Phenomenological implementations of TMD evolution

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Outline

- Introduction
- Unpolarized data
- Asymmetries
- Conclusions

Introduction

TMD phenomenology: data

- Tmd factorization has been proved for two kinds of processes:

DRELL-YAN

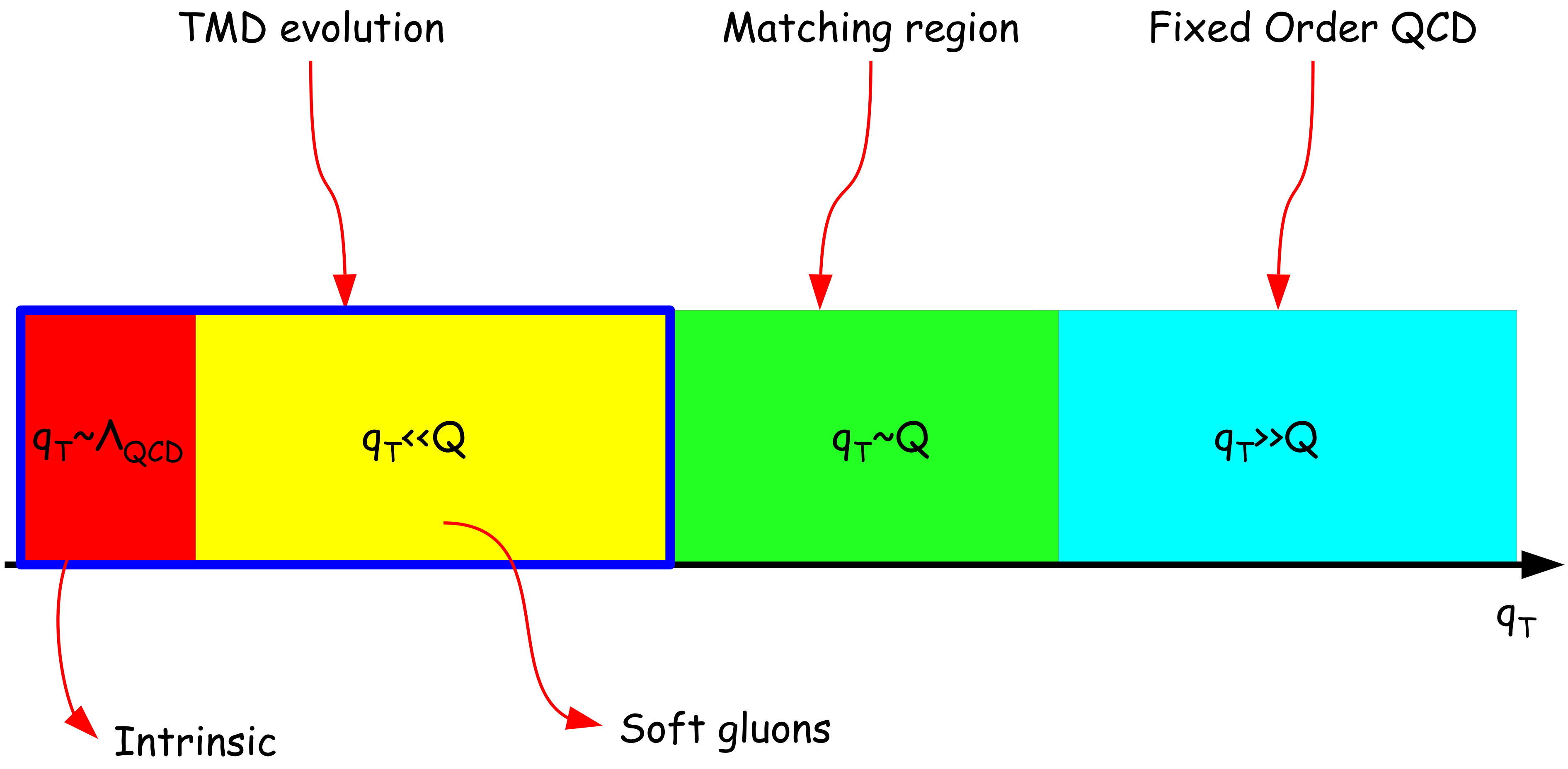
- $\sqrt{s}$~20-69 GeV; 1-7 TeV
- 4<Q<9; 10.5<Q<25 GeV; M_{Z_0}
- 0.1<P_T<tens GeV; 1-hundreds GeV

SIDIS

(JLAB,HERMES,COMPASS)

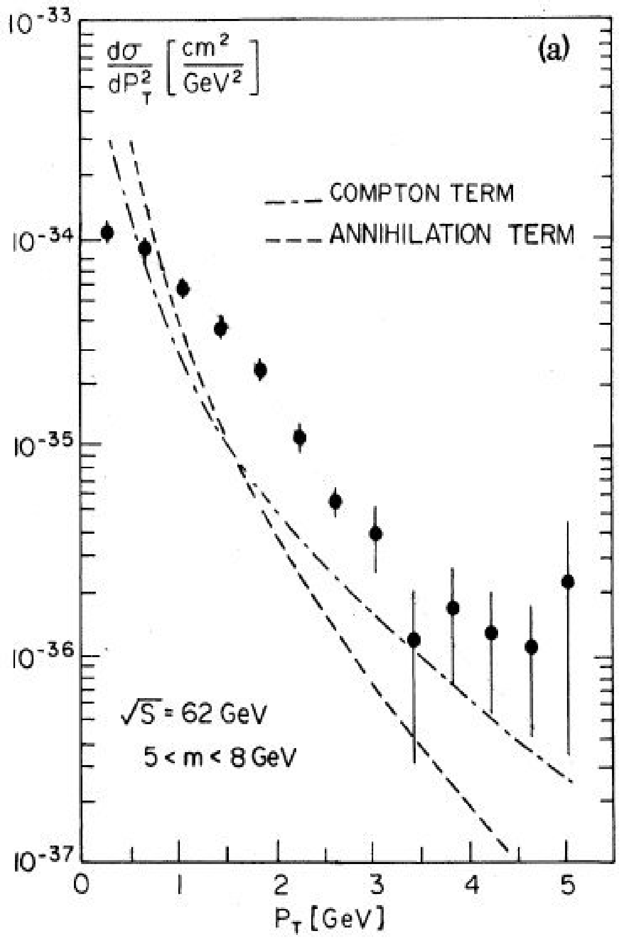
- $\sqrt{s}$~3.6-7-18 GeV
- 1<Q<4 GeV
- 0.1<P_T<few GeV

Resummation/TMD evolution



Resumation/TMD evolution

- Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T



$$q_T \to 0$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

Resummation/TMD evolution

- Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T

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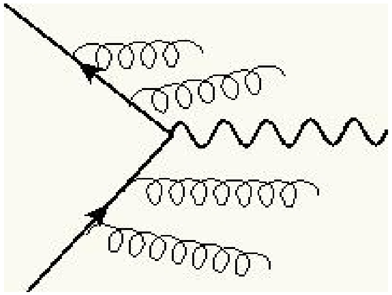
- These divergencies are cured by TMD evolution/resummation.....



$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{A(1)}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2}\right) \exp\left(-\frac{A^{(1)}}{4\pi}\alpha_s \ln^2(\frac{M^2}{q_T^2})\right)$$

DLLA approximation (Dokshitzer, Dyakonov, Troyan, 1980)

Resummation/TMD evolution

- Fixed order calculations cannot describe correctly DY/SIDIS data at small q_T

$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} = \frac{2C_F}{2\pi q_T^2}\alpha_s \ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

- These divergencies are cured by TMD evolution/resummation.....

- ...however DLLA does not take into account momentum conservation.
- The standard solution is to consider the TMD evolution in b_T space...

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1, x_2, b_T, Q) + Y(x_1, x_2, q_T, Q)$$

Resummation/TMD evolution

- We lose the control over q_T. Instead we have to deal with b_T!!

  Example in the CSS resummation scheme:

$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right] \sum_{i,k} C_{ji} \otimes f_i(x_1, C_1^2/b_T^2)\, C_{\bar{j}k} \otimes f_k(x_2, C_1^2/b_T^2)$$

$$S_j(b_T, Q) = -\int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[A_j(\alpha_s(\kappa)) \ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$\mu = \frac{C_1}{b_T}$$

At large b_T the scale μ becomes too small!

