# Improved effective TMD factorization for forward dijets in *pA* collisions


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supported by:
DEC-2011/01/B/ST2/03643
DE-FG02-93ER40771

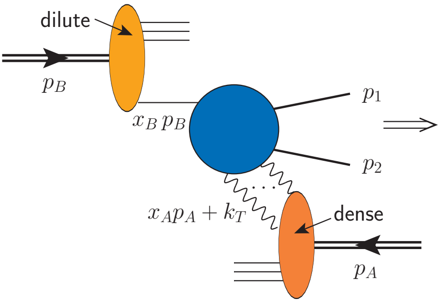


Low X 2015, Sandomierz, Poland

# Plan

1. Introduction
   - Hybrid approach for dijets in dilute-dense collisions
   - Regimes of dijet momentum imbalance
   - Effective TMD factorization in the jet back-to-back limit
2. Beynond the back-to-back limit: Improved Effective TMD factorization
3. Numerical studies (preliminary)
4. Summary

# Hybrid approach for dijets




forward dijets with transverse momentum imbalance:

$$\left|\vec{p}_{T1} + \vec{p}_{T2}\right| = \left|\vec{k}_T\right| = k_T$$

asymmteric kinematics:

$$x_B \gg x_A$$

- large-$x$ parton in hadron $B$ is treated as 'collinear' with standard PDFs
- small-$x$ partons witin hadron $A$ have internal transverse momentum $k_T$

## Three-scale problem

1. hard scale $P_T$ (of the order of the average transverse momentum of jets)
2. transverse momentum imbalance $k_T$
3. saturation scale $\Lambda_{\rm QCD} \ll Q_s$ (increasing with energy)

# Regimes of momentum imbalance

## Approaches and regimes

1. $Q_s \sim k_T \sim P_T$: Color Glass Condensate (CGC) regime

   [F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci. 60 (2010) 463]

   - quark wave functions convoluted with nonuniversal correlators of Wilson lines (eventually calculated from models)
   - valid in the saturation regime

2. $Q_s \ll k_T \sim P_T$: High Energy (or $k_T$) Factorization (HEF)

   [L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. 100 (1983) 1-150]
   [S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188]
   [J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3-30]

   - off-shell MEs convoluted with Unintegrated Gluon Distributions (UGDs)
   - does not apply in the saturation regime

3. $Q_s \sim k_T \ll P_T$: Effective (generalized) TMD factorization

   [F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

   - on-shell MEs convoluted with several UGDs expressable at large-$N_c$ by two universal UGDs
   - valid in the saturation regime

# Effective (generalized) TMD factorization (1)

[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

TMD approach to dijets:

$$\frac{d\sigma_{AB}}{dy_1 d^2p_{T1} dy_2 d^2p_{T1}} \sim \sum_{a,c,d} f_{a/B}\left(x_B, P_T^2\right) \sum_i \mathcal{F}_{ag}^{(i)}\left(x_A, k_T^2\right) H_{ag\to cd}^{(i)}$$

$H^{(i)}$ – hard on-shell factors, $f_{a/B}$ – collinear PDFs, $\mathcal{F}_{ag}^{(i)}$ – TMD Gluon Distributions:

$$\mathcal{F}_{qg}^{(1)} \sim \langle p_A|\,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle, \quad \mathcal{F}_{qg}^{(2)} \sim \langle p_A|\,\mathrm{Tr}\{F(\xi)\,\frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c}\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle,$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle p_A|\,\mathrm{Tr}\{F(\xi)\,\frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c}\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle, \quad \mathcal{F}_{gg}^{(2)} \sim \frac{1}{N_c}\,\langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\Box]\dagger}\right\}\mathrm{Tr}\left\{F(0)\,\mathcal{U}^{[\Box]}\right\}|p_A\rangle,$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p_A\rangle, \quad \mathcal{F}_{gg}^{(4)} \sim \langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[-]}\right\}|p_A\rangle,$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right\}|p_A\rangle, \quad \mathcal{F}_{gg}^{(6)} \sim \langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c}\,\frac{\mathrm{Tr}\mathcal{U}^{[\Box]}}{N_c}\,|p_A\rangle$$

The Wilson lines and loops are defined as:

$$\mathcal{U}^{[\pm]} = U(0,\pm\infty;0_T)U(\pm\infty,\xi^+;\xi_T) \quad \mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

where $U(a,b;x_T) = \mathcal{P}\exp\left[ig\int_a^b dx^+ A_a^-(x^+,x_T)t^a\right]$.

