Consequences of Pilgrim Dark Energy in $f(T, T_G)$cosmology

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Cosmic observations from Supernovae Ia (SNe Ia) (Perlmutter et al. 1999; Riess et al. 1998) have implied that the expansion of the universe is accelerating at the present stage.

A.G. Riess et al., Astron. J. **116, 1009 (1998).** doi:10.1086/300499

S. Perlmutter, Astrophys. J. **517, 565 (1999). doi:10.1086/307221**

Approaches to account for the late time cosmic acceleration fall into two representative categories: One is to introduce "dark energy" in the right-hand side of the Einstein equation in the framework of general relativity (for reviews on dark energy, see Copeland et al., 2006; Bamba et al., 2012). The other is to modify the left-hand side of the Einstein equation, called as a modified gravitational theory. e.g., f(R) gravity (for reviews, see Nojiri and Odintsov, 2011; Clifton et al., 2012).

Reviews on dark energy:
E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15,** 1753 (2006). doi:10.1142/S021827180600942X
K. Bamba, S. Capozziello, S. Nojiri, S.D. Odintsov, Astrophys. Space Sci. **342,** 155 (2012). doi:10.1007/s10509-012-1181-8

Reviews on modified gravity:
T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Phys. Rep. **513,** 1 (2012). doi:10.1016/j.physrep.2012.01.001
S. Nojiri, S.D. Odintsov, Phys. Rep. **505,** 59 (2011). doi:10.1016/j.physrep.2011.04.001

In recent years, the holographic dark energy (HDE) (Li, M.: *Phys. Lett. B* **603, 1 (2004)**), based on holographic principle is one of the interesting and powerful candidates for the DE. and its density is given by $\rho_\Lambda=3c^2M_p^2L^{-2}$

H. Wei (*Class.Quant.Grav.*29:175008,2012) revisited the idea of holographic dark energy and proposed the so-called "pilgrim dark energy (PDE)" based on the speculation that the repulsive force contributed by the phantom-like dark energy ($w < -1$) is strong enough to prevent the formation of the black hole. [e.g. Babichev et al., *PRL*, 93, 02112 (2004)]

If this speculation is true then the total energy in a box of size L could exceed the mass of a black hole of the same size i.e. $\rho_\Lambda L^3 \geq M_p^2 L$ that implies $\rho_\Lambda \geq M_p^2 L^{-2}$ ⇨ the first property of pilgrim dark energy

PDE requires $\rho_\Lambda \geq M_p^2 L^{-2}$

To implement this Wei (2012) introduced PDE as $\rho_\Lambda = 3n^2 M_p^{4-s} L^{-s}$ where n and s are both dimensionless constants.

Thus, $M_p^{4-s} L^{-s} \geq M_p^2 L^{-2}$ that implies $L^{2-s} \geq M_p^{s-2}$ that implies $L^{2-s} \geq l_p^{2-s}$ i.e. $L \geq l_p$ that is the reduced Plank length = 1.616×10^{-33} cm, which is extremely short length in fact.

Obviously, since $L > l_p$ in general, it is required that $s \leq 2$.

The second requirement of pilgrim dark energy is to be phantom-like, namely $w_\Lambda < -1$.

In order to obtain EoS for PDE, Wei (2012) had chosen $L = H^{-1}$.

If $\rho_\Lambda \alpha H^s$ then $w_\Lambda = -1 - \frac{s\dot{H}}{3H^2} < -1$ that implies $s < 0$.

To include the higher GB terms in *f (T) gravity and motivated from the f (R,G) model, recently f (T,T_G) has been constructed on the basis of T (old* quadratic torsion scalar) and T_G *(new quartic torsion scalar T_G that is the teleparallel equivalent of the Gauss-Bonnet term)*

By G. Kofinas, G. Leon and E. N. Saridakis, Class. Quantum Grav. 31 (2014) 175011

<u>Universe can result in dark-energy dominated, quintessence-like, cosmological constant-like or phantom-like solutions, according to the parameter choices</u>

$$T = 6H^2$$
$$T_G = 24H^2(\dot{H} + H^2),$$

$$H^2 = \frac{1}{3}(\rho_m + \rho_{DE})$$
$$\dot{H} = -\frac{1}{2}(\rho_m + \rho_{DE} + p_m + p_{DE})$$

Modified Friedmann equations

$$\rho_{DE} = \frac{1}{2}\left(6H^2 - f + 12H^2 f_T + T_G\, f_{T_G} - 24H^3 \dot{f}_{T_G}\right)$$
$$p_{DE} = \frac{1}{2}\left[-2(2\dot{H} + 3H^2) + f - 4(\dot{H} + 3H^2)f_T - 4H\dot{f}_T - T f_{T_G} + \frac{2}{3H}T_G\dot{f}_{T_G} + 8H^2\ddot{f}_{T_G}\right]$$

Energy and pressure of the effective dark energy sector

Matter and dark energy are conserved separately

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0$$
$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$$

Viewing the modified gravity model as an effective description of the underlying theory of DE,

and

considering the various versions of the HDE (M. Li, PLB, 603, 1-5 (2004)) as pointing in the direction of the underlying theory of DE,

it is interesting to study how the modified-gravity can describe the various forms of HDE densities as effective theories of DE models.