Not trivially connected to the physical region: $Q^2 \gg q_T^2 \simeq \Lambda_{QCD}^2$

- All TMD evolution schemes require a model or techniques to deal with this region

Example: CSS

- All the scales are freezed when we reach a non perturbative region:

$$b_T \longrightarrow b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad\qquad \mu = \frac{C_1}{b_T} \longrightarrow \mu_b = C_1/b_*$$

And then we define a non perturbative function for large b_T:

$$\frac{W_j(x_1, x_2, b_T, Q)}{W_j(x_1, x_2, b_*, Q)} = F_{NP}(x_1, x_2, b_T, Q)$$

$$W_j(x_1, x_2, b_T, Q) = \sum_{i,k} \exp\left[S_j(b_*, Q)\right] \left[C_{ji} \otimes f_i\left(x_1, \mu_b\right)\right] \left[C_{\bar{j}k} \otimes f_k\left(x_2, \mu_b\right)\right] F_{NP}(x_1, x_2, b_T, Q)$$

$b_*,\ \mu_b$ $\qquad\qquad$ b_T

$$C_1 = 2\exp(-\gamma_E)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS for DY processes

To perform phenomenological studies we need a non perturbative function.

$$F_{NP}(x_1, x_2, b_T, Q)$$

Davies-Webber-Stirling (DWS) $\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2;$

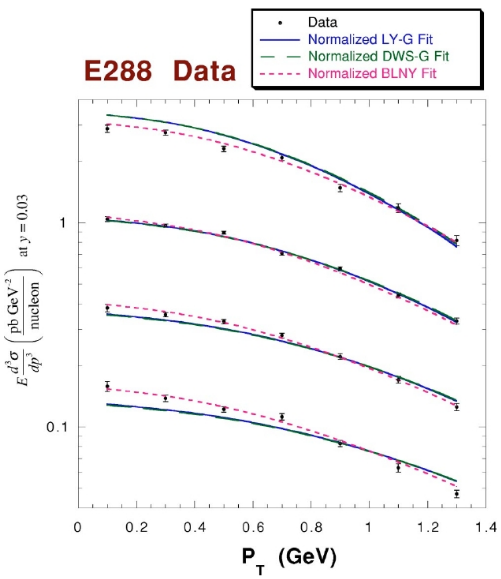
Ladinsky-Yuan (LY) $\exp\left\{\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 - [g_1 g_3 \ln(100 x_1 x_2)] b\right\};$

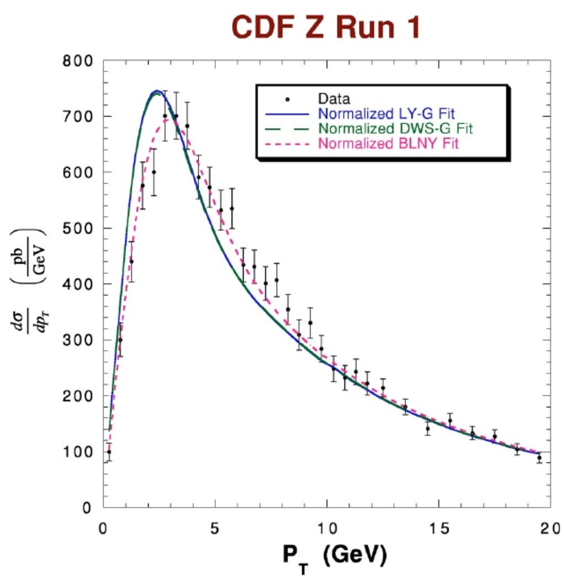
Brock-Landry-Nadolsky-Yuan (BLNY) $\exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$

Nadolsky et al., Phys.Rev. D67,073016 (2003)

CSS for DY processes

Nadolsky et al.* analyzed successfully low energy DY data and Z_0 production data using different parametrizations





Parameter	DWS-G fit	LY-G fit	BLNY fit
g_1	0.016	0.02	0.21
g_2	0.54	0.55	0.68
g_3	0.00	-1.50	-0.60
CDF Z Run-0 N_{fit}	1.00 (fixed)	1.00 (fixed)	1.00 (fixed)
R209 N_{fit}	1.02	1.01	0.86
E605 N_{fit}	1.15	1.07	1.00
E288 N_{fit}	1.23	1.28	1.19
DØ Z Run-1 N_{fit}	1.01	1.01	1.00
CDF Z Run-1 N_{fit}	0.89	0.90	0.89
χ^2	416	407	176
χ^2/DOF	3.47	3.42	1.48

$b_{max} = 0.5\ \text{GeV}^{-1}$

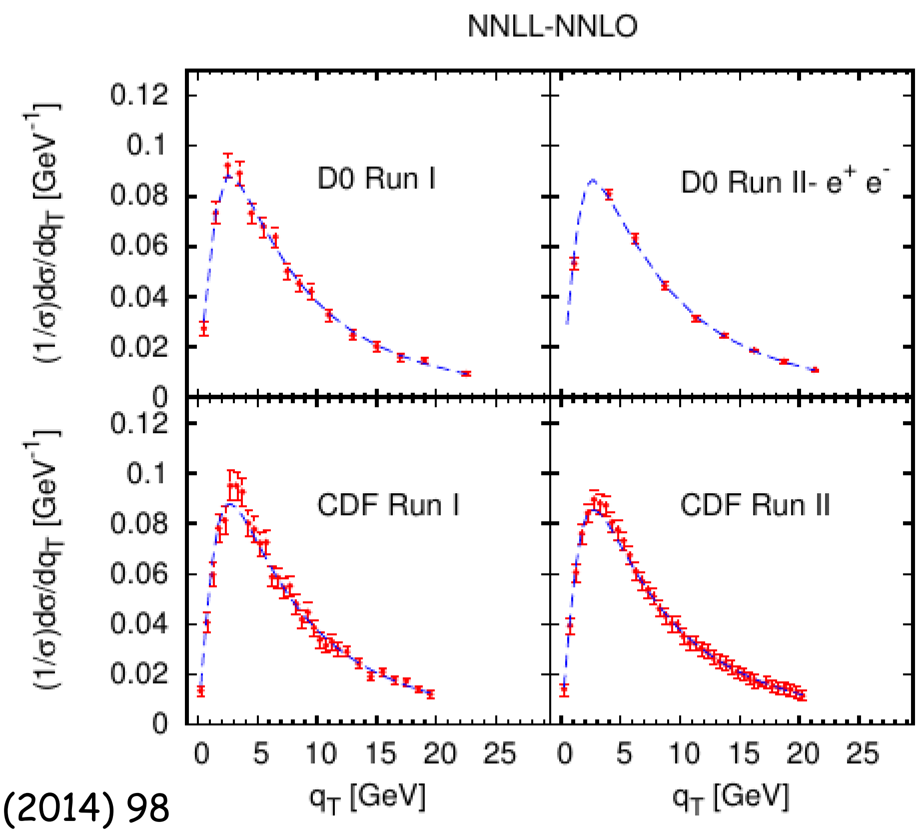
*Nadolsky et al., Phys.Rev. D67,073016 (2003)

Example II: SCET and DY

$$\tilde{F}_{q/N}(x, b_T; Q^2, Q) = \exp\left\{\int_{Q_i}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_F\left(\alpha_s(\bar{\mu}), \ln\frac{Q^2}{\bar{\mu}^2}\right)\right\} \left(\frac{Q^2 b_T^2}{4e^{-2\gamma_E}}\right)^{-D^R(b_T;Q_i)}$$

$$\times\, e^{h_\Gamma^R(b_T;Q_i) - h_\gamma^R(b_T;Q_i)} \sum_j \int_x^1 \frac{dz}{z} \hat{C}_{q\leftarrow j}(x/z, b_T; Q_i)\, f_{j/N}(z; Q_i)\, \tilde{F}_{q/N}^{\rm NP}(x, b_T; Q)\,,$$

$$\tilde{F}_{q/N}^{\rm NP}(x, b_T; Q) = e^{-\lambda_1 b_T}\left(1 + \lambda_2 b_T^2\right)$$



