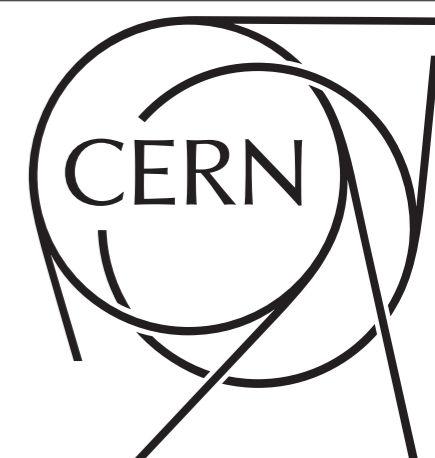


ELECTRO**W**EAK CORRECTIONS AND **C**OMPLEX-**M**ASS **S**CHEME

HUA-SHENG SHAO

CERN, PH-TH

2015.06.02



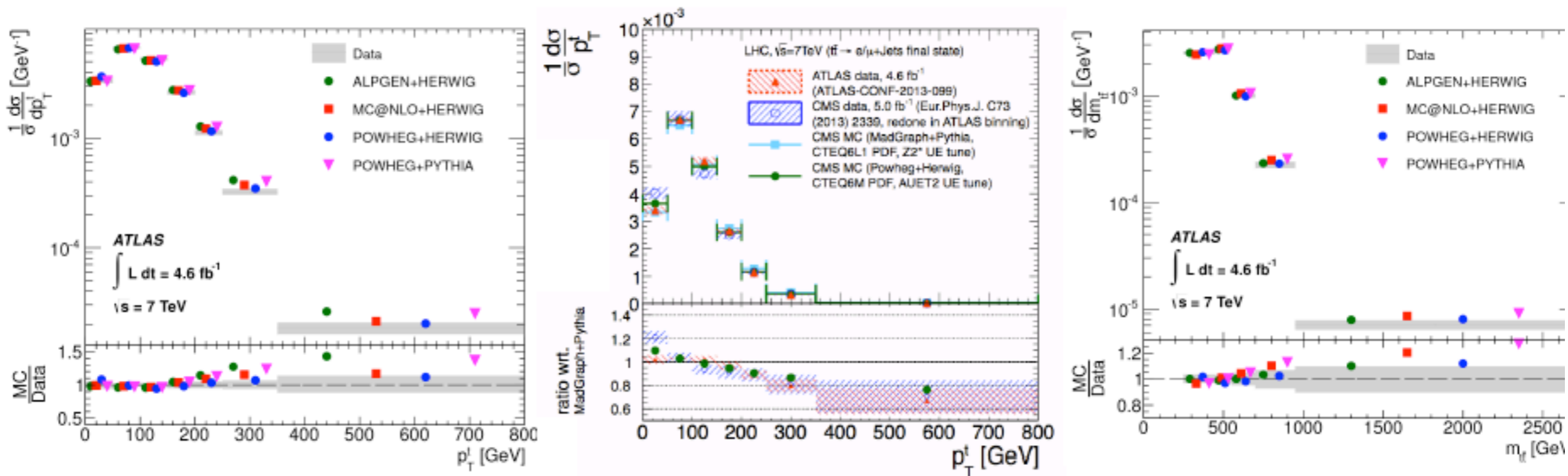
REVIEW THE STATUS OF MADGRAPH5_AMC@NLO-EW

EW OVERVIEW

	Yes	No
MadLoop	<p><i>EW corrections to any SM process w or w/o b mass in two schemes; Order splitting in mixed order case; Decay processes;</i></p>	<p><i>Complex-Mass Scheme</i></p>
MadFKS	<p><i>FKS QED subtraction; Order splitting in mixed order case; Scale and PDF uncer.; MC over helicity;</i></p>	<p><i>Virtual Trick; Quasi-collinear subtraction; On-shell subtraction; More cross check (tT,tTV);</i></p>

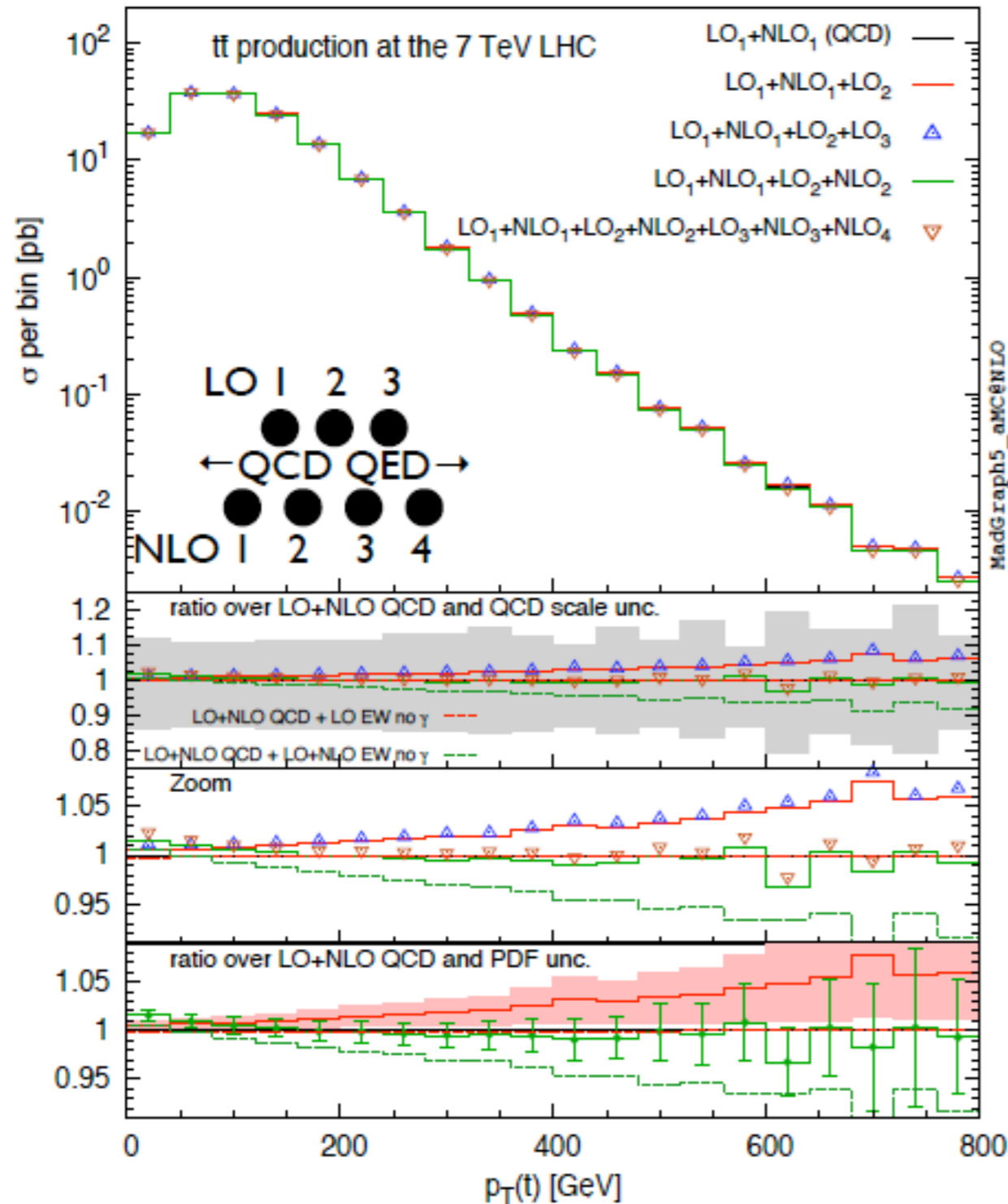
EW corrections to $t\bar{t}$ production

- ATLAS and CMS see some ‘anomaly’ on the top p_T distribution and $t\bar{t}$ invariant mass
- Data are softer than NLO QCD MonteCarlos (up to 30-40%)

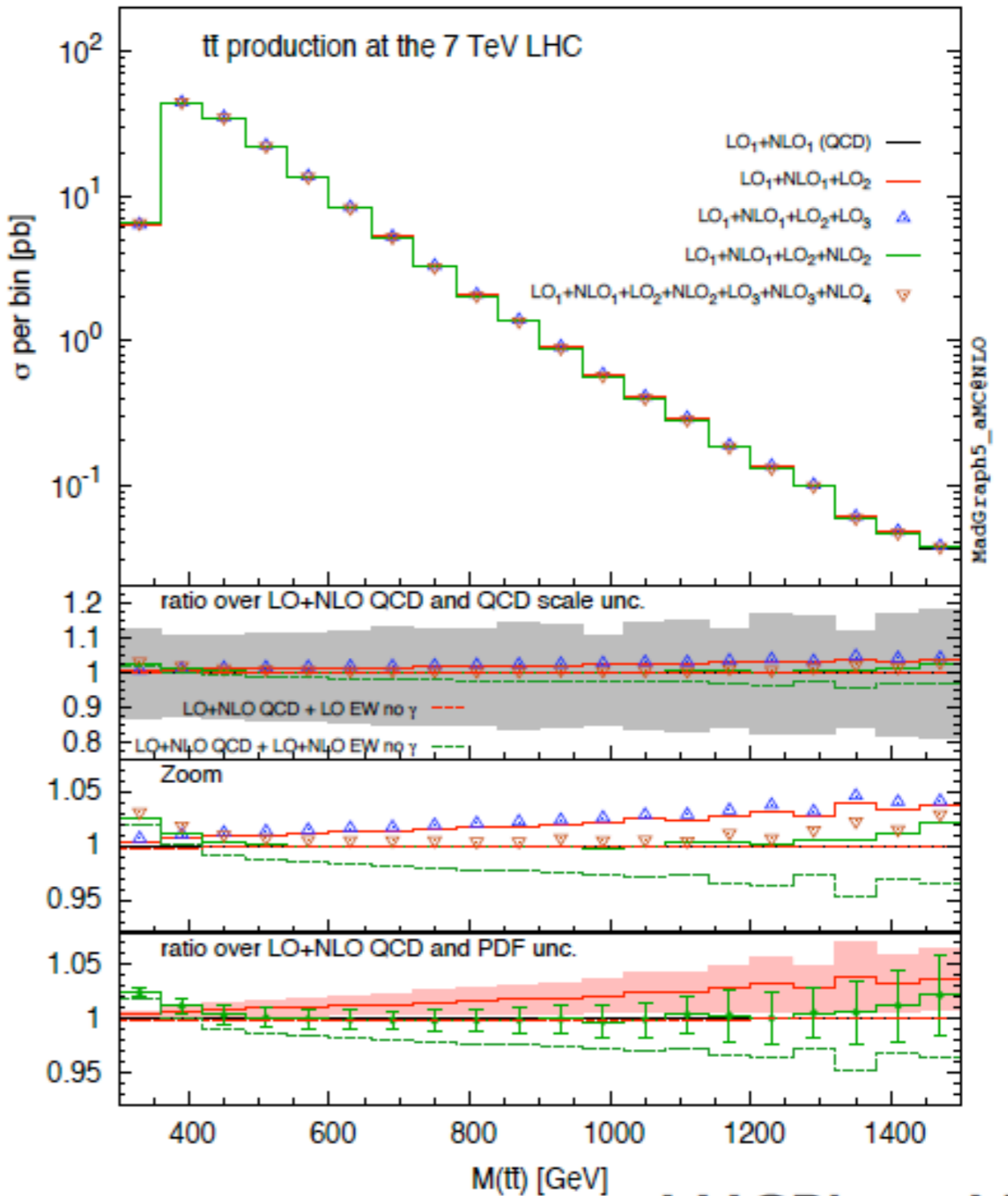


- Is it an EW effect?