This motivated studies to establish the different models of modified gravity according to the HDEs.

X. Wu , Z-H. Zhu, *Phys. Lett. B*, 660, 293 (2008)

M. R. Setare, *Int. J. Mod. Phys. D*, 17, 2219 (2008).

K. Karami, M.S. Khaledian, *JHEP* 03, 086 (2011).

W. Yang et al., *Mod. Phys. Lett. A* 26, 191 (2011).

L. N. Granda, *Int. J. Mod. Phys. D*, 18, 1749 (2009).

M.R. Setare, *Phys. Lett. B,* 644 , 99 (2007) .

Plan of the work: Two reconstruction approaches

One is to reconstruct $f(T,T_G)$ with a consideration of $\rho_\Lambda = \rho_{DE}$ leading to a differential equation on f and subsequent reconstruction of the EoS parameter for $f(T,T_G)$ with

1. Power law form of scale factor
2. Taking $H(t)=H_0+H_1/t$
3. Taking bouncing solution for scale factor

Second approach is to assume $f(T,T_G)$ in the form of a polynomial as

$f=b_0+b_1t+b_2t^2+b_3t^3$

and reconstructing Hubble parameter and w_Λ without any choice of scale factor.

Reconstruction through power law form of scale factor

Scale factor $a(t) = a_0 t^m$ m>1

$$H = \frac{m}{t}, \quad \dot{H} = -\frac{m}{t^2}$$

$$\rho_{DE} = \rho_{\Lambda} = 3n^2 m_p^{4-s} \left(\frac{t}{m}\right)^{-s}$$

$$T = \frac{6m^2}{t^2}, \quad T_G = \frac{24(-1+m)m^3}{t^4}; \quad \dot{T} = -\frac{12m^2}{t^3}; \quad \dot{T}_G = -\frac{96(-1+m)m^3}{t^5}$$

Hence from first modified Friedmann equation we get

$$\left(\frac{t^{2+s}}{4-4m}\right)\frac{d^2 f}{dt^2} + t^{1+s}\frac{5}{4}\left(\frac{m-2}{m-1}\right)\frac{df}{dt} + t^s f - 6m^2 t^{-2+s} + 6m^s n^2 = 0$$

$$f(t) = C_2 t^{\frac{5}{2}m - \frac{9}{2} + \frac{1}{2}\sqrt{65-74m+25m^2}} + C_1 t^{\frac{5}{2}m - \frac{9}{2} - \frac{1}{2}\sqrt{65-74m+25m^2}} + \frac{120\left(-\frac{8}{5}m^{1+s} + \frac{3}{5}m^{2+s} + m^s\right)n^2 t^{2-s} - 60\left(\left(s-\frac{4}{5}\right)m + \frac{1}{5}s^2 + \frac{4}{5} - \frac{9}{5}s\right)(m-1)m^2}{15t^2\left(\left(s-\frac{4}{5}\right)m + \frac{1}{5}s^2 + \frac{4}{5} - \frac{9}{5}s\right)\left(-\frac{5}{3}+m\right)}$$

Reconstructed f

$$f(T,T_G) = C_2\left(\frac{2T^{3/2}\sqrt{6}}{3G - 2T^2}\right)^{\frac{-5T^2}{3T_G-2T^2} - 9/2 + 1/2\sqrt{65 + \frac{148T^2}{3T_G-2T^2} + \frac{100T^4}{(3T_G-2T^2)^2}}}$$
$$+C_1\left(\frac{2T^{3/2}\sqrt{6}}{3T_G - 2T^2}\right)^{\frac{-5T^2}{3T_G-2T^2} - 9/2 - 1/2\sqrt{65 + \frac{148T^2}{3T_G-2T^2} + \frac{100T^4}{(3T_G-2T^2)^2}}}$$
$$-32805(T_G - 2/3T^2)^4((T_G - 2/3T^2)^4[3/5(\frac{-2T^2}{3T_G - 2T^2})^{2+s}$$
$$-8/5\left(\frac{-2T^2}{3T_G - 2T^2}\right)^{1+s} + \left(\frac{-2T^2}{3T_G - 2T^2}\right)^{s}]n^2\left(2\frac{T^{3/2}\sqrt{6}}{3T_G - 2T^2}\right)^{2-s}$$
$$+\frac{2}{45}((-2/3s^2 + 8/3s)T^2 + T_G(-9s + 4 + s^2)T^4 T_G)(3T_G - 2T^2)^{-4}$$
$$\times(8T^2 s + 3s^2 T_G - 2s^2 T^2 + 12T_G - 27sT_G)^{-1}(-4T^2 + 15T_G)^{-1}T^{-3}$$

