Lattice Gauge Theory: An Ox for QCD

Andreas S. Kronfeld February 12, 2009

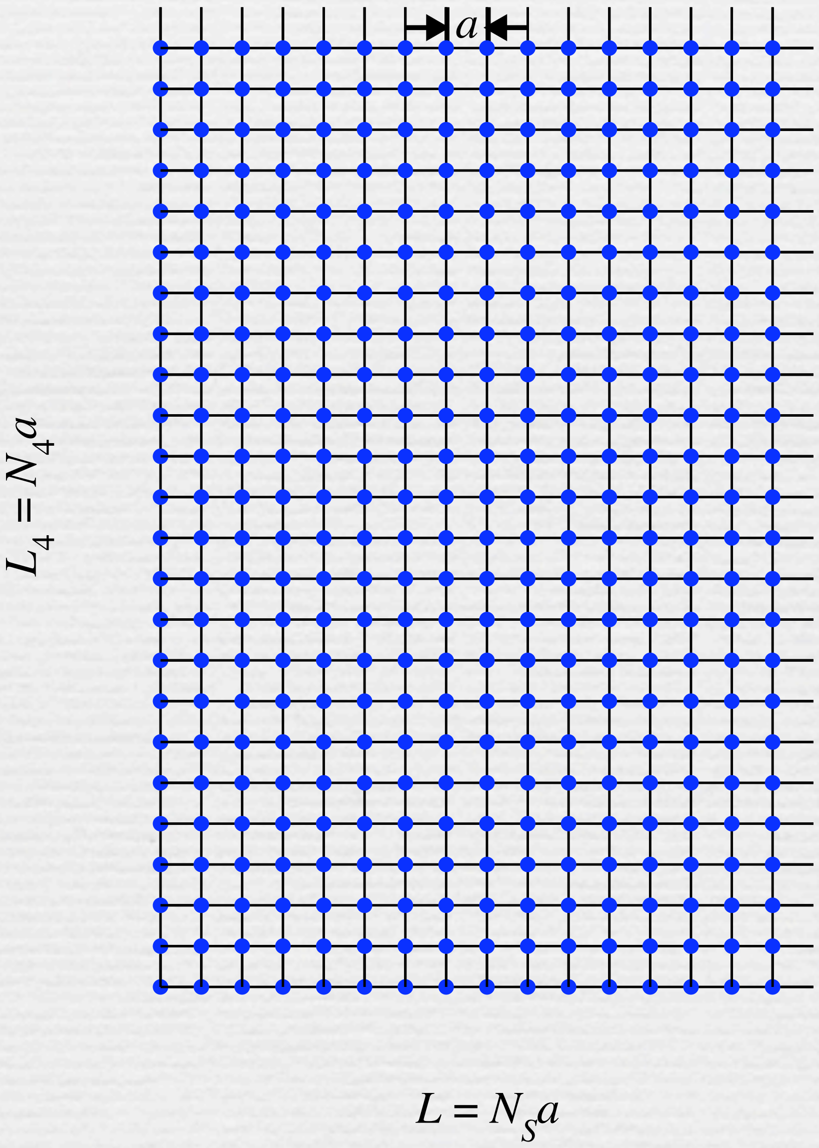
Aspen Winter Conference

Lattice QCD

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z}\int \boxed{\begin{array}{c}\mathcal{D}U\\ \text{MC}\end{array}}\boxed{\begin{array}{c}\mathcal{D}\psi\mathcal{D}\bar{\psi}\\ \text{hand}\end{array}}\exp\left(-S\right)\left[\bullet\right]$$

- Infinite continuum: uncountably many d.o.f.
- Infinite lattice: countably many; used to define QFT
- Finite lattice: can evaluate integrals on a computer; dimension ~ 10^8



Some Jargon

- QCD observables (quark integrals by hand):

$$\langle\bullet\rangle = \frac{1}{Z}\int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D}+m_f)\exp\left(-S_{\text{gauge}}\right)[\bullet]$$

- *Quenched* means replace det with 1.
- *Un*quenched means not to do that.
- *Partially* quenched doesn't mean "n_f too small" but $m_{\text{val}} \neq m_{\text{sea}}$, or even $\not{D}_{\text{val}} \neq \not{D}_{\text{sea}}$ ("mixed action").

Sea Quarks

- Staggered quarks, with rooted determinant, $O(a^2)$.
- Wilson quarks, $O(a)$:
  - tree or nonperturbatively $O(a)$ improved;
  - twisted mass term—auto $O(a)$ improvement.
- Ginsparg-Wilson (domain wall or overlap), $O(a^2)$:
  - $\not{D}\gamma_5 + \gamma_5\not{D} = 2a\not{D}^2$ implemented w/ $\mathrm{sign}(\not{D}_\mathrm{W})$.

- Many numerical simulations with sea quarks are called (perhaps misleadingly) “full QCD.”
  - $n_f = 2$: with *same* mass, omitting strange sea;
  - $n_f = 3$: may (or may not) imply 3 of *same* mass;
  - $n_f = 2+1$: strange sea + two as light as possible;
  - $n_f = 2+1+1$: add charmed sea to 2+1.
- “Full QCD” can also mean $m_{\mathrm{val}} = m_{\mathrm{sea}}$, $\not{D}_{\mathrm{val}} = \not{D}_{\mathrm{sea}}$.

Correlators

- Two-point functions for masses $\pi(t) = \bar{\psi}_u \gamma_5 S \psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-\boxed{m_{\pi_n}} t)$$

- Two-point functions for decay constants:

$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \boxed{\langle 0 | \hat{J} | \pi_n \rangle} \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

- Three-point functions for form factors, mixing:

$$\langle \pi(t) J(u) B^\dagger(0) \rangle \ = \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \boxed{\langle \pi_n | \hat{J} | B_m \rangle} \langle B_m | \hat{B}^\dagger | 0 \rangle$$

$$\times \exp[-m_{\pi_n}(t-u) - m_{B_m} u]$$

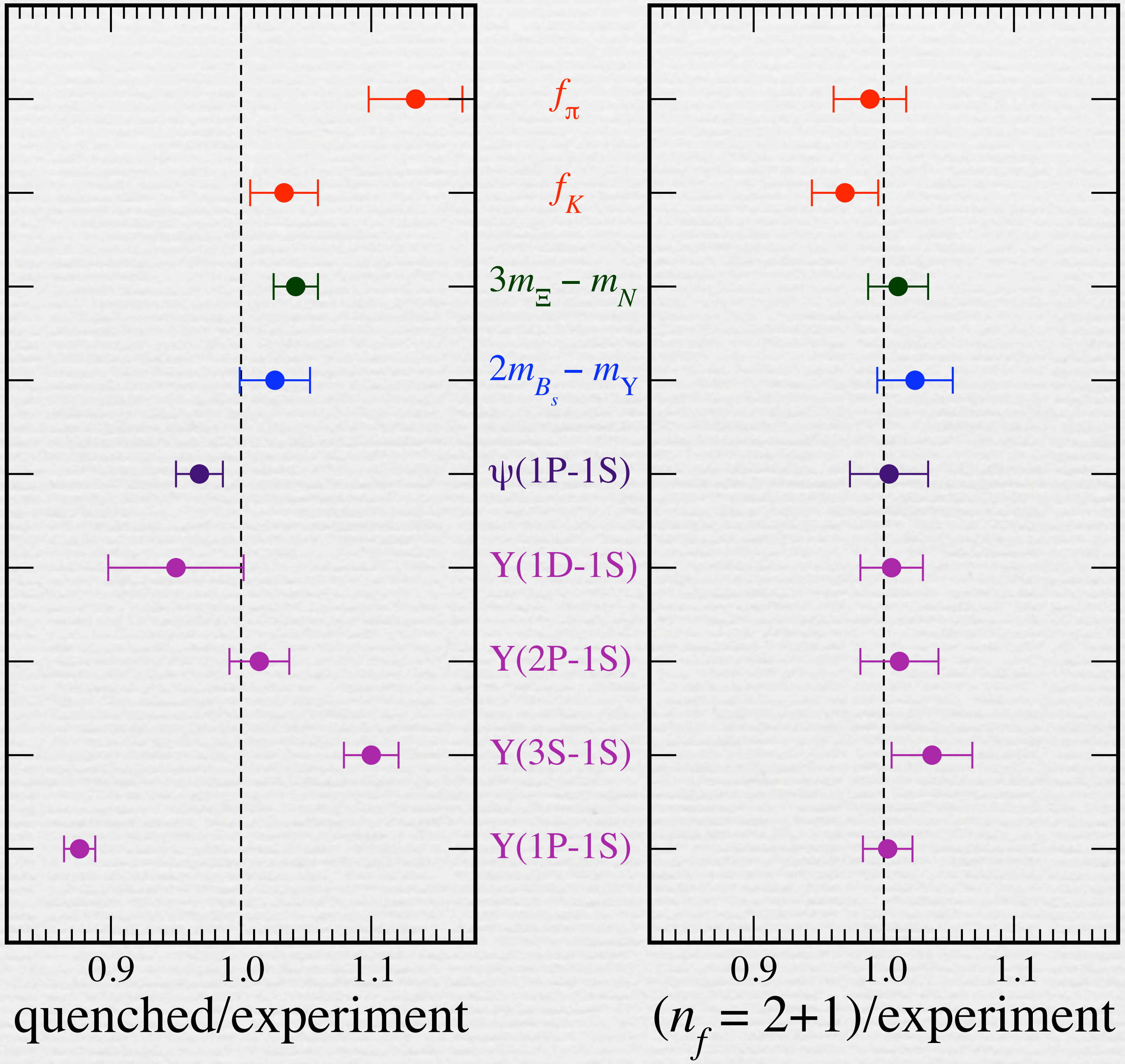
Scope of this talk

- Lattice QCD is now a broad field:
  - SM parameters and flavor physics;
  - nucleon properties and excited baryons;
  - hadron-hadron interactions;
  - QCD thermodynamics;
  - walking QCD—varying n_f, so $\beta(\alpha_s) \approx 0$.
- USQCD overview, arXiv:0807.2220.

Hadron Spectrum

2+1 Sea Quarks!

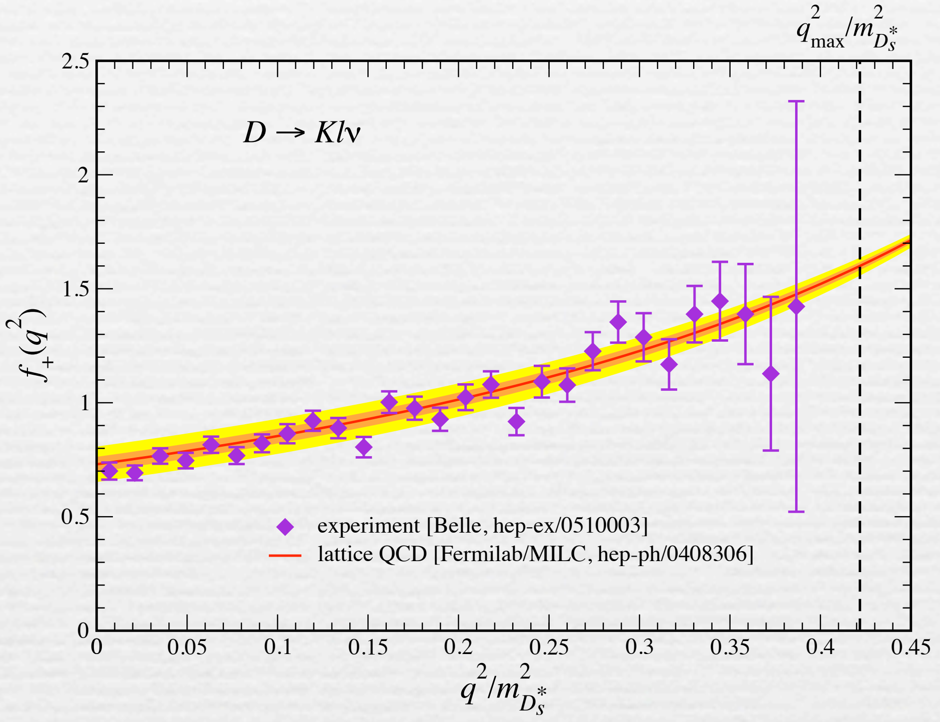
HPQCD, MILC, Fermilab Lattice, hep-lat/0304004



- a = 0.12 & 0.09 fm
- O(a^2) improved
- FAT7 smearing
- $2m_l < m_q < m_s$
- π, K, Υ(2S) input