EW corrections to $t\bar{t}$ production



Marco Zaro, 2-6-2015



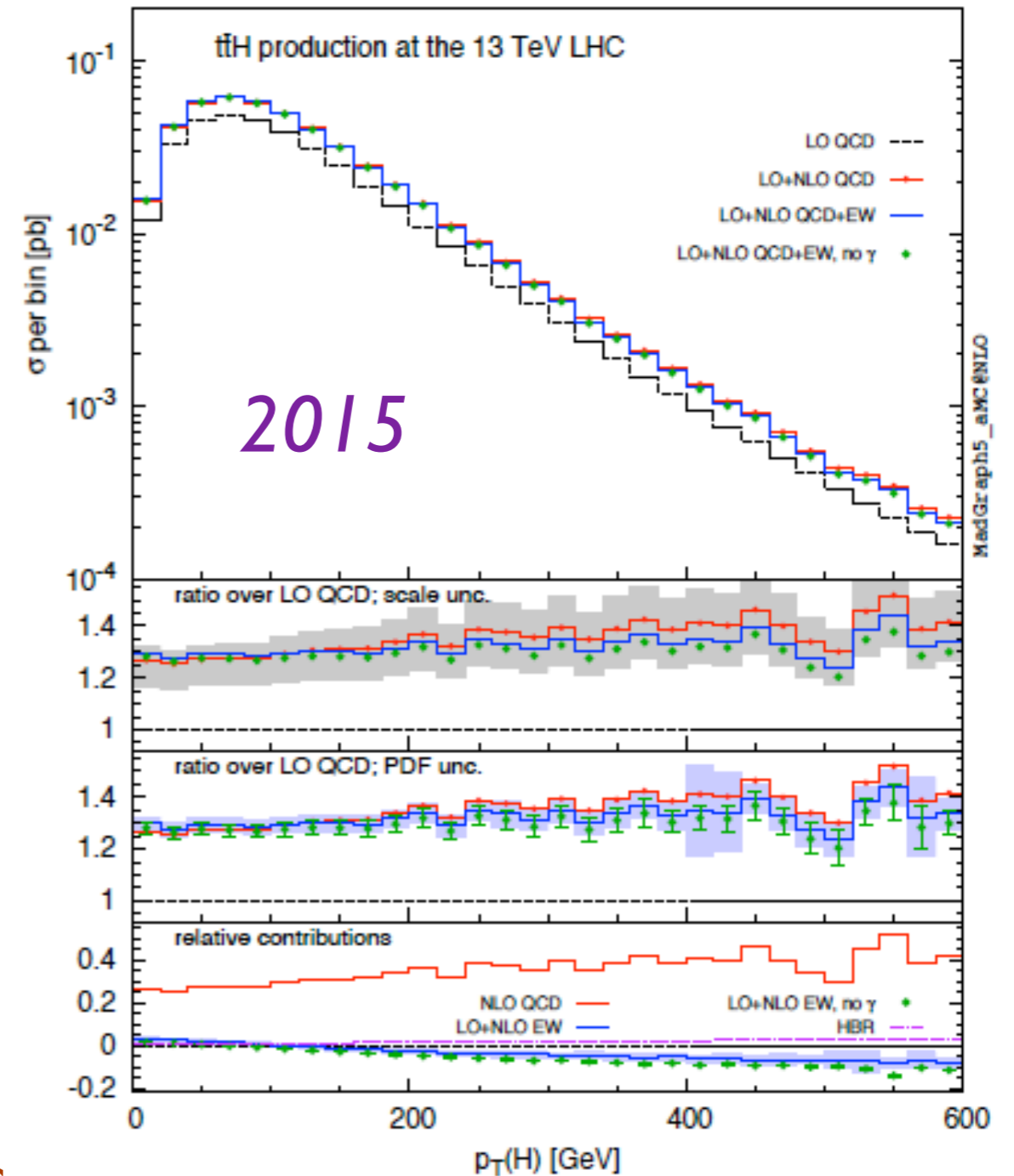
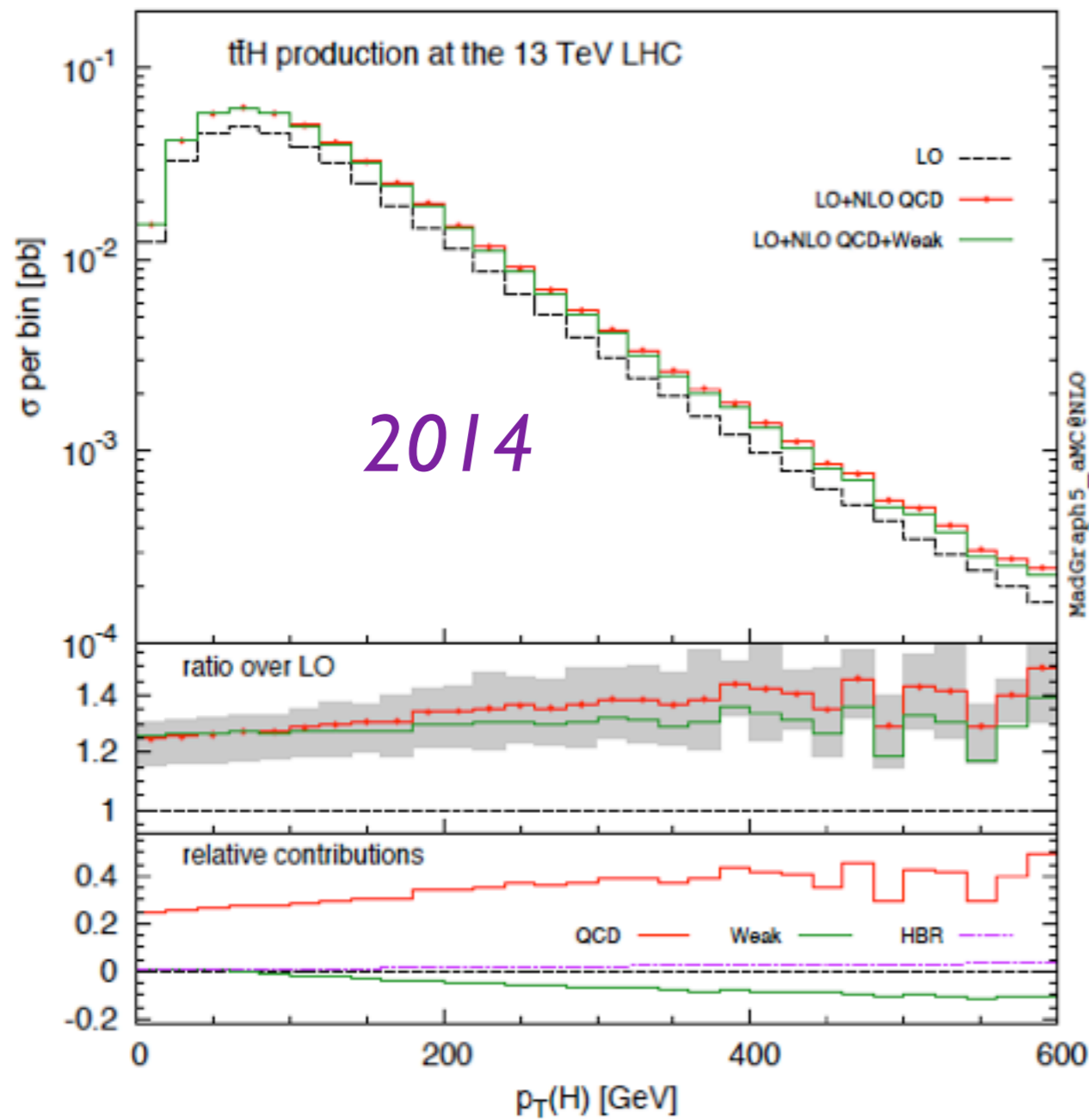
2

LHCPhenoNet

EW corrections to $t\bar{t}$ production

- EW corrections account at most -10% at large p_T , -5% at large mass
- Photon effect as large as EW corrections, but almost 100% uncertain
- Subleading corrections (LO_3 , $NLO_{3,4}$) very small

FROM WEAK TO ELECTROWEAK



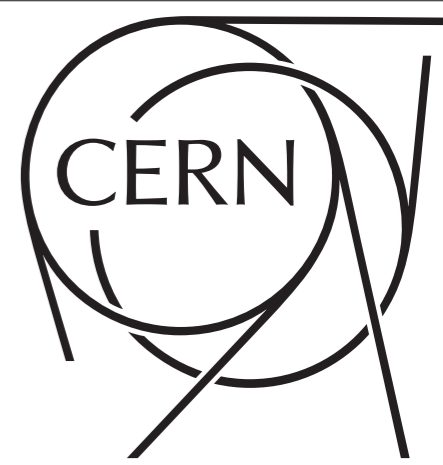
- **Improvements:** automated IR subtraction; photon emission; photon-induced process;

TO DO/XCHECK LIST

- **Drell-Yan**
- **W+jet(s)** Openloops group arXiv:1412.5157
- **Z+jet(s)**
- **Di-jet (, 3-jets** t3/t2 to extract α_S)
- **Single top**
- **tT+photon/jet** FB/charge asymmetry
- ...

We are ready !!!

Next Step: Matching to Parton Shower ?



COMPLEX-MASS SCHEME: CONCEPTS AND TECHNIQUES

- CMS

- A method to deal with unstable particle field in quantum field theories [A. Denner, S. Dittmaier etc]
- Unlike unstable-particle effective field theory [M. Beneke, A. Chapovsky, A. Signer, G. Zanderighi] , the introduction of complex mass for unstable particle makes it applicable to the whole phase space directly and hence suitable for automation.
- It reorganizes the bare Lagrangian \rightarrow same bare Lagrangian.
- Change mass (like W, Z, top, Higgs etc) to be complex $M^2 \rightarrow M^2 - i G M$ (M is mass, G is width).