Outcomes of Reconstruction Scheme

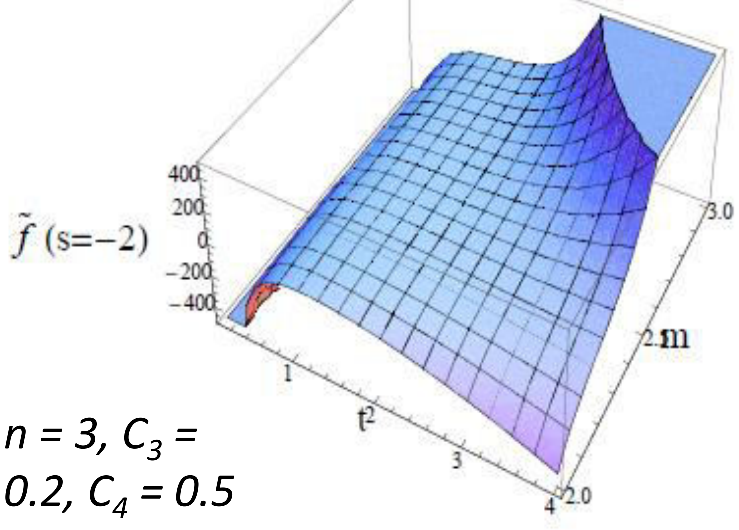
EoS resulting from s = −2 is in complete agreement with it as in this case wDE < −1, → −1 and never crosses −1.

Moreover, s = −2 is a choice that is in agreement with the prescription of Wei (2012), which states that if dH/dt < 0 (a requirement for cosmic acceleration and holds for our choice of scale factor) s < 0.

Hence, it is observed that the results stated for PDE in Einstein gravity are in close agreement with those in the framework of modified gravity under consideration.

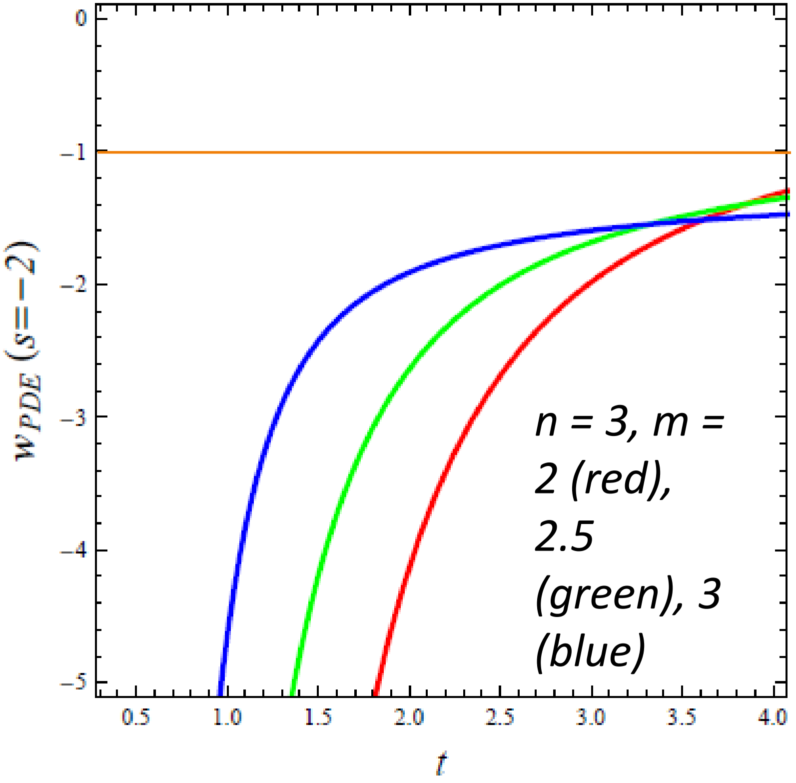
$$\tilde{f}(s=-2) = \frac{12(-1+m)\left(\frac{m^4}{5-3m}+\frac{n^2t^4}{13-7m}\right)}{m^2t^2} + t^{\frac{1}{2}\left(-9+5m+\sqrt{65+m(-74+25m)}\right)}\left[C_3 + C_4 \times t^{-\sqrt{65+m(-74+25m)}}\right]$$