D'Alesio, Echevarria, Melis, Scimemi JHEP1411 (2014) 98

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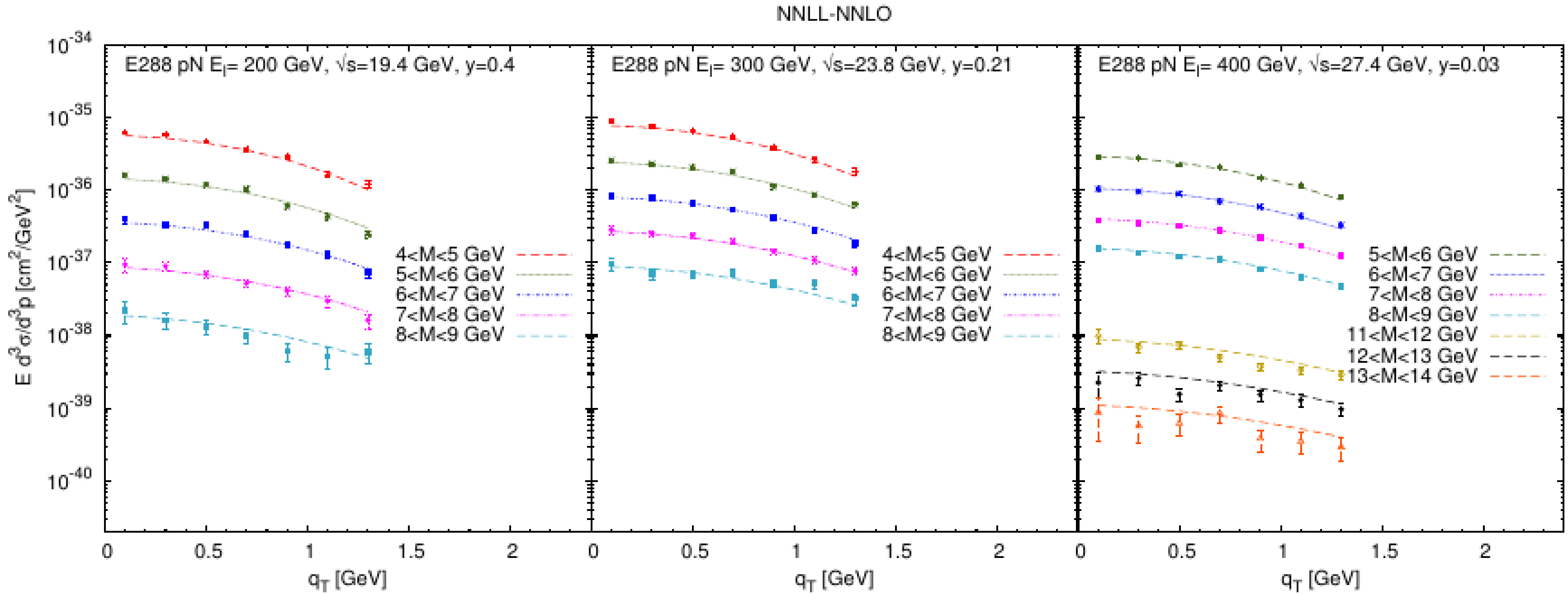


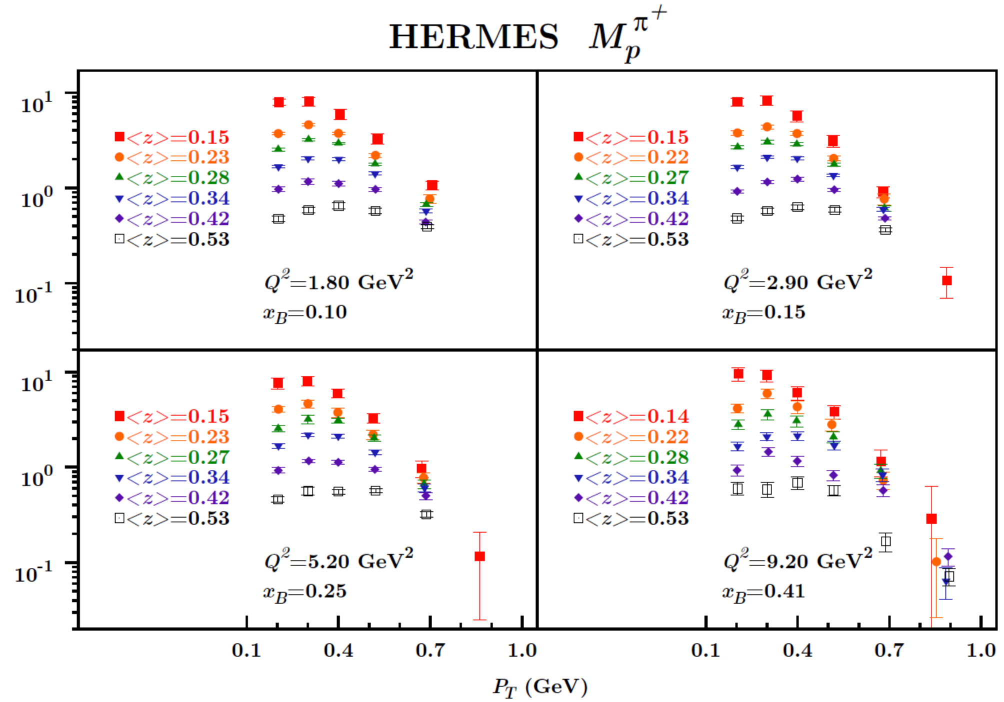
D'Alesio, Echevarria, Melis, Scimemi JHEP1411 (2014) 98

SIDIS phenomenology

SIDIS
(JLAB,HERMES,COMPASS)

- $\sqrt{s}$~3.6-7-18 GeV
- 1<Q<3.2 GeV
- 0.1<P_T<few GeV

- Multiplicity
- $\langle P_T^2 \rangle$
- Azimuthal asymmetries



Selection of multidimensional HERMES data

SIDIS phenomenology

- Simple phenomenological ansatz

$$f_{q/p}(x,k_\perp) = f(x)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$

SIDIS phenomenology

$$F_{UU} = \sum_q e_q^2 \, f_{q/p}(x_B) \, D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

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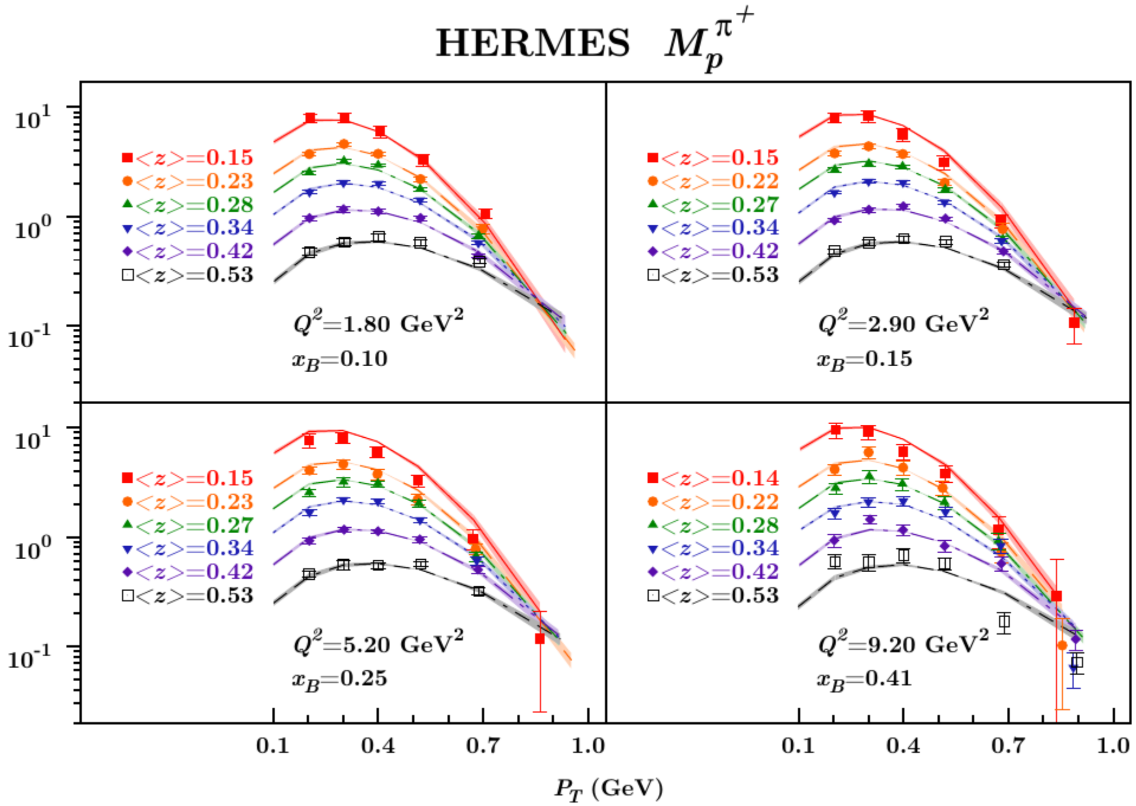


$$\langle k_\perp^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi^2_{\text{dof}} = 1.69$$

