



TMD Fragmentation Functions in e^+e^- and SIDIS processes

Phenomenological overview

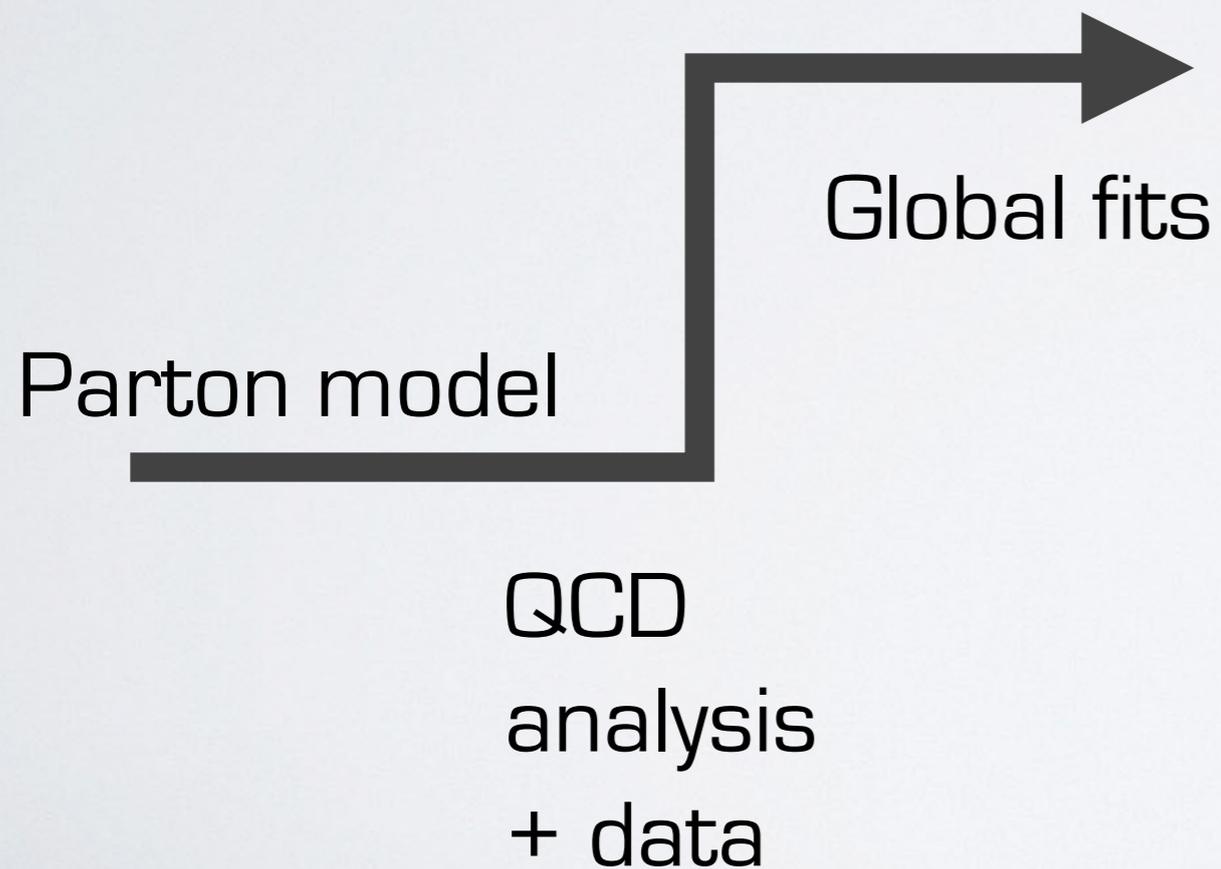
Marco Radici
INFN - Pavia



a phase transition in 3D studies

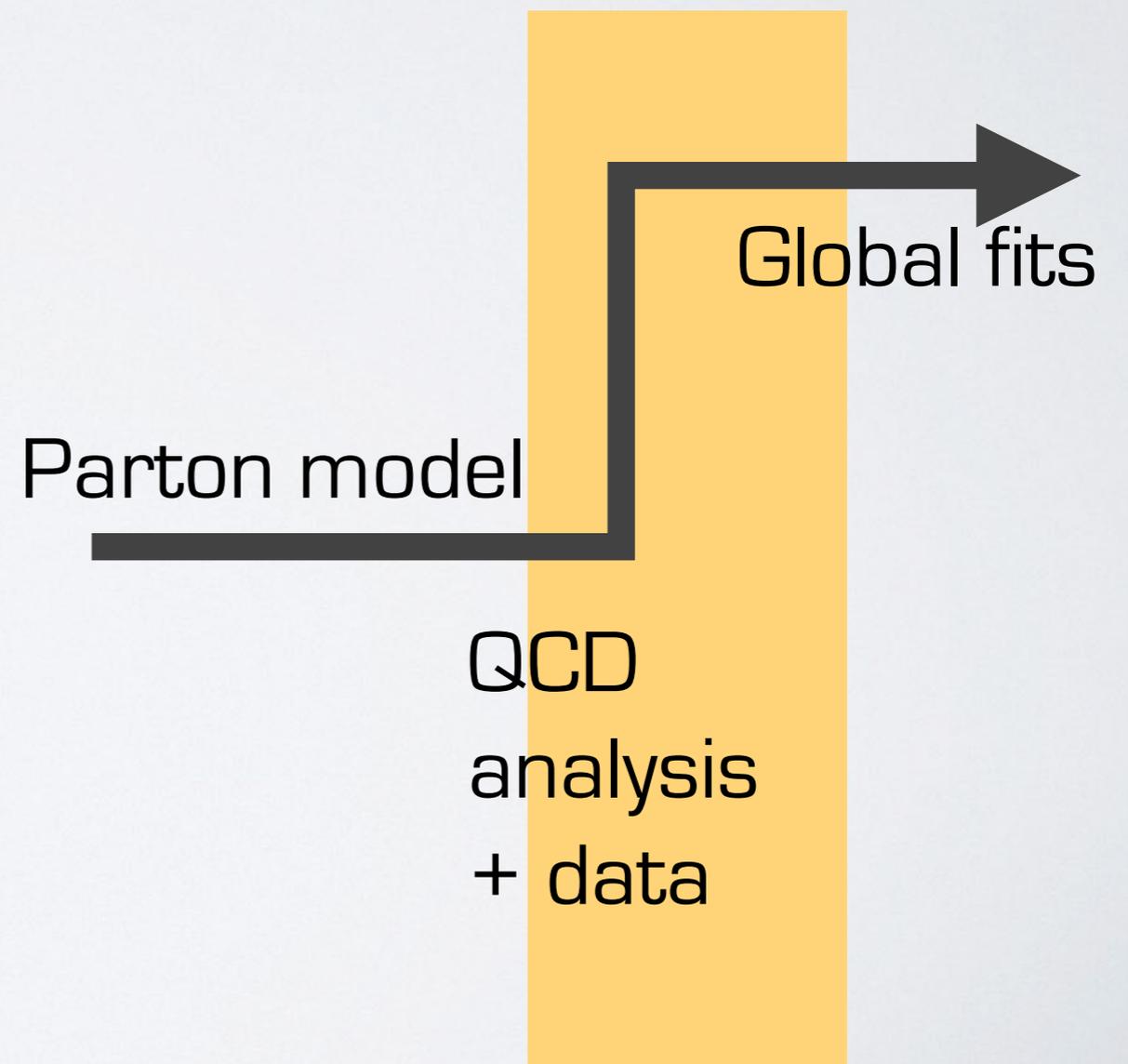
1D

(standard parton distribution functions - PDFs)



3D

(transverse momentum distributions - TMDs)



key results of QCD

factorization

universality

test TMDs by comparing different processes

evolution

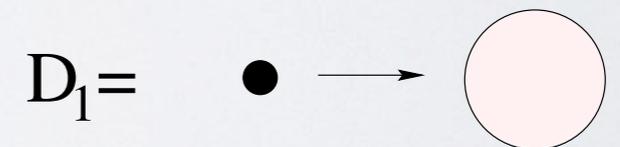
test TMDs by analyzing experiments at
at different energies

Single-hadron Fragmentation Functions (SiFF)

1. Status of collinear parametrization of $D_1^{q \rightarrow h}(z; Q^2)$

quark pol.

U	L	T
D_1		H_1^\perp



a new step

before 2007

- AKK (Albino, Kniehl, Kramer, 2005)
- BKK (Binnewies, Kniehl, Kramer, 1995)
- BFG (Bourhis, Fontannaz, Guillet, 1998)
- BFGW (Bourhis, Fontannaz, Guillet,, Werlen, 2001)
- CGRW (Chiappetta, Greco, Guillet, Rolli, Werlen, 1994)
- GRV (Glück, Reya, Vogt, 1993)
- KKP (Kniehl, Kramer, Potter, 2000)
- Kr (Kretzer, 2000)

after 2007 global fits error analysis

- AKK08 (Albino, Kniehl, Kramer, 2008)
- HKNS (Hirai, Kumano, Nagai, Sudoh, 2007)

- DSS (De Florian, Sassot, Stratmann, 2007)

- DSEHPS (De Florian, Sassot, Epele, Hernández-Pinto, Stratmann, P.R. D91 (2015) 014035)

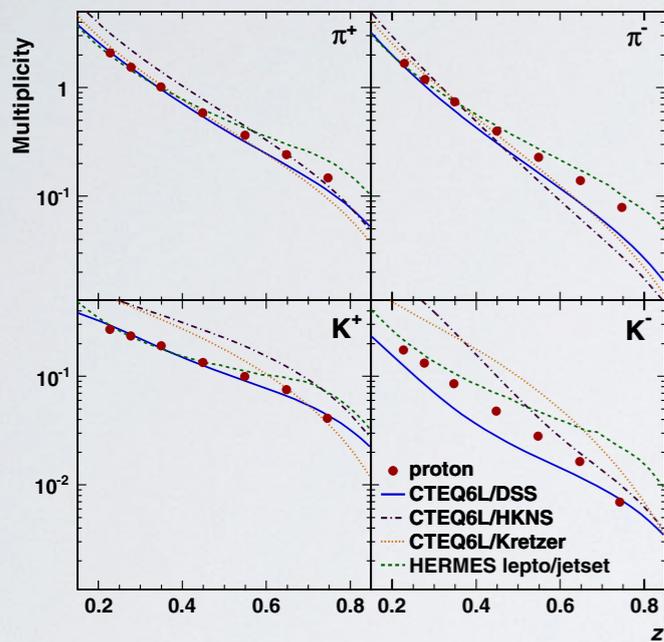
- more/better data for e^+e^- (Belle, BaBar), SIDIS (Hermes, Compass), LHC (Alice), RHIC (STAR)
- new error analysis
- global $\chi^2/\text{dof} \sim 2.2 \rightarrow 1.2$

2015

major update
for $q \rightarrow \pi$

is collinear description good ?

when fitting unintegrated multiplicities to extract TMD FF,
main assumption is that fitting integrated multiplicities gives
good collinear FF...

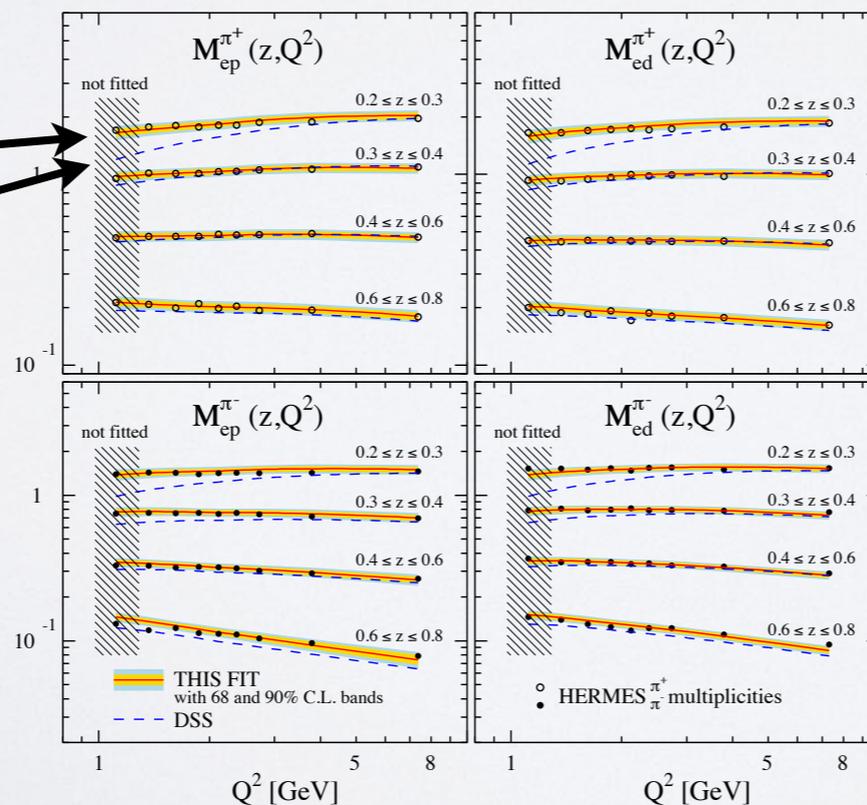


	$\chi^2/\text{d.o.f.}$			
	$Q^2 > 1.4 \text{ GeV}^2$	$Q^2 > 1.4 \text{ GeV}^2$ (no VM subtr.)	$Q^2 > 1.4 \text{ GeV}^2$ (with evolution)	$Q^2 > 1.6 \text{ GeV}^2$
global	2.86	3.90	3.55	2.29
$p \rightarrow K^-$	2.25	2.27	1.38	2.38
$p \rightarrow \pi^-$	3.39	6.58	5.03	2.70
$p \rightarrow \pi^+$	1.87	2.45	2.74	1.16
$p \rightarrow K^+$	0.89	0.85	1.13	0.59
$D \rightarrow K^-$	4.26	4.22	2.81	4.45
$D \rightarrow \pi^-$	5.05	8.66	7.96	3.42
$D \rightarrow \pi^+$	3.33	4.61	5.19	2.29
$D \rightarrow K^+$	1.80	1.57	2.17	1.31

MSTW08 +
DSS 2007

from
Signori *et al.*,
JHEP **1311** (13) 194

DSEHPS 2015
DSS 2007

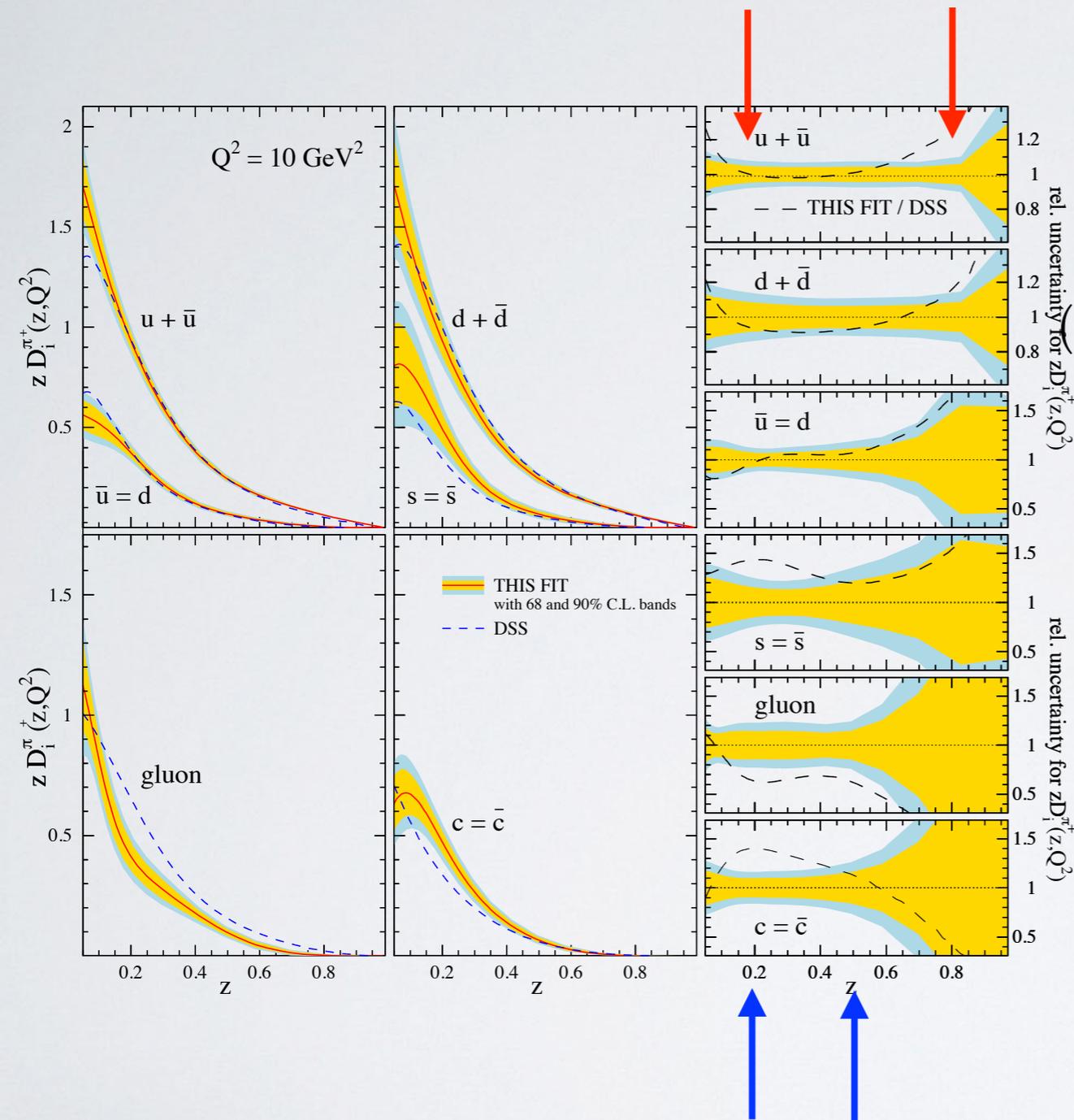


MSTW +
DSEHPS 2015

HERMES SIDIS multiplicities
 $\chi^2 \sim 2.86 \rightarrow 1.37$

is collinear description good ?