For m>2.2, f→0 as t→0



$n = 3$, $C_3 = 0.2$, $C_4 = 0.5$

f(s = −2) increases with cosmic time from very low value, becomes flat for a glimpse of time interval and then decreases for 2 < m < 2.2.
For 2.2 < m <2.5, it increases but approaches to zero after short interval of time. For m>2.5 , f(s = −2) increases with cosmic time from very low value, becomes flat for a glimpse of time interval and then increases.



Plot of the reconstructed EoS parameter

Reconstruction scheme for unification of matter dominated and accelerated phases

$$H(t) = H_0 + \frac{H_1}{t} \qquad a(t) = C_1 e^{H_0 t} t^{H_1} \Rightarrow \rho_\Lambda = 3n^2 t^{-s}\left(H_0 t + H_1\right)^s$$

(Nojiri, S., Odintsov, S.D.: Phys. Rev. D **74, 086005 (2006).**)

In the early universe $t \ll t_0$ and $H(t) \approx \frac{H_1}{t}$

For the early universe we have

$$\tilde{f} = \frac{1}{t^7}\left[\frac{48n^2(-t)^{7-s}}{8+(-14+s)s} - 3t^5 + C_1 t^{-\sqrt{41}} + C_2 t^{\sqrt{41}}\right]$$

Since t<<t0, we have the limiting case

$$\tilde{f} \sim C_1 t^{-(7+\sqrt{41})}, \qquad T \sim \frac{6H_1^2}{t^2}, \qquad T_G \sim \frac{24H_1^3(H_1-1)}{t^4}$$

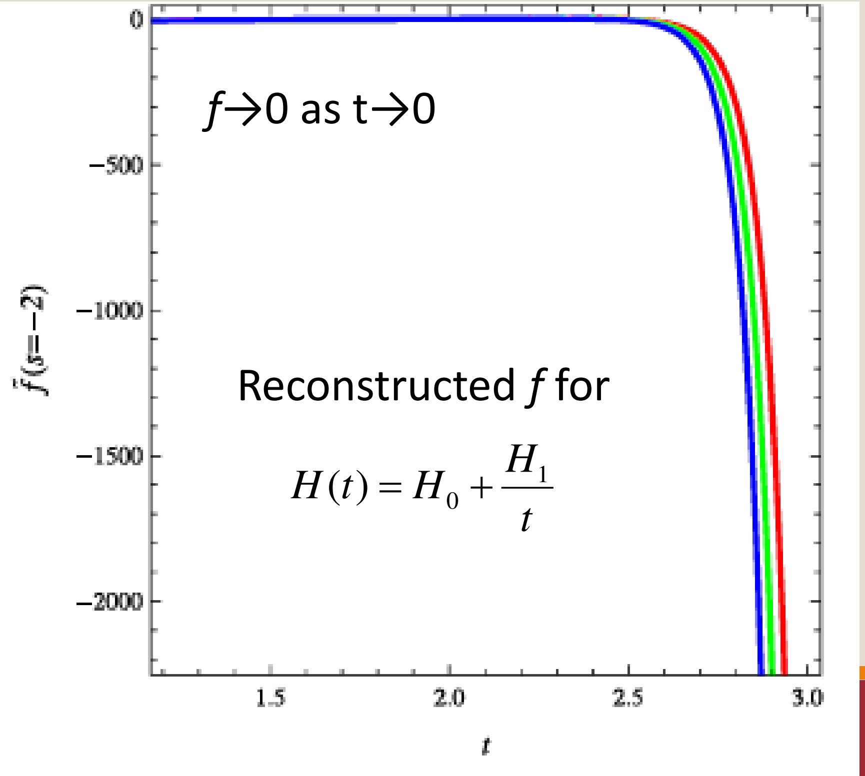
$$f(T, T_G) = C_1\left[-\frac{T_G}{24}\sqrt{\frac{6}{T}}\right]^{7+\sqrt{41}}$$

