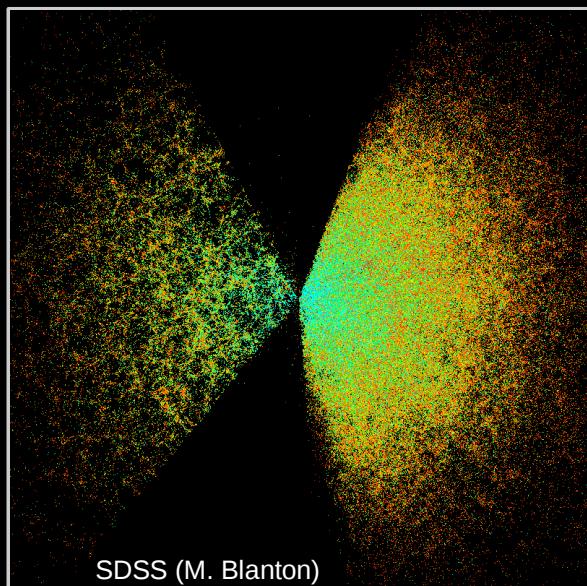


# Dark Matter Theory

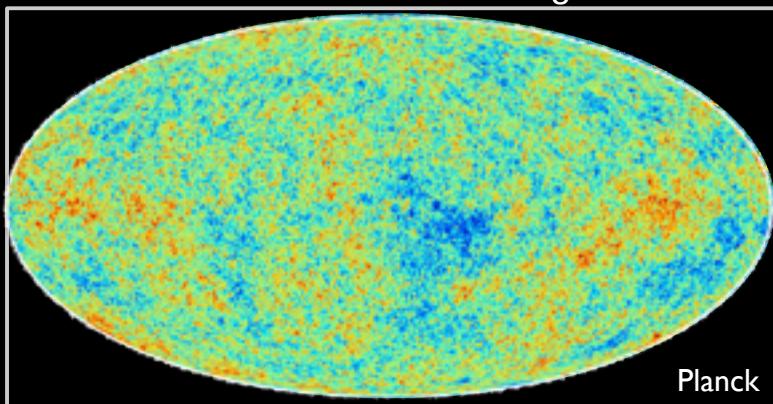
*Paolo Gondolo  
University of Utah*

# Evidence for cold dark matter

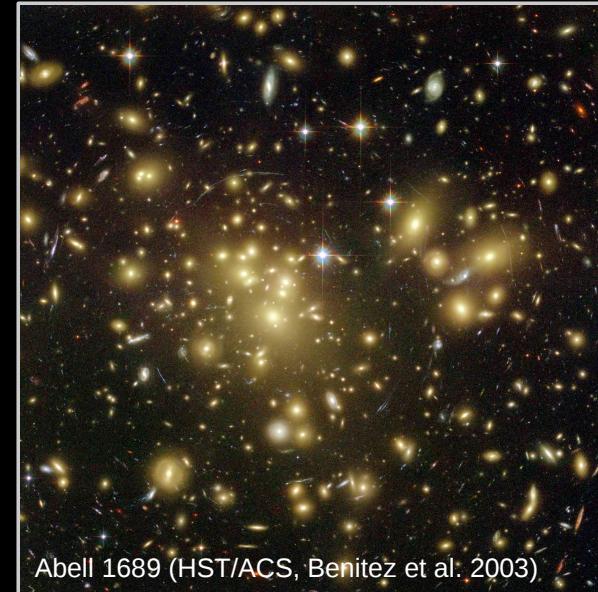
Large Scale Structure



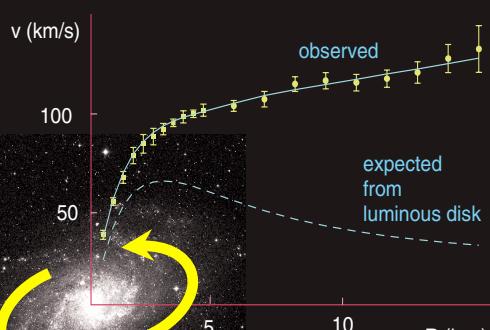
Cosmic Microwave Background



Galaxy Clusters



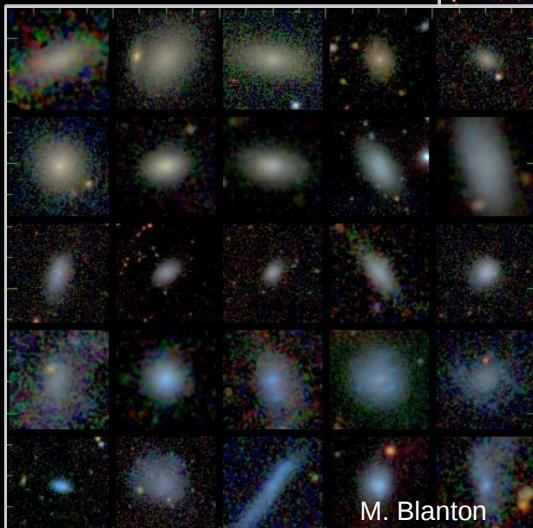
Supernovae



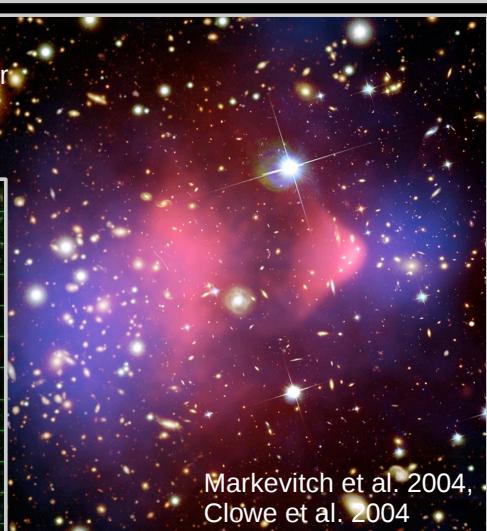
Galaxies



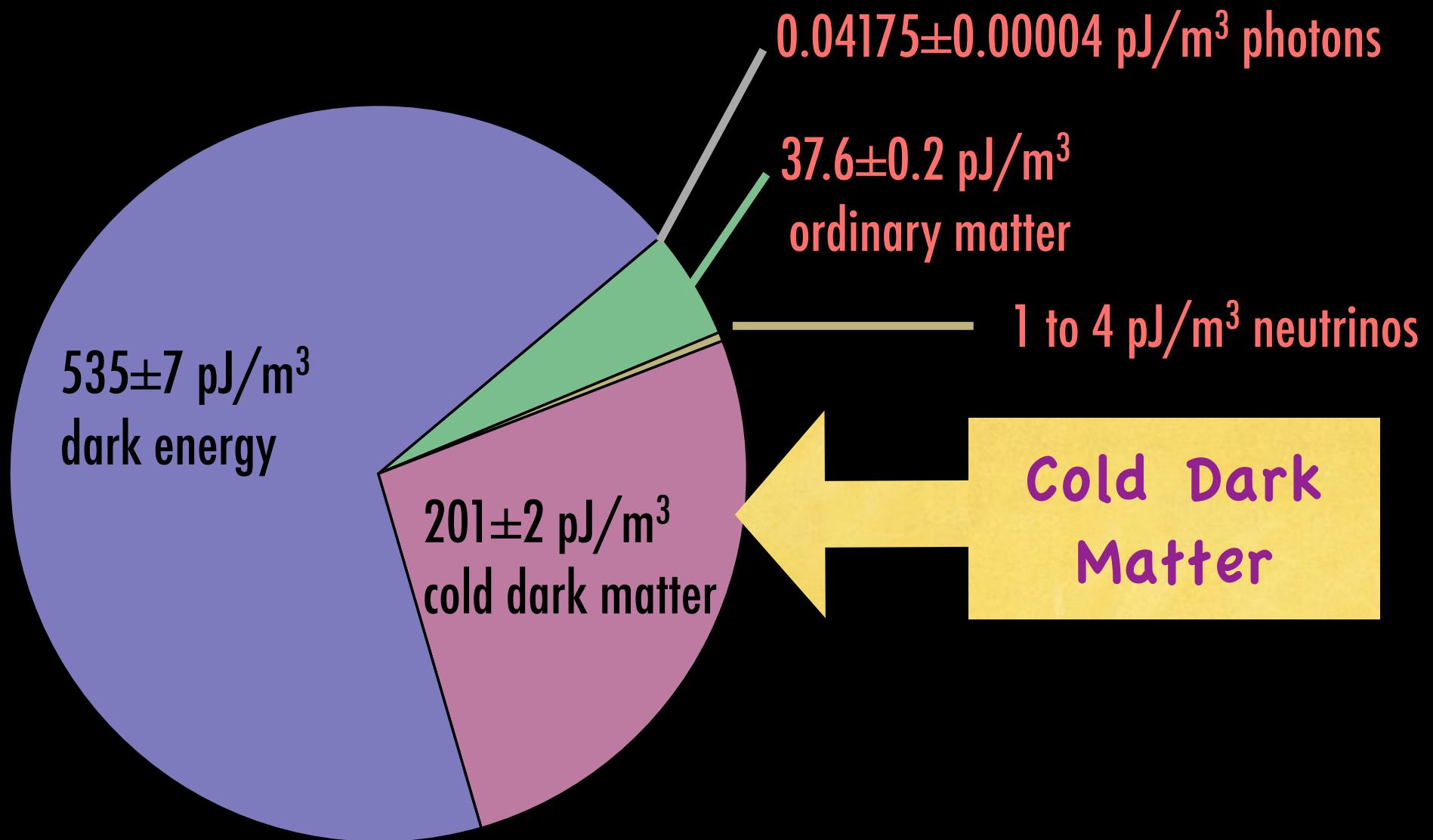
Dwarf Galaxies



Bullet Cluster



# The observed energy content of the universe



matter  $p \ll \rho$

radiation  $p = \rho/3$

vacuum  $p = -\rho$

Planck (2015)  
 $TT,TE,EE + lowP + lensing + ext$

$1 \text{ pJ} = 10^{-12} \text{ J}$

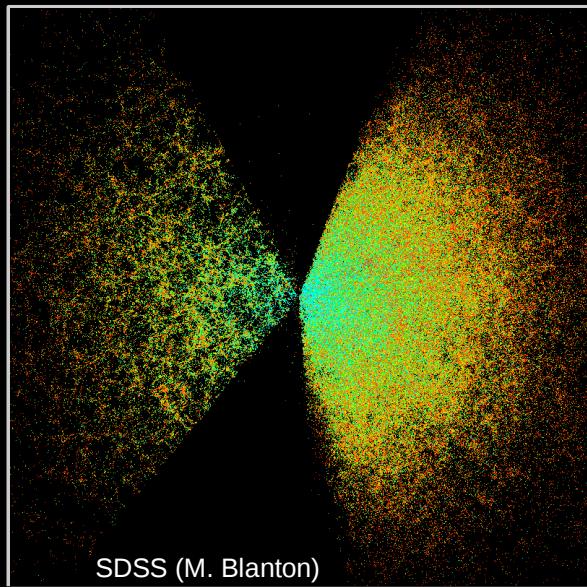
$\rho_{\text{crit}} = 1.68829 h^2 \text{ pJ/m}^3$

# **From CMB fluctuations to galaxies**

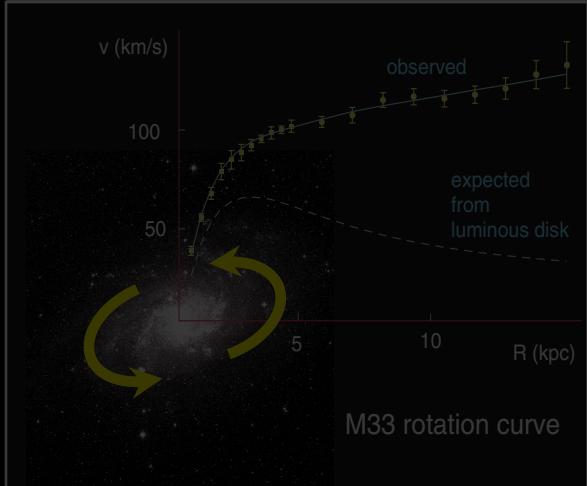
*Galaxy formation*

# From CMB fluctuations to galaxies

Large Scale Structure

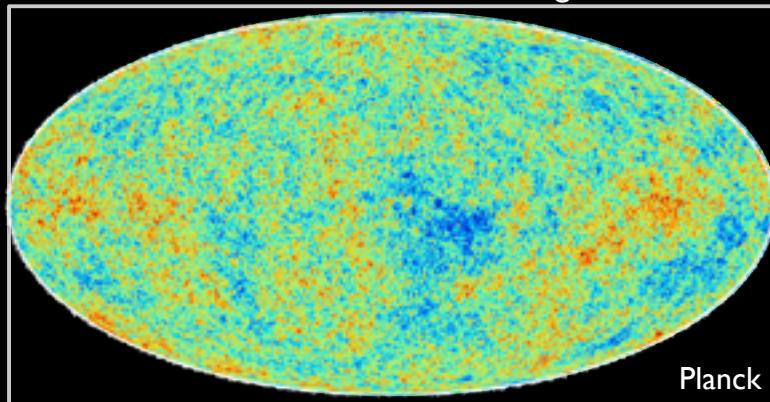


SDSS (M. Blanton)



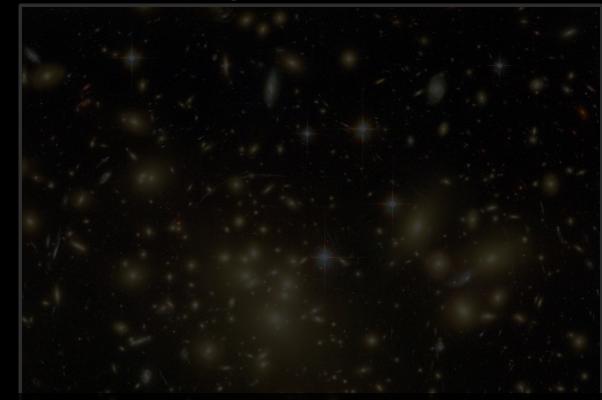
M33 rotation curve

Cosmic Microwave Background

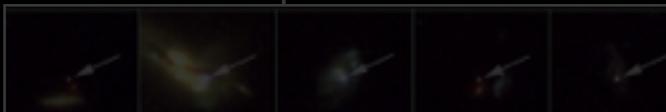


Planck

Galaxy Clusters



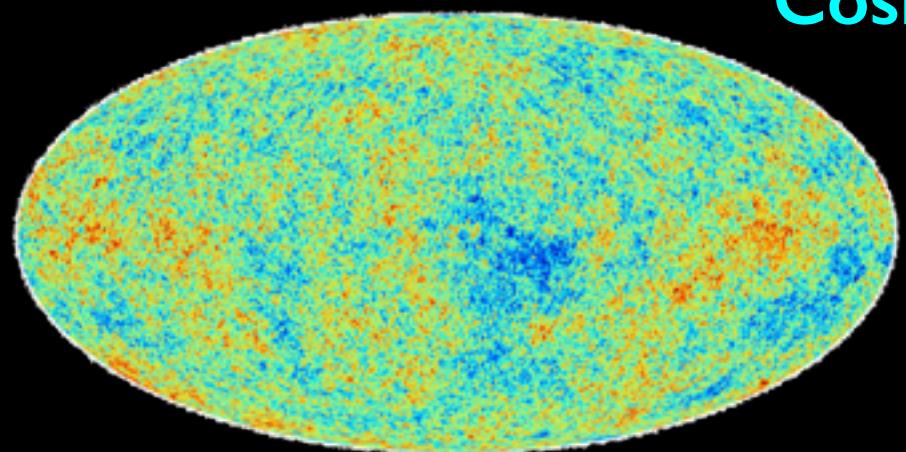
Supernovae



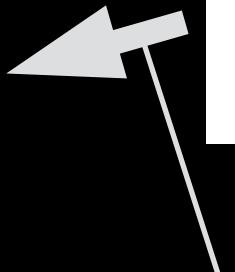
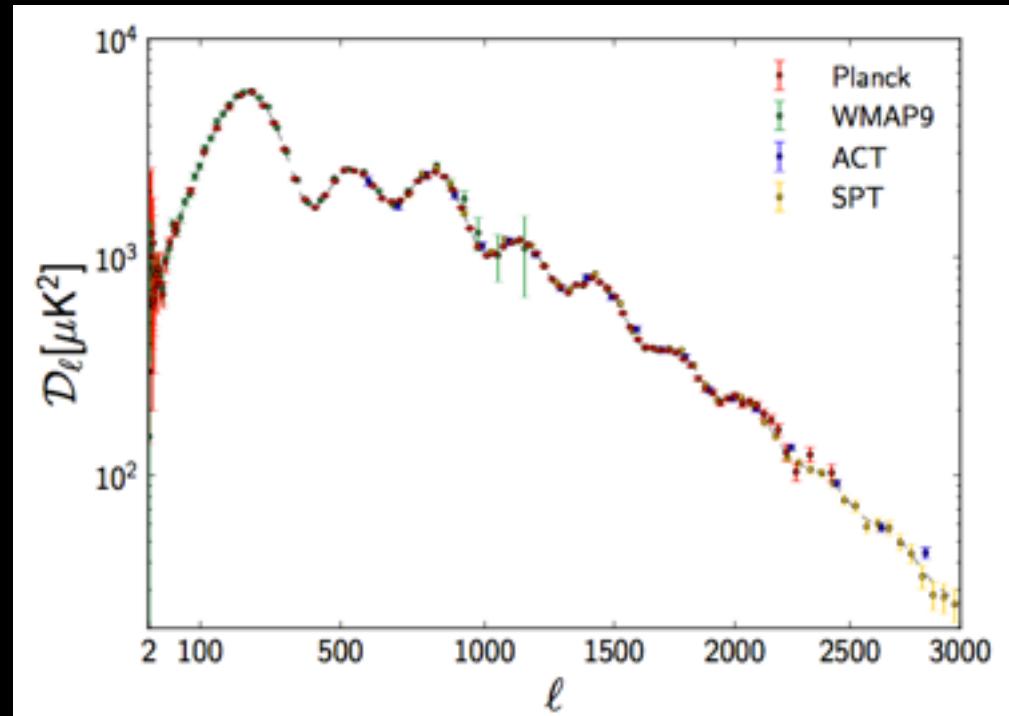
An invisible mass makes the Cosmic Microwave Background fluctuations grow into galaxies (CMB and matter power spectra, or correlation functions)

# Evidence for cold dark matter

## Cosmic Microwave Background fluctuations



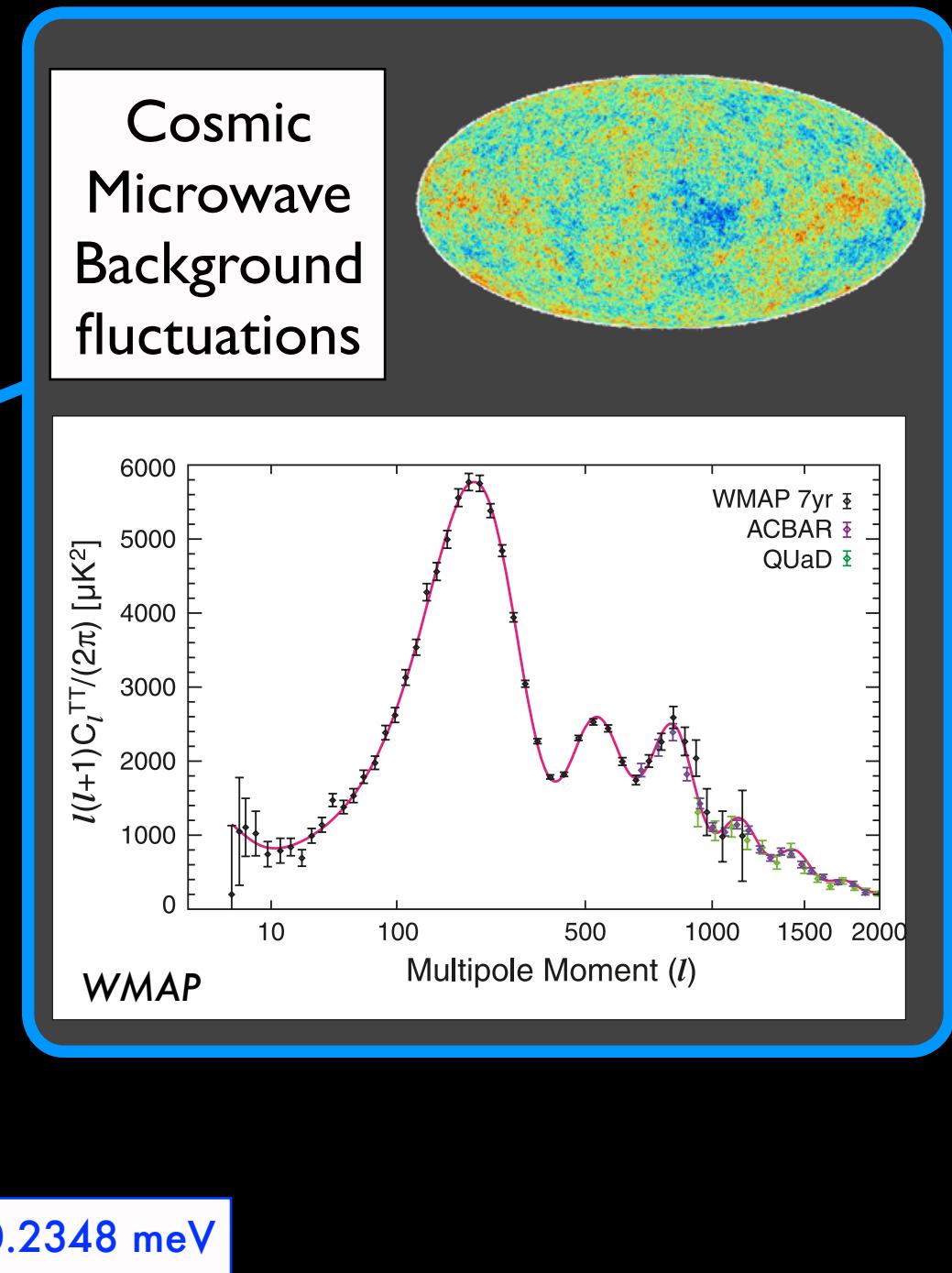
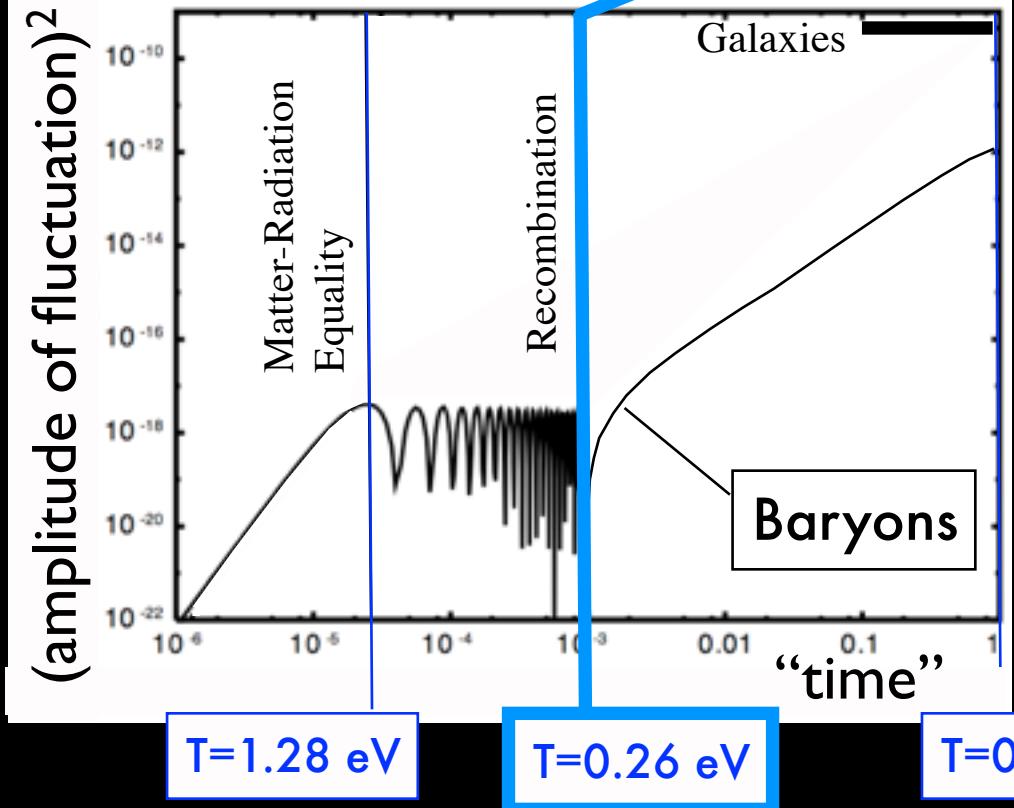
Parameter	<i>Planck+WP+highL+BAO</i>	
	Best fit	68% limits
$\Omega_b h^2$	0.022161	$0.02214 \pm 0.00024$
$\Omega_c h^2$	0.11889	$0.1187 \pm 0.0017$
$100\theta_{\text{MC}}$	1.04148	$1.04147 \pm 0.00056$
$\tau$	0.0952	$0.092 \pm 0.013$
$n_s$	0.9611	$0.9608 \pm 0.0054$
$\ln(10^{10} A_s)$	3.0973	$3.091 \pm 0.025$
$\Omega_\Lambda$	0.6914	$0.692 \pm 0.010$
$\sigma_8$	0.8288	$0.826 \pm 0.012$
$z_{\text{re}}$	11.52	$11.3 \pm 1.1$
$H_0$	67.77	$67.80 \pm 0.77$
Age/Gyr	13.7965	$13.798 \pm 0.037$
$100\theta_*$	1.04163	$1.04162 \pm 0.00056$
$r_{\text{drag}}$	147.611	$147.68 \pm 0.45$



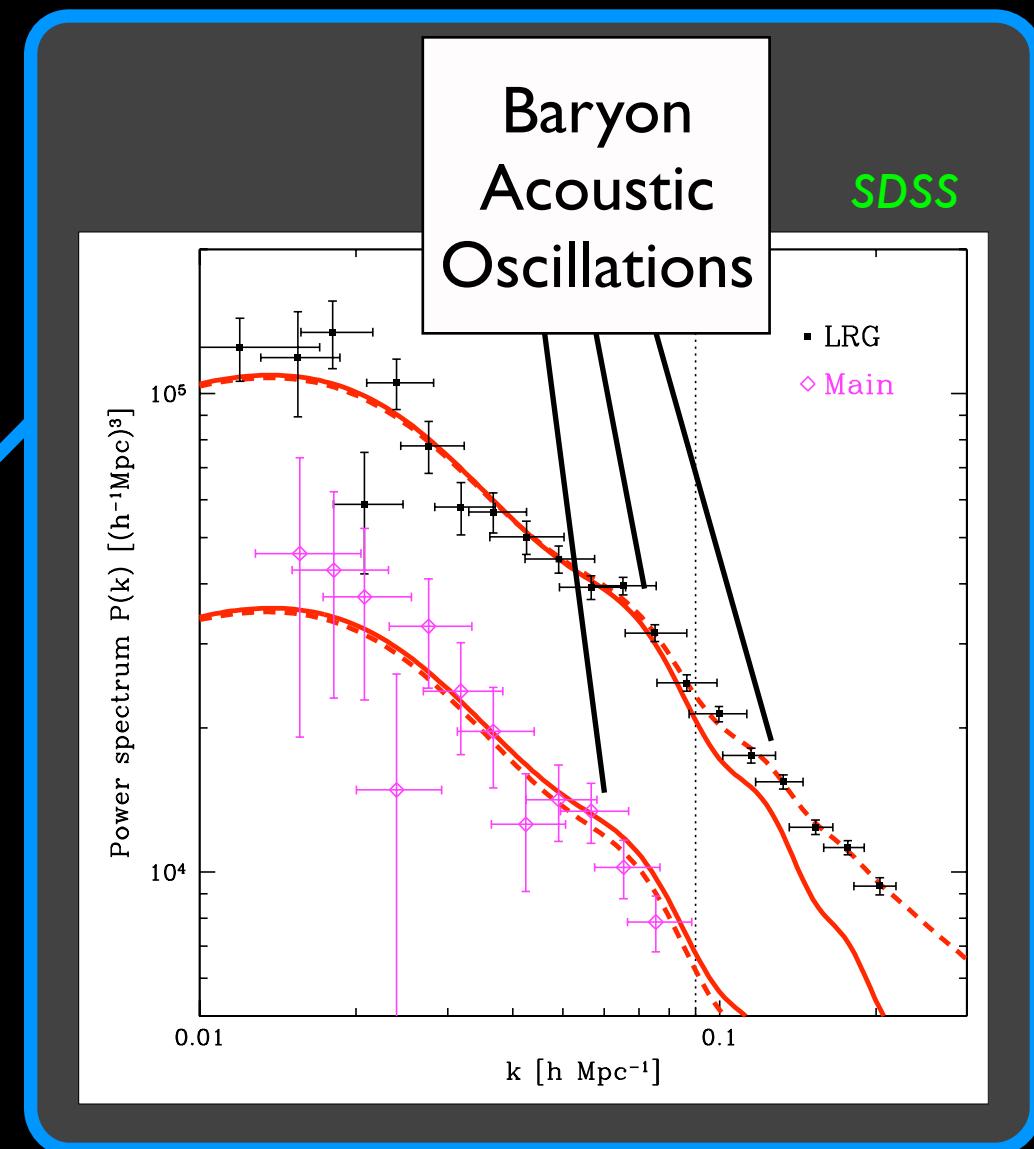
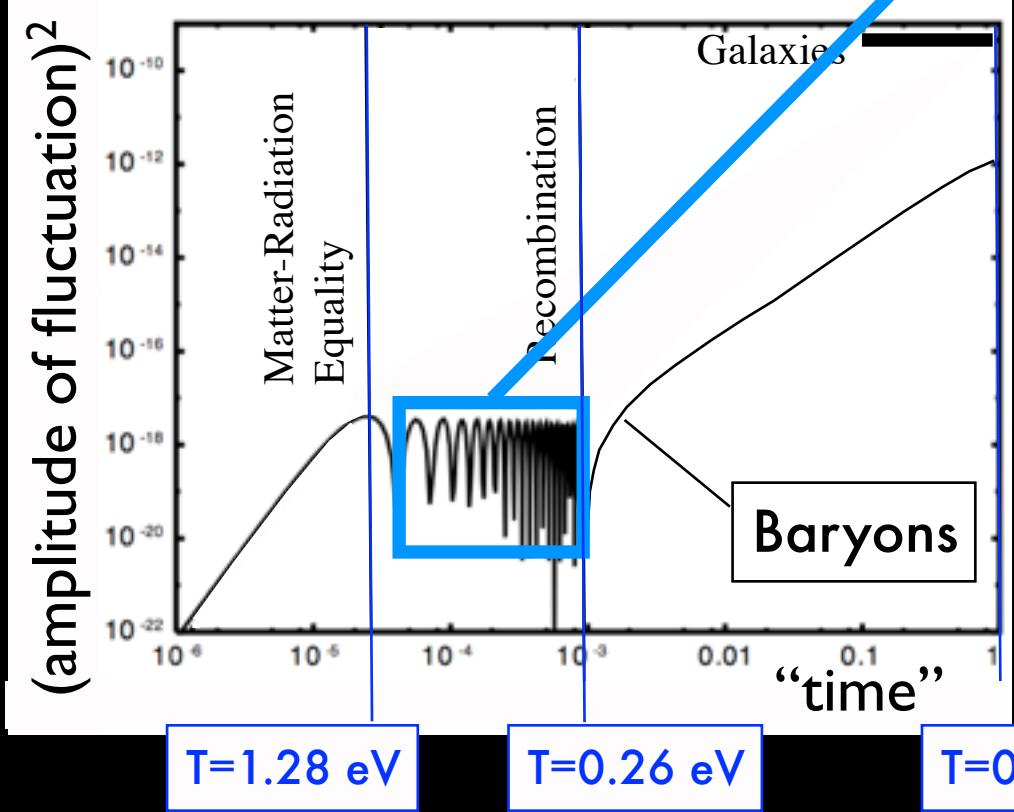
linear perturbation theory

general relativity and statistical mechanics at  $10^4 \text{ K} \sim 1 \text{ eV/k}$

# From CMB fluctuations to galaxies

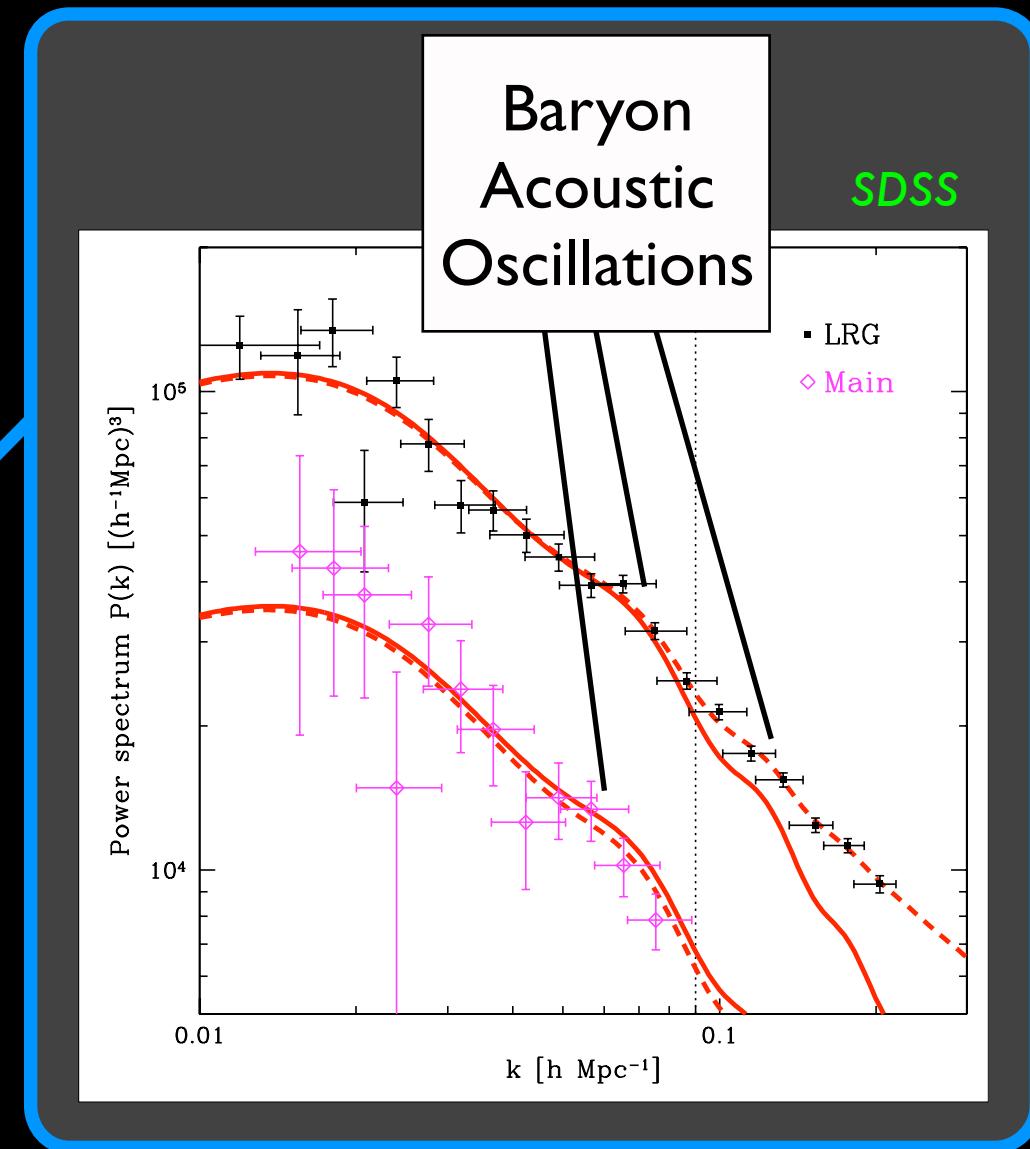
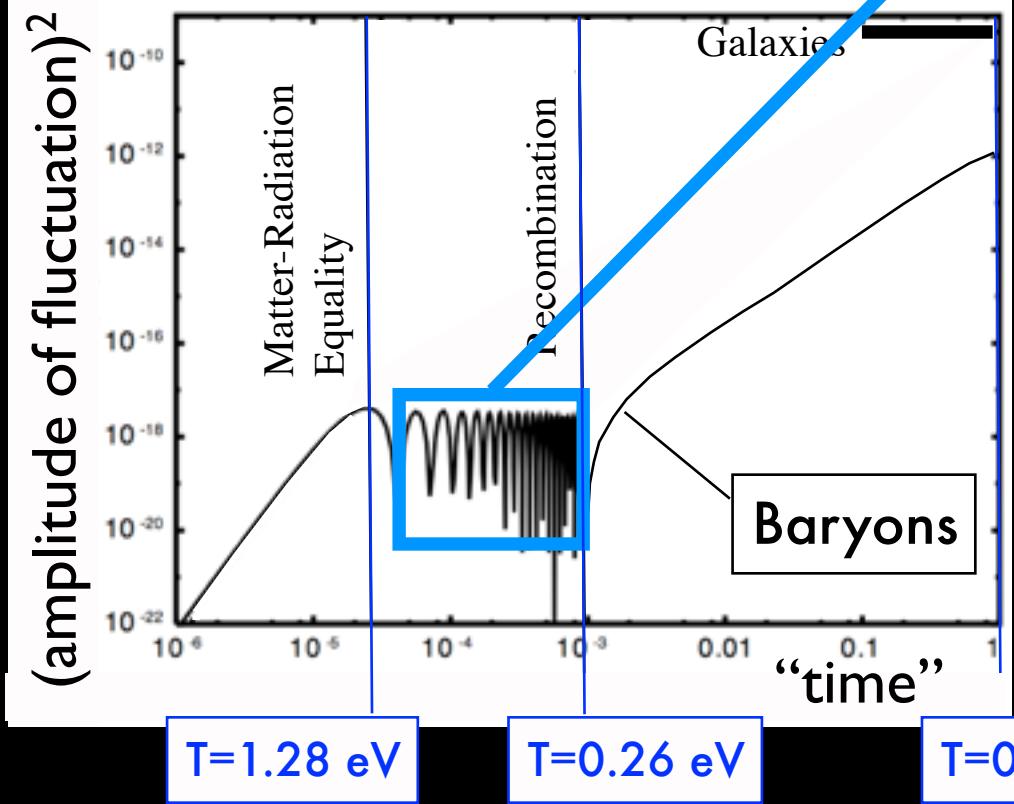


# From CMB fluctuations to galaxies



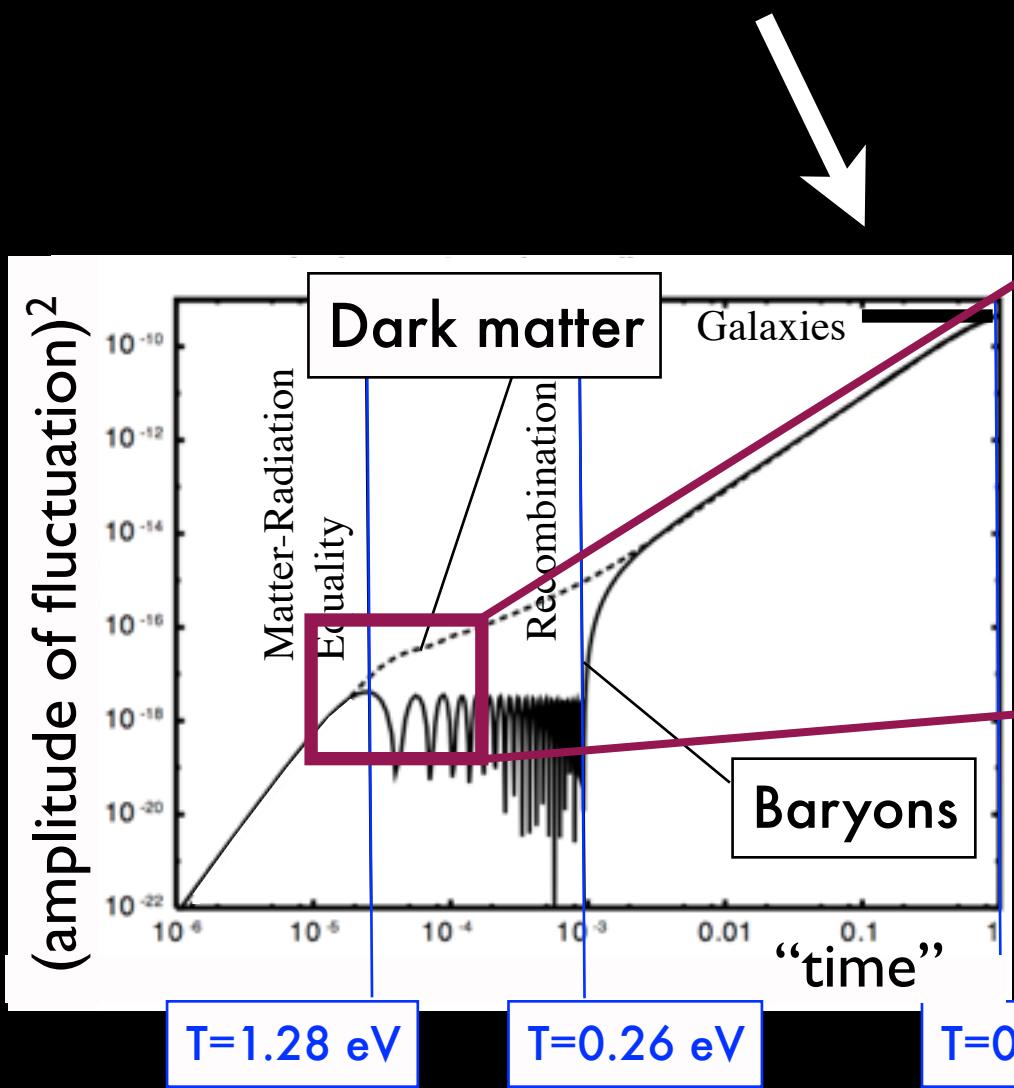
# From CMB fluctuations to galaxies

Fluctuations are too small to gravitationally grow into galaxies in the given 13 billion years.