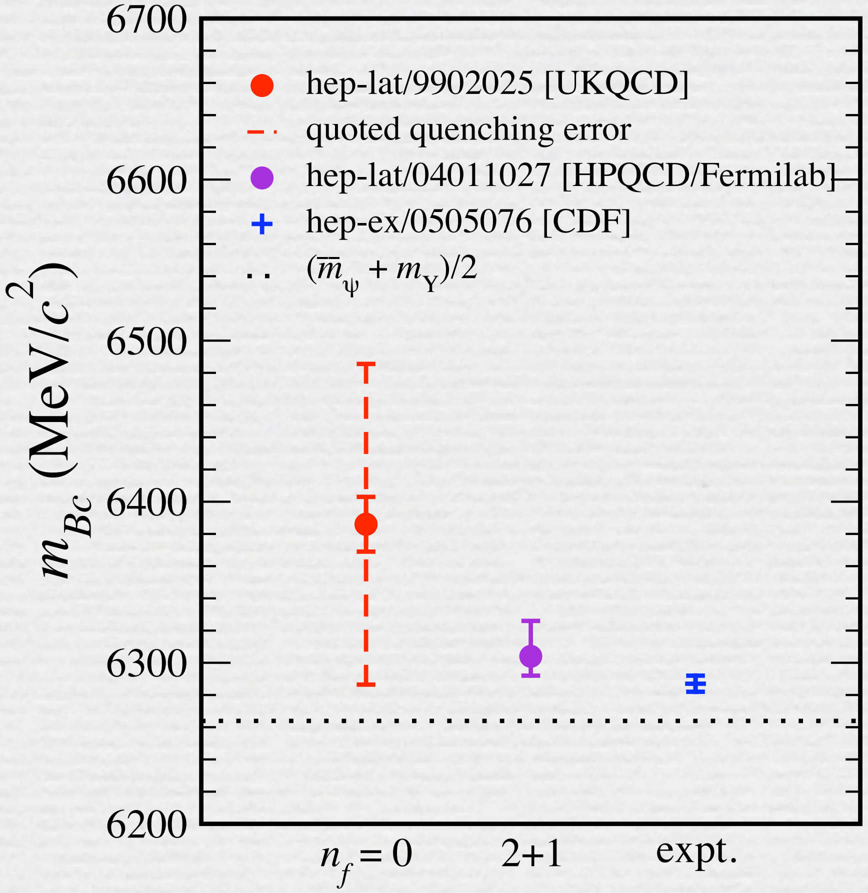
Predictions

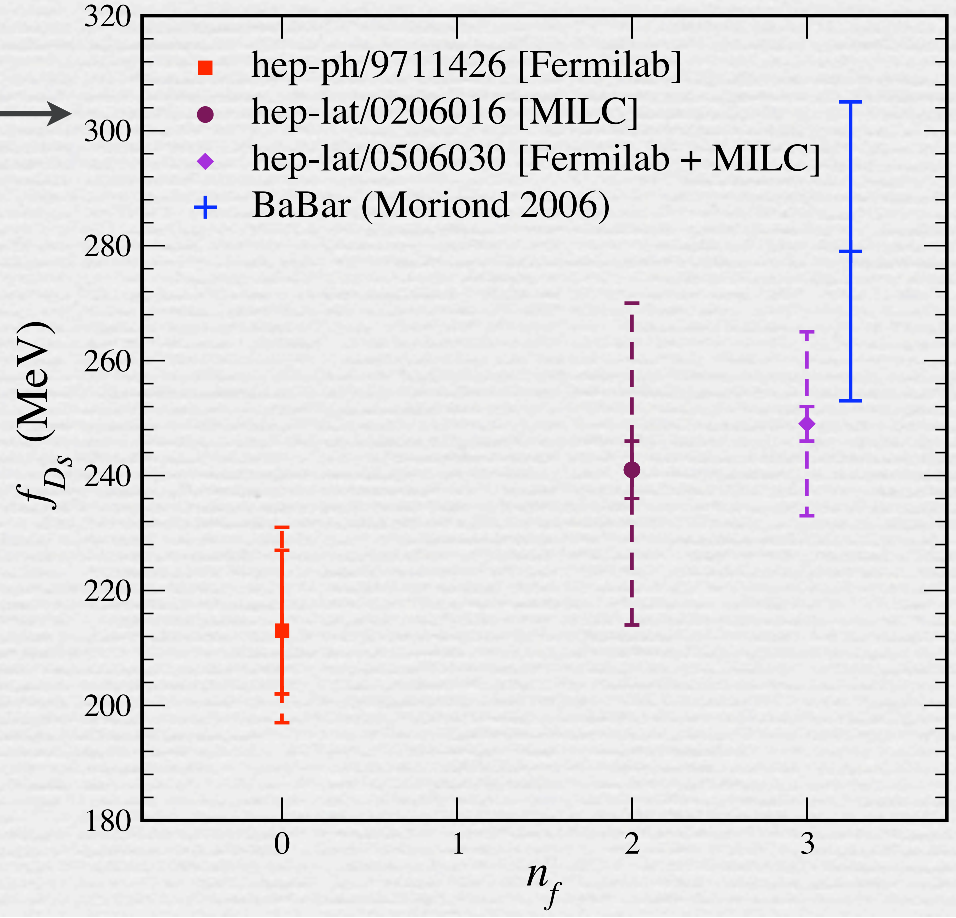
- Semileptonic form factor for $D \to Kl\nu$
- Mass of B_c meson
- Charmed decay constants



2004

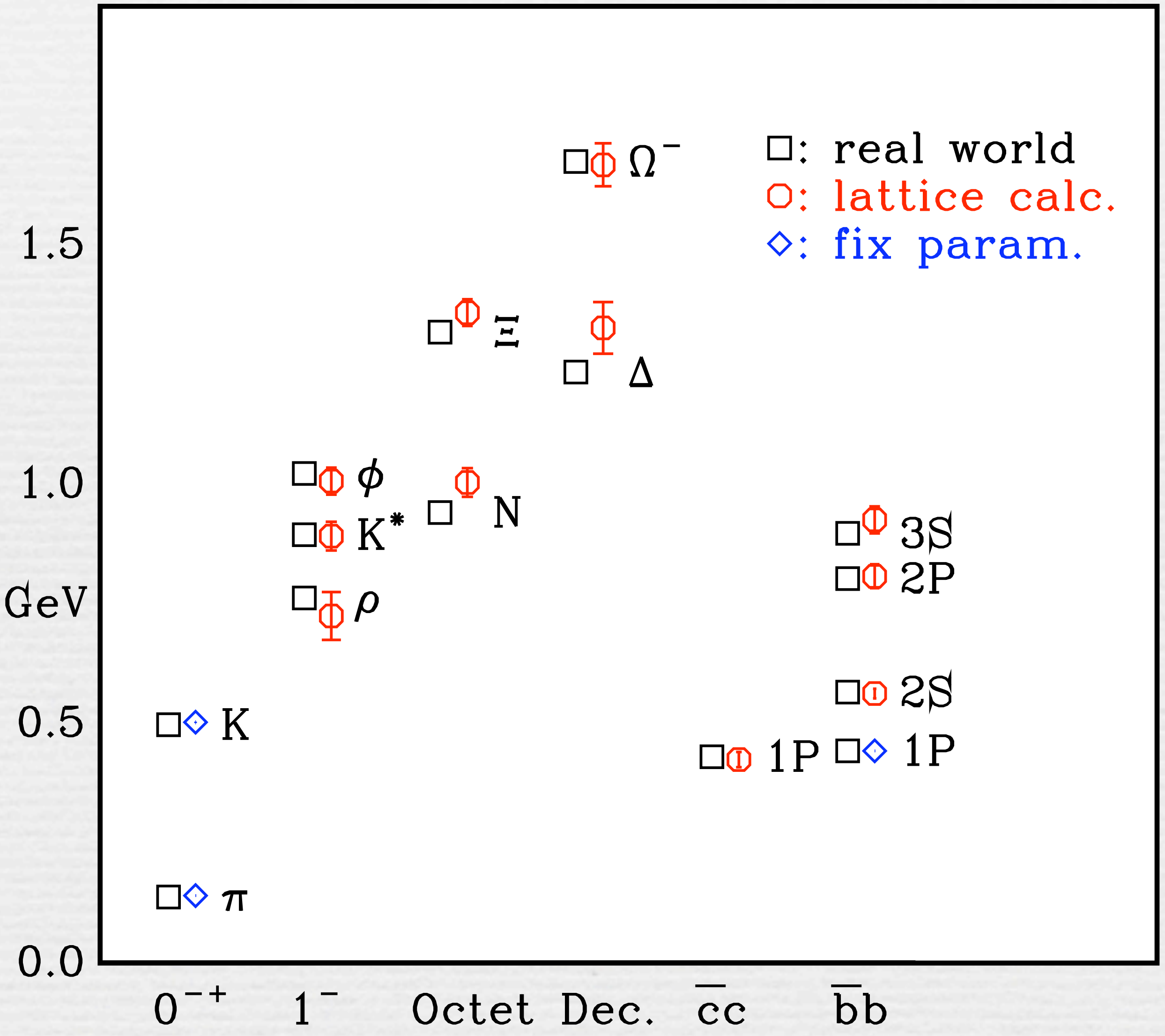
2005





Hadron Spectrum 1

MILC Collaboration, cf. arXiv:0711.0021



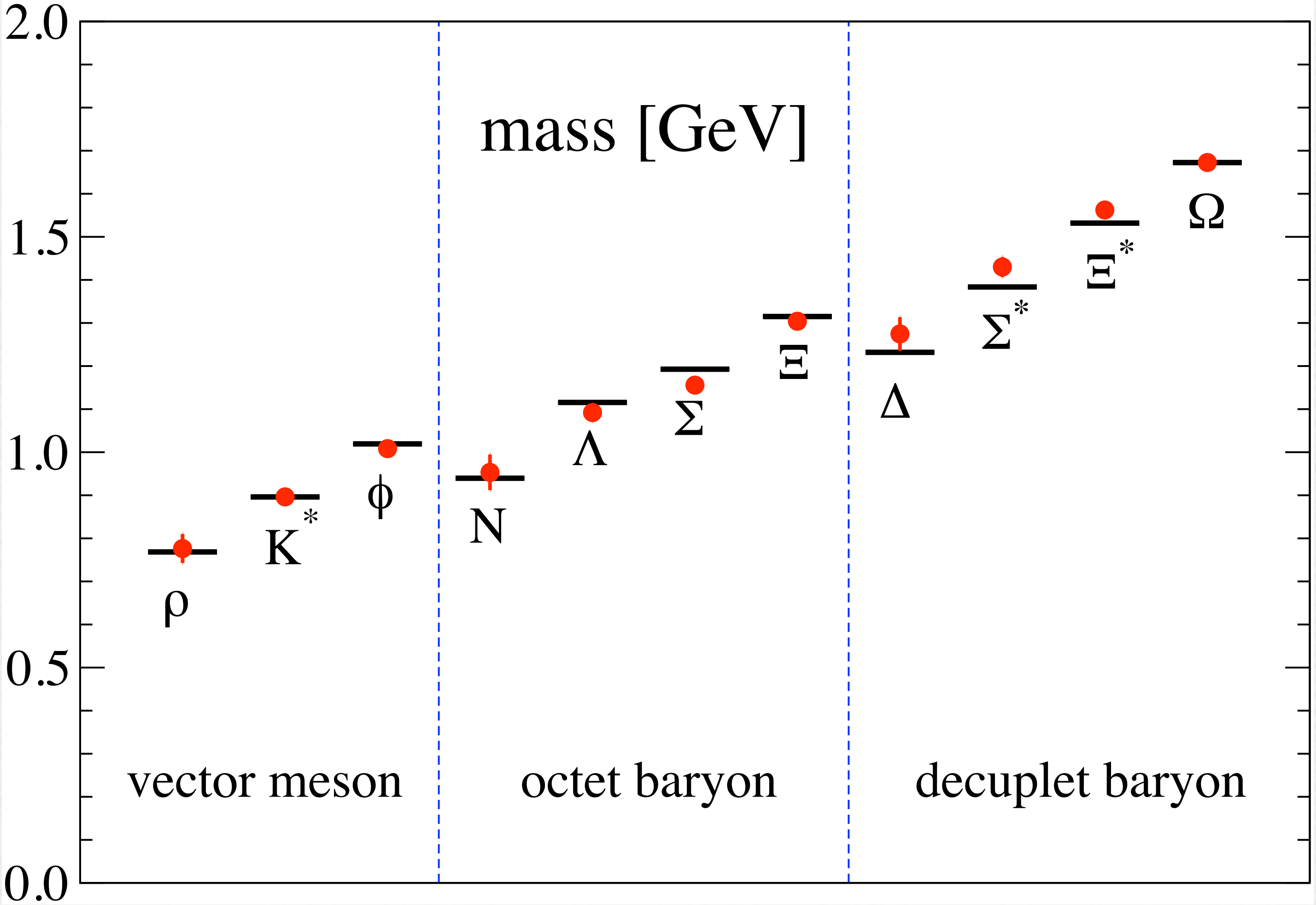
- a = 0.12 & 0.09 fm
- O(a^2) staggered
- FAT7 smearing
- $2m_l < m_q < m_s$
- π, K, Υ(1P) input

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 2

PACS-CS Collaboration, *PRD* 79, 034503 (2009).