De Florian *et al.*, P.R. D91 (2015) 014035

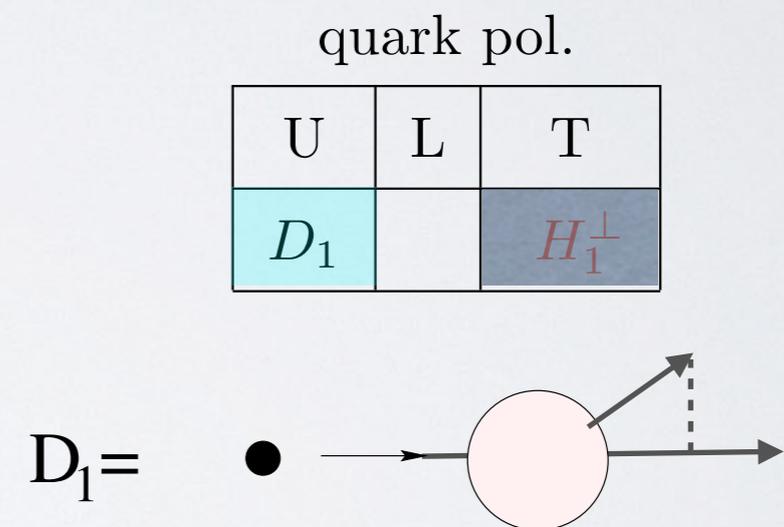


caveat

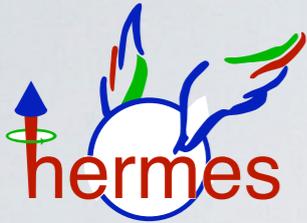
- major improvement for total up & down channels:
rel. uncertainty $\lesssim 10\%$ for $0.2 < z < 0.8$
- for other channels, improvement upon DSS only for $0.2 < z < 0.5$
- Compass data for SIDIS multiplicities for deuteron target only
- Kaon fragmentation data not included

Single-hadron **F**ragmentation **F**unctions (**SiFF**)

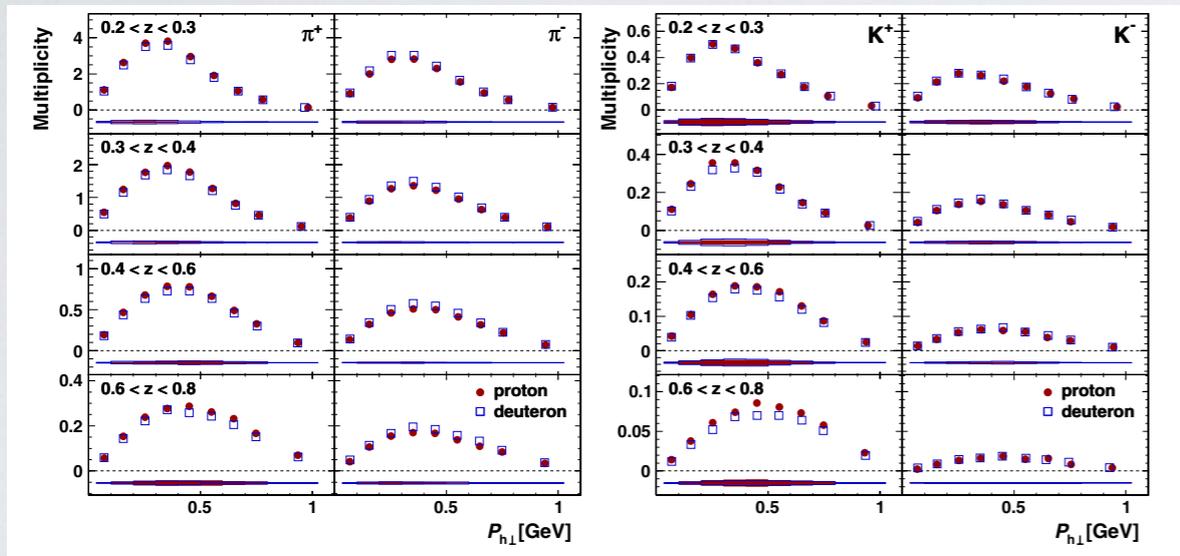
2. Multidimensional description of $D_1^q(z, \mathbf{p}_T; Q^2)$ through **SIDIS** multiplicities



recent data on SIDIS multiplicities



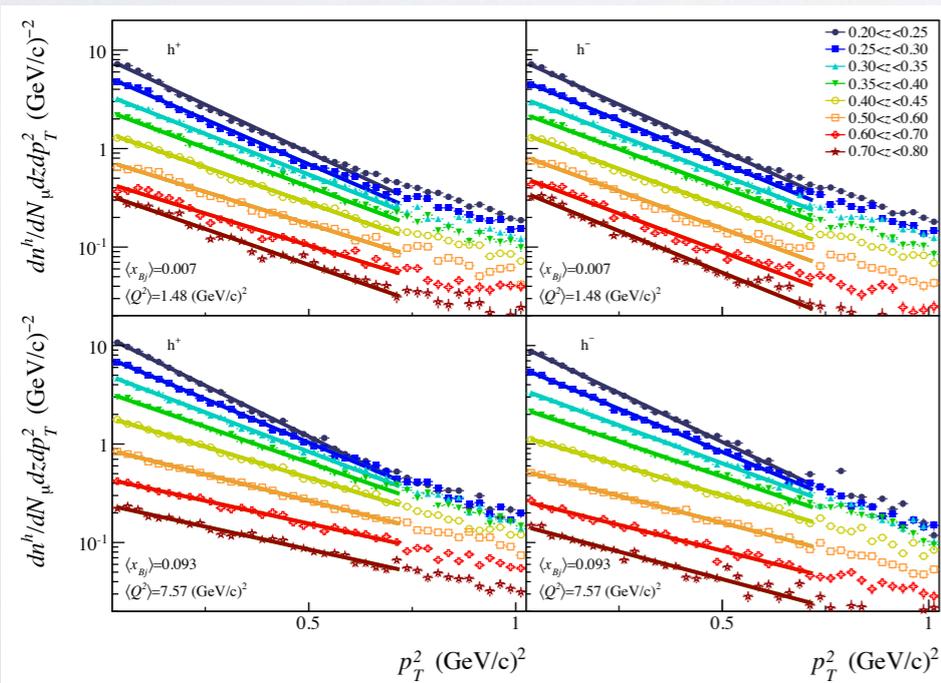
Airapetian *et al.*, P.R. D87 (13) 074029



- target: proton, deuteron
- final state: π^+ , π^- , K^+ , K^-
- 2688 points

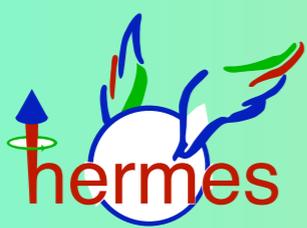


Adolph *et al.*, E.P.J. C73 (13) 2531

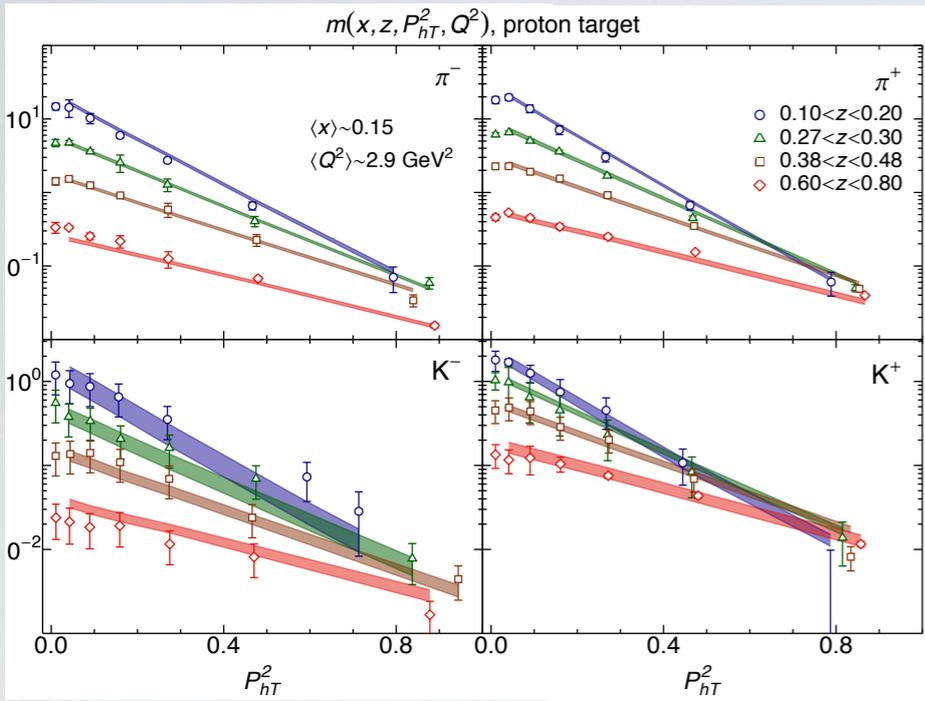


about 20000 data points (!)

- target: deuteron
- final state:
 - h^+ , h^- unidentified (run 2004)
 - π^+ , π^- , K^+ , K^- (run 2006)
- ongoing analysis



Pavia and Torino fits

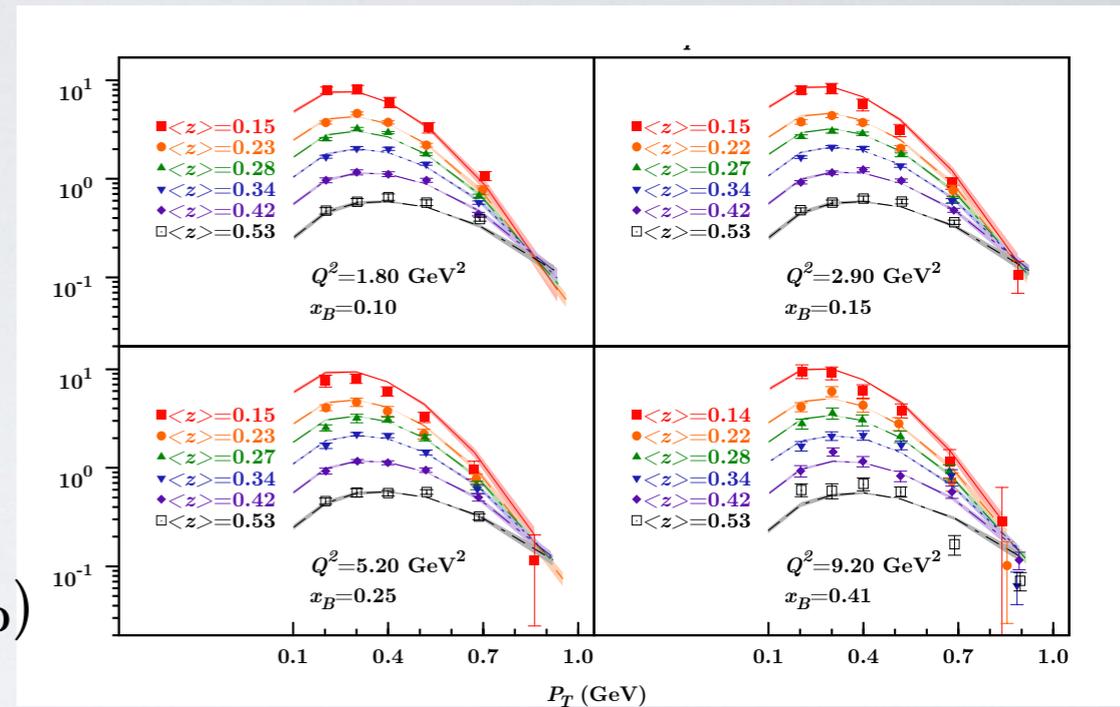


Signori *et al.*, JHEP **1311** (13) 194

almost
similar cuts

1538
points
($\approx 60\%$)

~ 550
points
($\approx 20\%$)



Anselmino *et al.*, JHEP **1404** (14) 005

- flavor dependent Gaussian widths, 7 parameters
- multiplicity = sum of Gaussians
- no QCD evolution
- error analysis: replica method

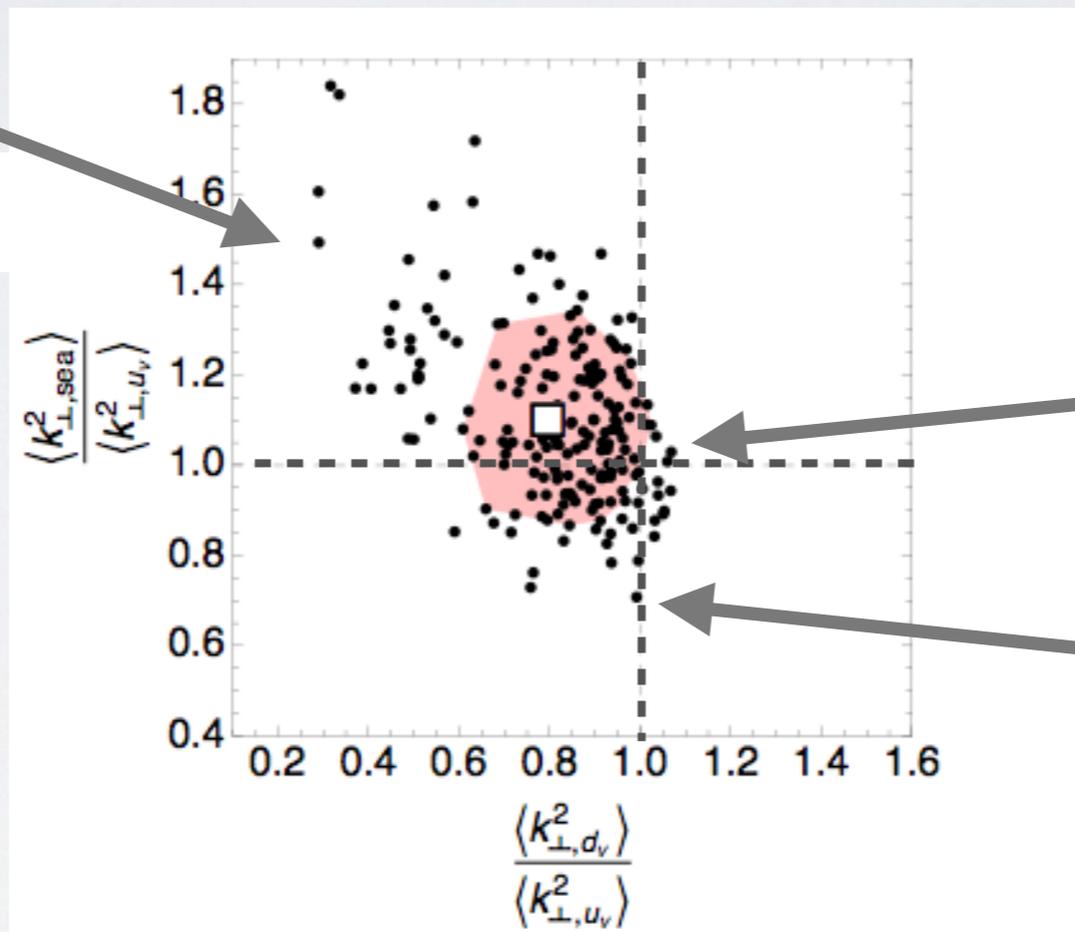
- simple Gaussians, 2 parameters
- only collinear DGLAP evolution

with flavor dep. $\chi^2 = 1.63 \pm 0.12$
with no flavor dep. 1.72 ± 0.11

replica 73
 $\chi^2/\text{dof} = 1.70$

sea / up

point of
no flavor dep.



