Classical gluon production amplitude in heavy ion collisions

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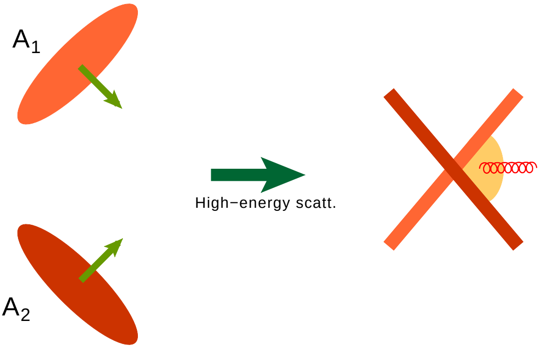
Outline

- Motivations for single-gluon cross-section in A-A collisions.
- Simpler case: p-A collisions.
- Simplified problem for A-A collisions: $1 \ll A_1 \ll A_2$
- Result for the g^3 amplitude.
- Sub-gauge conditions for light-cone propagator.

Result based on
JHEP 1503 (2015) 015 G.A.C., Y. Kovchegov, D. Wertepny

Goal: Single-gluon cross-section in A-A collisions

A_1 and A_2 are the number of nucleons in the two nuclei



Goal: Single-gluon cross-section in A-A collisions

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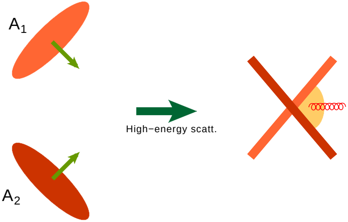
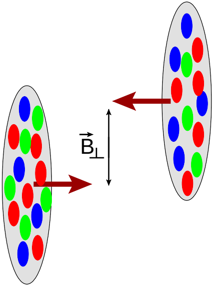


Motivations

- One would like to obtain the classical gluon produced in heavy-ion collisions: initial condition for Quark-Gluon-Plasma.
- Check validity of k_T-factorization formula with unintegrated gluon distributions employed in phenomenological applications.
  - Numerical simulations appear to rule out the k_T-factorization ansatz.

Set up of the calculation

- Resummation parameters: $\alpha_s^2 A_1^{1/3}$ and $\alpha_s^2 A_2^{1/3}$



- Resummation parameters are proportional to the saturation scale squared of each nucleus: $Q_{s1}^2 \sim \alpha_s^2 A_1^{1/3}$ and $Q_{s2}^2 \sim \alpha_s^2 A_2^{1/3}$

Set up of the calculation

Write quasi-classical single-gluon production cross section as

$$\frac{d\sigma}{d^2k\, d^2B\, d^2b} = \frac{1}{\alpha_s} f\left(\frac{Q_{s1}^2(\vec{B}_\perp - \vec{b}_\perp)}{k_T^2}, \frac{Q_{s2}^2(\vec{b}_\perp)}{k_T^2}\right)$$

- $\vec{B}_\perp$: impact parameter between the two nuclei;
- $\vec{b}_\perp$: transverse position of the produced gluon with respect to the center of the target nucleus;
- $\vec{k}_\perp$ is the transverse momentum of the produced gluon with $k_T = |\vec{k}_\perp|$.

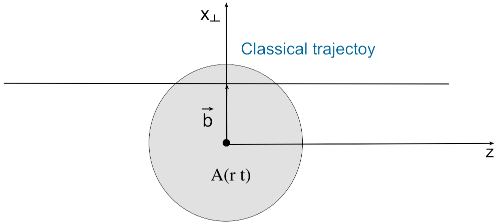
Set up of the calculation

Expansion of f in powers of $\alpha_s^2 A_1^{1/3}$ and $\alpha_s^2 A_2^{1/3} \Leftrightarrow Q_{s1}^2/k_T^2$ and Q_{s2}^2/k_T^2

$$f\left(\frac{Q_{s1}^2}{k_T^2}, \frac{Q_{s2}^2}{k_T^2}\right) = \sum_{n,m=1}^{\infty} c_{n,m} \left(\frac{Q_{s1}^2}{k_T^2}\right)^n \left(\frac{Q_{s2}^2}{k_T^2}\right)^m$$

- Analytic expression of function $f(Q_{s1}^2/k_T^2, Q_{s2}^2/k_T^2)$ is not known.
- Knowing analytic expression of function $f(Q_{s1}^2/k_T^2, Q_{s2}^2/k_T^2)$ would facilitate the inclusion of low-x evolution corrections.
- Coefficient $c_{1,n}$ is known: pA collisions
- Our goal is $c_{2,n}$: corresponds to LO contribution for case $1 \ll A_1 \ll A_2$.
- Idea: find a pattern to resum class of diagrams to get $c_{n,m}$.