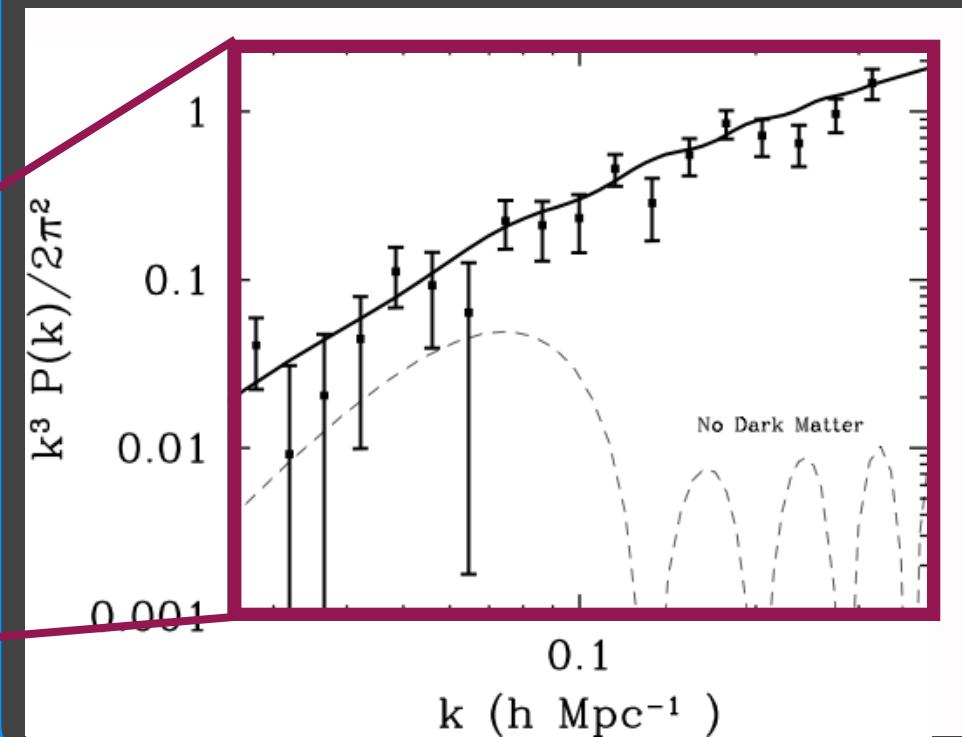


# From CMB fluctuations to galaxies

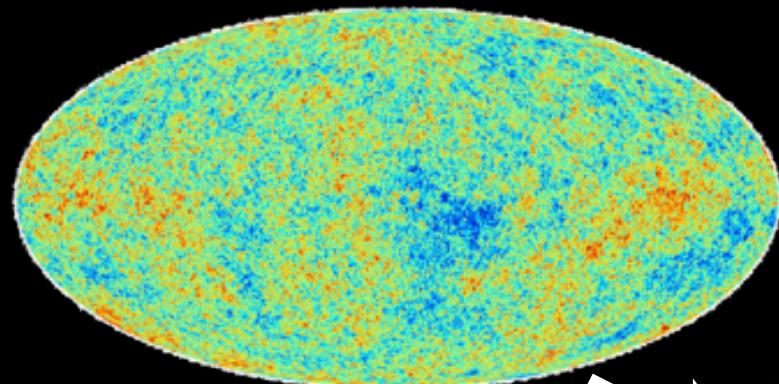
Fluctuation uncoupled to the plasma have enough time to grow



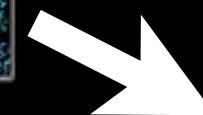
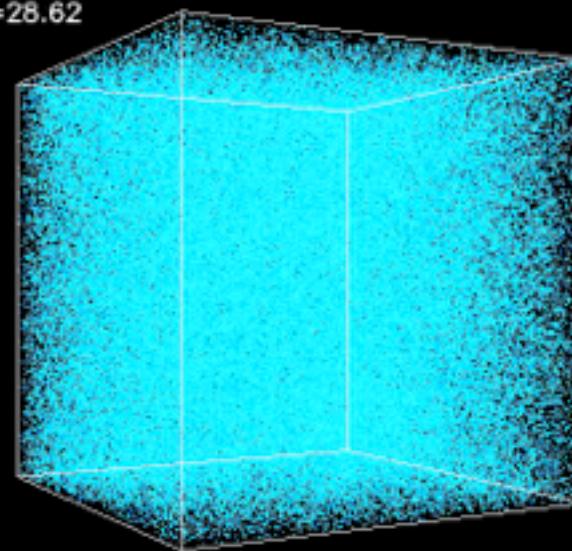
*Dark matter is non-baryonic  
More than 80% of all matter does not couple to the primordial plasma!* SDSS



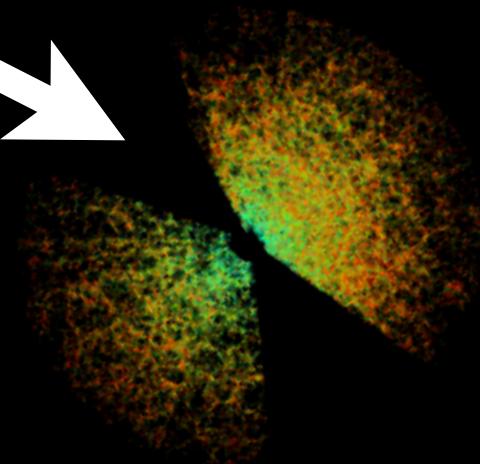
# From CMB fluctuations to galaxies



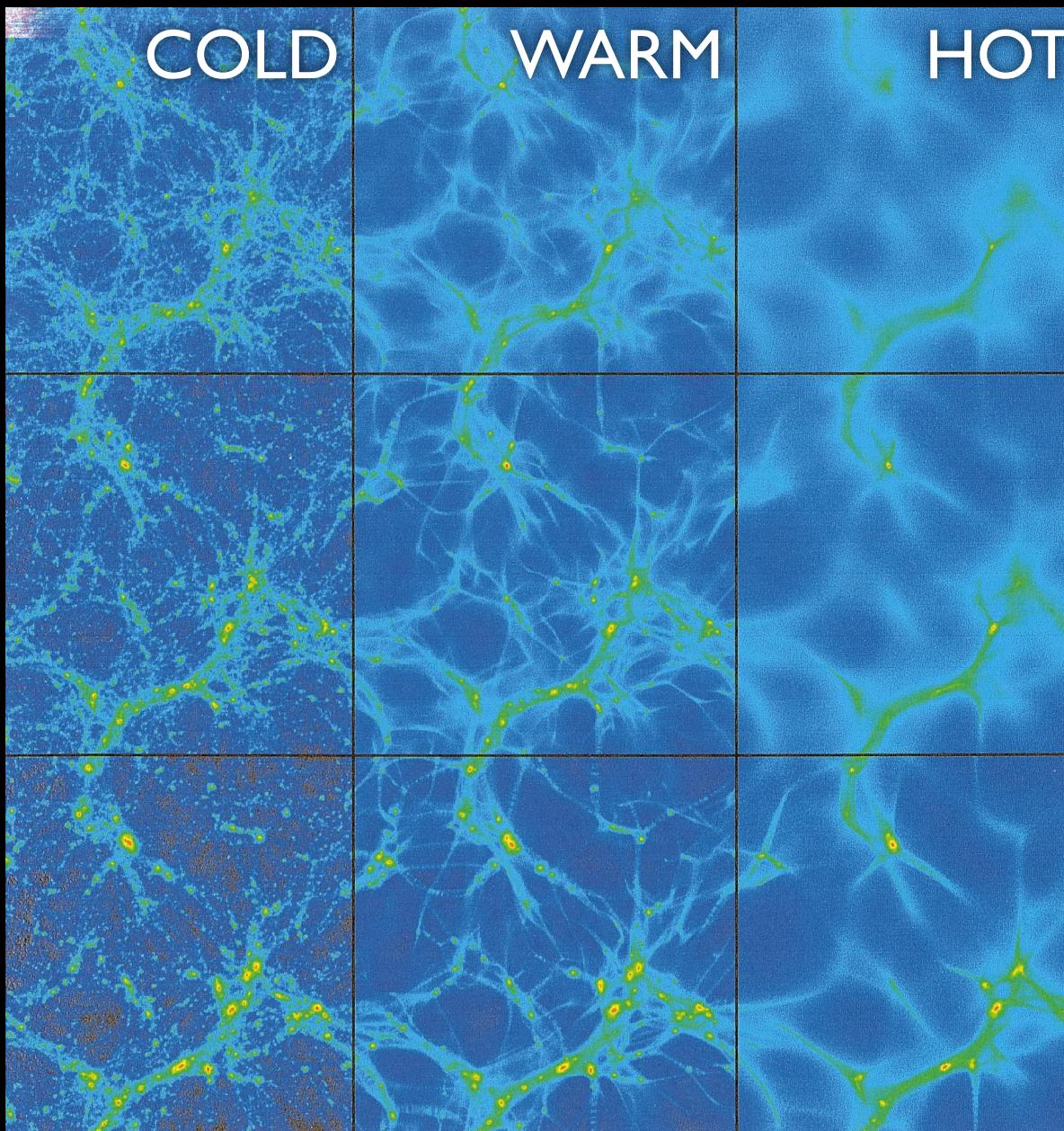
$Z=28.62$



*Kravtsov, Klypin*

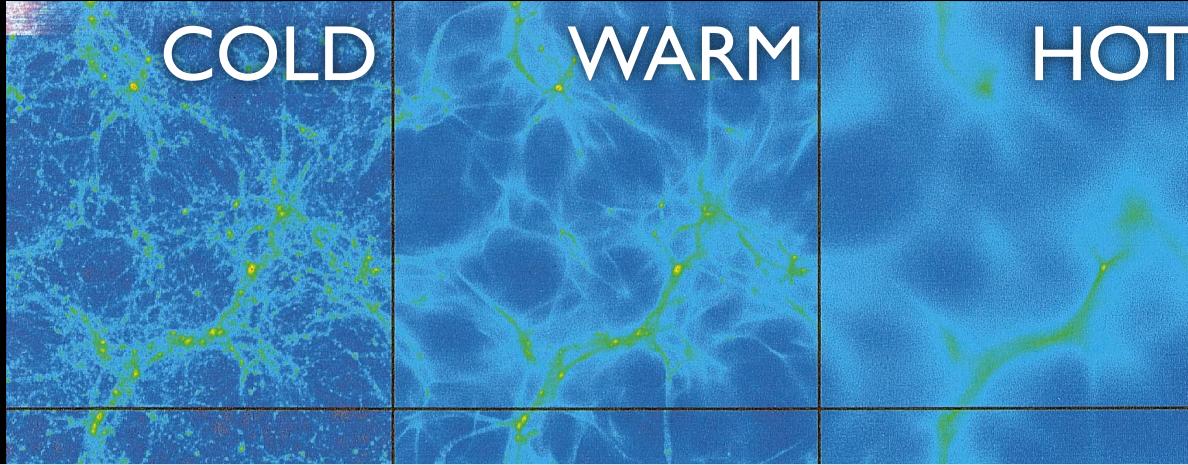


# Cold/warm/hot dark matter



Bode et al 2001

# Cold/warm/hot dark matter

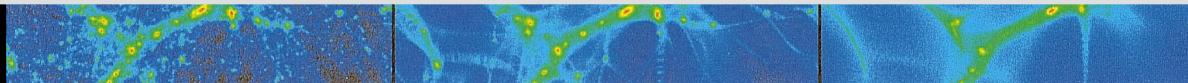


Fourier analysis of density fluctuations

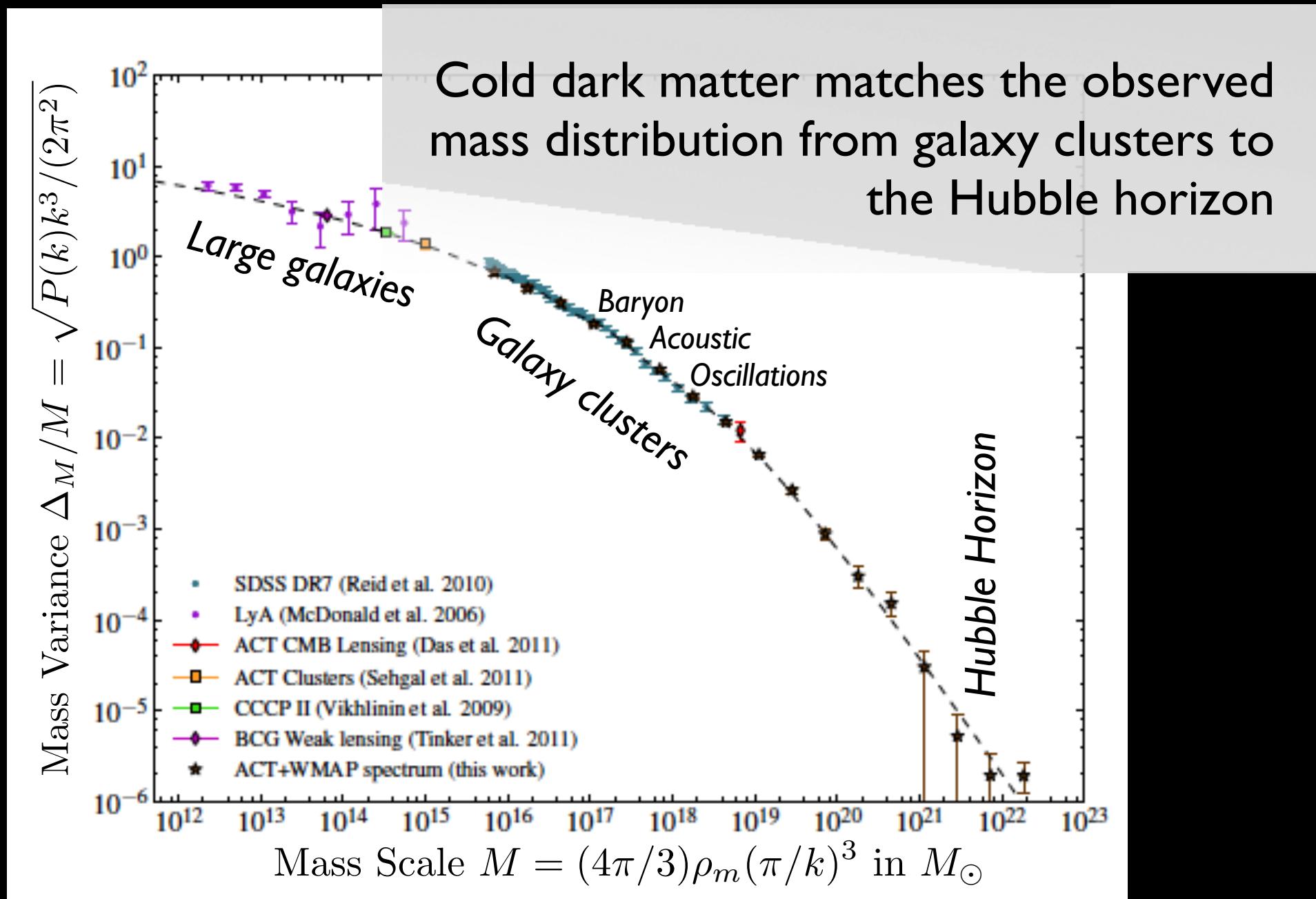
$$\frac{\delta\rho}{\rho} \equiv \frac{\rho(\mathbf{r}) - \bar{\rho}}{\bar{\rho}} = \int \delta_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{d^3k}{(2\pi)^3}$$

Matter power spectrum

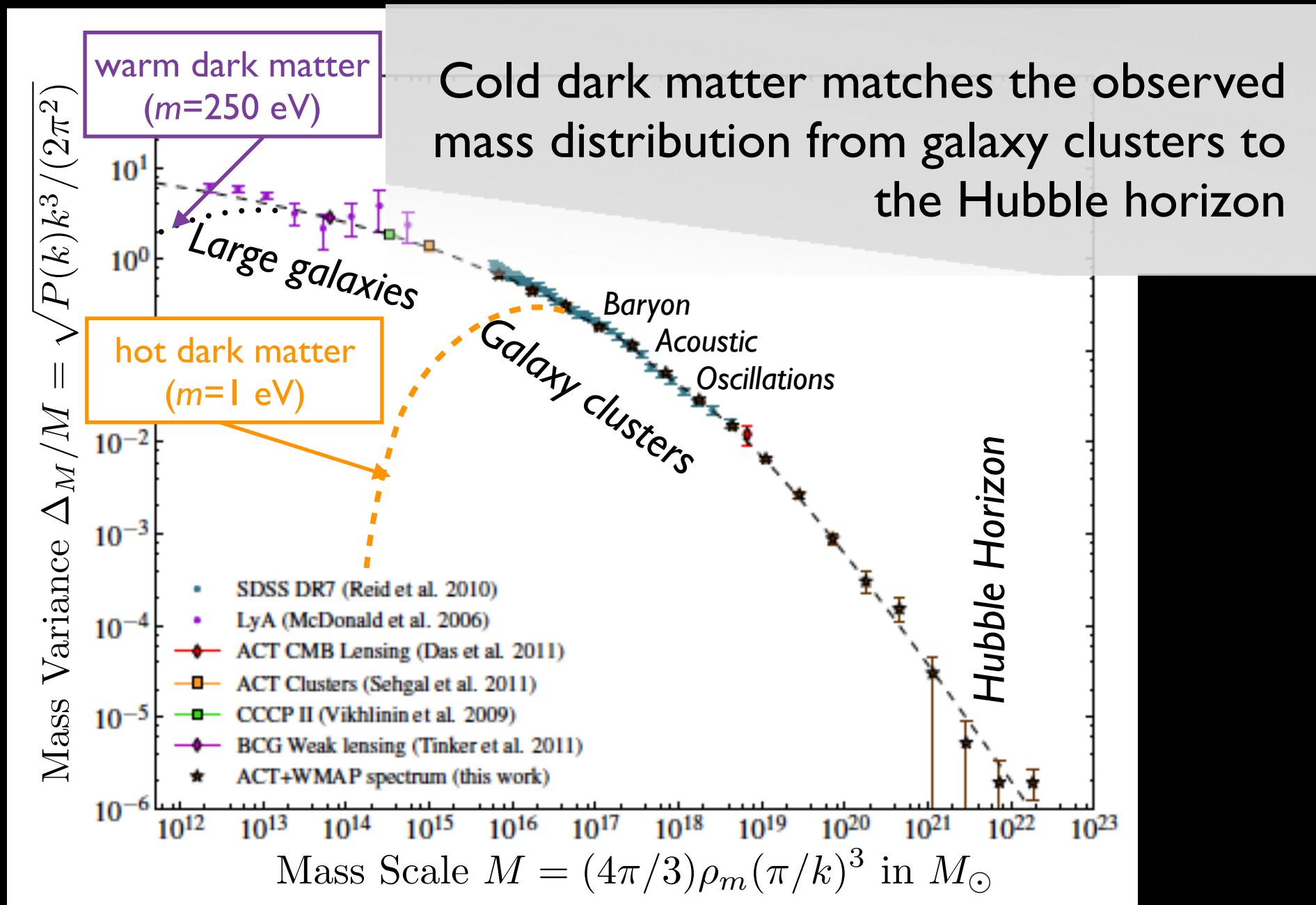
$$P(k) = \langle |\delta_{\mathbf{k}}|^2 \rangle$$



# From CMB fluctuations to galaxies



# Cold/warm/hot dark matter

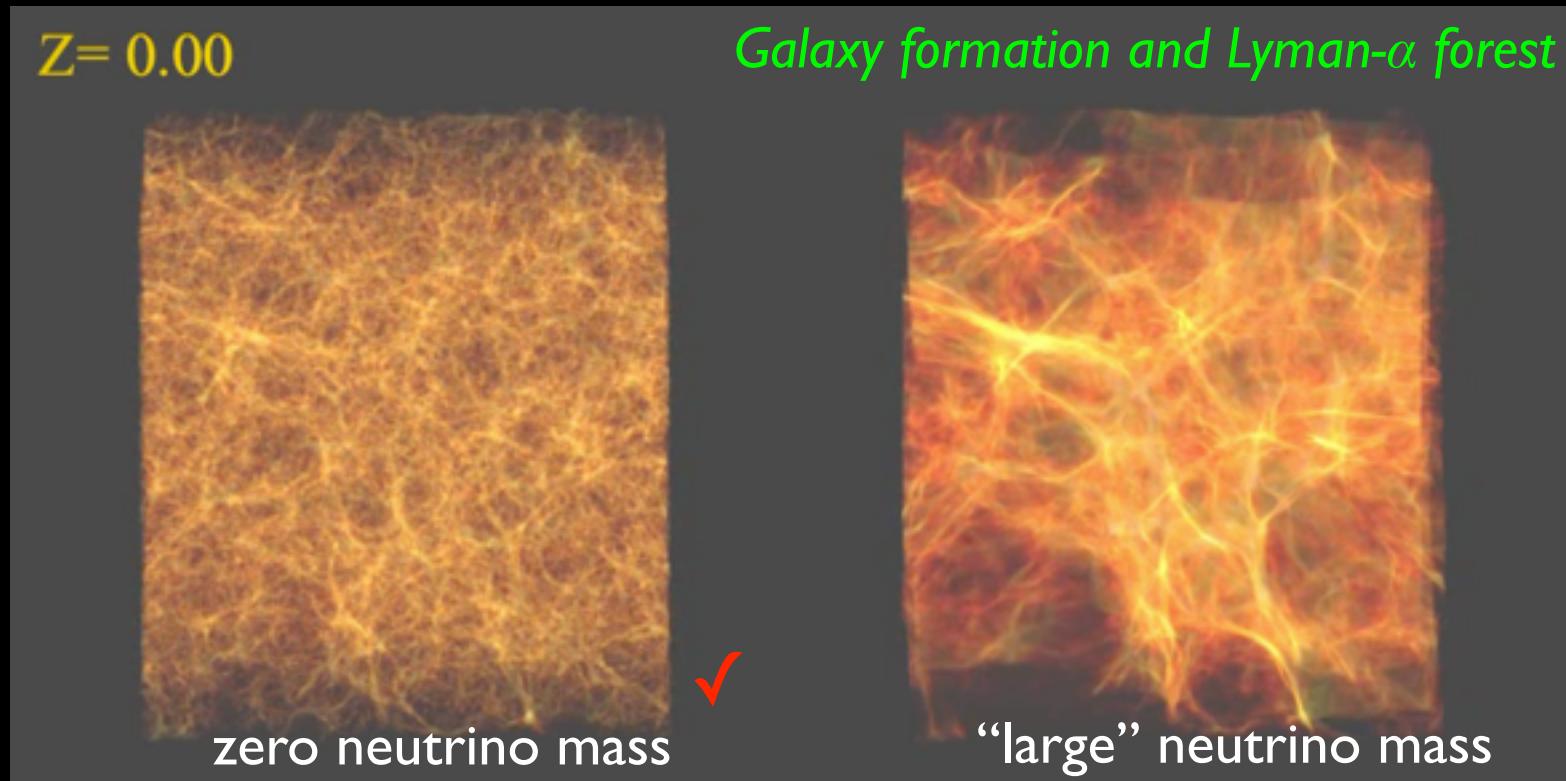


# Neutrinos as dark matter

Cosmology provides upper limits on neutrino masses

$$\sum m < 0.23 \text{ eV}$$

Future reach  
 $\sim 0.06 \text{ eV}$



# Neutrinos as dark matter

- Neutrino oscillations (largest  $\Delta m^2$  from SK+K2K+MINOS) place a lower bound on one of the neutrino masses,  $m_\nu > 0.048 \text{ eV}$
- Cosmology (CMB+LRG+ $H_0$ ) places an upper bound on the sum of the neutrino masses,  $\sum m_\nu < 0.44 \text{ eV}$
- Therefore neutrinos are *hot dark matter* ( $m_\nu \ll T_{\text{eq}} = 1.28 \text{ eV}$ ) with density  $0.0005 < \Omega_\nu h^2 < 0.0047$

*Detecting this Cosmic Neutrino Background (CNB) is a big challenge*

***Known neutrinos are hot dark matter***

# Neutrinos as dark matter

VOLUME 29, NUMBER 10

PHYSICAL REVIEW LETTERS

4 SEPTEMBER 1972

## An Upper Limit on the Neutrino Rest Mass\*

R. Cowsik† and J. McClelland

*Department of Physics, University of California, Berkeley, California 94720*

(Received 17 July 1972)

In order that the effect of gravitation of the thermal background neutrinos on the expansion of the universe not be too severe, their mass should be less than  $8 \text{ eV}/c^2$ .

Recently there has been a resurgence of interest in the possibility that neutrinos may have a finite rest mass. These discussions have been in the context of weak-interaction theories,<sup>1</sup> possible decay of solar neutrinos,<sup>2</sup> and enumerating the contributions of the  $\pi^0$  meson.<sup>3</sup>

and

$$n_{Bi} = \frac{2s_i + 1}{2\pi^2\hbar^3} \int_0^\infty \frac{p^2 dp}{\exp[E/kT(z_{eq})] - 1}. \quad (1b)$$

Here  $n_{Bi}$  is the number density of fermions of the  $i$ th kind,  $s_i$  is the number density of baryons.

Then  $m_\nu < 8 \text{ eV}/c^2$

from upper bound on  $\rho_\nu$

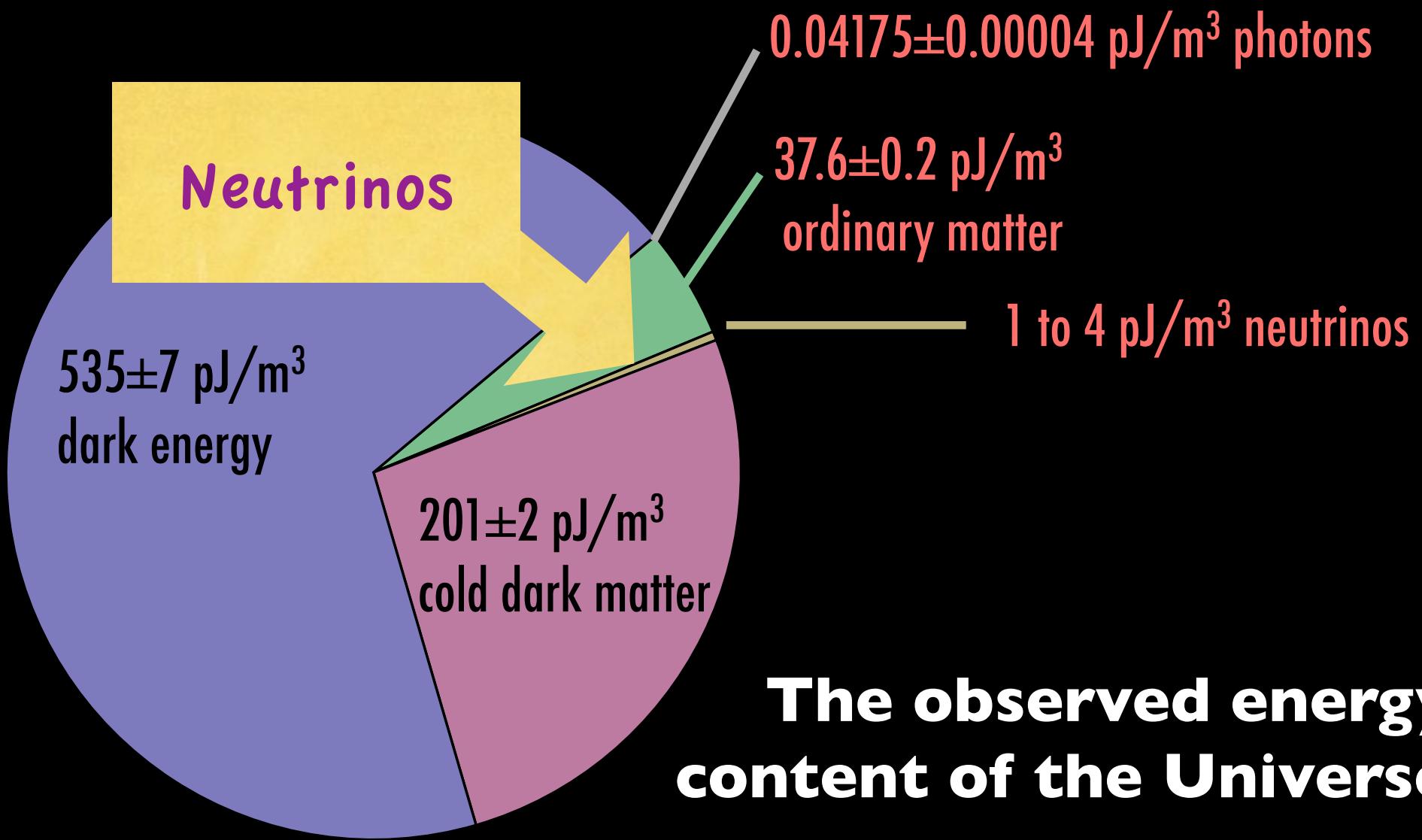
Now  $m_\nu < 0.44 \text{ eV}/c^2$  from  
upper bound on  $\delta\rho_\nu$

$$\rho_\nu = \frac{3\zeta(3)gT_\nu^3m_\nu}{8\pi^2} \quad m_\nu \gtrsim T_\nu$$

$$\rho_\nu = \frac{7\pi^2gT_\nu^4}{240} \quad m_\nu \lesssim T_\nu$$

$$T_\nu = (4/11)^{1/3}T_{\text{CMB}} = 168\mu\text{eV}/k$$

# Neutrinos as dark matter



**The observed energy content of the Universe**

matter  $p \ll \rho$

radiation  $p = \rho/3$

vacuum  $p = -\rho$

Planck (2015)  
TT,TE,EE+lowP+lensing+ext

$1 \text{ pJ} = 10^{-12} \text{ J}$

$\rho_{\text{crit}} = 1.68829 h^2 \text{ pJ/m}^3$

# The warning

“For any complex physical phenomenon there is a simple, elegant, compelling, wrong explanation.”



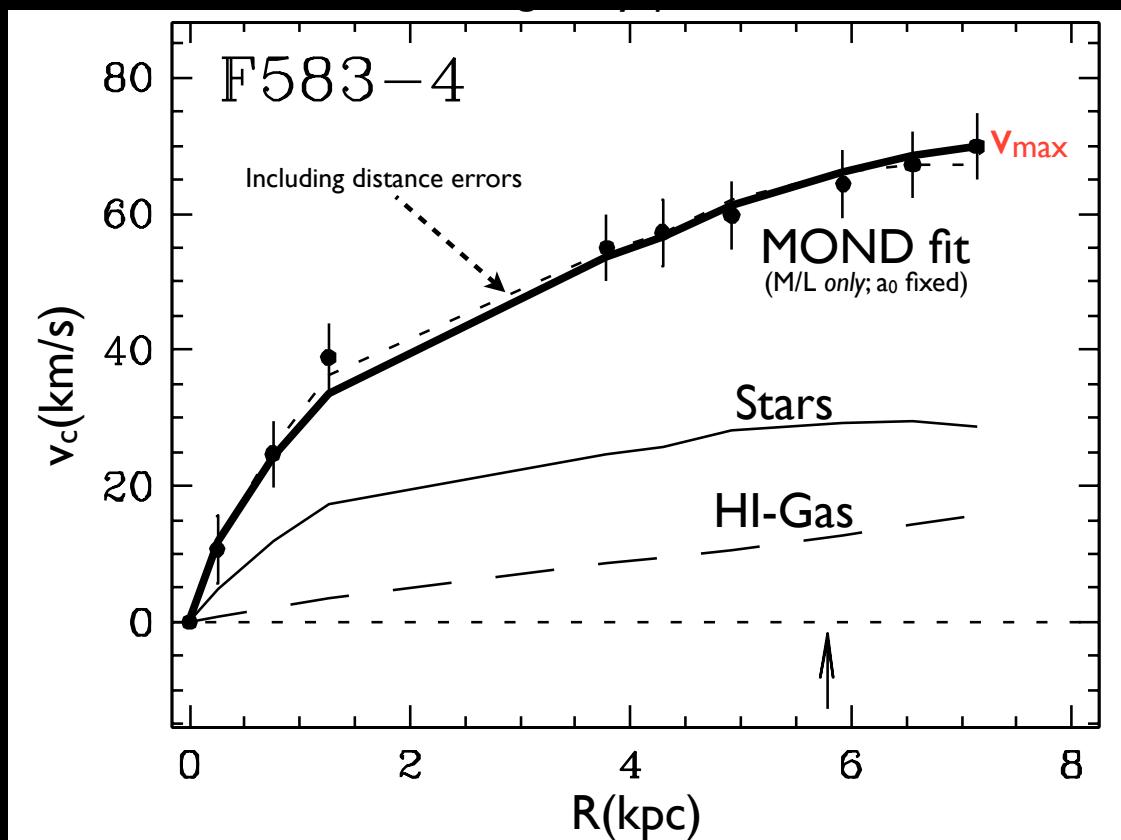
*Thomas Gold, 1920-2004,  
Austrian-born astronomer  
at Cambridge University  
and Cornell University*

**Cold dark matter or modified gravity?**

# Cold dark matter or modified gravity?

Modified Newtonian Dynamics  
or  
MOND

New constant of nature:  
universal acceleration  $a_0$



$$F=ma \text{ for } a \gg a_0$$

$$F=ma^2/a_0 \text{ for } a \ll a_0$$

# Cold dark matter or modified gravity?

- MOND ( $F=ma^2/a_0$  for  $a <$ universal  $a_0$ ) is only non-relativistic and so cannot be tested on cosmological scales
- TeVeS, MOND's generalization, contains new fields that could be interpreted as cold dark matter interacting only gravitationally. It does not reproduce the pattern of CMB peaks.
- There are other ideas, like conformal gravity, but are less studied

# Cold dark matter, *not* modified gravity

## The Bullet Cluster

*Symmetry argument: gas is at center, but potential has two wells.*



Gravitational potential  
from weak lensing

X-ray emitting hot gas  
(Chandra)

Galaxies in optical  
(Hubble Space  
Telescope)

# Cold dark matter, *not* modified gravity

Bekenstein's TeVeS  
does not reproduce  
the CMB angular  
power spectrum  
nor the matter  
power spectrum

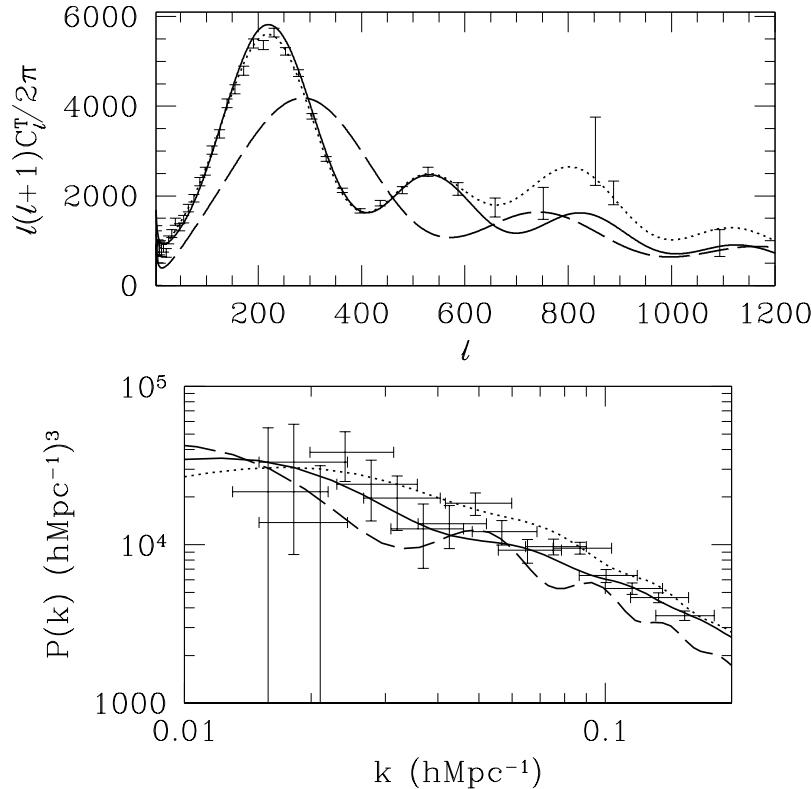
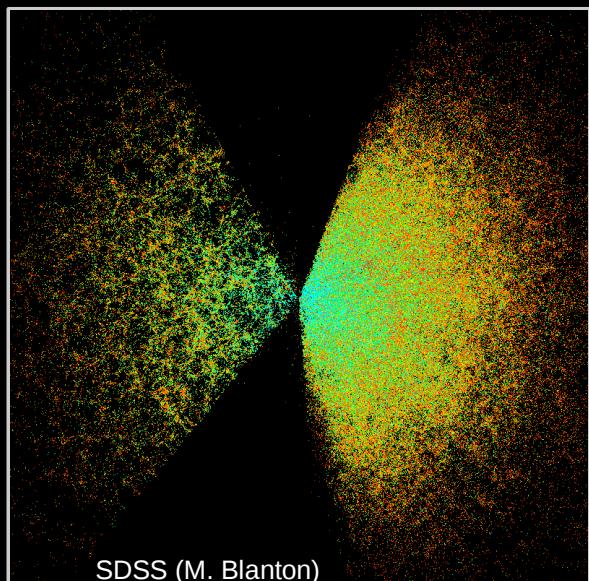


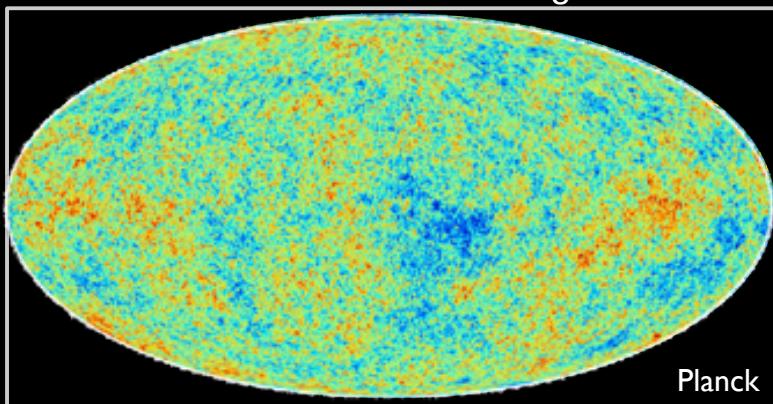
FIG. 4: The angular power spectrum of the CMB (top panel) and the power spectrum of the baryon density (bottom panel) for a MOND universe (with  $a_0 \simeq 4.2 \times 10^{-8} \text{ cm/s}^2$ ) with  $\Omega_\Lambda = 0.78$  and  $\Omega_\nu = 0.17$  and  $\Omega_B = 0.05$  (solid line), for a MOND universe  $\Omega_\Lambda = 0.95$  and  $\Omega_B = 0.05$  (dashed line) and for the  $\Lambda$ -CDM model (dotted line). A collection of data points from CMB experiments and Sloan are overplotted.