down / up

TMD PDF

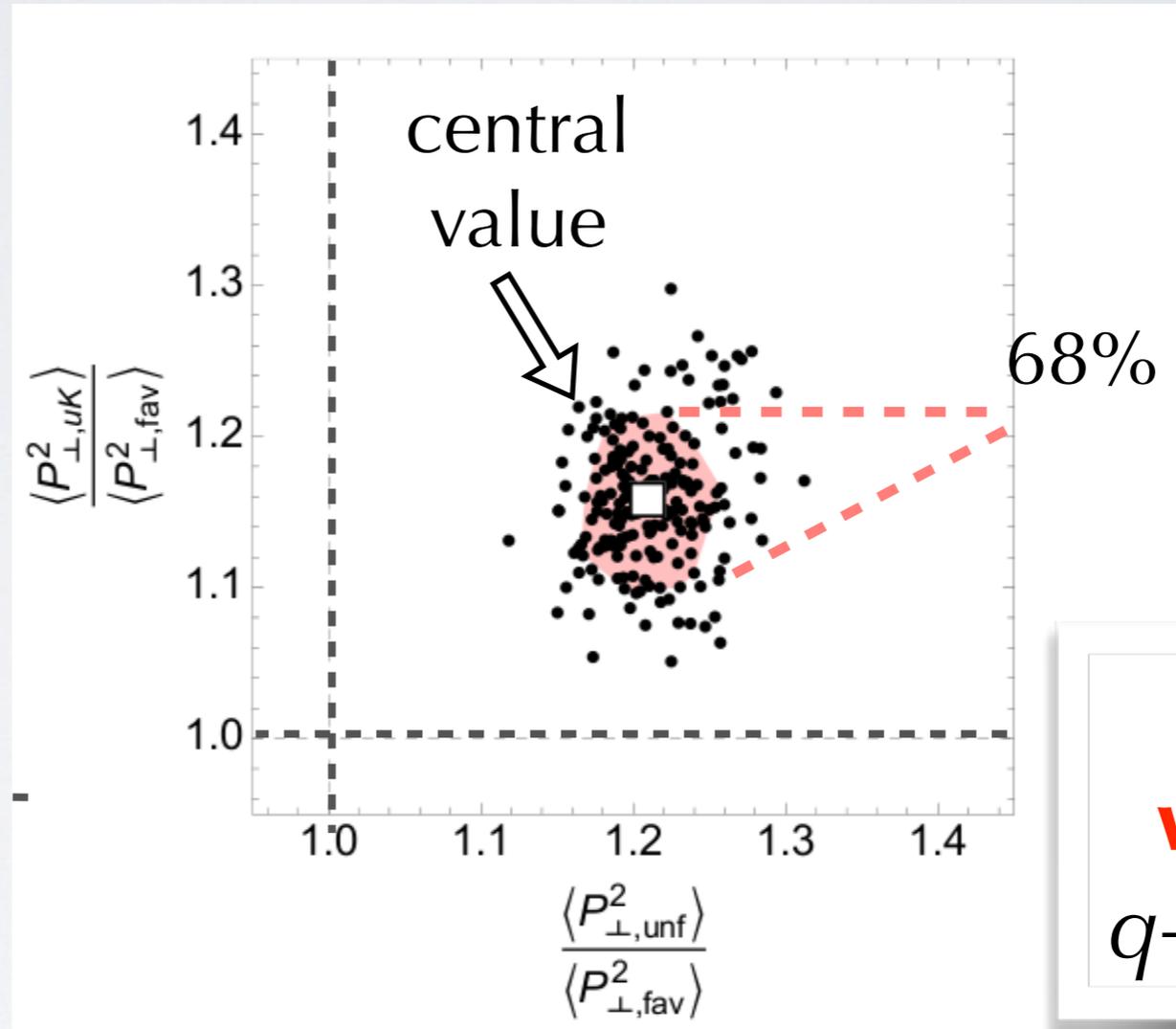
replica 130
 $\chi^2/\text{dof} = 1.77$

replica 186
 $\chi^2/\text{dof} = 1.38$

HERMES SIDIS data not enough selective

fit clearly prefers **TMD FF with** flavor dep.

favored $q \rightarrow K$
wider than
 favored $q \rightarrow \pi$



point of
no flavor dep.

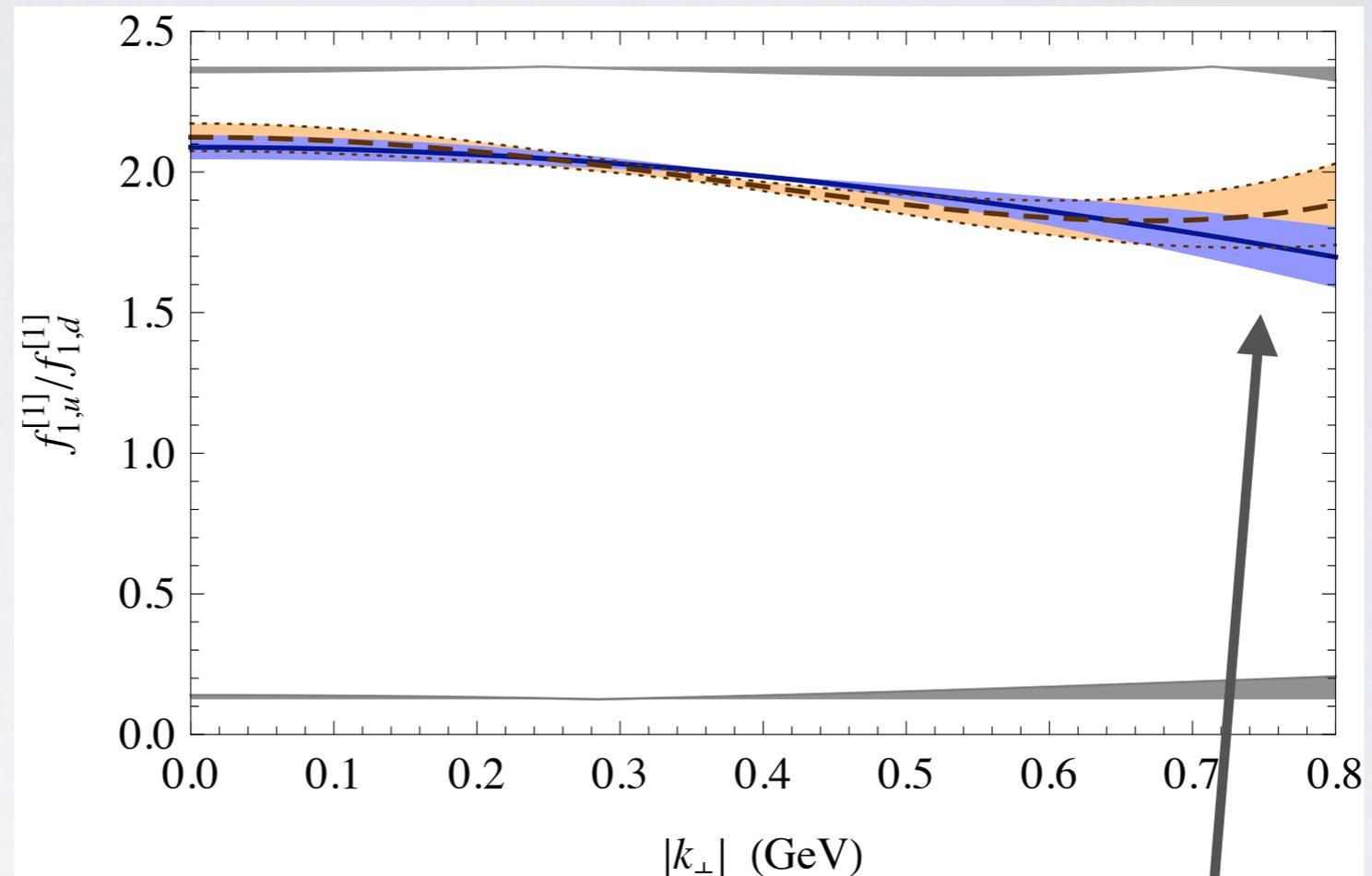
unfavored
wider than
 $q \rightarrow \pi$ favored

confirmed by Torino fit: if favored < unfavored
 \Rightarrow from $\chi^2 = 1.69$ to $\chi^2 = 1.60$