High-energy scattering in QCD

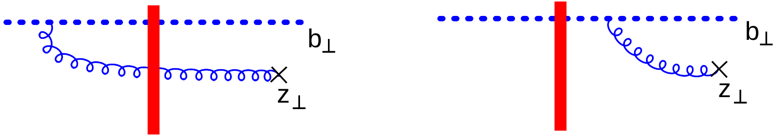


phase factor for the high-energy scattering: Wilson-line operator

$$U(x_\perp, v) = \mathrm{P}e^{\frac{-ig}{c\hbar}\int_{-\infty}^{+\infty} dt\ \dot{x}_\mu A^\mu(x(t))}$$

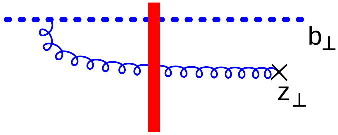
$$\mathrm{P}e^{\int_{-\infty}^{+\infty} dt A(t)} = 1 + \int_{-\infty}^{+\infty} dt\ A(t) + \int_{-\infty}^{+\infty} dt\ A(t) \int_{-\infty}^{t} dt'\ A(t')$$

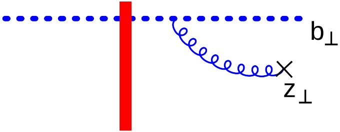
Simpler case: pA collision



- Power counting
  - Projectile: single nucleon $\Rightarrow \alpha_s^2 A_P^{1/3} \ll 1$
  - Target: $\Rightarrow (\alpha_s^2 A_T^{1/3})^N \sim 1$
  - $\Rightarrow$ the target reduces to a shock wave (red strip in the diagram).

Simpler case: pA collision





$$A(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\,g}{\pi}\, \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2}\, \left[U^{ab}_{\vec{z}_\perp} - U^{ab}_{\vec{b}_\perp}\right] \left(V_{\vec{b}_\perp} t^b\right)$$

The gluon production cross section is given by

$$\frac{d\sigma}{d^2k_T\, dy} = \frac{1}{2\,(2\pi)^3} \int d^2z\, d^2z'\, d^2b\, e^{-i\vec{k}_\perp\cdot(\vec{z}_\perp - \vec{z}'_\perp)} \left\langle A(\vec{z}_\perp, \vec{b}_\perp)\, A^*(\vec{z}'_\perp, \vec{b}_\perp)\right\rangle$$

Simpler case: pA collision

$$\frac{d\sigma}{d^2k_T\,dy} = \frac{\alpha_s\,C_F}{4\,\pi^4}\int d^2z\,d^2z'\,d^2b\,e^{-i\vec{k}_\perp\cdot(\vec{z}_\perp-\vec{z}'_\perp)}\,\frac{\vec{z}_\perp-\vec{b}_\perp}{|\vec{z}_\perp-\vec{b}_\perp|^2}\cdot\frac{\vec{z}'_\perp-\vec{b}_\perp}{|\vec{z}'_\perp-\vec{b}_\perp|^2}$$
$$\times\Big[S_G(\vec{z}_\perp,\vec{z}'_\perp)-S_G(\vec{b}_\perp,\vec{z}'_\perp)-S_G(\vec{z}_\perp,\vec{b}_\perp)+1\Big]$$

$$S_G(\vec{x}_\perp,\vec{y}_\perp)=\frac{1}{N_c^2-1}\left\langle U^{ab}_{\vec{x}_\perp}\,U^{\dagger\,ba}_{\vec{y}_\perp}\right\rangle$$

In the quasi-classical MV/Glauber–Mueller approximation

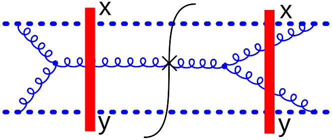
$$S_G(\vec{x}_\perp,\vec{y}_\perp)=\exp\left[-\frac{1}{4}\,(\vec{x}_\perp-\vec{y}_\perp)^2\,Q^2_{sG}\left(\frac{\vec{x}_\perp+\vec{y}_\perp}{2}\right)\,\ln\frac{1}{|\vec{x}_\perp-\vec{y}_\perp|\,\Lambda}\right]$$

- $Q^2_{sG}=4\pi\alpha_s^2T(\vec{b}_\perp)$ is the square of the gluon saturation scale.
- $T(\vec{b}_\perp)$ is the nuclear profile function.