cf. earlier work by CP-PACS

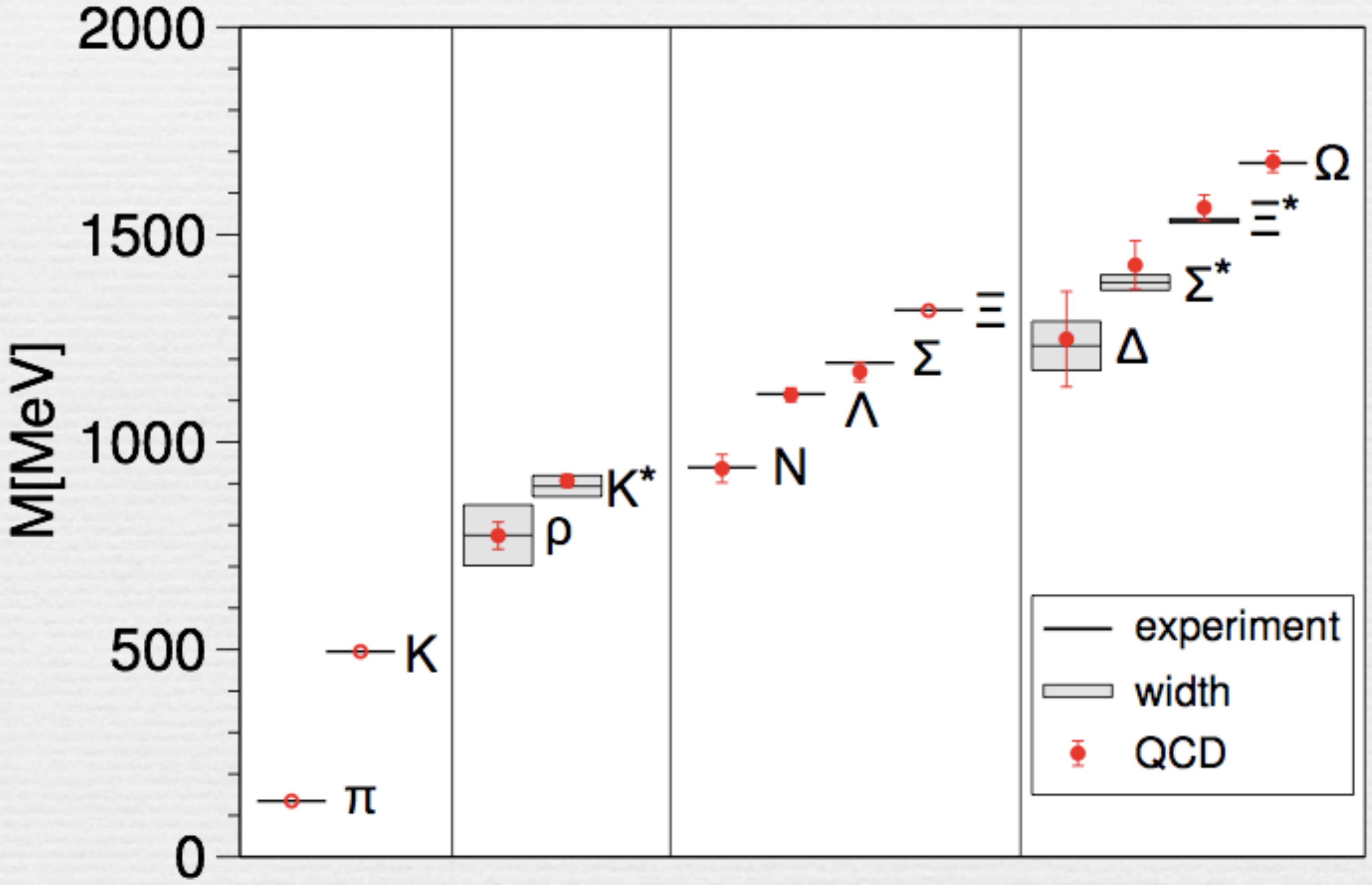


- a = 0.091 fm
- NP O(a) Wilson
- no smearing
- $m_q \approx 1.3 m_l$
- π, K, Ω input

QCD postdicts the low-lying hadron masses!

Hadron Spectrum 3

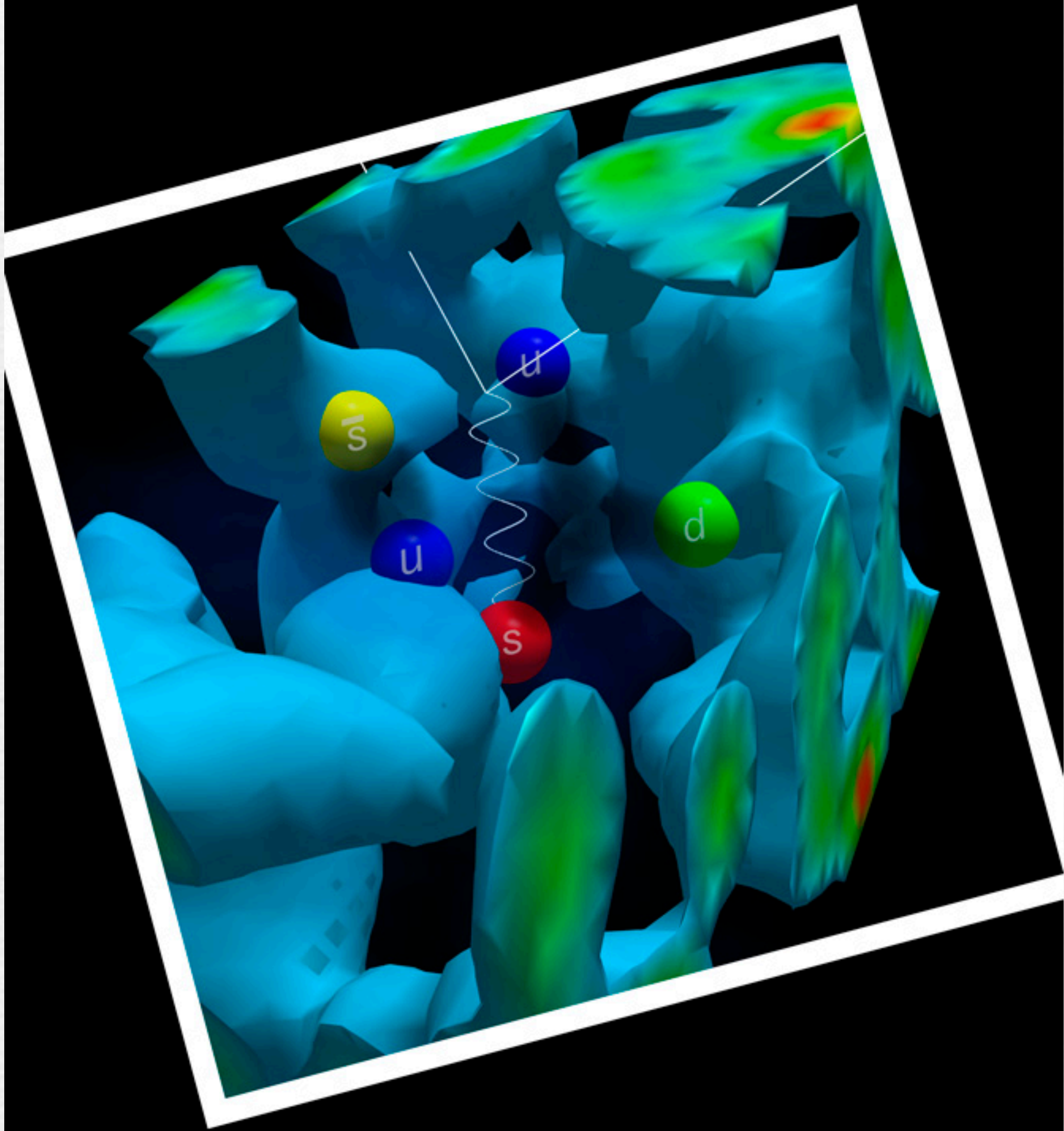
BMW Collaboration: *Science* **322**, 1224 (2008).



- a = 0.125, 0.085, & 0.065 fm
- tree O(a) Wilson
- 6× stout smearing
- $2m_l < m_q < 1.7m_s$
- π, K, Ξ input

QCD postdicts the low-lying hadron masses!

$$m = E/c^2$$



QCD Parameters

Quark Masses & α_s

- Light quark masses (MILC+HPQCD):

$$m_u = 1.9 \pm 0.2 \text{ MeV},$$
$$m_d = 4.6 \pm 0.3 \text{ MeV},$$
$$m_s = 88 \pm 5 \text{ MeV}.$$

with two-loop matching.

- Heavy quark masses (next slides).

- Strong coupling α_s (after that).

Charmed Quark Mass

HPQCD+Karlsruhe, arXiv:0805.2999

- Moments of charmonium correlators:

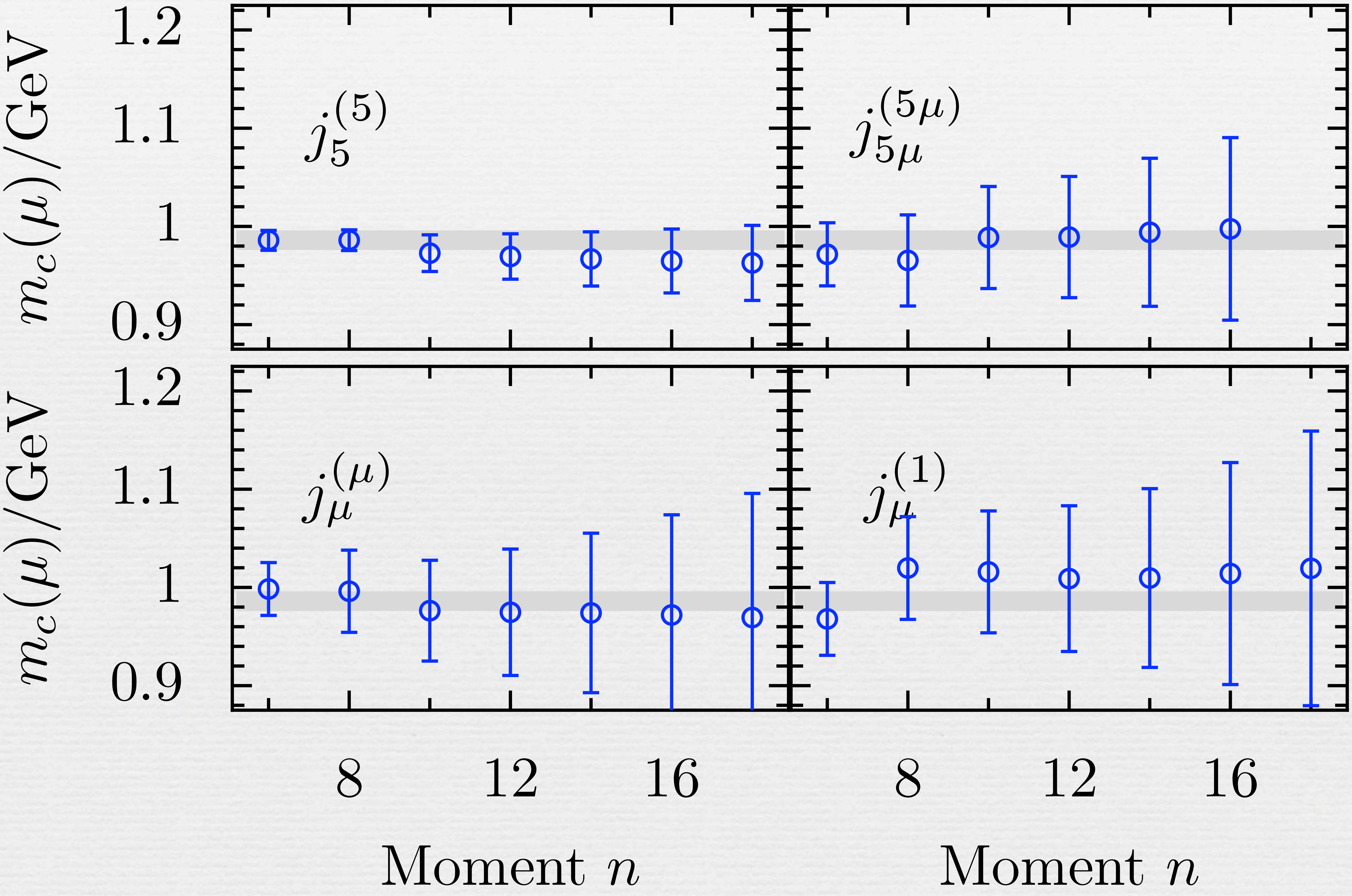
$$G_n = \sum_{\boldsymbol{x},t} t^n \langle m_c \bar{c}\gamma_5 c(\boldsymbol{x},t) m_c \bar{c}\gamma_5 c(\mathbf{0},0)\rangle$$

computed with lattice QCD *and* with continuum perturbation theory yield quark mass and α_s [Bochkarev & de Forcrand, hep-lat/9505025].

- Any channel would do; like using measurements of $e^+e^- \to$ hadrons for vector-vector channel.

- Several tactics to reduce discretization effects to $\mathrm{O}(\alpha_s(m_c a)^{2,4,\ldots})$.
- HPQCD: HISQ valence on 2+1 asqtad sea, with a = 0.15, 0.12, 0.09, 0.06 fm.
- Karlsruhe: PT for moments through α_s^3.
- From G_6 & G_8 of $j_5 j_5$: $m_c(m_c) = 1.268(9)$ GeV.
- Compare e^+e^- $j_\mu j_\mu$: $m_c(m_c) = 1.268(12)$ GeV.