evidence from lattice

valence picture of proton : $\#u / \#d = 2$

ratio of
number densities
(moments of f_1^q)
depends upon $|\mathbf{k}_\perp|$

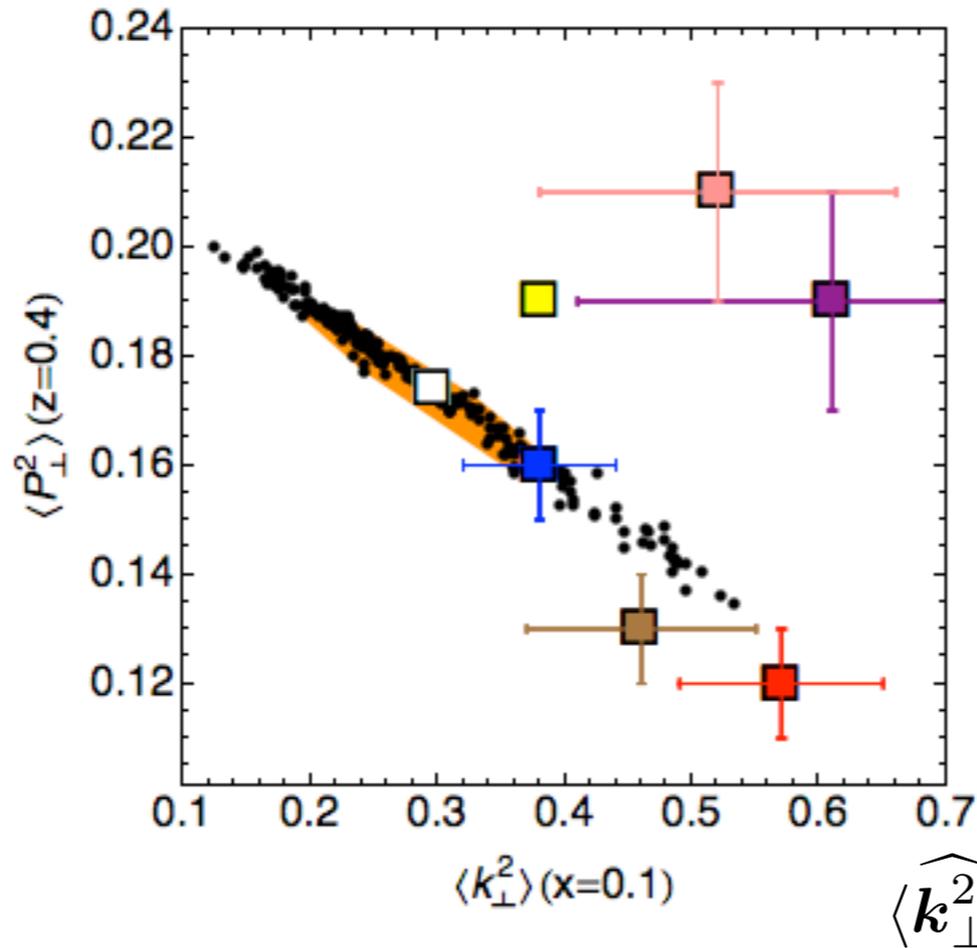


Musch *et al.*, P.R. D83 (11) 094507

“less” up at large $|\mathbf{k}_\perp|$

strong anticorrelation

TMD FF $\langle \widehat{P}_{\perp}^2 \rangle$



need combined fit
of  

 Signori *et al.*,
JHEP **1311** (13) 194

 Schweitzer, Teckentrup,
Metz
P.R.D**81** (2010) 094019

 Anselmino *et al.*, HERMES

 " " " , high z

 Anselmino *et al.*, COMPASS

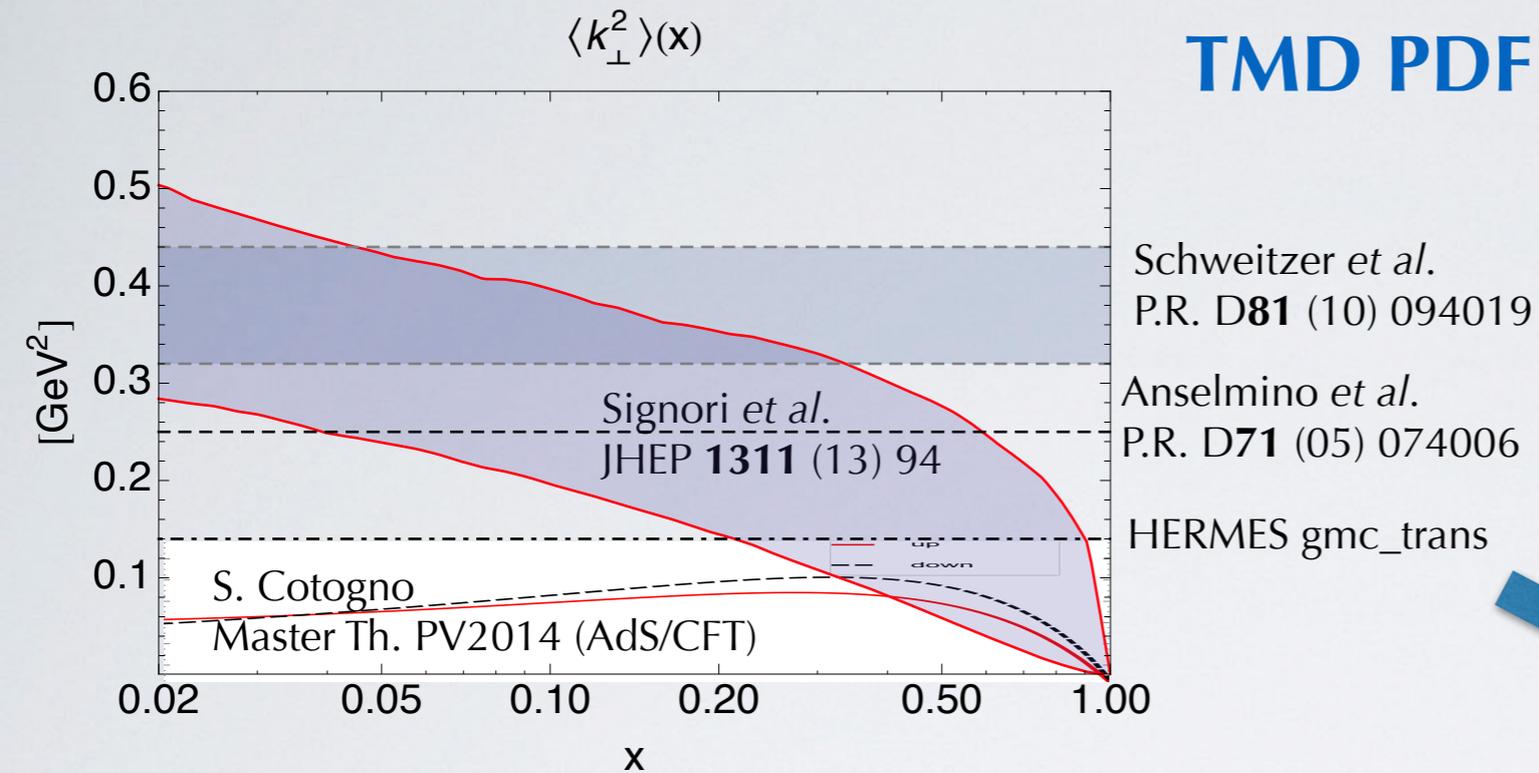
 " " , high z, y-norm

 Echevarria *et al.*,
P.R.D**89** (2014) 074013

JHEP **1404** (2014) 005

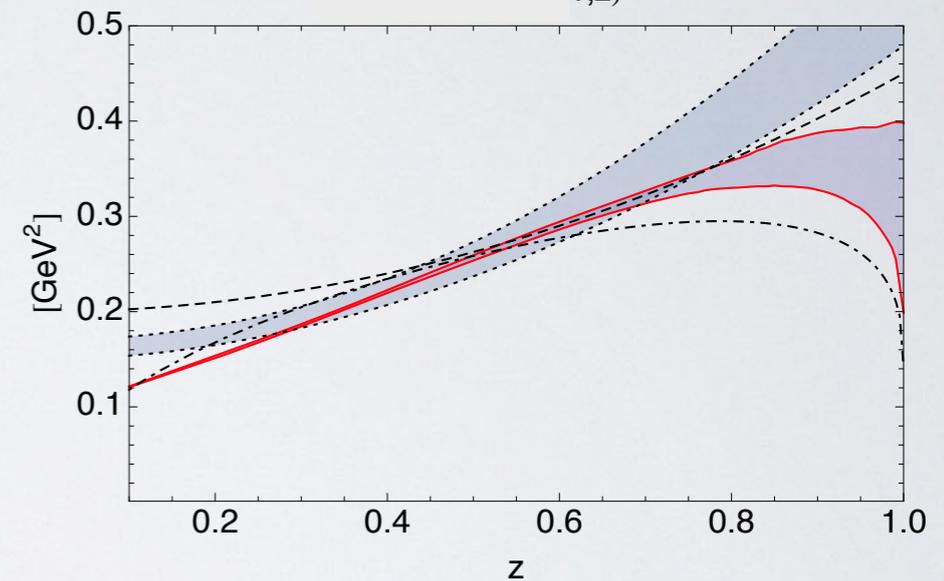
anticorrelation and 68% band

TMD PDF



observed

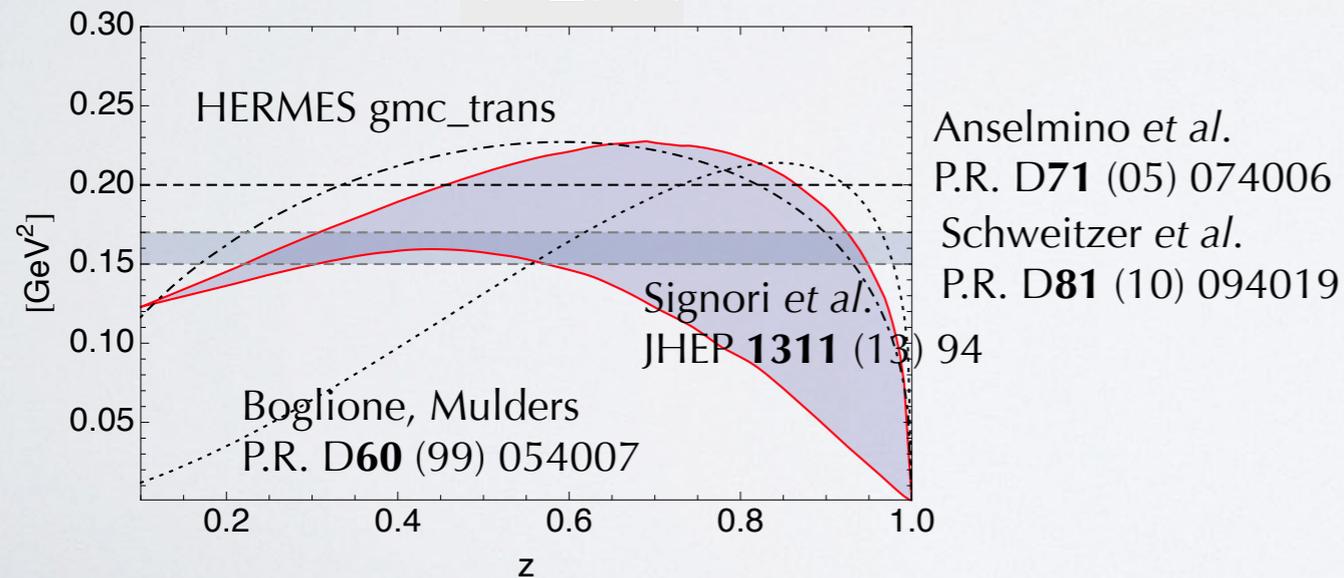
$$\langle P_{hT}^2 \rangle(x = 0.1, z)$$



$$\langle P_{hT}^2 \rangle = z^2 \langle k_{\perp}^2 \rangle + \langle P_{\perp}^2 \rangle$$

TMD FF

$$\langle P_{\perp}^2 \rangle(z)$$



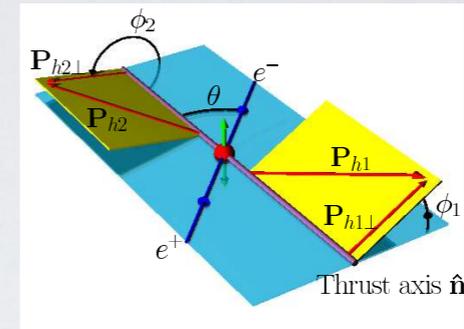
several $\{k_{\perp}, P_{\perp}\}$
give same P_{hT}

- limited Q^2 range \Rightarrow weak sensitivity to QCD evolution
- strong anticorrelation between average transverse momenta in **TMD PDF** & **TMD FF**

need e^+e^- data for **TMD FF**

issues with e^+e^- measurements

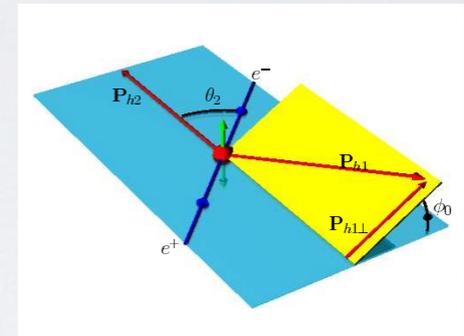
- which frame ? “thrust axis”



TMD factorization ?

(see talk Vossen)

“fixed hadron”



- which kinematics ? recall

$$\frac{P_{1T}^2}{z_1^2} = q_T^2 + \mathcal{O}\left(\frac{1}{Q^2}\right) \quad q_T^2 \ll Q^2$$

example:



$$Q^2 = 100 \text{ GeV}^2 \Rightarrow q_T^2 \approx 10 \text{ GeV}^2$$

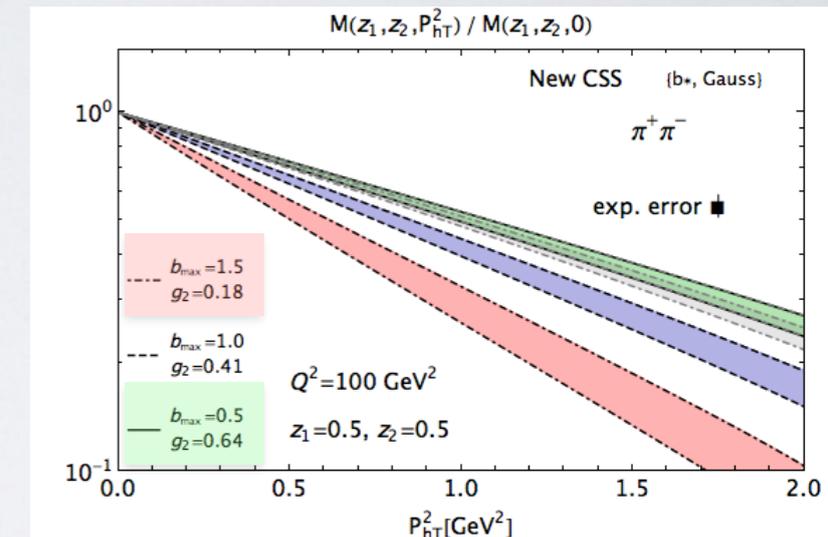
$$z_1=0.5 \leftrightarrow P_{1T}^2 \approx 2.5 \text{ GeV}^2$$

$$z_1=0.3 \leftrightarrow P_{1T}^2 \approx 1.2 \text{ GeV}^2 \dots$$

issues with e^+e^- measurements

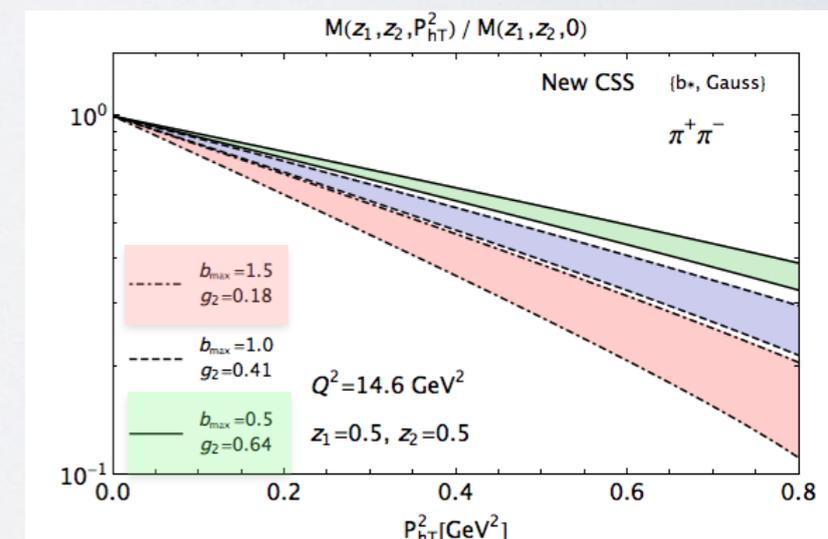
- virtuality fixed by c.m. energy $Q = \sqrt{s}/2$
⇒ need   and  to test evolution

- at  scale, a 7% error can discriminate evolution schemes and nonpert. evolution parameters (with full z dependence; see talk Signori)



Bacchetta *et al.*, arXiv:1508.00402

- at  scale, discriminate intrinsic parameters of TMD FF at input hadronic scale



issues with e^+e^- measurements

- double ratios of like-sign and unlike-sign final hadrons to kill false asymmetries

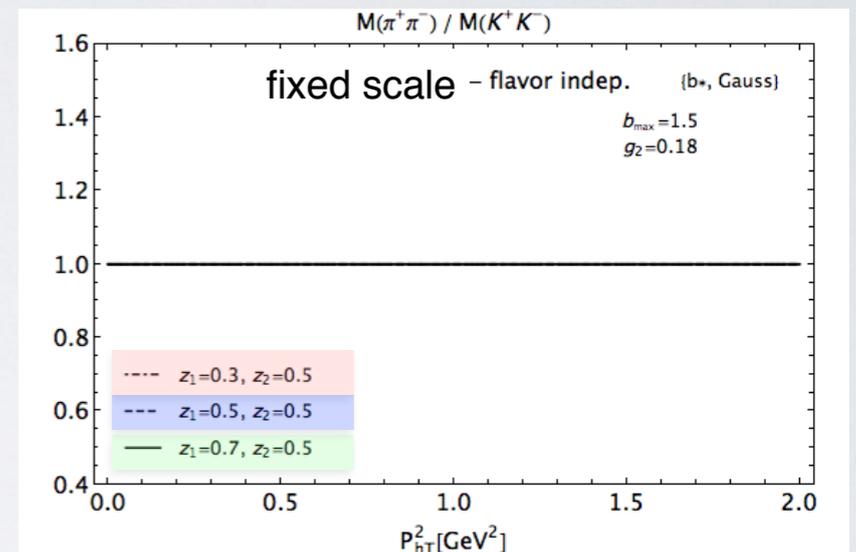
- at  scale, use also ratios of multiplicities for $\pi\pi$, KK , πK

to constrain
flavor dep. of
intrinsic parameters
in TMD FF at input scale
(see talk Signori)

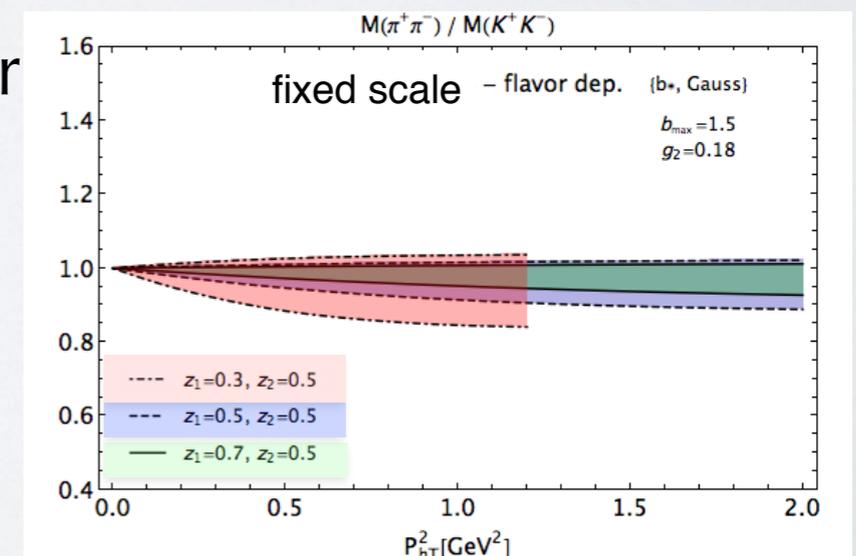
no flavor

$\pi^+\pi^- / K^+K^-$

with flavor



Bacchetta *et al.*, arXiv:1508.00402

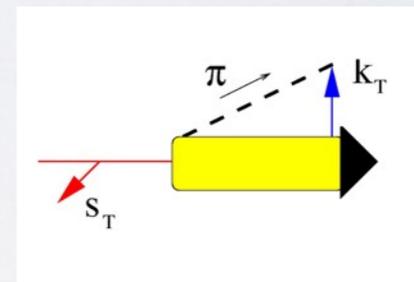


Single-hadron Fragmentation Functions (SiFF)

3. The Collins function $H_1^{\perp q}(z, \mathbf{p}_T; Q^2)$

quark pol.

