The pretzelosity of the nucleon

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- **My understanding of the pretzelosity**
- **Why we measure it?**
- **How we measure it?**

Leading-Twist TMD PDFs

Nucleon Spin

Quark Spin

Nucleon Polarization		Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally Polarized (L)	Quark polarization: Transversely Polarized (T)
	U	f_1		$h_1^{\perp}$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^{\perp}$ Long-Transversity
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} Trans-Helicity	h_1 Transversity $h_{1T}^{\perp}$ Pretzelosity

What is "Pretrelosity" ?

- Pretzelosity: one of the eight leading twist transverse dependent parton distributions (TMDs).
- The quark-quark correlator up to the leading twist

$$\Phi(x, \mathbf{p}_\perp) = \frac{1}{2}\{f_1 \not{n}_+ - f_{1T}^\perp \frac{\epsilon_\perp^{ij} p_\perp^i S_\perp^j}{M_N} \not{n}_+$$
$$+(S_\| g_{1L} + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} g_{1T})\gamma_5 \not{n}_+ + h_{1T} \frac{[\not{S}_\perp, \not{n}_+]\gamma_5}{2}$$
$$+(S_\| h_{1L}^\perp + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} h_{1T}^\perp) \frac{[\not{p}_\perp, \not{n}_+]\gamma_5}{2M_N} + i h_1^\perp \frac{[\not{p}_\perp, \not{n}_+]}{2M_N}\}. (7)$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. **B 461**, 197 (1996), Erratum-ibid. **B 484**, 538 (1997). K. Goeke, A. Metz, and M. Schlegel, Phys. Lett. **B 618**, 90 (2005).

What is "Pretrelosity"?

$$\frac{p_\perp^x p_\perp^y}{M_N^2} h_{1T}^\perp(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp \cdot \boldsymbol{\xi}_\perp)} \times \langle PS^y | \bar{\psi}(0) i\sigma^{1+}\gamma_5 \psi(0, \xi^-, \xi_\perp) | PS^y \rangle, \quad (12)$$

$|PS^y\rangle$: the hadronic state with a polarization in the y direction.

- Some properties of pretzelosity:
  1. It is chiral-odd, and needs a chiral-odd partner in the SIDIS.
  2. There is no gluon analog of pretzelosity.
  3. In a large class of models, it is the difference of helicity and transversity, and hence a measure for relativistic effects.

H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

“Pretrel” or “Brezel”

“Mahua(麻花)”: the Chinese Preztel

How to get a clear picture of nucleon?

- PDFs are physically defined in the IMF (infinite-momentum frame) or with space-time on the light-cone.
- Whether the physical picture of a nucleon is the same in different frames?

A physical quantity defined by matrix element is frame-independent, but its physical picture is frame-dependent.

The Wigner Rotation

for a rest particle $(m,\vec{0}) = p^{\mu}$ $(0,\vec{s}) = \mathrm{w}^{\mu}$

for a moving particle $\mathrm{L}(p)p = (m,\vec{0})$ $(0,\vec{s}) = \mathrm{L}(p)w/m$

$\mathrm{L}(p)$ = ratationless Lorentz boost

Wigner Rotation

$\vec{s}, p_{\mu} \rightarrow \vec{s'}, p'_{\mu}$

$\vec{s'} = R_w(\Lambda, p)\vec{s}$ $p' = \Lambda p$

$R_w(\Lambda, p) = \mathrm{L}(p')\Lambda \mathrm{L}^{-1}(p)$ a pure rotation

E.Wigner,
Ann.Math.40(1939)149

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame
Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^{+} + m)\chi^{\uparrow}(F) - q^{R}\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^{+} + m)\chi^{\downarrow}(F) + q^{L}\chi^{\uparrow}(F)].$$

What is Δq measured in DIS

- Δq is defined by $\Delta q\ s_\mu = \langle p,s|\bar{q}\gamma_\mu\gamma_5 q|p,s\rangle$

$$\Delta q = \langle p,s|\bar{q}\gamma^+\gamma_5 q|p,s\rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 \mathrm{d}x\left[q^\uparrow(x) - q^\downarrow(x)\right]$$

- Using conventional Dirac spinors

$$\Delta q = \int \mathrm{d}^3\vec{p} M_q\left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p})\right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin or quark spin in the infinite momentum frame, not that in the rest frame of the proton

The proton spin crisis

& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

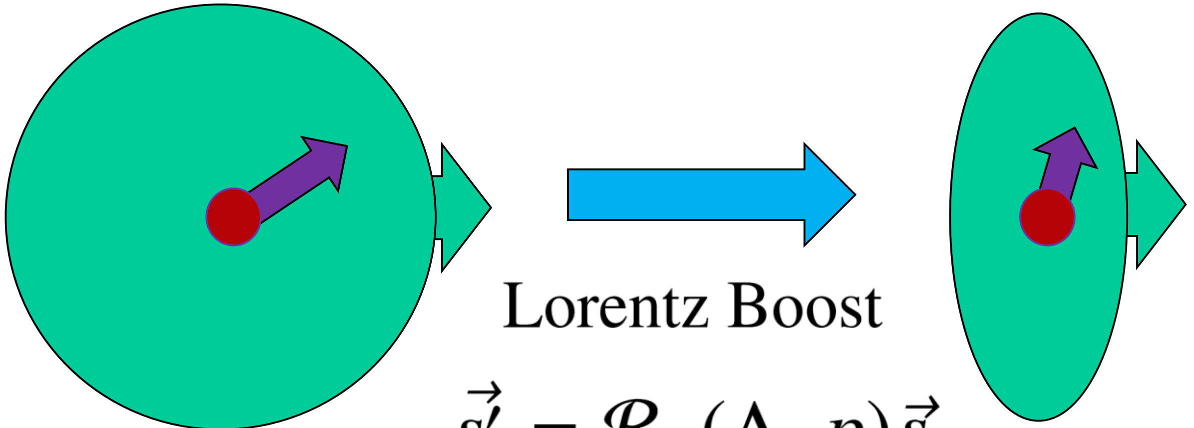
B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