# Effective (generalized) TMD factorization (2)

This can be compared with the CGC results in the back-to-back limit ($k_T \ll P_T$).

Assumming large $N_c$ and factorizability of traces in the MEs this leads to an effective factorization.

All, UGDs survivng in the large $N_c$ can be expressed by only two:

$$\mathcal{F}_{gg}^{(2)}\left(x,k_T^2\right) \sim \int \frac{d^2q_T}{q_T^2}\,\mathcal{F}_{gg}^{(3)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(1)}\left(x,k_T^2\right) \sim \int \frac{d^2q_T}{q_T^2}\,\mathcal{F}_{qg}^{(1)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(2)}\left(x,k_T^2\right) \sim \int \frac{d^2q_T}{q_T^2}\,\left(q_T-k_T\right)\cdot q_T\,\mathcal{F}_{qg}^{(1)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,|k_T-q_T|^2\right)$$

$$\mathcal{F}_{gg}^{(1)}\left(x,k_T^2\right) \sim \int \frac{d^2q_Td^2q'_T}{q_T^2}\,\mathcal{F}_{gg}^{(3)}\left(x,q_T^2\right)\mathcal{F}_{qg}^{(1)}\left(x,q'^2_T\right)\mathcal{F}_{qg}^{(1)}\left(x,\left|k_T-q_T-q'_T\right|^2\right)$$

1. Weizsacker-Williams (WW): $\mathcal{F}_{gg}^{(3)} = xG_1 \sim \langle p_A|\,\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}|p_A\rangle$
2. dipole: $\mathcal{F}_{qg}^{(1)} = xG_2 \sim \langle p_A|\,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\}\,|p_A\rangle$

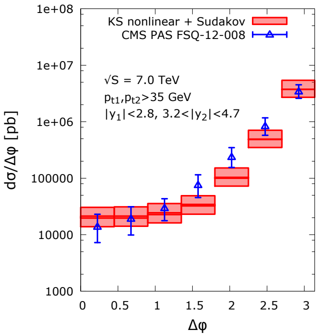
# Comments

- The previous approach is valid only in the correlation limit (back-to-back).
- The decorrelation region for inclusive dijets is nicely described by the HEF:

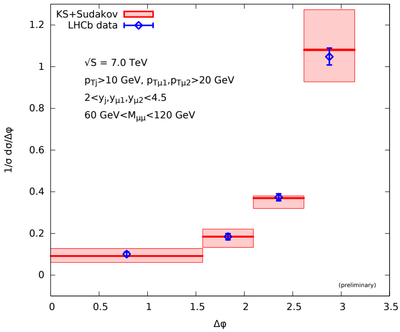
[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]

[A. van Hameren, PK, K. Kutak, arXiv:1505.02763]

forward-central dijets




forward $Z_0$ + jet




- Key observation: HEF can be derived from the dilute limit of CGC. Thus the interpolation formula can be conjectured by introducing off-shellness to the hard factors.

# Improved TMD factorization (1)

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, arXiv:150342]

Two main steps:

1. We revise the calculation of TMDs using color decomposition of amplitudes

$$\mathcal{M}^{a_1 \dots a_N}\left(\varepsilon_1^{\lambda_1}, \dots, \varepsilon_N^{\lambda_N}\right) = \sum_{\sigma \in S_{N-1}} \mathrm{Tr}\left(t^{a_1} t^{a_{\sigma_2}} \dots t^{a_{\sigma_N}}\right) \mathcal{M}\left(1^{\lambda_1}, \sigma_2^{\lambda_{\sigma 2}} \dots, \sigma_N^{\lambda_{\sigma N}}\right)$$

   $a_i$- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity $\lambda_i$, $S_{N-1}$ - set of noncyclic permutations.