- **CMS** beyond LO

- On-shell renormalization \rightarrow Complex renormalization

$$m_{cms}^2 = M^2 - i\Gamma M,$$

$$m_{cms}^2 = M_0^2 - \Sigma(m_{cms}^2),$$

$$\delta Z_{cms} = -\Sigma'(p^2)|_{p^2=m_{cms}^2}.$$

- **CMS** beyond LO

- On-shell renormalization \rightarrow Complex renormalization

$$m_{cms}^2 = M^2 - i\Gamma M,$$

$$m_{cms}^2 = M_0^2 - \Sigma(m_{cms}^2),$$

$$\delta Z_{cms} = -\Sigma'(p^2)|_{p^2=m_{cms}^2}.$$

BUT, is this the whole story ?

- **CMS** beyond LO

- On-shell renormalization \rightarrow Complex renormalization

$$m_{cms}^2 = M^2 - i\Gamma M,$$

$$m_{cms}^2 = M_0^2 - \Sigma(m_{cms}^2),$$

$$\delta Z_{cms} = -\Sigma'(p^2)|_{p^2=m_{cms}^2}.$$

BUT, is this the whole story ?

- Perturbation unitarity ? [Bauer, Gegelia etc; Denner, Lang]
- Relation between **CMS** and Narrow-Width Approximation ?
- Which level accuracy of width we need ?
- What cross-checks can we performed ?

ARCHITECTURES

- **Yes**

- Change Mass to Complex Mass
- Masses in one-loop integral can be complex -> **OneLoop**, **CutTools**, **IREGI**, **Golem95**, **PJFry++**

- **No**

- NLO width -> NLO accuracy -> **SMWidth** for SM
- Extending Feynman integral/UV CTs to second RS

A CLOSER LOOK

- Dyson summation $\frac{i}{p^2 - M_0^2 + \Sigma(p^2)}$

A CLOSER LOOK

- Dyson summation $\frac{1}{p^2 - M_0^2 + \Sigma(p^2)}$
- Complex renormalization

$$M^2 = M_0^2 - \text{Re}\Sigma(M^2 - i\Gamma M),$$

$$\Gamma = \frac{\text{Im}\Sigma(M^2 - i\Gamma M)}{M}.$$

A CLOSER LOOK

- Dyson summation $\frac{i}{p^2 - M_0^2 + \Sigma(p^2)}$
- Complex renormalization

$$\begin{aligned}
 M^2 &= M_0^2 - \text{Re}\Sigma(M^2 - i\Gamma M), \\
 \Gamma &= \frac{\text{Im}\Sigma(M^2 - i\Gamma M)}{M}.
 \end{aligned}
 \quad \rightarrow \quad
 \frac{\Gamma}{M} = \mathcal{O}(\alpha).$$

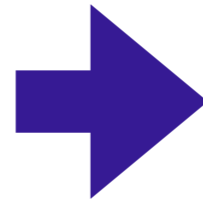
A CLOSER LOOK

- Dyson summation $\frac{i}{p^2 - M_0^2 + \Sigma(p^2)}$

- Complex renormalization

$$M^2 = M_0^2 - \text{Re}\Sigma(M^2 - i\Gamma M),$$

$$\Gamma = \frac{\text{Im}\Sigma(M^2 - i\Gamma M)}{M}.$$



$$\frac{\Gamma}{M} = \mathcal{O}(\alpha).$$

- Feynman integral: physical region

$$p_i^2 \rightarrow p_i^2 + i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} + i\varepsilon,$$

$$m_i \rightarrow m_i - i\varepsilon.$$

- non-physical region

$$p_i^2 \rightarrow p_i^2 - i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} - i\varepsilon,$$

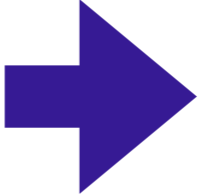
$$m_i \rightarrow m_i - i\varepsilon.$$

A CLOSER LOOK

• Dyson summation $\frac{i}{p^2 - M_0^2 + \Sigma(p^2)}$

• Complex renormalization

$$M^2 = M_0^2 - \text{Re}\Sigma(M^2 - i\Gamma M),$$

$$\Gamma = \frac{\text{Im}\Sigma(M^2 - i\Gamma M)}{M}.$$


$$\frac{\Gamma}{M} = \mathcal{O}(\alpha).$$

• Feynman integral: physical region

$$p_i^2 \rightarrow p_i^2 + i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} + i\varepsilon,$$

$$m_i \rightarrow m_i - i\varepsilon.$$

non-physical region

$$p_i^2 \rightarrow p_i^2 - i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} - i\varepsilon,$$

$$m_i \rightarrow m_i - i\varepsilon.$$

1st RS

$$\log\left(-\frac{p^2 + i\varepsilon}{m^2 - i\varepsilon}\right) = \log\frac{p^2}{m^2} - i\pi$$

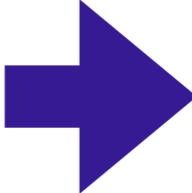
$$\log\left(-\frac{p^2 - i\varepsilon}{m^2 - i\varepsilon}\right) = \log\frac{p^2}{m^2} + i\pi$$

A CLOSER LOOK _{*i*}

• Dyson summation $\frac{1}{p^2 - M_0^2 + \Sigma(p^2)}$

• Complex renormalization

$$M^2 = M_0^2 - \text{Re}\Sigma(M^2 - i\Gamma M),$$

$$\Gamma = \frac{\text{Im}\Sigma(M^2 - i\Gamma M)}{M}.$$


$$\frac{\Gamma}{M} = \mathcal{O}(\alpha).$$

• Feynman integral: physical region

$$p_i^2 \rightarrow p_i^2 + i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} + i\varepsilon,$$

$$m_i \rightarrow m_i - i\varepsilon.$$

non-physical region

$$p_i^2 \rightarrow p_i^2 - i\varepsilon,$$

$$s_{ij} \rightarrow s_{ij} - i\varepsilon,$$

$$m_i \rightarrow m_i - i\varepsilon.$$

1st RS

$$\log\left(-\frac{p^2 + i\varepsilon}{m^2 - i\varepsilon}\right) = \log\frac{p^2}{m^2} - i\pi$$

$$\log\left(-\frac{p^2 - i\varepsilon}{m^2 - i\varepsilon}\right) = \log\frac{p^2}{m^2} + i\pi$$

2nd RS

$$\log\left(-\frac{p^2 - i\varepsilon}{m^2 - i\varepsilon}\right) = \log\frac{p^2}{m^2} - i\pi$$

- Narrow-Width Approximation:

- XS level $\sigma(pp \rightarrow Z \rightarrow \ell^+ \ell^-) = \sigma(pp \rightarrow Z) \frac{\Gamma(Z \rightarrow \ell^+ \ell^-)}{\Gamma_Z^{\text{tot}}}$

- ME level $\mathcal{A}(e^+ e^- \rightarrow Z \rightarrow \ell^+ \ell^-) = \mathcal{A}(e^+ e^- \rightarrow Z) \frac{i}{p^2 - M_Z^2} \mathcal{A}(Z \rightarrow \ell^+ \ell^-)$

- Relation between **CMS** and Narrow-Width Approximation ?

- Non-resonance region: **CMS** = NWA + higher-order effect

- Resonance region: **CMS** = NWA + finite-width effect

$$\frac{1}{(p^2 - M^2)^2 + \Gamma^2 M^2} = \frac{\pi}{\Gamma M} \delta(p^2 - M^2) + \mathcal{O}(1)$$

NON-RESONANCE REGION



NON-RESONANCE REGION

- Born



$$\begin{aligned}
 &= \mathcal{A}_{\text{CMS,Born,p}}^\mu \left(-g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} - (1 - \xi) i \frac{\xi \Gamma_W M_W}{q^2 - \xi M_W^2} \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} + \mathcal{O}(\alpha^2) \right) \mathcal{A}_{\text{CMS,Born,d}}^{\nu*} \\
 &\times \left[\frac{1}{(q^2 - M_W^2)} - i \frac{\Gamma_W M_W}{(q^2 - M_W^2)^2} + \mathcal{O}(\alpha^2) \right]
 \end{aligned}$$

NON-RESONANCE REGION

- Born



$$= \mathcal{A}_{\text{CMS,Born,p}}^\mu \left(-g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} - (1 - \xi) i \frac{\xi \Gamma_W M_W}{q^2 - \xi M_W^2} \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} + \mathcal{O}(\alpha^2) \right) \mathcal{A}_{\text{CMS,Born,d}}^{\nu*}$$

$$\times \left[\frac{1}{(q^2 - M_W^2)} - i \frac{\Gamma_W M_W}{(q^2 - M_W^2)^2} + \mathcal{O}(\alpha^2) \right]$$

- Virtual/UV



$$\sim \mathcal{A}_{\text{CMS,Born,p}}^\mu (-g_{\mu\nu}) \mathcal{A}_{\text{CMS,Born,d}}^\nu \frac{i \Gamma_W M_W}{(q^2 - M_W^2)^2}$$

NON-RESONANCE REGION

- Born



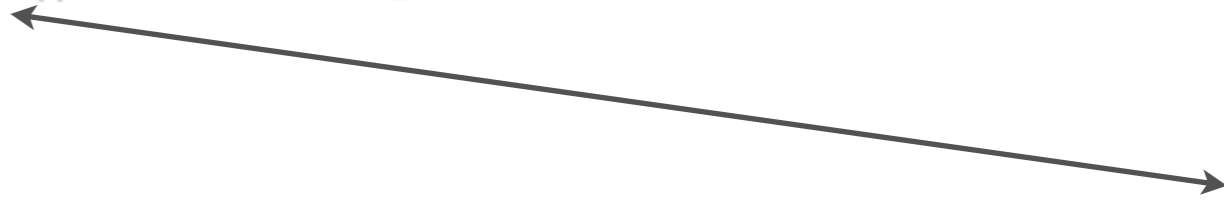
$$= \mathcal{A}_{\text{CMS,Born,p}}^\mu \left(-g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} - (1 - \xi) i \frac{\xi \Gamma_W M_W}{q^2 - \xi M_W^2} \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} + \mathcal{O}(\alpha^2) \right) \mathcal{A}_{\text{CMS,Born,d}}^{\nu*}$$

$$\times \left[\frac{1}{(q^2 - M_W^2)} - i \frac{\Gamma_W M_W}{(q^2 - M_W^2)^2} + \mathcal{O}(\alpha^2) \right]$$

- Virtual/UV



$$\sim \mathcal{A}_{\text{CMS,Born,p}}^\mu (-g_{\mu\nu}) \mathcal{A}_{\text{CMS,Born,d}}^\nu \frac{i \Gamma_W M_W}{(q^2 - M_W^2)^2}$$



