Cosmology : Understanding the Early and Late Universe

B. C. Paul[1]

[1]Physics Department, North Bengal University, Siliguri,Pin: 734013
e-mail : bcpaul@iucaa.ernet.in

October 13, 2015

Table of contents

Gravity Equation in Cosmology

1915, GR : Einstein's Field Equation :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \quad (1)$$

$$GRAVITY \Longleftrightarrow MATTER$$

Universe is homogeneous and isotropic. RW line element

$$ds^2 = -dt^2 + a(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin^2\theta d\phi^2) \right] \quad (2)$$

The energy momentum tensor : $T^{\mu}_{\mu} = Diagonal(-\rho, p, p, p)$

Einstein's Field Equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \quad (3)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G p \quad (4)$$

The conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (5)$$

where $H = \frac{\dot{a}}{a}$ represents Hubble parameter.

Raychaudhuri Equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3p\right) \tag{6}$$

A static universe is not permitted as one needs matter with negative pressure. To accommodate a static Universe, Einstein modified Field equation by introducing a repulsive term Λ, thus

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \; T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{7}$$

The Raychaudhuri Equation becomes

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + 3p\right) + \frac{\Lambda}{3} \tag{8}$$

A static Universe thus can be accommodated nicely with

$$\rho = \frac{3k}{a_o^2} - \Lambda$$

$$p = \Lambda - \frac{k}{a_o^2}$$

In 1920 Friedmann obtained expanding solution

$$a(t) \sim t^{\frac{2}{3\omega}}$$

for the EoS $p = \omega\rho$. But it was of academic interest during that time.

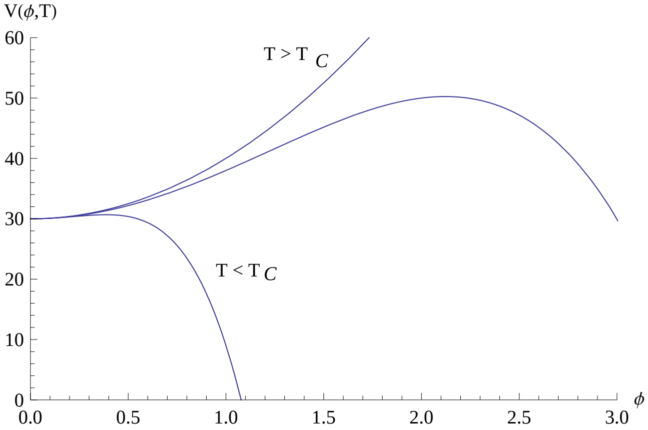
Observational Astronomy

- 1927, Hubble's discovery $\rightarrow$ Universe is expanding.
- Non-static Model of the universe $\rightarrow$ Bigbang Cosmology.
- A number of issues namely, (i) Horizon Problem (ii) Flatness Problem, (iii) Singularity problem, (iii) small scale inhomogeneity problems cropped up when early universe probed with perfect fluid model.

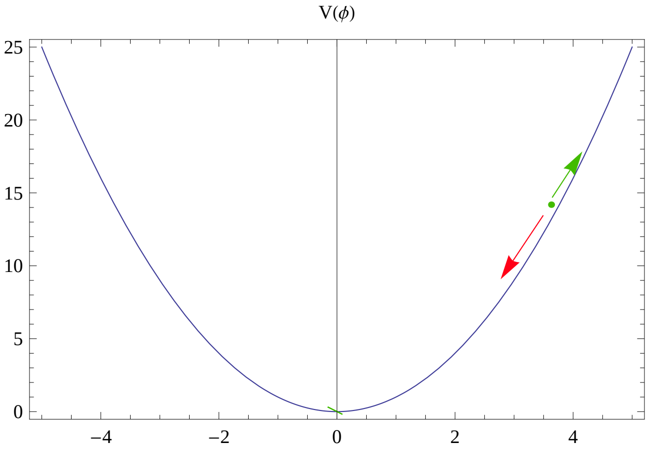
In the Early Universe GUT era, Size of the universe 10^{-27} cm

- Size now $a_p = (3x10^10)(4x10^17)(10^{-27}) = 10cm$
- But the present size ' $10^{28}cm$.
- N =a factor of 10^{27} is missing
- How to include this number in the early era so that the present universe can be accommodated ?