B.-Q. Ma, J.Phys.G 17 (1991) L53-L58

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

An intuitive picture to understand the spin puzzle



$$\vec{s}' = \mathcal{R}_w(\Lambda, p)\vec{s}$$

Rest Frame

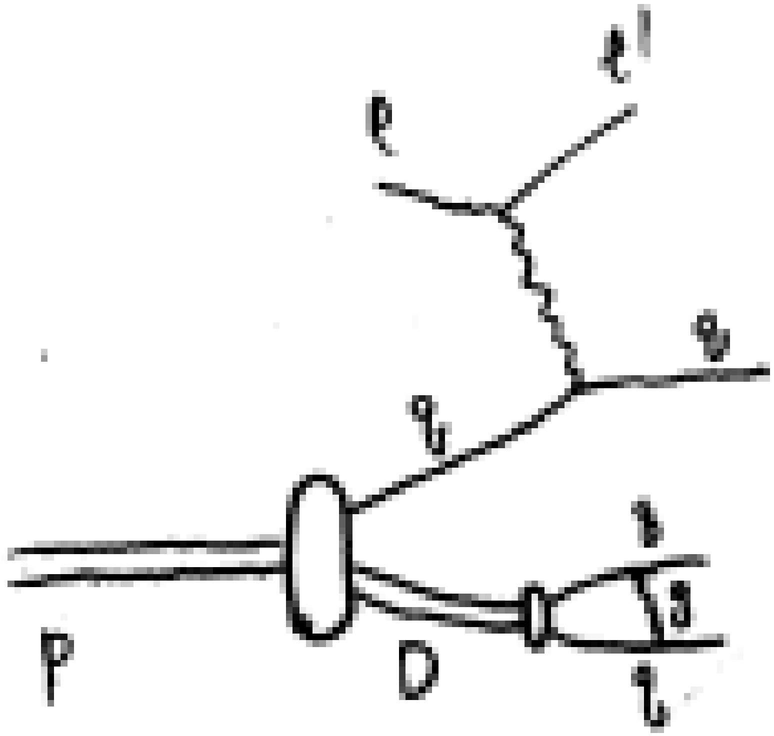
$$\Sigma \vec{s} = \vec{S}_p$$

Infinite Momentum Frame

$$\Sigma \vec{s}' \neq \vec{S}_p$$

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I.Schmidt, J.Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model



A relativistic quark-diquark model

- The unpolarized distribution of quark q in hadron h can be written as

$$q(x) = c_q^S a_S(x) + c_q^V a_V(x),$$

where $a_D(x)$ is

$$a_D(x) \propto \int [\mathrm{d}^2 \mathbf{k}_\perp] |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V),$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$\phi(x, \mathbf{k}_\perp) = A_D \exp\left\{-\frac{1}{8\alpha_D^2}\left[\frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}\right]\right\},$$

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I.Schmidt, J.Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [\mathrm{d}^2 \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

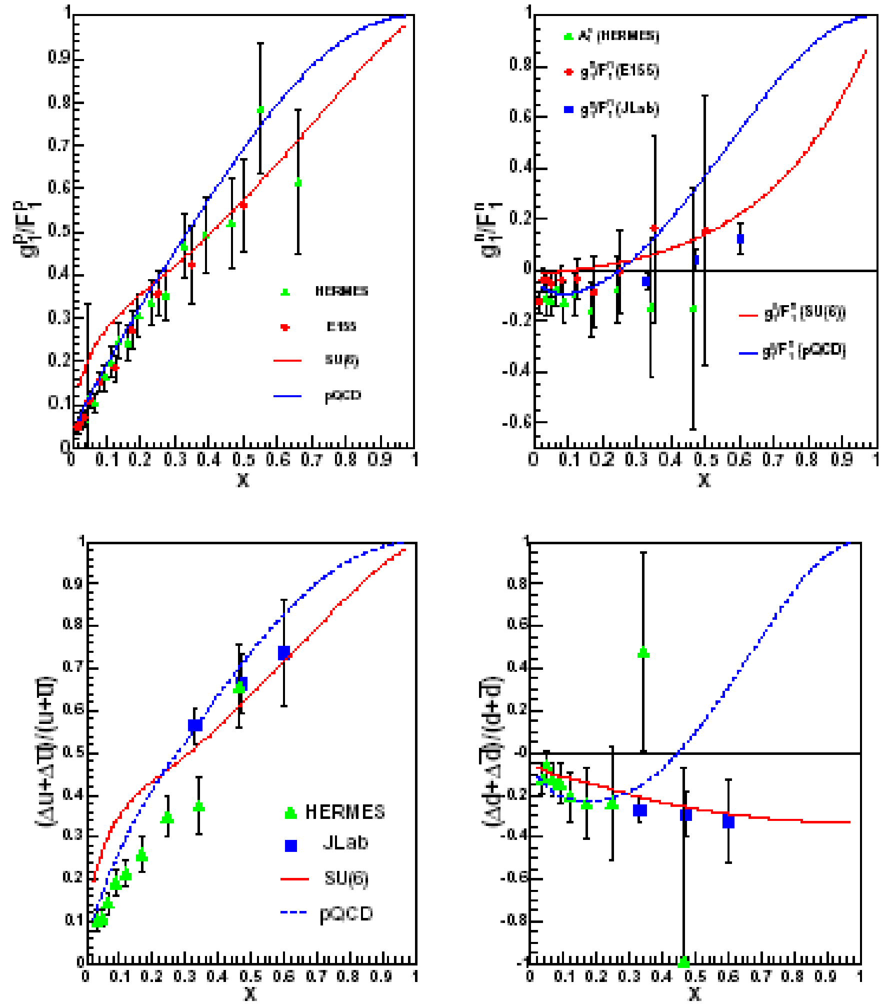
Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

Different predictions in two models

- Helicity distribution
  - SU(6) quark-diquark model:
    $\Delta u(x)/u(x) \to 1$ as $x \to 1$.
    $\Delta d(x)/d(x) \to -\frac{1}{3}$ as $x \to 1$.
  - pQCD based counting rule analysis:
    $\Delta u(x)/u(x) \to 1$ as $x \to 1$.
    $\Delta d(x)/d(x) \to 1$ as $x \to 1$.