U	L	T
D_1		H_1^{\perp}



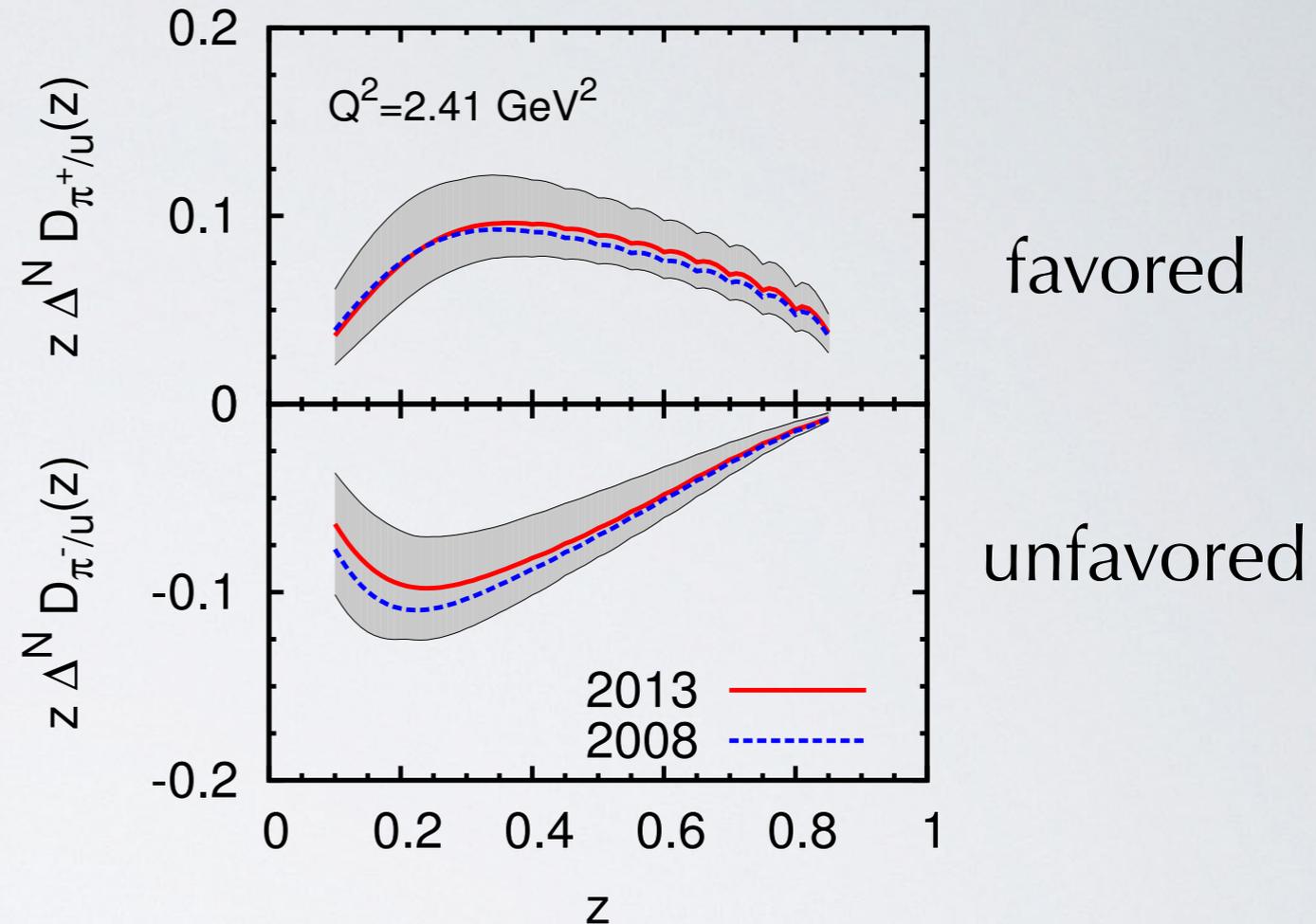
Torino fit 2013

Anselmino *et al.*,
P.R. D87 (13) 094019

$$\Delta^N D_{h/q}(z) = 4 H_1^{\perp(1/2)q \rightarrow h}(z)$$

$$H_1^{\perp(n)}(z) = \int d\mathbf{P}_{\perp} \frac{1}{2} \left(\frac{\mathbf{P}_{\perp}^2}{z^2 m_h^2} \right)^n H_1^{\perp}(z, \mathbf{P}_{\perp}^2)$$

combined fit of
SIDIS Collins effect
and
e+e- asymmetry
(9 fit parameters)



- Gaussian ansatz with **TMD PDF** $\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$ (0.57 ± 0.08)
- TMD FF** $\langle P_{\perp}^2 \rangle = 0.20 \text{ GeV}^2$ (0.12 ± 0.01)

- only DGLAP evolution of collinear part of TMD

- extracted transversity \approx independent extraction
based on collinear fact.

different from
multiplicity analysis



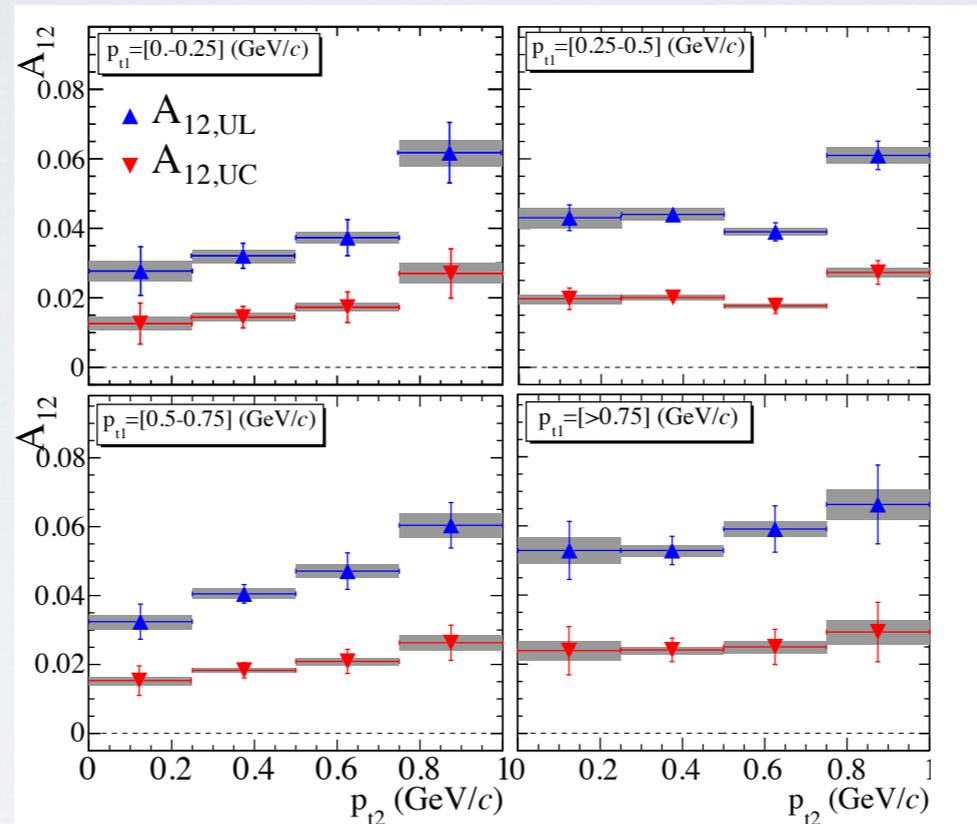
recent data on Collins effect

$$e^+e^- \rightarrow \pi\pi X$$

binning in $\{z, P_{hT}\}_i \quad i=1,2$

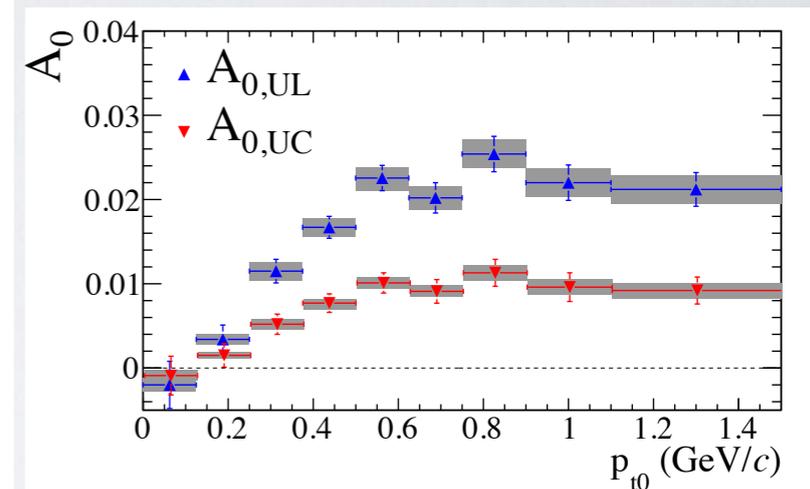
(see talk by Garzia)

“thrust axis”



Lees *et al.*, P.R. D90 (14) 052003

“fixed hadron”



Lees *et al.*, arXiv:1506:05864

binning in $\{z_1, z_2\}$ also for $K\pi$ and KK

fit of Kang, Prokudin, Sun, Yuan

Kang et al.,
P.R. D91 (15) 071501
arXiv:1505.05589

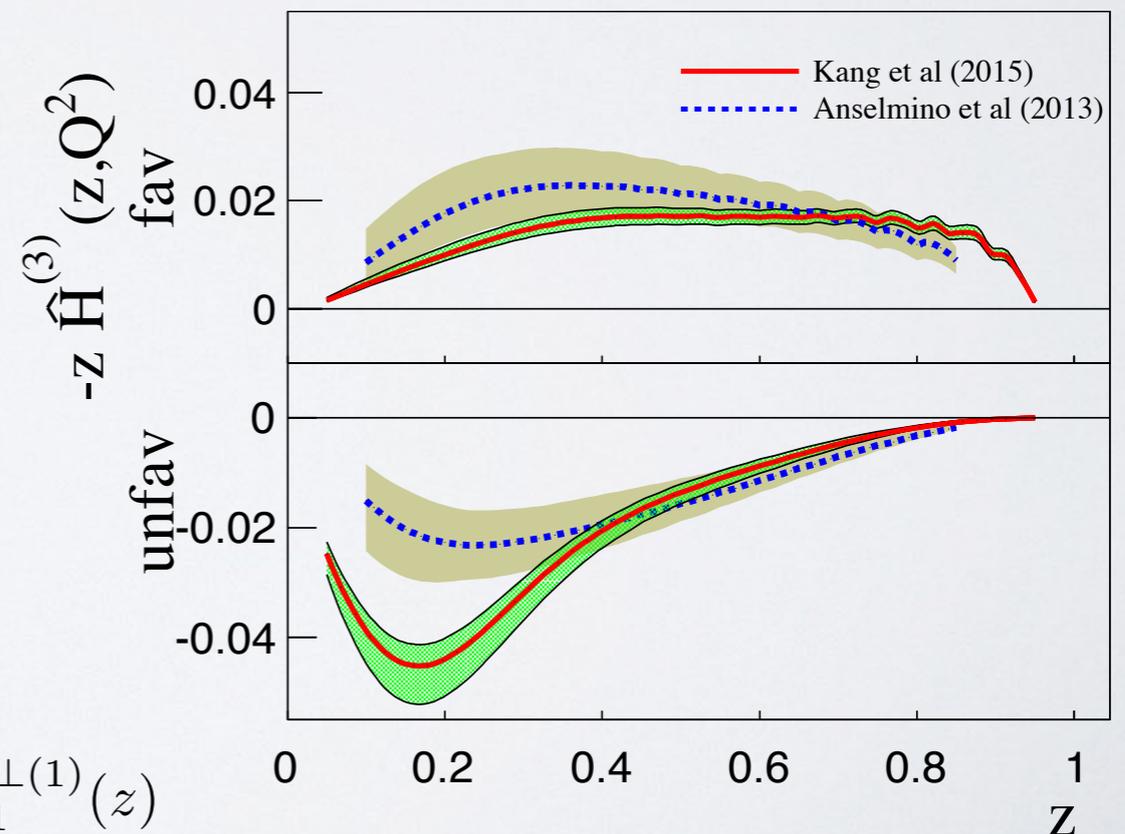
first combined fit of SIDIS Collins effect
and e^+e^- asymmetry
with TMD evolution



- NLO+NLL formalism, but only homogeneous evo eqs. for H_1^\perp
- 13 parameters, global $\chi^2/\text{dof} = 0.88$

Comparison with Torino fit

favoured < - unfavored ?



$$-z \hat{H}^{(3)}(z) = \frac{1}{2m_h} H_1^{\perp(1)}(z)$$

$e^+e^- \rightarrow \pi\pi X$
 “fixed hadron” method

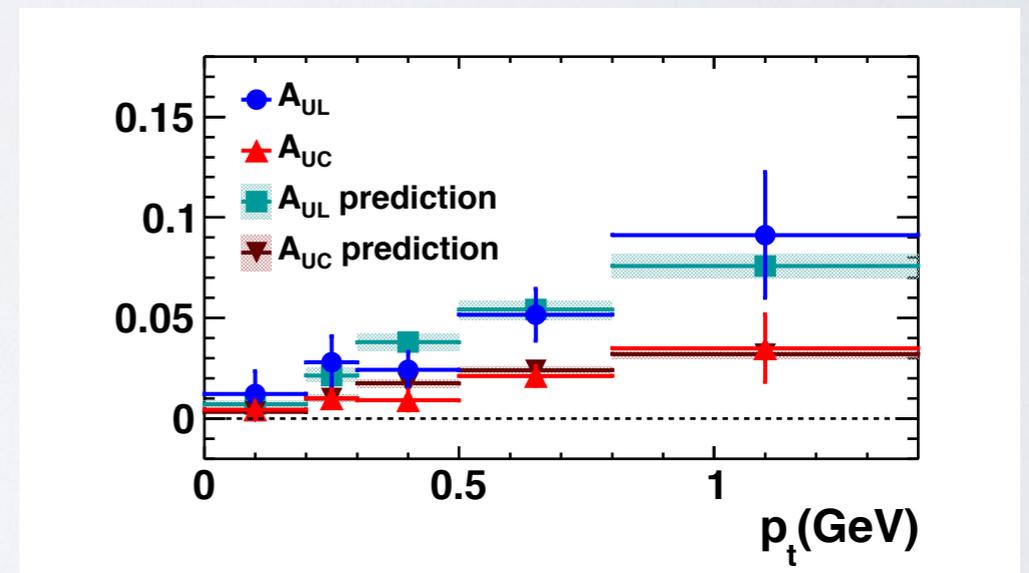
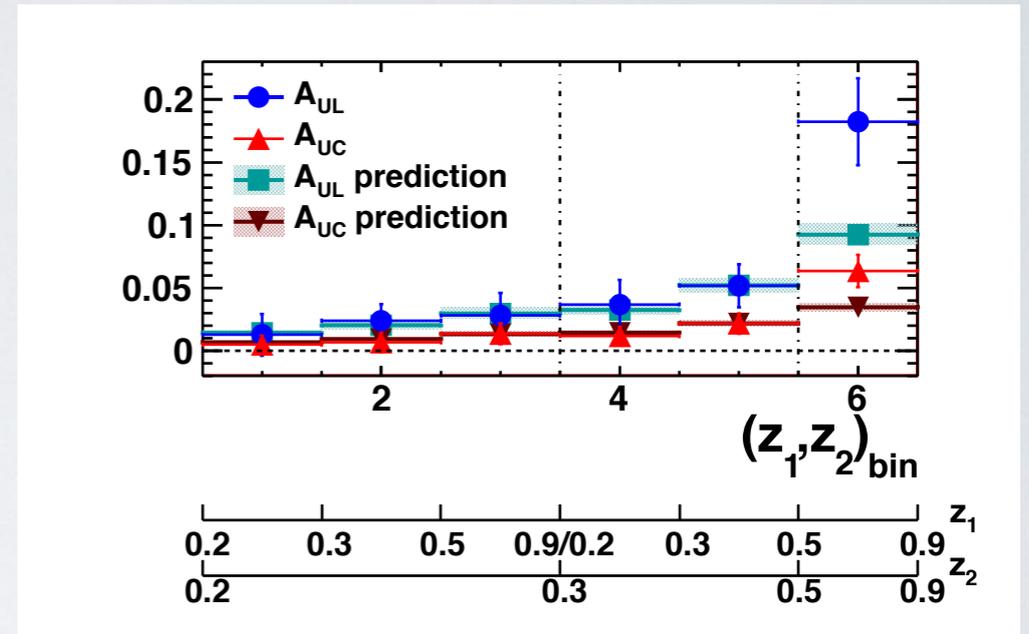
binning in $\{z_1, z_2\}$

binning in $\{z, P_{hT}\}$

(see talk by Garzia)

prediction from
 Kang, Prokudin, Sun, Yuan

Kang *et al.*, arXiv:1505.05589



Ablikim *et al.*, arXiv:1507.06824

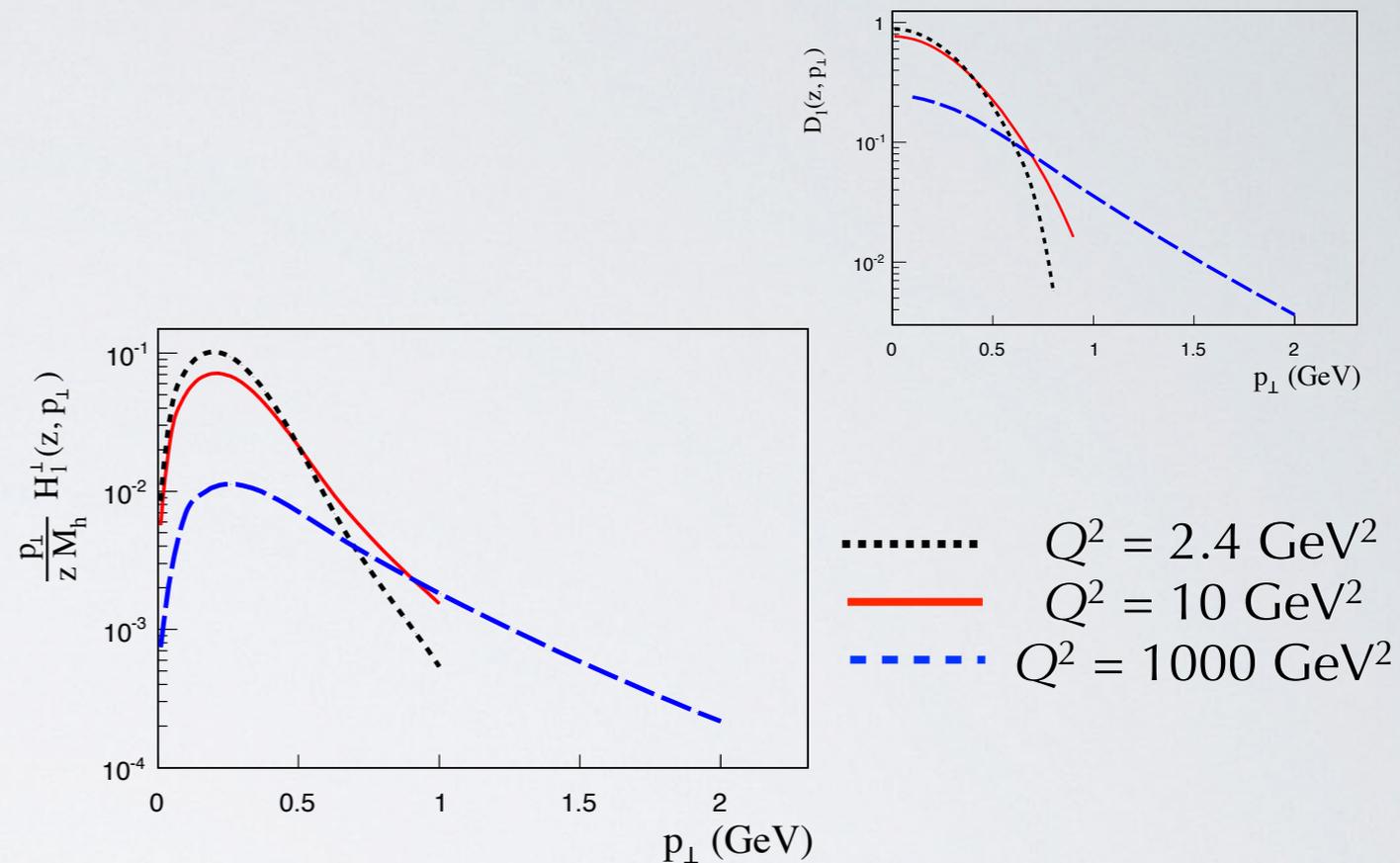
is Collins function under control ?