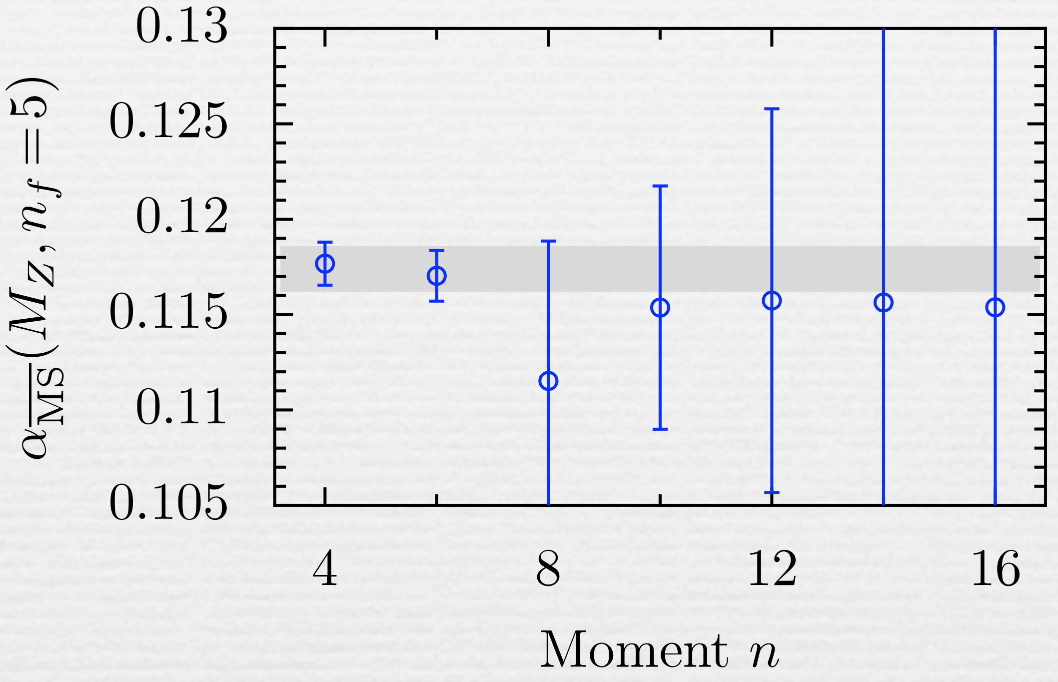
μ = 3 GeV



$m_c(\mu)/\mathrm{GeV}$
1.2
1.1
1
0.9
$j_5^{(5)}$
$j_{5\mu}^{(5\mu)}$
$j_\mu^{(\mu)}$
$j_\mu^{(1)}$
8
12
16
Moment n

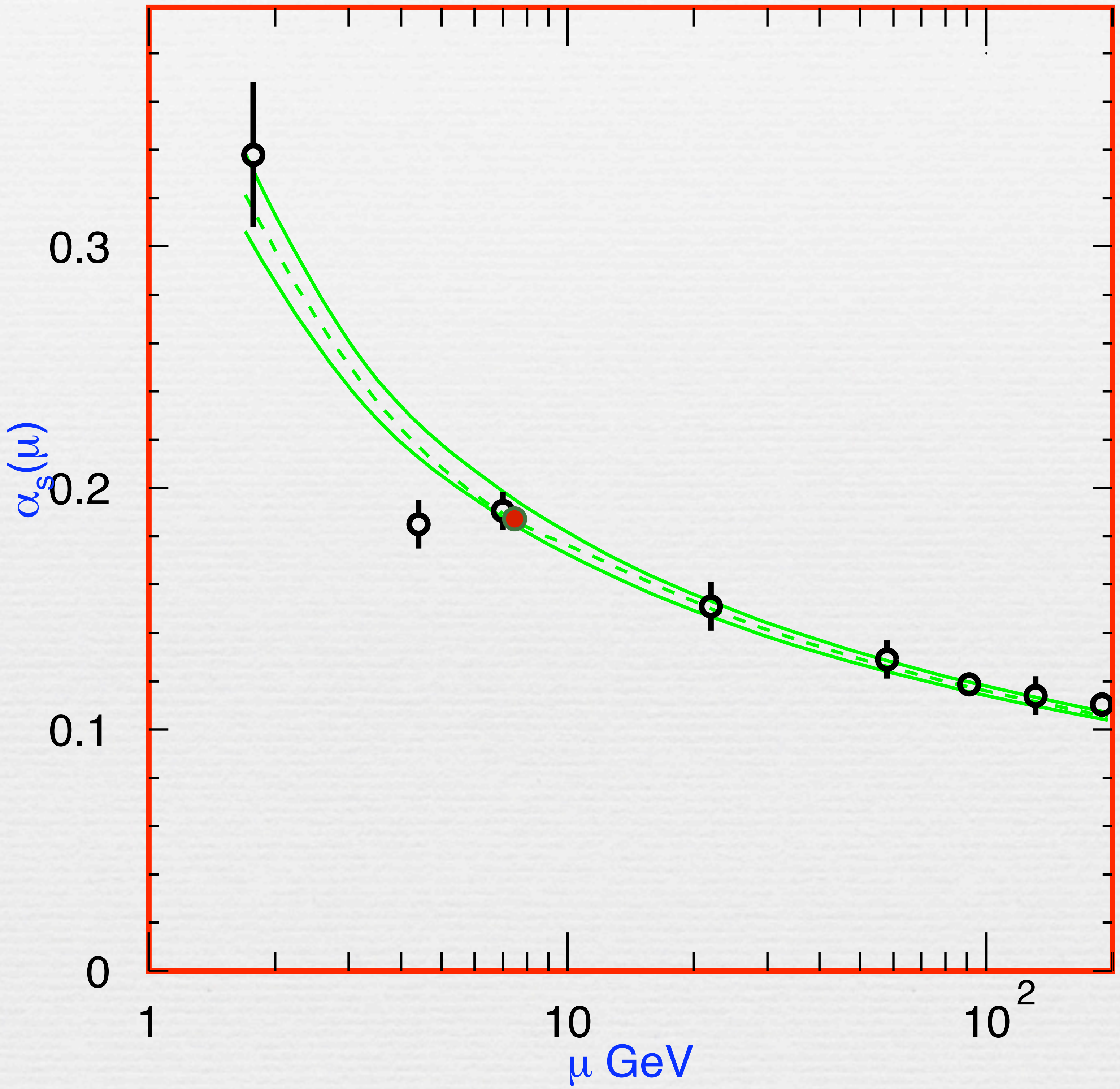
Strong Coupling α_s

- Charmonium moments:
  - $\alpha_s = 0.1174(12)$



- Wilson loops:
  - $\alpha_s = 0.1183(8)$, HPQCD, arXiv:0807.1687;
  - $\alpha_s = 0.1192(11)$, Maltman, arXiv:0807.2020;
  - $\alpha_s = 0.1185(9)$, PDG non-lat average (2008).

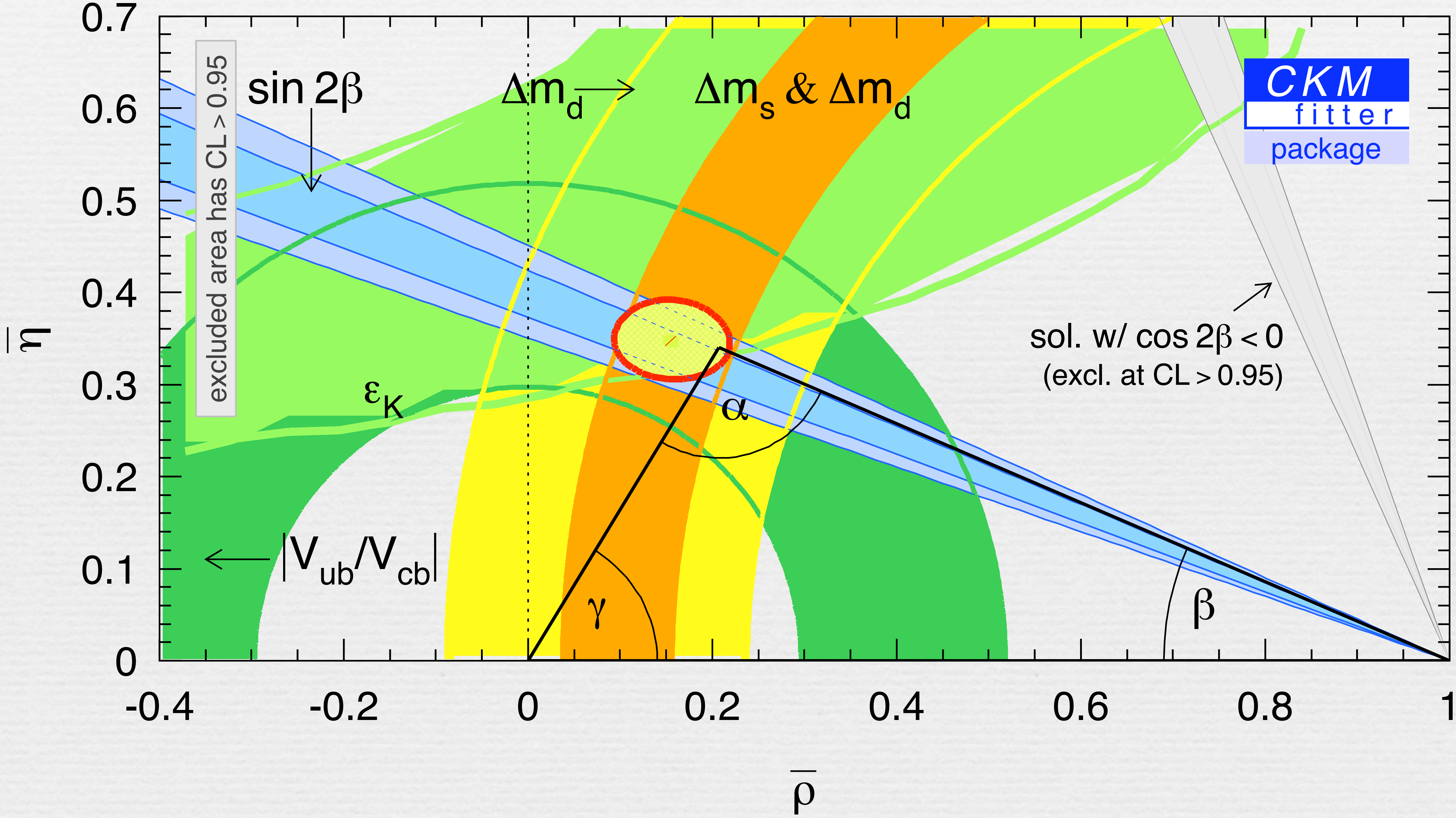
PDG 2008



QCD of hadrons = QCD of partons

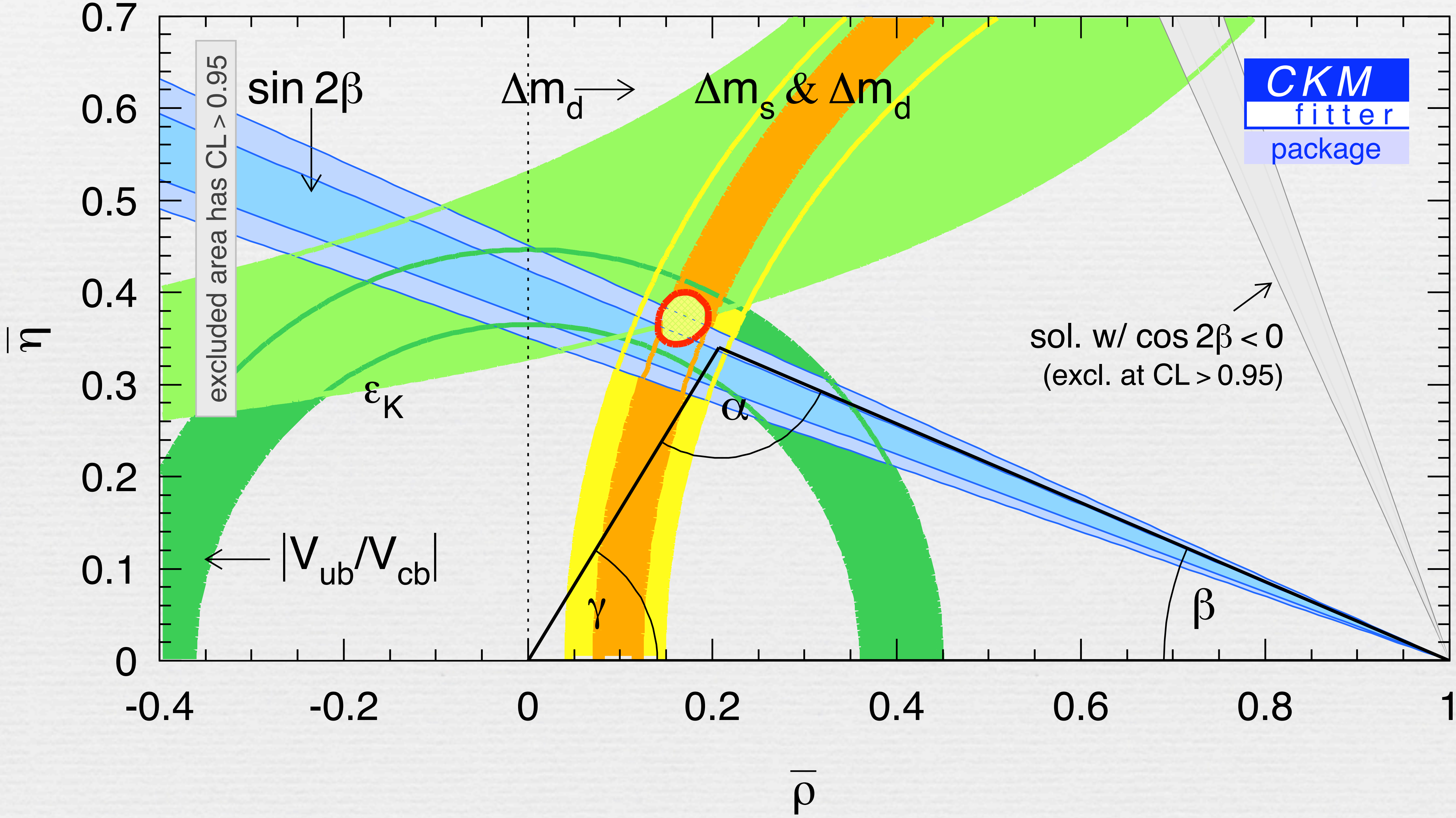
Flavor Physics

CKM UT Now



Plot from Ruth Van de Water

CKM UT 2014



Plot from Ruth Van de Water

Scope of this talk

- Neutral meson mixing: K, B, B_s.
- Semileptonic form factors:
  - $K \to \pi l\nu$ for $|V_{us}|$: RBC+UKQCD, 2007
  - $D \to Kl\nu$, $D \to \pi l\nu$: Fermilab+MILC, 2004
  - $B \to D^* l\nu$ for $|V_{cb}|$; $B \to \pi l\nu$ for $|V_{ub}|$
- Leptonic decay constants: f_π, f_K, f_D, f_{Ds}, f_B.

