Two-Loop Off-Shell QCD Amplitudes in FDR

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in collaboration with R. Pittau

Problem Sketch

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- Different regularizations ⇒ different renormalization schemes.
- Different renormalization schemes ⇒ different renormalization constants.
- Related by coupling shift.

$$Z_A \neq Z_B \Rightarrow$$

$$\alpha_B = \alpha_A \left[1 + c_1\, \alpha_A + c_2\, \alpha_A^2\right]$$

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- Two natural categories, global- and sub-vacuum.

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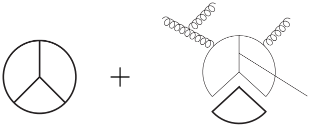

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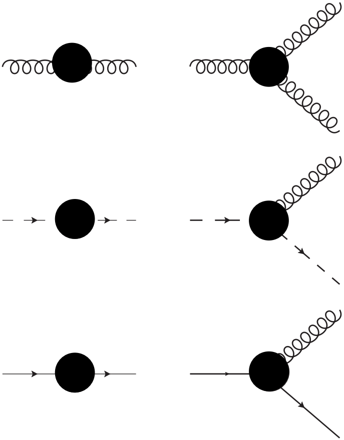

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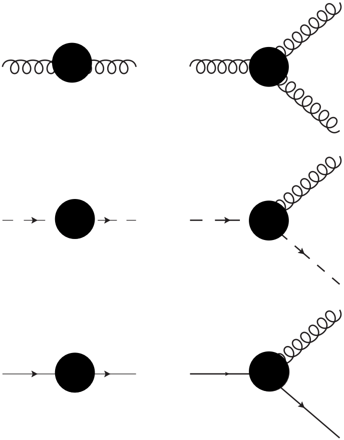

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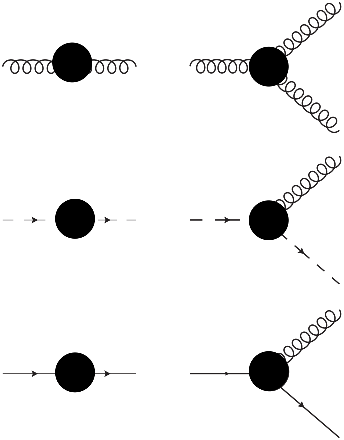

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Universality check in fermionic and bosonic sectors.

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- Corresponds to a choice of renormalization constants/CTs:

$$\Rightarrow \text{CTs} = \;\; ?$$

Renormalize DR *à la* FDR

- Use the defining expansion to relate DR integral to FDR integral:

$$\int d^n q_1 d^n q_2 \, J = \left(\int [d^4 q_1][d^4 q_2] \, J \right) + \left(\int d^n q_1 d^n q_2 \, V[J] \right)$$

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- We use this to calculate the CTs, and then extract the renormalization constants.

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Start with one loop as needed for order by order renormalization.

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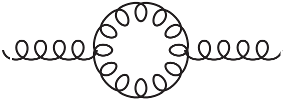
 $= -V \Bigg[$  $+ \cdots \Bigg].$

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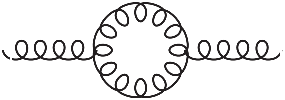
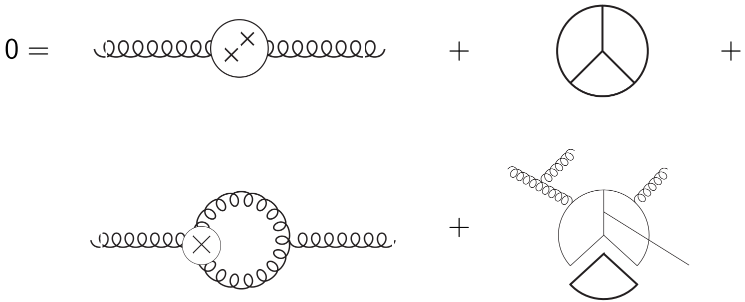
$= -V \Big[$  $+ \cdots \Big].$

One-loop Renormalization constants

$$Z_i = \frac{A}{\epsilon} + \text{universal constant}$$

Structure of 2-Loop FDR Renormalization



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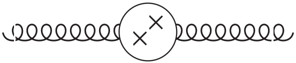
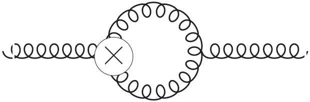
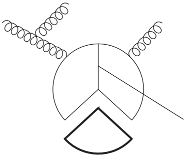
$0 =$  $+$  $+$