The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle\, p,\uparrow |\bar{q}_{\lambda}\gamma^{\perp}\gamma^{+} q_{-\lambda}| p,\downarrow\,\rangle$$

$$\delta q(x) = \int [\mathrm{d}^2 \boldsymbol{k}_{\perp}]\, \tilde{M}_q(x,\boldsymbol{k}_{\perp})\, \Delta q_{\mathrm{RF}}(x,\boldsymbol{k}_{\perp})$$

$$\tilde{M}_q(x,\boldsymbol{k}_{\perp}) = \frac{(k^{+}+m)^2}{(k^{+}+m)^2 + \boldsymbol{k}_{\perp}^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right).$$

$$L_q(x) = \int [d^2k_\perp] M_L(x,k_\perp) \Delta q_{QM}(x,k_\perp)$$

$$M_L(x,k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

The Melosh-Wigner Rotation in "Pretzelosity"

$$g_1^q(x,k_\perp) - h_1^q(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp)\ .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretzelosity} = \Delta \mathrm{q} - \delta \mathrm{q} = -\mathrm{L_q}$$

$$\mathrm{Pretzelosity} = -\int [\mathrm{d}^2\mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q}_{\mathrm{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

New Sum Rule of Physical Observables

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) \; .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretzelosity} = \Delta \mathrm{q} - \delta \mathrm{q} = -\mathrm{L_q}$$

$$\mathrm{Pretzelosity} = -\int [\mathrm{d}^2 \mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q_{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

A Simple Relation

- The difference of helicity and transversity is the first moment of pretzelosity.

$$h_{1T}^{\perp(1)qv}(x,\mathbf{p}_\perp) \equiv \frac{p_\perp^2}{2M_N^2} h_{1T}^{\perp qv}(x,\mathbf{p}_\perp) = g_1^{qv}(x,\mathbf{p}_\perp) - h_1^{qv}(x,\mathbf{p}_\perp),$$

- This relation has already been obtained in
  H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355. B. Pasquini, S. Cazzaniga and S. Boffi, Phys. Rev. **D 78**, 034025 (2008).
- But this relation is not fully satisfied in
  A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. **D 78**, 074010 (2008).

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子（$W_D(D=V,\ S)$）
g_{1L}	$[(x\mathscr{M}_D+m_q)^2-p_\perp^2]/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
g_{1T}	$2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
h_1	$(x\mathscr{M}_D+m_q)^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1L}^\perp$	$-2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1T}^\perp$	$-2M_N^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$

$\mathscr{M}_D^2=\frac{m_q^2+p_\perp^2}{x}+\frac{m_D^2+p_\perp^2}{1-x}$是旁观双夸克的不变质量。

Preztelosity in SIDIS

- Pretzelosity can be measured through $\sin(3\phi_h - \phi_S)$ asymmetry in the SIDIS process, where the cross section can be written as

$$\frac{d^6\sigma_{UT}}{dxdyd\phi_S dzd^2\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2}\{(1-y+\frac{1}{2}y^2)F_{UU} \\ +S_\perp \sin(3\phi_h - \phi_S)(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)} + \ldots\}, \quad (23)$$

with $F_{UU} = \mathcal{F}[\omega_1 f_1 D_1]$, $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{F}[\omega_2 h_{1T}^{\perp} H_1^{\perp}]$

- The $\sin(3\phi_h - \phi_S)$ asymmetry

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{\frac{2\alpha^2}{sxy^2}(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)}}{\frac{2\alpha^2}{sxy^2}(1-y+\frac{1}{2}y^2)F_{UU}}. \quad (24)$$

Quantities in Calculation

- DFs and FFs to be parametrized:

	x dependence	z dependence	TM dependence
f_1	well known	—	not so clear
$h_{1T}^{\perp}$	not known	—	not known
D_1	—	known	not so clear
$H_1^{\perp}$	—	a little known	not clear

- Theoretical understanding: non-perturbative, model calculation, cannot give the exact value so far.
- Transverse momentum dependence: not so clearly yet, usually parametrized in a Gaussian form.
- D_1 and $H_1^{\perp}$: Gaussian parametrization given by
  S. Kretzer, *et al.*, Eur. Phys. J. **C 22**, 269 (2001).
  M. Anselmino, *et al.*, arXiv:0807.0173.

Approach 0 to TMDs

- Starting with the equation

$$h_{1T}^{\perp(uv)}(x) = \left[f_1^{(uv)}(x) - \frac{1}{2}f_1^{(dv)}(x)\right]\hat{W}_S(x) - \frac{1}{6}f_1^{(dv)}(x)\hat{W}_V(x),$$
$$h_{1T}^{\perp(dv)}(x) = -\frac{1}{3}f_1^{(dv)}(x)\hat{W}_V(x), \quad (25)$$

where $\hat{W}_D(x) = \int d^2\mathbf{p}_\perp \varphi^2(x,\mathbf{p}_\perp)W_D(x,\mathbf{p}_\perp)/\int d^2\mathbf{p}_\perp \varphi^2(x,\mathbf{p}_\perp)$

- $f_1(x)$: CTEQ6L as an input. $h_{1T}^{\perp}(x)$: from Eq. 25
- Transverse momentum dependence: Gaussian form.
- How to fit the Gaussian width? $p_{av}/k_{av} \approx 2$?
H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

Approach 1 to TMDs

- Model calculation.

$$f_1^{(uv)}(x, \mathbf{p}_\perp) = \frac{1}{16\pi^3} \times (\frac{1}{3}\sin^2\theta\varphi_V^2 + \cos^2\theta\varphi_S^2),$$
$$f_1^{(dv)}(x, \mathbf{p}_\perp) = \frac{1}{8\pi^3} \times \frac{1}{3}\sin^2\theta\varphi_V^2.$$

$$h_{1T}^{\perp(uv)}(x, \mathbf{p}_\perp) = -\frac{1}{16\pi^3} \times (\frac{1}{9}\sin^2\theta\varphi_V^2 W_V - \cos^2\theta\varphi_S^2 W_S),$$
$$h_{1T}^{\perp(dv)}(x, \mathbf{p}_\perp) = -\frac{1}{8\pi^3} \times \frac{1}{9}\sin^2\theta\varphi_V^2 W_V.$$