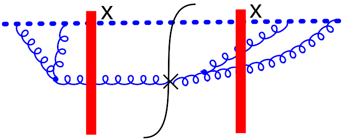
Simplified problem for AA collision: $1 \ll A_1 \ll A_2$

- Consider $1 \ll A_1 \ll A_2 \quad \Rightarrow \quad Q_{s1} \ll Q_{s2}$
- Nucleus A_1 is considered as a dilute system.
- Nucleus A_2 is densely packed diagrammatically represented as a red strip: shock wave.

$$\alpha_s^2 A_2^{1/3} \sim 1 \qquad \alpha_s^2 A_1^{1/3} \lesssim 1$$



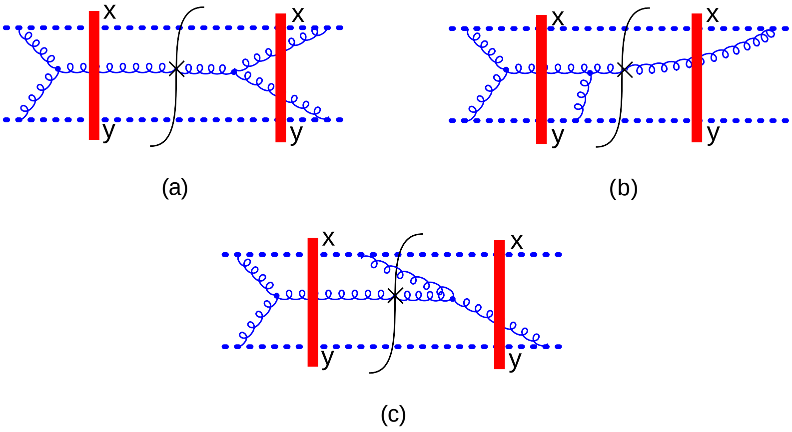
(i)



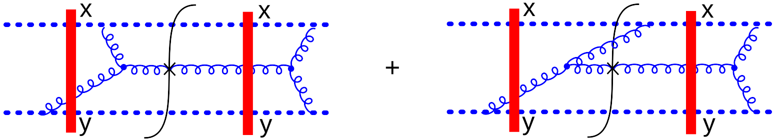
(ii)

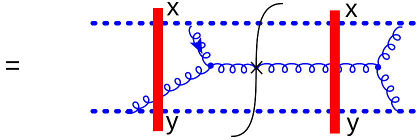
- Contribution from classical field: $A^\mu \sim \frac{1}{g} \quad \Rightarrow \quad \langle A_\mu A^\mu \rangle \sim \frac{1}{\alpha_s}$
- Power counting of diagram (i): $\frac{1}{\alpha_s}(\alpha_s^2 A_1^{1/3})^2$
- Power counting of diagram (ii): $\frac{1}{\alpha_s}\alpha_s^4 A_1^{1/3}$