NON-RESONANCE REGION

- Born



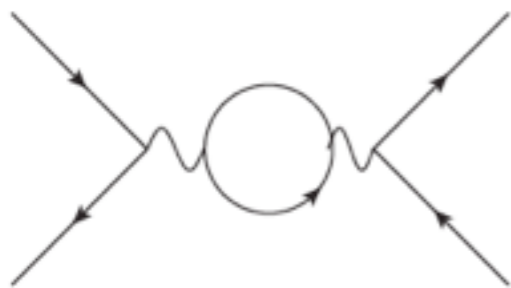
a parameter from W propagator

$$= \mathcal{A}_{\text{CMS,Born,p}}^\mu \left(-g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} - (1 - \xi) i \frac{\xi \Gamma_W M_W}{q^2 - \xi M_W^2} \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} + \mathcal{O}(\alpha^2) \right) \mathcal{A}_{\text{CMS,Born,d}}^{\nu*}$$

$$\times \left[\frac{1}{(q^2 - M_W^2)} - i \frac{\Gamma_W M_W}{(q^2 - M_W^2)^2} + \mathcal{O}(\alpha^2) \right]$$

W width exp. from the imaginary part: optical theorem

- Virtual/UV



\sim

$$\mathcal{A}_{\text{CMS,Born,p}}^\mu (-g_{\mu\nu}) \mathcal{A}_{\text{CMS,Born,d}}^\nu \frac{i \Gamma_W M_W}{(q^2 - M_W^2)^2}$$

NON-RESONANCE REGION

- Born



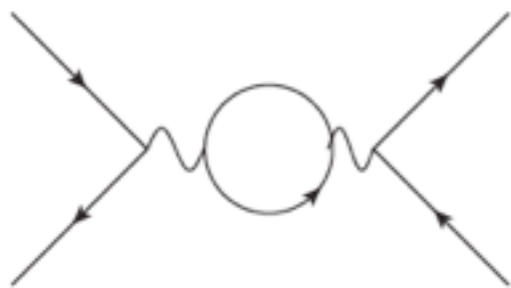
a parameter from W propagator

$$= \mathcal{A}_{\text{CMS,Born,p}}^\mu \left(-g_{\mu\nu} + (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} - (1 - \xi) i \frac{\xi \Gamma_W M_W}{q^2 - \xi M_W^2} \frac{q_\mu q_\nu}{q^2 - \xi M_W^2} + \mathcal{O}(\alpha^2) \right) \mathcal{A}_{\text{CMS,Born,d}}^{\nu*}$$

$$\times \left[\frac{1}{(q^2 - M_W^2)} - i \frac{\Gamma_W M_W}{(q^2 - M_W^2)^2} + \mathcal{O}(\alpha^2) \right]$$

W width exp. from the imaginary part: optical theorem

- Virtual/UV



$$\sim \mathcal{A}_{\text{CMS,Born,p}}^\mu (-g_{\mu\nu}) \mathcal{A}_{\text{CMS,Born,d}}^\nu \frac{i \Gamma_W M_W}{(q^2 - M_W^2)^2}$$

- We need the exact (LO) width and the correct RS to guarantee it !
- It only contributes to the imaginary part. For a $2 > n$ ($n \leq 2$) process, this piece will cancel in ME2. -> We need trick to check a $2 > 2$ process !

NON-RESONANCE REGION

- Cross-check method

$2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

D.3 $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$

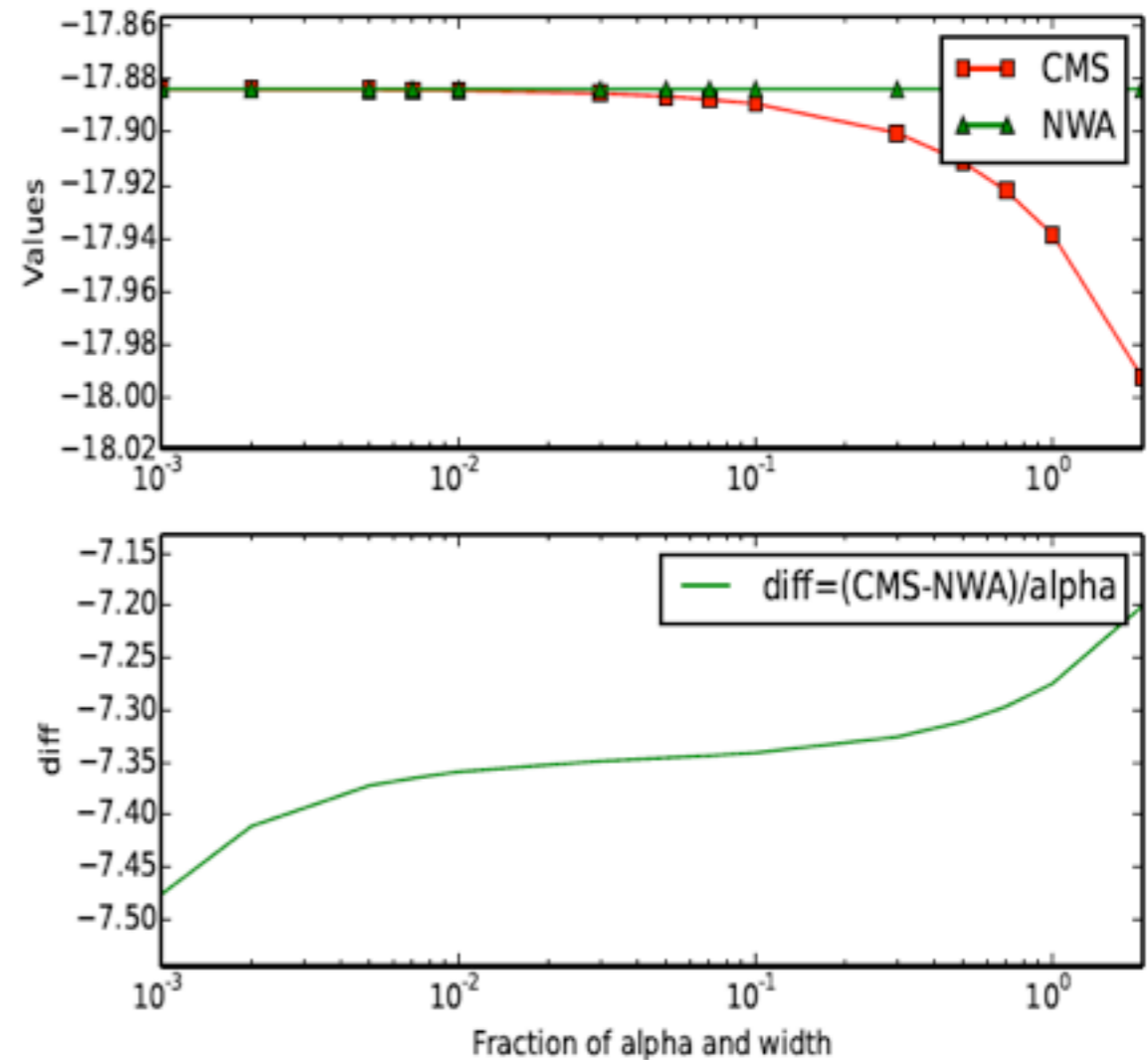


Figure 16: Cross checks for $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$ in the non-resonance region with the correct LO width Γ_t^{LO} .

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

D.3 $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$

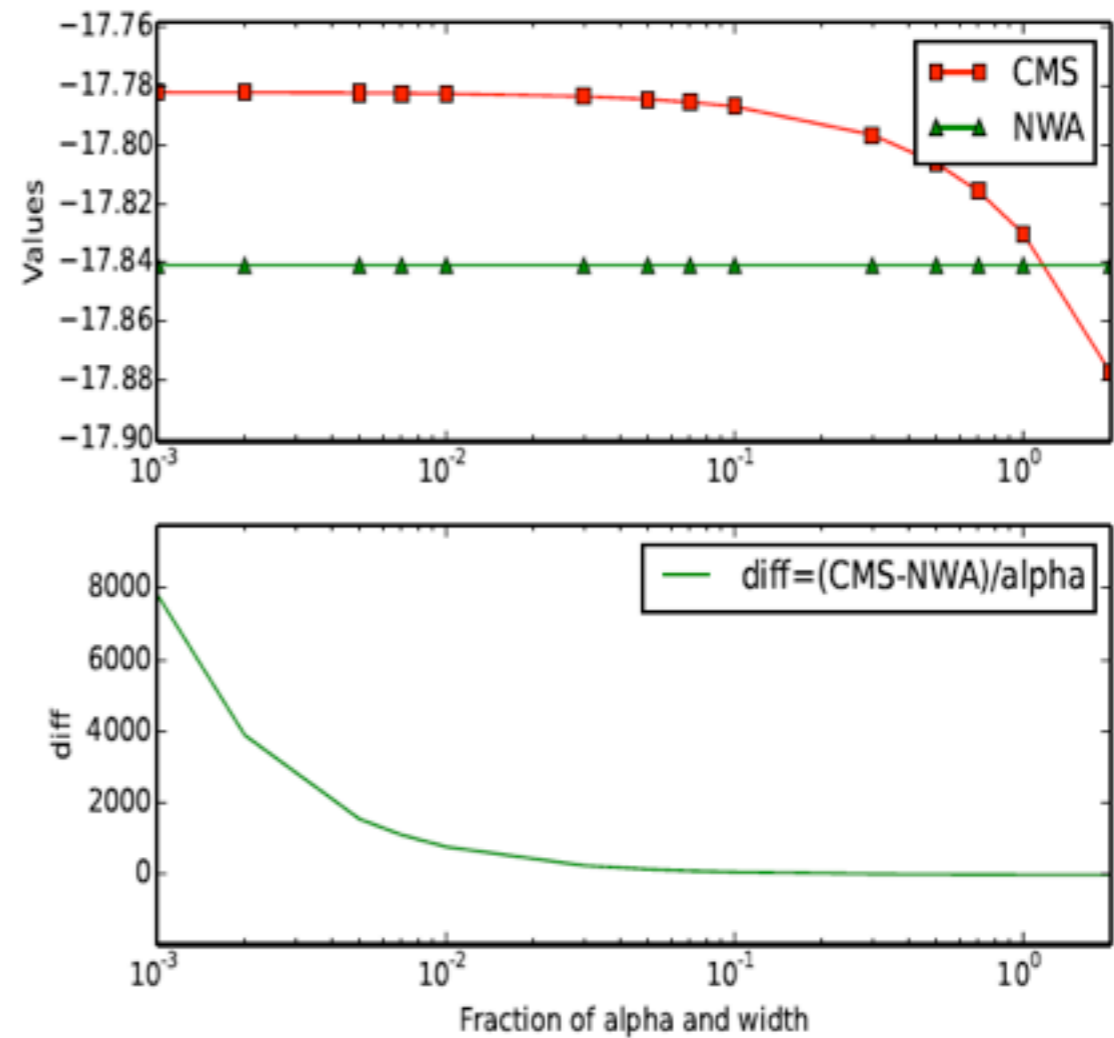


Figure 17: Cross checks for $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$ in the non-resonance region with the wrong LO width, i.e. $\Gamma_t = 1.2\Gamma_t^{\text{LO}}$.