- $\varphi_D(x, \mathbf{p}_\perp)$: adopting the BHL form:

$$\varphi_D(x, \mathbf{p}_\perp) = A_D \exp\{-\frac{1}{8\alpha_D^2}[\frac{m_q^2 + p_\perp^2}{x} + \frac{m_D^2 + p_\perp^2}{1-x}]\},$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Approach 2 to TMDs

- Staring with the equation (an unintegrated version)

$$
\begin{aligned}
h_{1T}^{\perp(uv)}(x,\mathbf{p}_\perp) &= \left[f_1^{(uv)}(x,\mathbf{p}_\perp)-\frac{1}{2}f_1^{(dv)}(x,\mathbf{p}_\perp)\right]W_S(x,\mathbf{p}_\perp) \\
&\quad -\frac{1}{6}f_1^{(dv)}(x,\mathbf{p}_\perp)W_V(x,\mathbf{p}_\perp), \\
h_{1T}^{\perp(dv)}(x,\mathbf{p}_\perp) &= -\frac{1}{3}f_1^{(dv)}(x,\mathbf{p}_\perp)W_V(x,\mathbf{p}_\perp). \qquad (27)
\end{aligned}
$$

- $f_1(x,\mathbf{p}_\perp)$: a Gaussian form

$$
f_1(x,\mathbf{p}_\perp)=f_1(x)\frac{\exp(-p_\perp^2/p_{av}^2)}{\pi p_{av}^2}, \qquad (28)
$$

with CTEQ6L parametrization for $f_1(x)$.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

$h_{1T}^{\perp(1)}(x)$ and $f_1(x)$

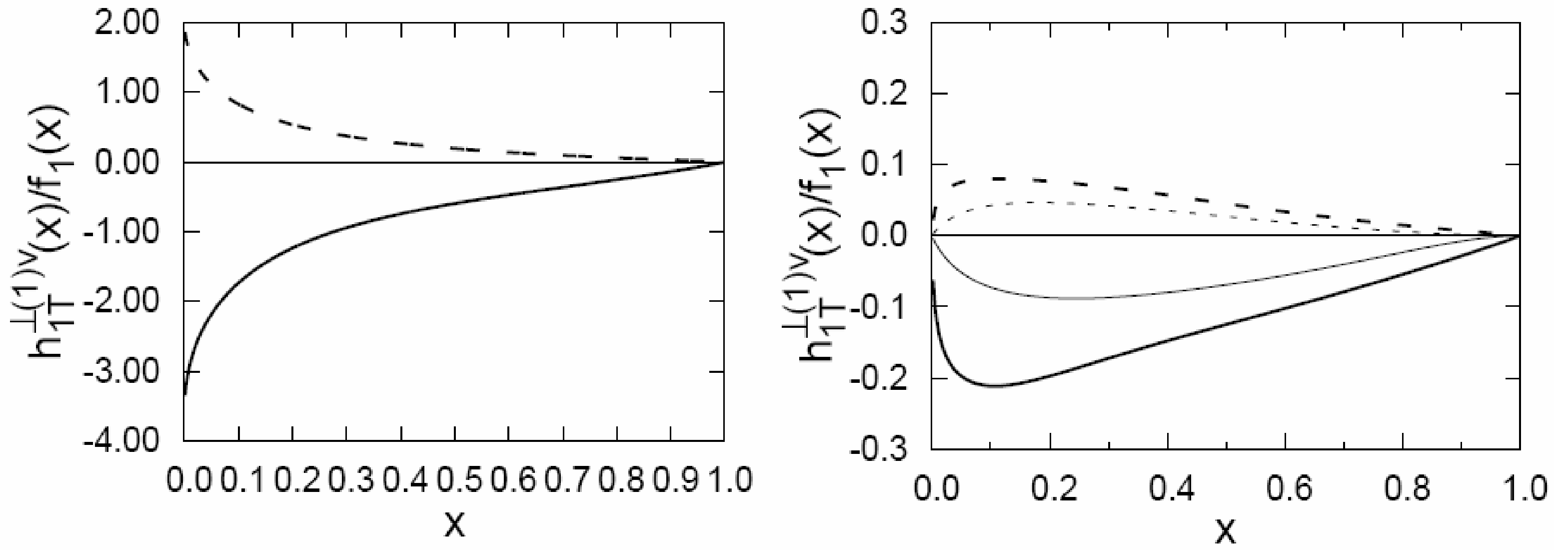


Figure: The ratio $h_{1T}^{\perp(1)(x)}/f_1(x)$. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the u quark, and dashed curves for the d quark. Only valence quarks are considered.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Results at HERMES kinematics.