$|V_{cb}|$

alia et Jack Laiho *et al.*, arXiv:0808.2519

- $|V_{us}|$, $|V_{ub}|$, and $|V_{cb}|$ are the three real parameters of the CKM matrix.
- $|V_{cb}|$ normalizes the unitarity triangle: enters all flavor physics.
- Inclusive $b \to cl\nu$: OPE + PT + measured moments.
- Exclusive $B \to D^* l\nu$: (zero recoil) form factor:

$$\mathcal{F}(1) = h_{A_1}(1), \quad \langle D^*|\mathcal{A}_\mu|B\rangle = i\sqrt{2m_{D^*}2m_B}\bar{\varepsilon}^*_\mu h_{A_1}(1)$$

- Previous quenched calculation (2001):

$$\mathcal{F}(1) = 0.913\underset{\text{stats}}{{}^{+0.024}_{-0.017}} \underset{\text{match}}{\pm 0.016} \underset{a}{{}^{+0.003}_{-0.014}} \underset{\chi\text{PT}}{{}^{+0.000}_{-0.016}} \underset{n_f=0}{{}^{+0.006}_{-0.014}}$$

  used till now with HFAG $|V_{cb}|\mathcal{F}(1)$ to get $|V_{cb}|$.

- Three double ratios, devised so that all uncertainties scale with $\mathcal{F}-1$, not $\mathcal{F}$.

- Update to 2+1 sea quarks with a *single* ratio— more direct & much less computer time.

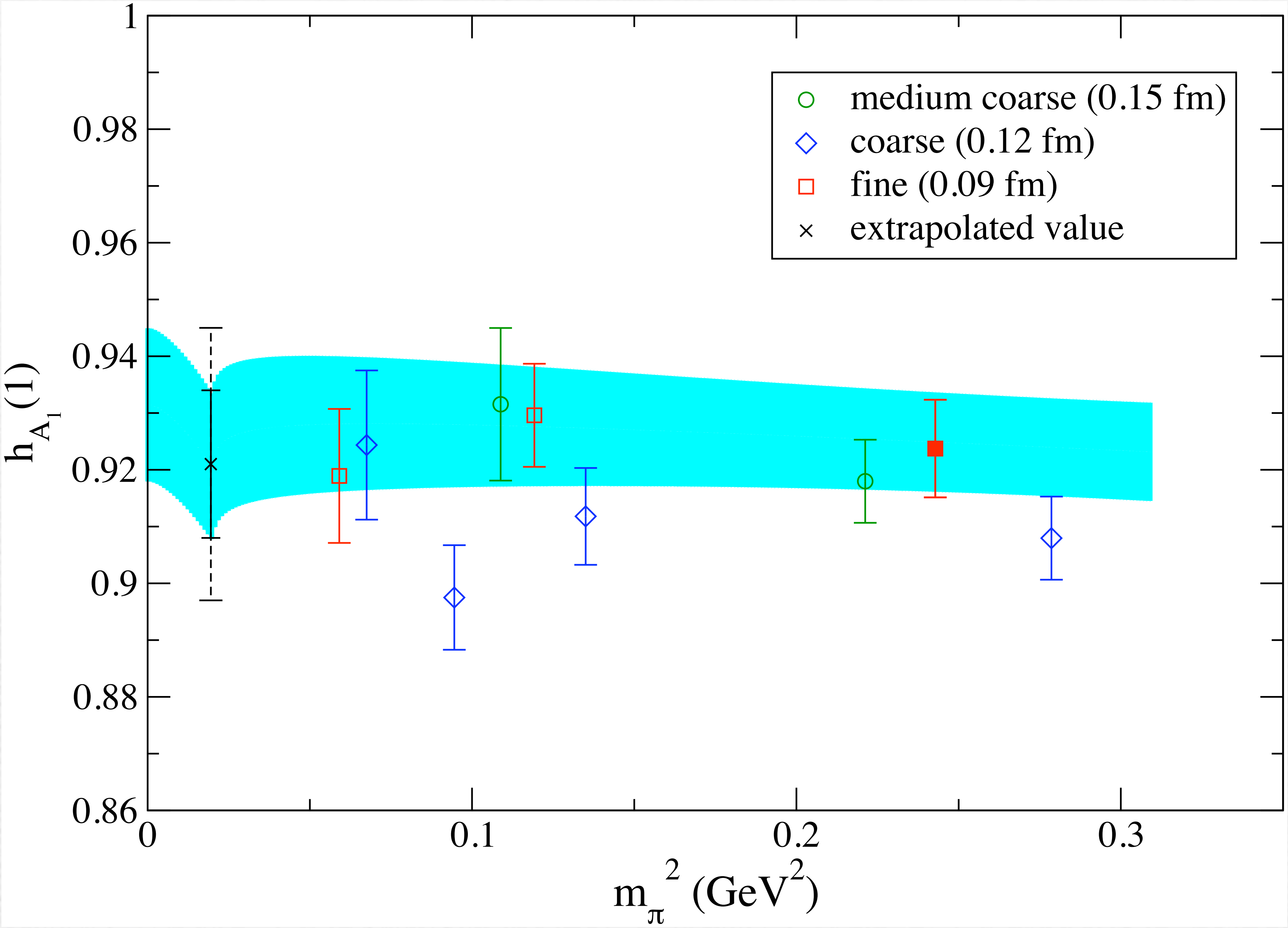
- Also introduces ratios of matrix elements to disentangle chiral extrapolation from heavy-quark discretization effects:

$$\mathcal{R}_{\mathrm{val}}(m_x,\hat{m}',m_s',a) := \frac{h_{A_1}(m_x,\hat{m}',m_s',a)}{h_{A_1}(m_x^{\mathrm{fid}},\hat{m}',m_s',a)},$$

$$\mathcal{R}_{\mathrm{sea}}(\hat{m}',m_s',a) := \frac{h_{A_1}(m_x^{\mathrm{fid}},\hat{m}',m_s',a)}{h_{A_1}(m_x^{\mathrm{fid}},\hat{m}^{\mathrm{fid}},m_s^{\mathrm{fid}},a)}.$$

- Reconstruct

$$\begin{aligned} h_{A_1} &= h_{A_1}(m_x^{\mathrm{fid}},\hat{m}^{\mathrm{fid}},m_s^{\mathrm{fid}},a\to 0) \\ &\quad \times \mathcal{R}_{\mathrm{val}}(m_x,\hat{m}',m_s',a)\times \mathcal{R}_{\mathrm{sea}}(\hat{m}',m_s',a) \end{aligned}$$



$$\underset{}{\mathcal{F}(1) = 0.921} \pm \underset{\text{stats}}{0.013} \pm \underset{g_{D^*D\pi}}{0.008} \pm \underset{\chi\text{PT}}{0.008} \pm \underset{\text{match}}{0.014} \pm \underset{m_Q}{0.007}$$

$|V_{ub}|$

alia et Ruth Van de Water, arXiv:0811.3640

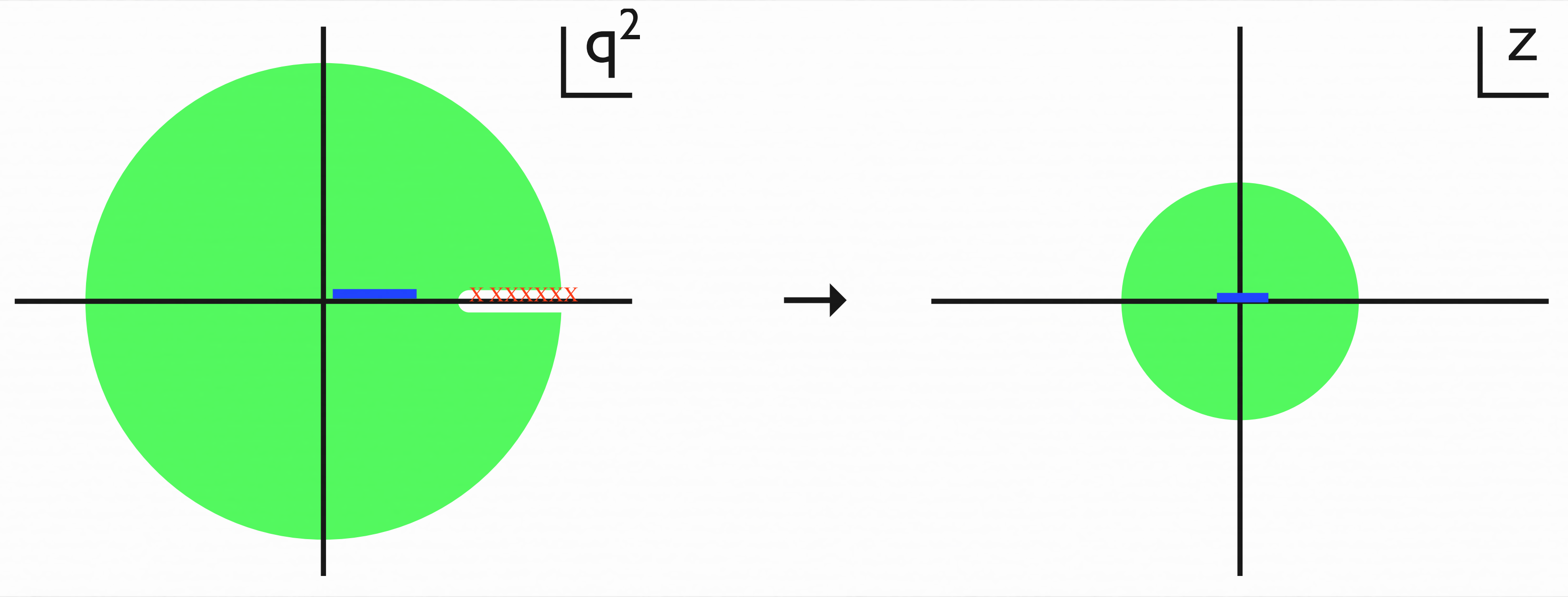
- $|V_{us}|$, $|V_{ub}|$, and $|V_{cb}|$ are the three real parameters of the CKM matrix.
- $|V_{ub}|$ gives a tree constraint comparable to sin 2β.
- Inclusive $b \to ul\nu$: keep control of OPE (or shape functions, or ...) in region with no charm.
- Exclusive $B \to \pi l\nu$: form factor $f_+(q^2)$

$$\langle\pi|\mathcal{V}_\perp^\mu|B\rangle = (p_B + p_\pi)_\perp^\mu f_+(q^2), \quad q\cdot p_\perp = 0$$

- Problem to determine $|V_{ub}|$:
  - lattice best when p_π small, so $q^2 \approx q^2_{\mathrm{max}}$,
  - but event rate highest when $q^2 \approx 0$.
- Until now: find least bad q^2 of both worlds, or introduce *Ansatz* for q^2 dependence.
- Here: a model *independent* simultaneous fit.

- Let $z = \dfrac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$, $\quad t_\pm = (m_B \pm m_\pi)^2$, $\quad t_- < t_0 < t_+$

  inspired by unitarity.



- For $B \to \pi l \nu$ kinematics $-0.34 < z < 0.22$.