INFLATION : 1981, Guth proposed a temperature dependent phase transition to introduce early inflation.



Later Linde (1983) proposed Chaotic Model



Thereafter a huge number of early Inflationary model in the last 35 years have been constructed.

Despite its very impressive achievements, the inflationary paradigm leaves some questions unanswered.

- when and how the universe entered the inflationary phase
- nature of the inflaton field
- the state of the universe prior to the commencement of inflation

Several alternative possibilities :

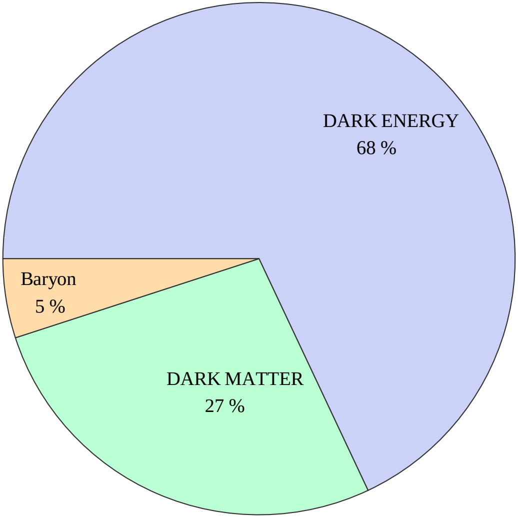
- The universe quantum mechanically tunnelled into an inflation
- The universe was dominated by radiation (or some other form of matter) prior to inflation and might therefore have encountered a singularity in its past.
- The universe underwent a non-singular bounce prior to inflation. Before the bounce the universe was contracting
- The universe existed eternally in a quasi-static state, out of which inflationary expansion emerged.

Recent Astronomical and Cosmological Observations

The recent observation is that the present universe is expanding at a rate much faster than it is expected from the theoretical framework for cosmological model building. The density parameter $(\Omega = \frac{8\pi G}{3H^2} \sum_{i=1}^{n} \Omega_i)$.
If we assume three components then
$\Omega = \sum_{i=1}^{3} \Omega_i = \Omega_1 + \Omega_2 + \Omega_3$.

- The baryonic matter
- Dark Matter
- Dark Energy
- A pie Chart is shown :

Energy Budget of the Universe



- Usual scalar fields in the particle physics zoo are not enough to understand the recent accelerating phase of the universe.

Several proposal came up

- Modification of the gravitational sector adding scalar curvature terms
- Modification of the matter sector (Exotic matter) e.g., *Phantom, Tachyon, Chaplygin Gas, quintom* etc.

Emergent Universe

- A universe which is ever existing, large enough so that space-time may be treated as classical entities.
- No time like singularity
- The universe in the infinite past is in an almost static state but it eventually evolves into an inflationary stage

Advantage of an Emergent Universe Model

An emergent Universe (EU) model, if developed in a consistent way then it is capable of solving the well known conceptual issues of the Big Bang model

Four Fold advantages :

- Exists forever
- No Quantum Gravity Region
- Free from Bigbang problems like HP, FP etc.
- Accommodates late time acceleration

Alternative Model

In 1967, Harrisson found a cosmological solution in a closed model with radiation and a positive cosmological constant

$$a(t) = a_i \left(1 + e^{\frac{\sqrt{2}t}{a_i}}\right)^{\frac{1}{2}} \tag{10}$$

as $t \rightarrow -\infty$, the universe goes over asymptotically to an Einstein Static Universe, the expansion is given by a finite number of e-foldings

$$N_o = \ln \frac{a_o}{a_i} = \frac{t_o}{\sqrt{2}a_i} \tag{11}$$

.

- Size of the universe is determined by Λ
- Problem : No graceful exit from the deSitter phase.

- Considered a dynamical scalar field to obtain EU in a closed universe ($k = +1$). In the model a minimally coupled scalar field ϕ with a self interacting potential $V(\phi)$ was considered.
- In the case the initial size a_i of the universe is determined by the KE of the field.
- To understand we consider a model consisting of ordinary matter and minimally coupled homogeneous scalar field.

