Evolution of double parton distributions - fighting with initial conditions

Krzysztof Golec-Biernat

IFJ PAN and Rzeszow University

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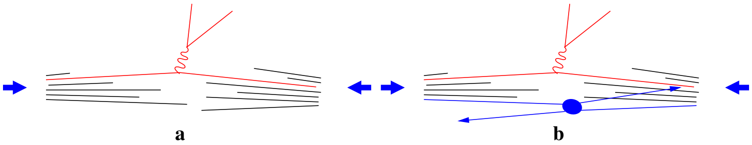
Double parton scattering

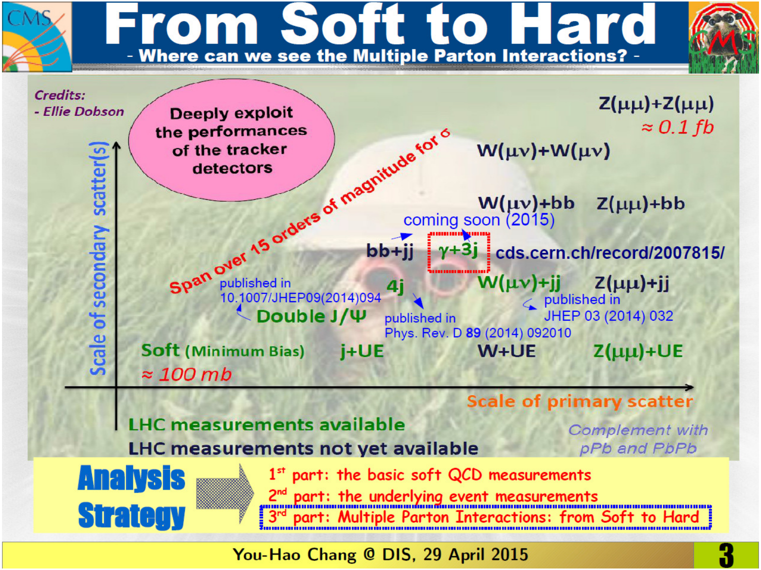
- At the LHC multiparton interactions (MPI) become increasingly important.
- If no hard scale is involved, MPI are crucial for modeling of underlying event.
- It two hard scales are involved, Q_1 and Q_2, double parton scattering.



$$pp \to X_{hard} + Y_{hard} + soft$$

- Double parton distribution functions in addition to single PDFs.

(M. Diehl, D. Ostermeier, A. Schafer, JHEP 1203 (2012) 089).


From Soft to Hard
- Where can we see the Multiple Parton Interactions? -
Credits:
- Ellie Dobson
Deeply exploit the performances of the tracker detectors
Span over 15 orders of magnitude for σ
Scale of secondary scatter(s)
Z(μμ)+Z(μμ)
≈ 0.1 fb
W(μν)+W(μν)
W(μν)+bb
Z(μμ)+bb
coming soon (2015)
bb+jj
γ+3j
cds.cern.ch/record/2007815/
published in
10.1007/JHEP09(2014)094
4j
W(μν)+jj
Z(μμ)+jj
published in
JHEP 03 (2014) 032
Double J/Ψ
published in
Phys. Rev. D 89 (2014) 092010
Soft (Minimum Bias)
≈ 100 mb
j+UE
W+UE
Z(μμ)+UE
Scale of primary scatter
LHC measurements available
LHC measurements not yet available
Complement with pPb and PbPb
Analysis Strategy
1st part: the basic soft QCD measurements
2nd part: the underlying event measurements
3rd part: Multiple Parton Interactions: from Soft to Hard
You-Hao Chang @ DIS, 29 April 2015
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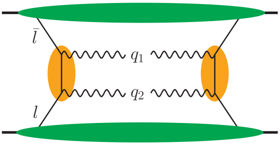
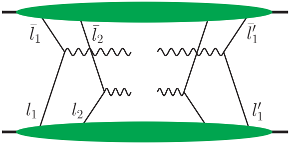
Collinear factorization cross sections

- Single versus double parton scattering





- Single parton scattering - single PDFs

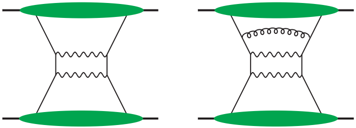
$$\frac{d\sigma_{AB}^{SPS}}{dx\, d\overline{x}} = \sum_{ff'} D_f(x, Q)\, \sigma_{ff'}^{\mathrm{AB}}(Q)\, D_{f'}(\overline{x}, Q)$$

- Double parton scattering - double parton distributions DPDFs

$$\frac{d\sigma_{AB}^{DPS}}{dx_1\, dx_2\, dx'_1\, dx'_2} = \sum_{flav} \int_{\mathbf{q}} D_{f_1 f_2}(x_1, x_2, Q_1, Q_2, \mathbf{q})\, \sigma^{A}_{f_1 f'_1}(Q_1)\, \sigma^{B}_{f_2 f'_2}(Q_2)\, D_{f'_1 f'_2}(x'_1, x'_2, Q_1, Q_2, -\mathbf{q})$$