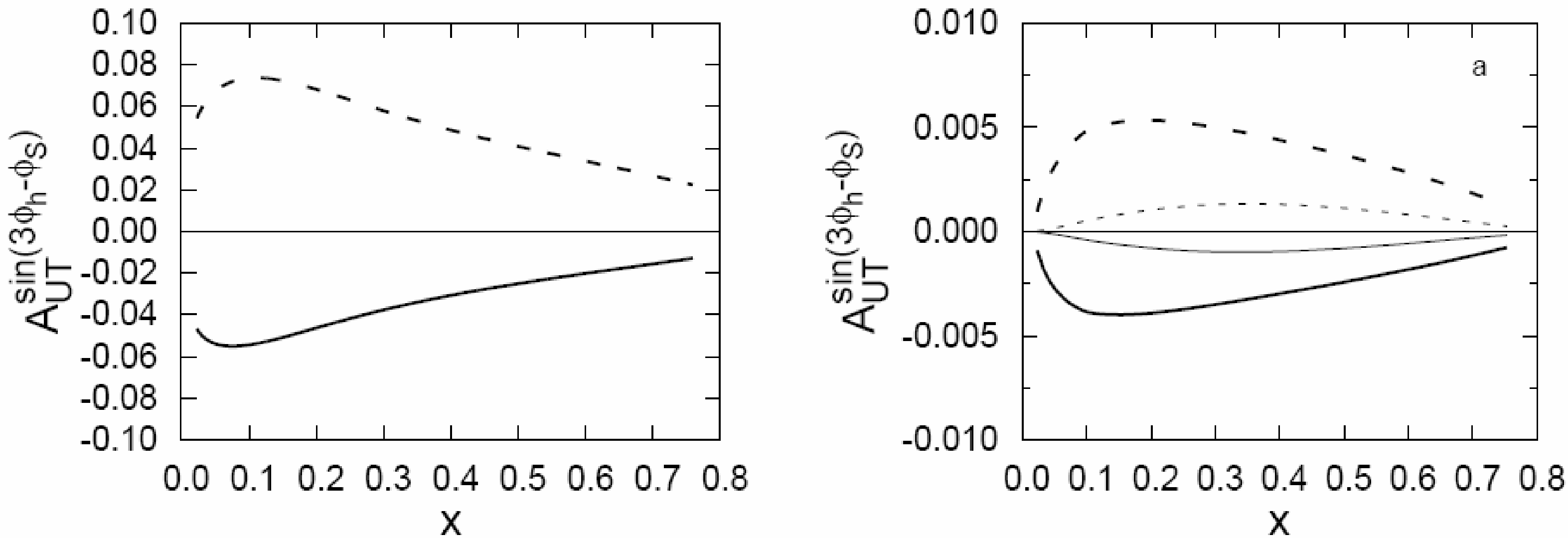


Figure: The results for HERMES kinematics with a proton target. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the π^+ production, and dashed curves for the π^- production.

Results at COMPASS kinematics.

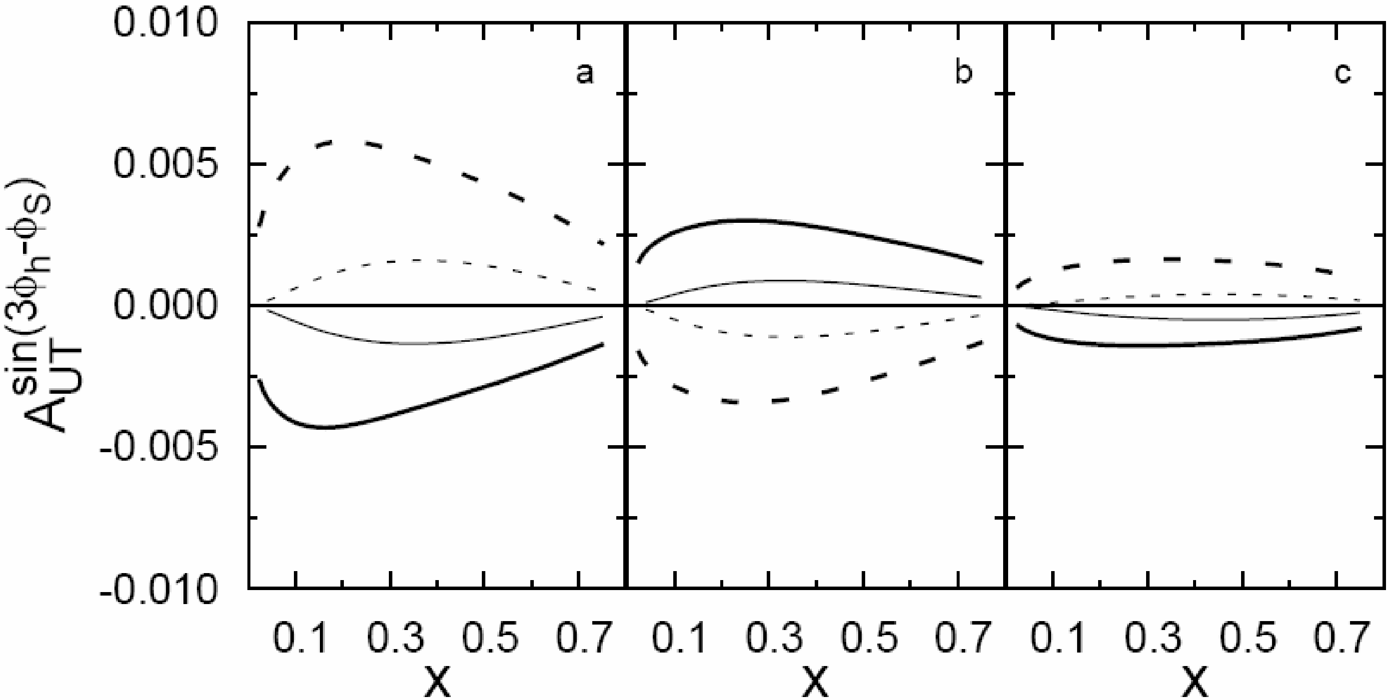


Figure: The results for COMPASS kinematics. a) proton target, b) neutron target, and c) deuteron target.

Results at JLab kinematics.