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

D.3 $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$

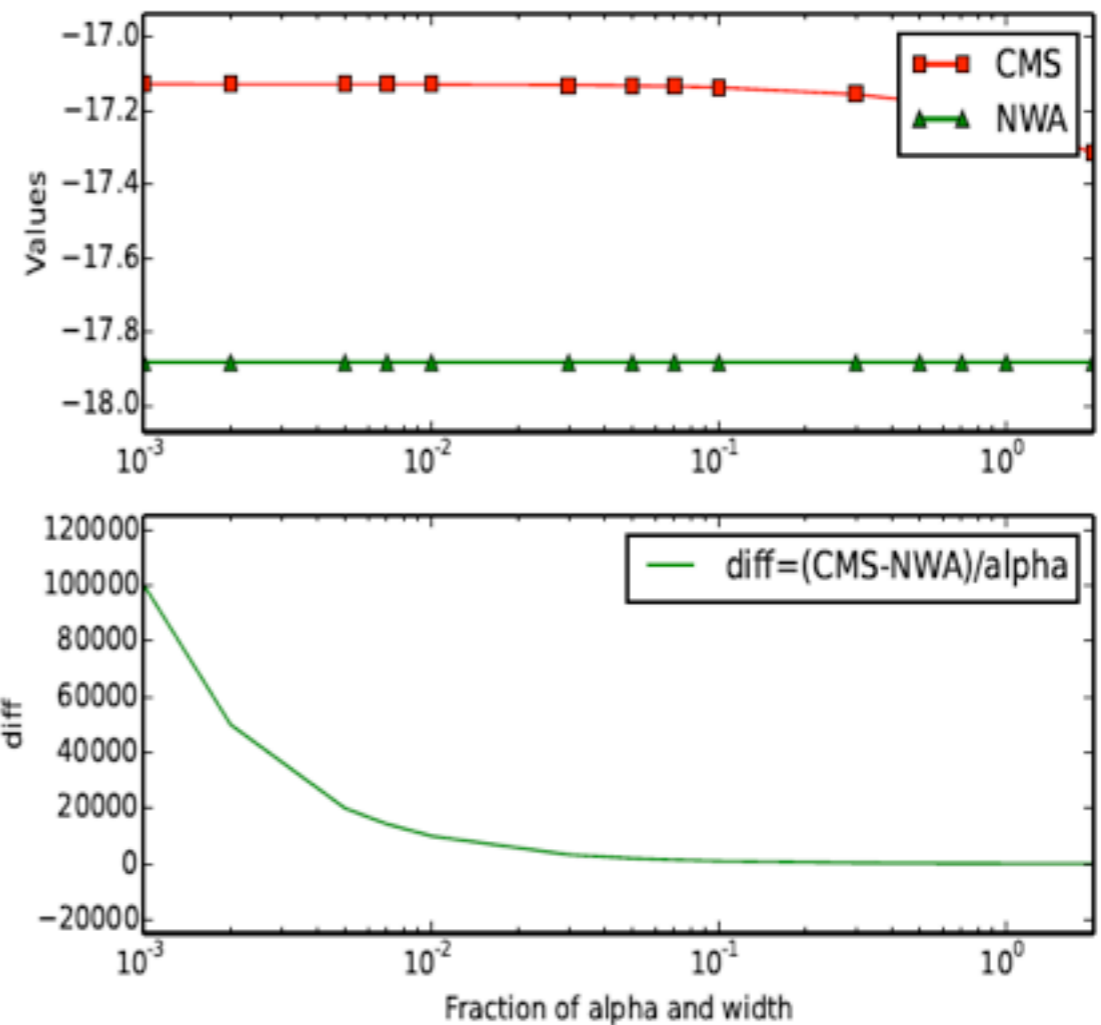


Figure 18: Cross checks for $e^+ \nu_e \rightarrow t \bar{b} \rightarrow W^+ b \bar{b}$ in the non-resonance region with the correct LO width but using the normal logarithms.

NON-RESONANCE REGION

- Cross-check method

$2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\delta Z_e=0, \delta s_w=0} \\ &\quad + 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{(\Gamma_W=0 \text{ in } W \text{ propagator only})}), \end{aligned}$$

$$\text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 = 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*},$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}}. \tag{D.2}$$

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\delta Z_e} \\ &+ 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\Gamma} \end{aligned}$$

$$\text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 = 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*},$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}}.$$

D.1 $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$

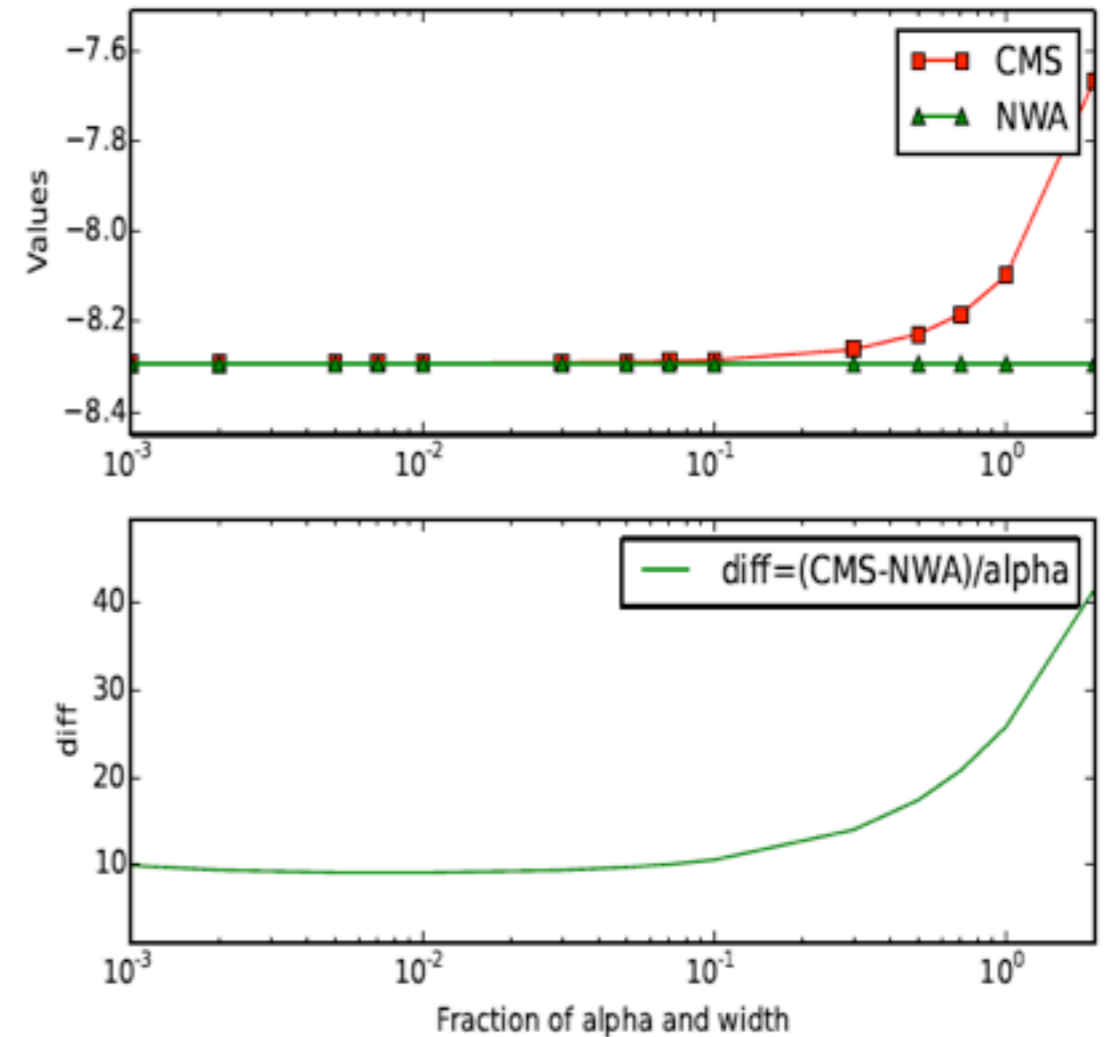


Figure 3: Cross checks for $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$ in the non-resonance region with the correct LO width Γ_W^{LO} .

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\delta Z_e=} \\ &+ 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\Gamma_V}) \end{aligned}$$

$$\text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 = 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*},$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}}.$$

D.1 $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$

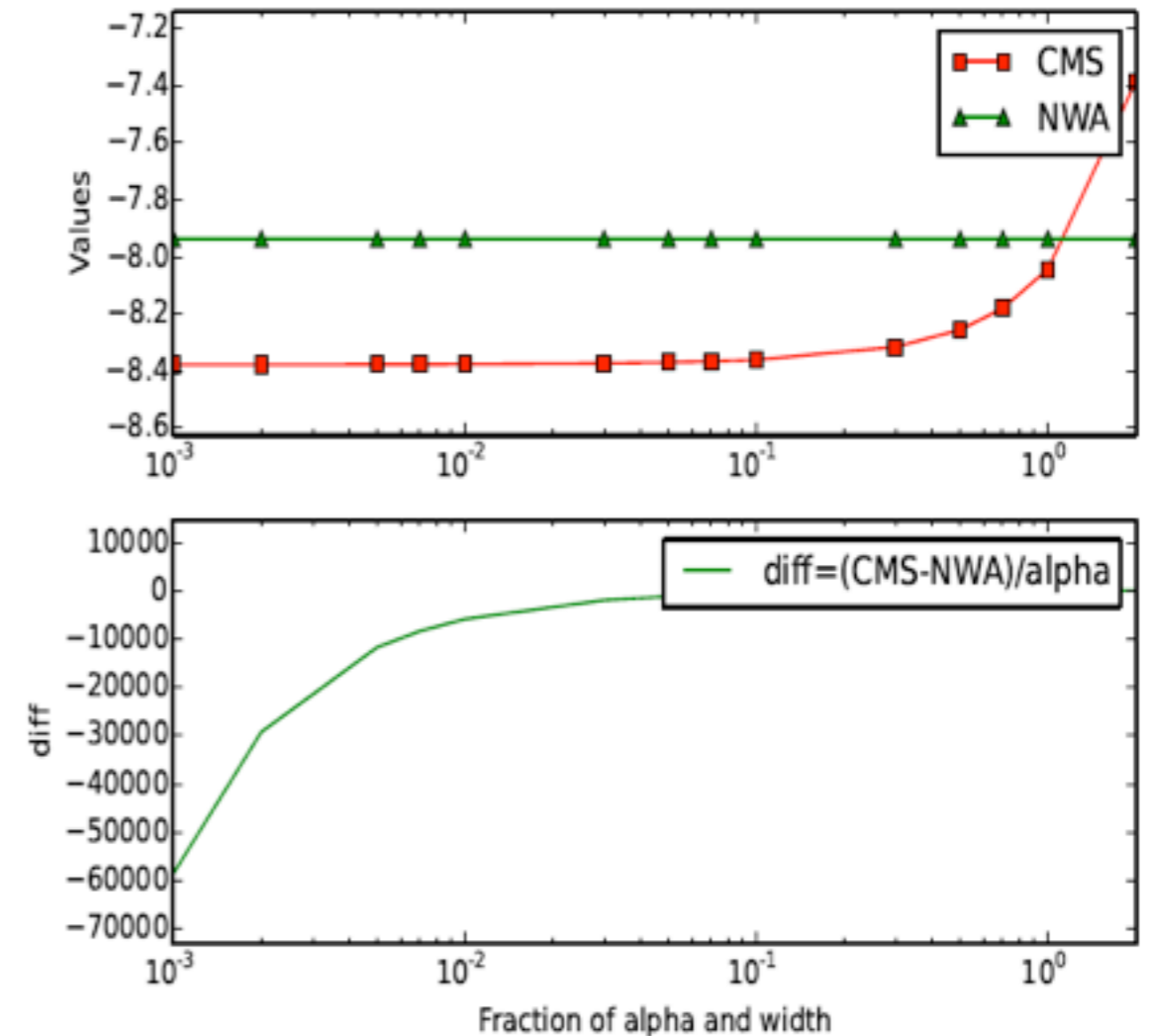


Figure 4: Cross checks for $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$ in the non-resonance region with the wrong LO width, i.e. $\Gamma_W = 1.2\Gamma_W^{\text{LO}}$.