# Evidence for cold dark matter

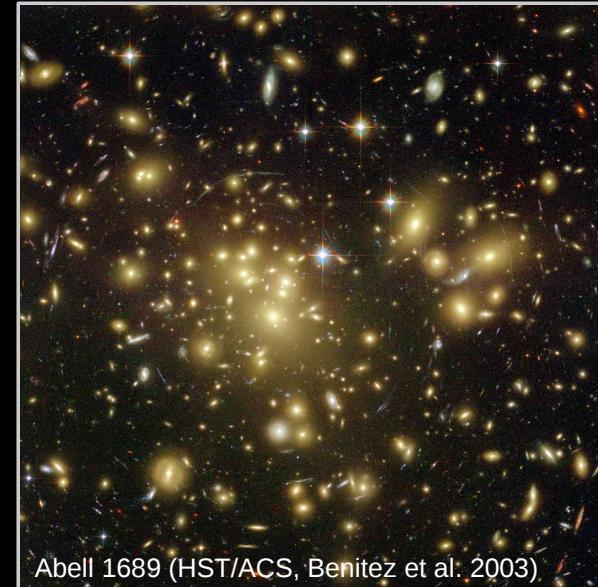
Large Scale Structure



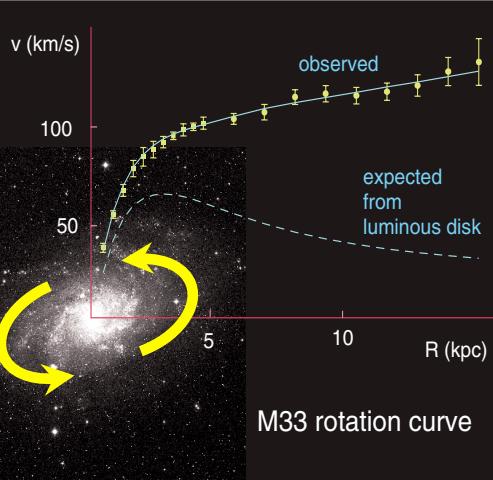
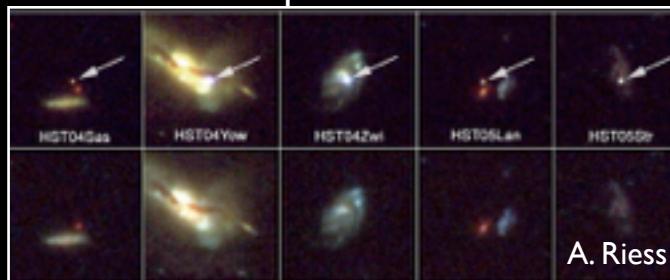
Cosmic Microwave Background



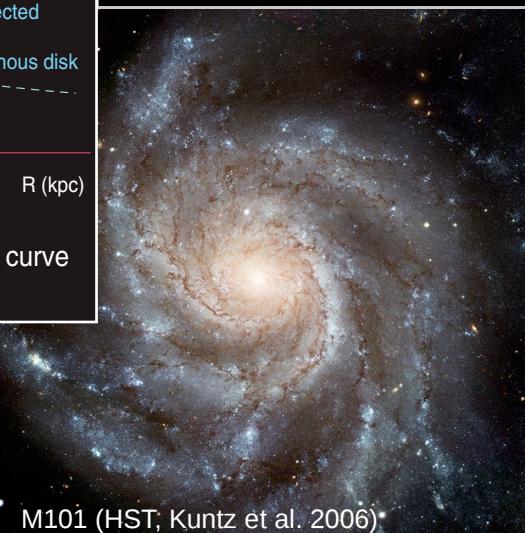
Galaxy Clusters



Supernovae

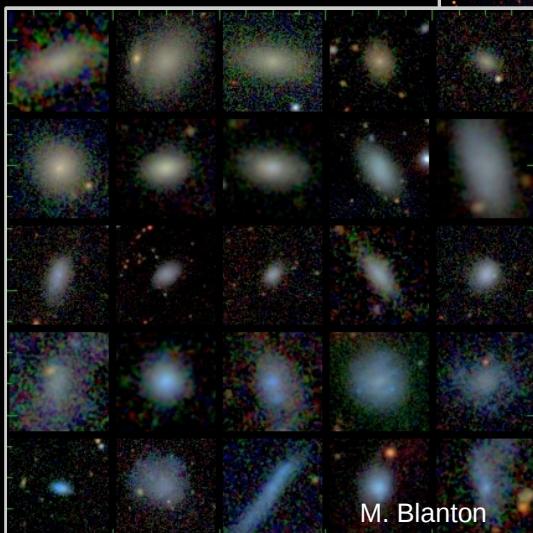


Galaxies



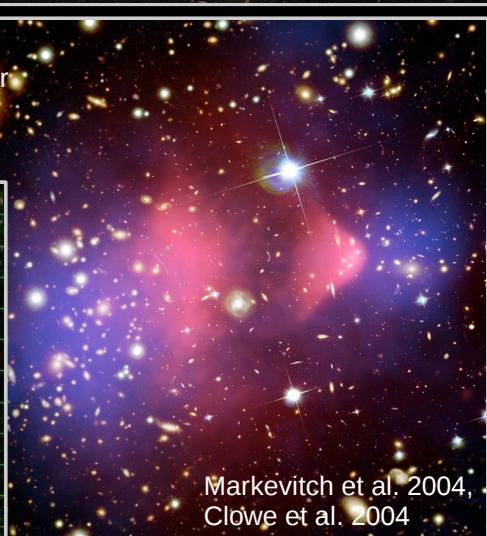
M101 (HST; Kuntz et al. 2006)

Dwarf Galaxies



M. Blanton

Bullet Cluster



Markevitch et al. 2004,  
Clowe et al. 2004

# Is cold dark matter an elementary particle?

## IS HINCHLIFFE'S RULE TRUE? \*

Boris Peon

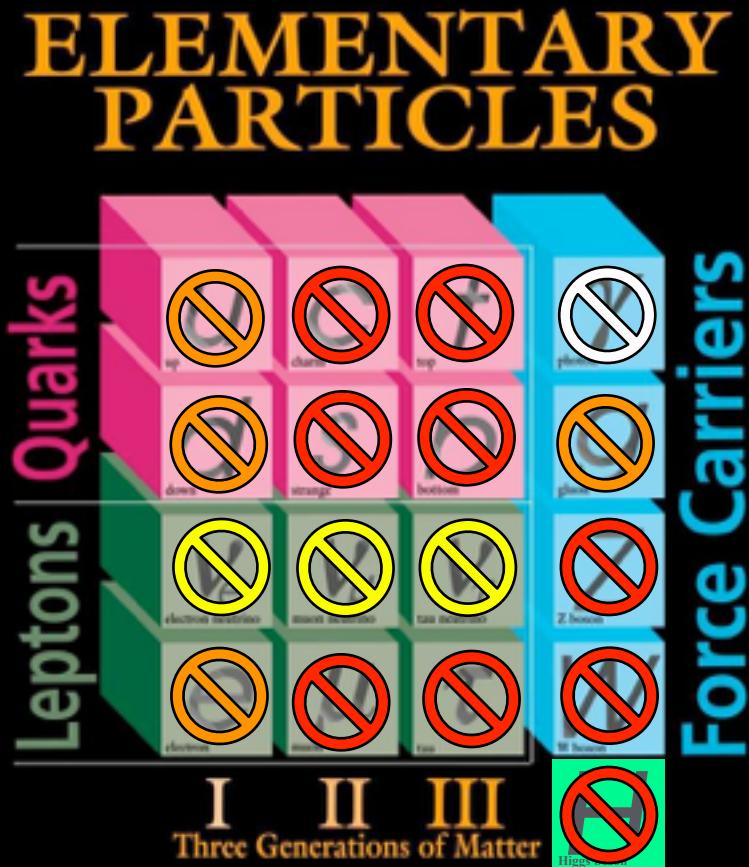
### Abstract

Hinchliffe has asserted that whenever the title of a paper is a question with a yes/no answer, the answer is always no. This paper demonstrates that Hinchliffe's assertion is false, but only if it is true.

# What particle model for dark matter?

- It should have the cosmic cold dark matter density
- It should be stable or very long-lived ( $\gtrsim 10^{24}$  yr)
- It should be compatible with collider, astrophysics, etc. bounds
- Ideally, it would be possible to detect it in outer space and produce it in the laboratory
- For the believer, it would explain any claim of dark matter detection (annual modulation, positrons, gamma-ray line, etc.)

# Which particle is cold dark matter?



🚫 is the particle of light

🚫 couples to the plasma

🚫 disappears too quickly

🚫 is hot dark matter

*No known particle can be cold dark matter!*

# Particle dark matter

Thermal relics

in thermal equilibrium in the early universe

neutrinos, neutralinos, other WIMPs, ....

Non-thermal relics

never in thermal equilibrium in the early universe

axions, WIMPZILLAs, solitons, ....

# Particle dark matter

## Hot dark matter

- relativistic at kinetic decoupling (start of free streaming)
- big structures form first, then fragment

light neutrinos

## Cold dark matter

- non-relativistic at kinetic decoupling
- small structures form first, then merge

neutralinos, axions, WIMPZILLAs, solitons

## Warm dark matter

- semi-relativistic at kinetic decoupling
- smallest structures are erased

sterile neutrinos, gravitinos

# Particle dark matter

- SM neutrinos (hot)
  - lightest supersymmetric particle (cold)
  - lightest Kaluza-Klein particle (cold)
  - sterile neutrinos, gravitinos (warm)
  - Bose-Einstein condensates, axions, axion clusters (cold)
  - solitons (Q-balls, B-balls, ...) (cold)
  - supermassive wimpzillas (cold)
- thermal relics
- non-thermal relics
- 
- The diagram illustrates the classification of particle dark matter. It is organized into two main categories: 'thermal relics' and 'non-thermal relics'. The 'thermal relics' category is further divided into 'hot' and 'cold'暗物质粒子。The 'non-thermal relics' category is also divided into 'warm' and 'cold'暗物质粒子。A large brace on the right side groups the 'cold'暗物质粒子 under each category. A smaller brace on the left side groups the 'cold'暗物质粒子 under each category. The 'hot'暗物质粒子 is grouped by itself. The 'warm'暗物质粒子 is grouped by itself.

Mass range

$10^{-22} \text{ eV}$  ( $10^{-56} \text{ g}$ ) B.E.C.s  
 $10^{-8} M_\odot$  ( $10^{+25} \text{ g}$ ) axion clusters

Interaction strength range

Only gravitational: wimpzillas  
Strongly interacting: B-balls

# Particle Dark Matter

Type Ia Candidates that exist

Type Ib Candidates in well-motivated frameworks

Type II All other candidates

# Particle Dark Matter

Type Ia Candidates that exist

Type Ib Candidates in well-motivated frameworks

- have been proposed to solve genuine particle physics problems, a priori unrelated to dark matter
- have interactions and masses specified within a well-defined particle physics model

Type II All other candidates

# Particle Dark Matter

Type Ia Candidates that exist

standard neutrinos

Type Ib Candidates in well-motivated frameworks

heavy neutrinos, axion, lightest supersymmetric  
particle (neutralino, sneutrino, gravitino, axino)

Type II All other candidates

maverick WIMP, WIMPZILLA, B-balls, Q-balls,  
self-interacting dark matter, string-inspired  
dark matter, etc.

# Heavy active neutrinos (4-th generation)

## PHYSICAL REVIEW LETTERS

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VOLUME 39

25 JULY 1977

NUMBER 4

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### Cosmological Lower Bound on Heavy-Neutrino Masses

Benjamin W. Lee<sup>(a)</sup>

*Fermi National Accelerator Laboratory,<sup>(b)</sup> Batavia, Illinois 60510*

and

Steven Weinberg<sup>(c)</sup>

*Stanford University, Physics Department, Stanford, California 94305*

(Received 13 May 1977)

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of  $2 \times 10^{-29} \text{ g/cm}^3$ , the lepton mass would have to be *greater* than a lower bound of the order of 2 GeV.

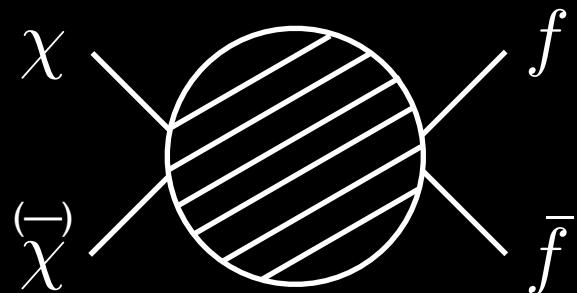
2 GeV/c<sup>2</sup> for  $\Omega_c=1$

Now 4 GeV/c<sup>2</sup> for  $\Omega_c=0.25$

# Cosmic density of heavy active neutrinos

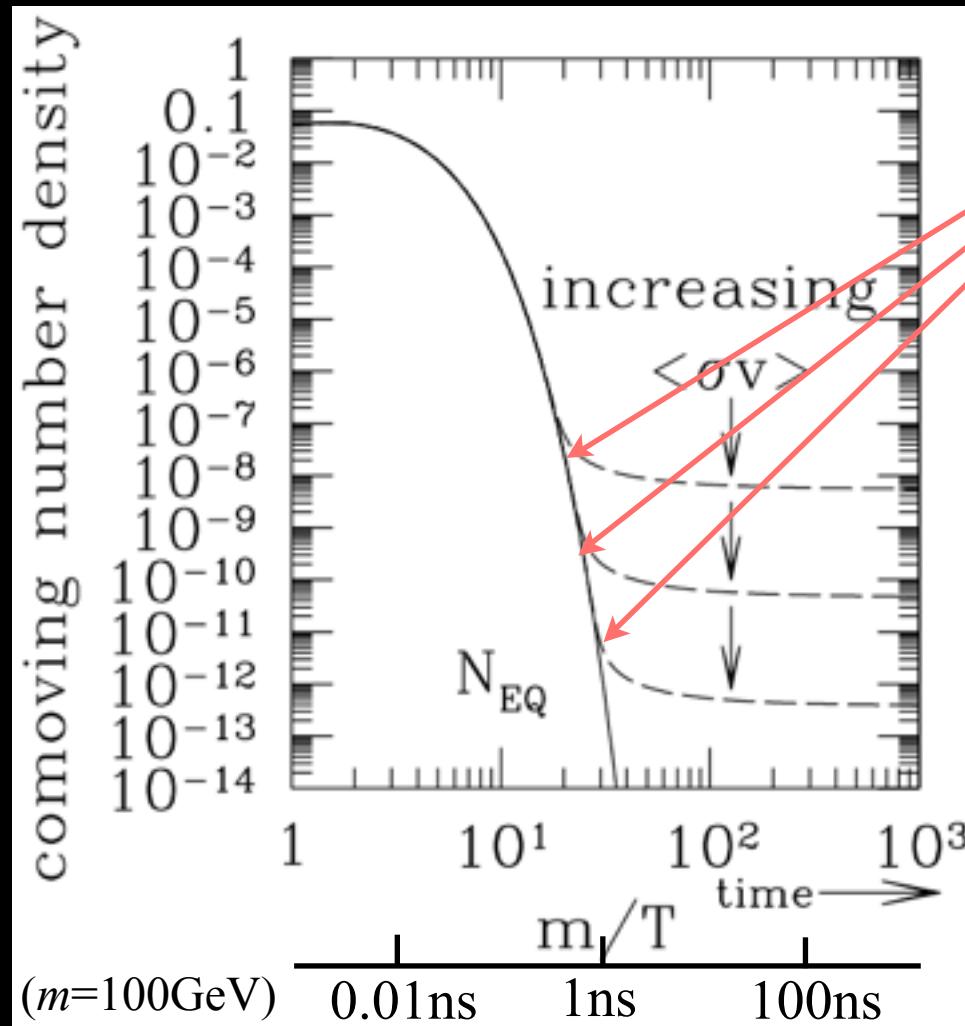
- At early times, heavy neutrinos are produced in  $e^+e^-$ ,  $\mu^+\mu^-$ , etc collisions in the hot primordial soup [*thermal production*].

$$e^+ + e^-, \mu^+ + \mu^-, \text{etc.} \leftrightarrow \chi + \overset{(-)}{\chi}$$



- Neutrino production ceases when the production rate becomes smaller than the Hubble expansion rate [*freeze-out*].
- After freeze-out, there is a constant number of neutrinos in a volume expanding with the universe.

# Cosmic density of heavy active neutrinos



freeze-out

$\Gamma_{\text{ann}} \equiv n\langle\sigma v\rangle \sim H$

annihilation rate

expansion rate

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27} \text{cm}^3/\text{s}}{\langle\sigma v\rangle_{\text{ann}}}$$

$$\Omega_\chi h^2 = \Omega_{\text{cdm}} h^2 \simeq 0.1143$$

$$\text{for } \langle\sigma v\rangle_{\text{ann}} \simeq 3 \times 10^{-26} \text{cm}^3/\text{s}$$

This is why they are called Weakly Interacting Massive Particles  
(WIMPless candidates are WIMPs!)

# Cosmic density of heavy active neutrinos

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle_{\text{ann}} (n^2 - n_{\text{eq}}^2)$$

density equation  
("Boltzmann equation")

*thermally averaged cross section times relative velocity*

$$\langle\sigma v\rangle_{\text{ann}} = \int_{4m^2}^{\infty} ds \frac{\sqrt{s - 4m^2} K_1(\sqrt{s}/T)}{16m^4 T K_2^2(m/T)} W(s)$$

*invariant annihilation rate (annihilations per unit time and unit volume)*

$$W_{12 \rightarrow \dots}(s) = 4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \sigma_{12 \rightarrow \dots}(s)$$

# Cosmic density of heavy active neutrinos

Enqvist, Kainulainen, Maalampi 1989

Dirac neutrino in 4-th generation lepton doublet

$$\begin{aligned}\mathcal{L} &= y_e \bar{\ell}_L \phi e_R + y_\nu \bar{\ell}_L \tilde{\phi} \nu_R \\ &= (\bar{\nu}_L \quad \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + (\bar{\nu}_L \quad \bar{e}_L) \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix} \nu_R \\ &= y_e (\bar{\nu}_L \phi^+ + \bar{e}_L \phi^0) e_R + y_\nu (\bar{\nu}_L \phi^0 - \bar{e}_L \phi^-) \nu_R\end{aligned}$$

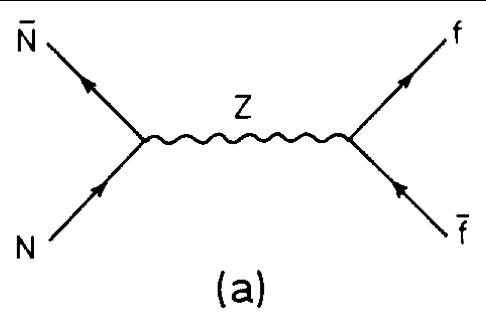
After electroweak symmetry breaking

$$\mathcal{L}_m = m_e \bar{e}_L e_R + m_\nu \bar{\nu}_L \nu_R$$

$$m_e = \frac{y_e v}{\sqrt{2}} \quad m_\nu = \frac{y_\nu v}{\sqrt{2}}$$

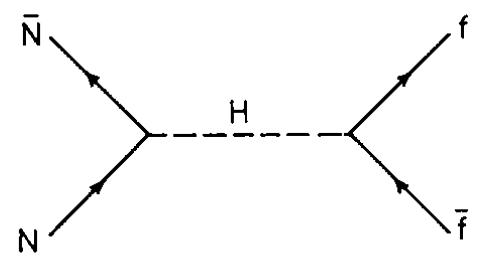
# Cosmic density of heavy active neutrinos

*Enqvist, Kainulainen, Maalampi 1989*



$$\sigma_Z(\bar{N}N \rightarrow \bar{f}f) = \frac{N_c}{4s} \frac{\pi\alpha^2}{x_W^2} \frac{\beta_f}{\beta_N} \frac{1}{16(1-x_W)^2} |D_Z|^2$$

$$\times \left[ \frac{1}{2} (v_f^2 + a_f^2) s^2 (1 + \frac{1}{3}\beta^2) + 2(v_f^2 - a_f^2) m_f^2 (s - 2m_N^2) \right]$$



$$\sigma_H(N\bar{N} \rightarrow \bar{f}f) = N_c \frac{\pi\alpha^2}{4sx_W^2} \frac{\beta_f}{\beta_N} |D_H|^2 \left( \frac{m_f m_N}{m_W^2} \right)^2 s^2 \beta^2,$$

$$\beta_f = \left( 1 - \frac{4m_f^2}{s} \right)^{1/2}, \quad \beta_N = \left( 1 - \frac{4m_N^2}{s} \right)^{1/2}. \quad |D_H|^2 = \frac{1}{(s - m_H^2)^2 + \Gamma_H^2 m_H^2}. \quad |D_Z|^2 = \frac{1}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}.$$

# Cosmic density of heavy active neutrinos

Enqvist, Kainulainen, Maalampi 1989

$$\sigma(\bar{N}N \rightarrow H^0 H^0) = \frac{g^4}{128\pi s} \frac{\beta_H}{\beta_N} \left( \frac{m_N}{m_W} \right)^4 (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$$

$$\sigma_1 = \left( \frac{1}{4} m_N^2 (s + 4m_H^2) - 4m_N^4 \right) R + \left( \frac{1}{2}s - m_H^2 + 4m_N^2 \right) L - \frac{1}{2},$$

$$\sigma_2 = \frac{9}{2} \left( \frac{m_H}{m_N} \right)^4 |D_H|^2 m_N^2 s \beta_N^2,$$

$$\sigma_3 = - (4m_N^2 s \beta_N^2 + m_H^4) \frac{L}{2m_H^2 - s} - \frac{1}{4},$$

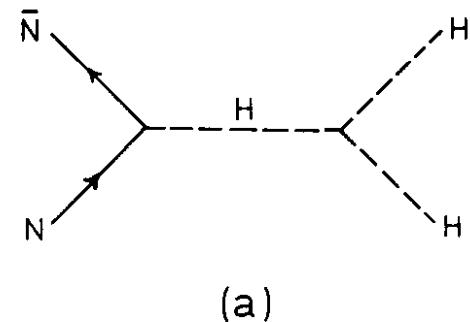
$$\sigma_4 = - 3 \left( \frac{m_H}{m_N} \right)^2 (s - m_H^2) |D_H|^2 m_N^2 [1 + (2s\beta_N^2 + (2m_H^2 - s))L].$$

$$L \equiv - \frac{1}{2s\beta_N\beta_H} \ln \left( \frac{2m_H^2 - s + s\beta_N\beta_H}{2m_H^2 - s - s\beta_N\beta_H} \right)$$

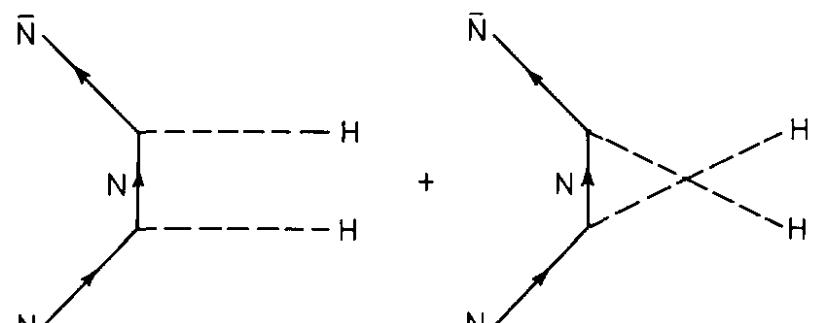
$$\beta_i = \left( 1 - \frac{4m_i^2}{s} \right)^{1/2} \quad (i = N, H),$$

$$R \equiv [m_H^4 + m_N^2 s \beta_H^2]^{-1},$$

$$|D_H|^2 = \frac{1}{(s - m_H^2)^2 + \Gamma_H^2 m_H^2}.$$



(a)



(b)

# Cosmic density of heavy active neutrinos

Enqvist, Kainulainen, Maalampi 1989

$$\sigma_{LL} = G_{LL},$$

$$\sigma_{ZZ} = \frac{1}{8}|D_Z|^2 m_W^4 G_{ZZ},$$

$$\sigma_{HH} = \frac{1}{4}|D_H|^2 m_W^4 G_{HH},$$

$$\sigma_{LZ} = \frac{1}{2}(s - m_Z^2)|D_Z|^2 m_W^2 G_{LZ},$$

$$\sigma_{LH} = \frac{1}{2}(s - m_H^2)|D_H|^2 m_W^2 G_{LH},$$

$$G_{LL} = \frac{1}{12}(\hat{s}^2 + 20\hat{s} - 24) + \left(\frac{1}{6}\hat{s} - \frac{5}{3}\right)m_N^2 - \frac{3}{2}\hat{m}_N^4 + P_1\hat{L}$$

$$- \frac{1}{2}(2 - \hat{m}_N^2 - \hat{m}_N^4)^2 \hat{R} - \hat{m}_L^2 \left[ \frac{1}{2}\hat{s} - 1 - 3\hat{m}_N^2 + 2P_2\hat{L} + \frac{1}{2}P_1\hat{R} \right]$$

$$- \hat{m}_L^4 \left[ \frac{3}{2} - 3(\hat{s} - 2 - 4\hat{m}_N^2)\hat{L} - \frac{1}{2}P_2\hat{R} \right]$$

$$+ \hat{m}_L^6 [4\hat{L} - (\frac{1}{2}\hat{s} - 1 - 2\hat{m}_N^2)\hat{R}] - \frac{1}{2}\hat{m}_L^8 \hat{R},$$

$$G_{ZZ} = \frac{2}{3}(\hat{s} - \hat{m}_N^2)(\hat{s}^3 + 16\hat{s}^2 - 68\hat{s} - 48), \quad (A.13)$$

$$G_{HH} = \hat{m}_N^2(\hat{s} - 4\hat{m}_N^2)(\hat{s}^2 - 4\hat{s} + 12), \quad (A.14)$$

$$G_{LZ} = -\frac{1}{3}(\hat{s}^3 + 18\hat{s}^2 - 28\hat{s} - 24 - (\hat{s}^2 + 6\hat{s} + 8)\hat{m}_N^2 - 6(\hat{s} - 2)\hat{m}_N^4)$$

$$+ 4(8\hat{s} + 4 - (10\hat{s} + 4)\hat{m}_N^2 + (\hat{s} + 2)\hat{m}_N^4 + (\hat{s} - 2)\hat{m}_N^6)\hat{L}$$

$$+ \hat{m}_L^2 [\hat{s}^2 - 4\hat{s} - 4 - (4\hat{s} - 8)\hat{m}_N^2$$

$$+ 4(4\hat{s}^2 - 5\hat{s} - 6 + (\hat{s}^2 - 5\hat{s} - 2)\hat{m}_N^2 - 3(\hat{s} - 2)\hat{m}_N^4)\hat{L}]$$

$$+ \hat{m}_L^4 [2(\hat{s} - 2) - 4(\hat{s}(\hat{s} - 4) - 3(\hat{s} - 2)\hat{m}_N^2)\hat{L}] - \hat{m}_L^6 [(4\hat{s} - 8)\hat{L}], \quad (A.15)$$

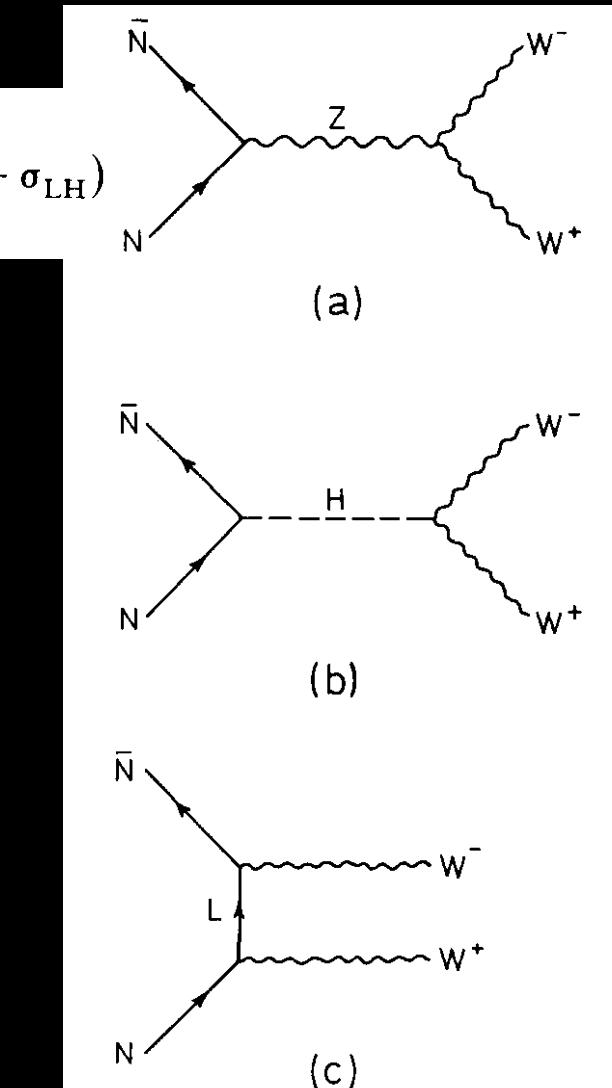
$$G_{LH} = \hat{m}_N^2 \left\{ -\hat{s}^2 + 2\hat{s} - 8 + 2(\hat{s} + 2)\hat{m}_N^2 + 4(2\hat{s} - 4 - (3\hat{s} - 2)\hat{m}_N^2 + (\hat{s} + 2)\hat{m}_N^4)\hat{L} \right.$$

$$+ \hat{m}_L^2 [-2(\hat{s} + 2) + 4(\hat{s}^2 - \hat{s} + 2 - (2\hat{s} + 4)\hat{m}_N^4)\hat{L}]$$

$$\left. + \hat{m}_L^4 [(4\hat{s} + 8)\hat{L}] \right\},$$

$$P_1 = 4(\hat{s} - 2) + 4\hat{s}\hat{m}_N^2 + (\hat{s} - 6)\hat{m}_N^4 - 4\hat{m}_N^6,$$

$$P_2 = 4\hat{s} - 5 + (2\hat{s} - 6)\hat{m}_N^2 - 6\hat{m}_N^4,$$

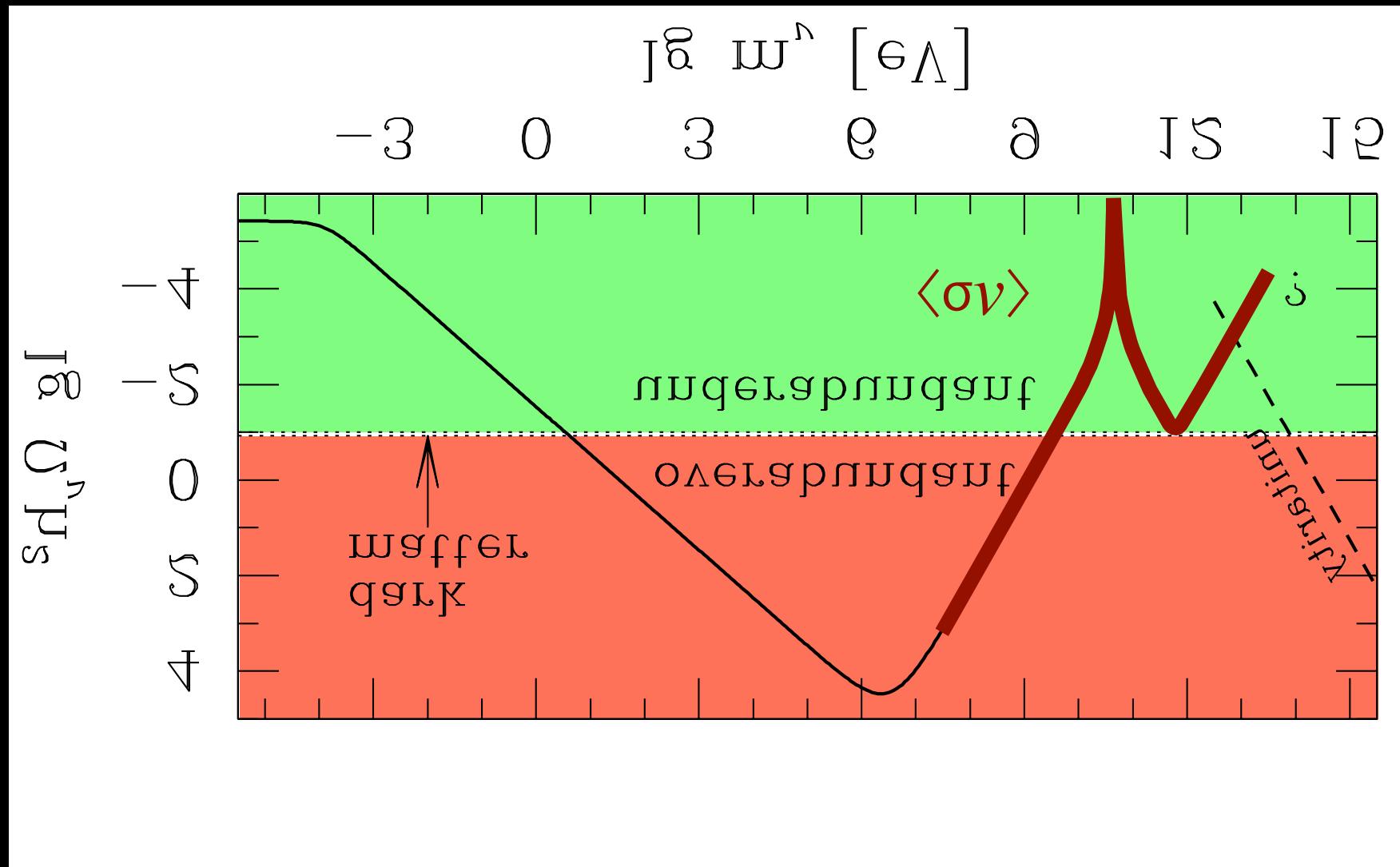


$$\hat{L} \equiv -\frac{1}{2\hat{s}\beta_N\beta_W} \ln \left( \frac{2 - \hat{s} + 2\hat{m}_N^2 - 2\hat{m}_L^2 + \hat{s}\beta_N\beta_W}{2 - \hat{s} + 2\hat{m}_N^2 - 2\hat{m}_L^2 - \hat{s}\beta_N\beta_W} \right),$$

$$\hat{R} \equiv \left[ (1 - \hat{m}_N^2)^2 - \hat{m}_L^2 (2 - \hat{s} + 2\hat{m}_N^2) + \hat{m}_L^4 \right]^{-1}.$$

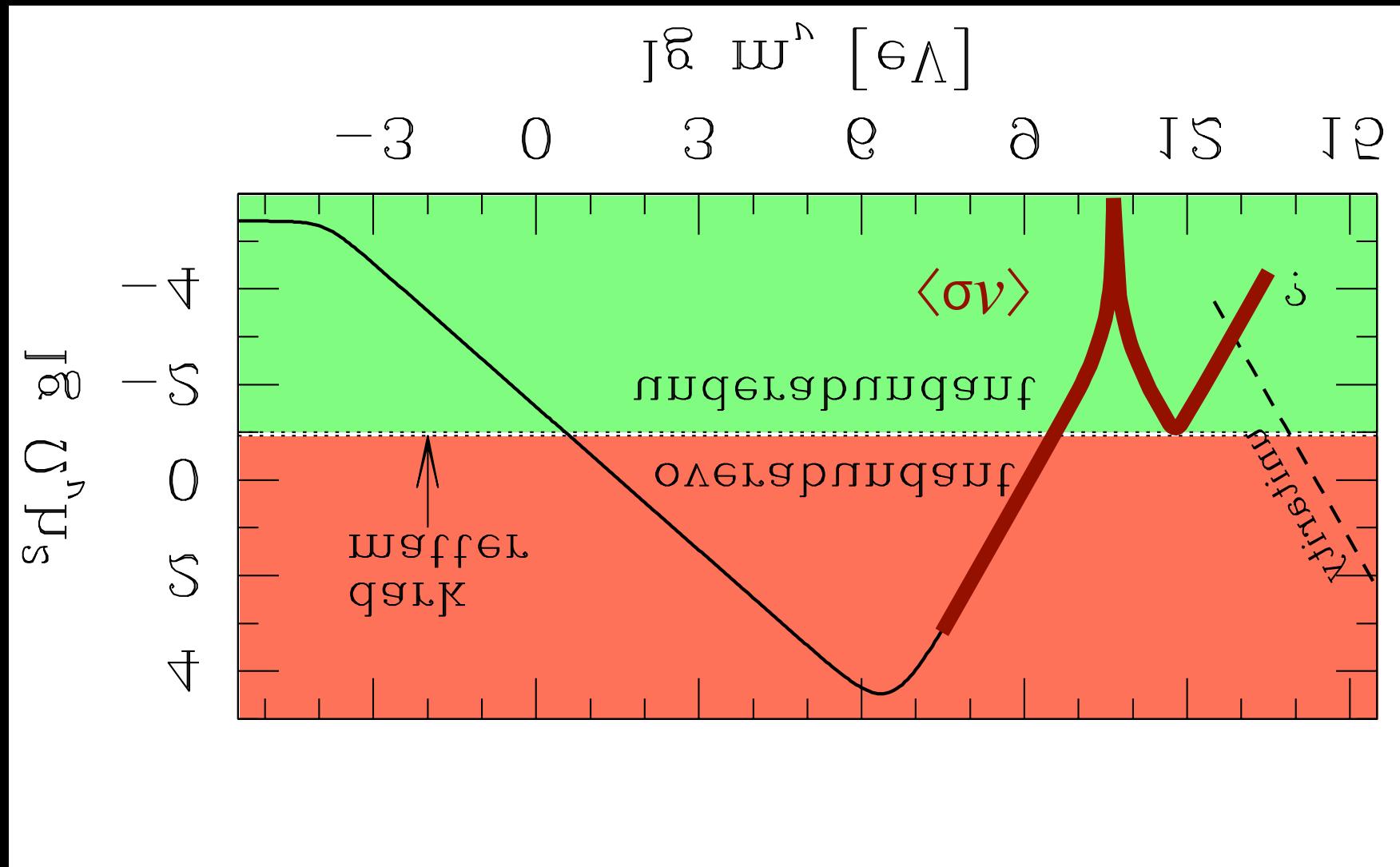
# Cosmic density of massive neutrinos

Fourth-generation Standard Model neutrino



# Cosmic density of massive neutrinos

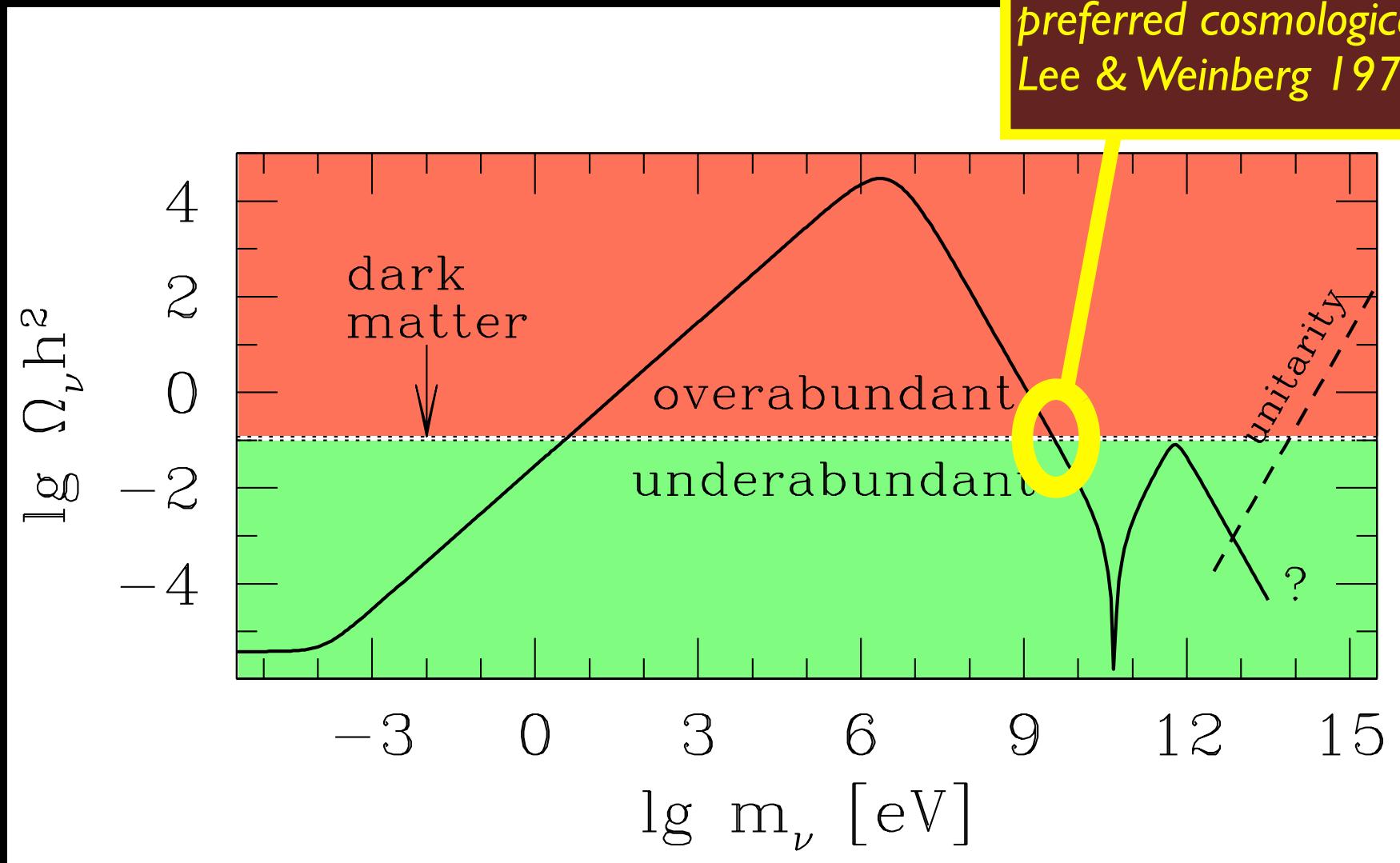
Fourth-generation Standard Model neutrino



# Cosmic density of massive neutrinos

Fourth-generation Standard Model neutrino

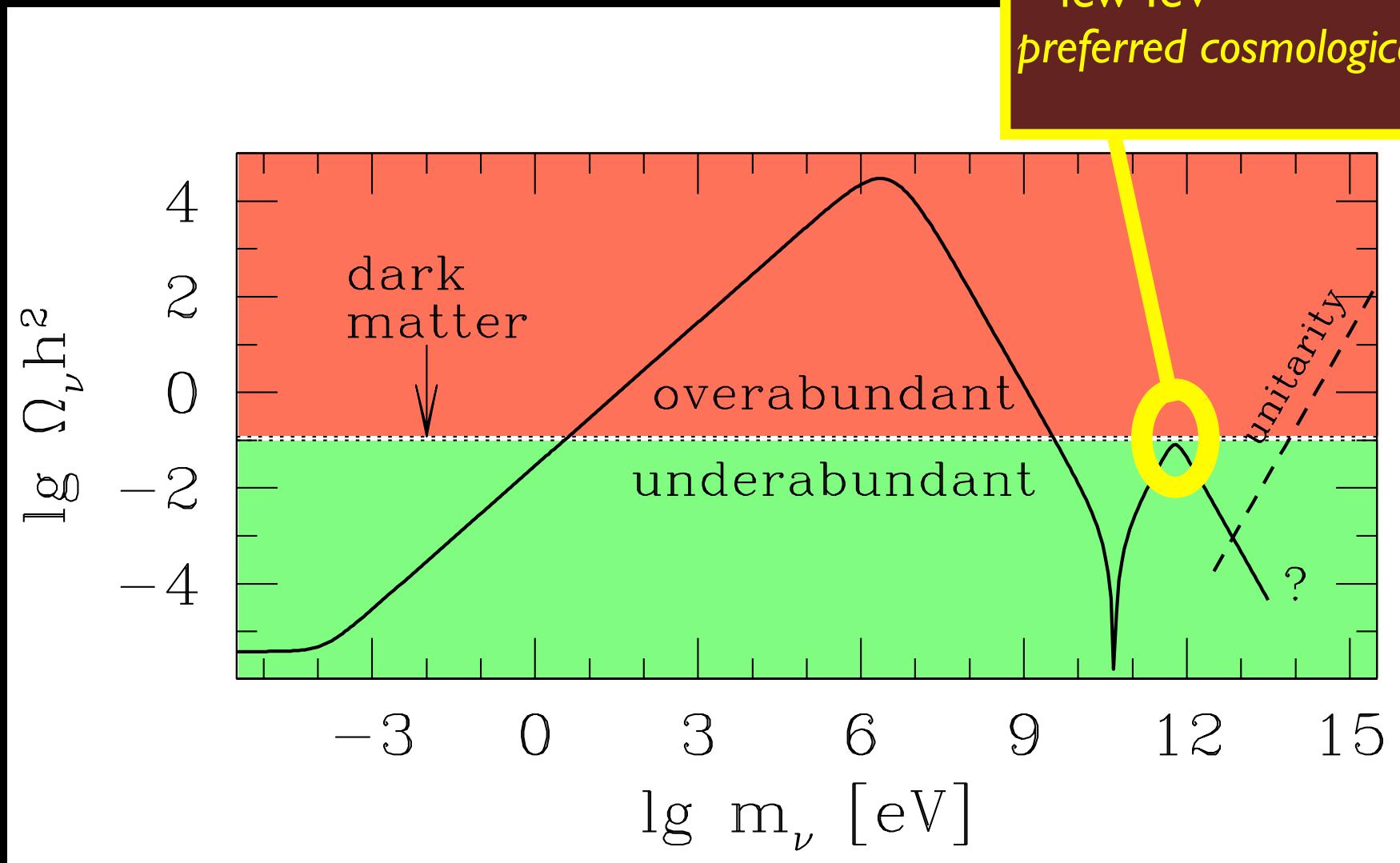
~ few GeV  
preferred cosmological mass  
Lee & Weinberg 1977