- Unitarity guarantees convergent expansion in $z(t)$:

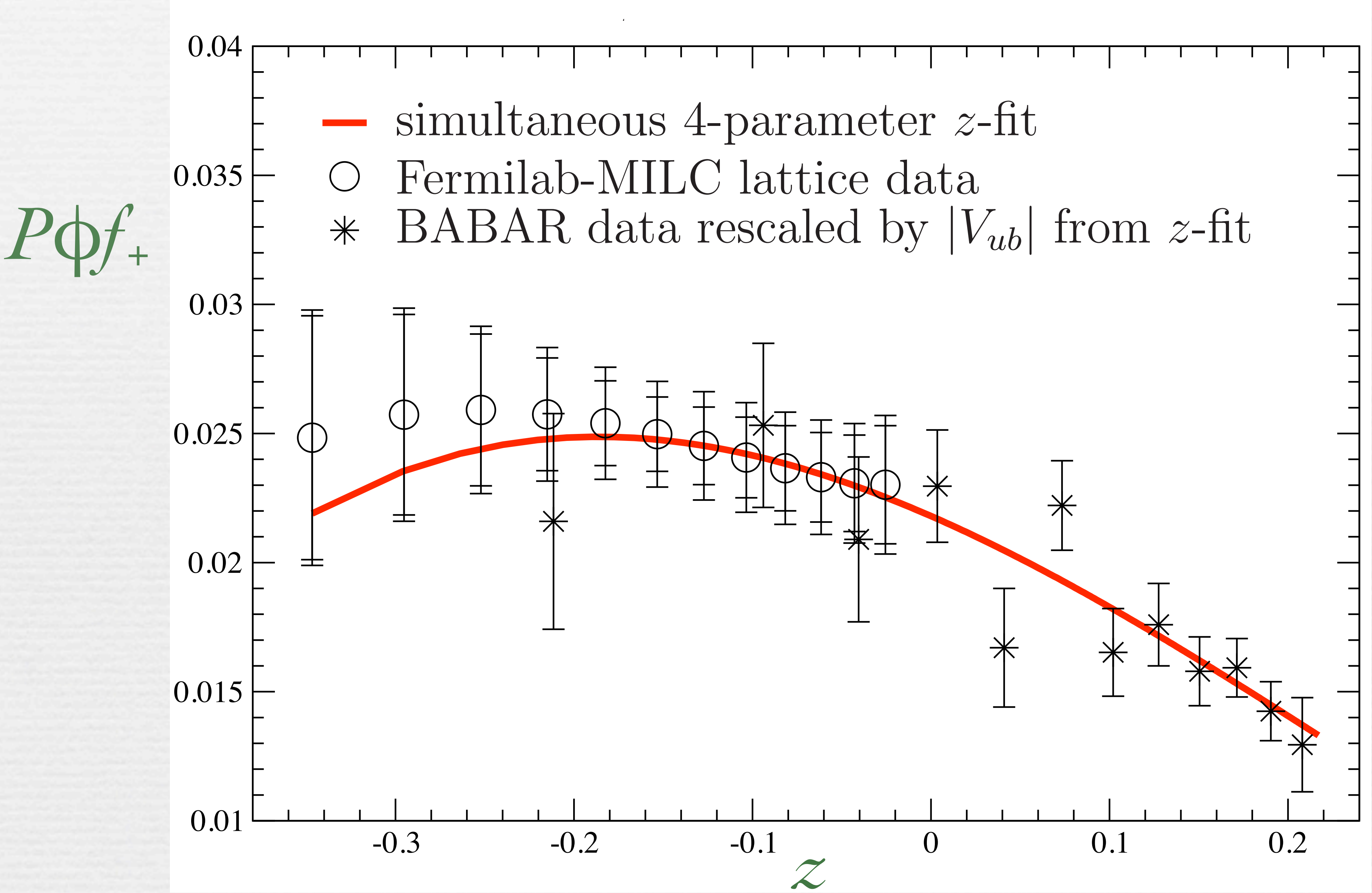
$$P(t)\phi(t,t_0)f(t) = \sum_{k=0}^{\infty} a_k z^k, \quad \sum_{k=0}^{N} a_k^2 \leq 1$$

B^* pole

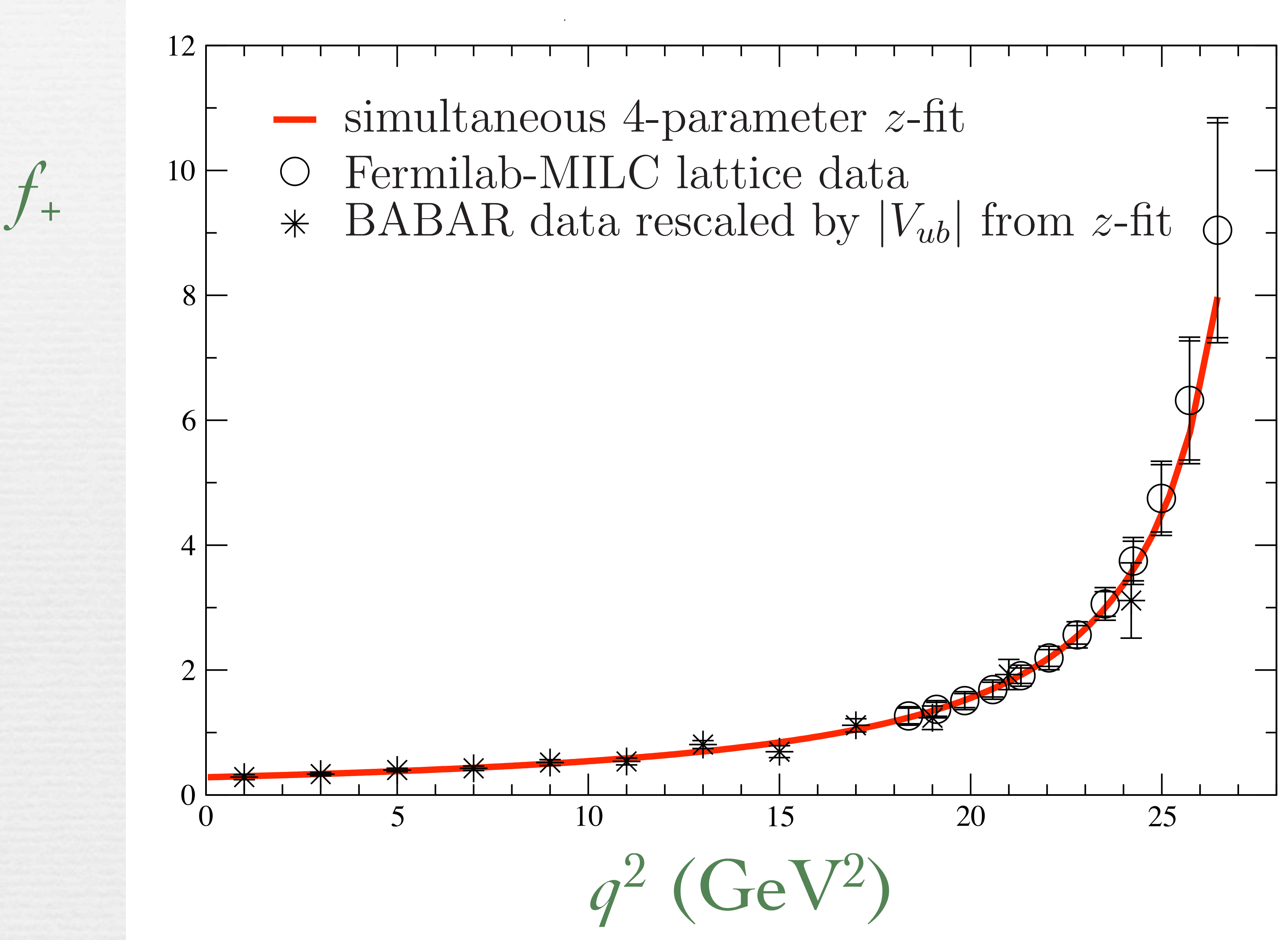
asymptotics

- New approach
  - fit lattice & expt separately: compare a_k/a_0;
  - fit lattice & expt together, yielding $|V_{ub}|$.

Lattice QCD + 12-bin BaBar measurement.



4 fit parameters: $|V_{ub}|$, a_0, a_1, a_2.



Fermilab Lattice + MILC

$|V_{cb}|$ & $|V_{ub}|$

- Using $\mathcal{F}(1)$ to get $|V_{cb}|$:

  $$10^3|V_{cb}| = 38.7(9)(10)$$

  with latest HFAG.

- Compared to inclusive:

  $$10^3|V_{cb}| = 41.6(8)$$

  from HFAG/ICHEP08.

- Final z-fit to get $|V_{ub}|$:

  $$10^3|V_{ub}| = 3.38(36)$$

  with BaBar 12-bin data.

- Compared to inclusive:

  $$10^3|V_{ub}| = (3.76\text{–}4.87) \pm 0.35$$

  from HFAG/ICHEP08.

Being sorted out for CKM 2008 report.

f_{D_s} Puzzle

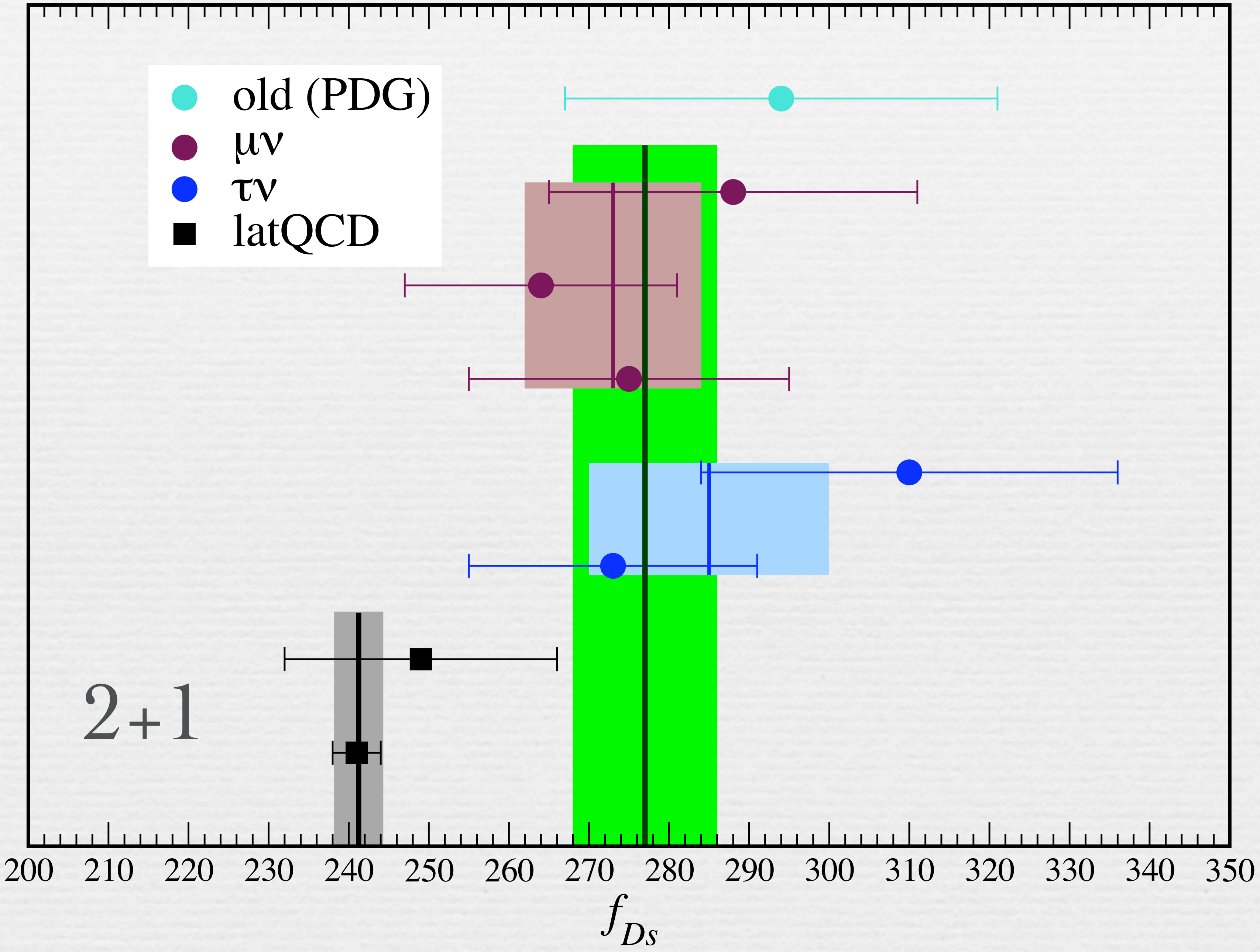
f_D and f_{D_s}

- These are thought of as tests of (lattice) QCD.
- Experiments (recently) yield $|V_{cd}|f_D$ and $|V_{cs}|f_{D_s}$:
  - $|V_{cx}|$ from CKM unitarity.
- First *un*quenched calculations [Fermilab/MILC] agreed, at 7% level, with first good measurements (CLEO for D, BaBar for D_s).

$D_s \to l\nu$

- $D_s \to l\nu$ should be the easiest leptonic decay for lattice QCD.
- A simple matrix element $\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle = i f_{D_s} p_\mu$.
- No light valence quarks.
- Counting experiment at CLEO, B factories.
- New physics thought to be *very unlikely*.