   We conclude that there are only two independent TMDs $\Phi^{(i)}$, $i = 1, 2$ (being a combination of $\mathcal{F}_{qg}^{(1)}$'s ) needed for each channel.

2. We calculate off-shell color-ordered helicity amplitudes needed to construct a hard factor for each $\Phi^{(i)}$.

   Methods for gauge invariant off-shell amplitudes:

[E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135]

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029; JHEP 1301 (2013) 078]

[A. van Hameren, JHEP 1407 (2014) 138,] [PK, JHEP 1407 (2014) 128]

see Mirko's talk → [A. van Hameren, M. Serino, JHEP 1507 (2015) 010]

# Improved TMD factorization (2)

## Gauge invariant off-shell helicity amplitudes

In spinor formalism, the non-zero gauge invariant off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

$$\mathcal{M}_{g^*g\to gg}\left(1^*, 2^-, 3^+, 4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*2\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*, 2^+, 3^-, 4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*3\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*, 2^+, 3^+, 4^-\right) = 2g^2\,\rho_1\,\frac{\langle 1^*4\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

where $\langle ij\rangle = \langle k_i - |k_j+\rangle$ with spinors defined as $|k_i\pm\rangle = \frac{1}{2}\left(1 \pm \gamma_5\right) u\left(k_i\right)$. Modified spinor products involve only longitudinal component of the off-shell momentum $\langle 1^* i\rangle = \langle p_A\, i\rangle$. Similar expressions can be derived for quarks.

The hard factors $K^{(i)}$ corresponding to $\Phi^{(i)}$ are easy to obtain from color-ordered helicity amplitudes.

# Improved TMD factorization (3)

## Final result

$$\frac{d\sigma_{AB}}{dy_1 d^2p_{T1} dy_2 d^2p_{T1}} \sim \sum_{a,c,d} f_{a/B}\left(x_B, P_T^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A, k_T^2\right) K^{(i)}_{ag\to cd}$$

$$K^{(1)}_{qg\to gq} = -\frac{\bar{u}\left(\bar{s}^2+\bar{u}^2\right)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right) \qquad \Phi^{(1)}_{qg\to gq} = \mathcal{F}^{(1)}_{qg}$$

$$K^{(2)}_{qg\to gq} = -\frac{C_F}{N_c}\,\frac{\bar{s}\left(\bar{s}^2+\bar{u}^2\right)}{\bar{t}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{qg\to gq} = \frac{1}{N_c^2-1}\left(-\mathcal{F}^{(1)}_{qg} + N_c^2\mathcal{F}^{(2)}_{qg}\right)$$

$$K^{(1)}_{gg\to q\bar{q}} = \frac{1}{2N_c}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg} - \mathcal{F}^{(3)}_{gg}\right)$$

$$K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2C_F}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{gg\to q\bar{q}} = -N_c^2\mathcal{F}^{(2)}_{gg} + \mathcal{F}^{(3)}_{gg}$$

$$K^{(1)}_{gg\to gg} = \frac{N_c}{C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}} \qquad \Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$$K^{(2)}_{gg\to gg} = -\frac{N_c}{2C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}} \qquad \Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$\hat{s}, \hat{t}, \hat{u}$ – ordinary Mandelstam variables, $\hat{s}+\hat{t}+\hat{u}=k_T^2$
$\bar{s}, \bar{t}, \bar{u}$ – off-shell momentum $k_A = x_A p_A + k_T$ replaced by $x_A p_A$, $\bar{s}+\bar{u}+\bar{t}=0$

