The k-essence scalar field in the context of Supernova Ia Observations

Arka Moulik

Collaborators: Abhijit Bandyopadhyay, Debashis Gangopadhyay
Department of Physics
Ramakrishna Mission Vivekananda University
Belur Math, Howrah

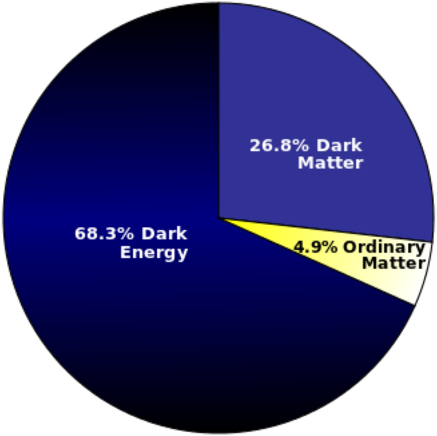
AAPCOS 2015

Introduction

- Present universe is undergoing a phase of accelerated expansion: $\boxed{\text{Observational evidences}}$
  1. SNe Ia data: Luminosity distance vs redshift measurements of type Ia Supernovae events
  2. Observation of Baryon Acoustic Oscillations (BAO)
  3. Observation of Cosmic Microwave Background Radiations (CMBR)
  4. Power spectrum of matter distributions in the universe
- Dark Energy: A general label useful in explaining observed accelerated expansion of the universe.

Introduction : Dark Energy

- Hypothetical unclusterd form of energy with negative pressure.
- Analysis of WMAP & Planck data: Roughly 68% of the content of the present universe consists of dark energy.



Introduction : Dark Energy

- The very nature and origin of dark energy still remains a mystery despite many years of research.
- Several theoretical models for dark energy exist
  - k-essence
  - $\Lambda - CDM$ model (plagued with fine tuning problem of particle physics)
  - Quintessence
  - $f(R)$ gravity
  - Scalar-tensor theories
  - Brane world models

K-essence

- Kinetic essence (k-essence) : A theory with non-canonical kinetic terms
- First proposed by Born and Infeld in order to get rid of the infinite self-energy of the electron.
- Later W.Heisenberg studied such non-linear scalar field theories in connection to physics of cosmic rays and meson production.
- Effective field theories arising from string theory (particularly in D-branes models) also have non-canonical kinetic terms.
- Cosmology witnessed scalar fields having non-canonical kinetic terms that drives inflation.
- $k-$essence models as dynamical dark energy were also constructed to solve the cosmic coincidence problem.
- Dark matter can also be described using k-essence or tachyon fields.

Our work

- Makes an attempt to establishes a phenomenological connection between a *k*-essence scalar field model and SNe Ia observations.
- A principal motivation of this work is to investigate whether the similar scalar fields can account for inflation as well as late time accelerated expansion.
- Plan:
  - Brief overview of the *k*-essence scalar field (ϕ) model involving the a kinetic term $F(X)$ with $X = (1/2)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  - Establishing connection between quantities involved in $k-essence$ model of dark energy (ϕ,$F(X)$) and the cosmological paramters *viz.* FRW-scale factors and its time detivatives
  - Methodology of analysis of SNe Ia data
  - Extraction of the relevant phenomenological parameters from data.
  - Obtaining phenomenological solutions for ϕ and $F(X)$.
  - Conclusions

k-essence scalar field model : framework

- Lagrangian: $L = -V(\phi)F(X)$
- $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$
- $V(\phi)$ is the potential.
- Energy density, $\rho = V(\phi)(F - 2XF_X)$
- Pressure, $p = L$

K-essence scalar field in FLRW background

- Observation: Universe is isotropic and homogeneous at large scales and is ever-expanding
- Spacetime Geometry described at this scale by FLRW metric: $ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$

  $r, \theta, \varphi \rightarrow$ comoving coordinates
  $t \rightarrow$ time measured by a comoving observer
  $a(t) \rightarrow$ scale factor with $\dot{a} > 0$ (expanding universe), $K \rightarrow$ curvature constant

- In a FLRW background filled with an ideal perfect fluid (ρ, p) the Einstein's field equations give Friedmann equaitons:

  $$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}} \qquad \boxed{\dot{H} + 4\pi G(\rho + p) - K/a^2 = 0}$$

- Continuity equation: $\boxed{\dot{\rho} + 3H(\rho + p) = 0}$ (K-independent)

K-essence scalar field in FLRW background

- Homogeneous ϕ: $\phi(t, \vec{x}) \rightarrow \phi(t)$
- $X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \rightarrow \frac{1}{2} \dot{\phi}^2$
- $\dot{\rho} + 3H(\rho + p) = 0$ gives EOM of k-essence field as

$$\boxed{(F_X + 2XF_{XX})\ddot{\phi} + 3HF_X\dot{\phi} + (2XF_X - F)\frac{V_\phi}{V} = 0}$$

$$V_\phi = dV/d\phi,\ F_X = dF/dX,\ F_{XX} = d^2F/dX^2$$

- Constant $V(\phi)$: $V_\phi = 0$ gives the scaling relation:

$$\boxed{XF_X^2 = Ca^{-6}}$$

Implies presence of relevant scales in the theory
$\Rightarrow$ Fundamental aspect of $k-$essence model with constant potential V.