The Klein-Gordon equation for scalar field

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \quad (12)$$

$$\dot{\rho} + 3H(1+\omega)\rho = 0 \quad (13)$$

using EOS $p = \omega\rho$. Now the Raychaudhuri equation can be written as

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\frac{1}{2}(1+3\omega) + \dot{\phi}^2 - V(\phi)\right] \quad (14)$$

First integral gives Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] - \frac{k}{a^2} \tag{15}$$

It leads to

$$\dot{H} = -4\pi G\left[\dot{\phi}^2 + (1+\omega)\rho\right] + \frac{k}{a^2} \tag{16}$$

Now, an accelerating universe ($\ddot{a} > 0$) demands

$$\dot{\phi}^2 + \frac{1}{2}(1+3\omega)\rho < V(\phi) \tag{17}$$

For a positive minimum

$$H_i = 0 \Rightarrow \frac{1}{2}\dot{\phi}_i^2 + V_i + \rho_i = \frac{3k}{8\pi G a_i^2} \tag{18}$$

where t_i may be infinte.

The Einstein static universe is characterized by $k = 1$ and $a = a_i = constant$. We therefore obtain

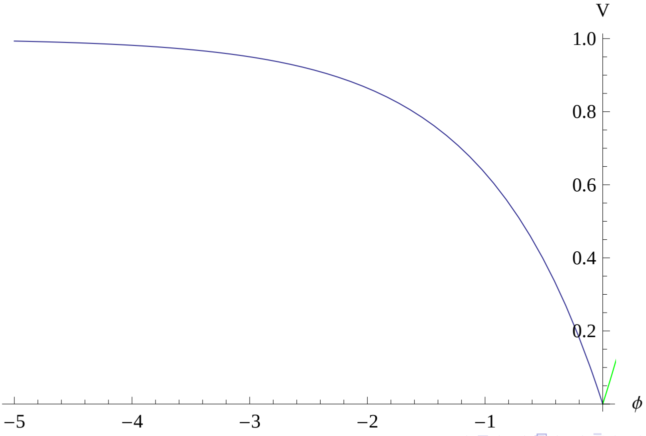
$$\frac{1}{2}(1-\omega)\rho_i + V_i = \frac{1}{4\pi G a_i^2} \tag{19}$$

$$(1+\omega)\rho_i + \dot{\phi}_i^2 = \frac{1}{4\pi G a_i^2} \tag{20}$$

- If the KE of the scalar field vanishes, there must be matter to obtain a static universe.
- If only scalar field with non-zero KE, then the field rolls at a constant speed along the flat potential.

- A simple potential for EU model with scalar field only is given by a potential having the characteristics $V(\phi) \to V_i$ as $\phi \to \infty$ and $t \longrightarrow -\infty$. But drops towards a minimum at a finite value ϕ_f.

Form of the Potential $V - V_f = (V_i - V_f)\left[Exp[\frac{\phi-\phi_i}{\alpha}] - 1\right]^2$



- Determination of the parameters for the Potential:
- Consider spacetime filled with a minimally coupled scalar field with potential

$$V(\phi) = \left(A_o e^{B_o \phi} - C_o\right)^2 + D_o \tag{21}$$

- A_o, B_o, C_o, D_o are constants to be determined by the specific properties of the EU model
- $$V' = 2A_o B_o \left(A_o e^{B_o \phi} - C_o\right) e^{B_o \phi} \tag{22}$$
$$V" = 2A_o B_o^2 \left(2A_o e^{B\phi} - C_o\right) e^{B_o \phi} \tag{23}$$
- Potential has a maximum at $\phi_o = \frac{1}{B_o}\left(\ln \frac{C_o}{A_o}\right)$ with $V|_{\phi=\phi_o} = D_o$.
- Minimum is set at the origin choosing $A_o = C_o$ and $D_o = 0$, thus $V(\phi) = A_o \left(e^{B_o \phi} - 1\right)^2$

