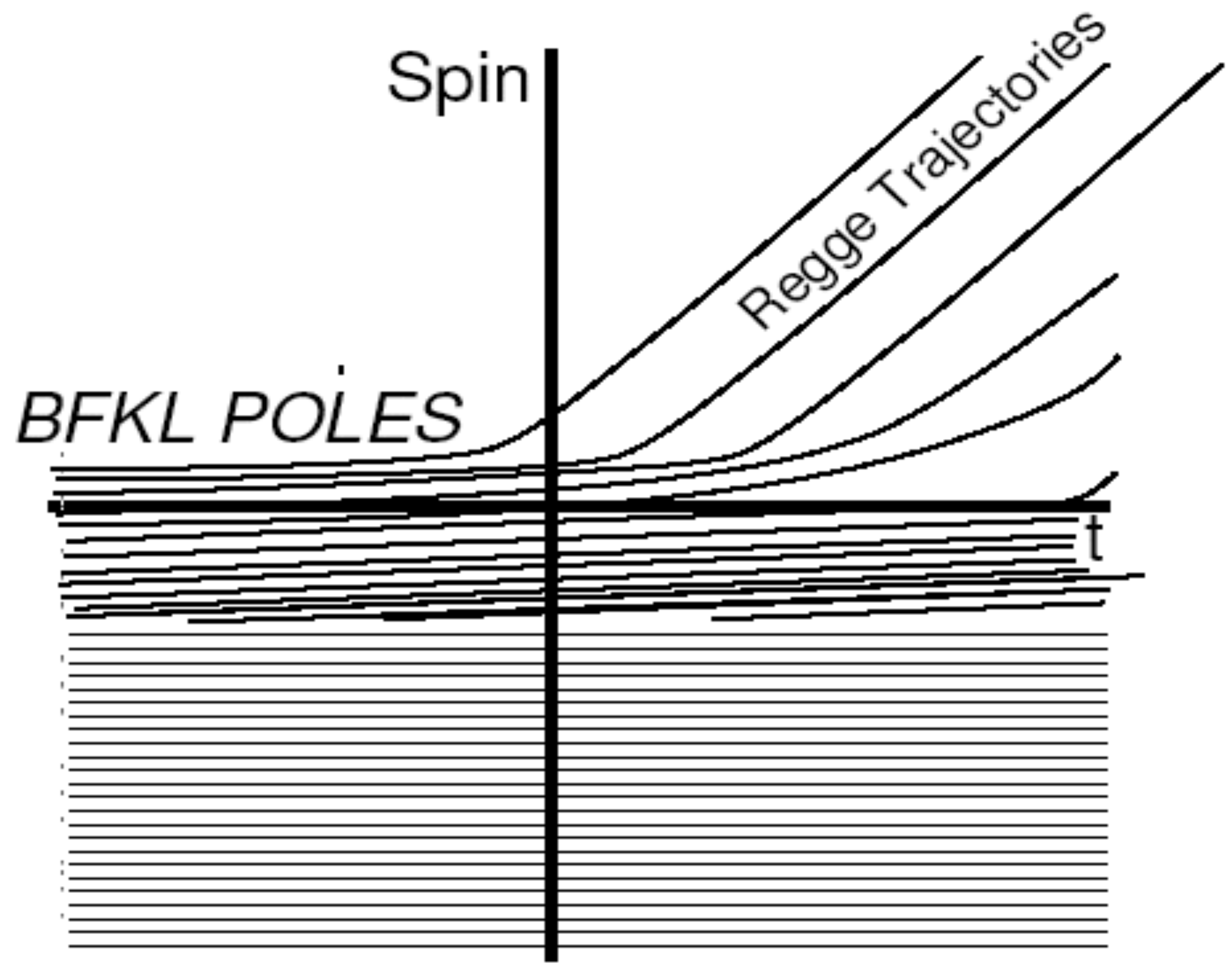




Contour plots of Im$[\chi]$ as a function of $1/x$ vs Q^2 (Gev) for conformal, hardwall, and softwall models. These plots are all in the forward limit, but the impact parameter representation can tell us about the onset of non-linear eikonal effects. The similar behavior for the softwall implies a similar conclusion about confinement vs saturation.

Pomeron in QCD

Running UV, Confining IR (large N)



The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

CFT correlate function – coordinate representation

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle$$

OPE: $$\phi(x_1)\phi_2(x_2) \simeq \sum_k C_{1,2;k}(x_{12}, \partial_1)\mathcal{O}_k(x_1)$$

Bootstrap: s-channel OPE = t-channel OPE

unitarity, positivity, locality, analyticity, etc.

Dynamics: $\mathcal{O}_{(\Delta,j)_k}(x)$ Conformal Dimension, Spin

$$A(s,b) = \int dz \int dz' \int_{-i\infty}^{i\infty} \frac{d\Delta}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dj}{2\pi} \; \mathcal{A}(\Delta, j, z, z') \;\; \widetilde{s}^{\,j} \;\; \mathcal{Y}_\Delta(L_{(b,z,z')})$$

AdS/CFT:

$$\mathcal{A}(\Delta, j, z, z') = \Phi_1(z)\Phi_2(z)\Phi_3(z')\Phi_4(z') \times \frac{1+e^{-i\pi j}}{\sin \pi j} \times A(\Delta, j)$$

Dynamics:

$$A(\Delta, j) \sim \frac{1}{\Delta - \Delta(j,\lambda)}$$

$$A(s,b) = \int dz dz' \, \Pi \Phi_i \sum_{j=0,2,\cdots} \beta(j) \, \widetilde{s}^{\,j} \;\; \mathcal{Y}_{\Delta(j)}(L_{(z,z',b)})$$

Anomalous Dimension: $\gamma(j,\lambda) \equiv \Delta(j,\lambda) - j - 2$

In the limit $\lambda \to \infty$, only $j = 2$ survives.