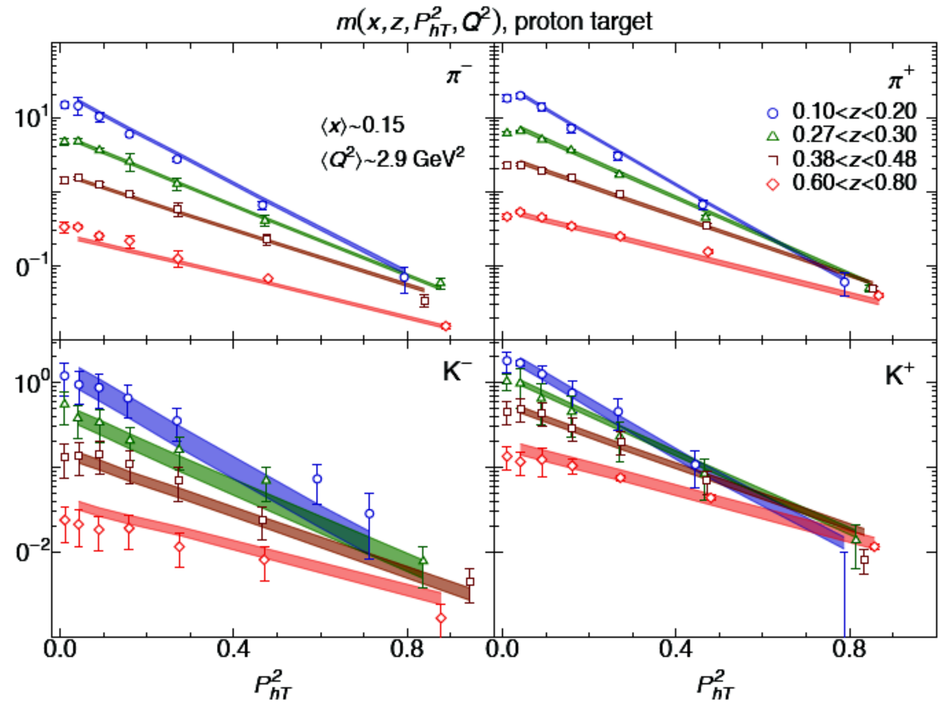
Anselmino et al. JHEP 1404 (2014) 005

SIDIS phenomenology

- Gaussians but flavor dependent, x dependent, z dependent....

$$f_{q/p}(x,k_\perp) = f(x)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$\langle \boldsymbol{k}_{\perp,q}^2\rangle(x) = \langle \widehat{\boldsymbol{k}_{\perp,q}^2}\rangle \frac{(1-x)^\alpha\, x^\sigma}{(1-\hat{x})^\alpha\, \hat{x}^\sigma} \qquad \langle \boldsymbol{P}_{\perp,q\to h}^2\rangle(z) = \langle \widehat{\boldsymbol{P}_{\perp,q\to h}^2}\rangle \frac{(z^\beta+\delta)\,(1-z)^\gamma}{(\hat{z}^\beta+\delta)\,(1-\hat{z})^\gamma}$$



proton target global χ^2 / d.o.f. = 1.63 ± 0.12

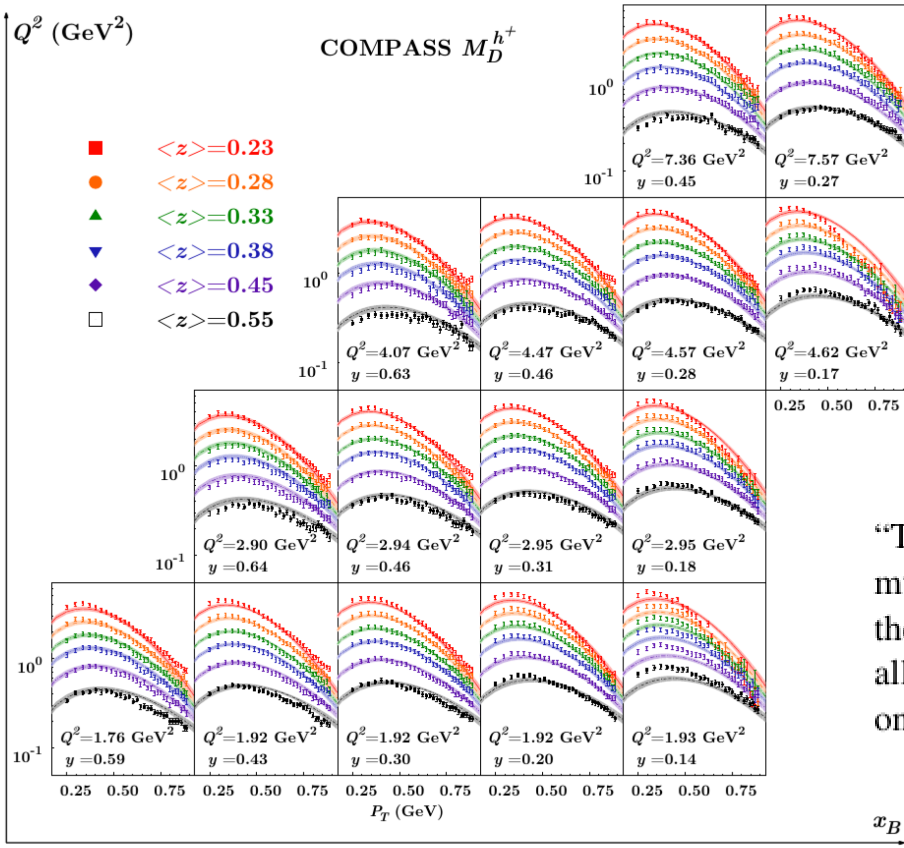
no flavor dep. 1.72 ± 0.11

Signori et al. JHEP 1311 (2013) 194

SIDIS phenomenology

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\, \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



$$\langle k_\perp^2\rangle = 0.60 \pm 0.14\ \mathrm{GeV}^2$$

$$\langle p_\perp^2\rangle = 0.20 \pm 0.02\ \mathrm{GeV}^2$$

$$\chi^2_{\mathrm{dof}} = 3.42$$

$$N_y = A + B\,y$$

"The point-to-point systematic uncertainty in the measured multiplicities as a function of p_T^2 is estimated to be 5% of the measured value. The systematic uncertainty in the overall normalization of the p_T^2-integrated multiplicities depends on z and y and can be as large as 40%".

Erratum Eur.Phys.J. C75 (2015) 2, 94

Anselmino et al. JHEP 1404 (2014) 005

SIDIS + TMD Evolution: EIKV

- TMD evolution (CSS-like version)

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \exp\left\{\frac{1}{2} S^{CSS}(b_*, \mu_b)\right\} \exp\left\{-g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right)\right\}$$

- Approximations

$$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij}\delta(1-z)$$

At LO; PDF and FF at LO

- Simple parametrizations for the non-perturbative part:

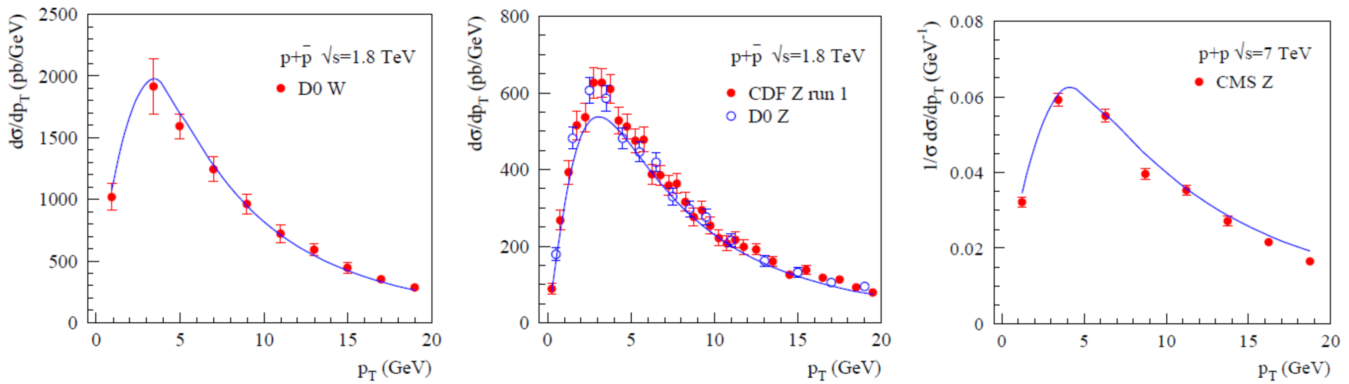
$$F_{NP}(b_T, Q)^{\rm pdf} = \exp\left[-b_T^2\left(g_1^{\rm pdf} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T, Q)^{\rm ff} = \exp\left[-b_T^2\left(g_1^{\rm ff} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