And then something funny happened (end 2007)...



$\chi^2/\text{dof} = 0.67$

BaBar

CLEO

Belle

CLEO πν

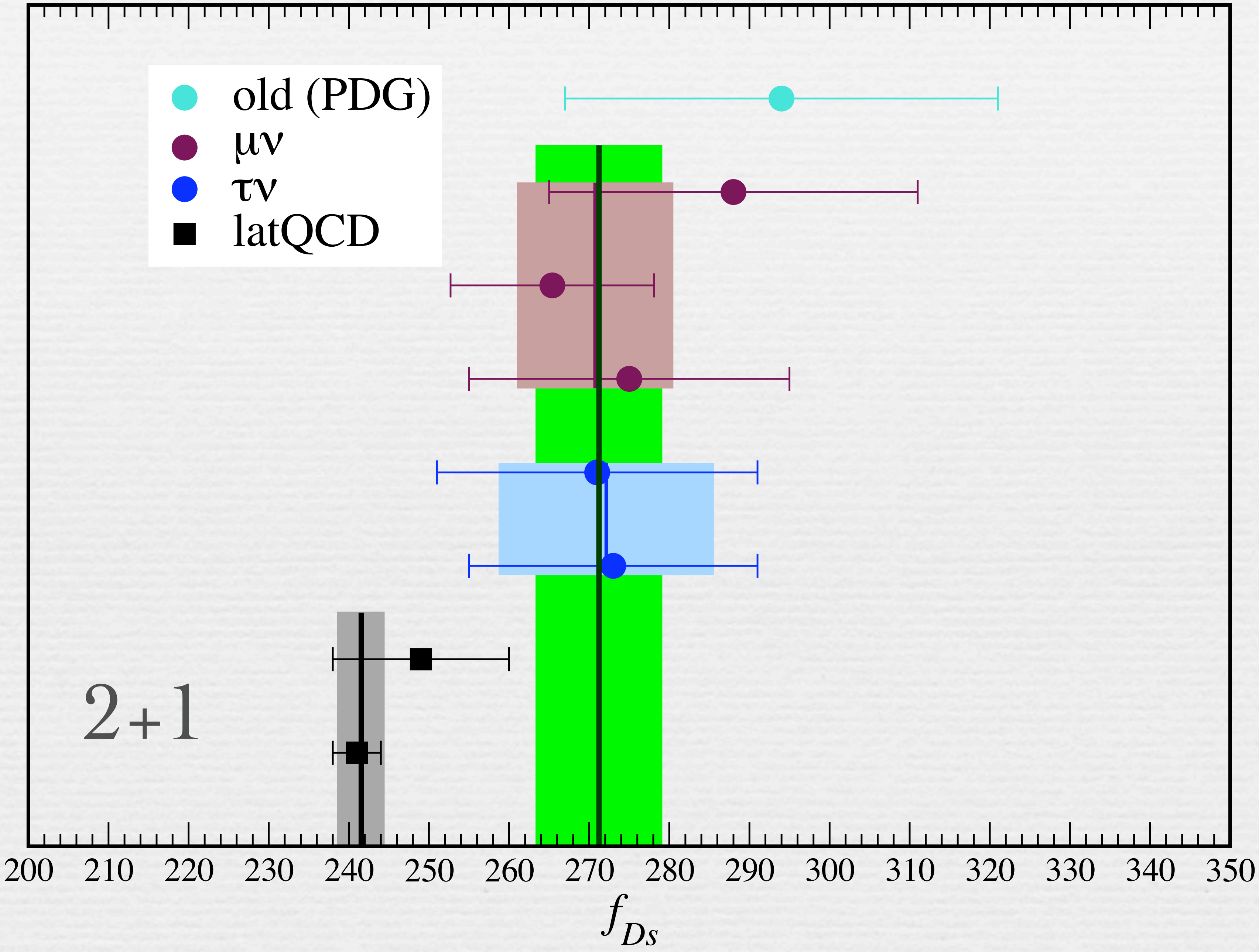
CLEO eνν

Fermilab/MILC

HPQCD

a 3.8σ discrepancy, or 2.7σ ⊕ 2.9σ.

Updates from FPCP (CLEO) and Lat'08 ...



χ^2/dof = 0.13

BaBar

CLEO

Belle

CLEO πν

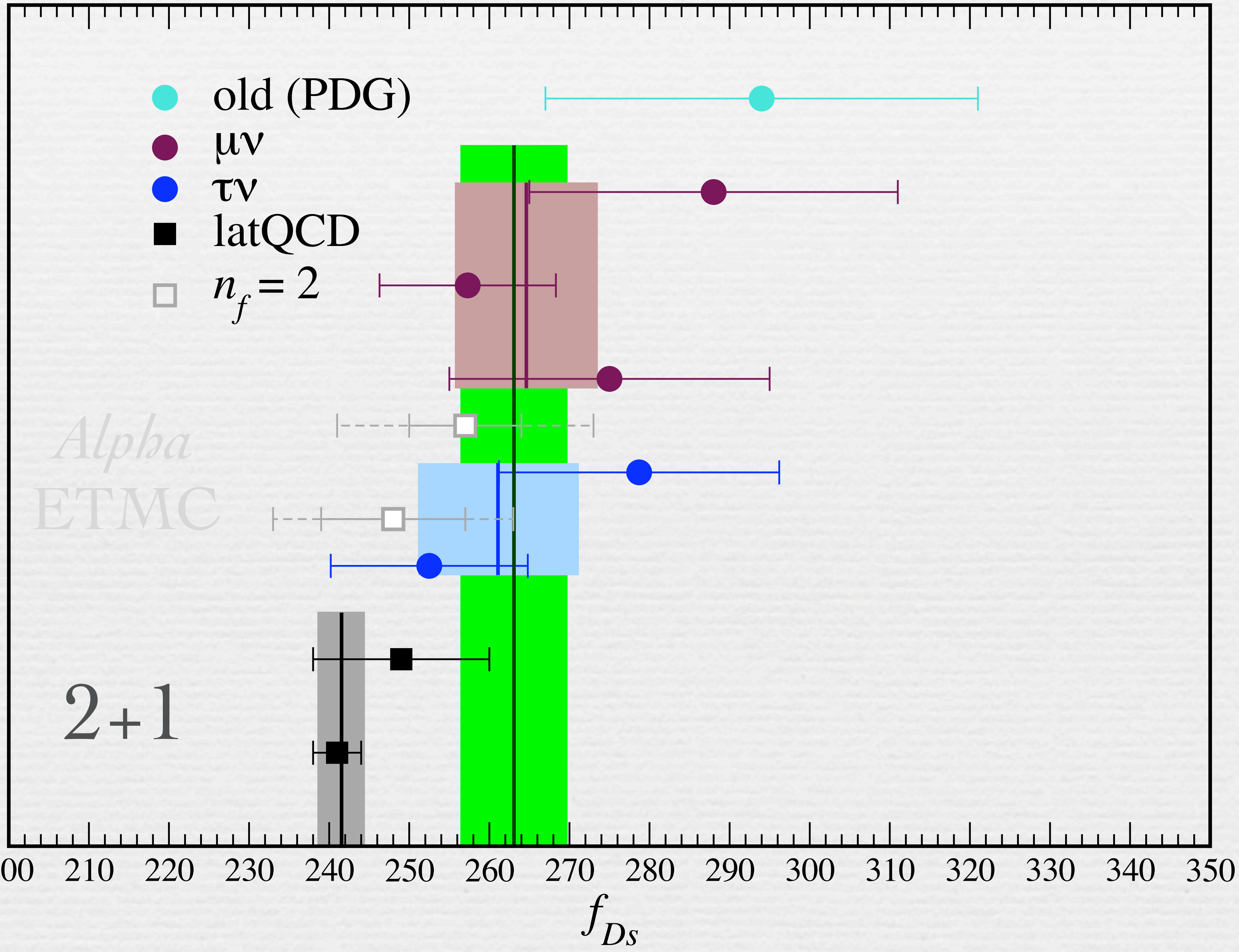
CLEO eνν

Fermilab/MILC

HPQCD

a 3.6σ discrepancy, or 2.9σ ⊕ 2.2σ.

With CLEO's papers of January 12, 2009



$\chi^2/\text{dof} = 0.73$

BaBar

CLEO

Belle

CLEO $\pi\nu$

CLEO e$\nu\nu$

Fermilab/MILC

HPQCD

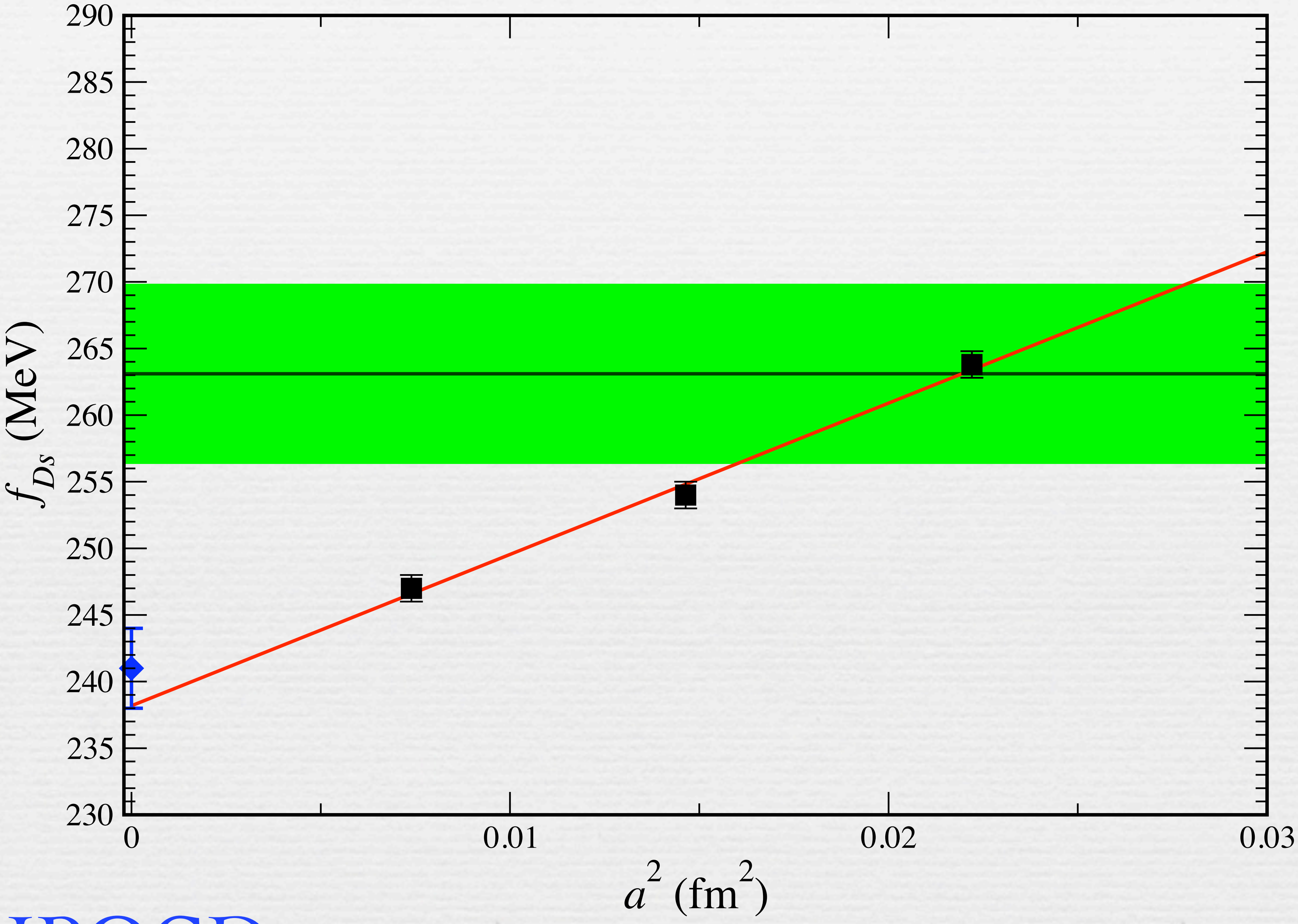
a 3.0σ discrepancy, or $2.5\sigma \oplus 1.9\sigma$.

A Puzzle

$$B(D_s \to \ell\nu) = \frac{m_{D_s}\tau_{D_s}}{8\pi} f_{D_s}^2 \, |G_F V_{cs}^* m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2$$

❧ Experimental errors?	❧ Unlikely: stats limited.
❧ Radiative corrections?	❧ No: 1–2%
❧ CKM?	❧ No: need $\lvert V_{cs}\rvert > 1.1$.
❧ Lattice QCD?	❧ Let's see.