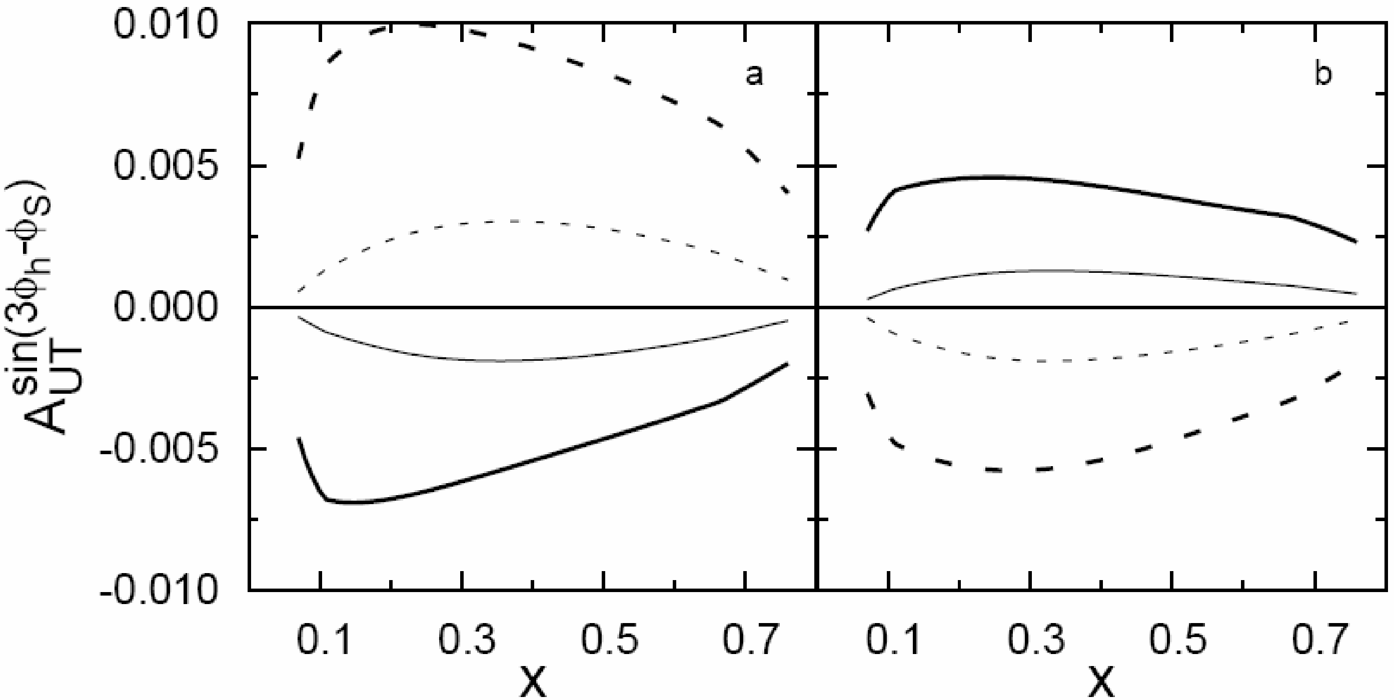


Figure: The results for JLab kinematics. a) proton target and b) neutron target.

Avakian's work

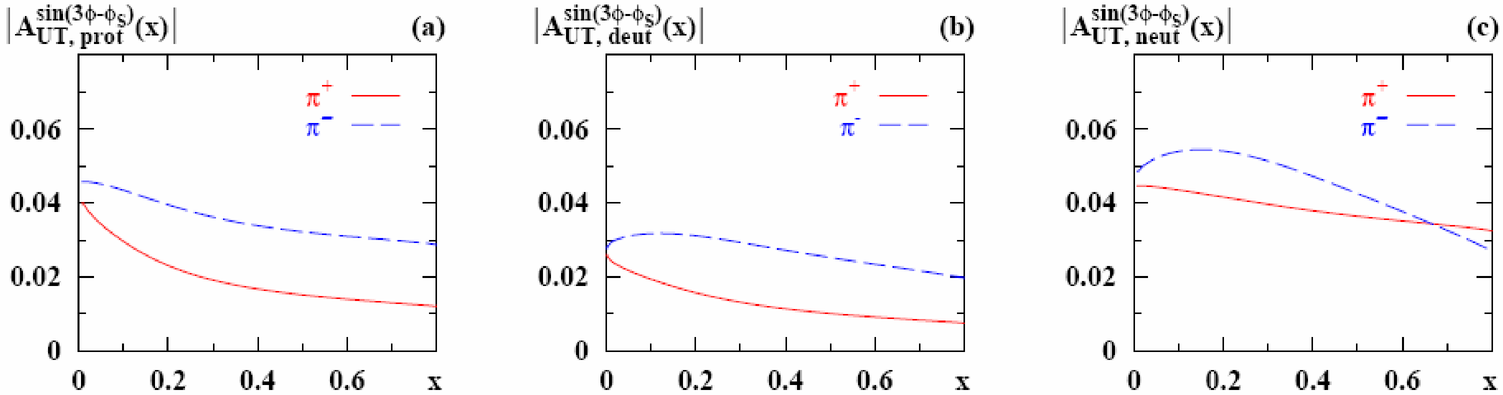


Figure: The predictions on the $\sin(3\phi_h - \phi_S)$ asymmetry at JLab kinematics. a) proton target, b) deuteron target and c) neutron target.

Much larger than our result!

H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, PRD 78, 114024 (2008) .

Boffi's work

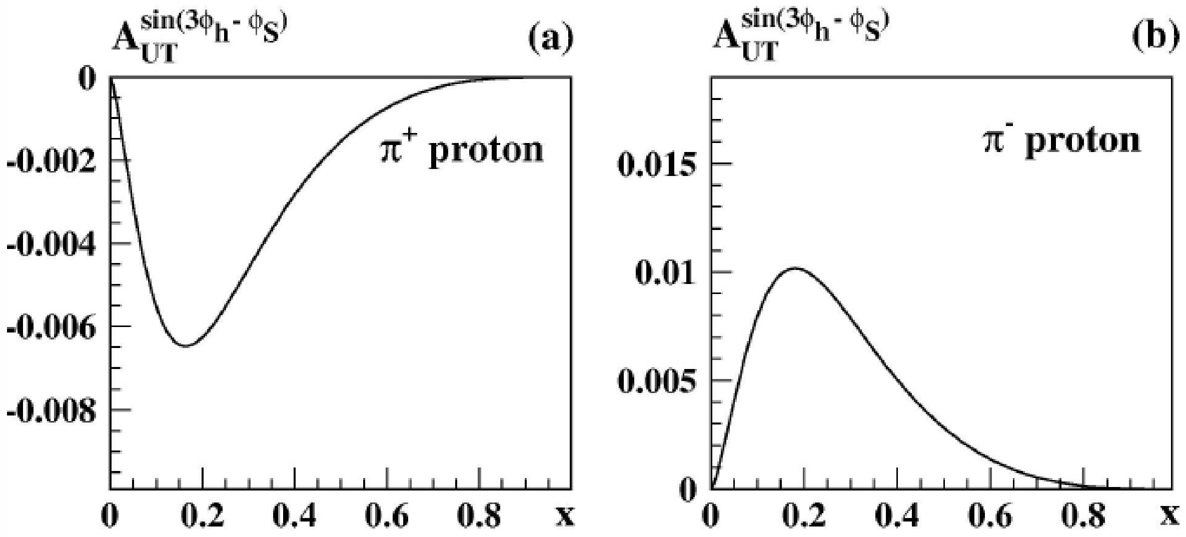


Figure: The $\sin(3\phi_h - \phi_S)$ asymmetry on a proton target.