# Cosmic density of massive neutrinos

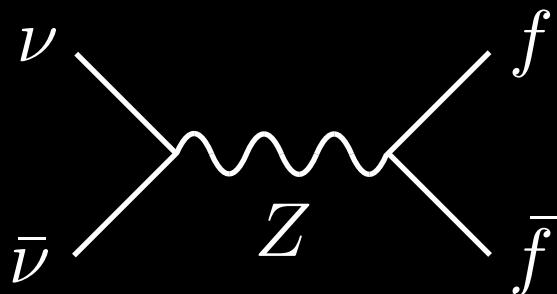
Fourth-generation Standard Model neutrino

~ few TeV  
*preferred cosmological mass*

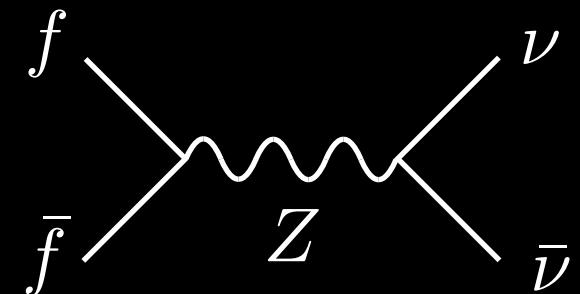


# Connection to colliders

Annihilation  $\nu\bar{\nu} \rightarrow f\bar{f}$



Production  $f\bar{f} \rightarrow \nu\bar{\nu}$



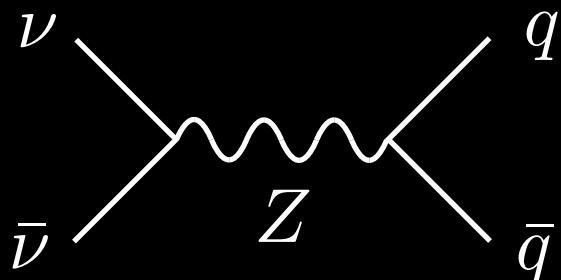
Inverse reaction

For example, a  $\sim 4$  GeV/c $^2$  dark matter neutrino would be copiously produced in resonant Z boson decays

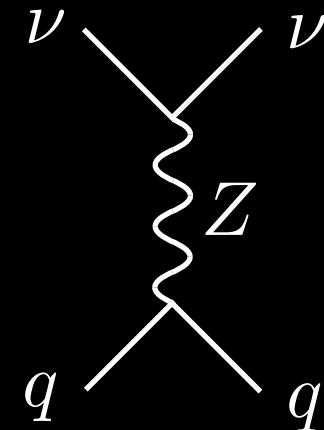
Excluded by LEP bound  $Z \rightarrow \nu\bar{\nu}$

# Connection to direct detection

Annihilation  $\nu\bar{\nu} \rightarrow q\bar{q}$



Scattering  $\nu q \rightarrow \nu q$



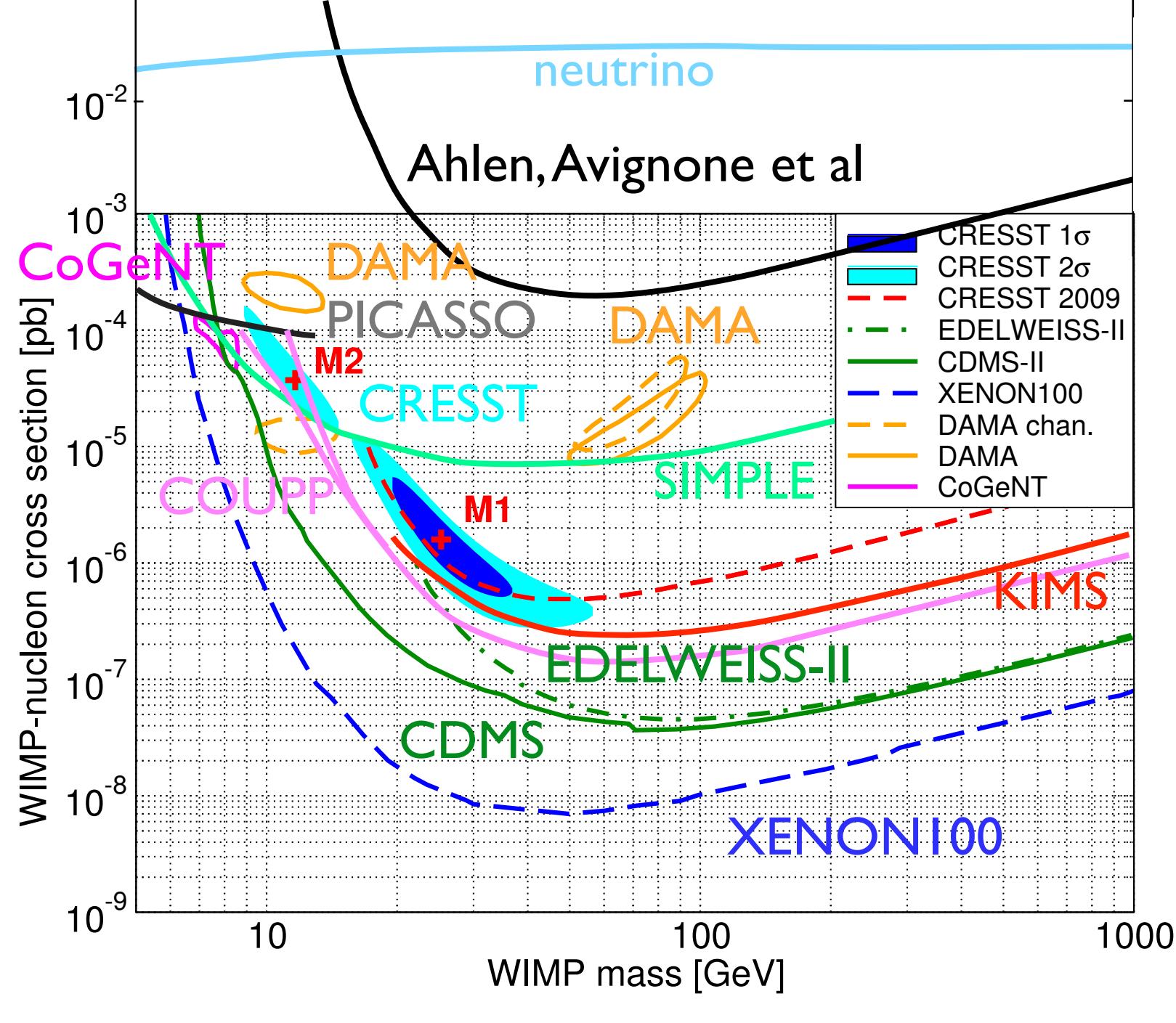
Crossing

For example, for a  $\sim 4$  GeV/c $^2$  dark matter neutrino, the scattering cross section is

$$\sigma_{\nu n} \simeq 0.01 \frac{\langle \sigma v \rangle}{c} \simeq 10^{-38} \text{ cm}^2$$

Excluded by direct searches

# Spin-



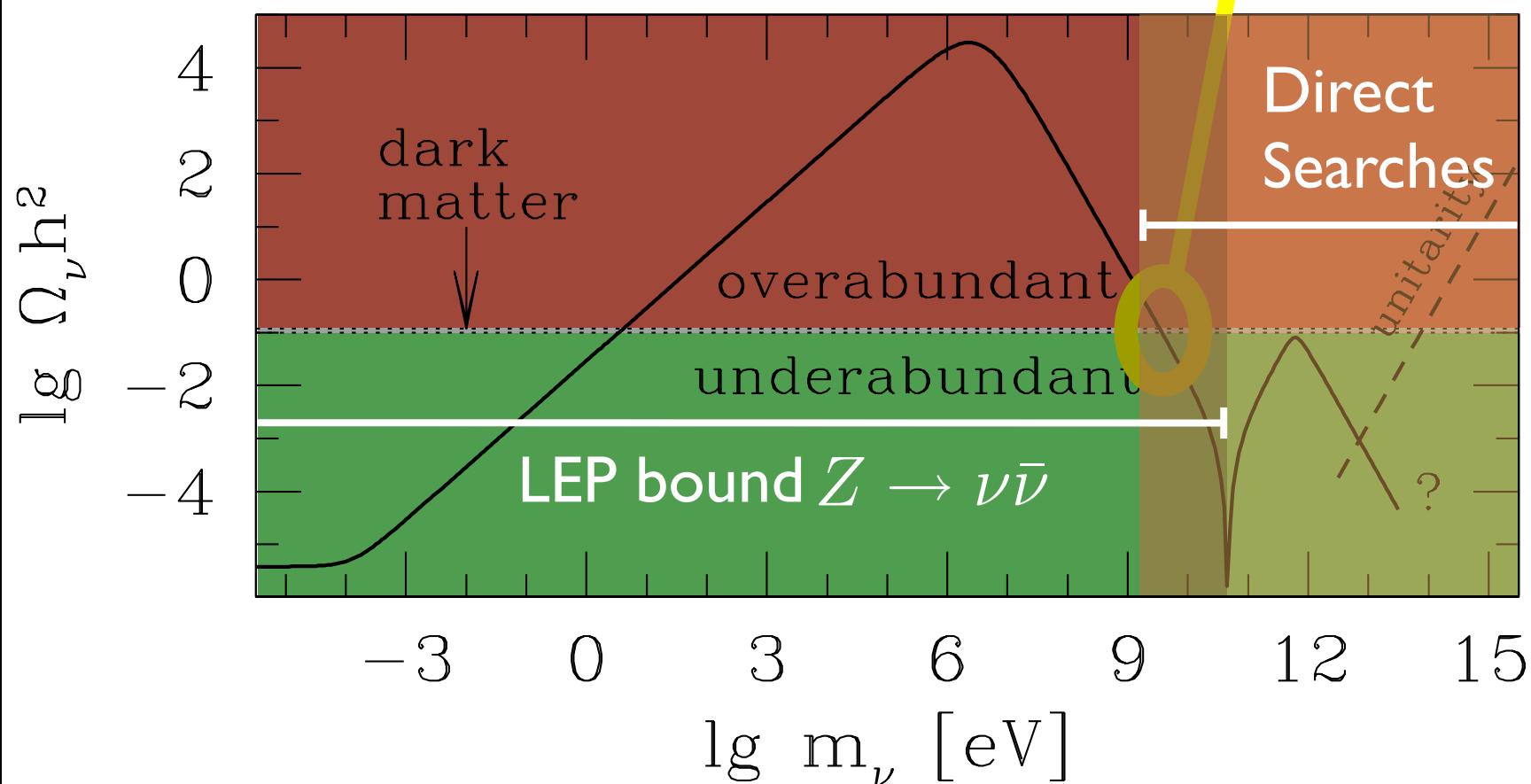
$$1 \text{ pb} = 10^{-36} \text{ cm}^2$$

# Cosmic density of massive neutrinos

Fourth-generation Standard Model neutrino

Excluded as dark matter (1991)

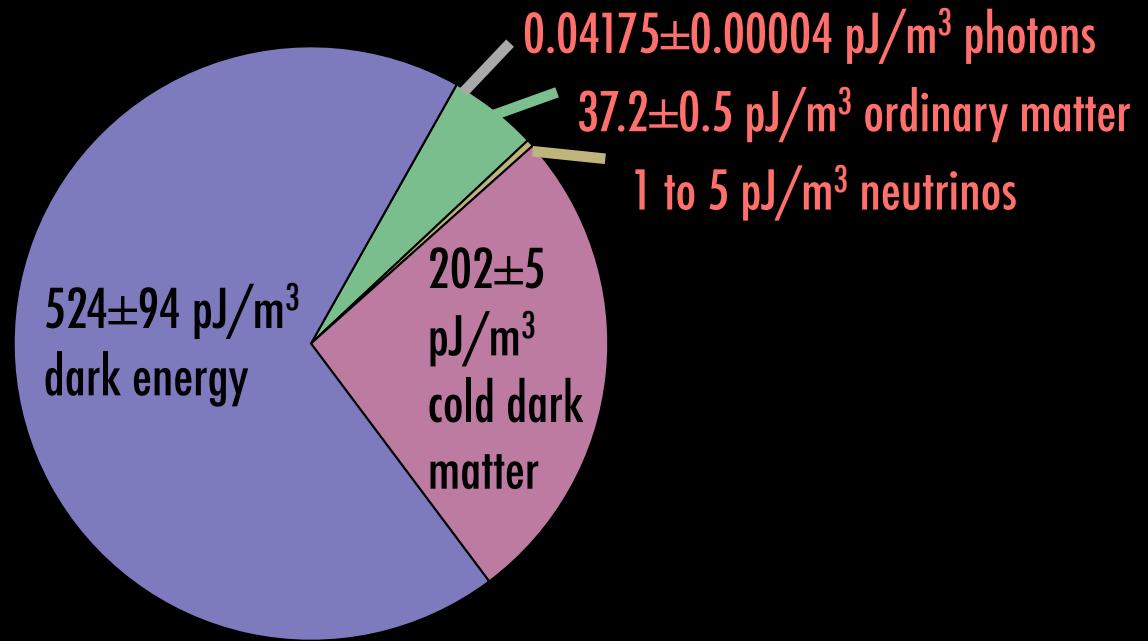
~ few GeV  
preferred cosmological mass  
Lee & Weinberg 1977



# *The Magnificent WIMP* *(Weakly Interacting Massive Particle)*

- One naturally obtains the right cosmic density of WIMPs

*Thermal production in hot primordial plasma.*



- One can experimentally test the WIMP hypothesis

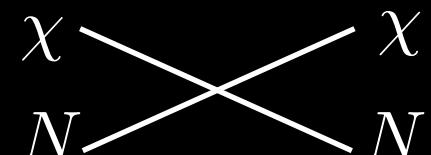
*The same physical processes that produce the right density of WIMPs make their detection possible*

# The magnificent WIMP

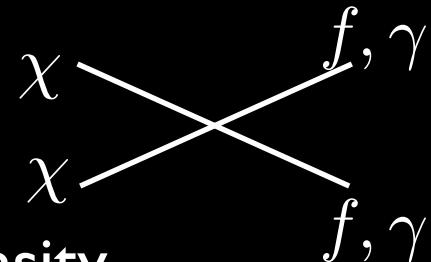
To first order, three quantities characterize a WIMP

- Mass  $m$ 
  - Simplest models relate mass to cosmic density:  $1-10^4 \text{ GeV}/c^2$

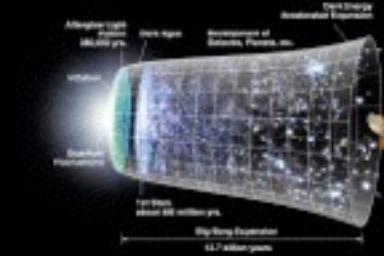
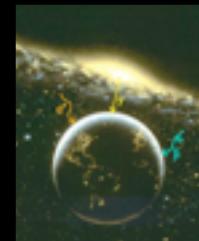
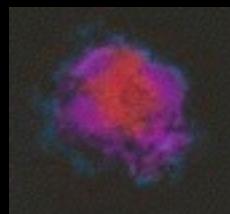
- Scattering cross section off nucleons  $\sigma_{\chi N}$ 
  - Usually different for protons and neutrons
  - Spin-dependent or spin-independent governs scaling to nuclei



- Annihilation cross section into ordinary particles
  - $\sigma \approx \text{const}/v$  at small  $v$ , so use  $\sigma v$
  - Simplest models relate cross section to cosmic density



*Indirect detection*



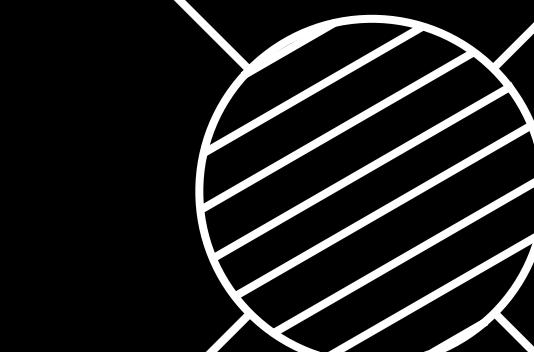
*Cosmic density*

Annihilation

$\chi$

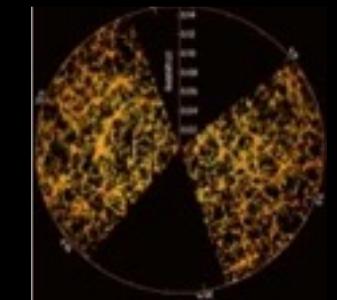
$f$

## The power of the WIMP hypothesis



Scattering

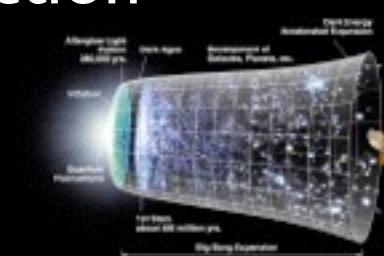
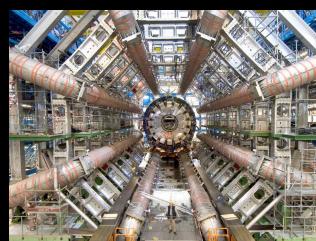
*Direct detection*



*Large scale structure*

Production

*Colliders*



*Cosmic density*

# **Minimalist dark matter**

# Minimalist dark matter

*do not confuse with minimal dark matter*

“*Higgs portal scalar dark matter*”

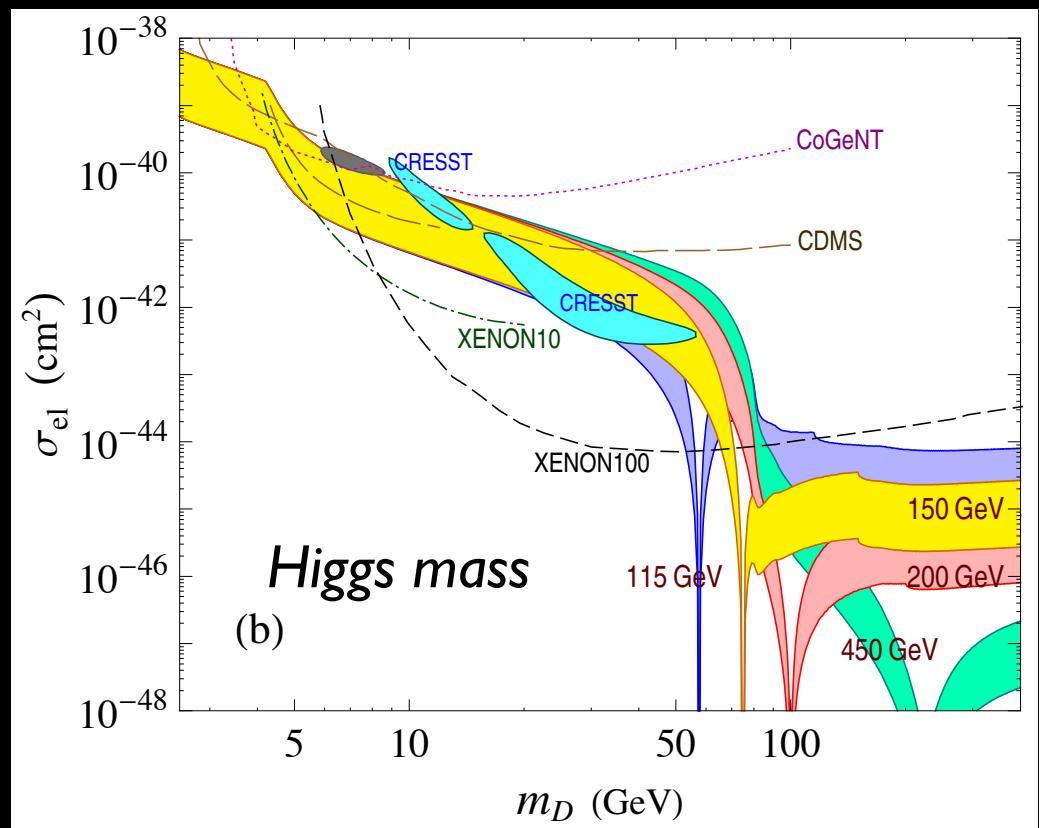
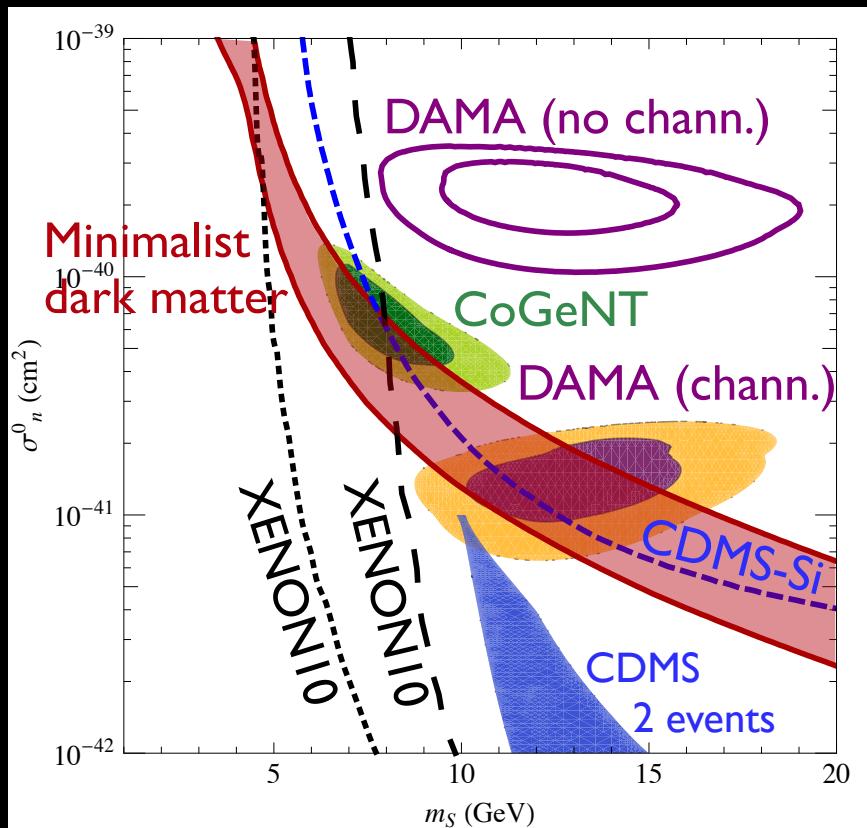
Gauge singlet scalar field  $S$ , stabilized by  $Z_2$  symmetry ( $S \rightarrow -S$ )

$$\mathcal{L}_S = \frac{1}{2} \partial^\mu S \partial_\mu S - \frac{1}{2} \mu_S^2 S^2 - \frac{\lambda_S}{4} S^4 - \lambda_L H^\dagger H S^2$$

*Silveira, Zee 1985*  
*Andreas, Hambye, Tytgat 2008*

# Minimalist dark matter

*do not confuse with minimal dark matter*



Andreas, Arina, Hambye, Ling, Tytgat 2010

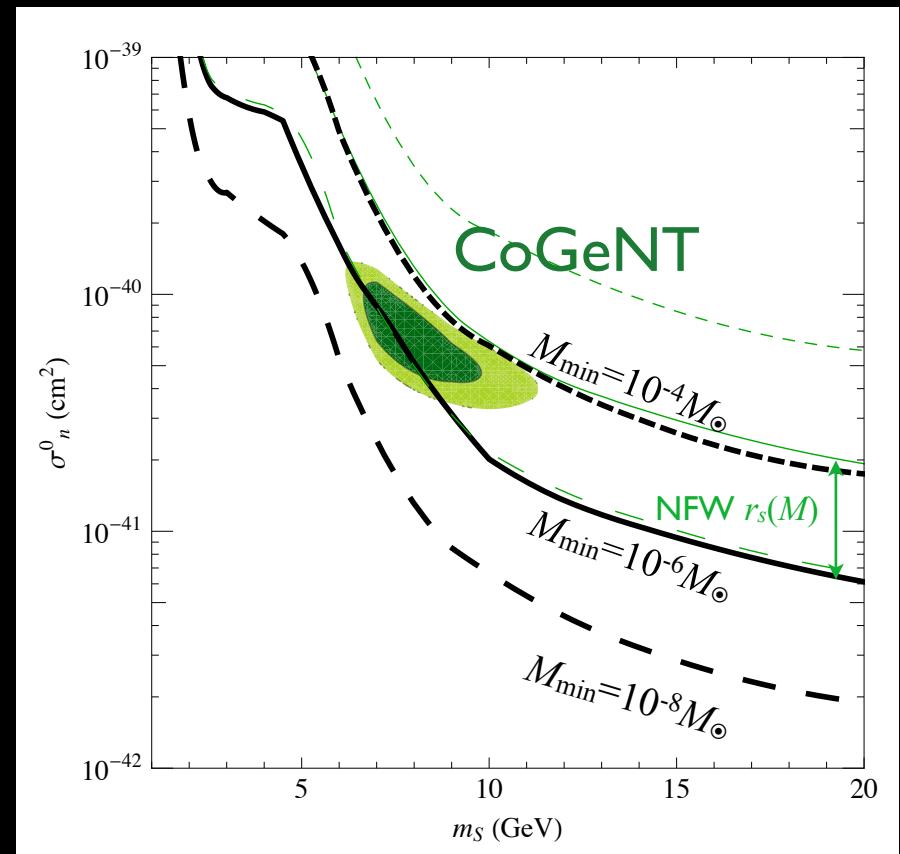
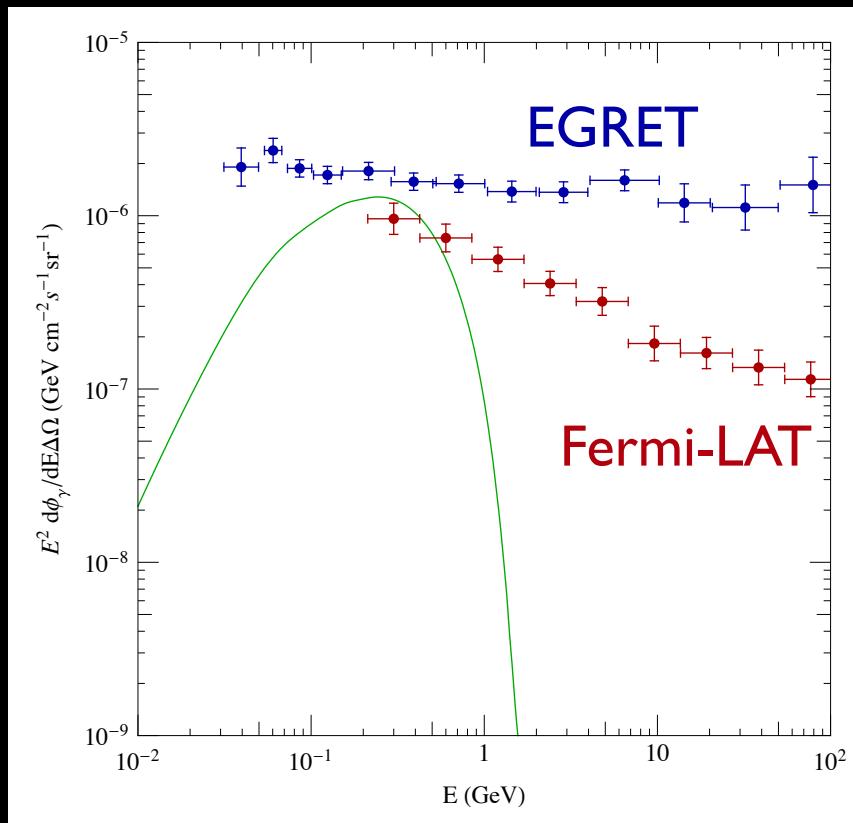
He, Tandean 2011

# Minimalist dark matter

*do not confuse with minimal dark matter*

## Constraints from diffuse Galactic gamma-rays

*Very sensitive to unknown  
properties of small dark subhalos*

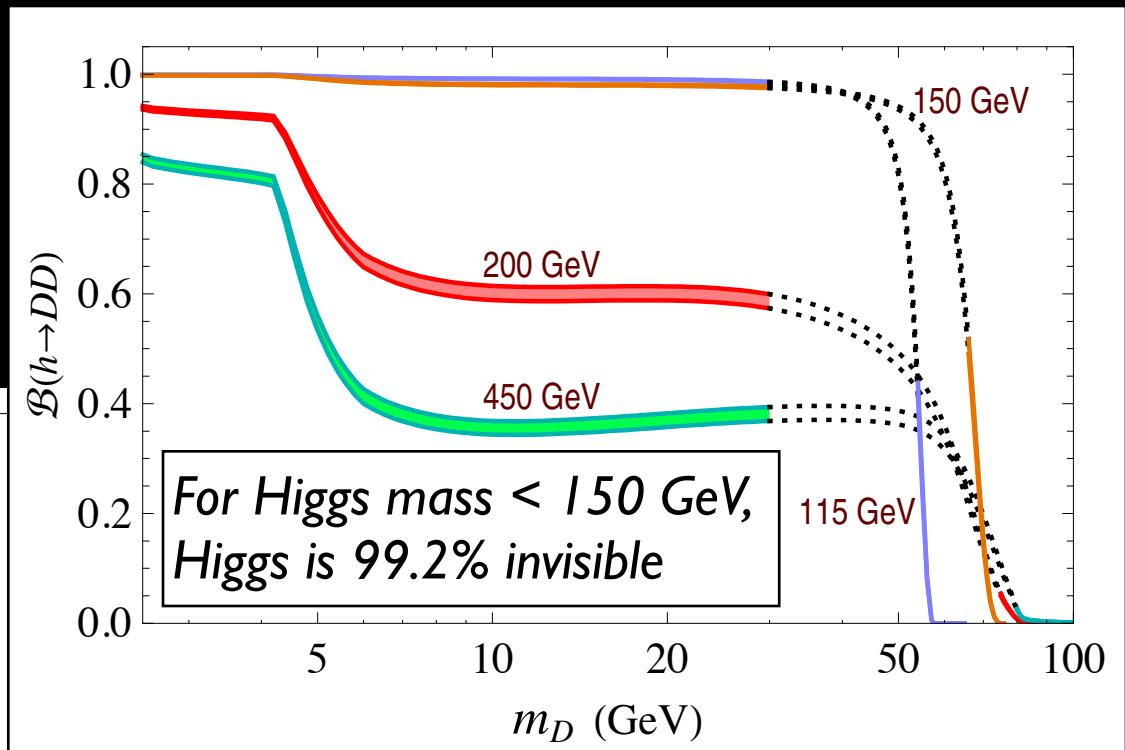
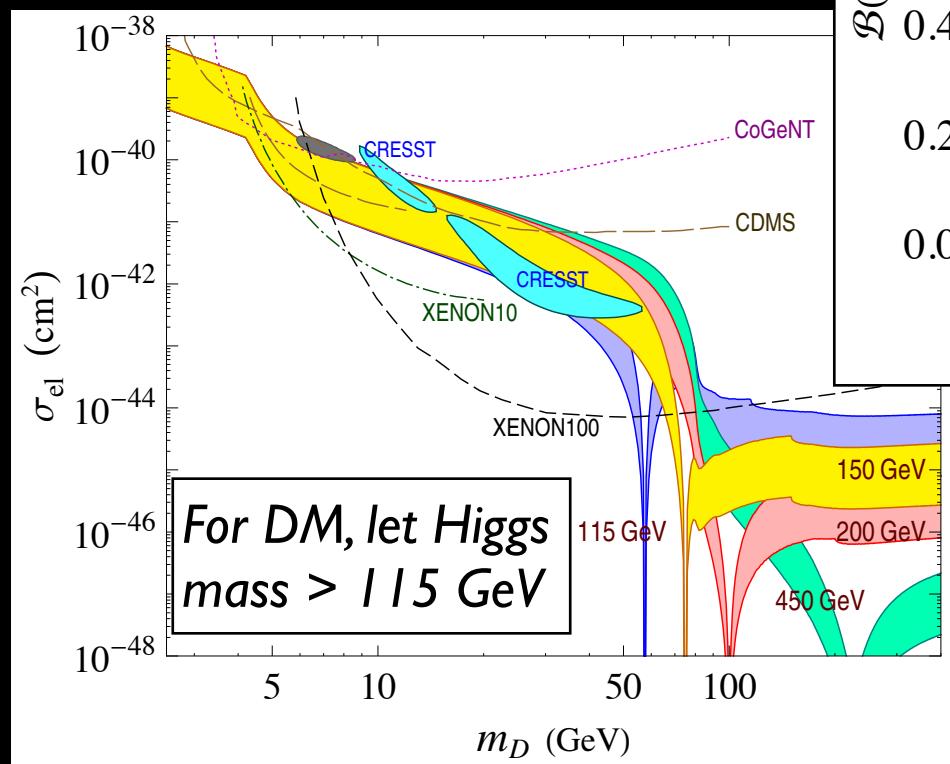


*Arina, Tytgat 2010*

# Minimalist dark matter

*do not confuse with minimal dark matter*

Constraints from the LHC: a 125 Higgs is not 99.2% invisible

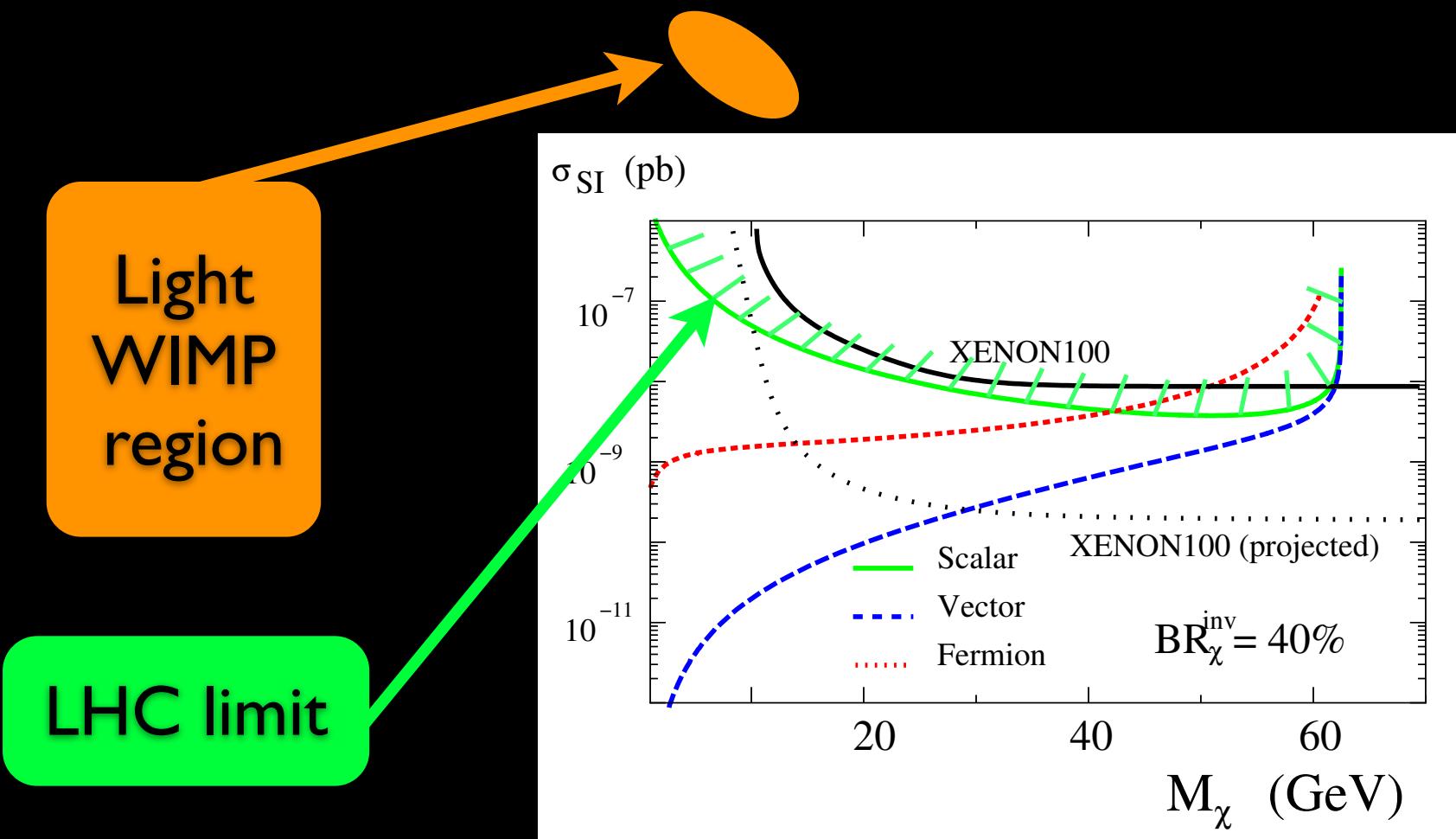


*He, Tandean 2011*

# Minimalist dark matter

*do not confuse with minimal dark matter*

Constraints from the LHC: a 125 GeV Higgs is not 99.2% invisible



# Minimalist dark matter

arxiv:1306.4710

## Update on scalar singlet dark matter

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Kimmo Kainulainen<sup>‡</sup>

*Department of Physics, P.O.Box 35 (YFL), FIN-40014 University of Jyväskylä, Finland and  
Helsinki Institute of Physics, P.O. Box 64, FIN-00014 University of Helsinki, Finland*

Christoph Weniger<sup>§</sup>

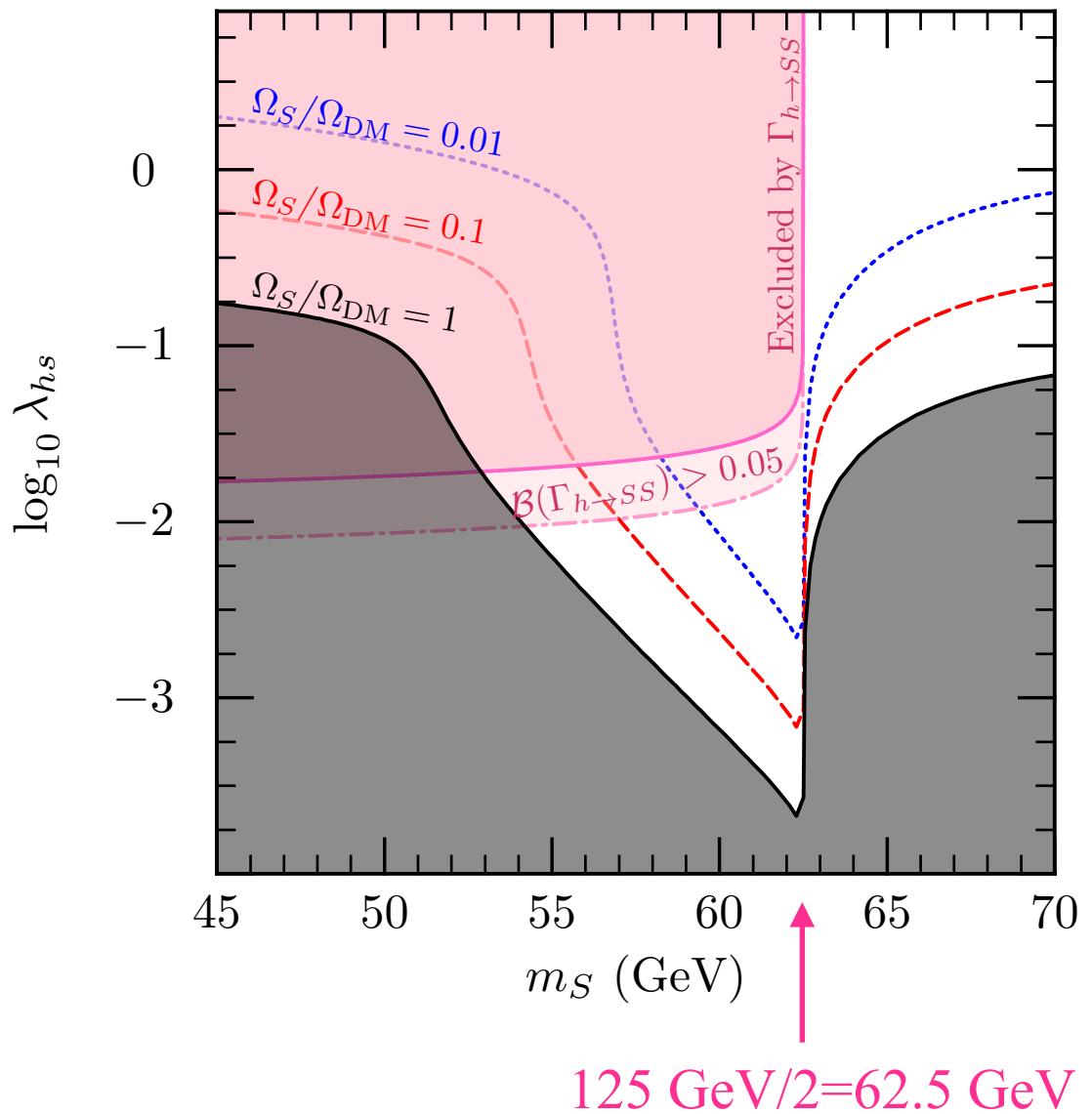
*GRAPPA Institute, University of Amsterdam, Science Park 904, 1098 GL Amsterdam, Netherlands*