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} | \delta Z_e \\ &+ 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*} | \delta Z_e \end{aligned}$$

$$\text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 = 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*},$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}}.$$

D.1 $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$

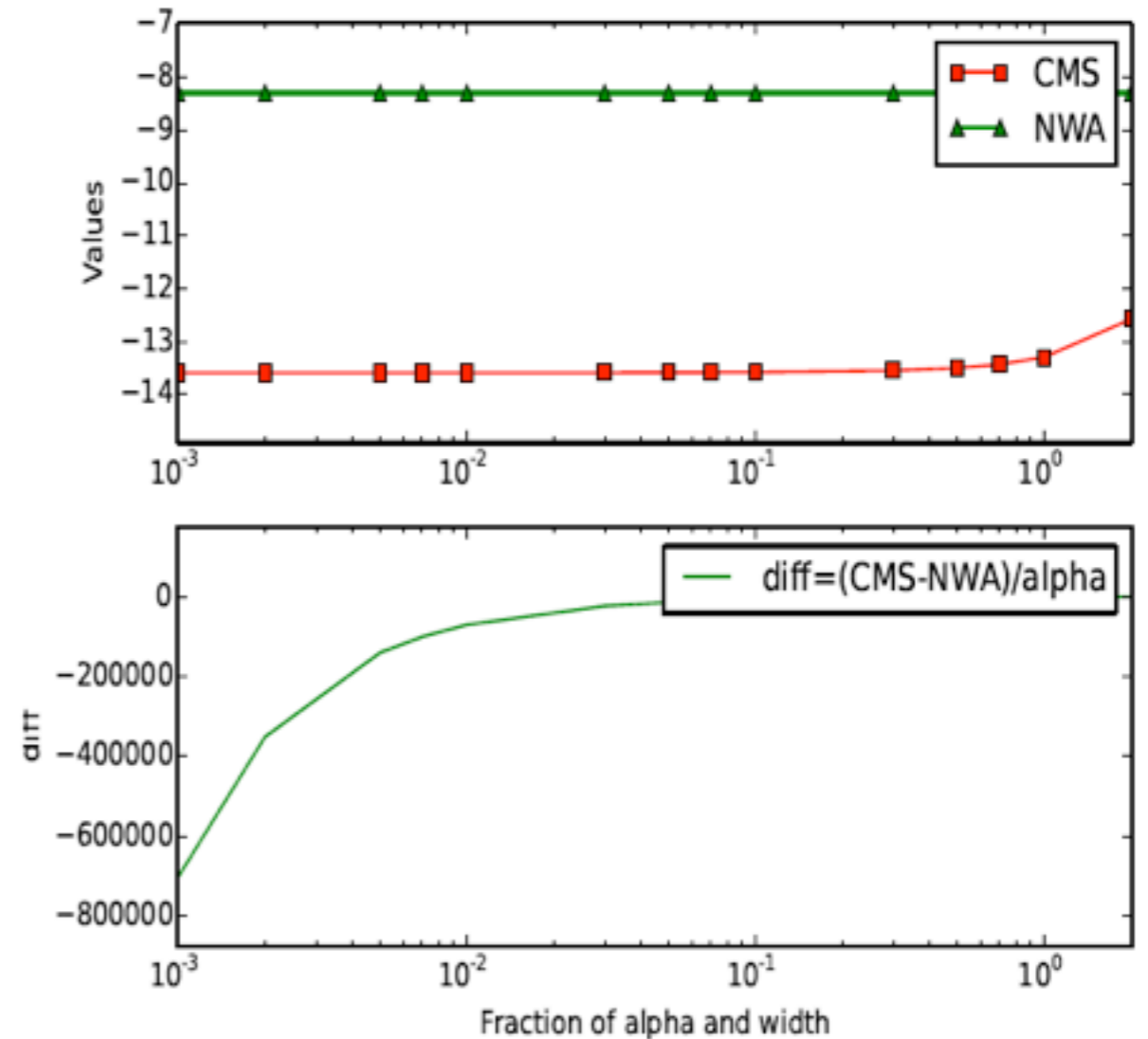


Figure 5: Cross checks for $e^+ \nu_e \rightarrow W^+ \rightarrow \mu^+ \nu_m$ in the non-resonance region with the correct LO width but using the normal logarithms.

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\delta} \\ &+ 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*}) \end{aligned}$$

$$\begin{aligned} \text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}, \\ \text{diff} &= (\text{CMS} - \text{NWA}) / \alpha_e \end{aligned}$$

D.2 $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-$

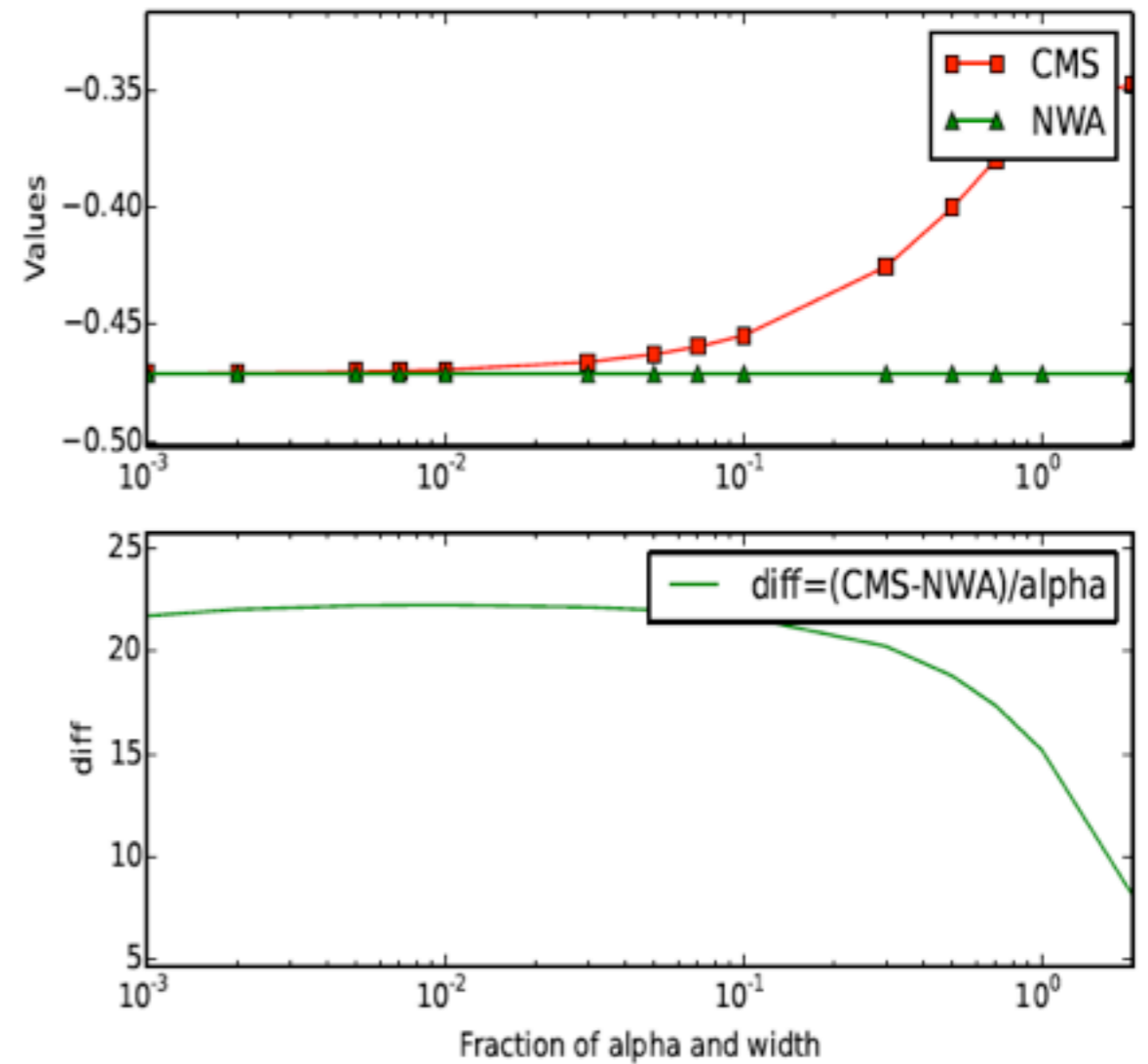


Figure 10: Cross checks for $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-$ in the non-resonance region with the correct LO width Γ_Z^{LO} .