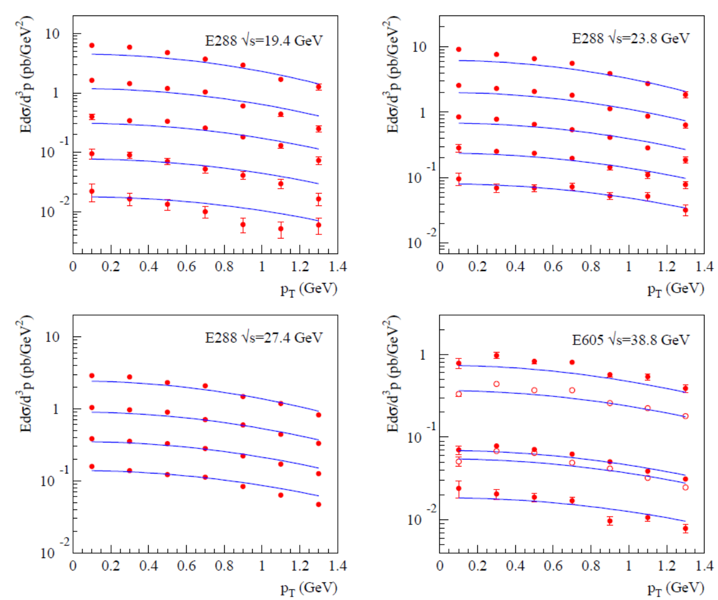
SIDIS + TMD Evolution: EIKV

- Fit DY data and SIDIS data....



Z and W-Boson Production

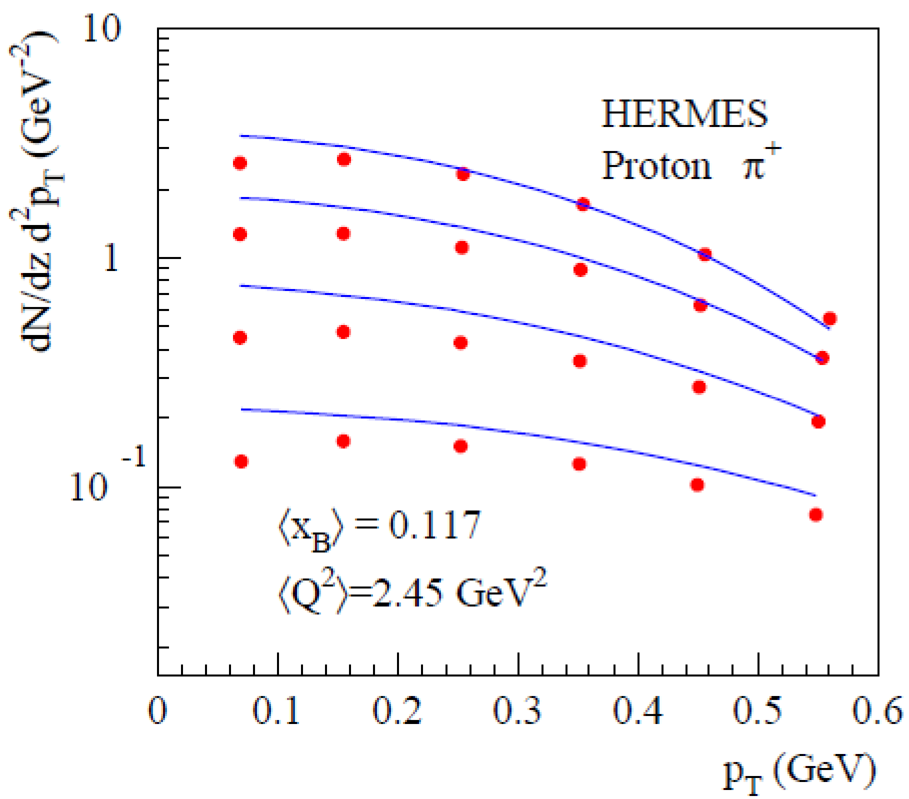
Low energy DY



Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

SIDIS + TMD Evolution: EIKV

- CSS (and therefore TMD evolution) can describe DY data
- What about HERMES/COMPASS SIDIS data?



Echevarria, Idilbi, Kang, Vitev
Phys .Rev. D89 (2014) 074013

- Global Fit DY+SIDIS
- TMD evolution
- Wilson Coefficient, PDF and FF at LO
- No full multidimensional data analysis
- No χ^2 provided

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

SIDIS + TMD Evolution: SIYY

- Fit of the DY data at NLL-NLO using a new non perturbative function:

$$S_{NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda\right)$$

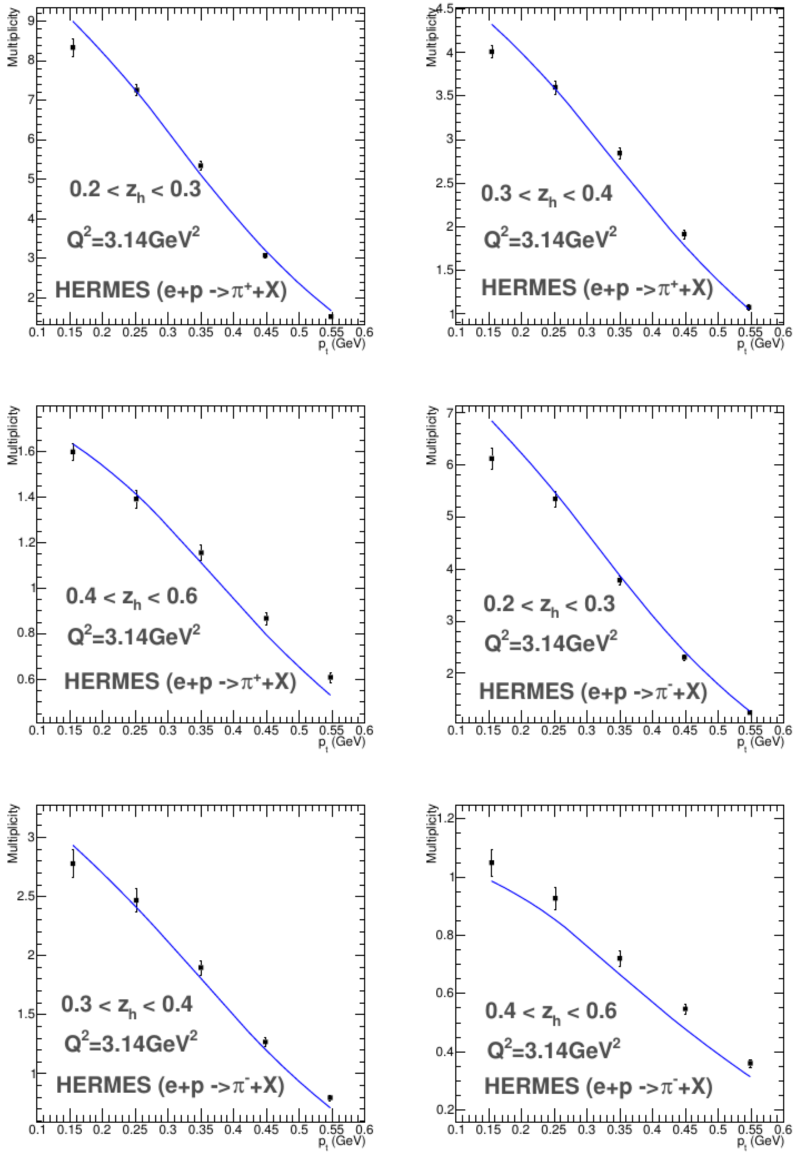
- Similar parametrization for SIDIS:

$$S_{NP}^{(DIS)} = \frac{g_1}{2} b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 (x_0/x_B)^\lambda + \frac{g_h}{z_h^2} b^2$$