QCD evolution equations for single PDFs

- Quark-gluon emissions: $\Lambda_{QCD} < k_\perp < Q$

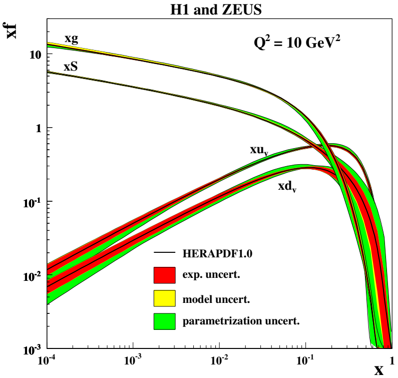


- PDFs are scale dependent: $D_f(x, Q), \quad f \in \{q, \overline{q}, g\}$
- DGLAP evolution equations:

$$\frac{\partial}{\partial \ln Q^2} D_f(x, Q) = \frac{\alpha_s}{2\pi} \sum_{f'} \int_x^1 du \, \mathcal{P}_{ff'}(x/u) \, D_{f'}(u, Q)$$

- Initial conditions, $D_f(x, Q_0)$, are very well known from global fits.

PDFs from global fits



QCD evolution equations for DPDFs

- Quark-gluon emissions: $\Lambda_{QCD} < k_\perp < Q$

- DGLAP type evolution equations in LLA with additional splitting terms

$$
\begin{aligned}
\frac{\partial}{\partial \ln Q^2} D_{f_1 f_2}(x_1, x_2, Q) &= \frac{\alpha_s}{2\pi} \sum_{f'} \Bigg\{ \int_{x_1}^{1-x_2} du\, \mathcal{P}_{f_1 f'}(x_1/u)\, D_{f' f_2}(u, x_2, Q) \\
&+ \int_{x_2}^{1-x_1} du\, \mathcal{P}_{f_2 f'}(x_2/u)\, D_{f_1 f'}(x_1, u, Q) \\
&+ \mathcal{P}_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q) \Bigg\}
\end{aligned}
$$

- This is for equal hard scales, $Q_1 = Q_2 \equiv Q$, and $\mathbf{q} = \mathbf{0}$.

Evolution for $Q_2 > Q_1$

- Evolve from Q_0 to $Q = Q_1 = Q_2$ with the previous equations.
- Evolve second parton from Q_1 to Q_2 using DGLAP equations with fixed (x_1, Q_1)

$$\frac{\partial}{\partial \ln Q_2^2} D_{f_1 f_2}(x_1, x_2, Q_1, Q_2) = \int_{x_2}^{1-x_1} du\, \mathcal{P}_{f_2 f'}(x_2/u)\, D_{f_1 f'}(x_1, u, Q_1, Q_2)$$

- For both evolutions we need initial conditions at initial scale Q_0,

$$D_{f_1 f_2}(x_1, x_2) \qquad \text{and} \qquad D_f(x)$$

Sum rules

- Evolution equations preserve momentum and valence quark number sum rules.
- For single PDFs:

$$\sum_{f\in\{q,\bar{q},g\}} \int_0^1 dx\, x\, D_f(x) = 1$$

$$\int_0^1 dx\,(D_q(x) - D_{\bar{q}}(x)) = N_q$$

- For double PDFs:

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2) = (1-x_2)\, D_{f_2}(x_2)$$

$$\int_0^{1-x_2} dx_1 \left\{ D_{q f_2}(x_1, x_2) - D_{\bar{q} f_2}(x_1, x_2) \right\} = (N_q - \delta_{q f_2} + \delta_{\bar{q} f_2})\, D_{f_2}(x_2)$$

- Impose them on initial conditions at initial scale Q_0.

Existing initial conditions

- Most popular do not exactly obey sum rules (J. Gaunt, W. J. Stirling, JHEP 1106, 048 (2011))

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, \frac{(1-x_1-x_2)^2}{(1-x_1)^{2+n_1}(1-x_2)^{2+n_2}}$$

- Initial conditions which obey sum rules (E. Lewandowska, KGB, PRD 90, 094032 (2014))

$$\begin{aligned} D_{f_1 f_2}(x_1, x_2) &= \frac{1}{1-x_2} D_{f_1}\left(\frac{x_1}{1-x_2}\right) D_{f_2}(x_2) \\ D_{qq}(x_1, x_2) &= \frac{1}{1-x_2}\left\{D_q\left(\frac{x_1}{1-x_2}\right) - \frac{1}{2}\right\} D_q(x_2) \\ D_{q\bar{q}}(x_1, x_2) &= \frac{1}{1-x_2}\left\{D_q\left(\frac{x_1}{1-x_2}\right) + \frac{1}{2}\right\} D_{\bar{q}}(x_2) \end{aligned}$$

No parton exchange symmetry, negative D_{qq} for large x.