Sample of diagrams



Retarded Propagator

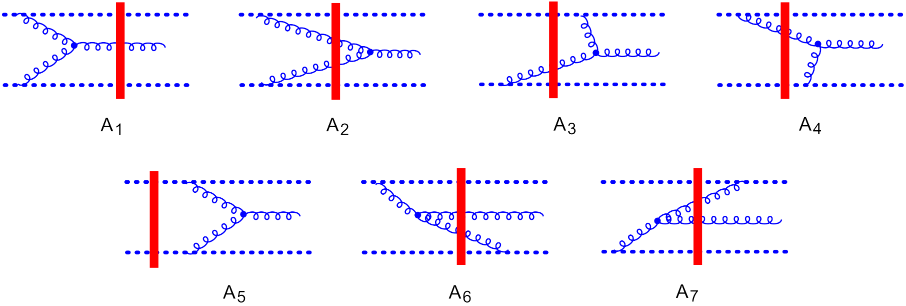




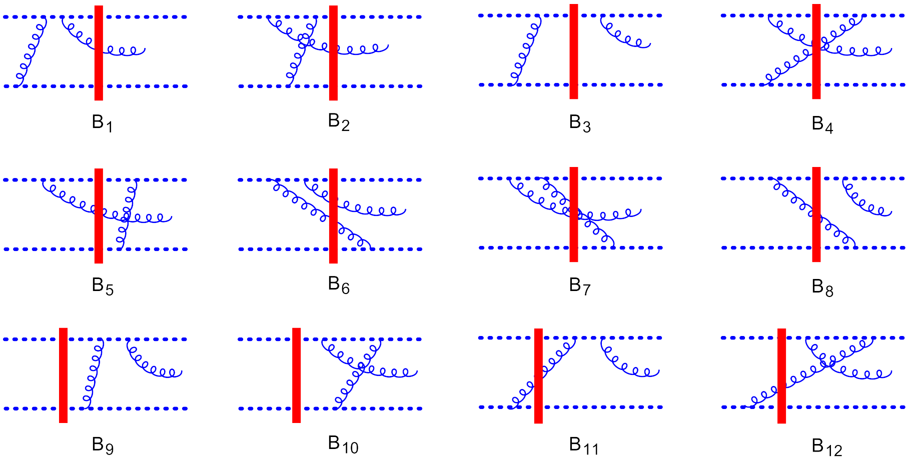
$$D^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+} \qquad k \cdot \eta = k^+$$

$$\frac{-iD^{\mu\nu}(k)}{k^2 + i\epsilon} + 2\pi\theta(-k^+)\delta(k^2)D^{\mu\nu}(k) = \frac{-iD^{\mu\nu}(k)}{k^2 + i\epsilon k^+}$$

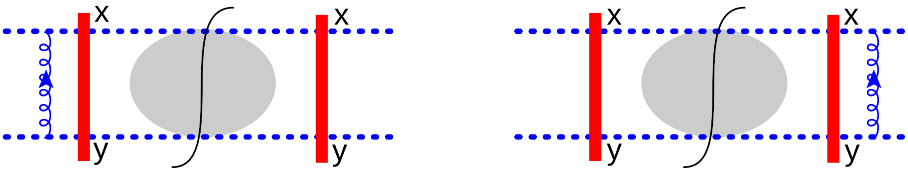
3-gluon vertex diagrams



Box-type diagrams



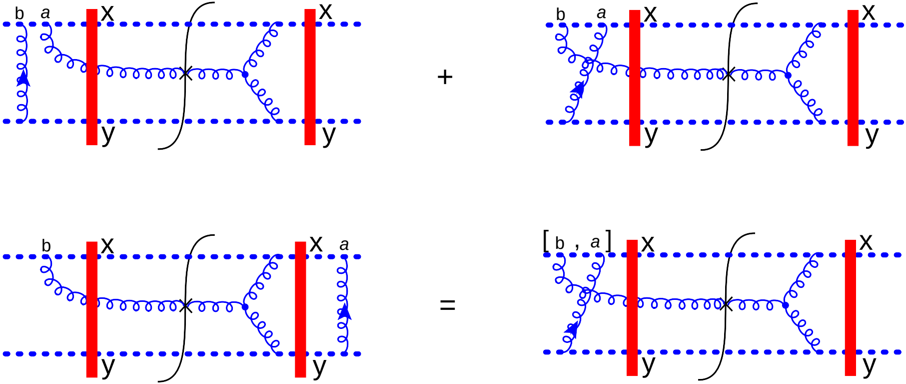
shaded area represents any possible interaction.