- Extraction of g_h

P. Sun, J. Isaacson, C.P. Yuan, F. Yuan
Arxiv: 1406.3073

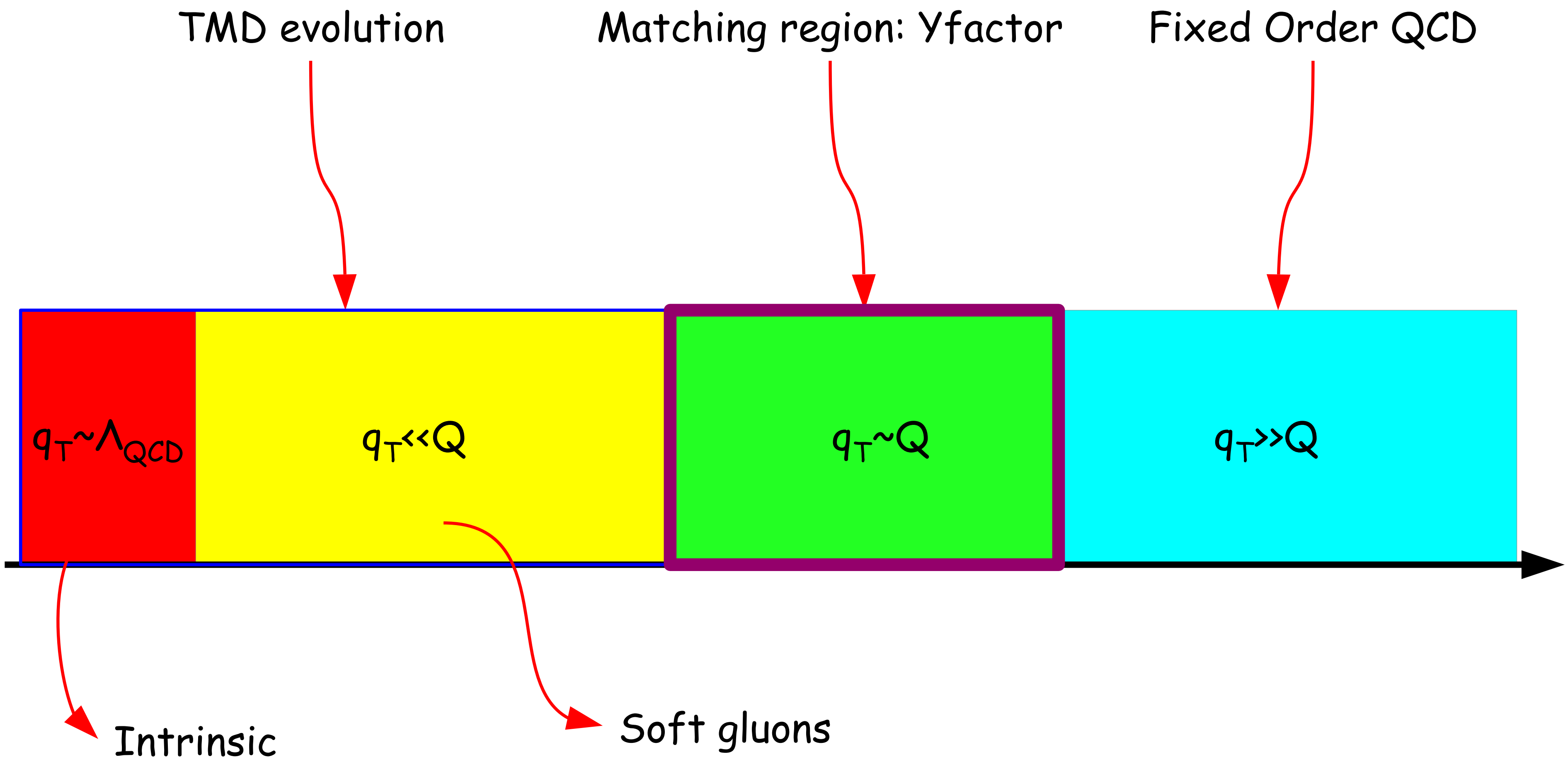
SIDIS + TMD Evolution: SIYY



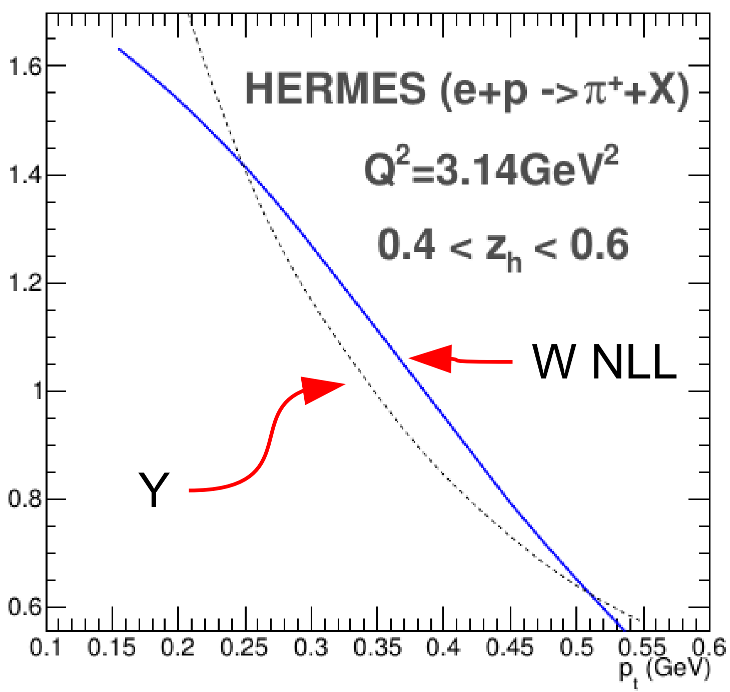
- CSS evolution at NLL-NLO
- Fit of an unknown number/selection of of data. Total χ^2=180....
- Normalization factor 2
- No Y factor....