Much smaller than Aviank's result, but a little larger than ours.

S. Boffi, A. V. Efremov, B. Pasquini, and P. Schweitzer, arXiv:0903.1271.

A short summary

- Results are sensitive to different transverse momentum approaches.
- The asymmetry is not an increasing function of x.
- The asymmetry is too small, up to a maximum less than 1%. A great challenge for a direct measurement.
- Can we enhance the asymmetry? We observe that the asymmetry is an increasing function of $\mathbf{p}_\perp^2$, but $\mathbf{p}_\perp^2$ cannot be manipulated directly.
- A compromise method is to select large $P_{h\perp}$ events instead, $\mathbf{P}_{h\perp} = z(\mathbf{p}_\perp - \mathbf{k}_\perp)$. We can exclude most small $p_\perp$ events.

The Necessity of Polarized p pbar Collider

The polarized proton antiproton Drell-Yan process

is ideal to measure

the pretzelosity distributions of the nucleon.

PHYSICAL REVIEW D **82,** 114022 (2010)

Probing the leading-twist transverse-momentum-dependent parton distribution function $h_{1T}^{\perp}$ via the polarized proton-antiproton Drell-Yan process

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Probing Pretzelosity in pion p Drell-Yan Process

COMPASS pion p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

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PHYSICS LETTERS B

Single spin asymmetry in πp Drell–Yan process

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Azimuthal asymmetries in lepton-pair production at a fixed-target experiment using the LHC beams (AFTER)

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unpolarized and single polarized *pp* and *pd* processes. We conclude that it is feasible to measure these azimuthal asymmetries, consequently the three-dimensional or transverse momentum dependent parton distribution functions (3dPDFs or TMDs), at this new AFTER facility.

Three QCD spin sums for the proton spin

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\nabla) \psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a + \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D}) \psi + \int d^3x \vec{x} \times (\vec{E} \times \vec{B})$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{J}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D}_{pure}) \psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a_{phys} + \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}_{phys}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

X.-S.Chen, X.-F.Lu, W.-M.Sun, F.Wang, T.Goldman, PRL100(2008)232002

Angular momentum of quarks and gluons from generalized form factors

$$\vec{J}_q = \int d^3x\, \psi^\dagger [\vec{\gamma}\gamma_5 + \vec{x} \times (-i\vec{D})]\psi$$

$$\vec{J}_g = \int d^3x [\vec{x} \times (\vec{E} \times \vec{B})]$$

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]U(P),$$

$$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)].$$

X. Ji, PRL78(1997)611

Angular momentum of quarks and gluons on the light-cone

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NUCLEAR PHYSICS B

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Light-cone representation of the spin and orbital angular momentum of relativistic composite systems ☆

Stanley J. Brodsky [a,*], Dae Sung Hwang [b], Bo-Qiang Ma [c,d,e], Ivan Schmidt [f]

Angular momenta of quarks and gluons on the light-cone

$$\langle J^i \rangle = \frac{1}{2}\epsilon^{ijk} \int \mathrm{d}^3x \, \langle T^{0k} x^j - T^{0j} x^k \rangle$$

$$= A(0)\langle L^i \rangle + [A(0) + B(0)]\bar{u}(P)\frac{1}{2}\sigma^i u(P).$$

$$\langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle [A(0) + B(0)].$$

One can define individual quark and gluon contributions to the total angular momentum from the matrix elements of the energy–momentum tensor [9]. However, this definition is only formal; $A_{q,g}(0)$ can be interpreted as the light-cone momentum fraction carried by the quarks or gluons $\langle x_{q,g} \rangle$. The contributions from $B_{q,g}(0)$ to J_z cancel in the sum. In fact, we shall show that the contributions to $B(0)$ vanish when summed over the constituents of each individual Fock state.

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311

Sum rules of quarks and gluons on the light-cone

$$A_{\mathrm{f}}(0) + A_{\mathrm{b}}(0) = F_1(0) = 1,$$

which corresponds to the momentum sum rule.

$$B(0) = B_{\mathrm{f}}(0) + B_{\mathrm{b}}(0) = 0,$$

which is another example of the vanishing of the anomalous gravitomagnetic moment.

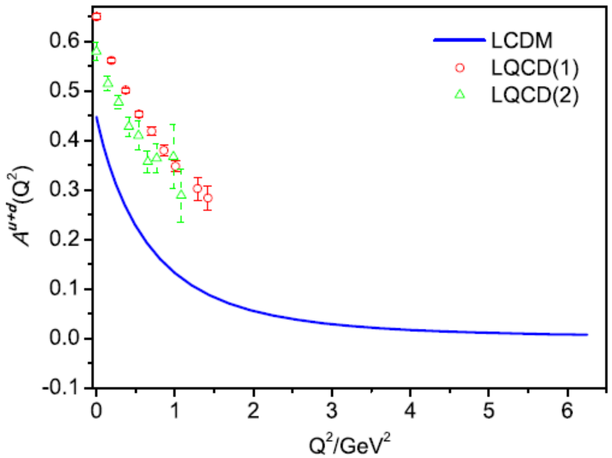
A and B are called gravitational form factors