TMD evo effects from

Kang, Prokudin, Sun, Yuan

Kang *et al.*, arXiv:1505.05589

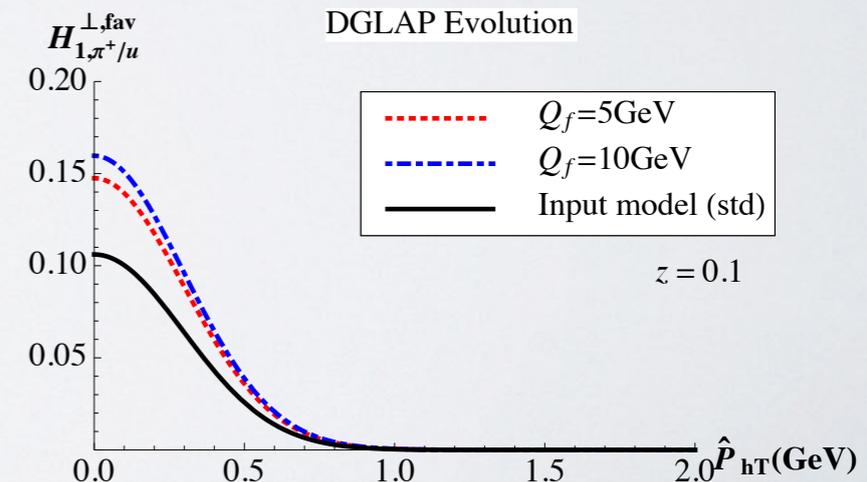
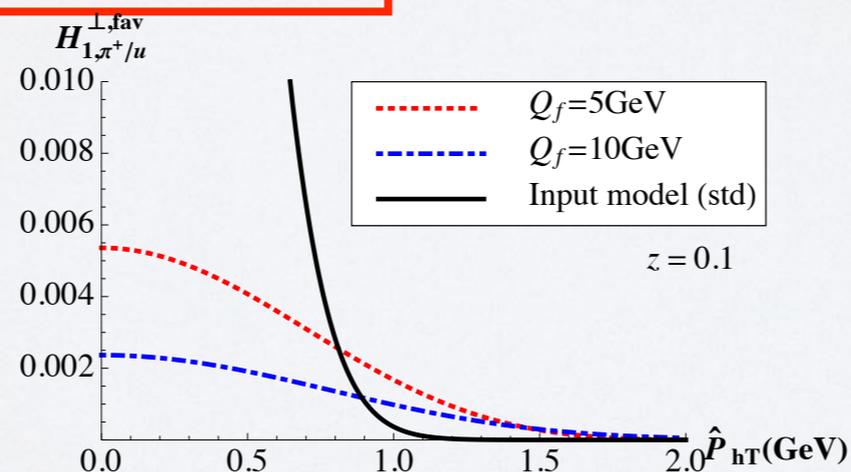
**large broadening only
at large Q^2**



Echevarria, Idilbi, Scimemi

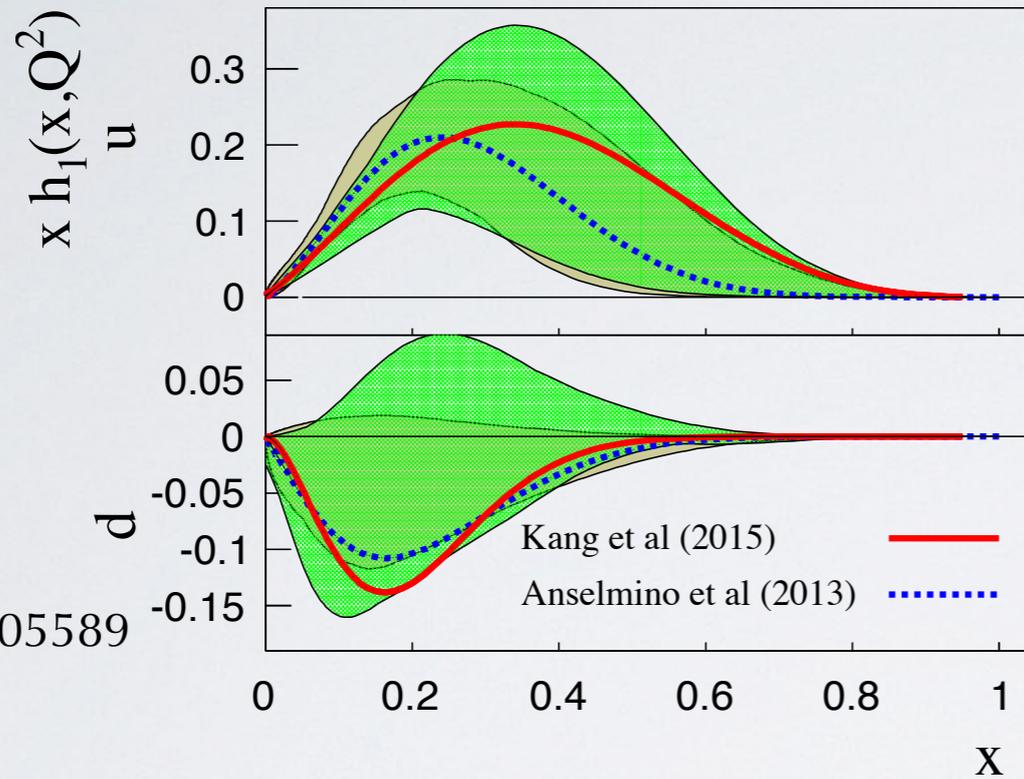
Echevarria *et al.*,
P.R. D90 (14) 014003

large reduction

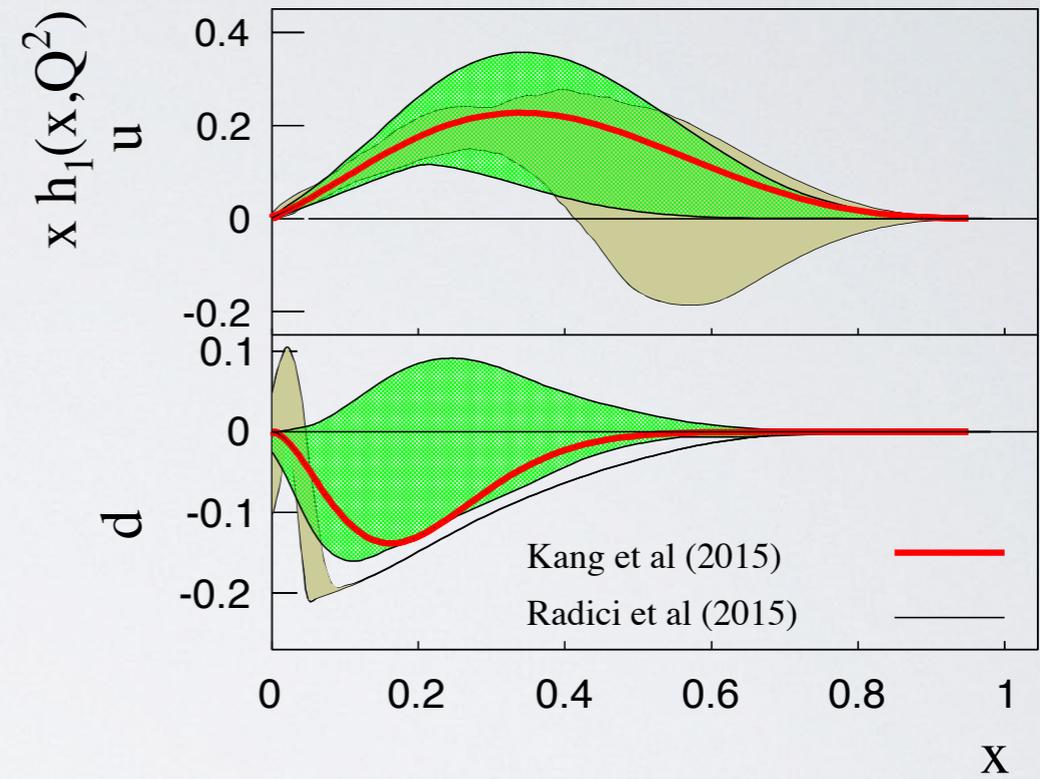


extraction of transversity

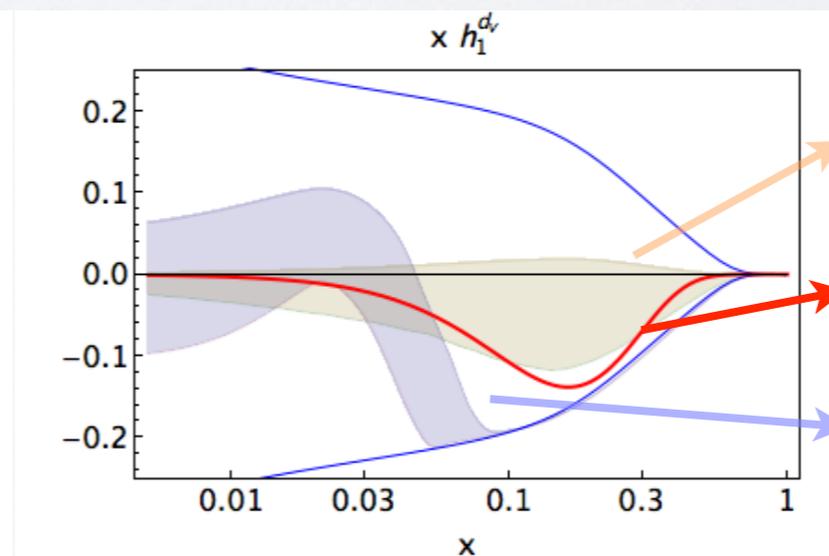
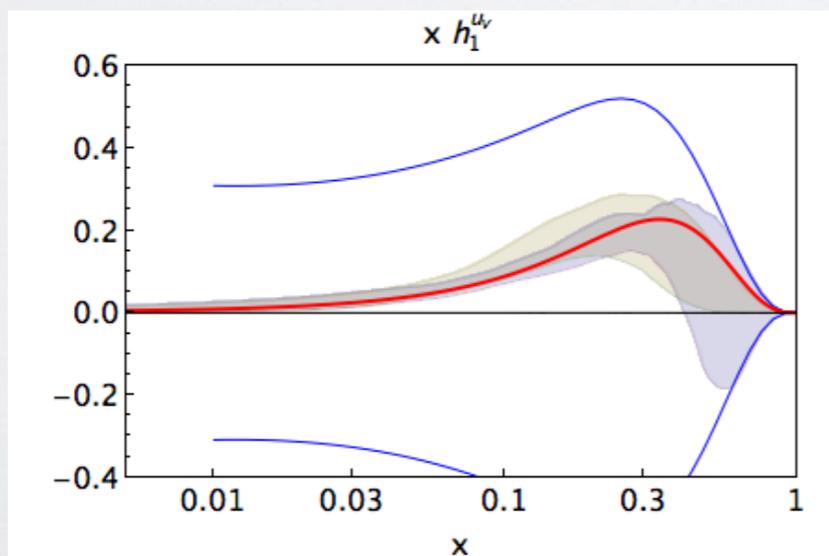
Kang *et al.* ↔ Torino



Kang *et al.* ↔ Pavia (DiFF)



Kang *et al.*,
arXiv:1505.05589



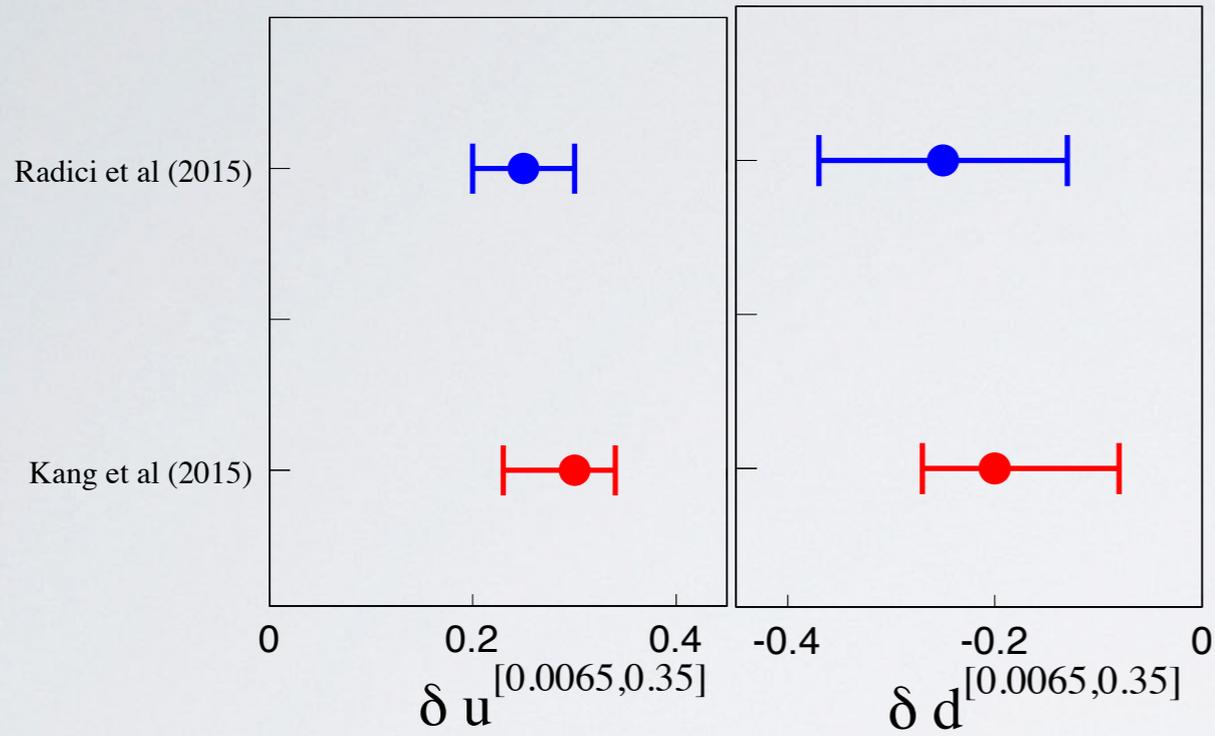
Torino 2013

Kang *et al.*

Pavia
(DiFF)