One of the simplest models of dark matter is that where a scalar singlet field  $S$  comprises some or all of the dark matter, and interacts with the standard model through an  $|H|^2 S^2$  coupling to the Higgs boson. We update the present limits on the model from LHC searches for invisible Higgs decays, the thermal relic density of  $S$ , and dark matter searches via indirect and direct detection. We point out that the currently allowed parameter space is on the verge of being significantly reduced with the next generation of experiments. We discuss the impact of such constraints on possible applications of scalar singlet dark matter, including a strong electroweak phase transition, and the question of vacuum stability of the Higgs potential at high scales.

$$V = \frac{1}{2} \mu_S^2 S^2 + \frac{1}{2} \lambda_{hS} S^2 |H|^2 .$$

$$m_S = \sqrt{\mu_S^2 + \frac{1}{2} \lambda_{hS} v_0^2} ,$$

# Minimalist dark matter



Invisible Higgs width

$$h \rightarrow S + S \quad h \quad \dots \dots \dots \quad S$$

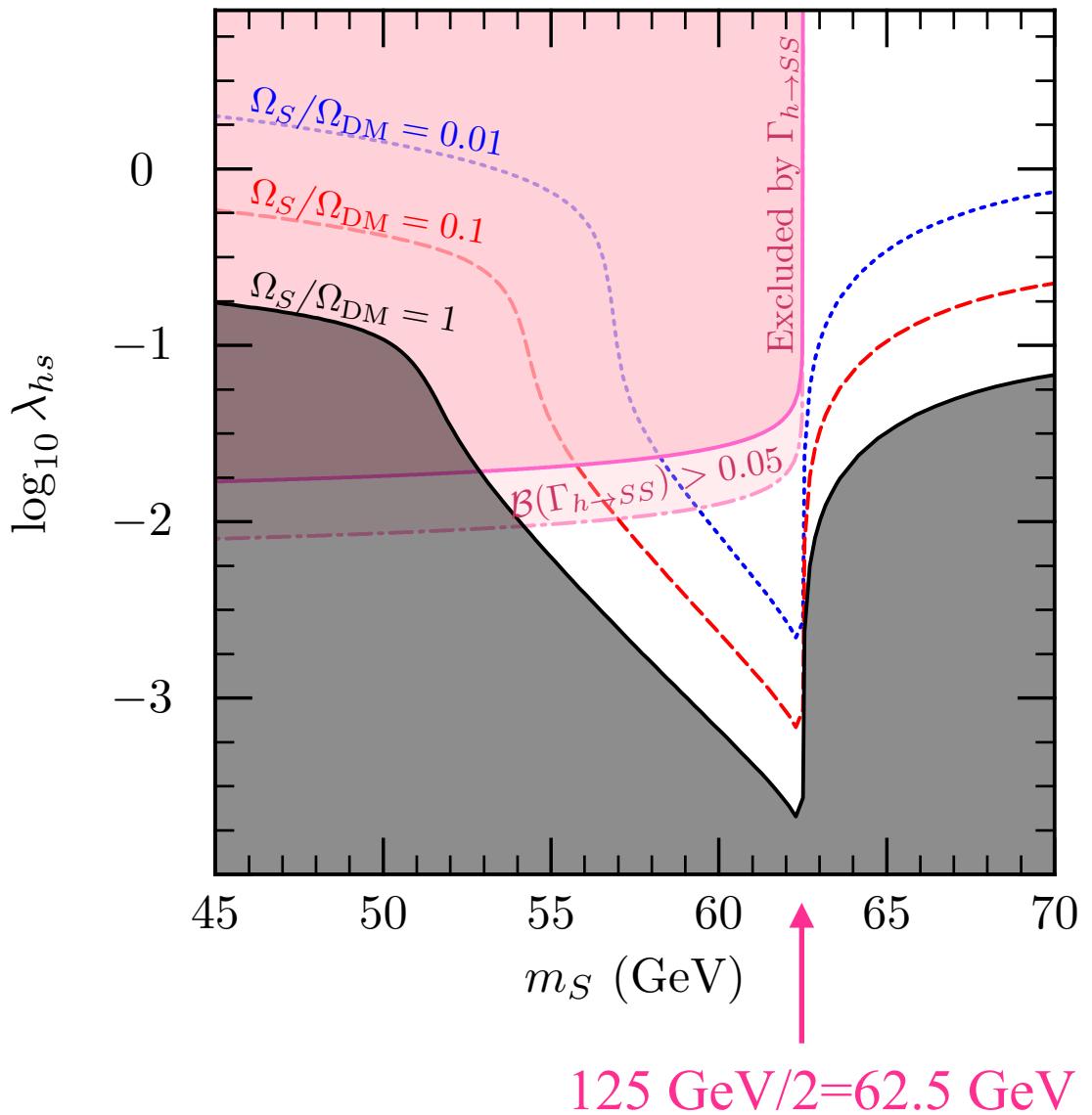
$$\Gamma_{\text{inv}} = \frac{\lambda_{hs}^2 v_0^2}{32\pi m_h} \left(1 - 4m_S^2/m_h^2\right)^{1/2}$$

LHC

$$\Gamma_{\text{vis}} = 4.07 \text{ MeV} \quad m_h = 125 \text{ GeV}$$

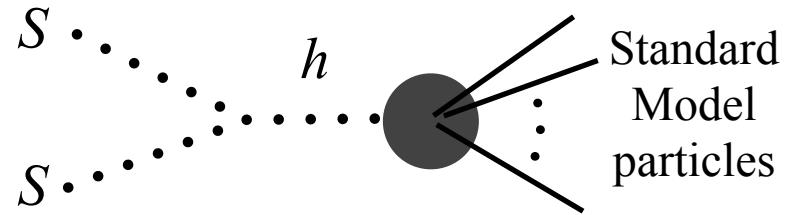
$$B(\Gamma_{h \rightarrow SS}) = \frac{\Gamma_{\text{inv}}}{\Gamma_{\text{vis}} + \Gamma_{\text{inv}}} < 0.19 \quad (2\sigma)$$

# Minimalist dark matter



## Cosmic density

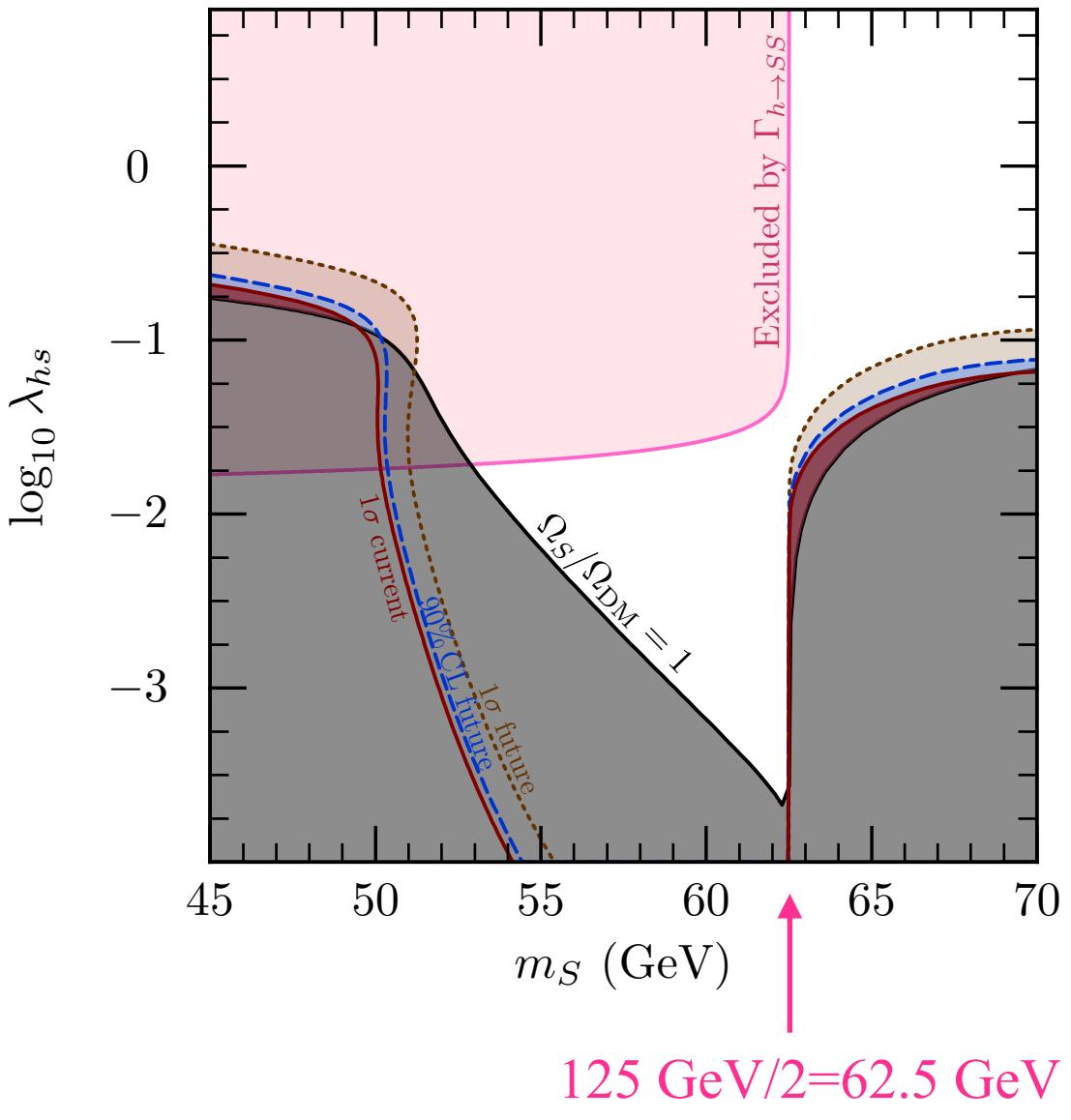
$S + S \rightarrow$  Standard Model particles



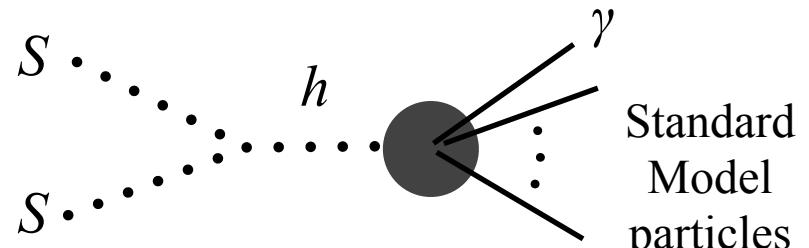
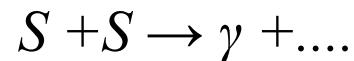
$$\sigma v_{\text{rel}} = \frac{2\lambda_{hs}^2 v_0^2}{\sqrt{s}} \frac{\Gamma_h(\sqrt{s})}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2(m_h)}$$

$$\langle \sigma v_{\text{rel}} \rangle = \int_{4m_S^2}^{\infty} \frac{s \sqrt{s - 4m_S^2} K_1(\sqrt{s}/T) \sigma v_{\text{rel}}}{16 T m_S^4 K_2^2(m_S/T)} ds$$

# Minimalist dark matter



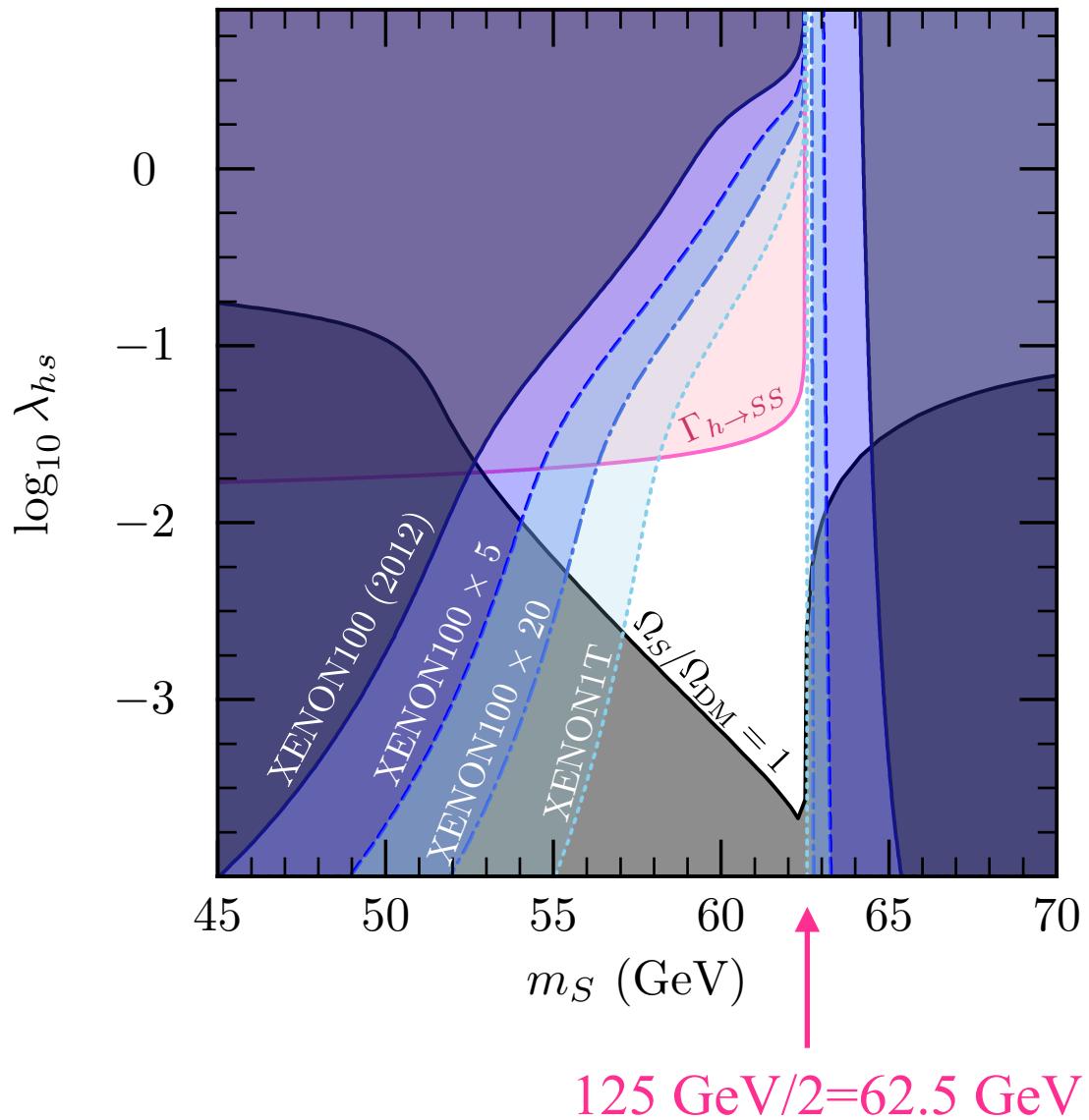
## Indirect detection



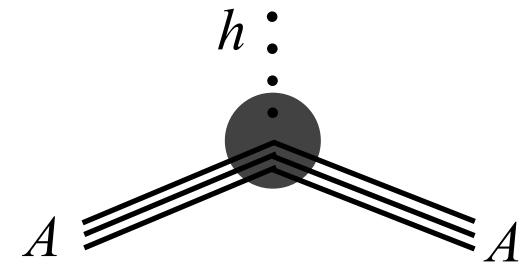
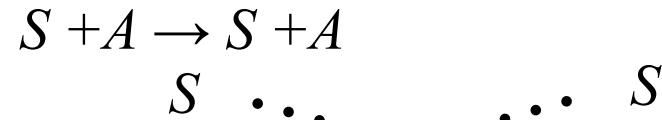
$$\frac{d\phi}{dE} = \frac{\langle \sigma v_{\text{rel}} \rangle}{8\pi m_S^2} \frac{dN_\gamma}{dE} \underbrace{\int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds \rho^2}_{\equiv J}$$

No gamma-rays in Fermi Observatory from dwarf spheroidal galaxies

# Minimalist dark matter



## Direct detection



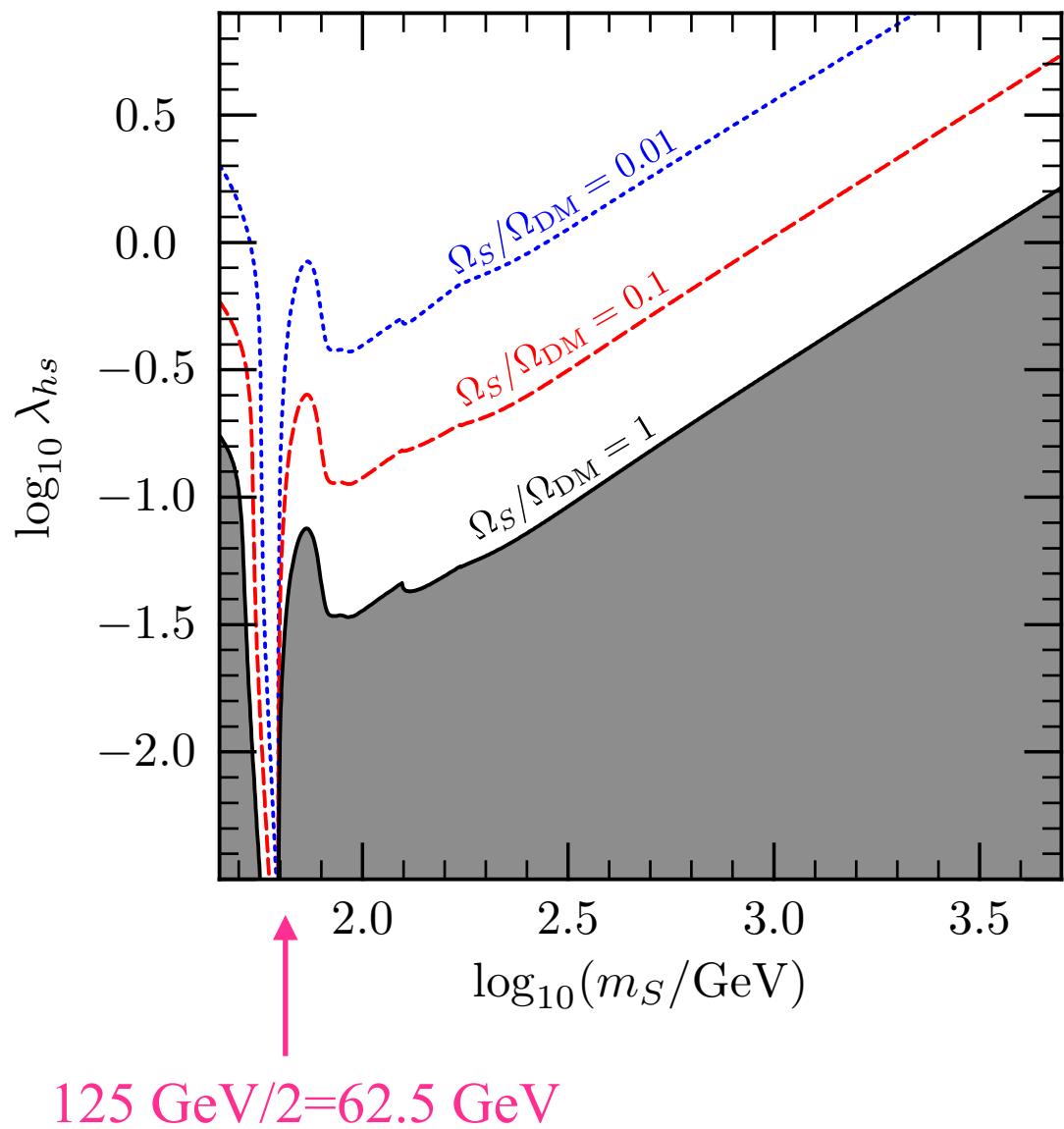
$$\sigma_{\text{SI}} = \frac{\lambda_{hS}^2 f_N^2}{4\pi} \frac{\mu^2 m_n^2}{m_h^4 m_s^2}$$

$$f_N = \sum_q f_q = \sum_q \frac{m_q}{m_N} \langle N | \bar{q} q | N \rangle$$

In the figure, limits from XENON experiments

# Minimalist dark matter

Heavier masses  
Cosmic density



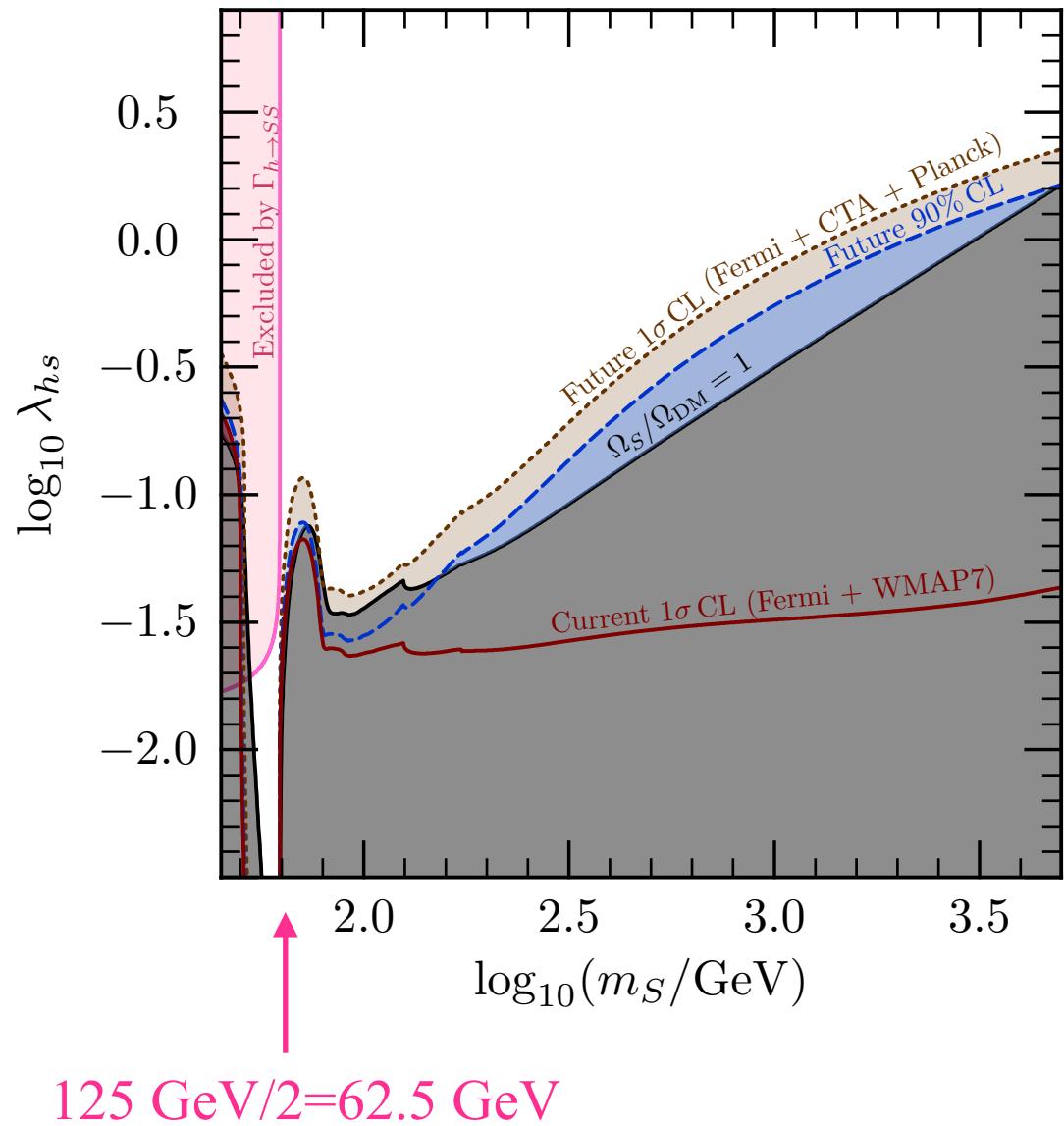
# Minimalist dark matter

Heavier masses

Cosmic density

Invisible Higgs width

Indirect detection



# Minimalist dark matter

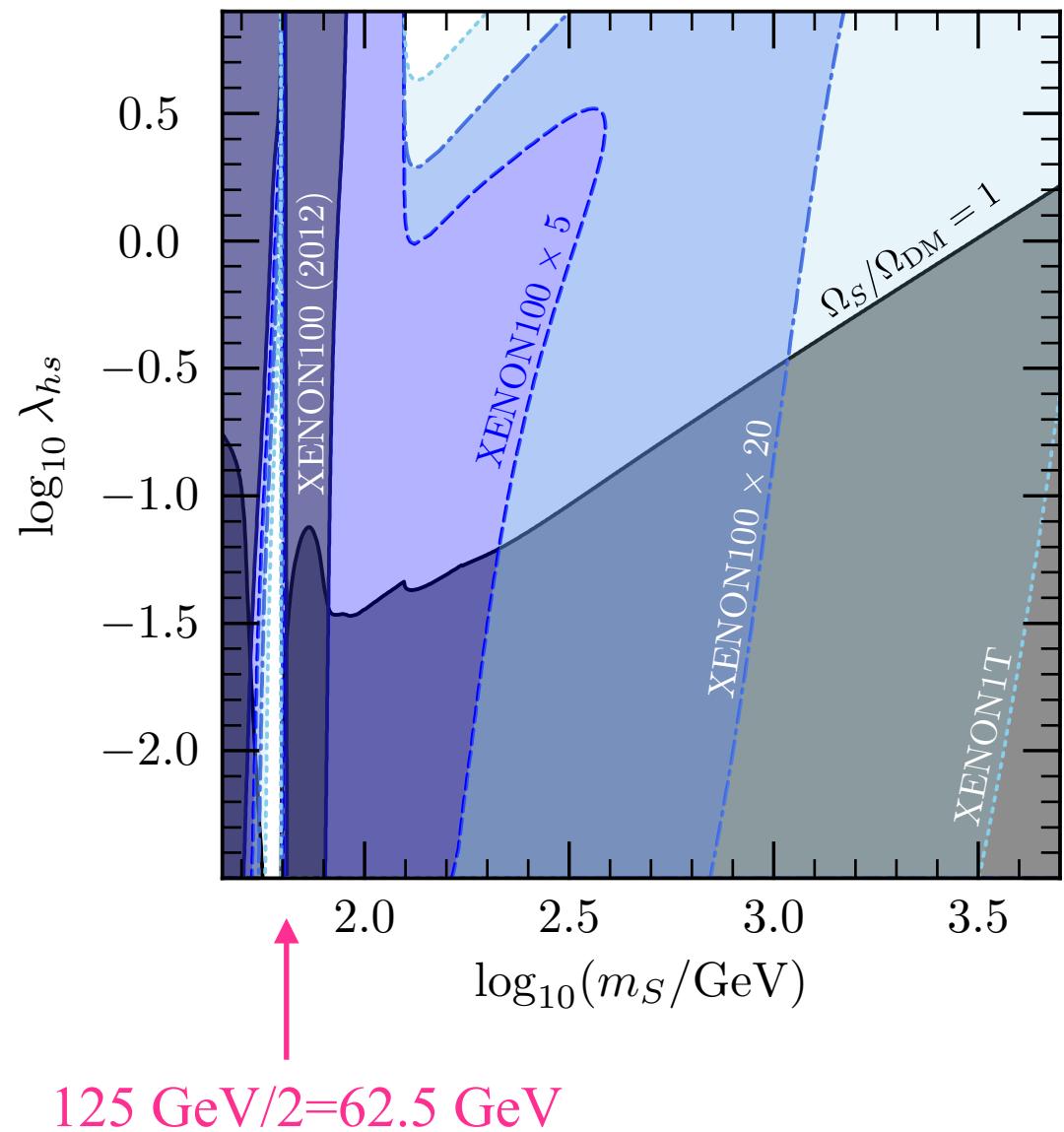
Heavier masses

Cosmic density

Invisible Higgs width

Indirect detection

Direct detection



# Particle Dark Matter

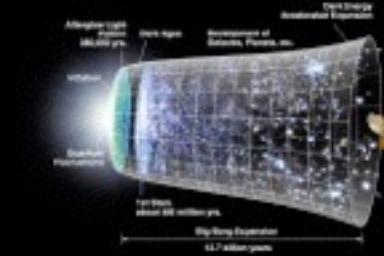
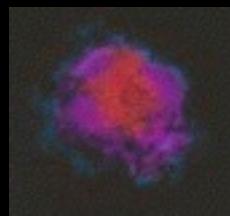
Type Ia Candidates that exist

Type Ib Candidates in well-motivated frameworks

- have been proposed to solve genuine particle physics problems, a priori unrelated to dark matter
- have interactions and masses specified within a well-defined particle physics model

Type II All other candidates

*Indirect detection*



*Cosmic density*

Annihilation

$\chi$

$f$

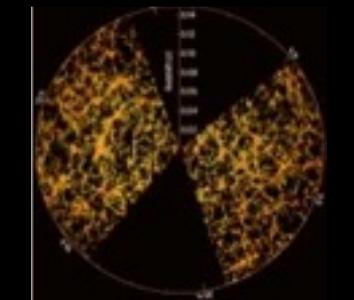
## The power of the WIMP hypothesis

$(-) \chi$

$(-) f$

Scattering

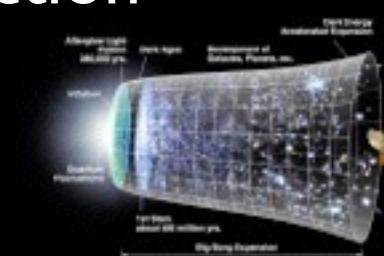
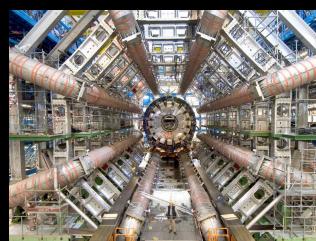
*Direct detection*



*Large scale structure*

Production

*Colliders*



*Cosmic density*

# **Supersymmetric dark matter**

# Supersymmetry

A supersymmetric transformation  $Q$  turns a bosonic state into a fermionic state, and viceversa.

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle$$

$$Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

$$\{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} = P_\mu \sigma^\mu_{\alpha\dot{\alpha}}, \{Q_\alpha, Q_\beta\} = \{Q_{\dot{\alpha}}^\dagger, Q_{\dot{\beta}}^\dagger\} = 0, [P^\mu, Q_\alpha] = [P^\mu, Q_{\dot{\alpha}}^\dagger] = 0$$

A *supersymmetric theory is invariant under supersymmetry transformations*

- bosons and fermions come in pairs of equal mass
- the interactions of bosons and fermions are related

# Supersymmetric Quantum Electrodynamics

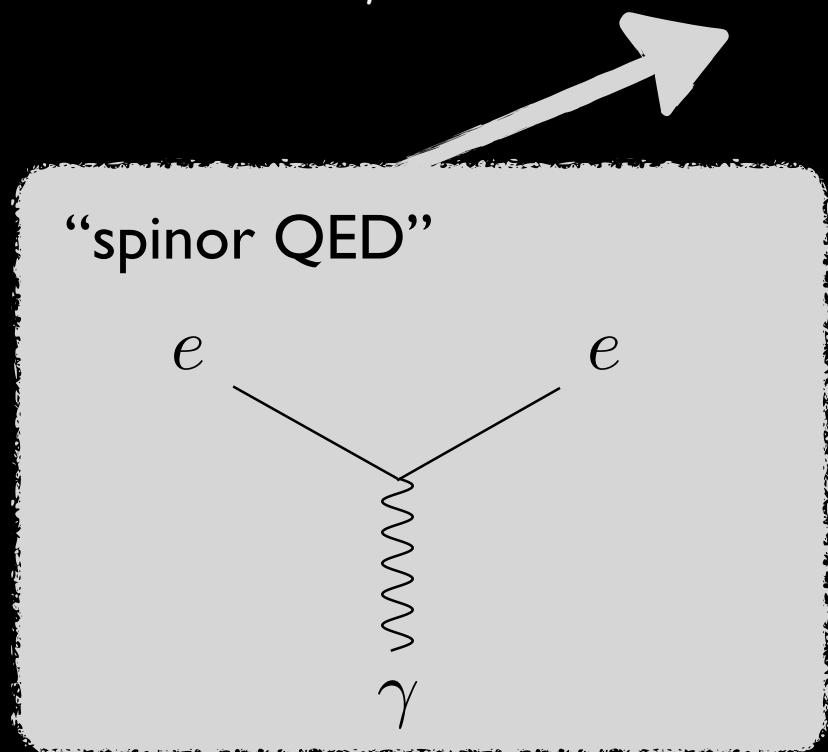
photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

*Start with non-supersymmetric QED*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu$$



# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu \\ & + \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L - iqA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L \\ & + \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R \\ & + \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda - \sqrt{2}q(\tilde{e}_L^*\bar{\lambda}e_L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.}) \\ & - \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L - \tilde{e}_R^*\tilde{e}_R)^2\end{aligned}$$

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

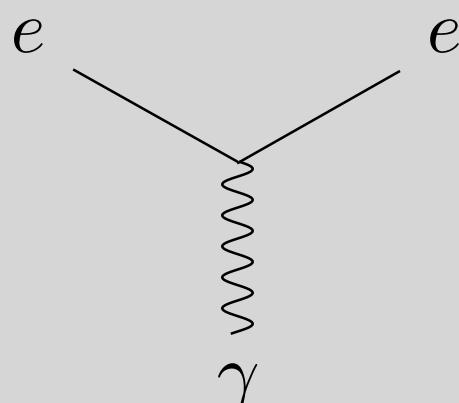
photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e[-q\bar{e}\gamma^\mu eA_\mu] \\ & + \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L - i\bar{q}A^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L \\ & + \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - i\bar{q}A^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R \\ & + \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda - \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L + \tilde{e}_R^*\tilde{e}_R) - \text{h.c.}\end{aligned}$$

“spinor QED”



# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

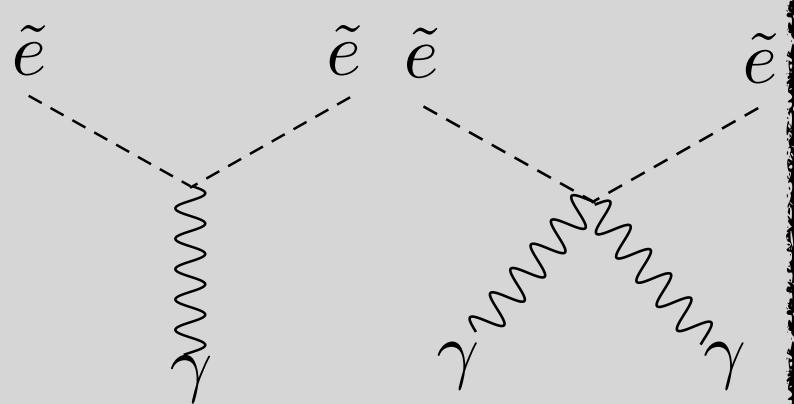
right-handed selectron  $\tilde{e}_R$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu$$

$$+ \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L \boxed{- iqA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L}$$

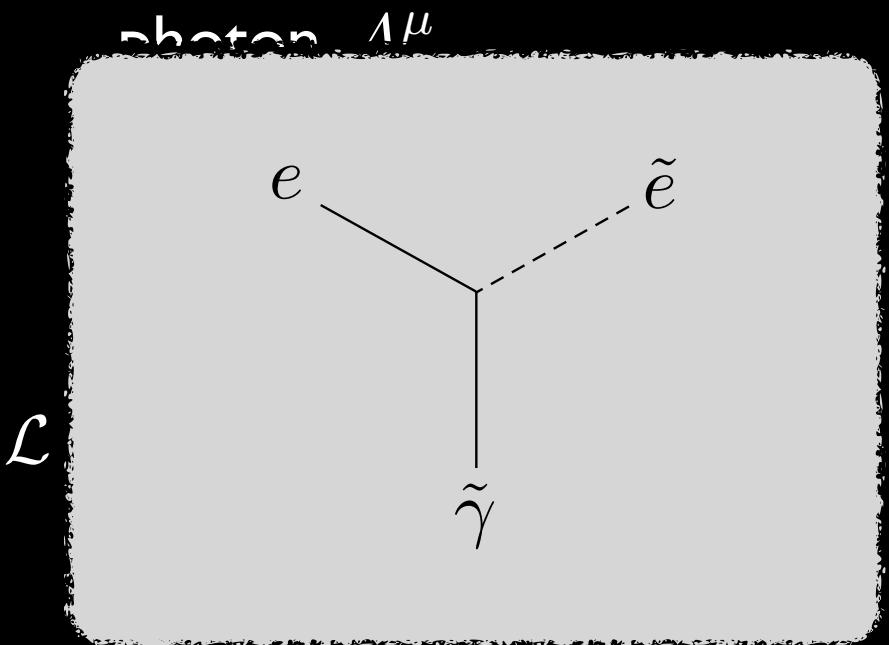
$$+ \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R \boxed{- iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R}$$

“scalar QED”



$$L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.})$$

# Supersymmetric Quantum Electrodynamics



photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu$$

$$qA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L$$

$$+ \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R$$

$$+ \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda \boxed{-\sqrt{2}q(\tilde{e}_L^*\bar{\lambda}e_L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.})}$$

$$- \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L - \tilde{e}_R^*\tilde{e}_R)^2$$

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-h.

right-

“D-term”

$$\mathcal{L} = -\frac{1}{4} F_\mu F^\mu + \partial^\mu \tilde{e}_I^* \partial_\mu \tilde{e}_I + \partial^\mu \tilde{e}_R^* \partial_\mu \tilde{e}_R + \frac{1}{2} \bar{\lambda} i \gamma^\mu \partial_\mu \lambda - \sqrt{2} q (\tilde{e}_L^* \bar{\lambda} e_L - \tilde{e}_R^* \bar{\lambda} e_R + \text{h.c.}) - \frac{1}{2} q^2 (\tilde{e}_L^* \tilde{e}_L - \tilde{e}_R^* \tilde{e}_R)^2$$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$q \bar{e} \gamma^\mu e A_\mu$$

$$\partial_\mu \tilde{e}_L - \tilde{e}_L \partial_\mu \tilde{e}_L^*] + q^2 A^\mu A_\mu \tilde{e}_L^* \tilde{e}_L$$

$$\partial_\mu \tilde{e}_R - \tilde{e}_R \partial_\mu \tilde{e}_R^*] + q^2 A^\mu A_\mu \tilde{e}_R^* \tilde{e}_R$$

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu \\ & + \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L - iqA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L \\ & + \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R \\ & + \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda - \sqrt{2}q(\tilde{e}_L^*\bar{\lambda}e_L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.}) \\ & - \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L - \tilde{e}_R^*\tilde{e}_R)^2\end{aligned}$$

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu \\ & + \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L - iqA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L \\ & + \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R \\ & + \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda - \sqrt{2}q(\tilde{e}_L^*\bar{\lambda}e_L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.}) \\ & - \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L - \tilde{e}_R^*\tilde{e}_R)^2 \left[ -m_L^2\tilde{e}_L^*\tilde{e}_L - m_R^2\tilde{e}_R^*\tilde{e}_R - \frac{1}{2}M\bar{\lambda}\lambda \right]\end{aligned}$$

“soft supersymmetry-breaking terms”

# Supersymmetric Quantum Electrodynamics

photon  $A^\mu$

left-handed electron  $e_L$

right-handed electron  $e_R$

photino  $\lambda$

left-handed selectron  $\tilde{e}_L$

right-handed selectron  $\tilde{e}_R$

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{e}i\gamma^\mu\partial_\mu e - m\bar{e}e - q\bar{e}\gamma^\mu eA_\mu \\ & + \partial^\mu\tilde{e}_L^*\partial_\mu\tilde{e}_L - m^2\tilde{e}_L^*\tilde{e}_L - iqA^\mu[\tilde{e}_L^*\partial_\mu\tilde{e}_L - \tilde{e}_L\partial_\mu\tilde{e}_L^*] + q^2A^\mu A_\mu\tilde{e}_L^*\tilde{e}_L \\ & + \partial^\mu\tilde{e}_R^*\partial_\mu\tilde{e}_R - m^2\tilde{e}_R^*\tilde{e}_R - iqA^\mu[\tilde{e}_R^*\partial_\mu\tilde{e}_R - \tilde{e}_R\partial_\mu\tilde{e}_R^*] + q^2A^\mu A_\mu\tilde{e}_R^*\tilde{e}_R \\ & + \frac{1}{2}\bar{\lambda}i\gamma^\mu\partial_\mu\lambda - \sqrt{2}q(\tilde{e}_L^*\bar{\lambda}e_L - \tilde{e}_R^*\bar{\lambda}e_R + \text{h.c.}) \\ & - \frac{1}{2}q^2(\tilde{e}_L^*\tilde{e}_L - \tilde{e}_R^*\tilde{e}_R)^2 - m_L^2\tilde{e}_L^*\tilde{e}_L - m_R^2\tilde{e}_R^*\tilde{e}_R - \frac{1}{2}M\bar{\lambda}\lambda\end{aligned}$$

**Softly-broken superQED**

# Minimal Supersymmetric Standard Model

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks ( $\times 3$ families)	$Q$	$(\tilde{u}_L \quad \tilde{d}_L)$	$(u_L \quad d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	$\bar{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\bar{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons ( $\times 3$ families)	$L$	$(\tilde{\nu} \quad \tilde{e}_L)$	$(\nu \quad e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	$\bar{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	$H_u$	$(H_u^+ \quad H_u^0)$	$(\tilde{H}_u^+ \quad \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	$H_d$	$(H_d^0 \quad H_d^-)$	$(\tilde{H}_d^0 \quad \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Names		spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon		$\tilde{g}$	$g$	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons		$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson		$\widetilde{B}^0$	$B^0$	$(\mathbf{1}, \mathbf{1}, 0)$

# Minimal Supersymmetric Standard Model

- Gauge interactions (covariant derivatives + D-terms)
- Superpotential (Yukawa terms + F-terms)

$$W = \epsilon_{ij}(-\hat{\mathbf{e}}_R^* \mathbf{Y}_E \hat{\mathbf{l}}_L^i \hat{H}_1^j - \hat{\mathbf{d}}_R^* \mathbf{Y}_D \hat{\mathbf{q}}_L^i \hat{H}_1^j + \hat{\mathbf{u}}_R^* \mathbf{Y}_U \hat{\mathbf{q}}_L^i \hat{H}_2^j - \mu \hat{H}_1^i \hat{H}_2^j)$$