- New analysis for gluons (KGB, E. Lewandowska, M. Serino, Z. Snyder, A. Staśto, arXiv:1507.08583)

Pure gluon case

- MSTW08 parameterization at $Q_0 = 1\ \mathrm{GeV}$:

$$D_g(x) = A\, x^{\delta-1}\, (1-x)^{\eta}\, (1+\epsilon\sqrt{x}+\gamma\, x)$$

with known parameters. Notice that

$$D_g(x) = \sum_{k=1}^{3} A_k\, x^{\alpha_k}\, (1-x)^{\eta}$$

- Solve momentum sum rule

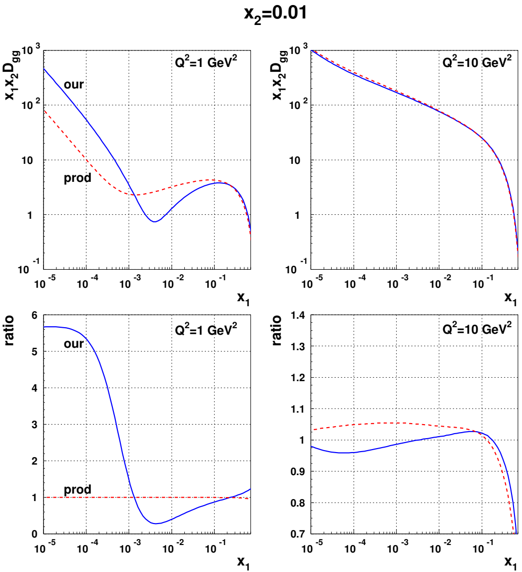
$$\int_0^{1-x_2} dx_1\, x_1\, D_{gg}(x_1, x_2) = (1-x_2)\, D_g(x_2)$$

for

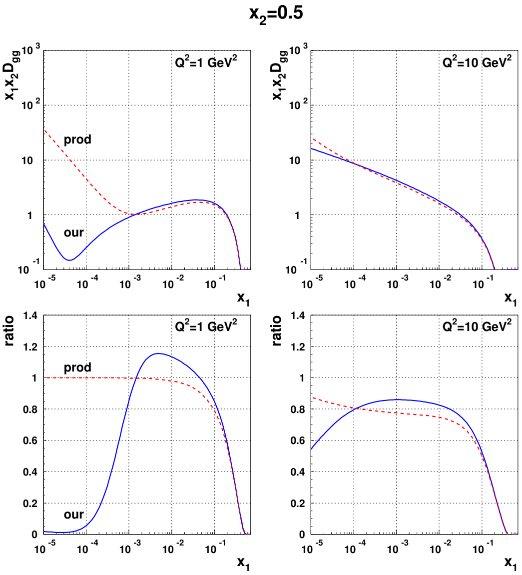
$$\sum_{k=1}^{3} \int_0^{1-x_2} dx_1\, x_1 \left[\bar{N}_k\, (x_1\, x_2)^{a_k}\, (1-x_1-x_2)^{b_k}\right] = (1-x_2) \sum_{k=1}^{3} \bar{A}_k\, {x_2}^{a_k}\, (1-x_2)^{a_k+b_k+1}$$

- From the comparison: $a_k = \alpha_k$ and $b_k = \eta - \alpha_k - 1$.

Numerical results for $x_2 = 10^{-2}$



Numerical results for $x_2 = 0.5$



Summary

- The initial double gluon distribution which obey momentum sum rule differs significantly from the product form

$$D_{gg}(x_1, x_2) = D_g(x_1) D_g(x_2)$$

for $x_{1,2} < 0.1$.

- However, evolution to relatively low scale, $Q^2 = 10\ \mathrm{GeV}^2$, washes out this effect.

- For $x_{1,2} > 0.1$, the product form with a multiplicative correction factor

$$\rho(x_1, x_2) = \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 (1 - x_2)^2}$$

agrees with the constructed double gluon distribution.

Outlook

- How to include double distributions for quarks and gluons which obey additionally valence quark number sum rule?
- Light-cone Fock expansion of the proton state in terms of partonic excitations

$$|\Psi\rangle = \sum_{N} \sum_{f_1 \ldots f_N} \int dx_1 \ldots dx_N \, \delta(1 - \sum_{k=1}^{N} x_k) \, \Psi_N(x_1 \ldots x_n; f_1 \ldots f_N) \, |x_1 \ldots x_N; f_1 \ldots f_N\rangle$$

  is the origin of the sum rules.
- Try to model wave functions Ψ_N to reconstruct the known single PDFs, $D_f(x)$, and then compute the double distributions, $D_{f_1 f_2}(x_1, x_2)$.
- Work in progress.