P. Sun, J. Isaacson, C.P. Yuan, F. Yuan
Arxiv: 1406.3073

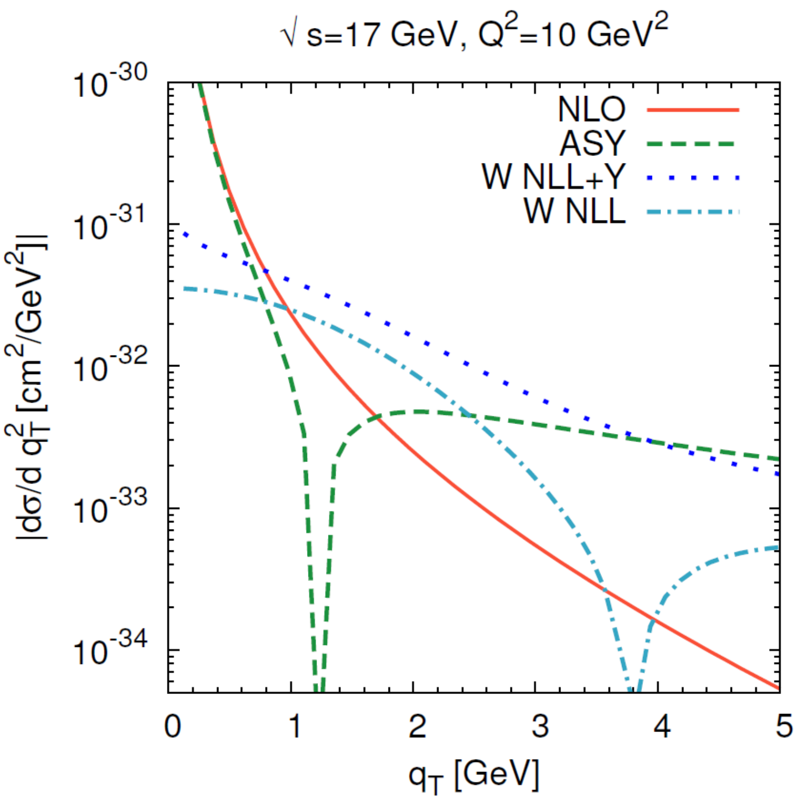
Resummation/TMD evolution



SIDIS-Y factor



Sun et al arXiv:1406.3073

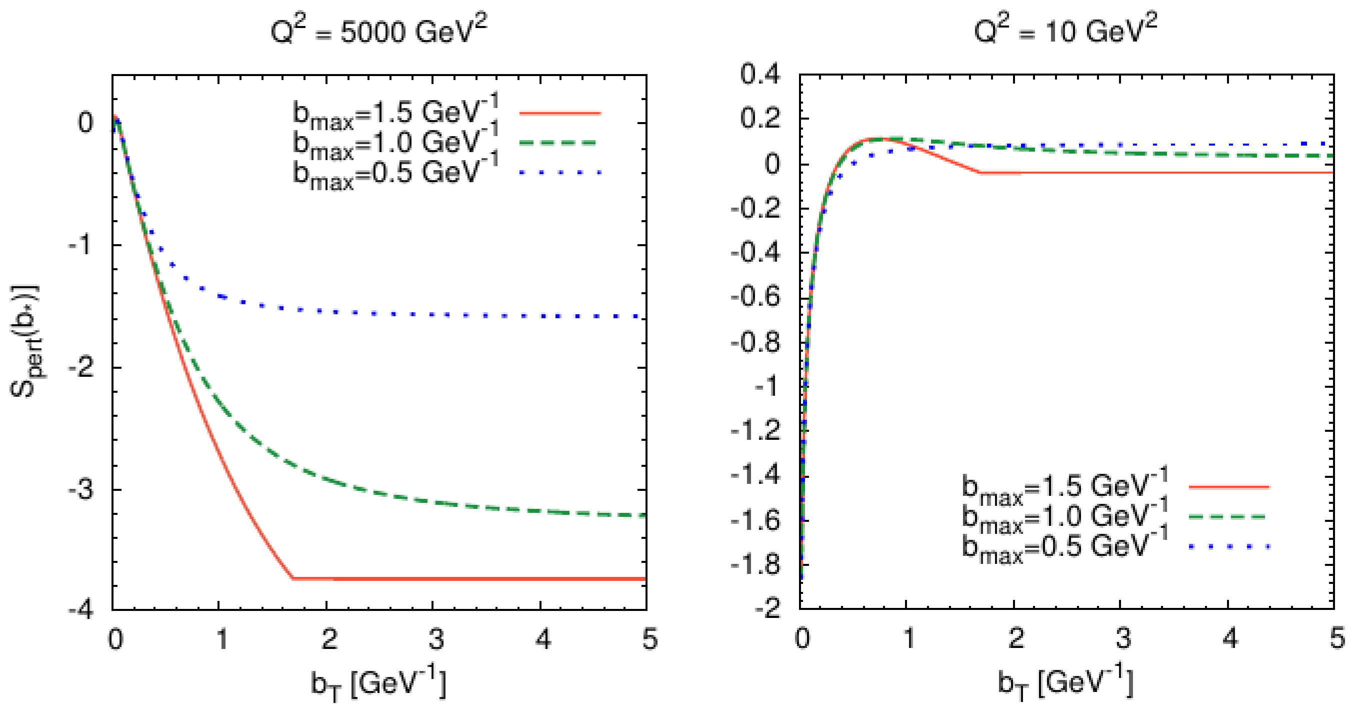


Boglione et al, JHEP 02 (2015) 095

$$Y = \frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} - \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2}$$

- Y factor is very large, larger or as large as the resummed cross section

Sudakov



Boglione et al, JHEP 02 (2015) 095

Conclusion I

- TMD evolution can describe DY data. The predictions are robust at the mass of Z and transverse momenta of $5<q_T<30$ GeV. Lower transverse momenta need a non perturbative modeling

- Present SIDIS data are at low energy therefore
  1) the non-pertubative behavior is dominant.
  2) the perturbative part could be not under control
     (i.e. Higher order terms may be large)
  3) The region of matching and that of resummation
     cannot be clearly separated.

The Sivers function from SIDIS data

Asymmetries

- Numerator and denominator have the same tmd evolution: many tmd effects cancel in the ratio (and many problems could disappear (???))

- Safe ratios do not mean safe extractions of TMDs... If you don't know the denominator you cannot extract the functions at numerator even if you can describe the asymmetry.

TMD evolution of the Sivers function

$$f_{1T}^{\perp}(x,k_\perp) = \frac{-1}{2\pi k_T}\int db_T b_T J_1(k_T b_T)\tilde{f}_{1T}^{\prime\perp}(x,b_T)$$

Object that evolves

$$\begin{aligned}\tilde{f}_{1T}^{\prime\perp}(x,b_T,Q,\zeta_F) \quad = \quad & \frac{m_p b_T}{2}\sum_j\int_x^1\frac{dx_1 dx_2}{x_1 x_2}\tilde{C}_{f/j}^{Siv}(x_1,x_2,b_*,\mu_b,\mu_b^2)T_{Fj}(x_1,x_2,\mu_b)\\ & \exp\left\{\int_{\mu_b}^{Q}\frac{d\kappa}{\kappa}\gamma_F(\kappa;1)-\ln\left(\frac{Q}{\kappa}\right)\gamma_K(\kappa)\right\}\\ & \exp\left\{-g_P^{Siv}(x,b_T)-g_K(b_T)\ln\left(\frac{Q}{Q_0}\right)\right\}\end{aligned}$$

$$\zeta_F = Q^2$$

Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2012)

TMD evolution of the Sivers function

$$\tilde{f}'^{\perp}_{1T}(x, b_T, Q, \zeta_F) = \frac{m_p b_T}{2} \sum_j \int_x^1 \frac{dx_1 dx_2}{x_1 x_2} \tilde{C}^{Siv}_{f/j}(x_1, x_2, b_*, \mu_b, \mu_b^2) T_{Fj}(x_1, x_2, \mu_b)$$
$$\exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{Q}{\kappa}\right) \gamma_K(\kappa) \right\}$$
$$\exp\left\{ -g_P^{Siv}(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

$$\zeta_F = Q^2$$

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$
$$\exp\left\{ \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{Q}{\kappa}\right) \gamma_K(\kappa) \right\}$$
$$\exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2012)

Alternative evolution equation

$$\frac{\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0} \equiv Q_0^2)} = \exp\left\{\int_Q^{Q_0} \frac{d\kappa}{\kappa}\left[\gamma_F(\kappa; 1) - \gamma_K(\kappa)\ln(Q/\kappa)\right]\right\}$$
$$\exp\left[-\int_{\mu_b}^{Q_0} \frac{d\kappa}{\kappa}\gamma_K(\kappa)\ln(Q/Q_0)\right]\exp\left[-g_K(b_T)\ln(Q/Q_0)\right]$$
$$= \tilde{R}(Q, Q_0, b_T)\exp\left[-g_K(b_T)\ln(Q/Q_0)\right]$$

$$\boxed{\tilde{F}(x, b_T, Q, Q^2) = \tilde{F}(x, b_T, Q_0, Q_0^2)\tilde{R}(Q, Q_0, b_T)\ \exp\left[-g_K(b_T)\ln(Q/Q_0)\right]}$$

Output function — Input function

Notice that:

$$\frac{\tilde{f}'^{\perp}_{1T}(x, b_T, Q, \zeta_F)}{\tilde{f}'^{\perp}_{1T}(x, b_T, Q_0, \zeta_{F0})} = \frac{\tilde{f}_1(x, b_T, Q, \zeta_F)}{\tilde{f}_1(x, b_T, Q_0, \zeta_{F0})} \equiv \frac{\tilde{F}(x, b_T, Q, \zeta_F)}{\tilde{F}(x, b_T, Q_0, \zeta_{F0})}$$

Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2012)