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \bar{\psi}_i \psi_j \quad \mathcal{L}_{\text{F-terms}} = \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

- Soft terms

$$\begin{aligned} V_{\text{soft}} = & \epsilon_{ij}(-\tilde{\mathbf{e}}_R^* \mathbf{A}_E \mathbf{Y}_E \tilde{\mathbf{l}}_L^i H_1^j - \tilde{\mathbf{d}}_R^* \mathbf{A}_D \mathbf{Y}_D \tilde{\mathbf{q}}_L^i H_1^j + \tilde{\mathbf{u}}_R^* \mathbf{A}_U \mathbf{Y}_U \tilde{\mathbf{q}}_L^i H_2^j - B \mu H_1^i H_2^j + \text{h.c.}) \\ & + H_1^{i*} m_1^2 H_1^i + H_2^{i*} m_2^2 H_2^i + \tilde{\mathbf{q}}_L^{i*} \mathbf{M}_Q^2 \tilde{\mathbf{q}}_L^i + \tilde{\mathbf{l}}_L^{i*} \mathbf{M}_L^2 \tilde{\mathbf{l}}_L^i + \tilde{\mathbf{u}}_R^* \mathbf{M}_U^2 \tilde{\mathbf{u}}_R + \tilde{\mathbf{d}}_R^* \mathbf{M}_D^2 \tilde{\mathbf{d}}_R \\ & + \tilde{\mathbf{e}}_R^* \mathbf{M}_E^2 \tilde{\mathbf{e}}_R + \frac{1}{2} M_1 \tilde{B} \tilde{B} + \frac{1}{2} M_2 (\tilde{W}^3 \tilde{W}^3 + 2 \tilde{W}^+ \tilde{W}^-) + \frac{1}{2} M_3 \tilde{g} \tilde{g}. \end{aligned}$$

124 parameters (cfr. 18 in SM)

From Martin hep-ph/9709356

# Minimal Supersymmetric Standard Model

Neutralinos are linear combinations of neutral gauginos and higgsinos

$$\tilde{\chi}_i^0 = N_{i1} \tilde{B} + N_{i2} \tilde{W}^3 + N_{i3} \tilde{H}_1^0 + N_{i4} \tilde{H}_2^0,$$

$$\mathcal{M}_{\tilde{\chi}_{1,2,3,4}^0} = \begin{pmatrix} M_1 & 0 & -\frac{g'v_1}{\sqrt{2}} & +\frac{g'v_2}{\sqrt{2}} \\ 0 & M_2 & +\frac{gv_1}{\sqrt{2}} & -\frac{gv_2}{\sqrt{2}} \\ -\frac{g'v_1}{\sqrt{2}} & +\frac{gv_1}{\sqrt{2}} & \delta_{33} & -\mu \\ +\frac{g'v_2}{\sqrt{2}} & -\frac{gv_2}{\sqrt{2}} & -\mu & \delta_{44} \end{pmatrix}$$

Charginos are linear combinations of charged gauginos and higgsinos

$$\begin{aligned} \tilde{\chi}_i^- &= U_{i1} \tilde{W}^- + U_{i2} \tilde{H}_1^-, \\ \tilde{\chi}_i^+ &= V_{i1} \tilde{W}^+ + V_{i2} \tilde{H}_2^+. \end{aligned}$$

$$\mathcal{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & gv_2 \\ gv_1 & \mu \end{pmatrix},$$

# Minimal Supersymmetric Standard Model

Squarks and sleptons are linear combinations of interaction eigenstates

$$\tilde{f}_{La} = \sum_{k=1}^6 \tilde{f}_k \Gamma_{FL}^{*ka},$$

$$\tilde{f}_{Ra} = \sum_{k=1}^6 \tilde{f}_k \Gamma_{FR}^{*ka}.$$

$$\mathcal{M}_{\tilde{u}}^2 = \begin{pmatrix} \mathbf{M}_Q^2 + \mathbf{m}_u^\dagger \mathbf{m}_u + D_{LL}^u \mathbf{1} & \mathbf{m}_u^\dagger (\mathbf{A}_U^\dagger - \mu^* \cot \beta) \\ (\mathbf{A}_U - \mu \cot \beta) \mathbf{m}_u & \mathbf{M}_U^2 + \mathbf{m}_u \mathbf{m}_u^\dagger + D_{RR}^u \mathbf{1} \end{pmatrix},$$

$$\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix} \mathbf{K}^\dagger \mathbf{M}_Q^2 \mathbf{K} + \mathbf{m}_d \mathbf{m}_d^\dagger + D_{LL}^d \mathbf{1} & \mathbf{m}_d^\dagger (\mathbf{A}_D^\dagger - \mu^* \tan \beta) \\ (\mathbf{A}_D - \mu \tan \beta) \mathbf{m}_d & \mathbf{M}_D^2 + \mathbf{m}_d^\dagger \mathbf{m}_d + D_{RR}^d \mathbf{1} \end{pmatrix}.$$

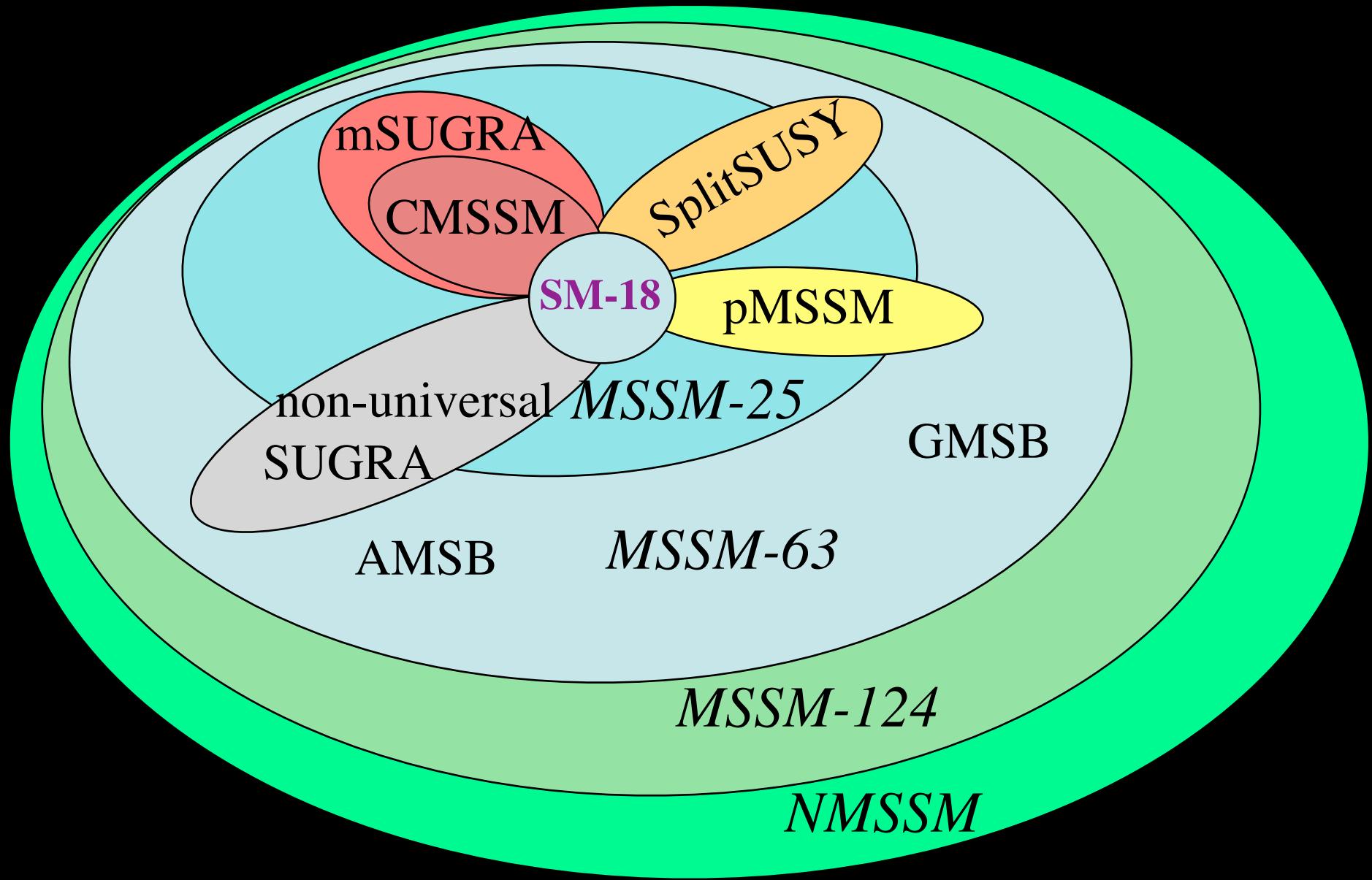
$$\mathcal{M}_{\tilde{\nu}}^2 = \mathbf{M}_L^2 + D_{LL}^\nu \mathbf{1}$$

$$\mathcal{M}_{\tilde{e}}^2 = \begin{pmatrix} \mathbf{M}_L^2 + \mathbf{m}_e \mathbf{m}_e^\dagger + D_{LL}^e \mathbf{1} & \mathbf{m}_e^\dagger (\mathbf{A}_E^\dagger - \mu^* \tan \beta) \\ (\mathbf{A}_E - \mu \tan \beta) \mathbf{m}_e & \mathbf{M}_E^2 + \mathbf{m}_e^\dagger \mathbf{m}_e + D_{RR}^e \mathbf{1} \end{pmatrix}.$$

$$D_{LL}^f = m_Z^2 \cos 2\beta (T_{3f} - e_f \sin^2 \theta_W),$$

$$D_{RR}^f = m_Z^2 \cos(2\beta) e_f \sin^2 \theta_W$$

# Intersections of supersymmetric models



# Supersymmetric dark matter

Neutralinos (the most fashionable/studied WIMP)

*Goldberg 1983; Ellis, Hagelin, Nanopoulos, Olive, Srednicki 1984; etc.*

Sneutrinos (also WIMPs)

*Falk, Olive, Srednicki 1994; Asaka, Ishiwata, Moroi 2006; McDonald 2007; Lee, Matchev, Nasri 2007; Deppisch, Pilaftsis 2008; Cerdeno, Munoz, Seto 2009; Cerdeno, Seto 2009; etc.*

Gravitinos (SuperWIMPs)

*Feng, Rajaraman, Takayama 2003; Ellis, Olive, Santoso, Spanos 2004; Feng, Su, Takayama, 2004; etc.*

Axinos (SuperWIMPs)

*Tamvakis, Wyler 1982; Nilles, Raby 1982; Goto, Yamaguchi 1992; Covi, Kim, Kim, Roszkowski 2001; Covi, Roszkowski, Ruiz de Austri, Small 2004; etc.*

# Supersymmetric superWIMPs

*Interaction scale with ordinary matter suppressed by large mass scale*

Axino dark matter ( $f_{\text{PQ}} \sim 10^{11} \text{ GeV}$ )

thermally and non-thermally produced in early universe

$$m_{\tilde{a}} \gtrsim 0.1 \text{ MeV}$$

scattering cross section with ordinary matter

$$\sigma \approx (m_W/f_{PQ})^2 \sigma_{\text{weak}} \approx 10^{-18} \sigma_{\text{weak}} \approx 10^{-56} \text{ cm}^2$$

Gravitino dark matter ( $m_{\text{Pl}} \sim 10^{19} \text{ GeV}$ )

thermally and non-thermally produced in early universe

$$m_{3/2} \approx 1 \text{ GeV} - 700 \text{ GeV}$$

scattering cross section with ordinary matter

$$\sigma \approx 10^{-72} \text{ cm}^2$$

# Neutralino dark matter

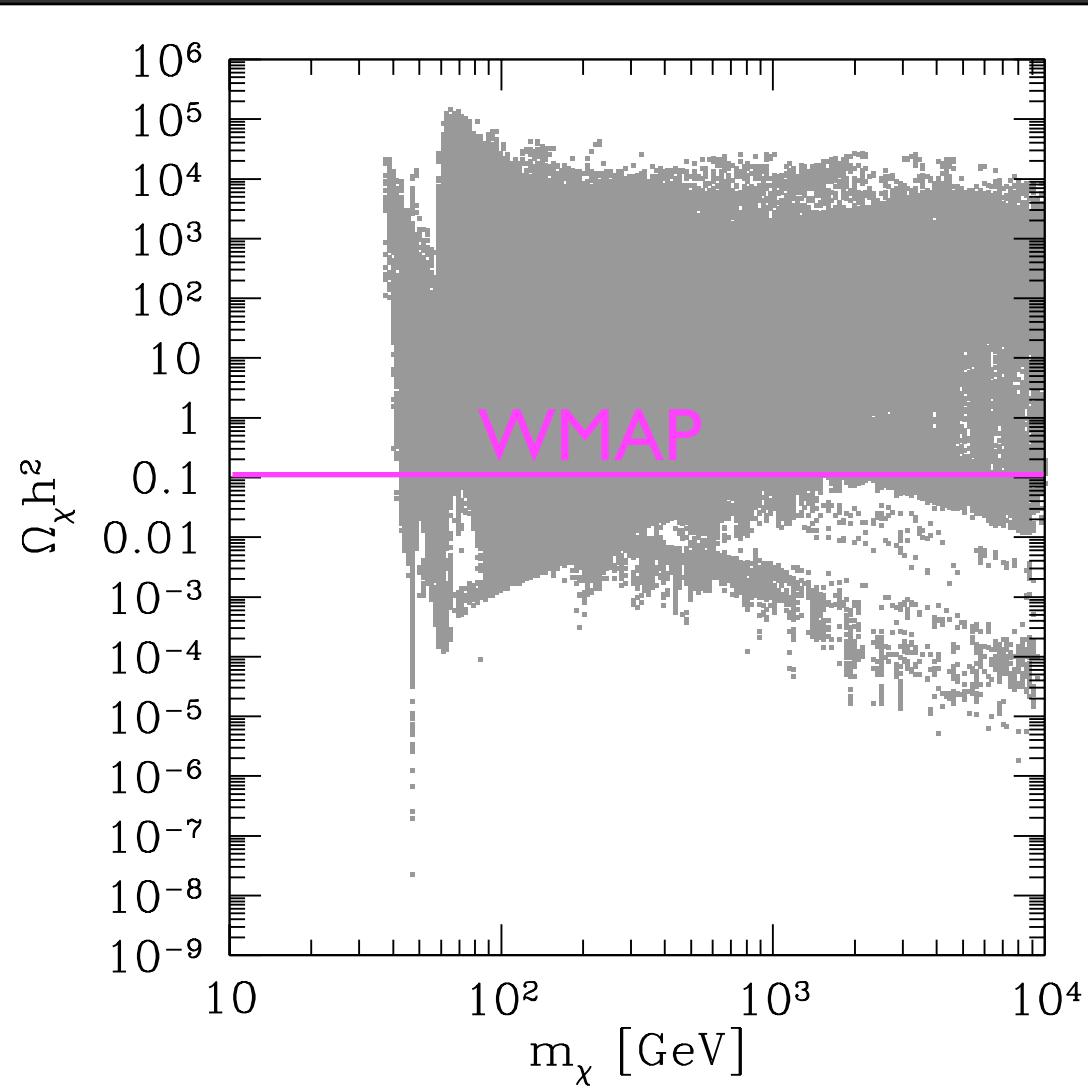
Process	Diagrams			
	s	t	u	p
$\chi_i^0 \chi_j^0 \rightarrow B_m^0 B_n^0$	$H_{1,2,3}^0, Z$	$\chi_k^0$	$\chi_l^0$	
$\chi_i^0 \chi_j^0 \rightarrow B_m^- B_n^+$	$H_{1,2,3}^0, Z$	$\chi_k^+$	$\chi_l^+$	
$\chi_i^0 \chi_j^0 \rightarrow f \bar{f}$	$H_{1,2,3}^0, Z$	$\tilde{f}_{1,2}$	$\tilde{f}_{1,2}$	
$\chi_i^+ \chi_j^0 \rightarrow B_m^+ B_n^0$	$H^+, W^+$	$\chi_k^0$	$\chi_l^+$	
$\chi_i^+ \chi_j^0 \rightarrow f_u \bar{f}_d$	$H^+, W^+$	$\tilde{f}'_{d_{1,2}}$	$\tilde{f}'_{u_{1,2}}$	
$\chi_i^+ \chi_j^- \rightarrow B_m^0 B_n^0$	$H_{1,2,3}^0, Z$	$\chi_k^+$	$\chi_l^+$	
$\chi_i^+ \chi_j^- \rightarrow B_m^+ B_n^-$	$H_{1,2,3}^0, Z, \gamma$	$\chi_k^0$		
$\chi_i^+ \chi_j^- \rightarrow f_u \bar{f}_u$	$H_{1,2,3}^0, Z, \gamma$	$\tilde{f}'_{d_{1,2}}$		
$\chi_i^+ \chi_j^- \rightarrow \bar{f}_d f_d$	$H_{1,2,3}^0, Z, \gamma$	$\tilde{f}'_{u_{1,2}}$		
$\chi_i^+ \chi_j^+ \rightarrow B_m^+ B_n^+$		$\chi_k^0$	$\chi_l^0$	
$\tilde{f}_i \chi_j^0 \rightarrow B^0 f$	$f$	$\tilde{f}_{1,2}$	$\chi_l^0$	
$\tilde{f}_{d_i} \chi_j^0 \rightarrow B^- f_u$	$f_d$	$\tilde{f}_{u_{1,2}}$	$\chi_l^+$	
$\tilde{f}_{u_i} \chi_j^0 \rightarrow B^+ f_d$	$f_u$	$\tilde{f}_{d_{1,2}}$	$\chi_l^+$	
$\tilde{f}_{d_i} \chi_j^+ \rightarrow B^0 f_u$	$f_u$	$\tilde{f}_{d_{1,2}}$	$\chi_l^+$	
$\tilde{f}_{u_i} \chi_j^+ \rightarrow B^+ f_u$		$\tilde{f}_{d_{1,2}}$	$\chi_l^0$	
$\tilde{f}_{d_i} \chi_j^+ \rightarrow B^+ f_d$	$f_u$		$\chi_l^0$	
$\tilde{f}_{u_i} \chi_j^- \rightarrow B^0 f_d$	$f_d$	$\tilde{f}_{u_{1,2}}$	$\chi_l^+$	
$\tilde{f}_{u_i} \chi_j^- \rightarrow B^- f_u$	$f_d$		$\chi_l^0$	
$\tilde{f}_{d_i} \chi_j^- \rightarrow B^- f_d$		$\tilde{f}_{u_{1,2}}$	$\chi_l^0$	
$\tilde{f}_{d_i} \tilde{f}_{d_j}^* \rightarrow B_m^0 B_n^0$	$H_{1,2,3}^0, Z, g$	$\tilde{f}_{d_{1,2}}$	$\tilde{f}_{d_{1,2}}$	$p$
$\tilde{f}_{d_i} \tilde{f}_{d_j}^* \rightarrow B_m^- B_n^+$	$H_{1,2,3}^0, Z, \gamma$	$\tilde{f}_{u_{1,2}}$		$p$
$\tilde{f}_{d_i} \tilde{f}_{d_j}^{*\prime} \rightarrow f_d'' \bar{f}_d'''$	$H_{1,2,3}^0, Z, \gamma, g$	$\chi_k^0, \tilde{g}$		
$\tilde{f}_{d_i} \tilde{f}_{d_j}^{*\prime} \rightarrow f_u'' \bar{f}_u'''$	$H_{1,2,3}^0, Z, \gamma, g$	$\chi_k^+$		
$\tilde{f}_{d_i} \tilde{f}_{d_j}' \rightarrow f_d f_d'$		$\chi_k^0, \tilde{g}$	$\chi_l^0, \tilde{g}$	
$\tilde{f}_{u_i} \tilde{f}_{d_j}^* \rightarrow B_m^+ B_n^0$	$H^+, W^+$	$\tilde{f}_{d_{1,2}}$	$\tilde{f}_{u_{1,2}}$	$p$
$\tilde{f}_{u_i} \tilde{f}_{d_j}^{*\prime} \rightarrow f_u'' \bar{f}_d'''$	$H^+, W^+$	$\chi_k^0, \tilde{g}$		
$\tilde{f}_{u_i} \tilde{f}_{d_j}' \rightarrow f_u'' \bar{f}_d'''$		$\chi_k^0, \tilde{g}$	$\chi_l^+$	

## Cosmic density

Thousands of annihilation (and coannihilation) processes

Use publicly-available computer codes, e.g. DarkSUSY, micrOMEGAs

# Neutralino dark matter: minimal supergravity

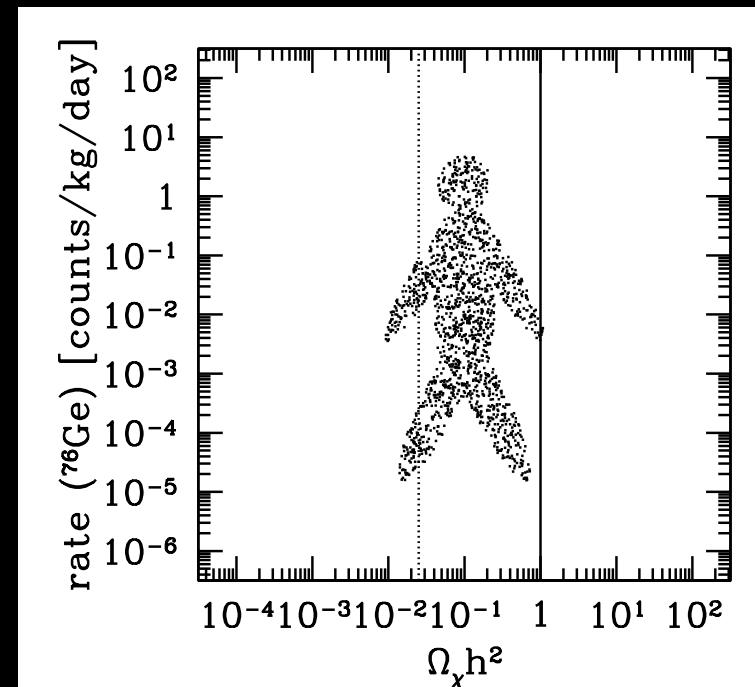


Range of  
 $\Omega_\chi h^2$  for  
millions of  
points in  
minimal  
supergravity  
(mSUGRA)

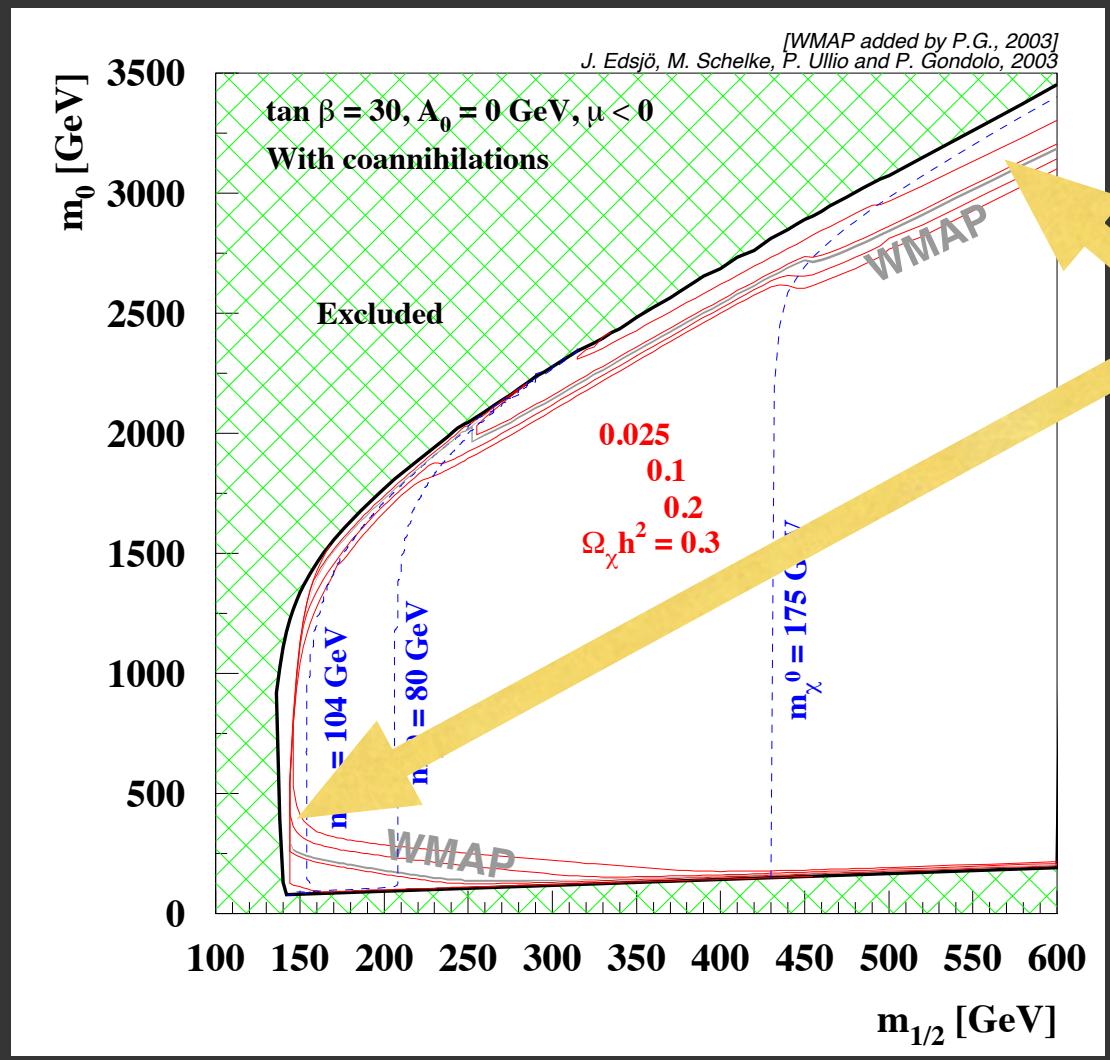
Ted Baltz 2005

# The density of points in parameter space

- Density of points depends on priors in parameters
- Priors describe our beliefs in the value of the model parameters
- What is a sensible prior for  $M_2$ , say?
  - Flat in  $M_2$ ? Flat in  $\log(M_2)$ ? Exponential in  $\arctan(M_2)$ ?
- Example: a scan in parameter space using an anthropic prior



# Neutralino dark matter: minimal supergravity

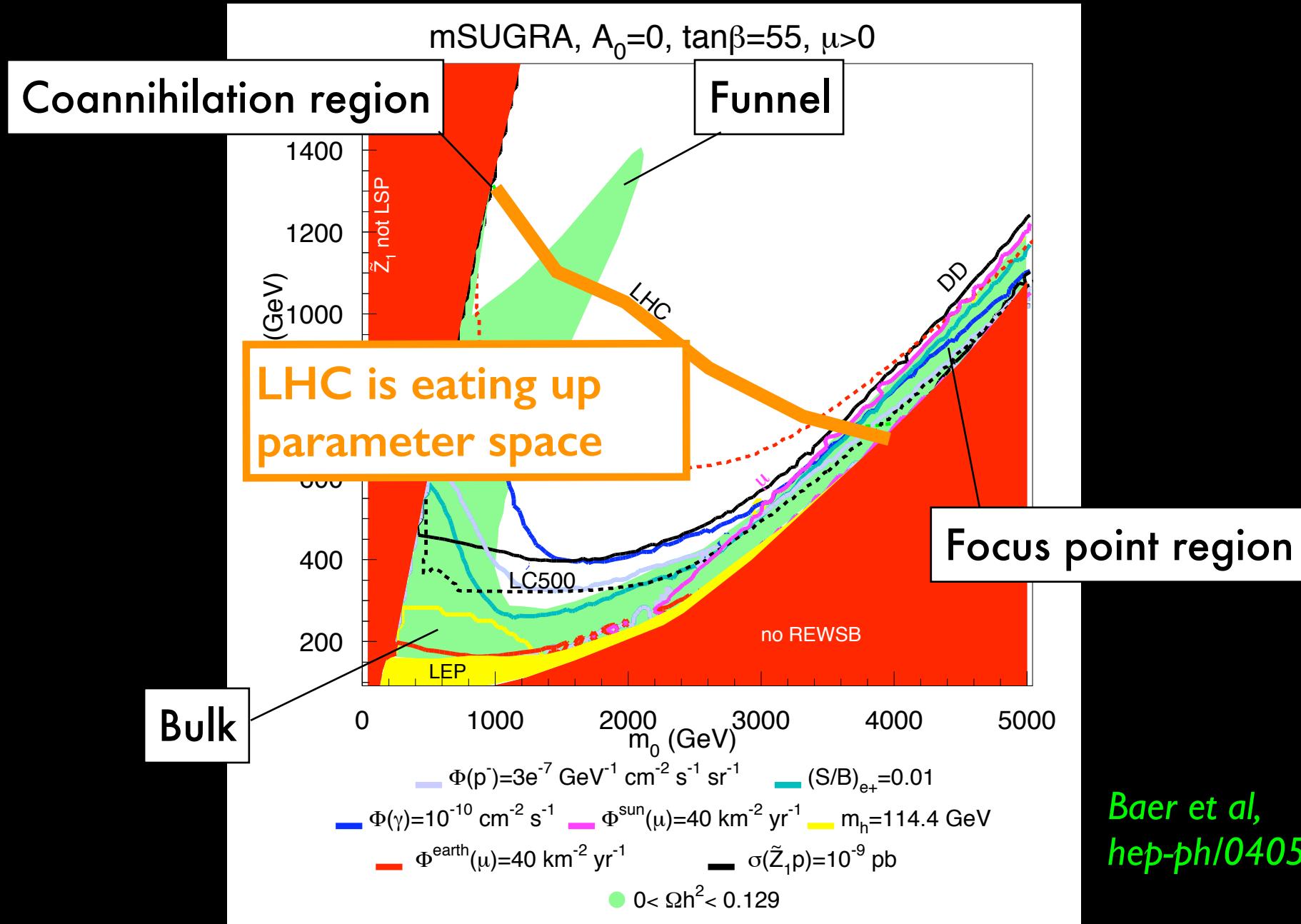


Narrow regions of  $\Omega_\chi h^2$  within the WMAP range in minimal supergravity (mSUGRA)

Edsjo et al 2003

# Neutralino dark matter: minimal supergravity

Only in special regions the density is not too large.



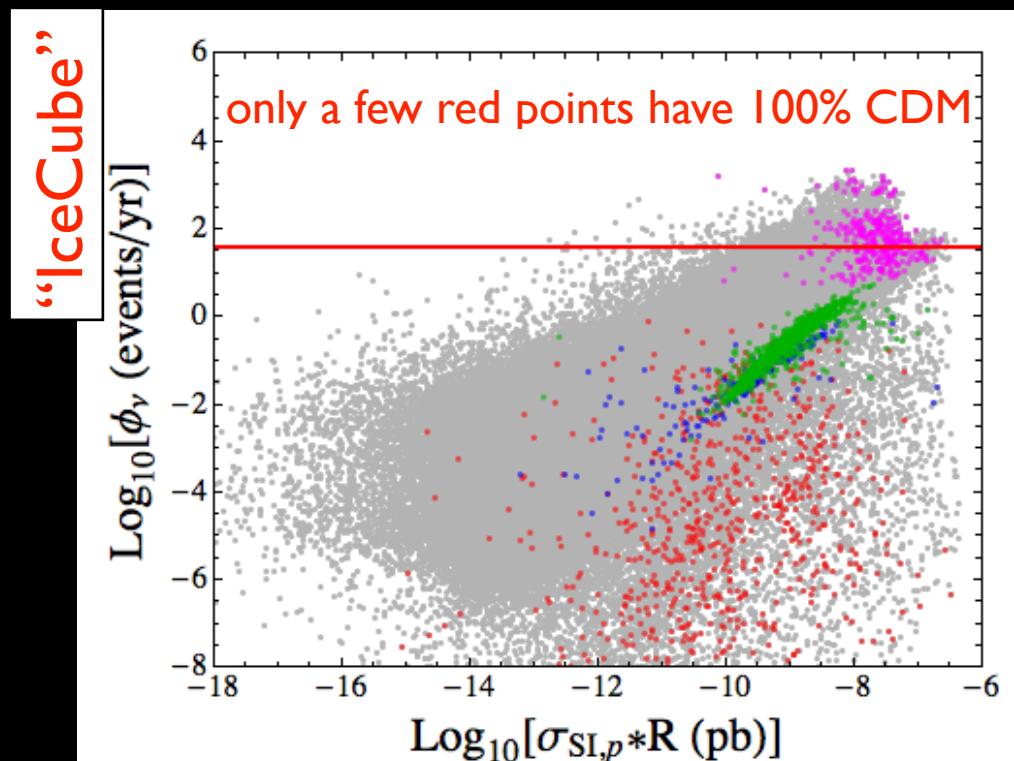
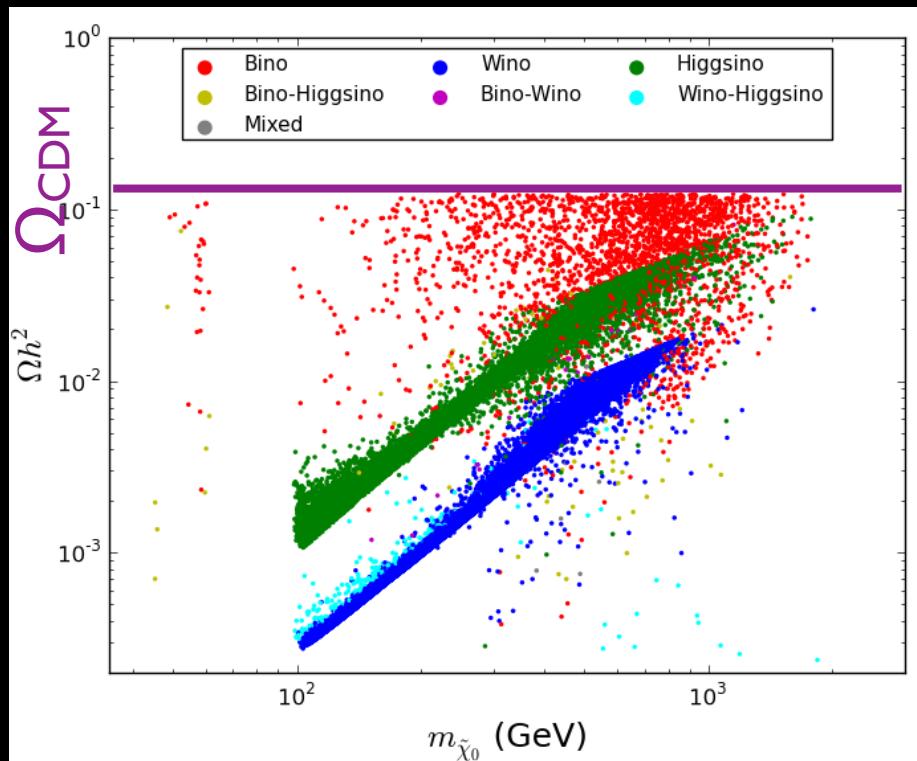
# Neutralino dark matter: impact of LHC

Cahill-Rowell et al 1305.6921

“the only pMSSM models remaining [with neutralino being 100% of CDM] are those with bino coannihilation”

pMSSM (phenomenological MSSM)

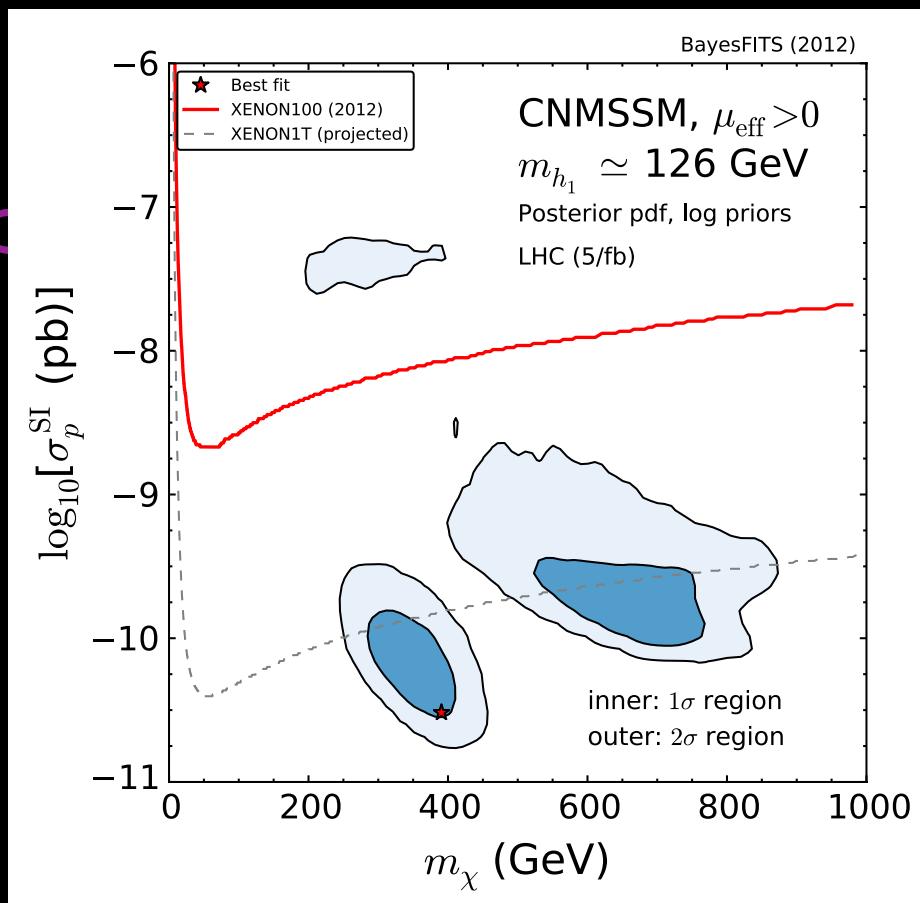
$\mu, m_A, \tan \beta, A_b, A_t, A_\tau, M_1, M_2, M_3,$   
 $m_{Q_1}, m_{Q_3}, m_{u_1}, m_{d_1}, m_{u_3}, m_{d_3},$   
 $m_{L_1}, m_{L_3}, m_{e_1}, m_{e_3}$   
(19 parameters)



# Neutralino dark matter: impact of LHC

Kowalska et al 1211.1693 [PRD 87(2013)115010]

CNMSSM: Alive and well!



NMSSM (Next-to-MSSM)

$$W = \lambda S H_u H_d + \frac{\kappa}{3} S^3 + (\text{MSSM Yukawa terms}),$$

$$\begin{aligned} V_{\text{soft}} = & m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \\ & + \left( \lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{H.c.} \right), \end{aligned}$$

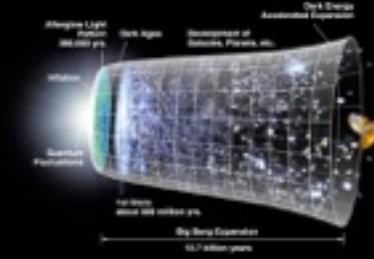
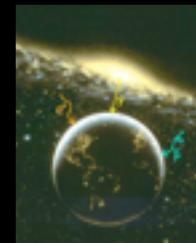
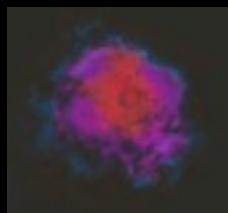
Constrained NMSSM

$m_0, m_{1/2}, A_0, \tan \beta, \lambda, \text{sgn}(\mu_{\text{eff}}),$   
*GUT & radiative EWSB*

Marginalized 2D posterior PDF  
of global analysis including LHC,  
WMAP,  $(g-2)_\mu$ ,  $B_s \rightarrow \mu^+ \mu^-$  etc.

# Axions

*Indirect detection*



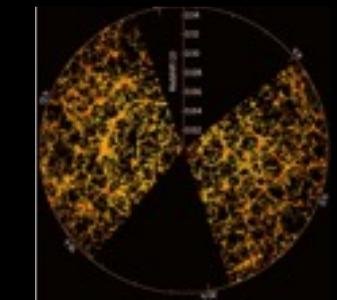
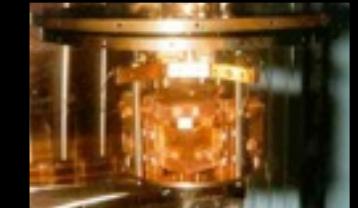
*Cosmic density*

*Annihilation*



$f$

*Direct detection*



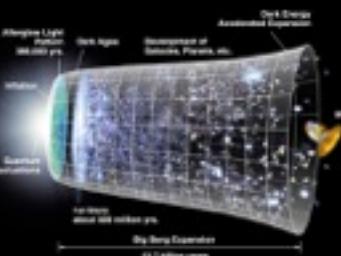
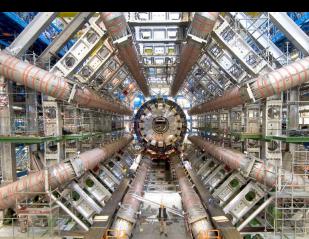
*Large scale structure*

# The power of the WIMP hypothesis

$\chi^-$

$(\bar{f})$

*Production*



*Cosmic density*

*Colliders*

# Axions as solution to the strong CP problem

## *The strong CP problem*

In QCD, the *neutron electric dipole moment*  $d_n$  should be  $\sim 10^{-16}$  ecm, but experimentally  $d_n < 1.1 \times 10^{-26}$  ecm

## *The Peccei-Quinn solution*

Introduce a new  $U(1)_{\text{PQ}}$  symmetry and a new field to break it spontaneously. The remaining pseudoscalar Goldstone boson is the axion. It acquires mass through QCD instanton effects.