At late time $t \gg t_0$ $H(t) \approx H_0$

$$T \sim 6H_0^2, \qquad T_G \sim 24H_0^4,$$

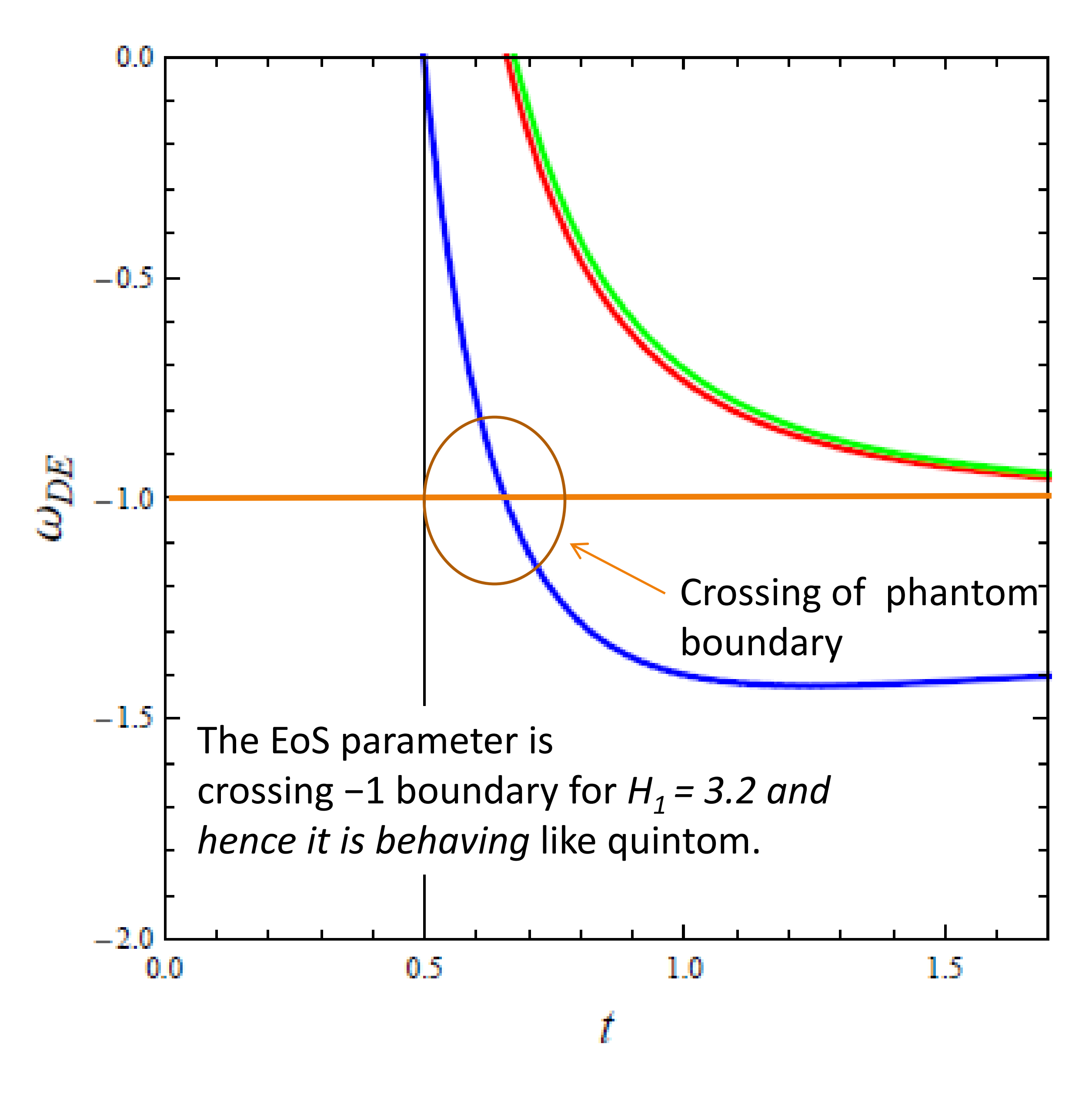
$$\dot{f}_{T_G} = \dot{T}_G f_{T_G,T_G} + \dot{T} f_{T_G,T}|_{H=H_0} = 0$$

$$f(T, T_G) = \sqrt{T}F\left(\frac{T_G}{\sqrt{T}}\right) - T + \frac{6^{1-s/2}n^2 m_p^{4-s}}{s-1}T^{s/2}$$



Behaviour of reconstructed EoS parameter for f (T,T_G)

- Reconstructed f (T,T_G) through consideration of PDE can attain phantom era when s = −2.
- This is consistent with the behavior of PDE.
- However, in f (T,T_G) gravity, it can go beyond phantom for our choice H(t) = H0 + H1/t.