NON-RESONANCE REGION

- Cross-check method
 $2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} \\ &\quad + 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}}) \end{aligned}$$

$$\begin{aligned} \text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*} \\ \text{diff} &= (\text{CMS} - \text{NWA}) / c \end{aligned}$$

D.2 $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-$

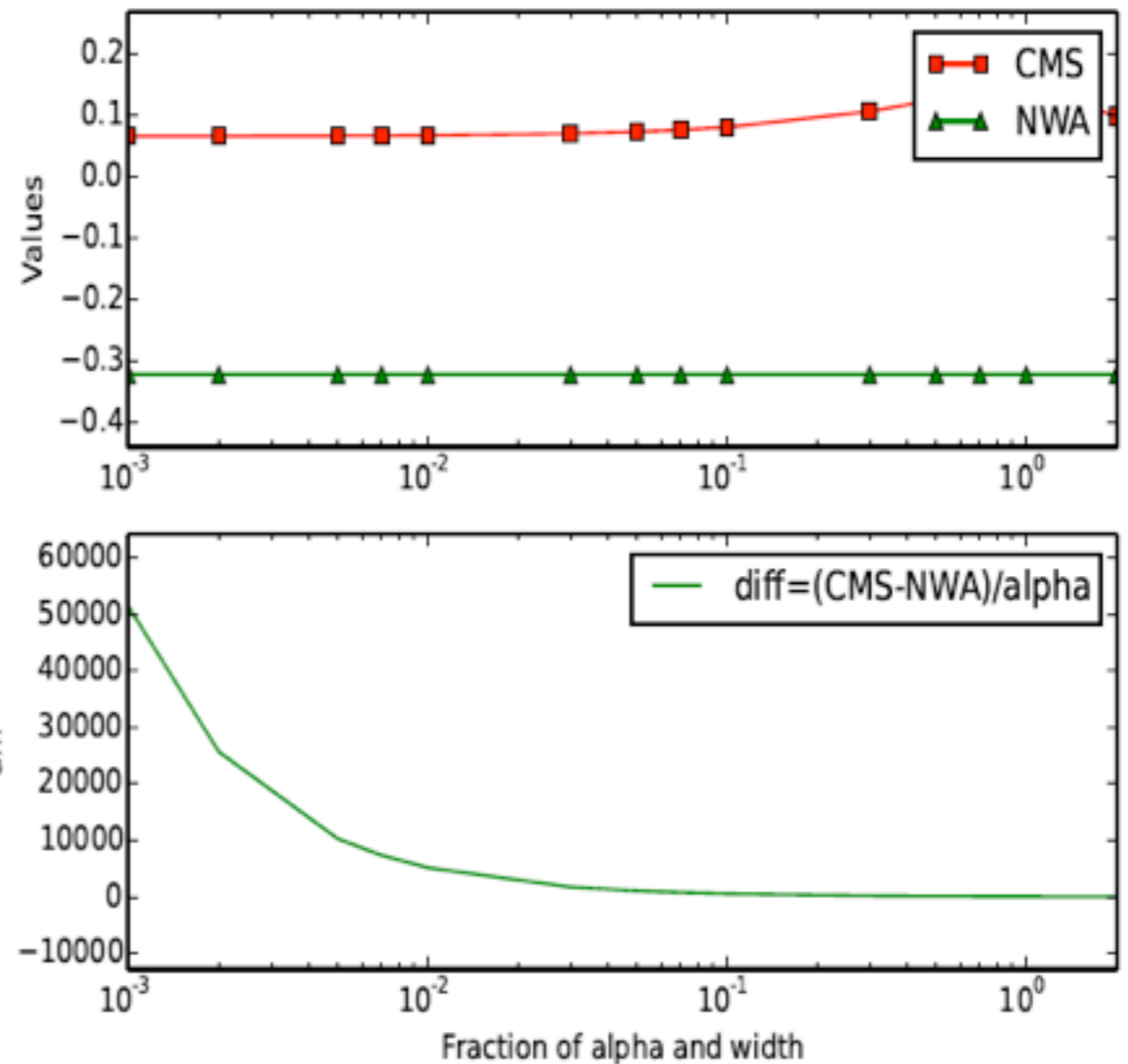


Figure 11: Cross checks for $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-$ in the non-resonance region with the wrong LO width, i.e. $\Gamma_Z = 1.2\Gamma_Z^{\text{LO}}$.

NON-RESONANCE REGION

- Cross-check method

$$2 > n \quad (n \geq 3):$$

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$$2 > 2:$$

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Re \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} \\ &\quad + 2\Re \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}}) \end{aligned}$$

$$\begin{aligned} \text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Re \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*} \\ \text{diff} &= (\text{CMS} - \text{NWA}) / \alpha \end{aligned}$$

$$\mathbf{D.2} \quad e^+ e^- \rightarrow Z/\gamma^* \rightarrow \mu^+ \mu^-$$

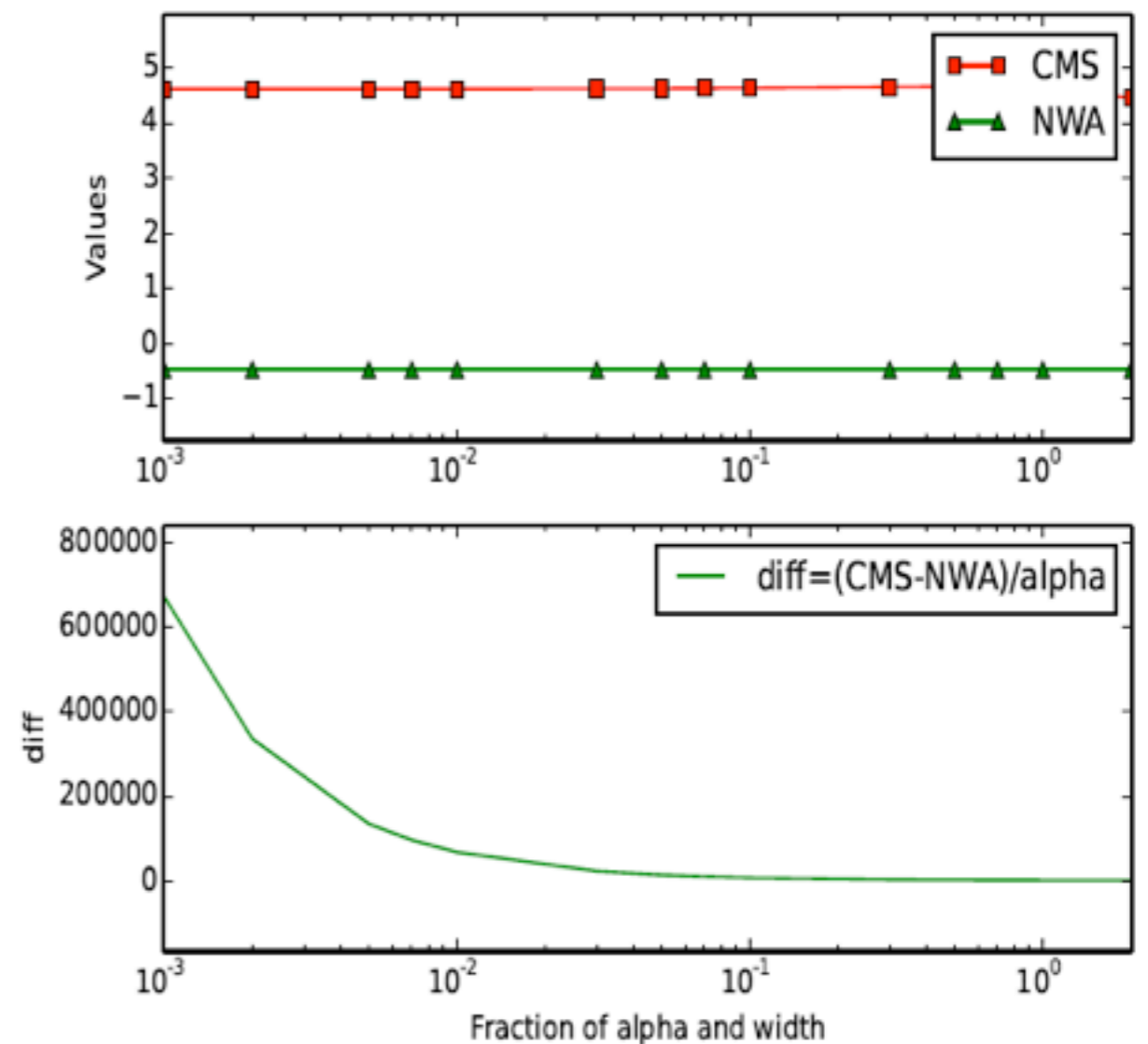


Figure 12: Cross checks for $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-$ in the non-resonance region with the correct LO width but using the normal logarithms.

NON-RESONANCE REGION

- Cross-check method

$2 > n$ ($n \geq 3$):

$$\text{CMS} = [2\Re(\mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*}) + |\mathcal{A}_{\text{Born}}^{\text{CMS}}|^2 - |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2] / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2)$$

$$\text{NWA} = 2\Re(\mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*}) / (\alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2),$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}},$$

$2 > 2$:

$$\begin{aligned} \text{CMS} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 &= 2\Im \mathcal{A}_{\text{Virtual}}^{\text{CMS}} \mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{\delta Z_e=0, \delta s_w=0} \\ &\quad + 2\Im \mathcal{A}_{\text{Virtual}}^{\text{Born}} (\mathcal{A}_{\text{Born}}^{\text{CMS}*} |_{(\Gamma_W=0 \text{ in } W \text{ propagator only})}), \end{aligned}$$

$$\text{NWA} \times \alpha_{\text{ew}} |\mathcal{A}_{\text{Born}}^{\text{NWA}}|^2 = 2\Im \mathcal{A}_{\text{Virtual}}^{\text{NWA}} \mathcal{A}_{\text{Born}}^{\text{NWA}*},$$

$$\text{diff} = (\text{CMS} - \text{NWA}) / \alpha_{\text{ew}}. \tag{D.2}$$

RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

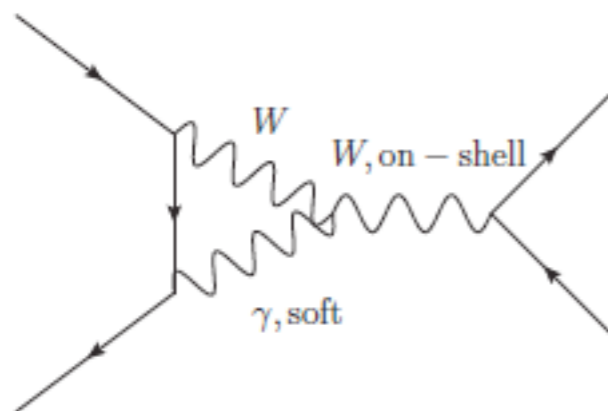
- A valuable cross-check in this region ? Yes !
 - XS level: difficult because of the spin correlation. Easier for a scalar resonance.
 - ME level: give a slight offshellness to avoid infinity in NWA. We have also cross checked in this region though it is much tricky:

RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

- A valuable cross-check in this region ? Yes !
- XS level: difficult because of the spin correlation. Easier for a scalar resonance.
- ME level: give a slight offshellness to avoid infinity in NWA. We have also cross checked in this region though it is much tricky:

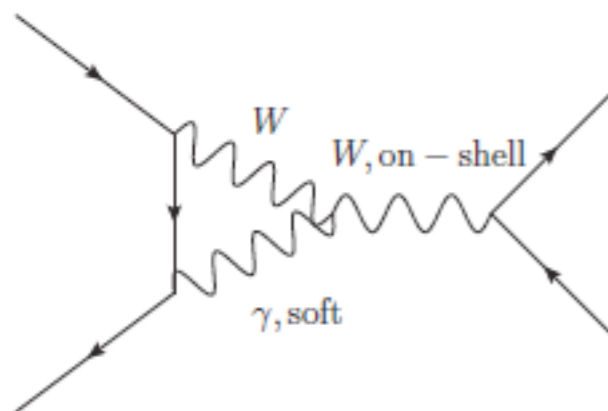


RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

- A valuable cross-check in this region ? Yes !
- XS level: difficult because of the spin correlation. Easier for a scalar resonance.
- ME level: give a slight offshellness to avoid infinity in NWA. We have also cross checked in this region though it is much tricky:



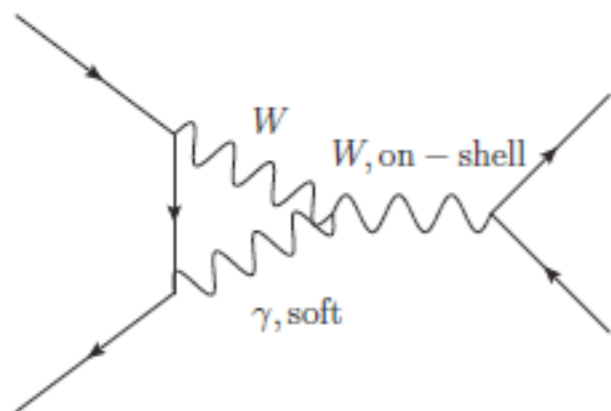
NWA: $\log \varepsilon$

RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

- A valuable cross-check in this region ? Yes !
- XS level: difficult because of the spin correlation. Easier for a scalar resonance.
- ME level: give a slight offshellness to avoid infinity in NWA. We have also cross checked in this region though it is much tricky:



NWA: $\log \varepsilon$

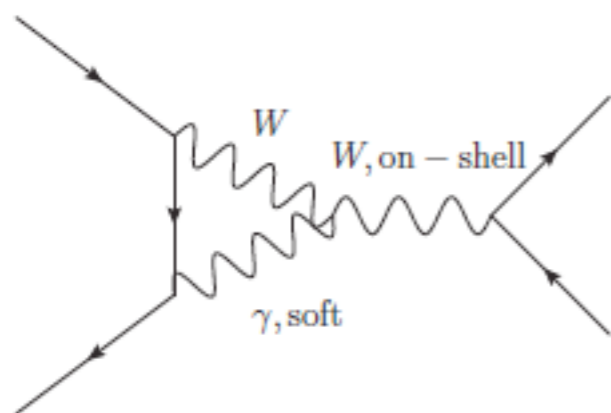
CMS: $\log \Gamma_W$

RESONANCE REGION

- NLO accuracy in this region \rightarrow NLO accuracy width

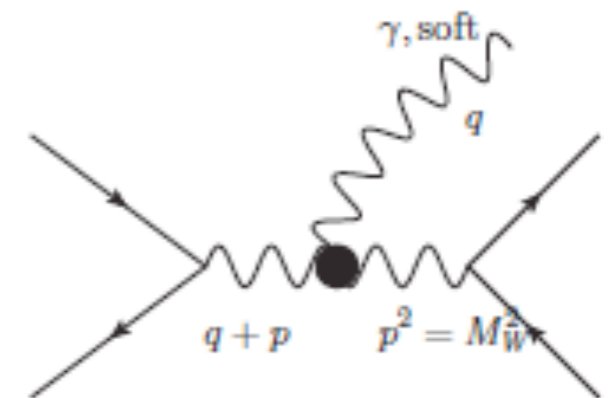
$$\frac{1}{p^2 - M^2 + i\Gamma M} \rightarrow \frac{1}{i\Gamma M}$$

- A valuable cross-check in this region ? Yes !
- XS level: difficult because of the spin correlation. Easier for a scalar resonance.
- ME level: give a slight offshellness to avoid infinity in NWA. We have also cross checked in this region though it is much tricky:



NWA: $\log \varepsilon$

CMS: $\log \Gamma_W$

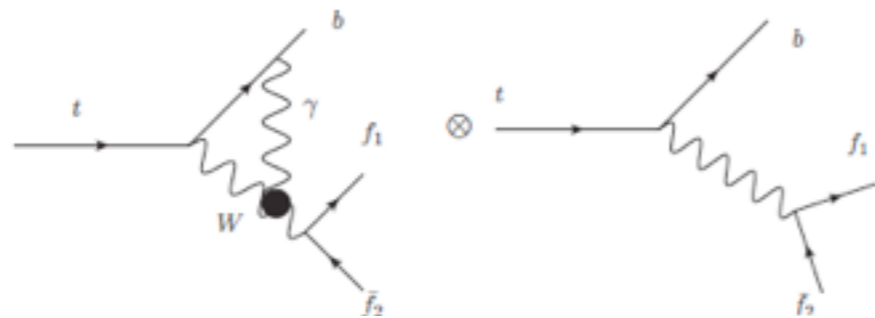


WIDTH

- Do we have to calculate the NLO-level top quark width with W decay ?
 - Not necessary !
 - NLO-level $t \rightarrow W+b$ in NWA
 - Finite width effect from lower-order diagrams (i.e. Born diagrams at NLO)

WIDTH

- Do we have to calculate the NLO-level top quark width with W decay ?
- Not necessary !
- NLO-level $t \rightarrow W+b$ in NWA
- Finite width effect from lower-order diagrams (i.e. Born diagrams at NLO)



$$\frac{C_0(0, m_t^2, M_W^2, 0, 0, M_W^2 - i\Gamma_W M_W) - C_0(0, m_t^2, M_W^2, 0, 0, M_W^2)}{i\pi^{2-\epsilon}\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)\Gamma(1-2\epsilon)^{-1}}$$

$$= -\frac{1}{2(m_t^2 - M_W^2)} \frac{1}{\epsilon_{\text{IR}}^2} + \frac{1}{2(m_t^2 - M_W^2)} \log\left(-\frac{\Gamma_W^2}{\mu^2}\right) \frac{1}{\epsilon_{\text{IR}}}$$

$$+ \frac{1}{2(m_t^2 - M_W^2)} \left(-\frac{\pi^2}{2} - \frac{1}{2} \log^2\left(-\frac{\Gamma_W^2}{\mu^2}\right)\right).$$

WIDTH

- Do we have to calculate the NLO-level top quark width with W decay ?
- Not necessary !
- NLO-level $t \rightarrow W+b$ in NWA
- Finite width effect from lower-order diagrams (i.e. Born diagrams at NLO)



$$\begin{aligned}
 & \frac{C_0(0, m_t^2, M_W^2, 0, 0, M_W^2 - i\Gamma_W M_W) - C_0(0, m_t^2, M_W^2, 0, 0, M_W^2)}{i\pi^{2-\epsilon}\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)\Gamma(1-2\epsilon)^{-1}} \\
 &= -\frac{1}{2(m_t^2 - M_W^2)} \frac{1}{\epsilon_{\text{IR}}^2} + \frac{1}{2(m_t^2 - M_W^2)} \log\left(-\frac{\Gamma_W^2}{\mu^2}\right) \frac{1}{\epsilon_{\text{IR}}} \\
 &+ \frac{1}{2(m_t^2 - M_W^2)} \left(-\frac{\pi^2}{2} - \frac{1}{2} \log^2\left(-\frac{\Gamma_W^2}{\mu^2}\right)\right). \\
 & \frac{\text{Real}^\pm(\Gamma_W) - \text{Real}^\pm(0)}{i\pi^{2-\epsilon}\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)\Gamma(1-2\epsilon)^{-1}} \\
 &= \left(-\frac{1}{\epsilon_{\text{IR}}} + \frac{\pi^2\epsilon}{6}\right) \times \left(-\frac{1}{2(m_t^2 - M_W^2)} \frac{1}{\epsilon_{\text{IR}}} + \frac{1}{2(m_t^2 - M_W^2)} \log\left(-\frac{\Gamma_W^2}{\mu^2}\right) - \epsilon \frac{\frac{\pi^2}{3} + \frac{\log^2\left(-\frac{\Gamma_W^2}{\mu^2}\right)}{4}}{m_t^2 - M_W^2}\right). \tag{3.100}
 \end{aligned}$$

WIDTH

$$\begin{aligned}
 \Gamma_W &= \Gamma_W^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f}), \\
 \Gamma_Z &= \Gamma_Z^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f}), \\
 \Gamma_t &= \Gamma_t^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f} + \delta_{\Gamma_W}).
 \end{aligned}
 \tag{5.1}$$

	Γ^{LO} [GeV]	δ_{α_S} (%)	δ_α (%)	δ_{m_f} (%)	δ_{Γ_W} (%)
W^\pm	2.10490	2.55	-3.51	-0.0238	-
Z	2.51376	2.61	-3.34	-0.0374	-
t	1.54624	-8.58	-1.41	-0.239	-1.58

Table 1: The widths calculated by SMWIDTH in $\alpha(M_Z)$ renormalization scheme.

	Γ^{LO} [GeV]	δ_{α_S} (%)	δ_α (%)	δ_{m_f} (%)	δ_{Γ_W} (%)
W^\pm	2.04808	2.55	-0.364	-0.0238	-
Z	2.44591	2.61	-0.197	-0.0374	-
t	1.50450	-8.58	1.68	-0.239	-1.54

Table 2: The widths calculated by SMWIDTH in G_μ renormalization scheme.

WIDTH

$$\begin{aligned}
 \Gamma_W &= \Gamma_W^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f}), \\
 \Gamma_Z &= \Gamma_Z^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f}), \\
 \Gamma_t &= \Gamma_t^{\text{LO}} (1 + \delta_{\alpha_S} + \delta_\alpha + \delta_{m_f} + \delta_{\Gamma_W}).
 \end{aligned}
 \tag{5.1}$$

	Γ^{LO} [GeV]	δ_{α_S} (%)	δ_α (%)	δ_{m_f} (%)	δ_{Γ_W} (%)
W^\pm	2.10490	2.55	-3.51	-0.0238	-
Z	2.51376	2.61	-3.34	-0.0374	-
t	1.54624	-8.58	-1.41	-0.239	-1.58

Table 1: The widths calculated by SMWIDTH in $\alpha(M_Z)$ renormalization scheme.

	Γ^{LO} [GeV]	δ_{α_S} (%)	δ_α (%)	δ_{m_f} (%)	δ_{Γ_W} (%)
W^\pm	2.04808	2.55	-0.364	-0.0238	-
Z	2.44591	2.61	-0.197	-0.0374	-
t	1.50450	-8.58	1.68	-0.239	-1.54

Table 2: The widths calculated by SMWIDTH in G_μ renormalization scheme.

EW OVERVIEW

	Yes	No
MadLoop	<p><i>EW corrections to any SM process w or w/o b mass in two schemes;</i></p> <p><i>Order splitting in mixed order case;</i></p> <p><i>Decay processes;</i></p> <p><i>Complex-Mass Scheme;</i></p>	??
MadFKS	<p><i>FKS QED subtraction;</i></p> <p><i>Order splitting in mixed order case;</i></p> <p><i>Scale and PDF uncer.;</i></p> <p><i>MC over helicity;</i></p>	<p><i>Virtual Trick;</i></p> <p><i>Quasi-collinear subtraction;</i></p> <p><i>On-shell subtraction;</i></p> <p><i>More cross check (tT,tTV);</i></p>