# Axions as solution to the strong CP problem

## *The strong CP problem*

Vacuum potentials  $A_\mu = i\Omega \partial_\mu \Omega^{-1}$  with  $\Omega \rightarrow e^{2\pi i n}$  as  $r \rightarrow \infty$

Vacuum state  $|\theta\rangle = \sum_n e^{-in\theta} |0\rangle$

New term in lagrangian  $\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$

$\mathcal{L}_\theta$  violates P and T but conserves C, thus produces a neutron electric dipole moment  $d_n \approx e(m_q/M_n^2)\theta$

Experimentally  $d_n < 1.1 \times 10^{-26}$  ecm so  $\theta < 10^{-9}$ - $10^{-10}$

Why  $\theta$  should be so small is the strong CP problem

# Axions as solution to the strong CP problem

## The Peccei-Quinn solution

Introducing a  $U(1)_{\text{PQ}}$  symmetry replaces

$$\theta_{\text{total}} = \theta + \arg \det M_{\text{quark}} \Rightarrow \theta(x) = a(x)/f_a$$

*static CP-violating angle*

axion

*dynamic CP-conserving field*

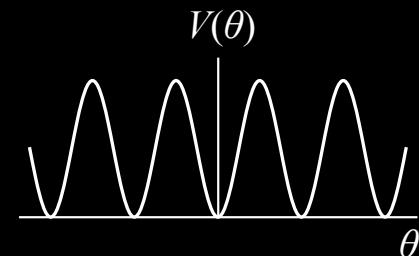
$$\text{New lagrangian } \mathcal{L}_a = -\frac{1}{2}\partial^\mu a\partial_\mu a + \frac{a}{f_a} \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} + \mathcal{L}_{\text{int}}(a)$$

Before QCD phase transition,  $\langle \theta \rangle$  can be anything

After QCD phase transition, instanton effects generate

$$V(\theta) = m_a^2 f_a^2 (1 - \cos \theta)$$

and  $\langle \theta \rangle = 0$  dynamically



Wilczek realized this leads to a very light pseudoscalar particle he called the “axion” after the name of a famous laundry detergent

# Axions



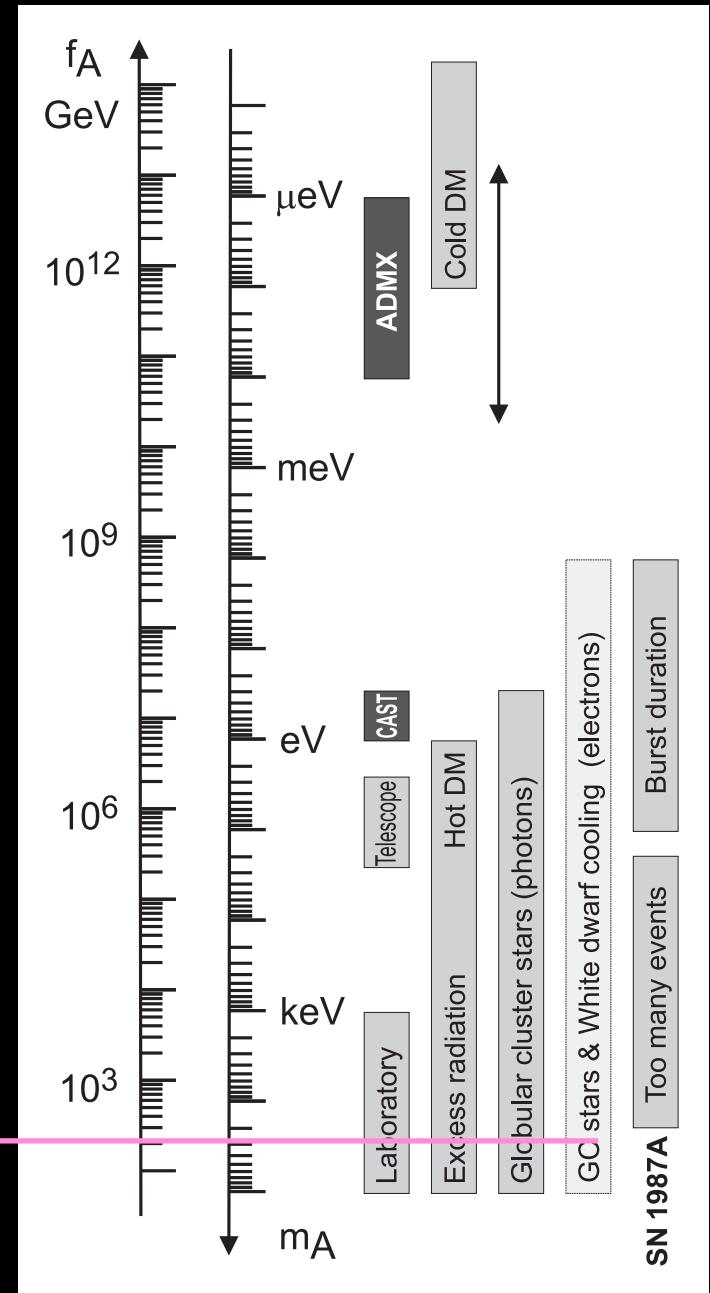
“Whenever you come up with a good idea,  
somebody tries to copy it.”

(Axion Commercial with Arthur Godfrey, 1968)

# Axions as solution to the strong CP problem

## Constraints from laboratory searches and astrophysics

Peccei & Quinn had 2 Higgs doublets and  $f_a \sim 200$  GeV (electroweak), with an axion-quark coupling too high and quickly excluded by laboratory searches



# Axions as solution to the strong CP problem

## Beyond Peccei-Quinn: the *invisible axion*

Kim (1979)

Shifman, Vainshtein, Zakharov (1980)

1 Higgs doublets, 1 Higgs singlet,  
1 exotic quark ( $SU(2)_w$ -singlet  $SU(3)_c$ -triplet)

$$\mathcal{L}_y = f \bar{Q}_L \sigma Q_R + f^* \bar{Q}_R \sigma^* Q_L$$

Judicious choice of  $U(1)_{\text{PQ}}$  charges

$$V(\varphi, \sigma) = -\mu_\varphi^2 \varphi^+ \varphi - \mu_\sigma^2 \sigma^* \sigma + \lambda_\varphi (\varphi^+ \varphi)^2 + \lambda_\sigma (\sigma^* \sigma)^2 + \lambda_{\varphi\sigma} \varphi^+ \varphi \sigma^* \sigma.$$

Axion not coupled to quarks at tree level

Zhitnitski (1980)

Dine, Fischler, Srednicki (1981)

2 Higgs doublets, 1 Higgs singlet

$$\mathcal{L}_Y = G_u (\bar{u} \bar{d})_L \phi_u u_R + G_d (\bar{u} \bar{d})_L \phi_d d_R + \text{h.c.}$$

Judicious choice of  $U(1)_{\text{PQ}}$  charges

$$V(\phi, \phi_u, \phi_d) = \lambda_u (|\phi_u|^2 - V_u^2)^2 + \lambda_d (|\phi_d|^2 - V_d^2)^2 + \lambda (|\phi|^2 - V^2)^2 + (a |\phi_u|^2 + b |\phi_d|^2) |\phi|^2 + c (\phi_u^i \epsilon_{ij} \phi_d^j \phi^2 + \text{h.c.}) + d |\phi_u^i \epsilon_{ij} \phi_d^j|^2 + e |\phi_u^* \phi_d|^2. \quad (5)$$

Axion-quark couplings suppressed by  $200 \text{ GeV}/\langle \phi \rangle \ll 1$

# Axions as solution to the strong CP problem

*Beyond Peccei-Quinn: the invisible axion*

Model-dependent axion-photon coupling

$$L_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a} (C - C') a \mathbf{E} \cdot \mathbf{B}$$

$$C' = \frac{2}{3} \frac{m_u m_d + 4m_d m_s + m_s m_u}{m_u m_d + m_d m_s + m_s m_u} = 1.93 \pm 0.04$$

$$C_{\text{DFSZ}} = \frac{8}{3}$$

$$C_{\text{KSVZ}} = 6Q^2$$

Model-dependent axion-fermion coupling

$$\mathcal{L}_{Aff} = \frac{C_f}{2f_A} \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f \partial_\mu \phi$$

$$C_e^{\text{DFSZ}} = \frac{\cos^2 \beta}{3}$$

$$C_e^{\text{KSVZ}} \ll 1$$

See e.g. Srednicki hep-th/0210172, Review of Particle Properties

# Axions as dark matter

## Hot

Produced thermally in early universe

*Important for  $m_a > 0.1 \text{ eV}$  ( $f_a < 10^8$ ), mostly excluded by astrophysics*

## Cold

Produced by coherent field oscillations around minimum of  $V(\theta)$   
*(Vacuum realignment)*

Produced by decay of topological defects

*(Axionic string decays)*

Still a very complicated and  
uncertain calculation!  
e.g. Harimatsu et al 2012

# Axion cold dark matter parameter space

Kim-Shifman-Vainshtein-Zakharov  
Dine-Fischler-Srednicki-Zhitnitski

$f_a$	Peccei-Quinn symmetry breaking scale
$N$	Peccei-Quinn color anomaly
$N_d$	Number of degenerate QCD vacua
	Couplings to quarks, leptons, and photons
$H_I$	Expansion rate at end of inflation
$\theta_i$	Initial misalignment angle
	Axionic string parameters

Assume  $N = N_d = 1$  and show results for KSVZ and HHCS string network

Thus 3 free parameters  $f_a$ ,  $\theta_i$ ,  $H_I$  and one constraint  $\Omega_a = \Omega_{\text{CDM}}$

# Cold axion production in cosmology

## Vacuum realignment

- Initial misalignment angle  $\theta_i$
- Coherent axion oscillations start at temperature  $T_1$

$$3H(T_1) = m_a(T_1)$$

Hubble expansion parameter  
*non-standard expansion histories*  
*differ in the function  $H(T)$*

$T$ -dependent axion mass  
*axions acquire mass through*  
*instanton effects at  $T < \Lambda \approx \Lambda_{\text{QCD}}$*

- Density at  $T_1$  is  $n_a(T_1) = \frac{1}{2}m_a(T_1)f_a^2\chi\langle\theta_i^2 f(\theta_i)\rangle$

Anharmonicity correction  $f(\theta)$   
*axion field equation has anharmonic terms*  $\ddot{\theta} + 3H(T)\dot{\theta} + m_a^2(T) \sin \theta = 0$

- Conservation of comoving axion number gives present density  $\Omega_a$

# Cold axion production in cosmology

## Axionic string decays

- Energy density ratio (string decay/misalignment)

$$\alpha \equiv \frac{\rho_a^{\text{str}}}{\rho_a^{\text{mis}}} = \frac{\xi \bar{r} N_d^2}{\zeta}$$

The diagram illustrates the components of the energy density ratio  $\alpha$ . It shows three boxes connected by arrows pointing to the right: a top-left box labeled '(String stretching rate)<sup>-2</sup>', a top-right box labeled 'Density enhancement from string decays', and a bottom-right box labeled 'Uncertainty in axion spectrum'. Arrows point from each of these boxes to the right side of the central equation.

Slow-oscillating strings (Davis-Battye-Shellard)

$$\bar{r} = \frac{1-\beta}{3\beta-1} \ln(t_1/\delta)$$

Fast-oscillating strings (Harari-Hagmann-Chang-Sikivie)

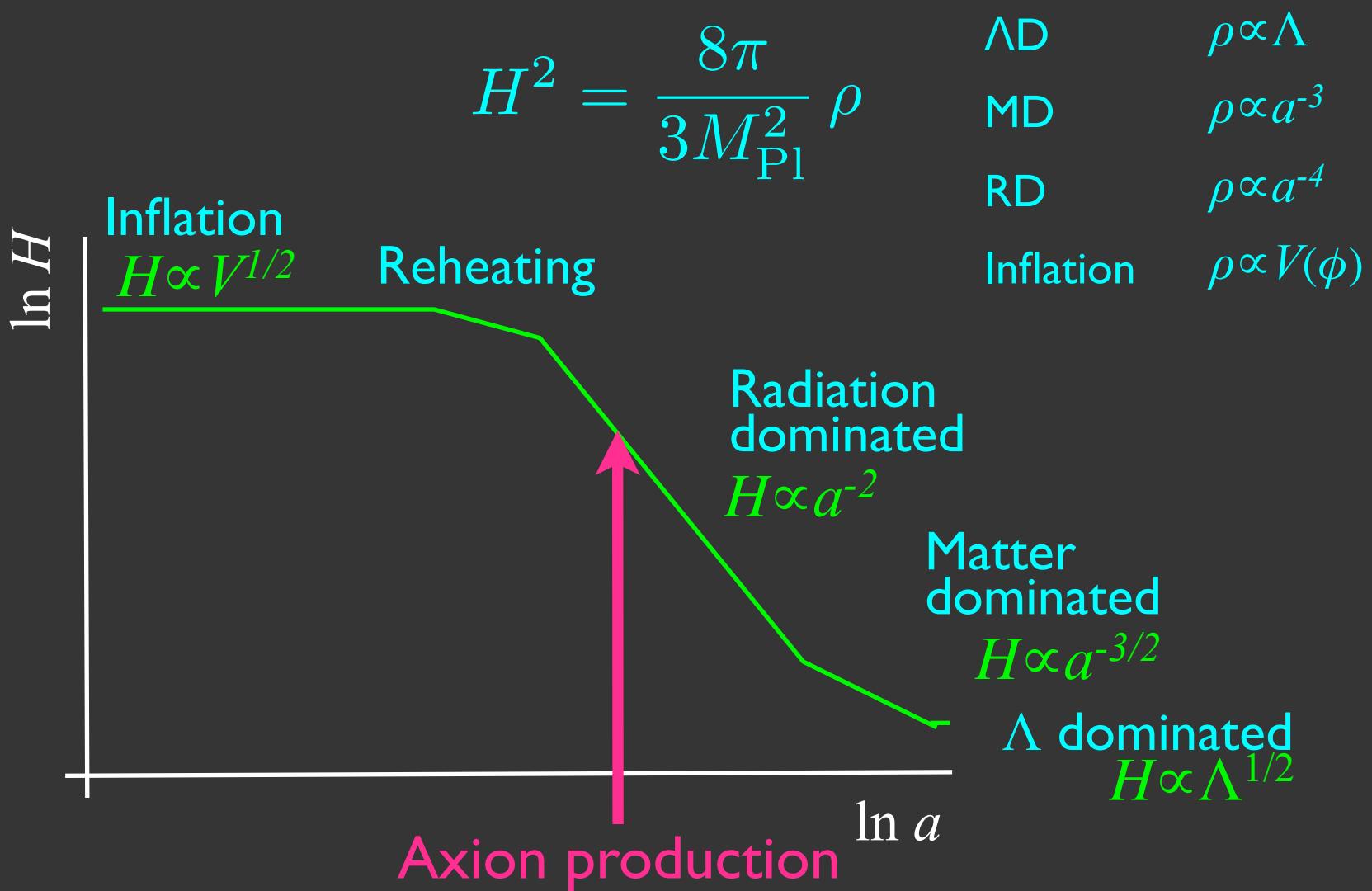
$$\bar{r} = \frac{1-\beta}{3\beta-1} 0.8$$

$$\xi = \frac{1}{4c^2} \left( 2 - 3\beta + \sqrt{(4c+0)\beta^2 - 12\beta + 4} \right)^2$$

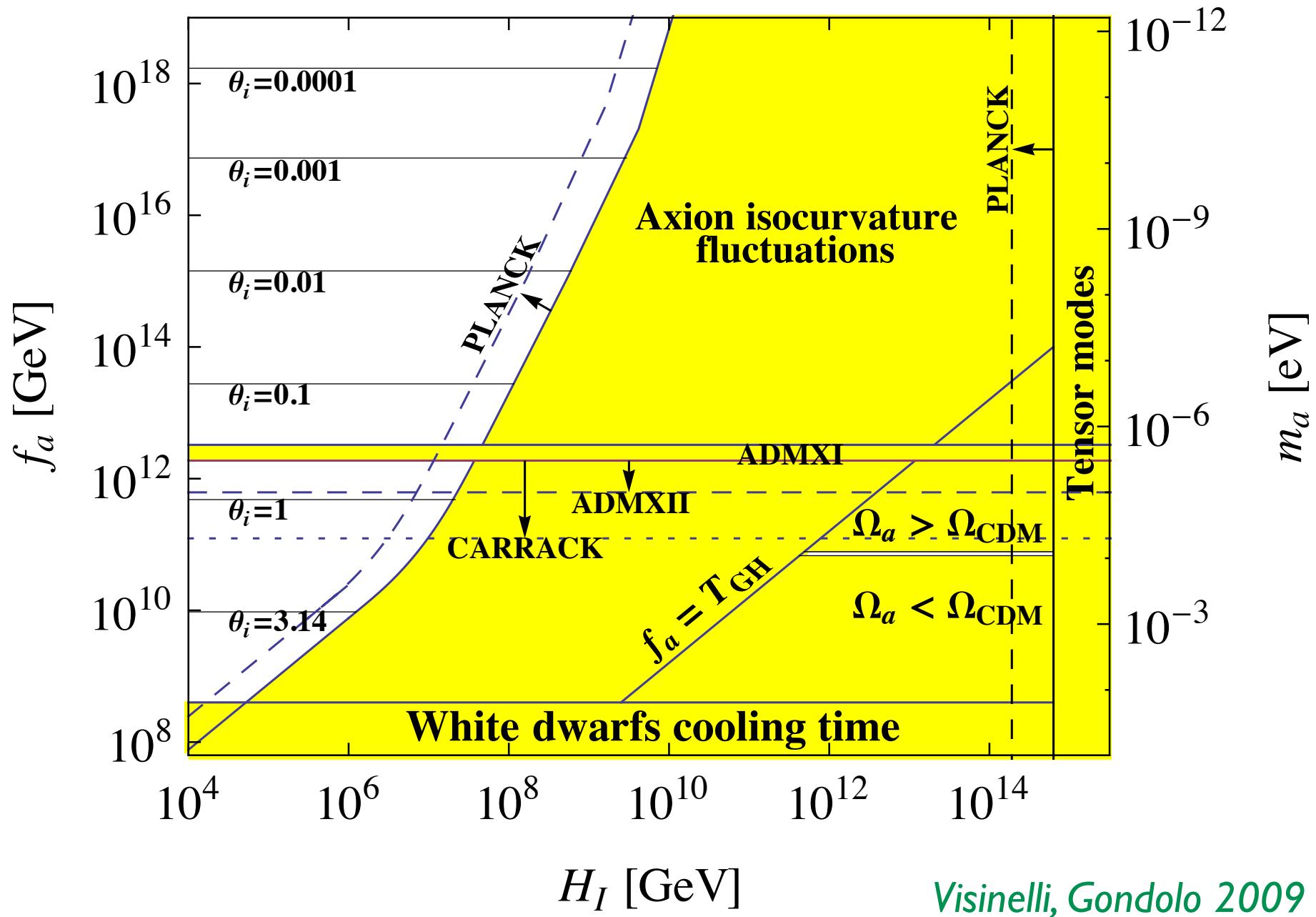
with  $a(t) \propto t^\beta$

$$c = (1 + 2\sqrt{\xi^{\text{std}}})/(4\xi^{\text{std}})$$

# Standard cosmology



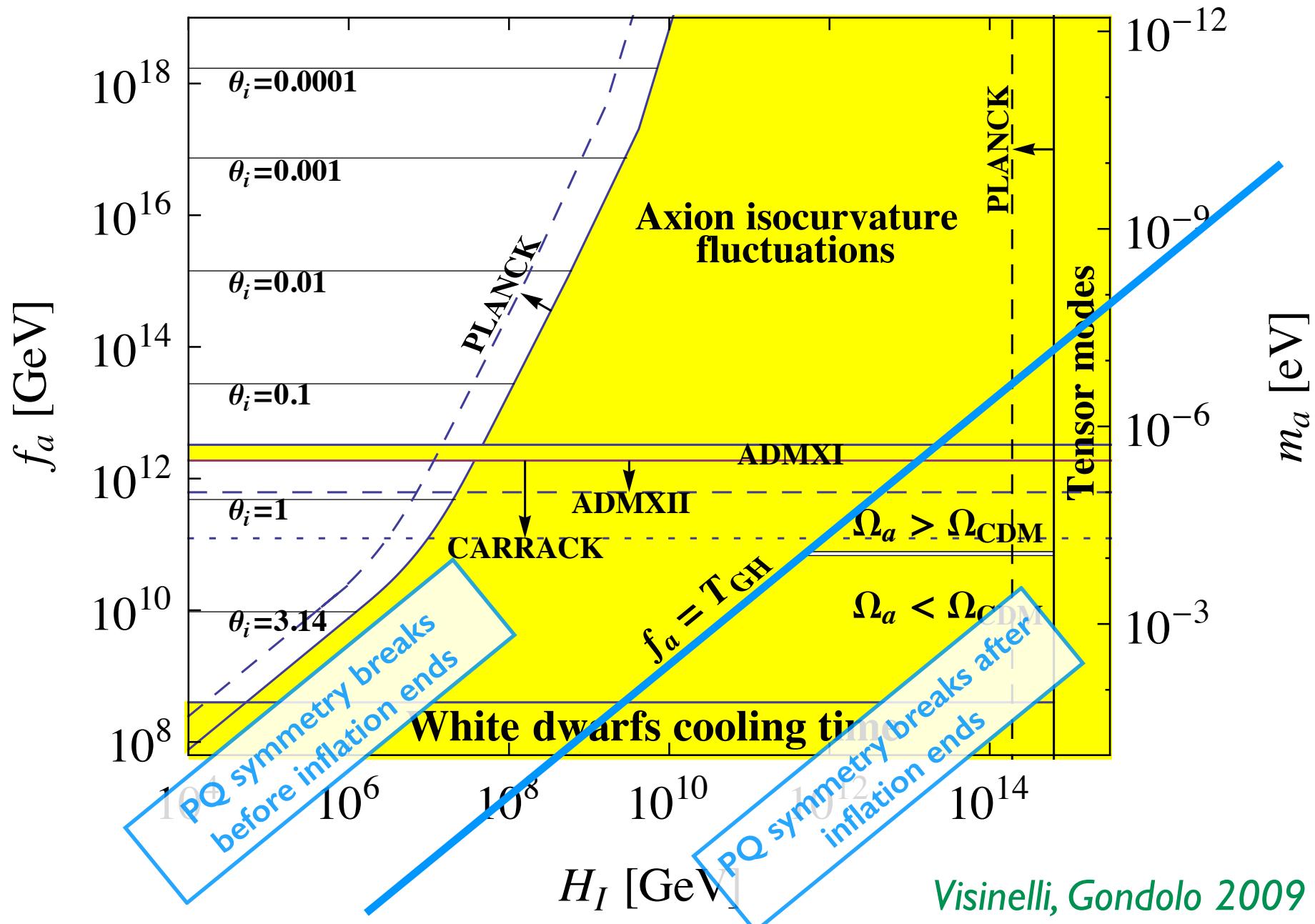
# Axion CDM - Standard cosmology



$H_I$  [GeV]

Visinelli, Gondolo 2009

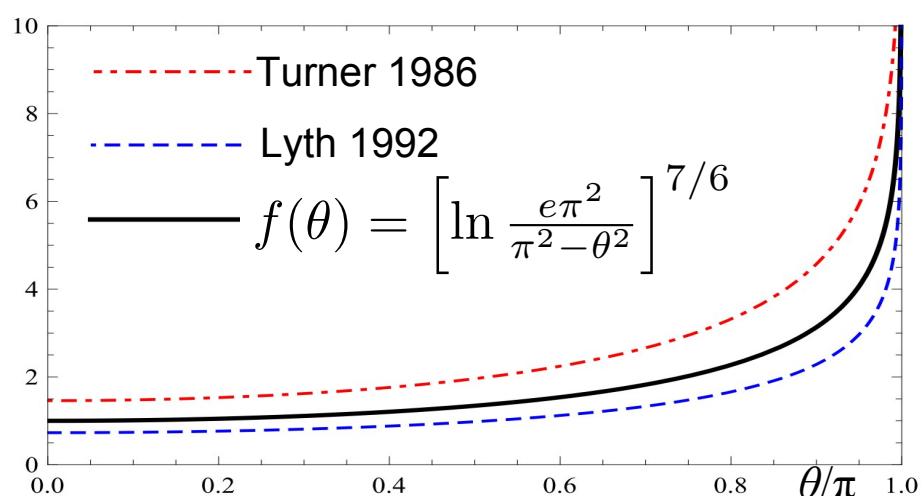
# Axion CDM - Standard cosmology



# Axion CDM - Standard cosmology

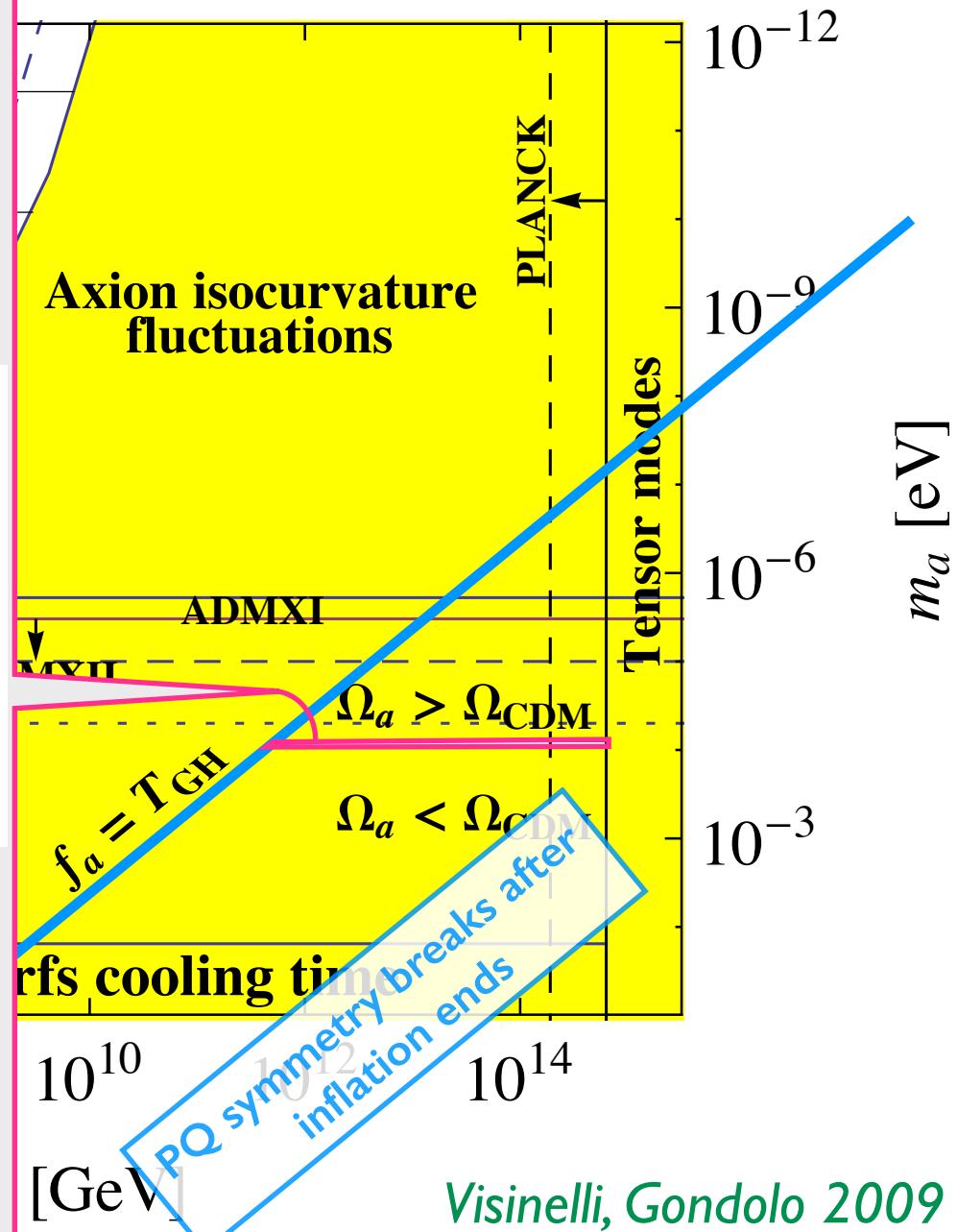
PQ symmetry breaks  
after inflation ends

- Average  $\theta_i$  over Hubble volume
- Anharmonicities are important



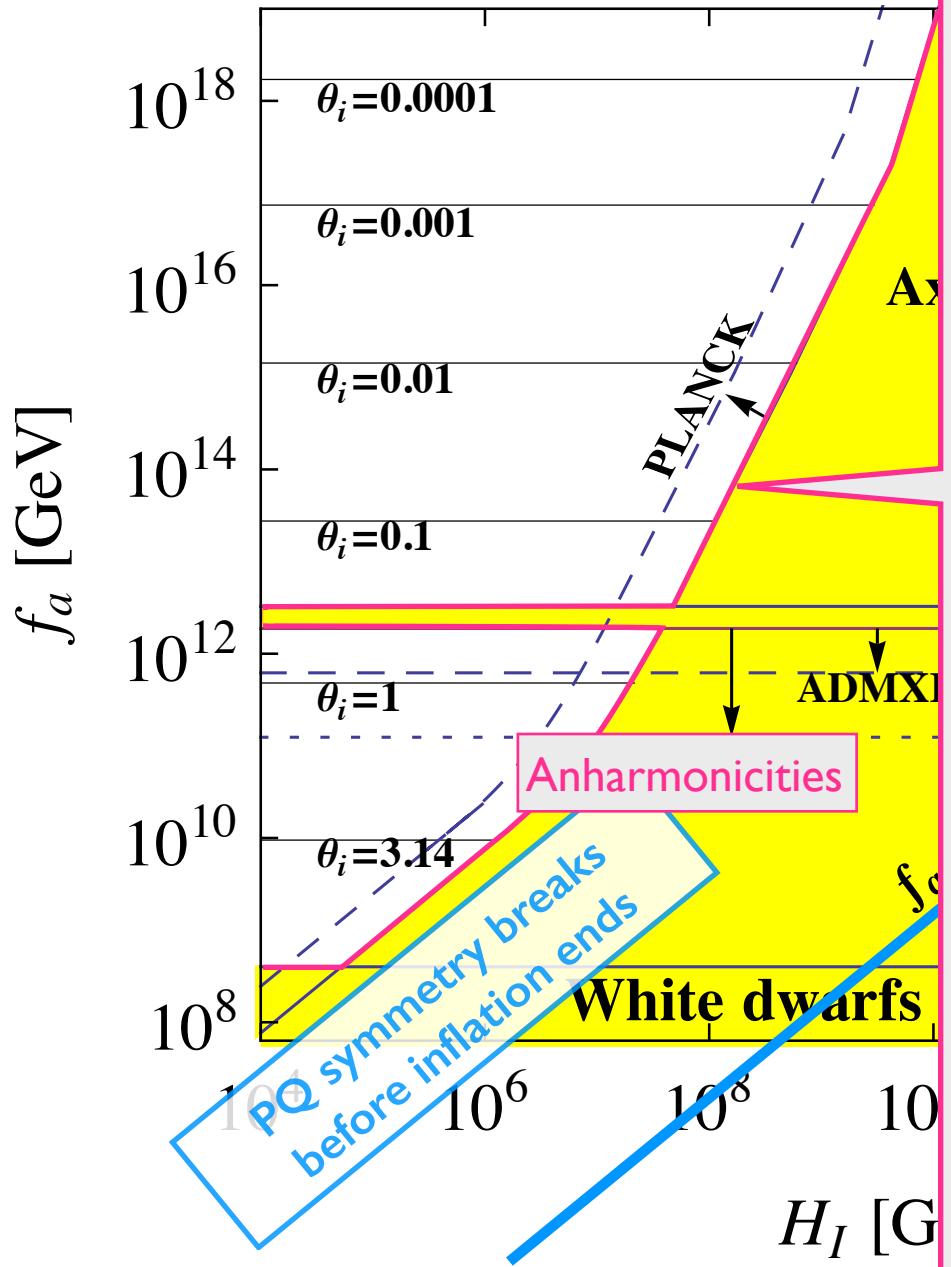
$$\langle \theta_i^2 f(\theta_i) \rangle = (2.96)^2$$

- String decay contribution is ~16% of vacuum realignment



Visinelli, Gondolo 2009

# Axion CDM - Standard cosmology



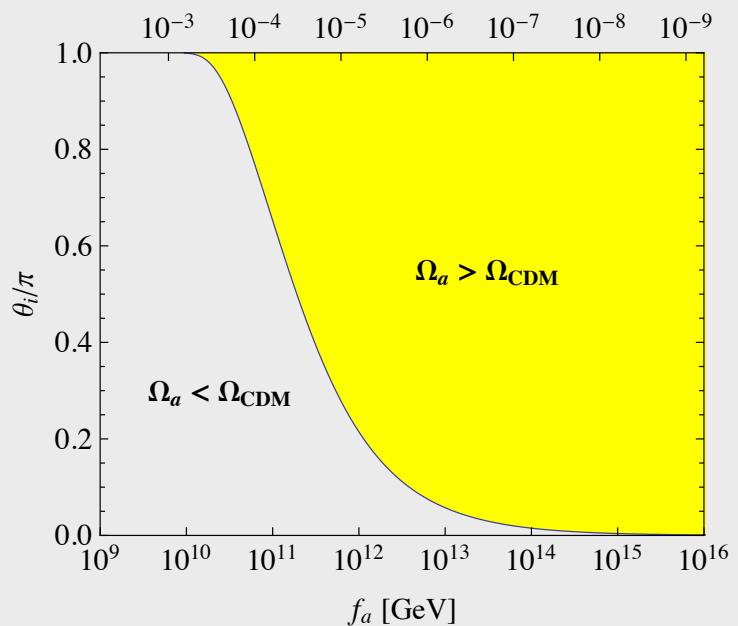
*PQ symmetry breaks  
before inflation ends*

- Constrained by non-adiabatic fluctuations

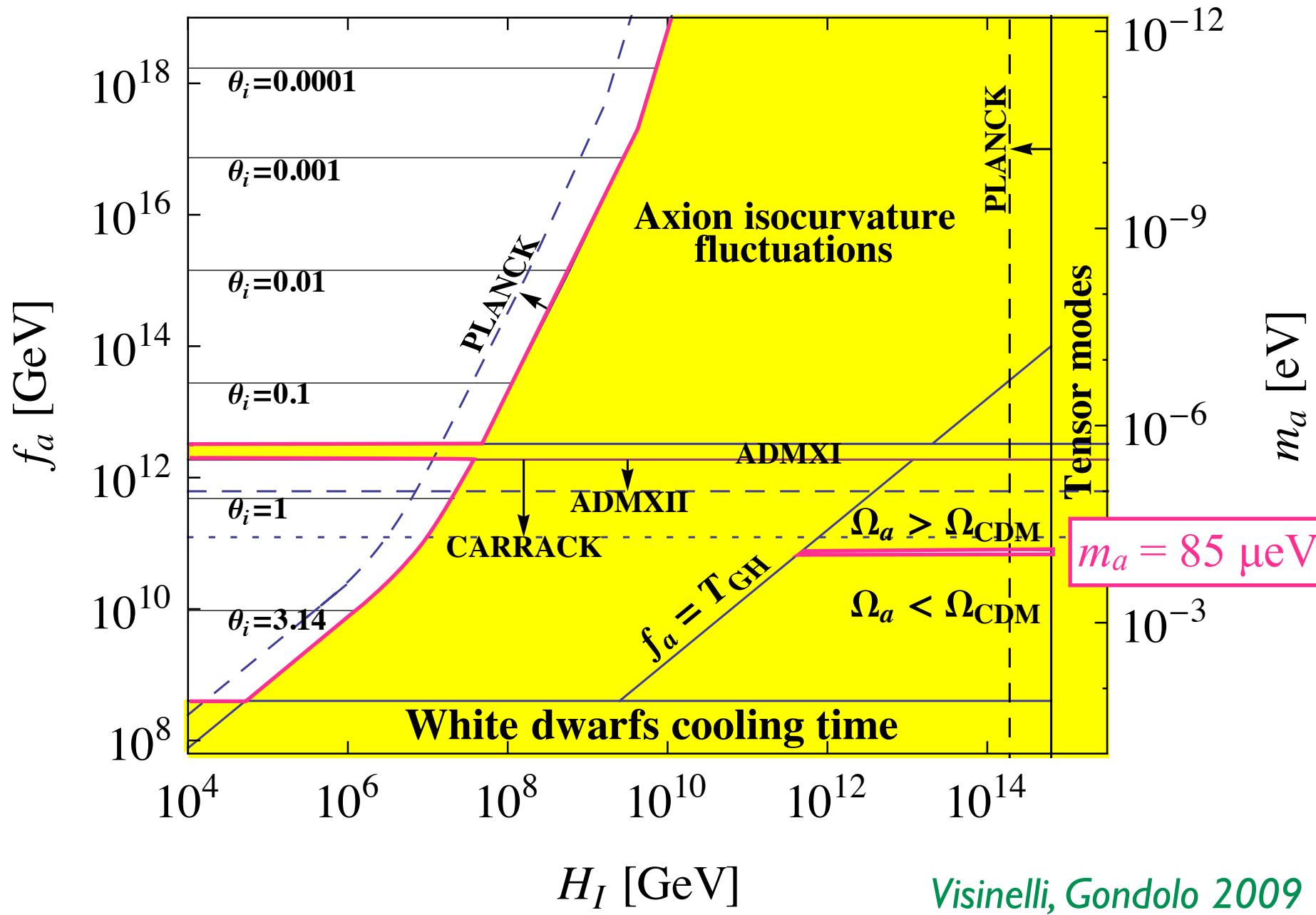
$$\frac{H_I}{\theta_i f_a} < 4.2 \times 10^{-5} \quad \text{WMAP7 96\% CL}$$

- Single value of  $\theta_i$  throughout Hubble volume

$$\langle \theta_i^2 f(\theta_i) \rangle = \left[ \theta_i^2 + \left( \frac{H_I}{2\pi f_a} \right)^2 \right] f(\theta_i)$$



# Axion CDM - Standard cosmology



$H_I$  [GeV]

Visinelli, Gondolo 2009

# **Sterile neutrinos**

# Active-sterile neutrino mixing

Standard model + right-handed neutrinos

$$-\mathcal{L}_m = y_\nu v \bar{\nu}_L \nu_R + \frac{1}{2} M \bar{\nu}_R^c \nu_R + \text{h.c.} = \frac{1}{2} \begin{bmatrix} \bar{\nu}_L^c & \bar{\nu}_R \end{bmatrix} \begin{bmatrix} 0 & y_\nu v \\ y_\nu v & M \end{bmatrix} \begin{bmatrix} \bar{\nu}_L \\ \bar{\nu}_R^c \end{bmatrix} + \text{h.c.}$$

Neutrino mass eigenstates are obtained by diagonalization

$$-\mathcal{L}_m = \frac{1}{2} m_a \bar{\nu}_a \nu_a + \frac{1}{2} m_s \bar{\nu}_s \nu_s$$

$$\begin{cases} \nu_a = \cos \theta \nu_L - \sin \theta \nu_R^c \\ \nu_s = \sin \theta \nu_L + \cos \theta \nu_R^c \end{cases}$$

 mixing angle  $\theta$

# Active-sterile neutrino mixing

If  $y_\nu v \ll M$ , then  $m_s \simeq M$ ,  $m_a \simeq \frac{y_\nu^2 v^2}{M} \ll M$ ,  $\theta \simeq \frac{y_\nu v}{M} \ll 1$

seesaw mechanism

$\nu_a$  are  $\approx$ LH, light, with tree-level couplings (active neutrinos)

$\nu_s$  are  $\approx$ RH, heavy, with no tree-level coupling (sterile neutrinos)