- By definition EU corresponds to a past-asymptotic Einstein Static model:
$$V(\phi \to -\infty) = \frac{2}{\kappa a_i^2}$$
where a_i is the radius of the initial static model. To determine an additional parameter recall that an Einstein Static universe filled with a single scalar field which satisfies
-
$$V(\phi) = \frac{2}{\kappa a_i^2} = \dot{\phi} \to A_o = \frac{2}{\kappa a_i^2} \tag{24}$$
- Thus we have now A_o, B_o, C_o, D_o are constants to be determined by the specific properties of the EU
$$V(\phi) = \frac{2}{\kappa a_i^2} \left(e^{B_o \phi} - 1 \right)^2 \tag{25}$$
- Basic properties of EU model have fixed three of the four parameters in the original potential. The parameter B_o will be fixed as follows :

- $$V' = \frac{4B_o}{\kappa a_i^2}\left(e^{B_o\phi} - 1\right) < 0 \quad for \quad -\infty < \phi < 0. \tag{26}$$

Ellis et al., CQG 21, 233 (2004)

Making use of R^2-modified gravity. The gravitational action

$$I = \int d^4x\sqrt{-g}[R + \alpha R^2] \quad (27)$$

Define a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad (28)$$

Here $\Omega^2 = 1 + 2\alpha R$, one obtains

$$\tilde{R} = \frac{1}{\Omega^2}[R - 6g^{\mu\nu}\nabla_\mu\nabla_\nu)(\ln\Omega) - 6g^{\mu\nu}\nabla_\mu(\ln\Omega)\nabla_\nu(\ln\Omega)] \quad (29)$$

Now set $\phi = \sqrt{3}\ln(1 + 2\alpha R)$ one obtains

$$I = \int d^4x\sqrt{-g}\left[\tilde{R} - \frac{1}{2}\tilde{g}_{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{4\alpha}\left(e^{-\frac{\phi}{\sqrt{3}}} - 1\right)^2\right] \quad (30)$$

Scalar field Potential

The corresponding potential in EU is the reflection of the potential that obtained from higher derivative gravity. The different parts of the potential are :

- Slow-rolling regime or intermediate pre-slow roll phase
- Scale factor grows (slow-roll phase)
- inflation is followed by a re-heating phase
- standard hot Big Bang evolution

EU Model in a spatially flat case

S Mukherjee, BCP, N K Dadhich, S D Maharaj, A Beesham, CQG 23, 6927 (2006)

Preamble : In looking for a model of emergent universe, the following features for the universe are assumed:

- The universe is isotropic and homogeneous at large scales.
- Spatially flat (WMAP results) :
- It is ever existing, No singularity
- The universe is always large enough so that classical description of space-time is adequate.
- The matter or in general, the source of gravity has to be described by quantum field theory.
- The universe may contain exotic matter (SEC violated)
- The universe is accelerating (Type Ia Supernovae data)

EOS

$$p = A\rho - B\sqrt{\rho} \tag{31}$$

The Einstein equations for a flat universe in RW-metric ($G = \frac{1}{8\pi}$)

$$\rho = 3\frac{\dot{a}^2}{a^2} \tag{32}$$

$$p = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \tag{33}$$

Making use of the EOS we obtain

$$2\frac{\ddot{a}}{a} + (3A+1)\frac{\dot{a}^2}{a^2} - \sqrt{3}B\frac{\dot{a}}{a} = 0 \tag{34}$$

On integration

$$a(t) = \left(\frac{3\kappa(A+1)}{2}\left(\sigma + \frac{2}{\sqrt{3}B}Exp\left[\frac{\sqrt{3}Bt}{2}\right]\right)\right)^{\frac{2}{3(A+1)}} \tag{35}$$

- If $B < 0$ Singularity
- If $B > 0$ and $A > -1$ Non-singular (EU)

Composition of the Emergent Universe

Using EOS, in $\frac{d\rho}{dt} + 3(\rho + p)\frac{\dot{a}}{a} = 0$ one obtains

$$\rho(a) = \frac{1}{(A+1)^2}\left(B + \frac{K}{a^{\frac{3(A+1)}{2}}}\right)^2 \tag{36}$$

where K is an integration constant.