Reconstruction scheme for bouncing scale factor

Bounncing scenario predicts a transitionary Universe, in which the Universe evolves from a contracting epoch (H < 0) to an expanding epoch (H > 0).

In GB gravity, bouncing solutions widely studied in literature:

Bamba, K., Nojiri, S., Odintsov, S. D.:Phys. Lett. B 731 257: arXiv:1401.7378 [gr-qc] (2014)

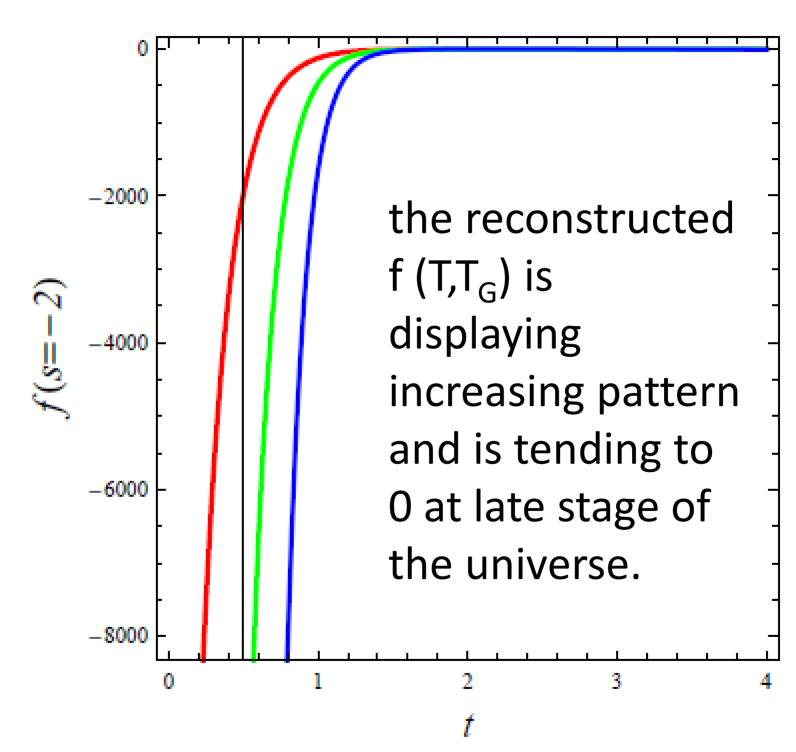
Bamba, K., Odintsov, S. D.:arXiv:1402.7114 [hep-th] (2014)

$$a(t) = a_0 + \alpha(t-t_0)^{2n}\,, \quad H(t) = \frac{2n\alpha(t-t_0)^{2n-1}}{a_0 + \alpha(t-t_0)^{2n}}\,, \quad n = 1, 2, 3..$$

For this choice of scale factor we have
The following diff. eq.

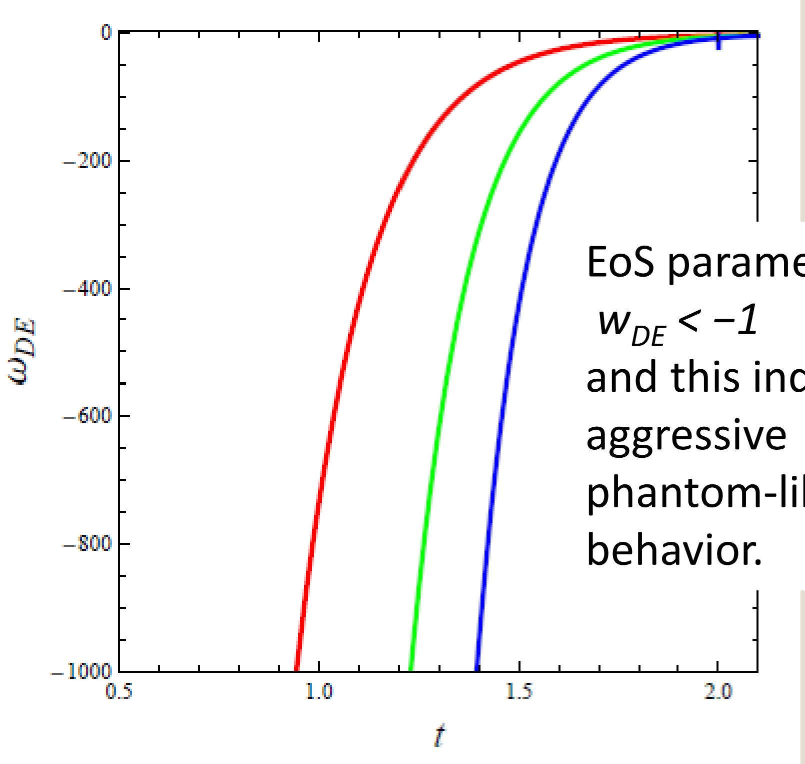
$$\begin{aligned}
&\frac{1}{4}\left[\frac{48n^2(t-t_0)^{-2+4n}\alpha^2}{(a_0+(t-t_0)^{2n}\alpha)^2} + \frac{(t-t_0)\left(a_0+(t-t_0)^{2n}\alpha\right)f'[t]}{a_0(-2+3n)-2(t-t_0)^{2n}\alpha} - \frac{2(t-t_0)\left(a_0+(t-t_0)^{2n}\alpha\right)f'[t]}{a_0-2a_0n+(t-t_0)^{2n}\alpha}\right.\\
&- \; 2f[t]\left((t-t_0)\left(\left(a_0^2(-2+3n)(-5+6n)-a_0(-20+3n(9+2n))(t-t_0)^{2n}\alpha+10(t-t_0)^{4n}\alpha^2\right)\right.\right.\\
&\times \; f'[t]-(t-t_0)\left(a_0(-2+3n)-2(t-t_0)^{2n}\alpha\right)\left(a_0+(t-t_0)^{2n}\alpha\right)f''[t]\big)\big)\left((-1+2n)\left(a_0(2-3n)\right.\right.\\
&\times \left.\left.\left. +2(t-t_0)^{2n}\alpha\right)^2\right)^{-1}\right] = 32^s n^2\left(\frac{(t-t_0)^{1-2n}\left(a_0+(t-t_0)^{2n}\alpha\right)}{n\alpha}\right)^{-s}.
\end{aligned}$$