tensor charge

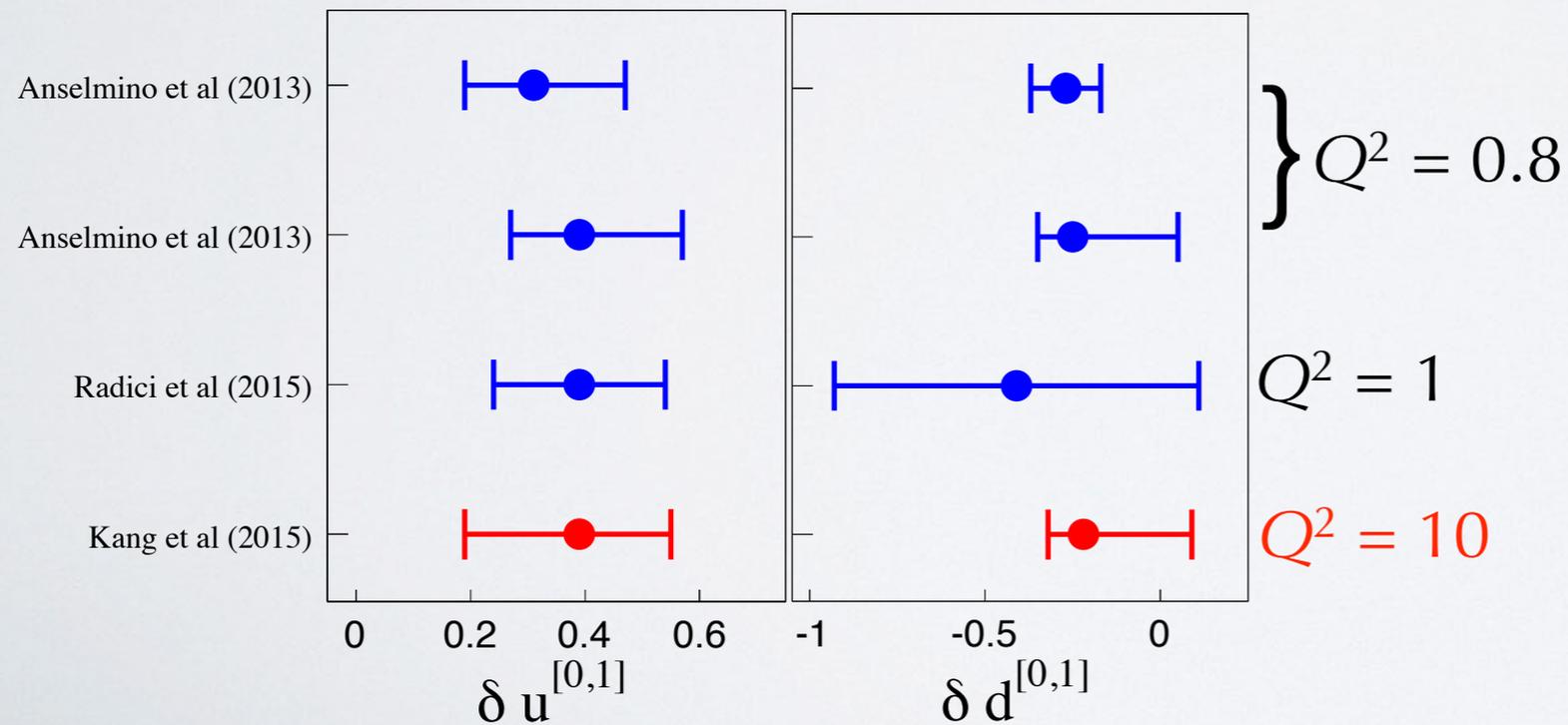
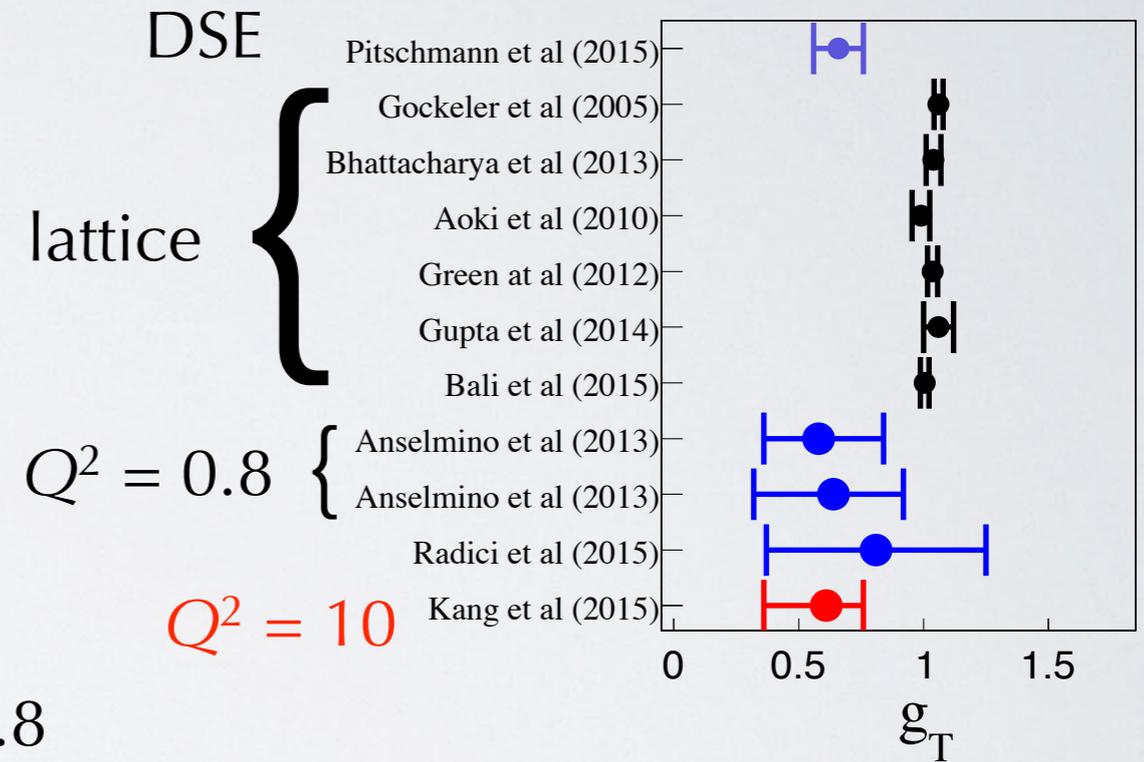
$$Q^2 = 10 \text{ GeV}^2$$



isovector tensor charge

$$g_T = \delta u - \delta d$$

$$Q^2 = 4 \text{ GeV}^2$$

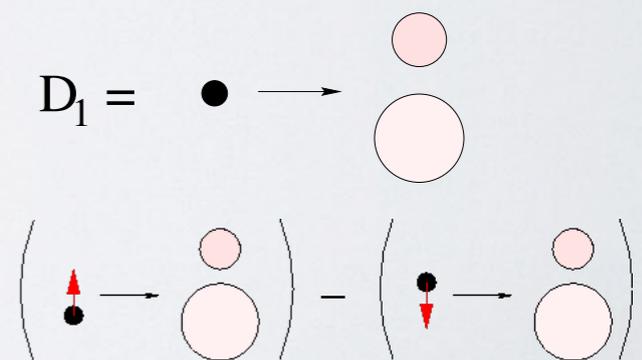


Kang *et al.*,
arXiv:1505.05589

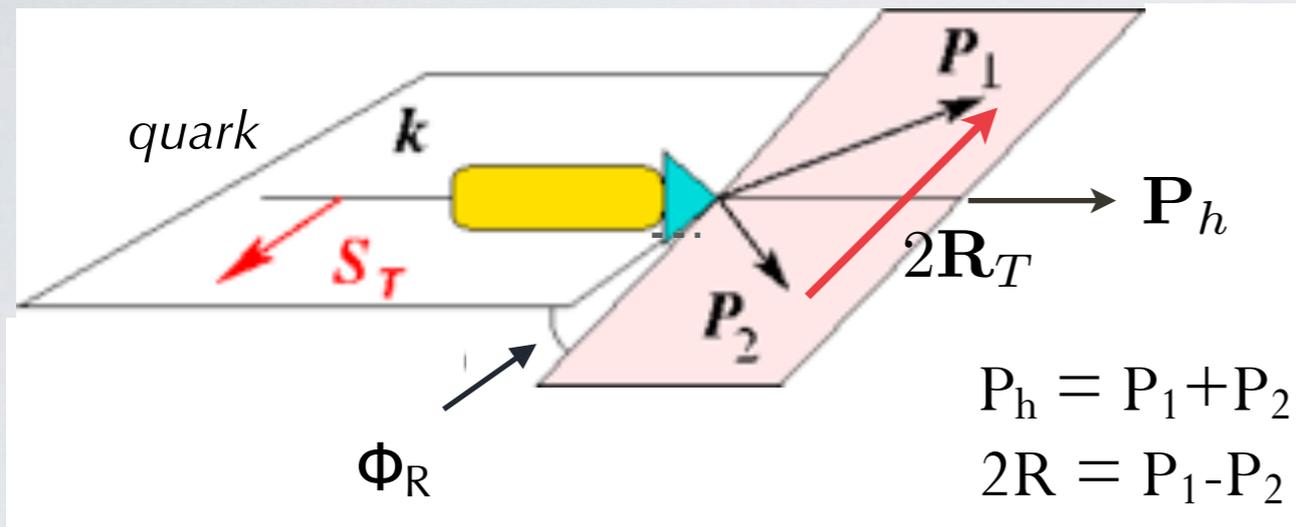
Di-hadron Fragmentation Functions (DiFF)

quark pol.

U	L	T
D_1		H_1^{\triangleleft}



the **DiFF** effect



$$S_T \leftrightarrow R_T$$

$$\int d\mathbf{P}_{hT} \leftrightarrow \mathbf{P}_{hT}=0 \rightarrow \mathbf{k}_T=0$$

collinear factorization

quark pol.

U	L	T
D_1		H_1^{\triangleleft}

2 DiFF ($z_1, z_2, \mathbf{R}_T^2; Q^2$)

DGLAP evolution

SIDIS

$$e p^\uparrow \rightarrow e' (\pi, \pi) X$$

Radici, Jakob, Bianconi
 P.R. D65 (02)
 Bacchetta & Radici,
 P.R. D67 (03)

$$d\sigma \sim d\sigma^0 + \sin(\Phi_R + \Phi_S) h_1 H_1^*$$

$e^+ e^-$

$$e^+ e^- \rightarrow (\pi^+ \pi^-) (\pi^+ \pi^-) X$$

Boer, Jakob, Radici
 P.R. D67 (03) 094003
 Artru & Collins,
 Z.Ph. C69 (96) 277

$$d\sigma \sim d\sigma^0 + \cos(\Phi_R + \Phi_{\bar{R}}) H_1^* \bar{H}_1^*$$

Ceccopieri, Radici, Bacchetta,
 P.L. B650 (07)

extraction of **DiFF**

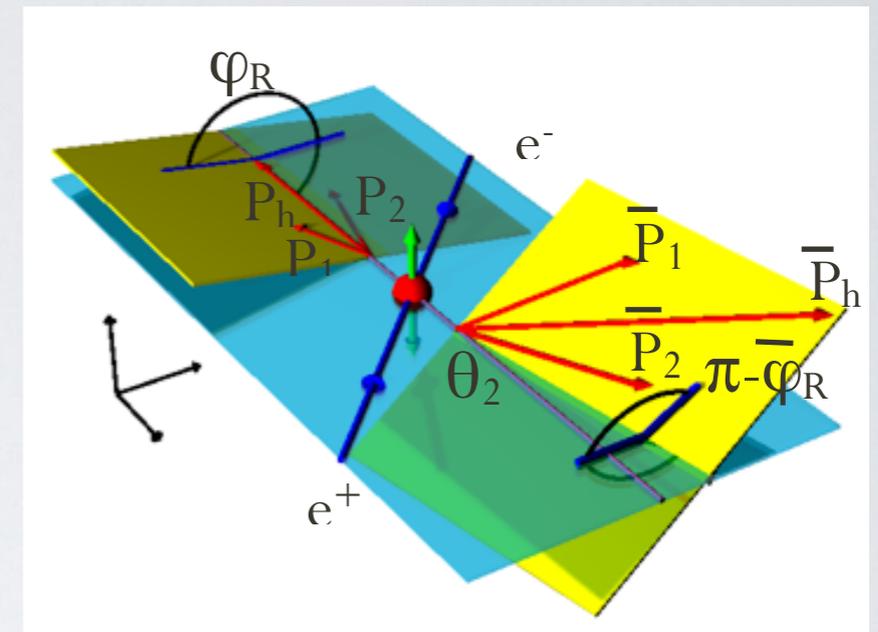
$$e^+ e^- \rightarrow (\pi^+ \pi^-) (\pi^+ \pi^-) X$$

first measurement of

$$A^{\cos(\Phi_R + \Phi_{\bar{R}})} \sim \frac{H_1^* \bar{H}_1^*}{D_1 \bar{D}_1}$$



Vossen *et al.*,
P.R.L. **107** (11) 072004



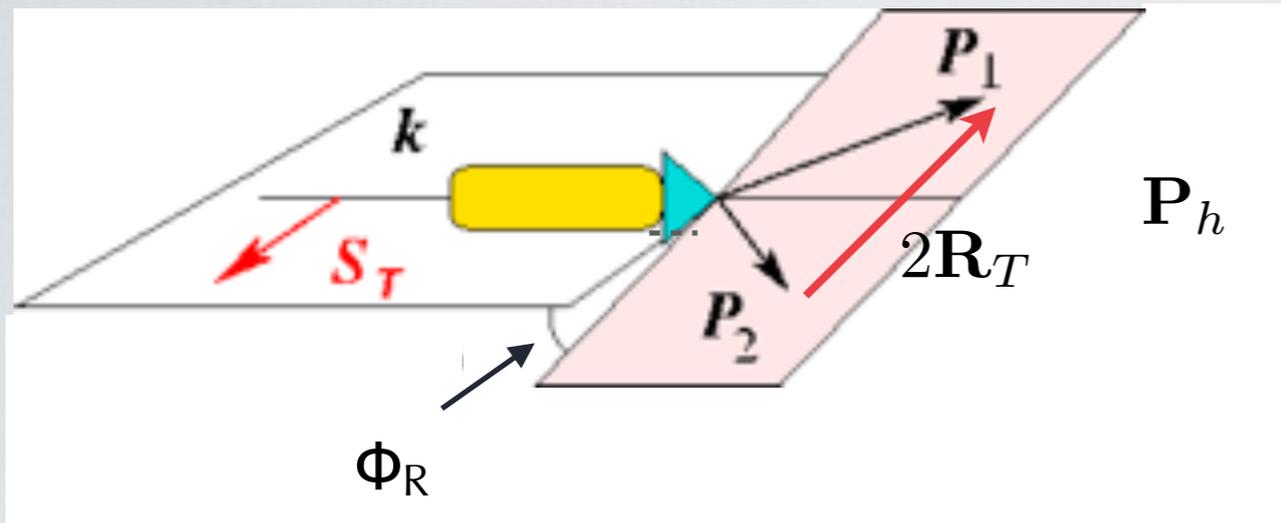
fit of $A^{\cos(\Phi_R + \Phi_{\bar{R}})} \rightarrow$ extract H_1^*

Courtoy *et al.*, P.R. D**85** (12) 114023

but D_1 fitted to PYTHIA (adapted to Belle kin.)

need di-hadron multiplicities

expansion of DiFF in partial waves

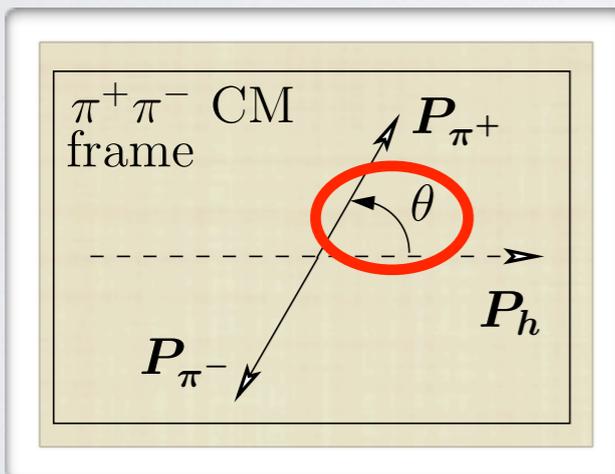


Bacchetta & Radici, P.R. D67 (03) 094002

for low pair invariant masses,
the system $(h_1, h_2)_L$ can be in
relative $L = 0$ (s) or 1 (p)
interference \rightarrow model T-odd H_1^*

$$\text{DiFF}(z_1, z_2, \mathbf{R}_T^2) \leftrightarrow \text{DiFF}(z=z_1+z_2, \zeta, M_h^2)$$

(h_1, h_2) c.m. frame



change of variable $\zeta = \frac{z_1 - z_2}{z} \longleftrightarrow \cos \theta$

expansion in $\cos \theta$

$$D_1^q(z, \zeta, M_h^2) \approx D_1^q(z, M_h^2) + D_{1,sp}^q(z, M_h^2) \cos \theta + \dots$$

$$H_1^{\triangleleft q}(z, \zeta, M_h^2) \approx H_{1,sp}^{\triangleleft q}(z, M_h^2) + H_{1,pp}^{\triangleleft q}(z, M_h^2) \cos \theta + \dots$$

these are really measured ... maybe ...