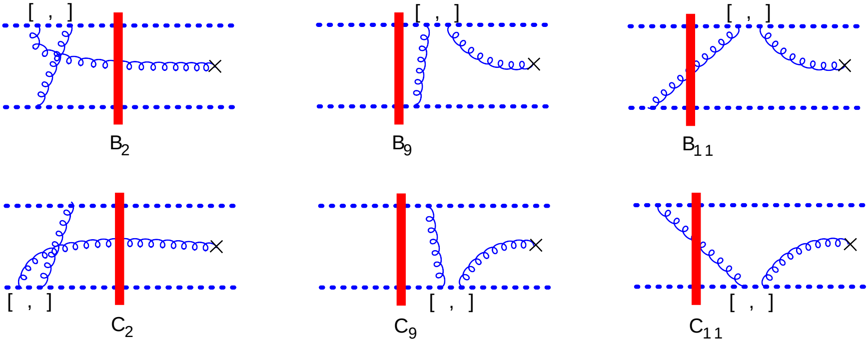
(a) (b)

- Sum of diagram (a) and (b) is zero.

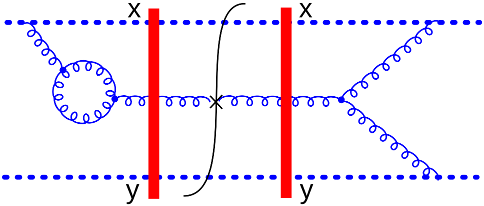
Commutator: three-gluon vertex



3-gluon vertex like diagrams



No quantum corrections

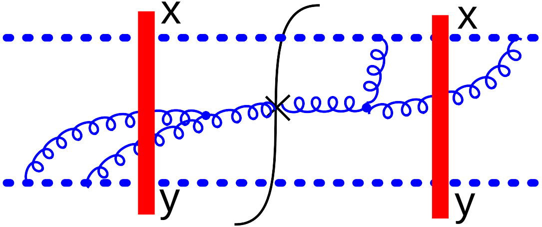


- The diagram is proportional to $\mathrm{tr}\{U_y U_y^\dagger t^a\} = \mathrm{tr}\{t^a\} = 0$

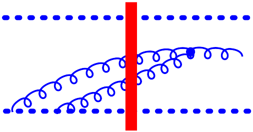
$$
\begin{aligned}
&\sum_{i=1}^{7} A_i + \sum_{i=1}^{12}{}' B_i + \sum_{i=1}^{12}{}' C_i \\
&= - \frac{g^3}{4\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
&\quad \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] \\
&\quad \times f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{1\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
&\quad + \frac{i\, g^3}{4\pi^3} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[U^{bd}_{\vec{b}_{1\perp}} \left(U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right) \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right. \\
&\quad \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \\
&\quad - \left(U^{bd}_{\vec{x}_\perp} - U^{bd}_{\vec{b}_{1\perp}} \right) U^{ce}_{\vec{b}_{2\perp}} \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right. \\
&\quad \left. \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \right] - \frac{i\, g^3}{4\pi^2} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
&\quad \times \left[(U^{bd}_{\vec{z}_\perp} - U^{bd}_{\vec{b}_{1\perp}}) \, U^{ce}_{\vec{b}_{2\perp}} \, \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\, \Lambda} - U^{bd}_{\vec{b}_{1\perp}} \, (U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}}) \, \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}|\, \Lambda} \right] \\
&\quad - \frac{i\, g^3}{4\pi^3} \int d^2x \, [U^{ab}_{x_\perp} - U^{ab}_{z_\perp}] \, f^{bde} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \, \text{Sign}(b_2^- - b_1^-)
\end{aligned}
$$

3-gluon vertex diagrams with one nucleon

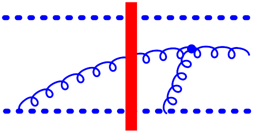
- Power counting: $\frac{1}{\alpha_s}\left(\alpha_s^2 A_1^{1/3}\right)^2$



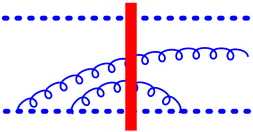
3-gluon vertex diagrams with one nucleon