As the lattice gets finer, the discrepancy grows:

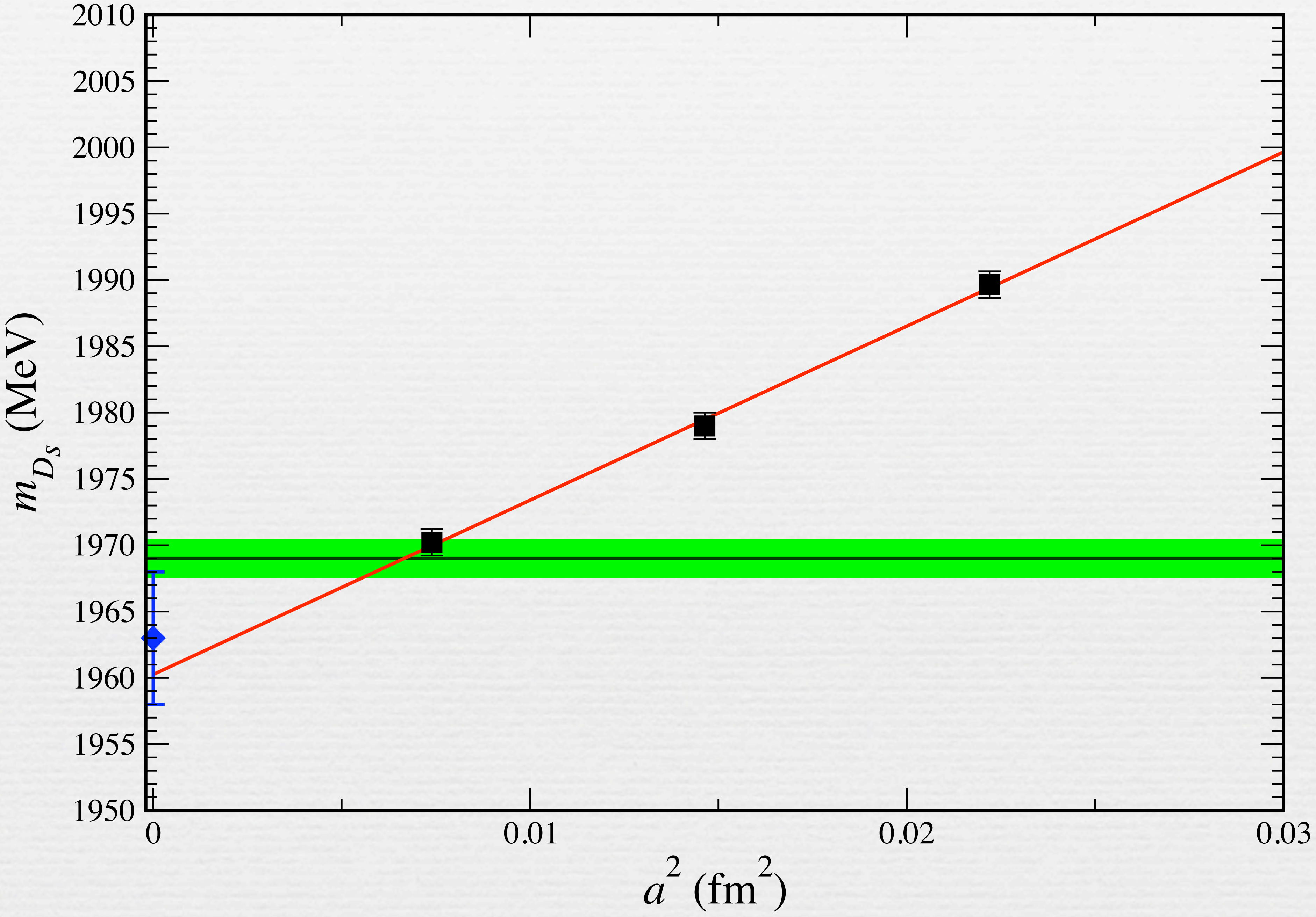


263.1 ± 6.7 MeV

slope is $O(\alpha_s m_c \Lambda a^2)$ as expected

HPQCD
241±3

linear in a^2: 239; quad in a^2: 242;
linear in a^4: 245.



If m_c (set from η_c) were retuned to flatten this, f_{D_s} (at $a \neq 0$) would not change much.

Error Budget

$\Delta_q = 2m_{Dq} - m_{\eta c}$

	f_K/f_π	f_K	f_π	f_{D_s}/f_D	f_{D_s}	f_D	Δ_s/Δ_d
r_1 uncerty.	0.3	1.1	1.4	0.4	1.0	1.4	0.7
a^2 extrap.	0.2	0.2	0.2	0.4	0.5	0.6	0.5
Finite vol.	0.4	0.4	0.8	0.3	0.1	0.3	0.1
$m_{u/d}$ extrap.	0.2	0.3	0.4	0.2	0.3	0.4	0.2
Stat. errors	0.2	0.4	0.5	0.5	0.6	0.7	0.6
m_s evoln.	0.1	0.1	0.1	0.3	0.3	0.3	0.5
m_d, QED, etc.	0.0	0.0	0.0	0.1	0.0	0.1	0.5
Total %	0.6	1.3	1.7	0.9	1.3	1.8	1.2

charmed sea $\ll 1\%$?

Other Results

arXiv:hep-lat/0610092 & arXiv:0706.1726 [hep-lat]

what	expt	HPQCD	
$m_{J/\psi} - m_{\eta c}$	118.1	111 ± 5[‡]	MeV
m_{Dd}	1869	1868 ± 7	MeV
m_{Ds}	1968	1962 ± 6	MeV
Δ_s/Δ_d	1.260 ± 0.002	1.252 ± 0.015	
f_π	130.7 ± 0.4	132 ± 2	MeV
f_K	159.8 ± 0.5	157 ± 2	MeV
f_D	205.8 ± 8.9[*]	207 ± 4	MeV

[*] CLEO arXiv:08062112 [‡] annihilation corrected

What if

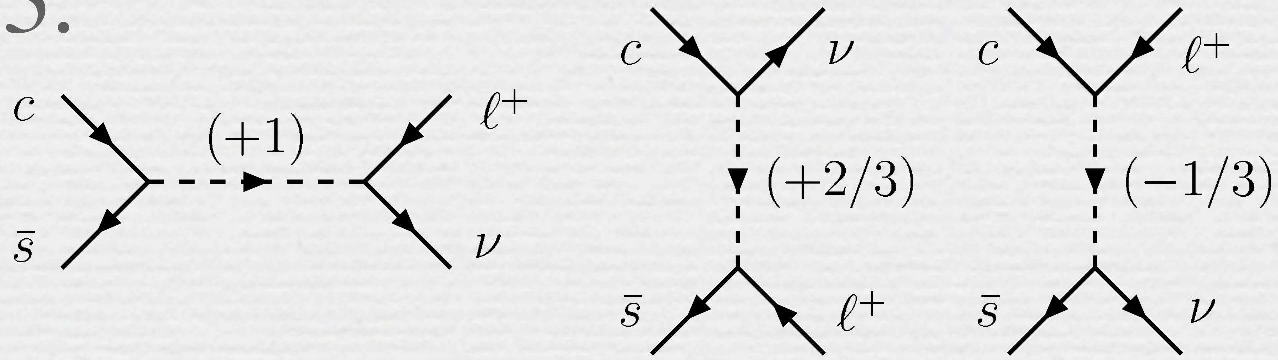
- … the discrepancy is real?
- Then it must be non-Standard physics.
- How wacky would a non-Standard model be?
- It turns out particles that are already being considered can do the trick.
- B.A. Dobrescu & ASK, arXiv: 0803.0512

New Particles

- Effective interactions

  $$\mathcal{L}_{\text{eff}} = \frac{C_A^\ell}{M^2}\left(\bar{s}\gamma_\mu\gamma_5 c\right)\left(\bar{\nu}_L\gamma^\mu\ell_L\right) + \frac{C_P^\ell}{M^2}\left(\bar{s}\gamma_5 c\right)\left(\bar{\nu}_L\ell_R\right) + \text{H.c.}$$

  can be induced by heavy particles of charge +1, +2/3, –1/3.

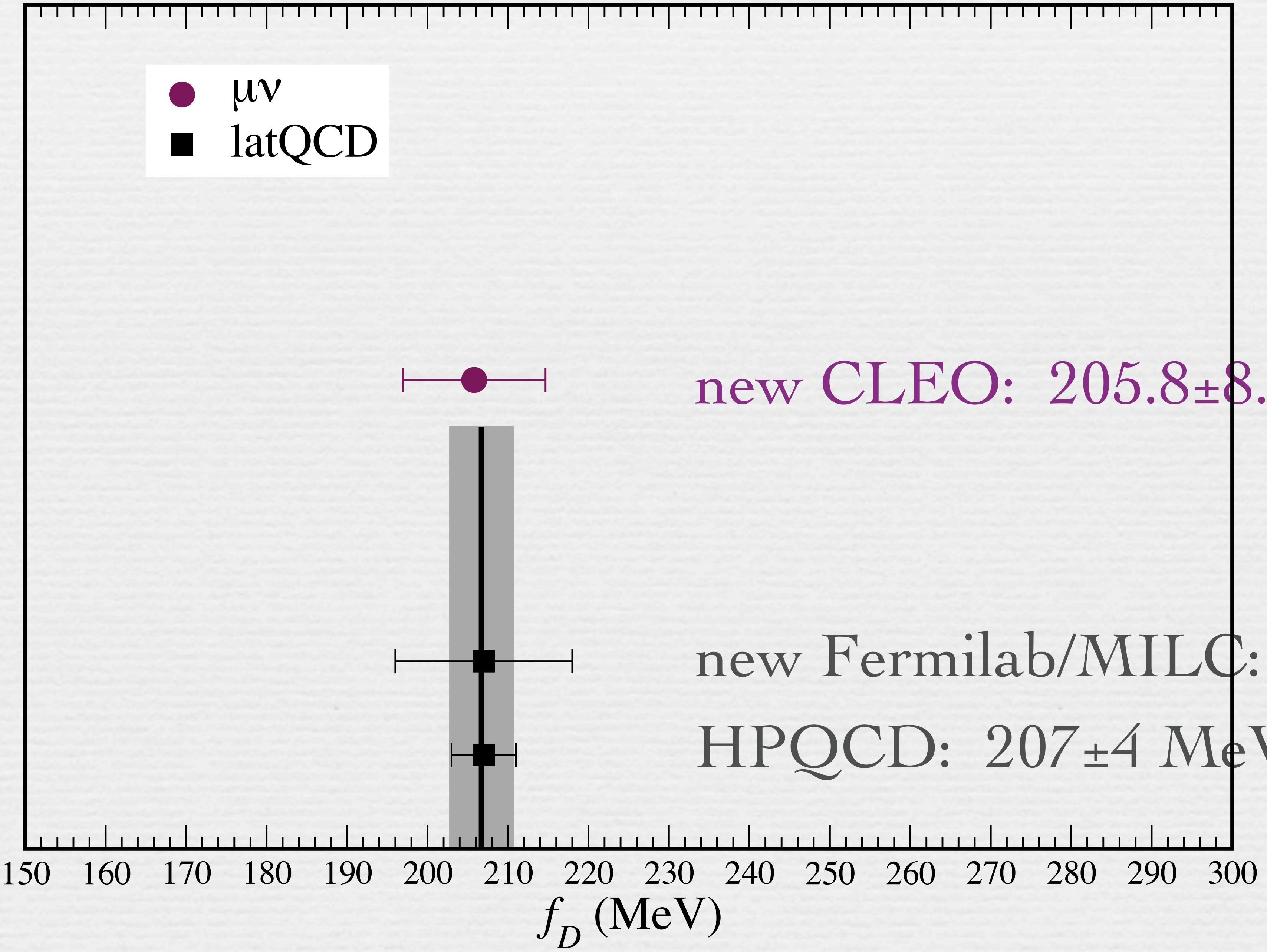


- Charged Higgs, new W'; leptoquarks.

Beyond SM

- New W’ boson: unlikely.
- Charged Higgs:
  - Model II destructively interference;
  - BAD & ASK found new model.
- Leptoquarks:
  - $J = 0$, (3, 1, –1/3), aka $\tilde{d}$, can explain the effect.

- Charged Higgs model predicts a similarly-sized deviation in $D \to l\nu$, now disfavored:



new CLEO: 205.8±8.9 MeV

new Fermilab/MILC: 207±11 MeV

HPQCD: 207±4 MeV

LHC

- The generic bounds on mass/coupling:

$$\frac{M}{(\mathrm{Re}\,C^{\ell}_{A,P})^{1/2}} \gtrsim \begin{cases} 710\ \mathrm{GeV}, \quad 920\ \mathrm{GeV} & \text{for } \ell = \tau \\ 850\ \mathrm{GeV}, \ 4500\ \mathrm{GeV} & \text{for } \ell = \mu \end{cases}$$

any non-Standard explanation of the effect is observable at the LHC.

- Leptoquarks: $gg \to \tilde{d}\bar{\tilde{d}} \to \ell_1^+ \ell_2^- j_c j_c$.

Conclusions

- Lattice QCD with 2+1 staggered sea quarks has provided many results since 2003;
  - now 2+1 Wilson and DWF sea too.
- Broad, and often precise, agreement with experiment in hadron masses, quarkonium splittings, decay properties.
- Precise agreement of α_s and heavy-quark masses, where pQCD also reliable.

- The outlier is f_{Ds}, which should be *easy*:
  - valence quarks aren't light;
  - PCAC normalization.
- Experimental *statistical* error is yardstick for discrepancy: with 2×(lattice error) still 2.4σ.
- CLEO done; BaBar & Belle could revisit; BES will go further in a few years.
- If new particle, LHC will make them.