The differential eq. is solved numerically:



the reconstructed $f\,(T,T_G)$ is displaying increasing pattern and is tending to 0 at late stage of the universe.

$a0 = 10.5$, $\alpha = 10.1$ and red, green and blue lines correspond to $n = 6, 7, 8$ respectively.



EoS parameter $w_{DE} < -1$ and this indicates aggressive phantom-like behavior.

Reconstruction by choosing $f(T,T_G)$ in the form of a polynomial

Instead of reconstructing $f(T,T_G)$ through PDE we assume $f(T,T_G)$ in the following form and subsequently reconstruct ρ_Λ:

$$f(T,T_G) \equiv f = b_0 + b_1\, t + b_2\, t^2 + b_3\, t^3 + \ldots$$

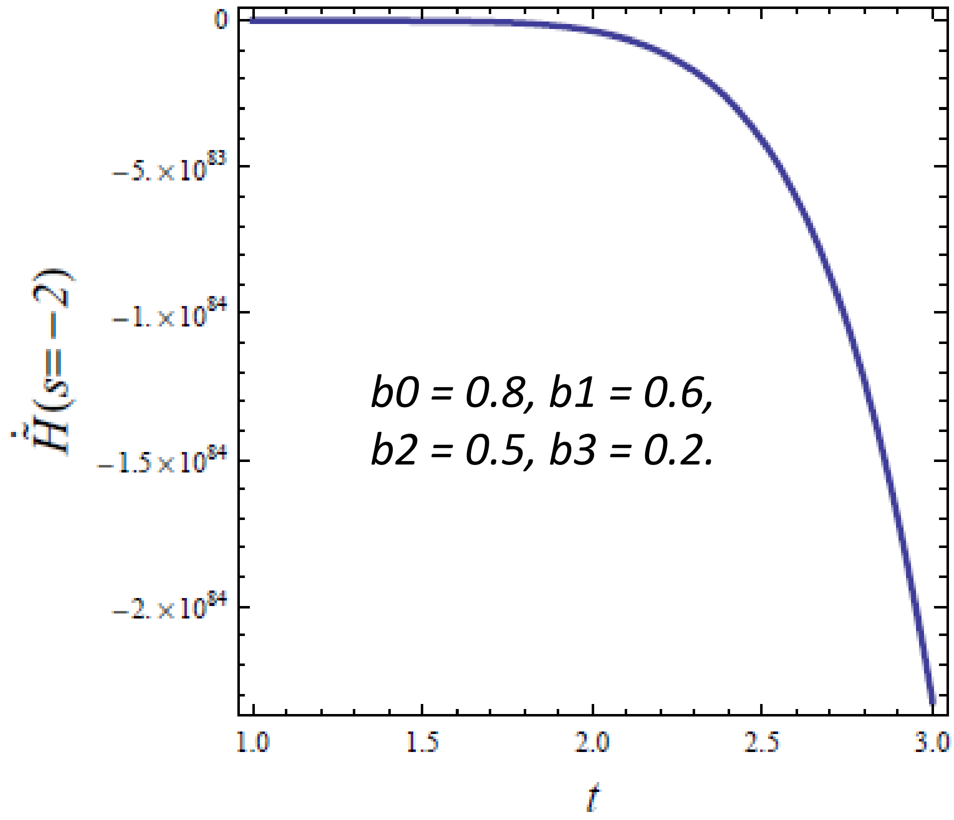
$$\rho_{DE} = \frac{1}{2}\Bigg[-b_0 - b_1 t - b_2 t^2 - b_3 t^3 + 6H^2 + \frac{(b_1+t(2b_2+3b_3t))H}{H'} + \frac{(b_1+t(2b_2+3b_3t))H\left(H^2+H'\right)}{2H'(2H^2+H')+HH''} - \frac{H\left(48(b_2+3b_3t)H\left(2H'\left(2H^2+H'\right)+HH''\right)-24(b_1+t(2b_2+3b_3t))\left(2H'^3+4H^3H''+6HH'H''+H^2\left(12H'^2+H^3\right)\right)\right)}{24(2H'(2H^2+H')+HH'')^2}\Bigg]$$