(see talk Vossen)

extraction of valence $h_1^{q_v}$ and future

$e^+ e^-$



$(\pi^+ \pi^-)$

Vossen *et al.*,
P.R.L. **107** (11) 072004

SIDIS



proton target

Airapetian *et al.*,
JHEP **0806** (08) 017

first point-by-point extraction of

$$u_v - 1/4 d_v$$

Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001



proton + deuteron

Adolph *et al.*, P.L. **B713** (12)

$$u_v - 1/4 d_v$$

$$u_v + d_v$$

fit of
 u_v and d_v

Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119



new proton data

Braun *et al.*,
E.P.J. Web Conf. **85** (15) 02018

new fit of u_v and d_v

Radici *et al.*, JHEP **1505** (15) 123

Future



data for $p p^\uparrow \rightarrow (\pi\pi) X$

Adamczyk *et al.*, arXiv:1504.00415

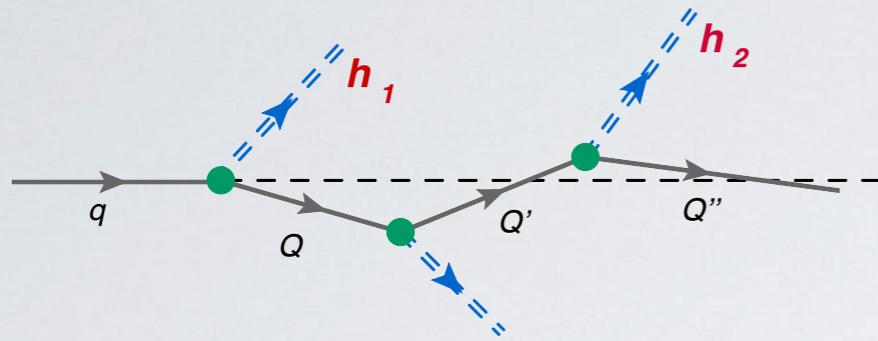
ongoing analysis
test of universality



data with $K\pi$, KK pairs \Rightarrow  data with KK and $K\pi$?

explore contribution of sea quarks to transversity

parallel model studies: the NJL-jet model

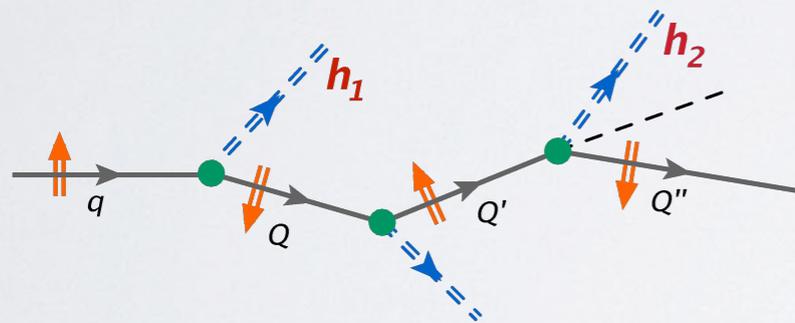


quark hadronization chain
NJL assumed at each vertex
DiFF = MC averages of proper multiplicities
results for $q=u,d,s$ and
 $h = \pi\pi, \pi K, KK$ pairs and
decay of vector mesons

Casey *et al.*, P.R. D**85** (12) 114049

P.R. D**86** (12) 114018

Matevosyan *et al.*, P.R. D**88** (13) 094022



recently extended to include effects of
transverse polarization of fragmenting quark
and
transverse momentum-spin correlations

Matevosyan *et al.*, P.L. **B731** (14) 208

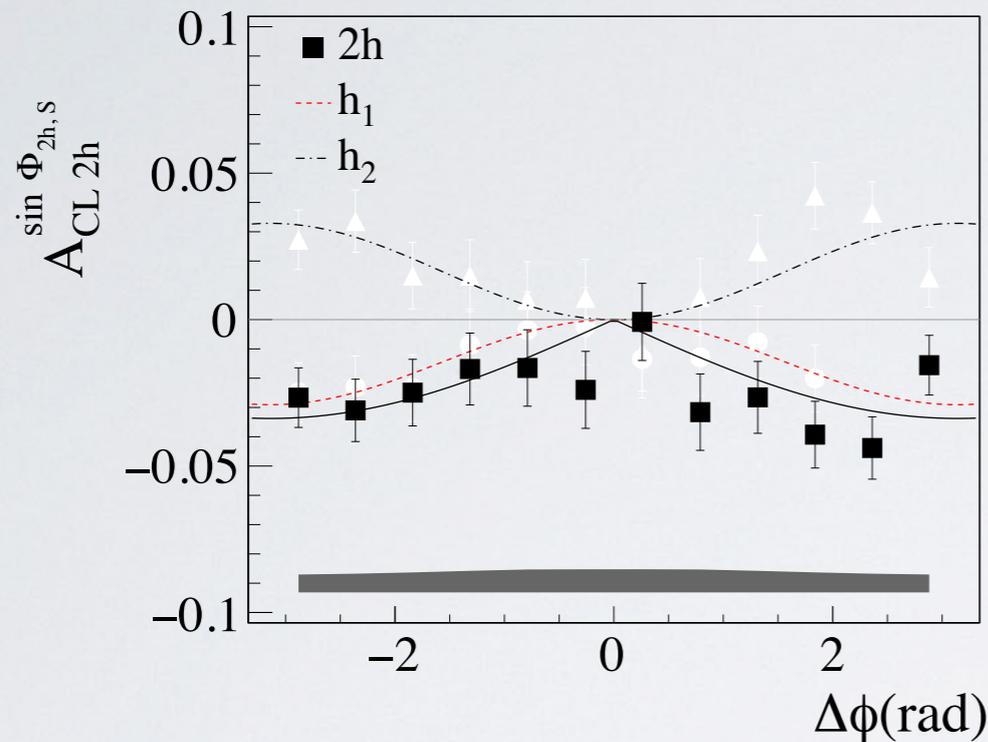
Kotzinian *et al.*, P.R.L. **113** (14) 062003

P.R. D**90** (14) 074006

Matevosyan *et al.*, arXiv:1502.02669

(see talk Matevosyan)

similar origin for Collins and DiFF effects ?



fit to Collins-like asymmetry for h_2

fit to Collins-like asymmetry for h_1

■ $\sin \varphi_{2h}$ asymmetry $\varphi_{2h} \leftrightarrow \mathbf{R} = \mathbf{p}_1 - \mathbf{p}_2$

$\Delta\varphi = \varphi_1 - \varphi_2$ $\varphi_1 \leftrightarrow \mathbf{p}_1$ $\varphi_2 \leftrightarrow \mathbf{p}_2$

Adolph *et al.*, arXiv:1507.07593

(see talk Bradamante)

in agreement with $3P^0$ string fragmentation model

Artru *et al.*, Z.Ph. **C73** (97) 527

supported by MC calculations of DiFF in NJL-jet model:

elementary Collins mechanism generates $\sin\varphi_R$ modulation

Matevosyan *et al.*, P.L. **B731** (14) 208

a detour from phenomenology to theory..

Gliske, Bacchetta, Radici,
P.R. D90 (14) 114027

general di-hadron SIDIS cross section
up to twist 3 for any partial wave

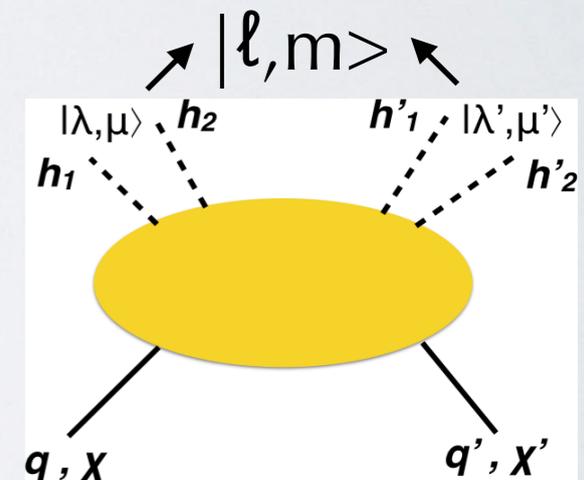
$$d\sigma_{XY} \sim \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell m}(\cos \theta) f_m(\cos \phi_h, \cos \phi_R) F_{XY}^{P_{\ell m} f_m}$$

$$F_{XY}^{P_{\ell m} f_m}(x, z, P_{h\perp}^2, M_h^2) = \text{TMD}(x, \mathbf{k}_\perp) \otimes_{\mathbf{k}_\perp, \mathbf{p}_\perp} \text{DiFF}^{\ell m}(z, M_h^2, |\mathbf{p}_\perp|)$$

(assuming factorization th. also for 2-hadron SIDIS)

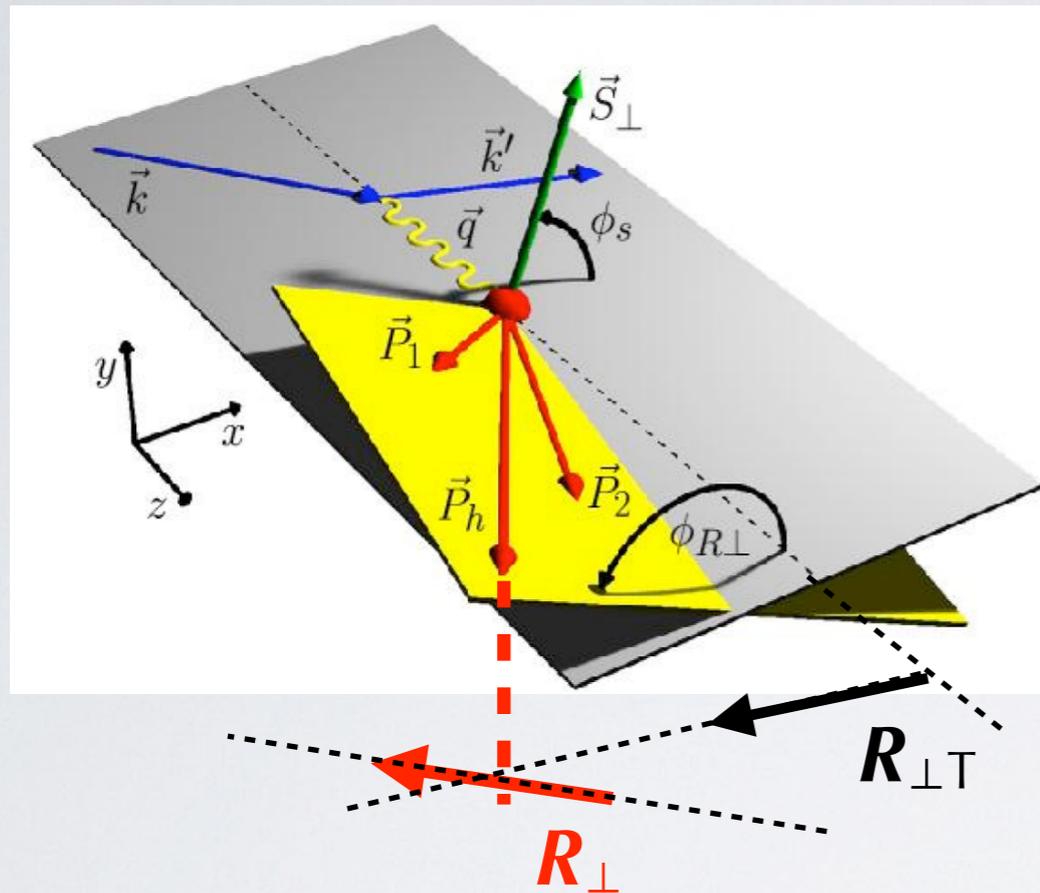
- $\ell = \lambda \oplus \lambda'$ total partial wave of final hadronic system
- ℓ_{\max} in principle not limited, because hadronic tensor not linear in R

- 3 classes of $\text{DiFF}^{\ell m}$:
 - $D^{\ell m} \leftrightarrow \chi = \chi' = 1/2 + \chi = \chi' = -1/2$
 - $G^{\ell m} \leftrightarrow \chi = \chi' = 1/2 - \chi = \chi' = -1/2$
 - $H^{\ell m} \leftrightarrow \chi \neq \chi'$



definition of azimuthal angles

Gliske, Bacchetta, Radici,
P.R. D90 (14) 114027



$$R_{\perp}^{\mu} = g_{\perp}^{\mu\nu} R_{\nu}$$

$$l_T^{\mu} = g_T^{\mu\nu} l_{\nu}$$

$$R_{\perp T}^{\mu} = g_T^{\mu\nu} R_{\perp\nu}$$

covariant definition

$$\cos \phi_{R_T} = -\frac{l_{\mu} R_{\perp\nu} g_T^{\mu\nu}}{\sqrt{l_T^2 R_{\perp T}^2}}$$

$$\sin \phi_{R_T} = -\frac{l_{\mu} R_{\perp\nu} \epsilon_T^{\mu\nu}}{\sqrt{l_T^2 R_{\perp T}^2}}$$

covariant definition

$$\rightarrow \text{TRF } R_{\perp T}|_{\text{TRF}} = \frac{z_2 P_{1T} - z_1 P_{2T}}{z} + \mathcal{O}\left(\frac{1}{Q^3}\right) \text{ by Artru \& Collins COMPASS}$$

$$= (\mathbf{R} - \mathbf{R} \cdot \hat{P}_h \hat{P}_h)_T + \left(\frac{1}{Q^3}\right) \text{ HERMES, BELLE}$$

$$\rightarrow \text{IMF } \gamma^*\text{-P Breit frame } \quad \quad \quad +\mathcal{O}\left(\frac{1}{Q^2}\right)$$

definition of azimuthal angles

Gliske, Bacchetta, Radici,
P.R. D90 (14) 114027

covariant
definition

→ TRF

$$\phi_{R_T} = \frac{(\mathbf{q} \times \boldsymbol{\ell}) \cdot \mathbf{R}_\perp}{|(\mathbf{q} \times \boldsymbol{\ell}) \cdot \mathbf{R}_\perp|} \arccos \frac{(\mathbf{q} \times \boldsymbol{\ell}) \cdot (\mathbf{q} \times \mathbf{R}_\perp)}{|\mathbf{q} \times \boldsymbol{\ell}| |\mathbf{q} \times \mathbf{R}_\perp|}$$

with this choice, if $\int d\mathbf{P}_{hT}$ then no di-hadron Sivers or Collins effects arise

$$\hookrightarrow \text{no } \mathbf{k}_T \Rightarrow \mathbf{k} \times \mathbf{P} \cdot \mathbf{S} = 0 \text{ and } \mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_q = 0$$

choice of

Kotzinian, Matevosyan, Thomas,
P.R.L. **113** (14) 062003
P.R. D90 (14) 074006
P.L. **B731** (14) 208

Matevosyan *et al.*,
arXiv:1502.02669

$$\mathbf{R}_T = \frac{\mathbf{P}_{1T} - \mathbf{P}_{2T}}{2} = \mathbf{R}_\perp + \frac{z_1 - z_2}{2} \mathbf{k}_T$$

non covariant choice: depending on
the frame, TMD effects can arise...

claim of Sivers effect and Collins-induced $\sin \varphi_R$ modulation in
di-hadron SIDIS even without sensitivity to quark \mathbf{k}_T ($\int d\mathbf{P}_{hT}$)
(see talk Kotzinian & Matevosyan)

are really Sivers and Collins-like effects ?