This provides us with indications about the components of energy density in EU.

$$\rho(a) = \Sigma_{i=1}^{3}\rho_i \quad and \quad p(a) = \Sigma_{i=1}^{3}p_i \tag{37}$$

where we denote

$$\rho_1 = \frac{B^2}{(A+1)^2}, \quad \rho_2 = \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad \rho_3 = \frac{K^2}{(A+1)^2}\frac{1}{a^{3(A+1)}} \tag{38}$$

$$p_1 == -\frac{B^2}{(A+1)^2}, \quad p_2 = \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad p_3 = \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \tag{39}$$

Composition of EU model

Comparing with the barotropic EoS given by $p_i = \omega_i \rho_i$ one obtains

- $\omega_1 = -1$
- $\omega_2 = \frac{A-1}{2}$
- $\omega_3 = A$

Table-I

A	$\omega_2 = \frac{1}{2}(A-1)$	$\omega_3 = A$	Composition
$\frac{1}{3}$	- $\frac{1}{3}$	$\frac{1}{3}$	DE, Exotic Matter, Radiation
-$\frac{1}{3}$	-$\frac{2}{3}$	-$\frac{1}{3}$	DE, Exotic Matter, Cosmic String
1	0	1	DE, Exotic Matter, Stiff fluid
0	-$\frac{1}{2}$	0	DE, Exotic Matter, Dust

Implementation of EU in Different Gravity theories

- Brane World Scenario
  A Banerjee, T Bandyopadhyay, S Chakraborty : (i) (2007) Grav. Cosmo 13, 290 (ii) (2008) GRG 26, 075017
- Phantom and Tachyon Field : U. Debnath (2008) CQG 25, 205019
- Non-linear Sigma model
  A Beesham, S V Chervon, S D Maharaj (2009) CQG 26, 075017
  A Beesham, S V Chervon, S D Maharaj, A Kubasov (2009) arXiv: 0904.0773
- EU with Gauss-Bonnet terms : B C Paul and S Ghose, (2010) *GRG* **42** 795
- Chiral field in EGB gravity
  S V Chervon, S D Maharaj, A Beesham, A Kubasov (2014) (i) arXiv:1405.7219 (ii) (2013) *Quantum Matter* **2** 388

Observational Aspects

- BCP, P Thakur, S Ghose (2010) *Mon. Not. Roy. Astron. Soc.* **407** 415
- BCP, S Ghose, P Thakur (2011) *Mon. Not. Roy. Astron. Soc.* **413** 686
- S Ghose, P Thakur, BCP (2012) *Mon. Not. Roy. Astron. Soc.* **421** 20

Model I : The two fluids model

Interacting fluids : ρ and ρ' which can exchange energies. One of the fluid say, ρ is dominated to begin with a non-linear EoS given by eq. (1) which leads to an emergent universe model as discussed above with no interaction. The other fluid is assumed to be important at a later epoch. The pressure of the former fluid

$$p = A\rho - B\rho^{1/2}. \tag{40}$$

The other fluid is considered to be barotropic

$$p' = \omega' \rho' \tag{41}$$

where ω' corresponds to EoS parameter. The Hubble parameter is

$$3H^2 = \rho + \rho'. \tag{42}$$

Let us assume that iteraction is operative at $t \geq t_i$ and they satisfy the conservation equations

$$\dot{\rho} + 3H(\rho + p) = -\alpha\rho H, \tag{43}$$

$$\dot{\rho'} + 3H(\rho' + p') = \alpha\rho H \tag{44}$$

where α represents a coupling parameter.
The energy density and pressure for the fluid of the first kind are given by

$$\rho = \frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{2KB}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}}, \tag{45}$$

$$p = -\frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{KB(A-1+\frac{\alpha}{3})}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{(A+\frac{\alpha}{3})K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}} \tag{46}$$

Considering interaction with pressureless dark fluid *i.e.*, $p' = 0$ ($\rho' \neq 0$),

$$\rho_{total} = \rho + \rho' = \frac{B^2}{A+1} + \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{K^2}{A+1}\frac{1}{a^{3(A+1)}}, \quad (47)$$

$$p_{total} = p = -\frac{B^2}{A+1} + \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \quad (48)$$

The equation of state parameter for the second fluid

$$\omega' = \frac{p'}{\rho'} = \frac{p_{total} - p}{\rho_{total} - \rho}. \quad (49)$$

In the limiting case as $\omega' \to 0$, we get $p_{total} = p$. An interesting case emerges when the coupling parameter $\alpha = 2$ and $A = \frac{1}{3}$. A universe with dark energy, exotic matter and radiation to begin with (*i.e.*, before the interaction sets in) transits to a matter dominated phase.