K-essence scalar field in FLRW background

- Using scaling relation in Friedmann equation we get $\dot{H} - \frac{K}{a^2} - 4\sqrt{2}\sqrt{C}\pi G V \frac{\dot{\phi}}{a^3} = 0$
- Rescale the field: $4\sqrt{2}\sqrt{C}\pi G V \phi \rightarrow \phi$
- Connection between scalar field and scale factor and its time derivatives

$$\boxed{\dot{H} - \frac{K}{a^2} - \frac{\dot{\phi}}{a^3} = 0}$$

- Using $X = (1/2)\dot{\phi}^2$ and scaling relation $XF_X^2 = Ca^{-6}$ we also get

$$\boxed{\frac{dF}{dt} = \sqrt{2C}\frac{1}{a^3}\frac{d\dot{\phi}}{dt}}$$

Extracting cosmological parameters from analysis of SNe Ia data

- We take a closed form parametrisation of the luminosity distance, d_L

$$d_L(\alpha, \beta, z) = \frac{c}{H_0}\left(\frac{z(1+\alpha z)}{1+\beta z}\right) \tag{1}$$

  c is the speed of light.
  $H_0 = 100\ h$ km s^{-1} Mpc^{-1} is the value of the Hubble parameter at the present epoch.

- Distance modulus μ is related to Luminosity distance D_L which we parametrize in terms of z as
  $\mu_{\mathrm{th}}(\alpha, \beta, z) = 5 \log_{10}\left[\frac{z(1+\alpha z)}{1+\beta z}\right] + \mu_0$

$$\mu_0 = 42.38 - 5 \log_{10} h$$
$$H_0 = 100\ h \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

- Observed quantities:
  Distance modulus $\mu_{\text{obs}}(z_i)$ of Supernova Ia events and corresponding redshifts z_i
- Collection of different compilations of SNe Ia data with 192 data points:
  HST+SNLS+ESSENCE SALT2 and MLCS data, UNION, UNION2 data.
- **Gemini Deep Deep Survey GDDS , SPICES and VDSS** surveys provide the values of the Hubble parameter at 15 different redshift values.

Extracting cosmological parameters from analysis of SNe Ia data

- Minimisation of χ^2 function:

$$\chi^2(\alpha,\beta) = \chi^2_{\rm SN}(\alpha,\beta) + \chi^2_{\rm OHD}(\alpha,\beta) \ .$$

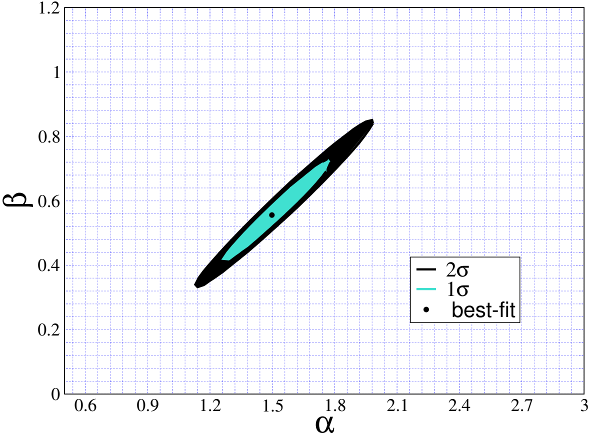
where

$$\chi^2_{\rm SN}(\alpha,\beta) = \sum_{i=1}^{192} \frac{(\mu_{obs}(z_i) - \mu_{th}(\alpha,\beta,z))^2}{\sigma_i^2}$$

and

$$\chi^2_{\rm OHD}(\alpha,\beta) = \sum_{i=1}^{15} \left[\frac{H(\alpha,\beta;z_i) - H_{\rm obs}(z_i)}{\Sigma_i}\right]^2 \ ,$$

- Minimisation is obtained at $\alpha = 1.50 \quad , \quad \beta = 0.55$
- $\chi^2_{min} = 204.94$.