Dynamics

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$Tr[F^2], \quad Tr[F_{\mu\rho}F_{\rho\nu}], \quad Tr[F_{\mu\rho}D_{\pm}^{S}F_{\rho\nu}], \quad Tr[Z^{\tau}], \quad Tr[D_{\pm}^{S}Z^{\tau}], \cdots$$

Super-gravity in the $\lambda \to \infty$:

$$Tr[F^2] \leftrightarrow \phi, \quad Tr[F_{\mu\rho}F_{\rho\nu}] \leftrightarrow G_{\mu\nu}, \quad \cdots$$

Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

Graviton Spectral Curve:

$$a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \rightarrow \quad \frac{1}{\Delta - \Delta(j)}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$Tr[F_{\pm\perp} D_{\pm}^{j-2} F_{\perp\pm}], \quad j = 2, 4, \cdots$$

Super-gravity in the $\lambda \rightarrow \infty$:

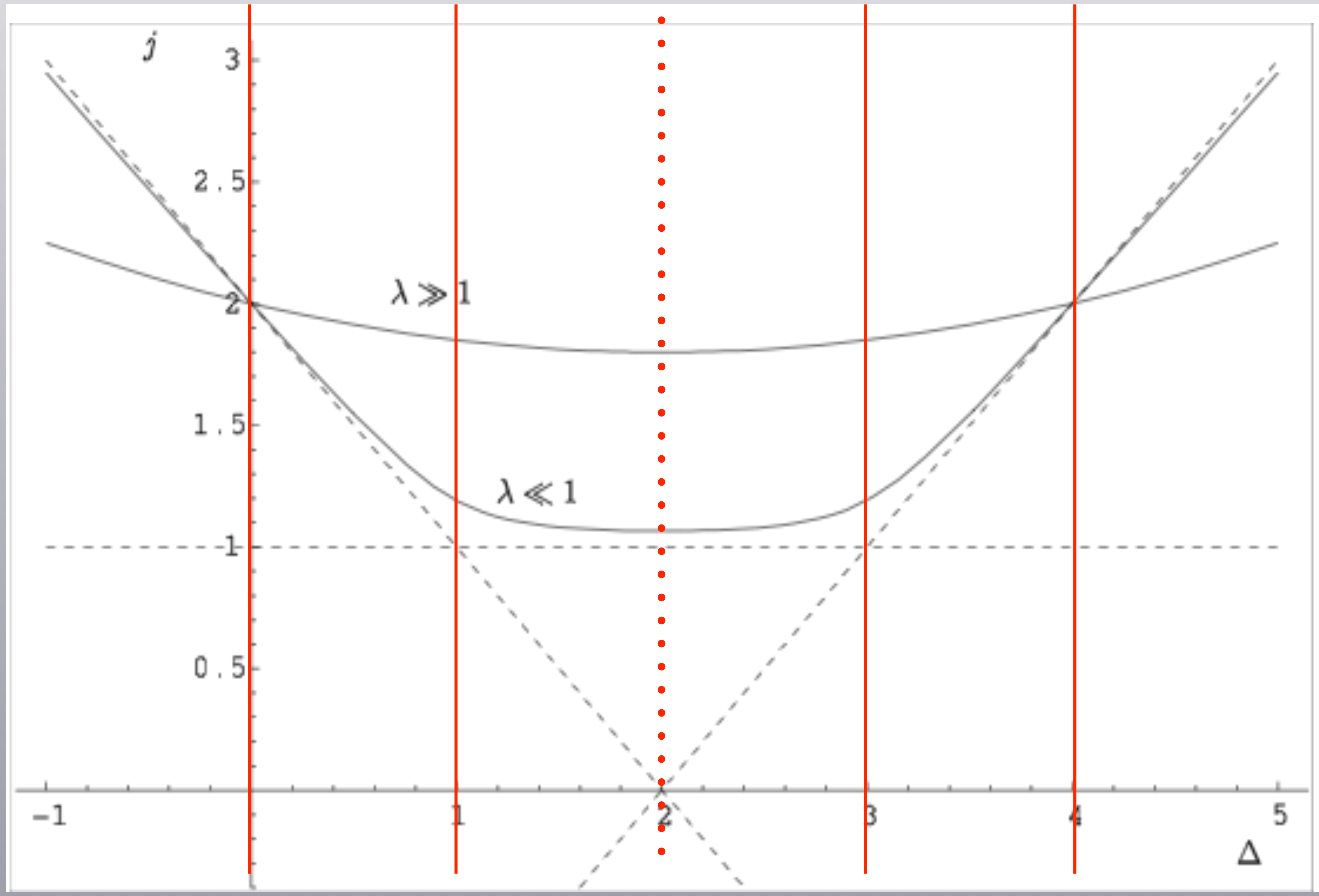
$$\Delta(2) = 4; \quad \Delta(j) = O(\lambda^{1/4}) \rightarrow \infty, \quad j > 2$$

Symmetry of Spectral Curve:

$$\Delta(j) \leftrightarrow 4 - \Delta(j)$$

ANOMALOUS DIMENSIONS:

$$\gamma(j,\lambda) = \Delta(j,\lambda) - j - 2$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Energy-Momentum Conservation built-in automatically.