Model II : The three fluids model

The original emergent universe model is composed of three non-interacting fluids. Here we bring interaction among themselves at $t \geq t_0$,

$$\dot{\rho_1} + 3H(\rho_1 + p_1) = -Q', \quad (50)$$

$$\dot{\rho_2} + 3H(\rho_2 + p_2) = Q, \quad (51)$$

$$\dot{\rho_3} + 3H(\rho_3 + p_3) = Q' - Q, \quad (52)$$

where Q and Q' represent the interaction terms.
Here $\rho_1 \longrightarrow$ DE density, $\rho_2 \longrightarrow$ DM and $\rho_3 \longrightarrow$ normal matter.

- $Q < 0 \Rightarrow$ energy transfer from DM sector to the other two constituents,
- $Q' > 0 \Rightarrow$ energy transfer from DE sector to the other two fluids
- $Q' < Q \Rightarrow$ energy loss from the matter sector.
- $Q = Q' \Rightarrow$ DE interacts only with the DM.

The equivalent effective uncoupled model is described by the conservation equations:

$$\dot{\rho}_1 + 3H(1 + \omega_1^{eff})\rho_1 = 0 \tag{53}$$

$$\dot{\rho}_2 + 3H(1 + \omega_2^{eff})\rho_2 = 0 \tag{54}$$

$$\dot{\rho}_3 + 3H(1 + \omega_3^{eff})\rho_3 = 0 \tag{55}$$

where the effective equation of state parameters are

$$\omega_1^{eff} = \omega_1 + \frac{Q'}{3H\rho_1}, \tag{56}$$

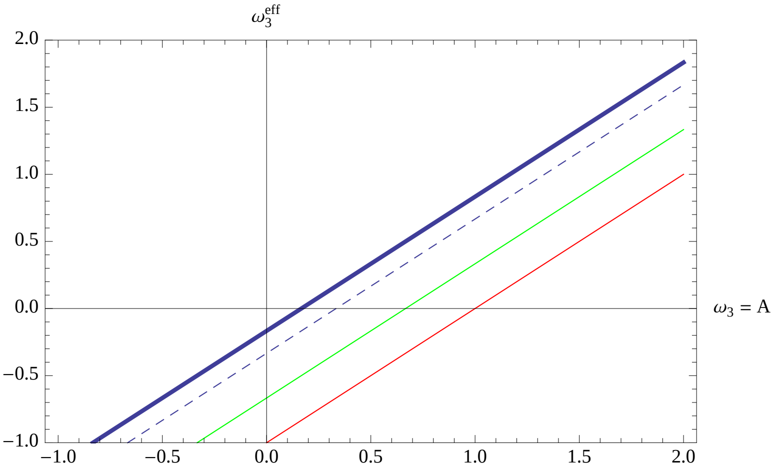
$$\omega_2^{eff} = \omega_2 - \frac{Q'}{3H\rho_2}, \tag{57}$$

$$\omega_3^{eff} = \omega_3 + \frac{Q - Q'}{3H\rho_3}. \tag{58}$$

Let us consider $Q - Q' = -\beta H \rho_3$, the effective state parameter for the normal fluid becomes

$$\omega_3^{eff} = \omega_3 - \frac{\beta}{3} \tag{59}$$

Plot the variation of effective equation of state parameter ω_3^{eff} with A which corresponds to ω_3 for different strengths of interaction determined by β. The strength of interaction is increased ω_3 which is actually determined by A for which $\omega^{eff} = 0$ (corresponds to matter domination). A universe with any A value is found to admit a matter domination phase which is not permitted in the absence of interaction.