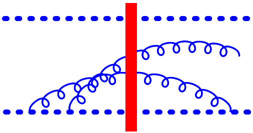
D_1



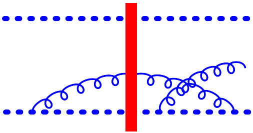
D_2



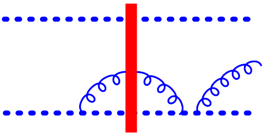
D_3



D_4

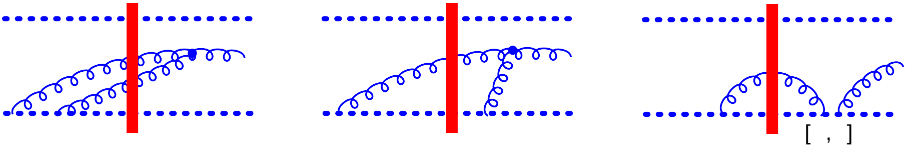


D_5



D_6

3-gluon vertex diagrams with one nucleon



D_1 D_2 D_6'

$$
\sum_{i=1}^{6}{}' D_i
$$

$$
= -\frac{g^3}{8\,\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right.
$$

$$
\left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{2\perp})}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right]
$$

$$
\times f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{2\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2
$$

$$
+ \frac{i\, g^3}{4\,\pi^3} \int d^2x f^{abc}\, U^{bd}_{\vec{b}_{2\perp}} \left[U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2 \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right.
$$

$$
\left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right)
$$

$$
+ \frac{i\, g^3}{4\,\pi^2} f^{abc}\, U^{bd}_{\vec{b}_{2\perp}} \left[U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\, \Lambda}
$$

Cross-checks

Setting all $U = 1$ and all $V = 1$ we have

$$\sum_{i=1}^{7} A_i = 0, \quad {\sum_{i=1}^{12}}' B_i = 0, \quad {\sum_{i=1}^{12}}' C_i = 0, \quad {\sum_{i=1}^{6}}' D_i = 0, \quad {\sum_{i=1}^{6}}' E_i = 0$$

as expected.

Gauge invariance

Light-cone coordinates: $x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}}$

Propagator in light-cone gauge $A^+ = 0$:

$$\langle A^\mu(x) A^\nu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{d^{\mu\nu}(k)}{k^2 + i\epsilon} e^{-ik\cdot(x-y)}$$

Light-cone propagator singularity:

$$d^{\mu\nu}(k) = g^{\mu\nu} - \frac{\eta^\mu k^\nu + \eta^\nu p^\mu}{k^+}$$

Sub-gauge condition will set the prescription for the $\frac{1}{k^+}$ singularity.

Sub-gauge conditions for light-cone propagator

- PV-sub-gauge: $\partial_\perp \cdot A_\perp(x^- = +\infty) + \partial_\perp \cdot A_\perp(x^- = -\infty) = 0$
  G.A.C., Y. Kovchegov, D. Wertepny, arXiv:1508.07962

$$D^{\mu\nu}_{PV}(x,y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \left(k^\mu\eta^\nu + k^\nu\eta^\mu\right)\mathrm{PV}\left\{\frac{1}{k^+}\right\}\right]$$

$\mathrm{PV}\left\{\frac{1}{k^+}\right\} \equiv \frac{1}{2}\left(\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\right)$

- sub-gauge: $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = +\infty) = 0$

$$D^{\mu\nu}_1(x,y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^+ - i\epsilon} - \frac{k^\nu\eta^\mu}{k^+ + i\epsilon}\right]$$

- sub-gauge: $\vec{\partial}_\perp \cdot \vec{A}_\perp(x^- = -\infty) = 0$

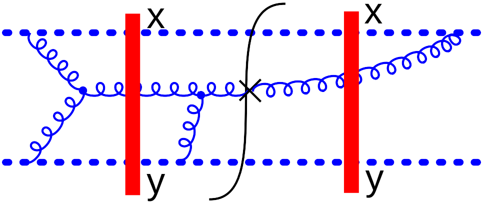
$$D^{\mu\nu}_2(x,y) \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{k^\mu\eta^\nu}{k^+ + i\epsilon} - \frac{k^\nu\eta^\mu}{k^+ - i\epsilon}\right]$$

Conclusions

- Result in transverse coordinate space for the g^3 amplitude have been presented.
- The result have been obtained using two different sub-gauge conditions which fix the prescription of the k^+ singularity in the light-cone propagator.
- This result is part of the analytic calculation of the single inclusive gluon production cross-section for Heavy-Light Ion collisions at the classical level.
- Similar calculation have been performed by Balitsky (2004).
- Check conformal invariance in transverse coordinate space of the final result.

Work in progress

- Sample of diagrams: g^5 amplitude



- Final goal:
  - Cross-section for gluon production in Nucleus-Nucleus collision.
  - Initial condition of Quark Gluon Plasma.