# Numerical studies (1)

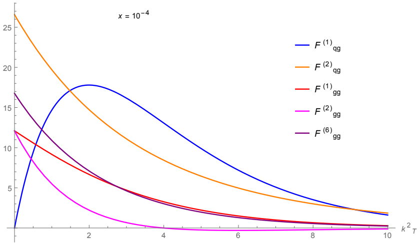
Gluon distributions in the Golec-Biernat-Wusthoff (GBW) model

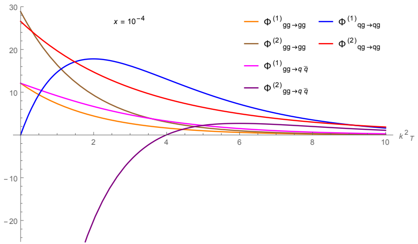
[K. Golec-Biernat, M. Wusthoff, Phys.Rev. D59 (1998) 014017]

As a first try we take the GBW model:

$$xG_2\left(x,k_T^2\right)=\mathcal{F}_{qg}^{(1)}\left(x,k_T^2\right)=\frac{N_c S_\perp}{2\pi^3\alpha_s}\,\frac{k_T^2}{Q_s^2\left(x\right)}\,\exp\left(-\frac{k_T^2}{Q_s^2\left(x\right)}\right)$$

Within the considered approximations the WW gluon distribution as well as the others can be calculated from $xG_2$.



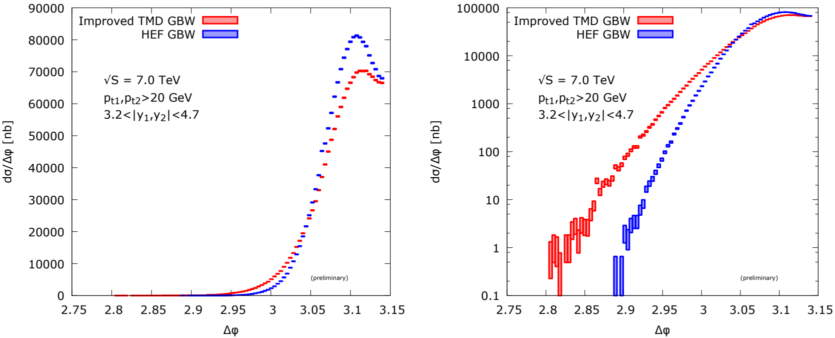



# Numerical studies (2)

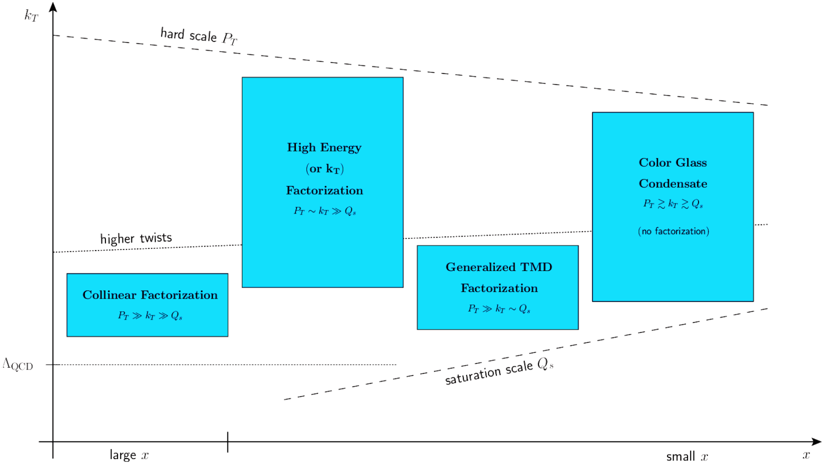
## Azimuthal decorrelations in GBW model

The improved TMD factorization has been implemented in the Monte Carlo C++ program `LxJet`.