ω_3^{eff}
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$\omega_3 = A$

Wormhole for EU

- Gravitational action for massive Gravity

$$S = -\frac{1}{8\pi}\int \left(\frac{1}{2}R + m^2 L\right)\sqrt{-g}\,d^4x + S_m \qquad (60)$$

- Massive Gravity Lagrangian

$$L = -\frac{1}{2}\left(K^2 - K^{\mu}_{\nu}K^{\nu}_{\mu}\right) + \frac{c_3}{3!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\sigma}K^{\mu}_{\alpha}K^{\nu}_{\beta}K^{\rho}_{\gamma} + \frac{c_4}{4!}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta} K^{\mu}_{\alpha}K^{\nu}_{\beta}K^{\rho}_{\gamma}K^{\sigma}_{\delta} \qquad (61)$$

where c_3, c_4 are constants and $K^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \gamma^{\mu}_{\nu}$, $\gamma^{\mu}_{\sigma}\gamma^{\sigma}_{\nu} = g^{\mu\nu}f_{\sigma\nu}$, and $f_{\sigma\nu}$ is symmetric tensor.

- For Euclidean time

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G\rho}{3} - \frac{m^2}{3}\left(4c_3 + c_4 - 6 + 3C\frac{3 - 3c_3 - c_4}{a}\right) - \frac{m^2}{3}\left(3C^2\frac{c_4 + 2c_3 - 1}{a^2} - C^3\frac{c_3 + c_4}{a^3}\right) \qquad (62)$$

Wormhole for EU

- Conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{63}$$

- The field equation can be rewritten as

$$\dot{a}^2 = V(a) \tag{64}$$

- $V(a) = -\frac{m^2}{3}\left[(4c_3 + c_4 - 6 + 3\Lambda)a^2 - \frac{c_3+c_4}{a} + X\right]$
-

$$\Lambda = \frac{8\pi G}{3m^2}\rho$$

$$X = 3(3 - 3c_3 - c_4)a + 3(c_4 + 2c_3 - 1)$$

$$p = A\rho - B\sqrt{\rho} \tag{65}$$

- we note existence of wormhole solution

$$\dot{a}^2 = 1 - \mu a^2 + \Sigma_{n=1}^{N} \frac{\nu_n}{a^{2n}}; \tag{66}$$

- The field equation can be rewritten as

$$\dot{a}^2 = \alpha - \beta a^2 - \frac{\gamma}{a^2} \tag{67}$$

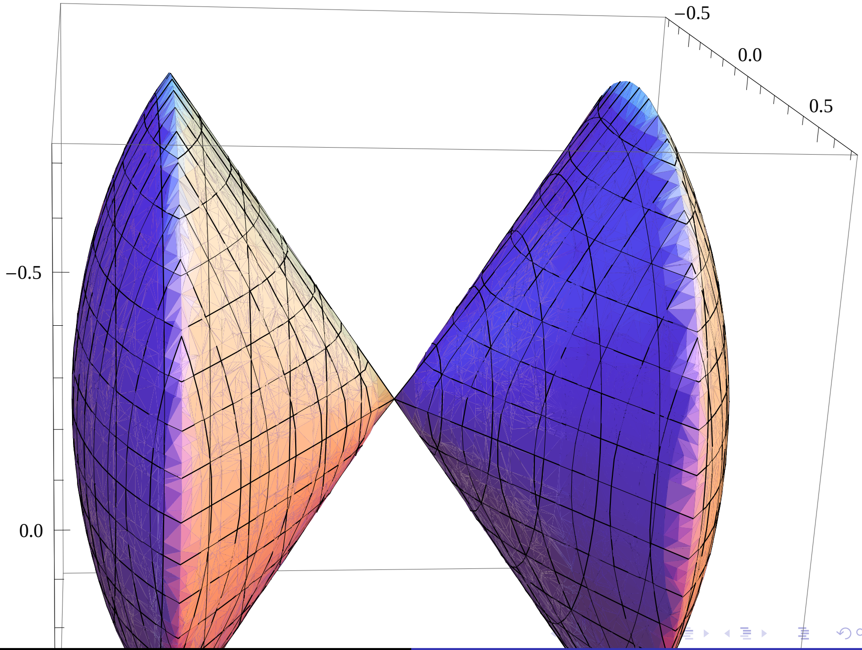
- where

$$\alpha = \frac{1}{2}\left(1 - \frac{4K}{B}\Lambda\right)$$

$$\beta = \frac{1}{2}(2\Lambda - 1) \quad \gamma = \left(\frac{K}{B}\right)^2 \Lambda$$