Determining $\phi(t)$ from the analysis of SNe Ia data

- Hubble parameter $H(z)$ is directly related to the luminosity distance through the relation
  $\frac{H}{H_0} = \left[\frac{d}{dz}\left(\frac{D_L(\alpha,\beta,z)}{1+z}\right)\right]^{-1}$
- From the equations $H = \dfrac{\dot{a}}{a}$ and $\dfrac{a_0}{a} = 1 + z$ we get
  $$dt = -\frac{dz}{(1+z)H}$$
- Introduce a dimensionless parameter
  $\tau = H_0\Big(t(z) - t_0\Big) = -H_0 \int_0^z \frac{dx}{(1+x)H(x)}$
- $\dot{H} - \frac{K}{a^2} - \frac{\dot{\phi}}{a^3} = 0$ becomes $\frac{d\phi}{d\tau} = \frac{a^3(\tau)}{H_0}\frac{dH}{d\tau} - a(\tau)K$

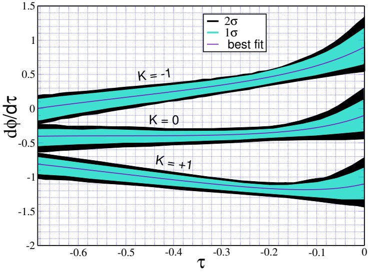


Figure : Plot of $d\phi/d\tau$ vs τ for $K = +1, 0, -1$. Solid lines represent the plots drawn for best fit values of α and β. The corresponding 1σ and 2σ variations are shown by shaded regions.

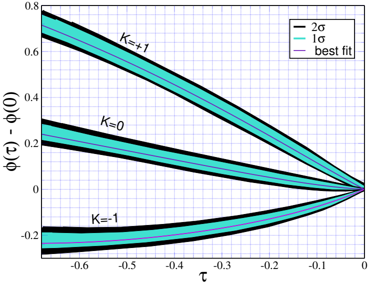


Figure : Plot of $\phi(\tau) - \phi(0)$ vs τ for $K = +1, 0, -1$. Solid lines represent the plots drawn for best fit values of α and β. The corresponding 1σ and 2σ variations are shown by shaded regions.

- The obtained time dependence of $\phi(\tau)$ can be fitted with

$$\phi(\tau) - \phi(0) \quad \sim \quad \lambda_1 \tau + \lambda_2 \tau^2 \tag{2}$$

to an appreciable extent of statistical precision.

- Minmization of the following $\tilde{\chi}^2$ function with respect to the parameters λ_1 and λ_2,

$$\tilde{\chi}^2 \quad = \quad \sum_{i=1}^{n} \left[\frac{\Phi_{\rm est}(\tau_i) - \Phi_{\rm fit}(\tau_i, \lambda_1, \lambda_2)}{\Delta\Phi_{\rm est}(\tau_i)} \right]^2 \tag{3}$$

- $\Phi_{\rm est}(\tau_i) \equiv \phi(\tau_i) - \phi(0)$.
- $\Phi_{\rm fit}(\tau_i, \lambda_1, \lambda_2) = \lambda_1 \tau_i + \lambda_2 {\tau_i}^2$.
- $\Delta\Phi_{\rm est}(\tau_i)$ is the average value of estimated 1σ uncertainty in $\Phi_{\rm est}(\tau_i)$.

Table : Values of λ_1, λ_2 and $\tilde{\chi}^2_{\rm minimum}/n$ for $K = 1, 0, -1$.

K	λ_1	λ_2	$\tilde{\chi}^2_{\rm minimum}/n$
$+1$	-1.22	-0.23	0.02
0	-0.23	0.208	0.03
-1	0.76	0.64	0.035

$F(X)$ as obtained from

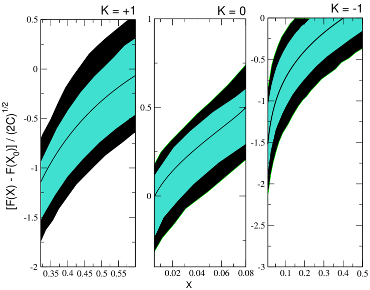


Figure : Plot of the quantity $[F(X) - F(X_0)]/\sqrt{2C}$ vs X with their 1σ and 2σ allowed ranges as obtained from analysis of observational data

Discussions and Conclusions

- **A model of $k-$essence incorporating a scaling relation can be accommodated within the luminosity distance-redshift data observed for type Ia Supernova.**
- **Existence of the scaling relation within the framework of k-essence model allows to establish contact between the spatially homogeneous k-essence scalar field, and the scale factor and the Hubble parameter associated with the expansion of the FRW universe.**
- **The time dependences of the scale factor and the Hubble parameter from the observed SNe Ia data allows one to find temporal behaviour of the $k-$essence field $\phi(\tau)$. This comes out to be $\phi(t) \sim \lambda_0 + \lambda_1\tau + \lambda_2\tau^2$.**

- The use of scaling relation helps to obtain the form of the function $F(X) - F(X_0)$ upto an arbitrary multiplicative constant occurring in the scaling relation itself.
- We know in case of homogeneous inflation the e.o.m of the inflaton field is given by $\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$. For slow-roll approximation $V(\phi)(\gg \dot{\phi}^2)$, H is approximately a constant and the last equation implies $\phi \sim Ae^{-\gamma t} \sim \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \cdots$.
- The temporal domains of the two scalar fields (k-essence and quintessence) are well separated. Therefore, similar scalar fields (so far as their time evolution is concerned) can account for inflation as well as dark energy driven accelerated expansion.
- Moreover, the scaling relation implies that for well behaved $F(X)$, X cannot go to zero. This is borne out by the above results.

THANK YOU