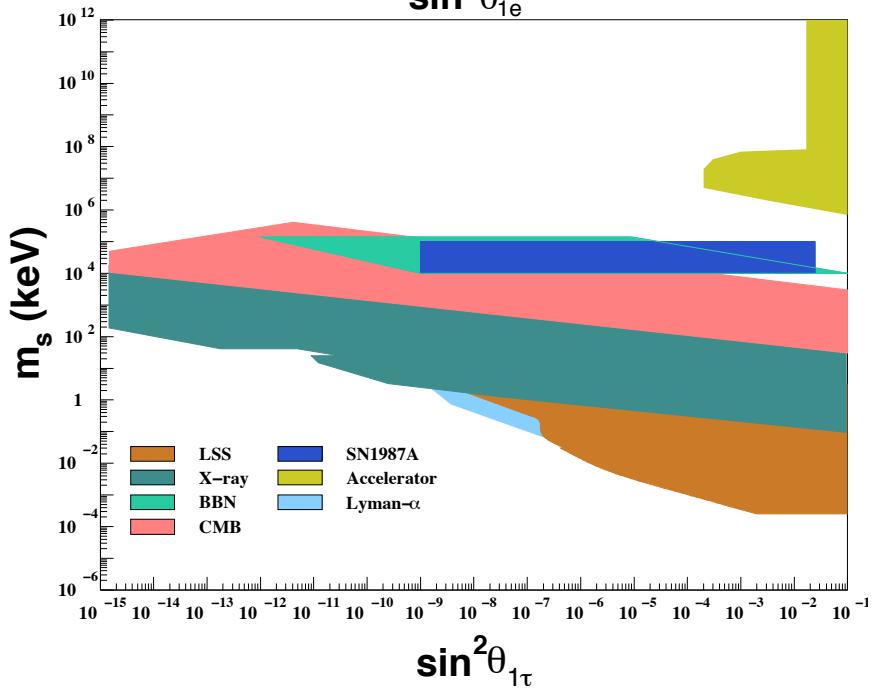
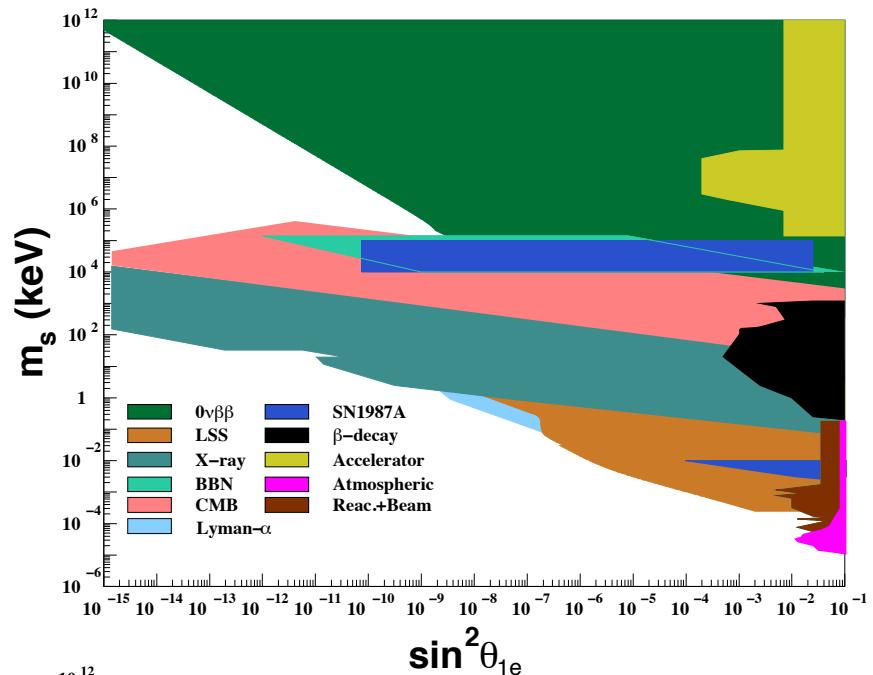
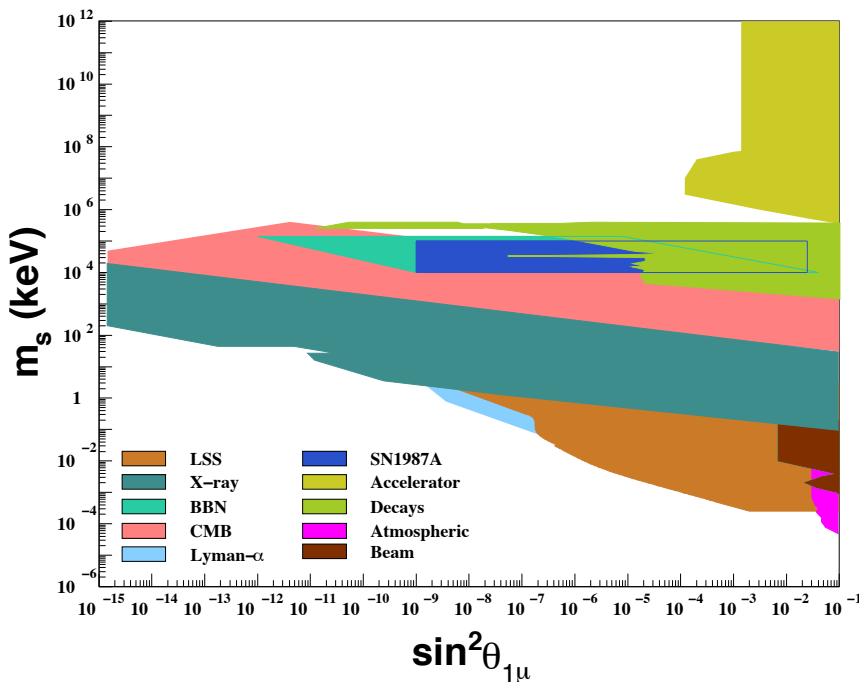
Neutrinos produced in weak interactions are left-handed, while mass eigenstates contain a (tiny) right-handed component

Oscillations between active and sterile neutrinos

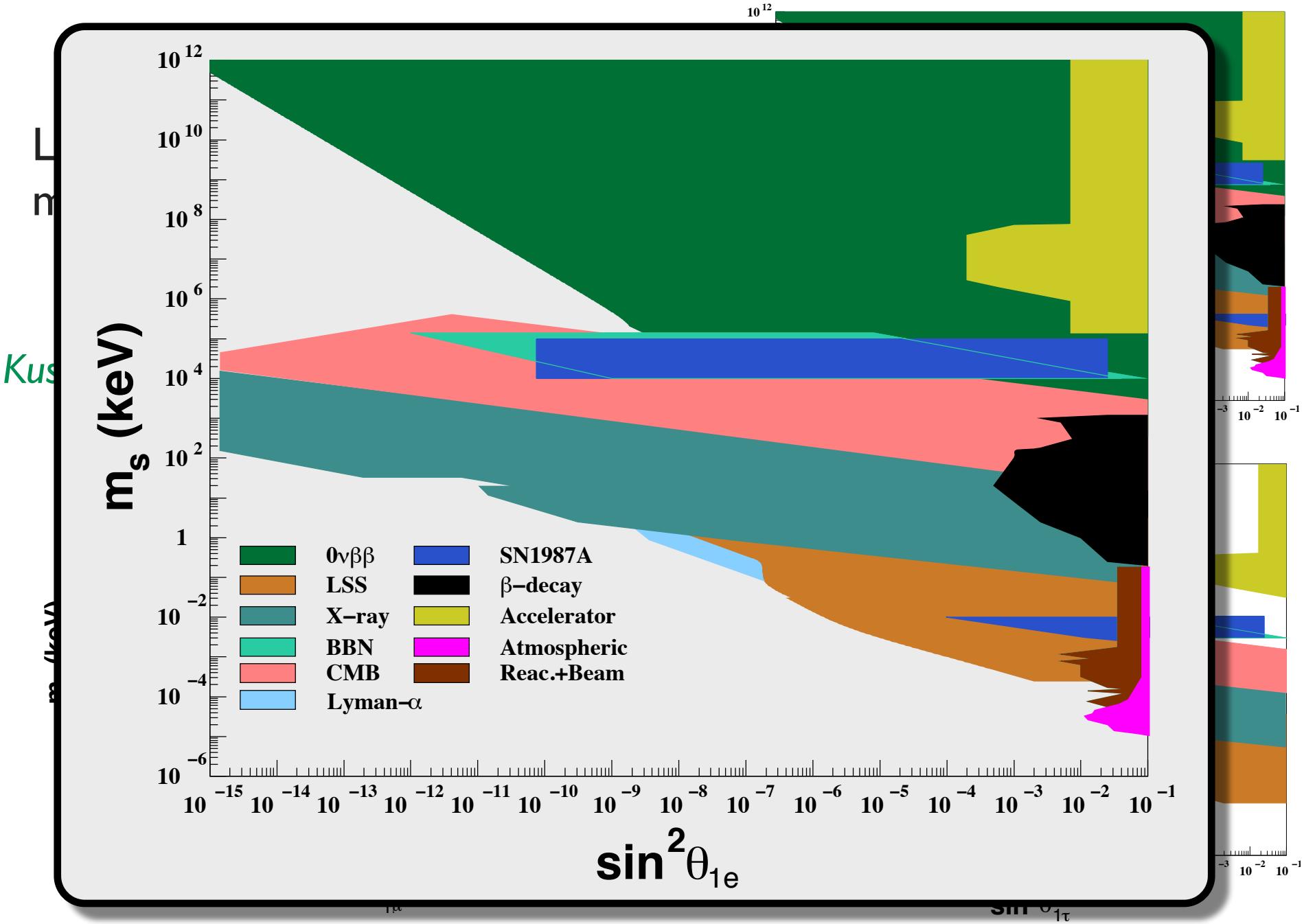
# Neutrino mixing

Limits on sterile neutrino mixing with  $\nu_e, \nu_\mu, \nu_\tau$

Kusenko 0906.2968

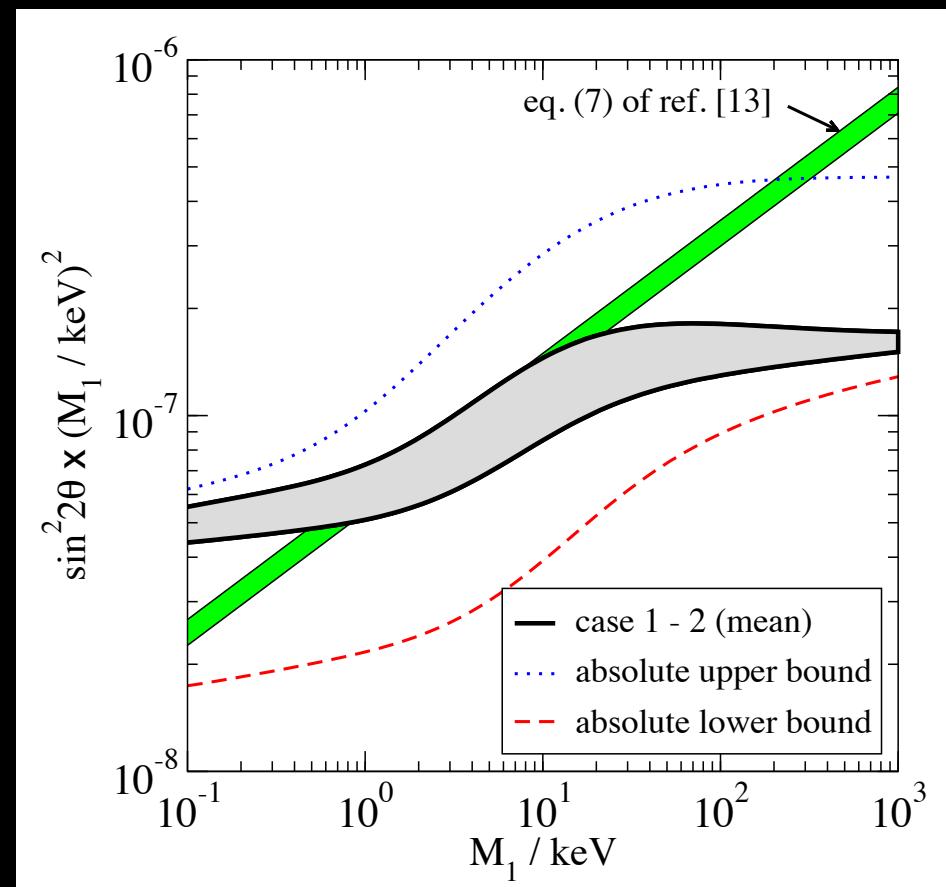


# Neutrino mixing



# Sterile neutrino dark matter

- Mass > 0.3 keV (Tremaine-Gunn bound)
- Sterile neutrinos are produced from oscillations of active neutrinos in the early universe ( $T \sim 100$  MeV) *Dodelson, Widrow 1994*
- In the presence of a large lepton asymmetry, oscillation production is enhanced *Shi, Fuller 1999*
- In a model with three generations of sterile neutrinos ( $\nu$ MSM), decay of the two heavy neutrinos can generate a lepton asymmetry then converted to baryon asymmetry, and the light sterile neutrino can be the dark matter  
*Laine, Shaposhnikov 2008*



*Asaka, Laine, Shaposhnikov 2007*

# Limits on sterile neutrino dark matter

The main decay mode of keV sterile neutrinos ( $\nu_s \rightarrow 3\nu$ ) is undetectable

Radiative decay of sterile neutrinos  $\nu_s \rightarrow \gamma \nu_a$

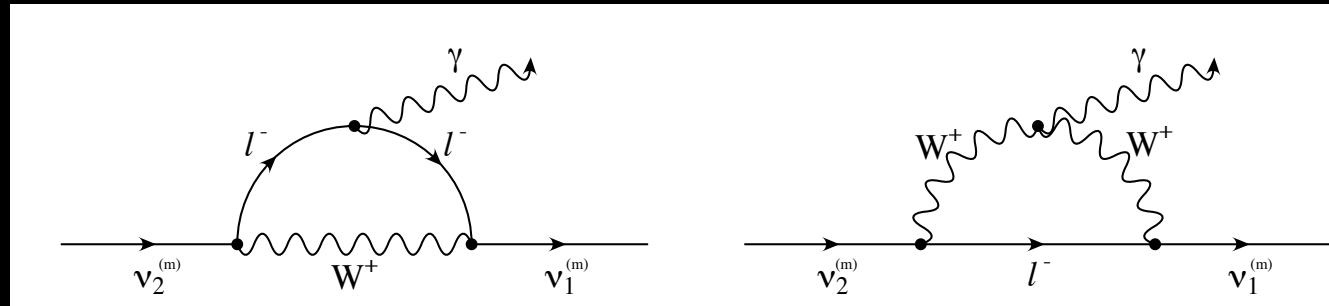


Figure from Kusenko 0906.2968

X-ray line

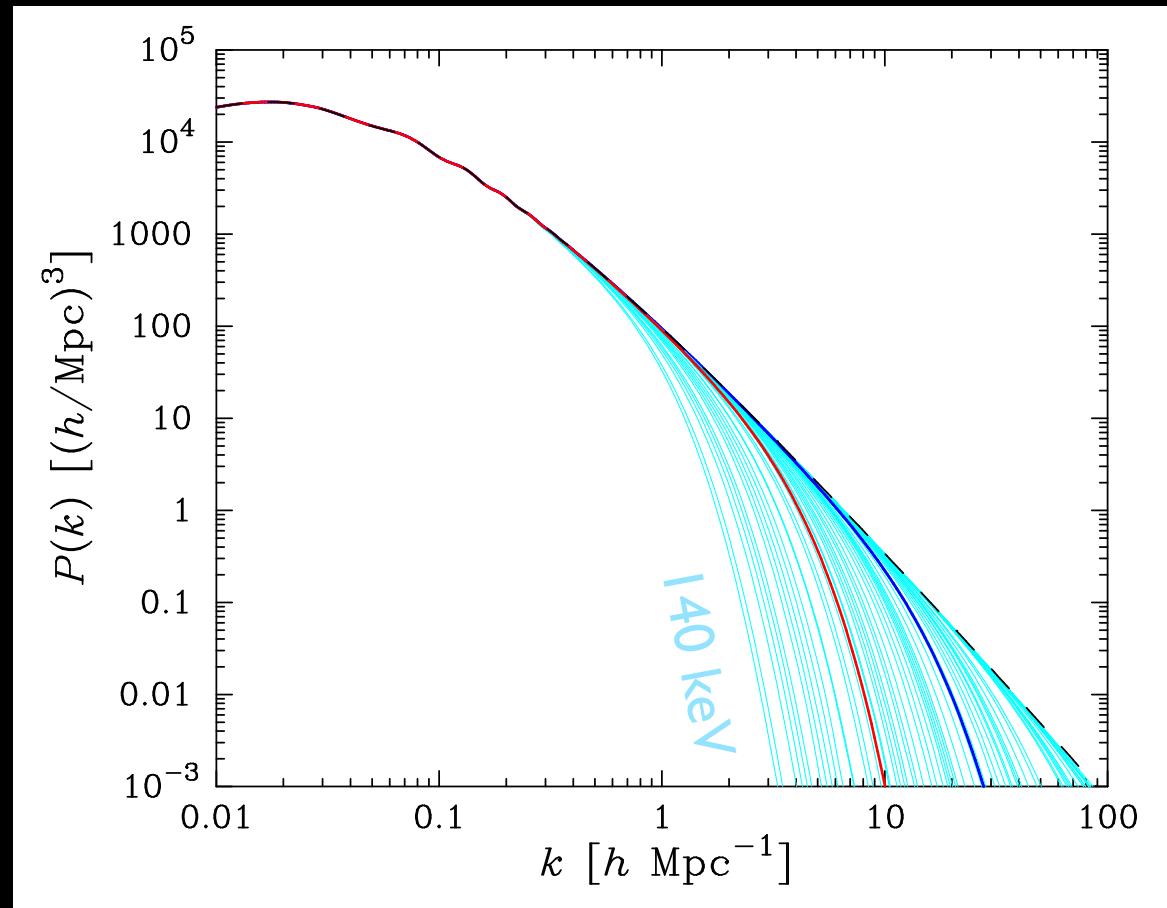
$$E_\gamma = \frac{1}{2} m_s$$

$$\begin{aligned}\Gamma_{\nu_s \rightarrow \gamma \nu_a} &= \frac{9}{256\pi^4} \alpha_{\text{EM}} G_F^2 \sin^2 \theta m_s^5 \\ &= \frac{1}{1.8 \times 10^{21} S} \sin^2 \theta \left( \frac{m_s}{\text{keV}} \right)^5\end{aligned}$$

# Limits on sterile neutrino dark matter

Sterile neutrinos are warm dark matter

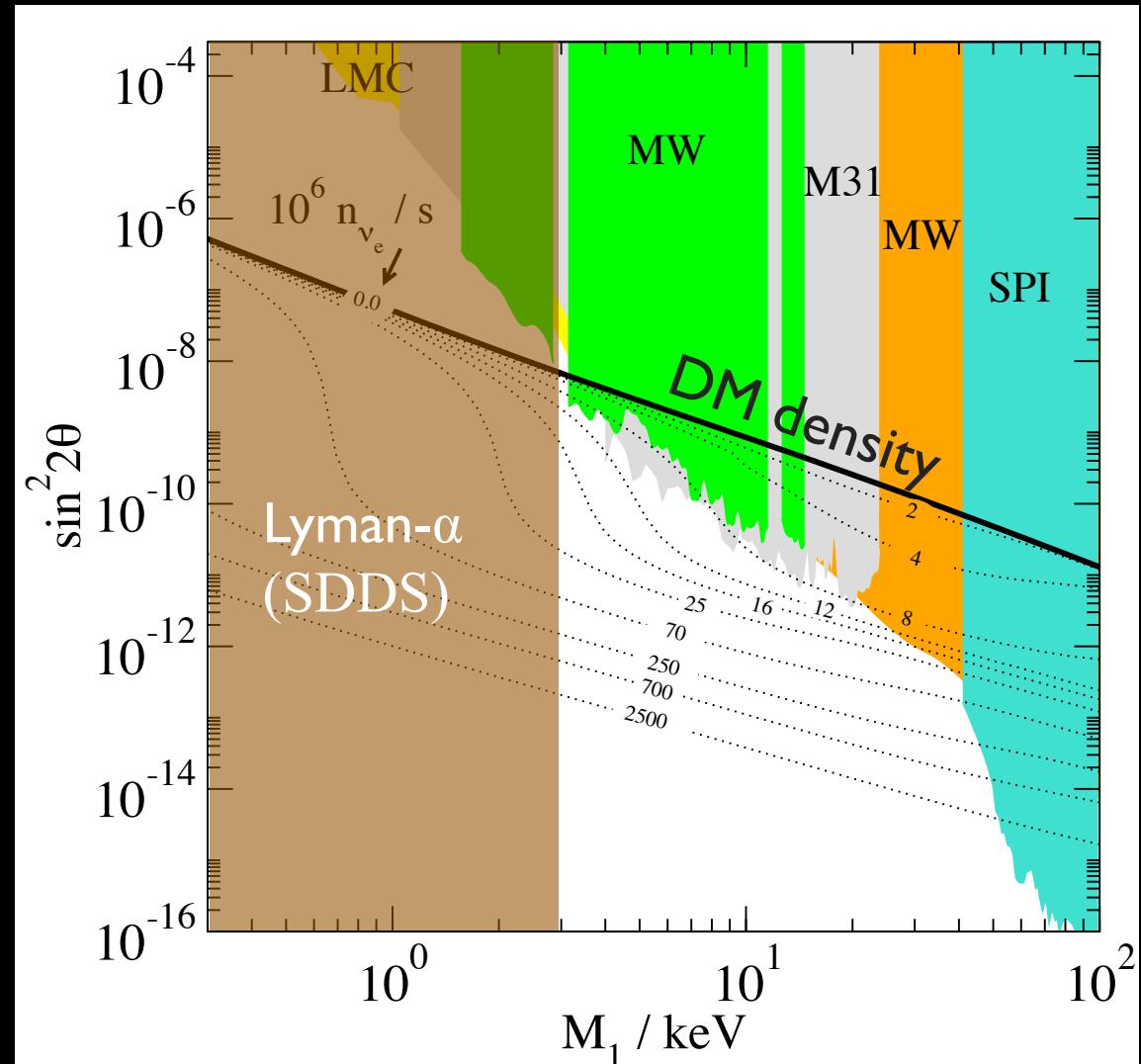
*Small scale  
structure is  
erased*



Abazajian 2005

# Limits on sterile neutrino dark matter

$\nu_{\text{MSM}}$

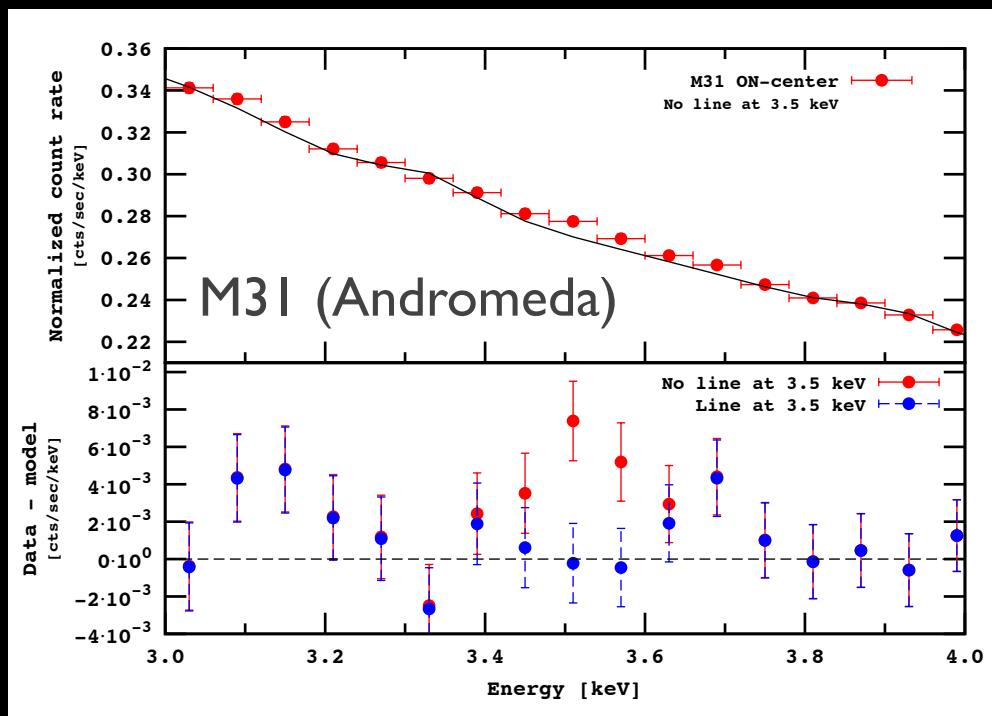


Laine, Shaposhnikov 2008

# X-rays from dark matter?

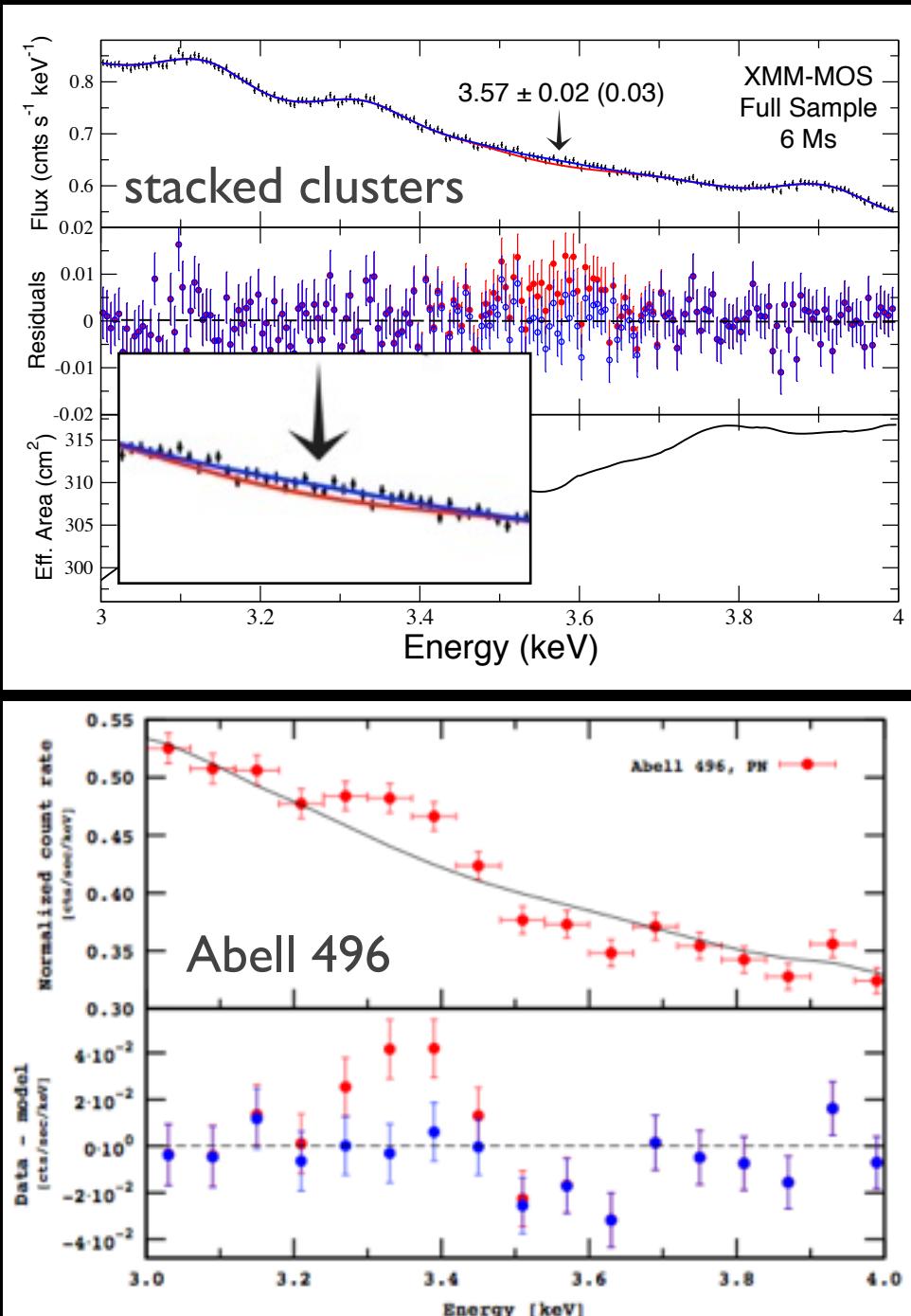
Bulbul et al 2014

An unidentified 3.5 keV X-ray line has been reported in galaxy clusters and in the Andromeda galaxy



Boyarsky et al 2014

Iakubovskyi et al 2015



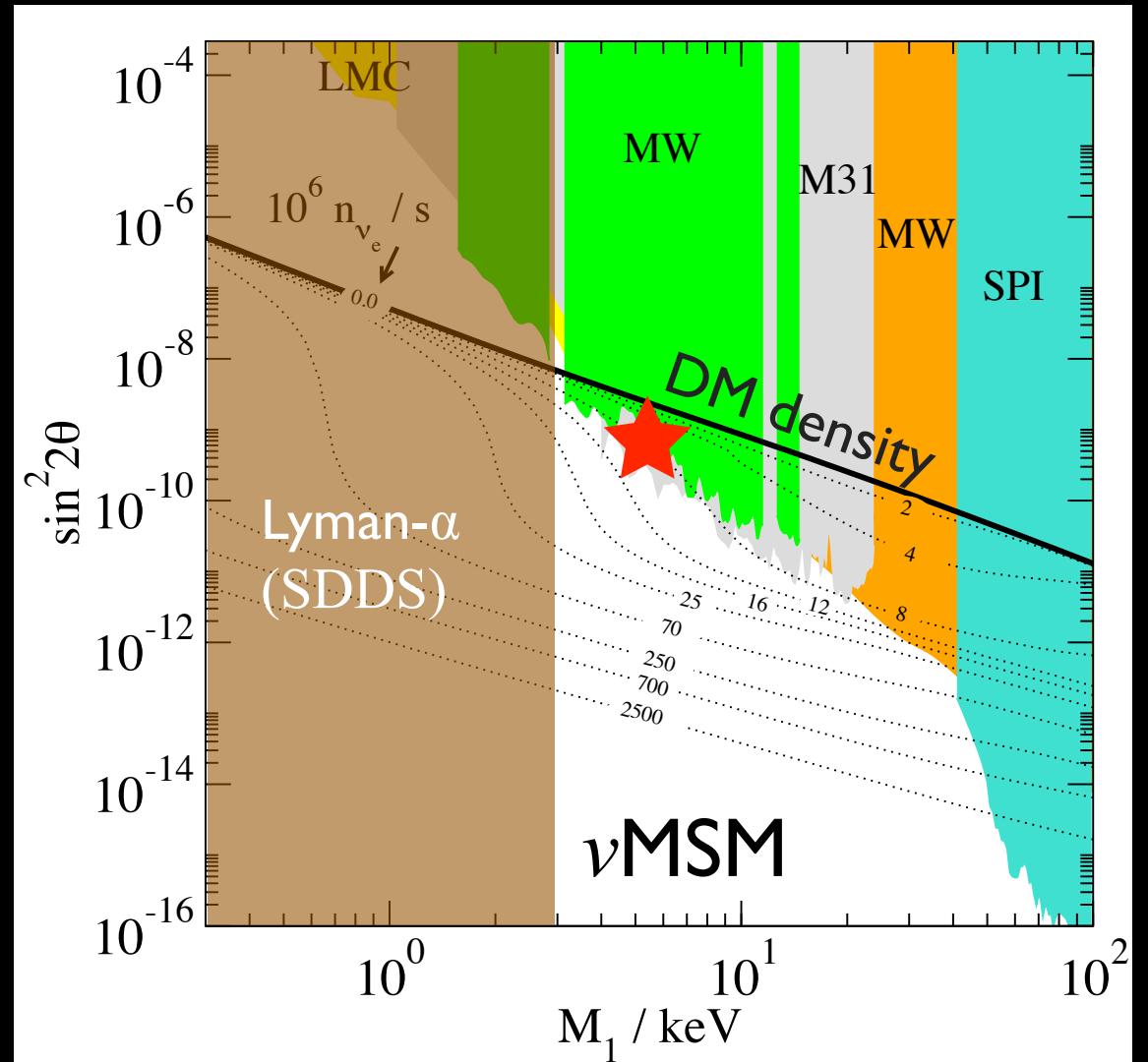
# X-rays from dark matter?

Radiative decay of  
sterile neutrinos  $\nu_s \rightarrow \gamma \nu_a$

X-ray line  $E_\gamma = \frac{1}{2} m_s$

$m_\nu = 7.1$  keV

$\sin^2(2\theta) = 7 \times 10^{-11}$



Laine, Shaposhnikov 2008

# **Particle dark matter flowchart**

# A NEW AND DEFINITIVE META-COSMOLOGY THEORY

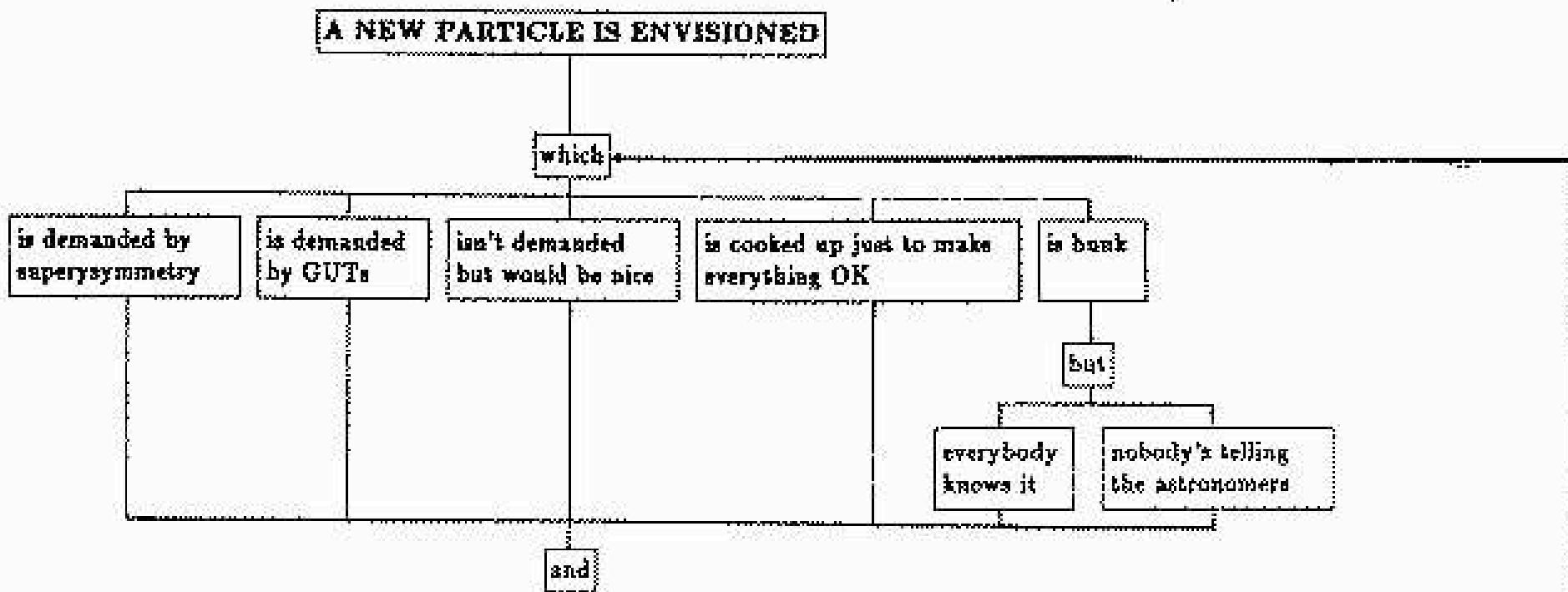
T. R. Lauer

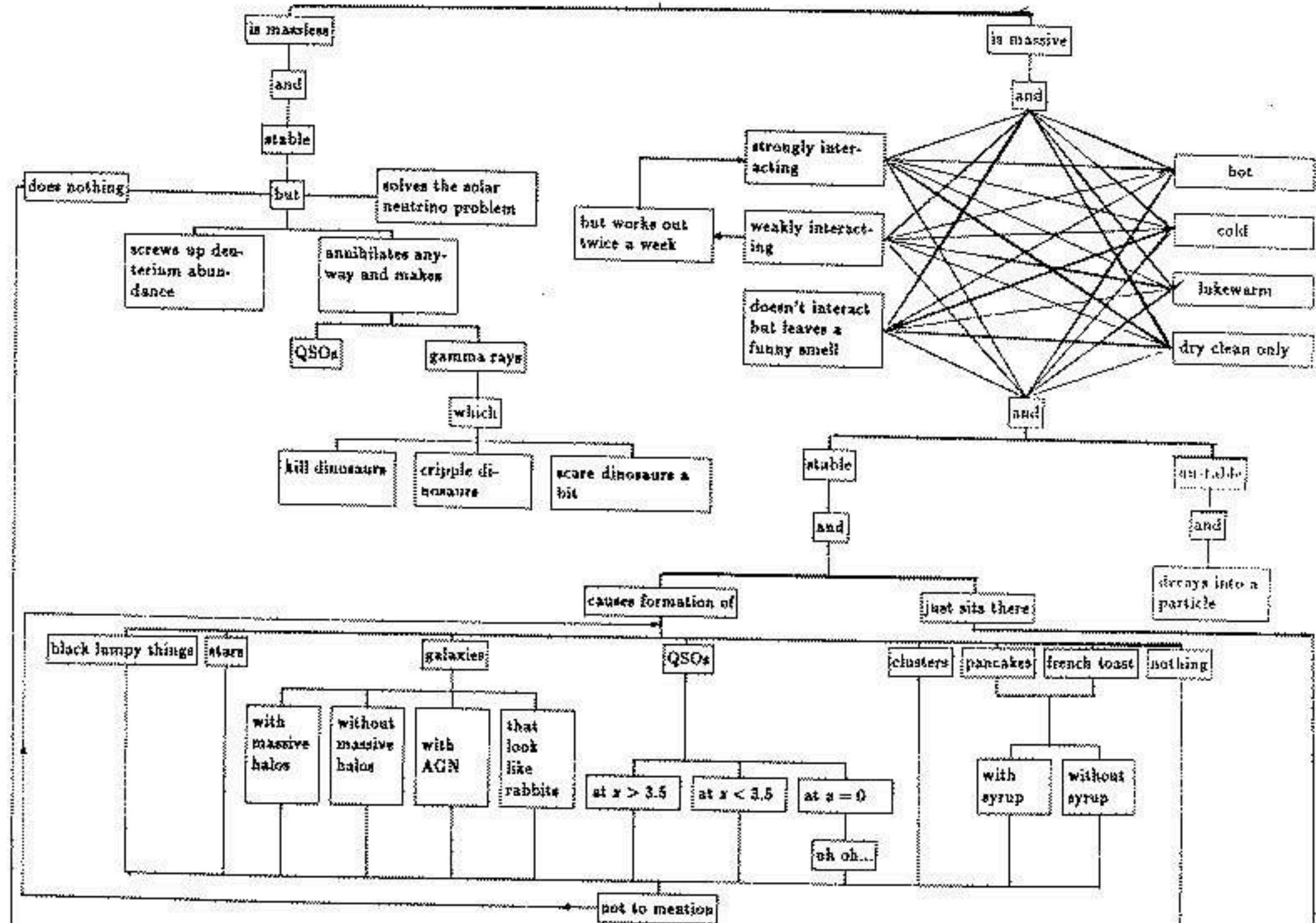
T. S. Statler

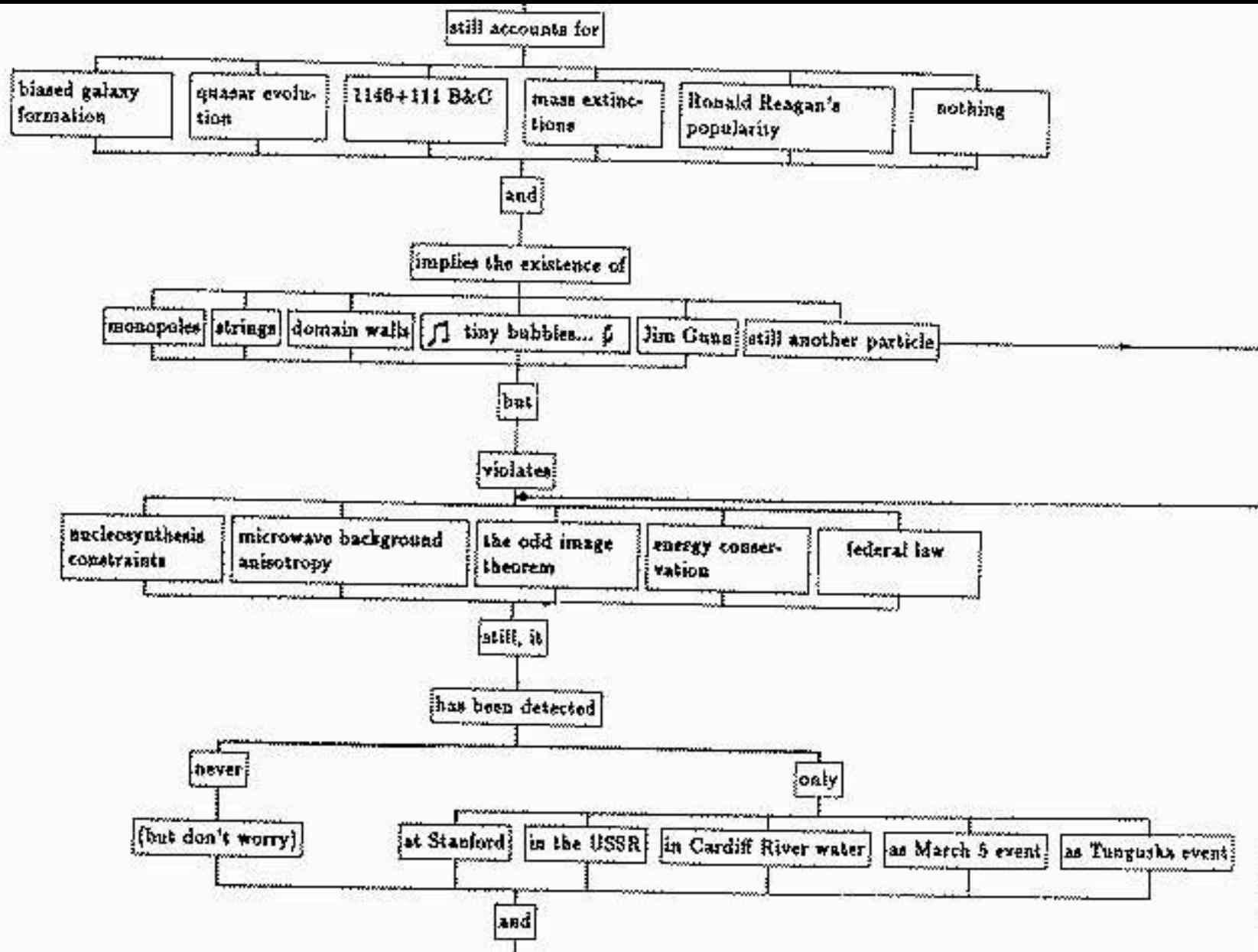
B. S. Ryden

D. H. Weinberg

*Department of Astrophysical Sciences, Princeton University*







can be detected

only

in the imagination

at Stanford

in the USSR

when ST Bies

with an accelerator coating

by borrowing the Pacific Ocean for

\$10<sup>10</sup>

\$10<sup>10</sup>

\$10<sup>11</sup>

if you have to  
ask, you can't  
afford it

just a second  
or two

a month

a year

a century,  
tops

of

but

that doesn't prevent  
a paper from being  
written by

which is then  
contradicted by

whose arguments are  
subsequently ripped  
off by

and eventually  
weasled out of by

Silk      Peebles      Press      Spergel      Ostriker      Turner  
↓           ↓           ↓           ↓           ↓           ↓  
Ed      Mike

who

ran endless N-  
body experi-  
ments

saw it  
in a  
dream

ran still more  
N-body experi-  
ments

Made an inspired but  
ultimately fallacious  
argument

did a completely an-  
alytic calculation but  
forgot about

BUT IT MAKES  $f_{\text{D}} = 1$

# A NEW AND DEFINITIVE META-COSMOLOGY THEORY

A NEW AND DEFINITIVE META-COSMOLOGY

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B. S. Ryden  
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