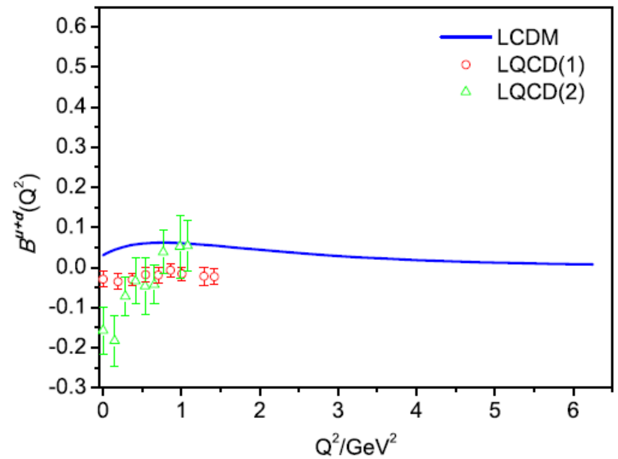
S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311

PHYSICAL REVIEW C **89**, 055202 (2014)

Generalized form factors of the nucleon in a light-cone spectator-diquark model

Tianbo Liu[1] and Bo-Qiang Ma[1,2,3,*]





- **We start from a quark model with total angular momentum from quarks, but we don't have a correct sum of angular momenta from generalized form factors.**
- **The definition of quark angular momentum as from generalized form factors is artificial.**

Arbitrary in defining angular momenta: what is C?

$$\langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle [A(0) + B(0)].$$

$$J_{q/g} = \frac{1}{2}[A_{q/g}(0) + B_{q/g}(0)] \pm C$$

as $$B(0) = B_{\mathrm{f}}(0) + B_{\mathrm{b}}(0) = 0,$$

so $$\langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle [A(0) + \cancel{B}(0)].$$

so $$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + \cancel{B}_{q,g}(0)].$$

But A(0) is the momentum fraction, not angular momentum

A simple QED system as thought experiment

PHYSICAL REVIEW D **91,** 017501 (2015)

Angular momentum decomposition from a QED example

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We investigate the angular momentum decomposition with a quantum electrodynamics example to clarify the proton spin decomposition debates. We adopt the light-front formalism where the parton model is well defined. We prove that the sum of fermion and boson angular momenta is equal to half the sum of the two gravitational form factors $A(0)$ and $B(0)$, as is well known. However, the suggestion to make a separation of the above relation into the fermion and boson pieces, as a way to measure the orbital angular momentum of fermions or bosons, respectively, is not justified from our explicit calculation.

DOI: 10.1103/PhysRevD.91.017501 PACS numbers: 14.20.Dh, 11.15.-q, 12.20.-m, 13.40.-f

A simple QED system as thought experiment: an electron

$$\psi^{\uparrow}_{\uparrow +} = \frac{k^1 - ik^2}{x(1-x)}\phi(x, \vec{k}_\perp),$$

$$\psi^{\uparrow}_{\uparrow -} = -\frac{k^1 + ik^2}{1-x}\phi(x, \vec{k}_\perp),$$

$$\psi^{\uparrow}_{\downarrow +} = \frac{1-x}{x} m\phi(x, \vec{k}_\perp),$$

$$\psi^{\uparrow}_{\downarrow -} = 0,$$

$$\phi(x, \vec{k}_\perp) = -\frac{\sqrt{2}e}{\sqrt{1-x}} \frac{x(1-x)}{\vec{k}_\perp^2 + (1-x)^2 m^2 + x\lambda^2}$$

A simple QED system as thought experiment: an electron

$$S_f = \frac{1}{2} - \frac{e^2}{16\pi^2},$$

$$S_b = \frac{3e^2}{16\pi^2\varepsilon} - \frac{e^2}{8\pi^2},$$

$$L = -\frac{3e^2}{16\pi^2\varepsilon} + \frac{3e^2}{16\pi^2}.$$

$$L_f = -\frac{e^2}{12\pi^2\varepsilon} + \frac{e^2}{12\pi^2},$$

$$L_b = -\frac{5e^2}{48\pi^2\varepsilon} + \frac{5e^2}{48\pi^2},$$

$$S_f + S_b + L_f + L_b = \frac{1}{2}.$$

A simple QED system as thought experiment: an electron

$$A_f(0) = 1 - \frac{e^2}{6\pi^2\varepsilon} + \frac{e^2}{8\pi^2}, \qquad A_b(0) = \frac{e^2}{6\pi^2\varepsilon} - \frac{e^2}{8\pi^2},$$

$$B_f(0) = \frac{e^2}{12\pi^2}, \qquad B_b(0) = -\frac{e^2}{12\pi^2}.$$

$$A_f(0) + A_b(0) = 1,$$

$$B_f(0) + B_b(0) = 0,$$

$$\frac{1}{2}[A(0) + B(0)] = S + L = \frac{1}{2},$$

A simple QED system as thought experiment: an electron

$$\frac{1}{2}[A_f(0) + B_f(0)] \neq S_f + L_f,$$

$$\frac{1}{2}[A_b(0) + B_b(0)] \neq S_b + L_b.$$

Therefore $J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]$ is unjustified.

T.Liu and B.-Q.Ma, Phy.Rev.D91 (2015) 017501, arXiv:1412.7775

Spectator diquark model calculation

Physics Letters B 741 (2015) 256–261

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Quark angular momentum in a spectator model

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We cannot identify the canonical angular momentums with half the sum of gravitational form factors:

$$\frac{1}{2}\left[A_q^S(0)+B_q^S(0)\right]=0.356\neq J_q^S,$$

$$\frac{1}{2}\left[A_q^V(0)+B_q^V(0)\right]=-0.038\neq J_q^V.$$

T.Liu and B.-Q. Ma, Phys.Lett.B741 (2015) 256, arXiv:1501.00062

Conclusions

- **The relativistic effect of parton transversal motions plays an significant role in spin-dependent quantities: helicity and transversity, five 3dPDFs or TMDs, GPDs, the Wigner distributions.**
- **It is still challenging to measure the quark orbital angular momentum: The pretzelosity with quark transversal motions is an important quantity for the spin-orbital correlation of the nucleon**
- **It is necessary to push forward theoretical explorations and experimental measurements of new quantities of the nucleon.**