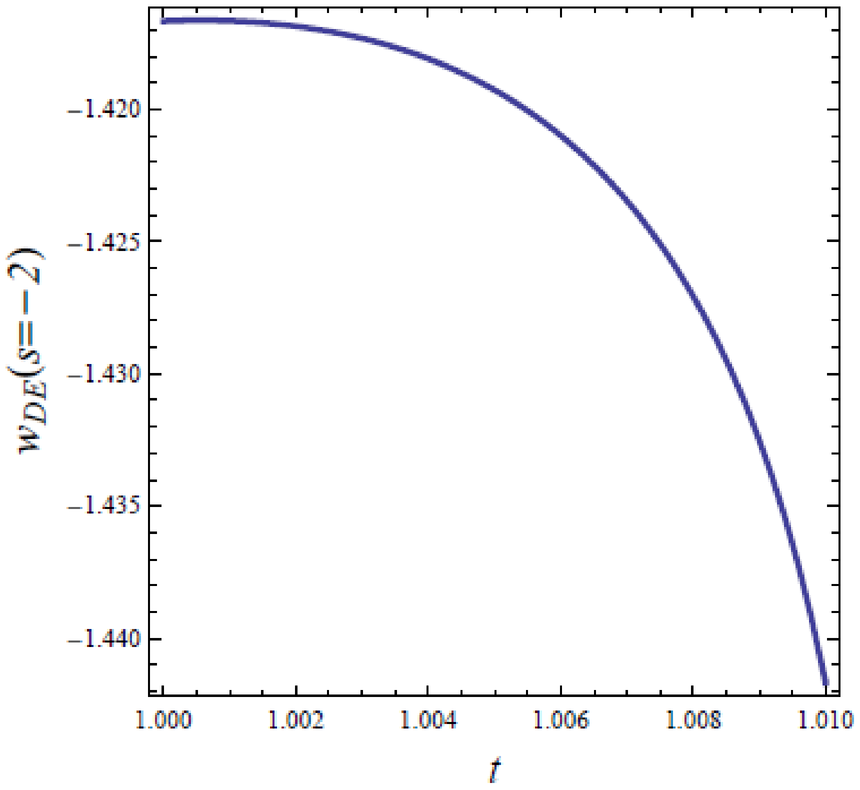
which, on setting equal to ρ_Λ gives rise to a differential equation on H that is solved numerically to generate the reconstruction of H and subsequently EoS parameter.

$$w_{DE} = -1 - \frac{2\dot{H}}{3H^2}$$

Negative time derivative of Hubble parameter is consistent with the accelerated expansion of the universe.

The aggressive phantom-like behavior of w_{DE} is consistent with the basic property of pilgrim dark energy.





Concluding remarks

1. PDE reconstructed $f(T,T_G)$ for different choices of scale factor has resulted in phantom-like solution for EoS parameter.
2. In most of the cases the reconstructed $f(T,T_G)$ is seen to tend to 0 as t tends to 0 and this is believed to be a sufficient condition for a model that is expected to be stable to small perturbations.
3. When $f(T,T_G)$ is assumed to be in the form of a polynomial and accordingly the density of PDE is reconstructed it is observed that reconstructed H has time derivative to be less than 0 that is required for accelerated expansion.
4. Reconstructed PDE has EoS behaving like phantom that is consistent with the basic property of the PDE.

Thank you