$$\frac{1}{2} < \Lambda < \frac{B}{4K}$$

- The above differential admits (i) $\dot{a}(\tau) < 0$ (ii)$\dot{a}(\tau) = 0$ (iii) $\dot{a}(\tau) < 0$
- It is found that in GTR with nonlinear EoS $\alpha < 0$ in flat universe. Consequently No WORMHOLE. However, in massive gravity $\alpha \geq 1$, WORMHOLE exists.


−0.5
0.0
0.5
−0.5
0.0

- The wormhole solution we obtain for EU model

$$a^2(\tau) = \frac{\frac{1}{4} - \frac{K}{B}\Lambda}{2\Lambda - 1} + a_o \; Cos\sqrt{2(2\Lambda - 1)} \;\; \tau \tag{68}$$

- where

$$a_o = \frac{\sqrt{\left(\frac{1}{4} - \frac{K}{b}\Lambda\right)^2 - 2\Lambda\left(\frac{K}{B}\right)^2}}{2\Lambda - 1}$$

$$\Lambda = \left(\frac{B}{A+1}\right)^2 \tag{69}$$

- Wormhole solution in GTR is not permitted in flat universe
- Wormhole solution in massive gravity theory is permitted in flat universe case.

Discussion

- Demonstrated the possibility of obtaining viable cosmological dynamics of the emergent universe.
  Two different cosmological models are presented:
- Model I, The interaction of fluids for emergent universe with a pressureless barotropic fluid is considered. The flow of energy from the fluids required to realize the emergent universe to a pressureless fluid which sets in at an epoch $t = t_i$.
- Model II, we consider interactions among the three fluids of the emergent universe at time $t = t_0$. Before this epoch the emergent universe can be realized without an interaction among the fluids. The problem with earlier cosmological realizations of the emergent universe was that once the EoS parameter A is fixed at a given value, the universe is unable to come out of the phase with a given composition of fluids. Here we overcome this problem by assigning an interaction among the fluids at the epoch t_0.

Discussion

- The early inflation can be accommodated with
  • scalar field in the standard model of particle physics.
  • modification of the gravitational action
  • a new graviatational action e.g., Brane World scenario, Viscous universe etc.
- The recent cosmic acceleration can be understood making use of
  • modification of the graviatational sector of the Einstein's Gravitational action
  • modification of the matter sector
  • a new graviatational action

- Demonstrated the possibility of obtaining viable cosmological dynamics of the emergent universe.
  Two different cosmological models are presented:
- Model I, The interaction of fluids for emergent universe with a pressureless barotropic fluid is considered. The flow of energy from the fluids required to realize the emergent universe to a pressureless fluid which sets in at an epoch $t = t_i$.
- Model II, we consider interactions among the three fluids of the emergent universe at time $t = t_0$. Before this epoch the emergent universe can be realized without an interaction among the fluids. The problem with earlier cosmological realizations of the emergent universe was that once the EoS parameter A is fixed at a given value, the universe is unable to come out of the phase with a given composition of fluids. Here we overcome this problem by assigning an interaction among the fluids at the epoch t_0.

Discussion ... ... contd.

- This feature represents a clear improvement over the earlier cosmological scenarios in an emergent universe where it is rather difficult to accommodate a pressureless matter fluid phase.
- A cosmological evolution of the observed universe through unified dynamics of associated matter and dark energy components thus becomes feasible in the emergent universe scenario.
- Consistency of the interacting fluid emergent universe scenario with the generalized second law of thermodynamics is also stuided.
- Further work is needed for a comparative analysis of the interacting fluid emergent universe cosmology with more popular current cosmological models. Detailed analysis of

THANK YOU