 $+$  $= 0$

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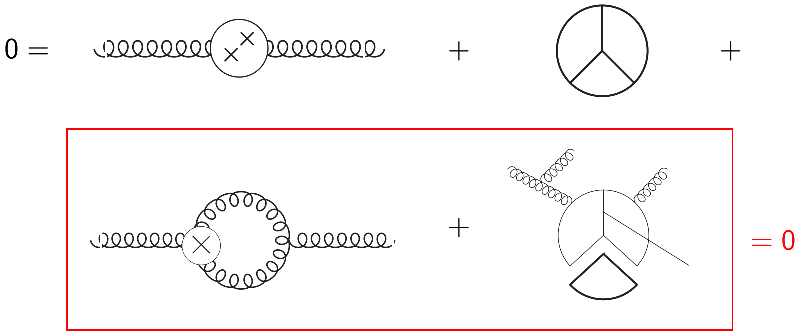

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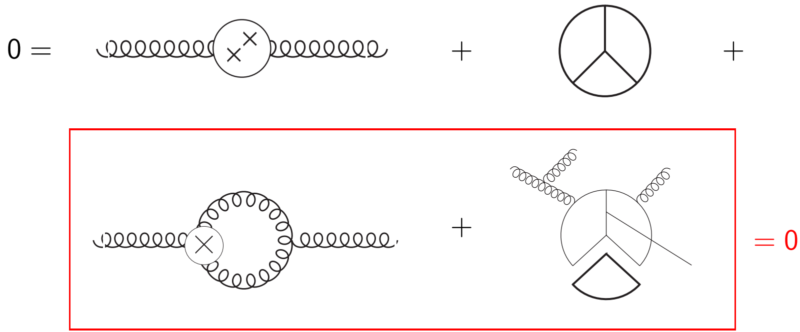

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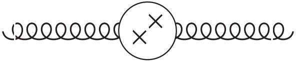
- Suggests way to calculate renormalization constants directly.
- Some extra terms needed in fermionic case.

2 Loop FDR Renormalization Constants



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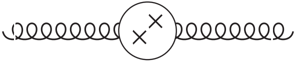

$$\sim \int d^n q_1 d^n q_2 \left(a \frac{1}{\bar{q}_1^4 \bar{q}_2^2 \bar{q}_{12}^2} + b \frac{1}{\bar{q}_1^4 \bar{q}_2^4} \right)$$

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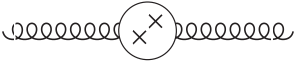

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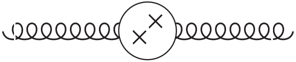
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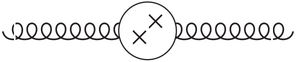 $\sim \int d^n q_1 d^n q_2 \left(a \frac{1}{\bar{q}_1^4 \bar{q}_2^2 \bar{q}_{12}^2} + b \frac{1}{\bar{q}_1^4 \bar{q}_2^4} \right)$

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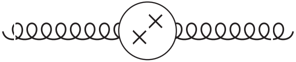

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- Z_i are universal.
- Finite part in terms of Clausen function:

$$f_{11} = \frac{i}{\sqrt{3}} \left(\text{Li}_2(e^{i\frac{\pi}{3}}) - \text{Li}_2(e^{-i\frac{\pi}{3}}) \right) = -1.17195361 \ldots .$$

Two-loop Coupling Constant Shift

Similar procedure to extract the $\overline{MS}$ renormalization constants, allowing us to find the coupling shift:

$$\alpha_{FDR} = \alpha_{\overline{MS}} \left[1 + c_1\, \alpha_{\overline{MS}} + (c_2 + c_3\, f_{11})\, \alpha^2_{\overline{MS}} \right].$$

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- We have used this to calculate three-loop beta function in FDR.

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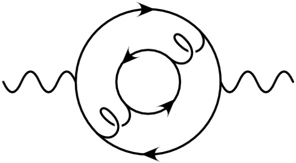

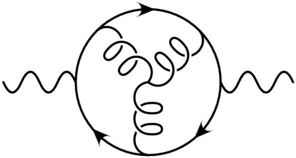

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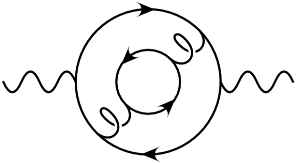
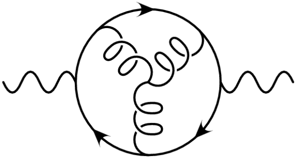
- Suggestive that analogous extra terms could make FDH consistent.





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- Next step is massive fermions.