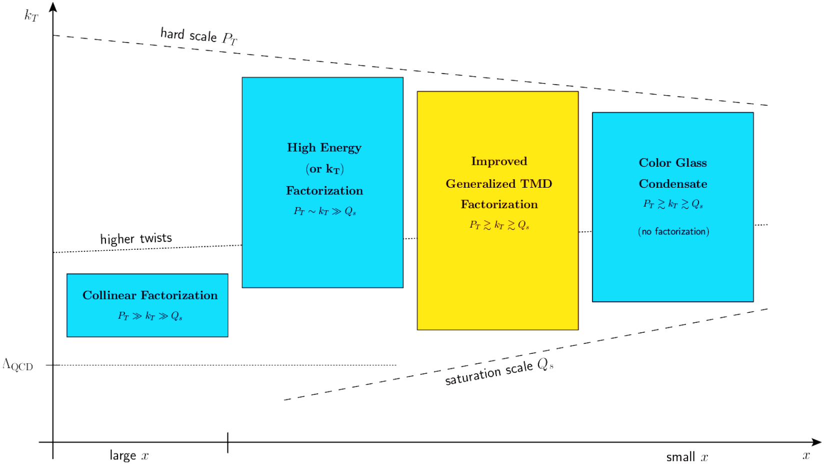
In the GBW model we get a suppresion in the correlation limit and enhancement in the decorrelation region in comparison to HEF (note, however, that the large-$k_T$ behaviour of this model is unphysical).



# Summary (1)


$k_T$
hard scale $P_T$
High Energy
(or $\mathbf{k_T}$)
Factorization
$P_T \sim k_T \gg Q_s$
Color Glass
Condensate
$P_T \gtrsim k_T \gtrsim Q_s$
(no factorization)
higher twists
Generalized TMD
Factorization
$P_T \gg k_T \sim Q_s$
Collinear Factorization
$P_T \gg k_T \gg Q_s$
$\Lambda_{\rm QCD}$
saturation scale $Q_s$
large $x$
small $x$
$x$

# Summary (1)



$k_T$
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# Summary (2)

- The numerical implementation is ready (still to be cross checked).
- First numerical results using five unintegrated gluon distributions using GBW input are available.
- More physical gluon distributions are under development.
- Future plans:
  - Sudakov-type resummation
  - NLO corrections

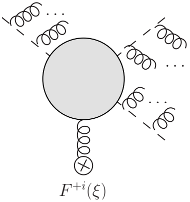
# Backup

# TMD gluon distributions (1)

TMD gluon distributions are defined through FT of matrix elements of nonlocal gauge invariant operators:

$$\phi(x, k_T) = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{i x_A p_A^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle p_A | \,\mathrm{Tr}\left\{ F^{+i}(\xi)\, [\xi, 0]_{C_1}\, F^{+i}(0)\, [0, \xi]_{C_2} \right\} | p_A \rangle$$

The Wilson lines $[\xi, 0]_{C_i}$ depend on the process-dependent paths $C_i$.

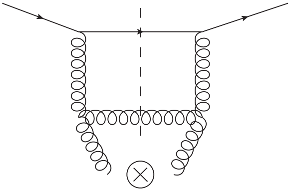



Wilson lines come from resummation of collinear gluons attached to the external lines. Then they are 'glued' by the color structure of the hard process to give $[\xi, 0]_{C_i}$.

# TMD gluon distributions (2)

Example: TMD for a particular diagram

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]



$$\langle p_A|\,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\left[\frac{\mathrm{Tr}\mathcal{U}^{[\square]\dagger}}{N_c}\mathcal{U}^{[+]}+\mathcal{U}^{[-]}\right]\}\,|p_A\rangle$$

with the following definitions of Wilson lines and loops:

$$\mathcal{U}^{[\pm]}=U(0,\pm\infty;0_T)U(\pm\infty,\xi^+;\xi_T)\quad \mathcal{U}^{[\square]}=\mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}=\mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

where $U(a,b;x_T)=\mathcal{P}\exp\left[ig\int_a^b dx^+A_a^-(x^+,x_T)t^a\right]$.