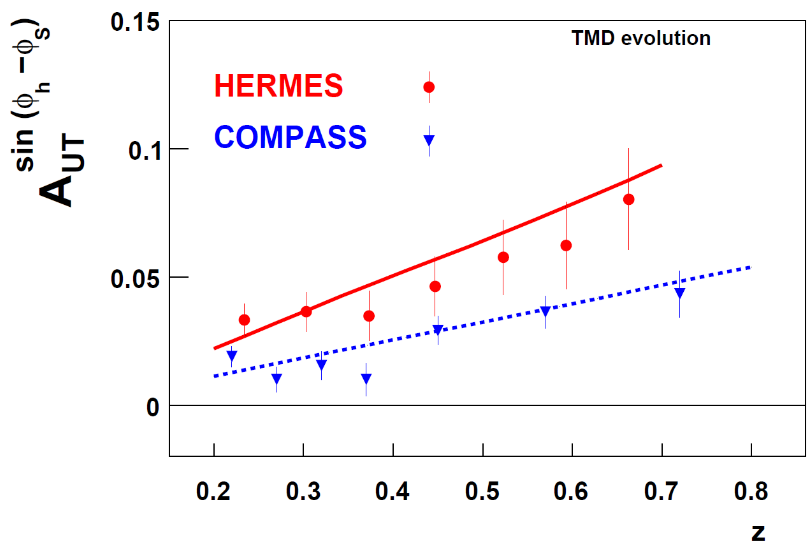
Sivers phenomenology

- Aybat-Roger-Prokudin: TMD EVO IO   No FIT   Qual. OK

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\tilde{F}(x,b_T,Q_0,Q_0^2) = f(x,Q_0)\exp\left[-\frac{\langle k_\perp^2\rangle}{4}b_T^2\right]$$

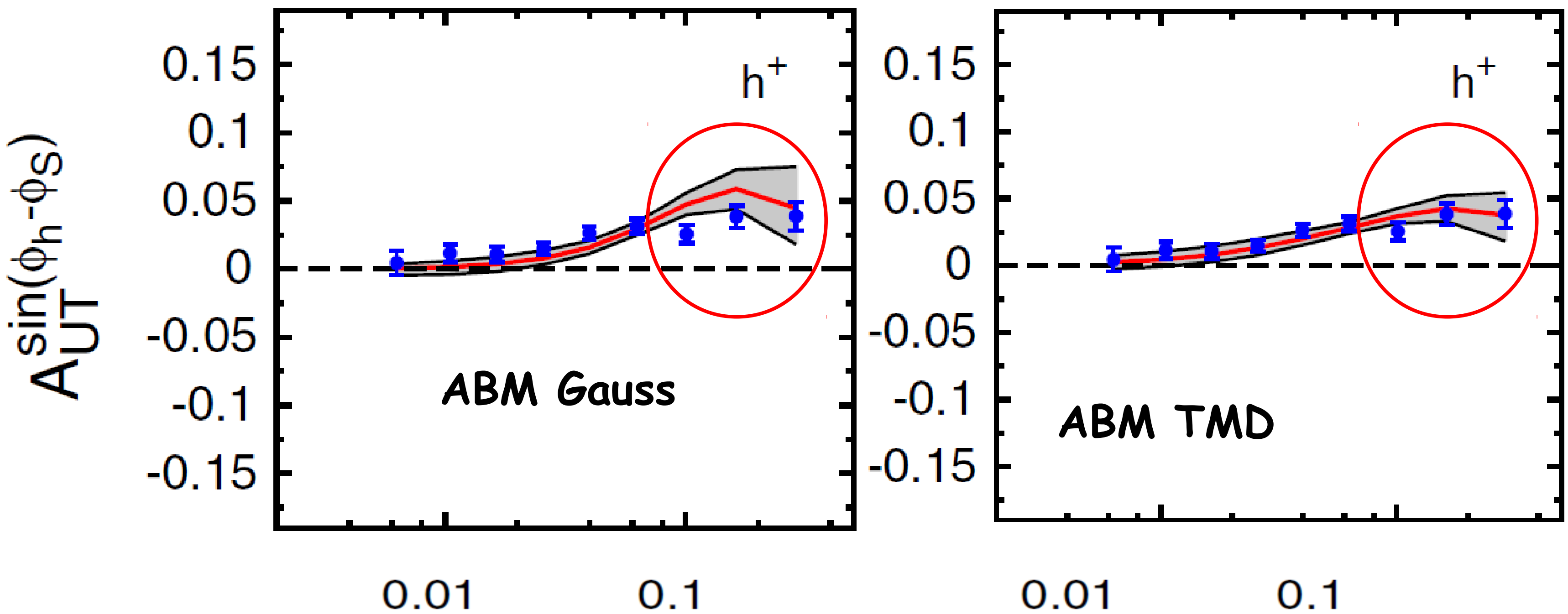
$$g_K(b_T) = \frac{1}{2}g_2 b_T^2$$ g_2 from DY



Aybat, Prokudin, Rogers, Phys.Rev.Lett. 108 (2012) 242003

Sivers phenomenology

➢ Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
➢ Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
➢ Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02



Anselmino, Boglione, Melis Phys.Rev. D86 (2012) 014028

Sivers phenomenology

➢ Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
➢ Anselmino-Boglione-Melis: Gaussian	FIT	χ^2=1.26
➢ Anselmino-Boglione-Melis: TMD EVO IO	FIT	χ^2=1.02
➢ EIKV: TMD Evo a la CSS+ C at LO	FIT	χ^2=1.3

$$F_{UT}^{\sin(\phi_h-\phi_s)} = \frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(P_{h\perp} b/z_h) \sum_q e_q^2\, T_{q,F}(x_B, x_B, c/b_*) D_{h/q}(z_h, c/b_*)$$
$$\times \exp\left\{-\int_{c^2/b_*^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\} \exp\left\{-b^2\left(g_1^{\rm ff} + g_1^{\rm sivers} + g_2 \ln\frac{Q}{Q_0}\right)\right\}$$

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

Sivers phenomenology

➢Aybat-Roger-Prokudin: TMD EVO IO	No FIT	Qual. OK
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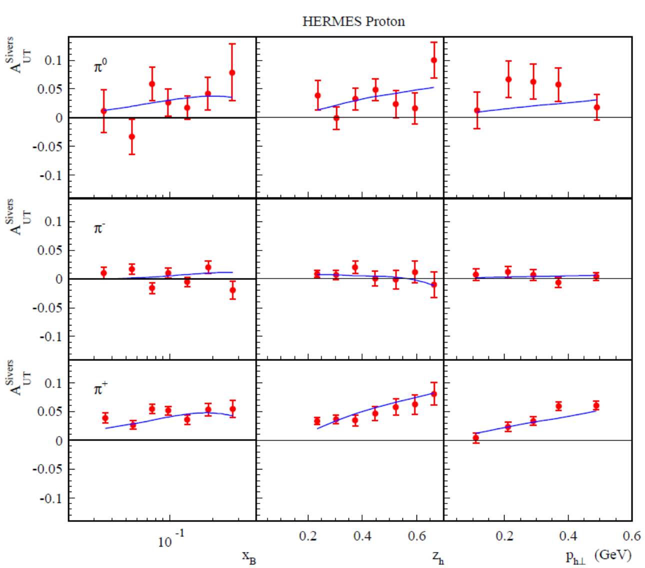

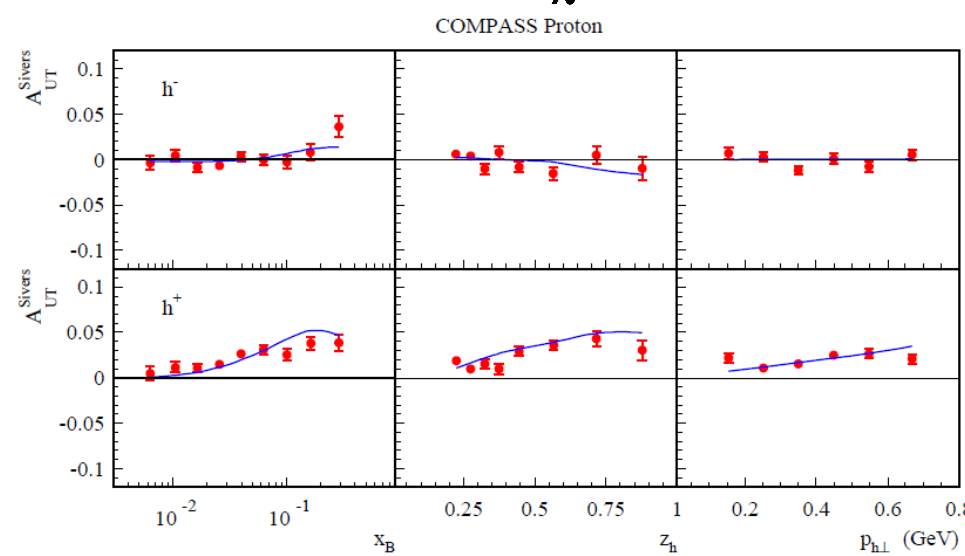

Echevarria, Idilbi, Kang, Vitev Phys.Rev. D89 (2014) 074013

Sivers phenomenology

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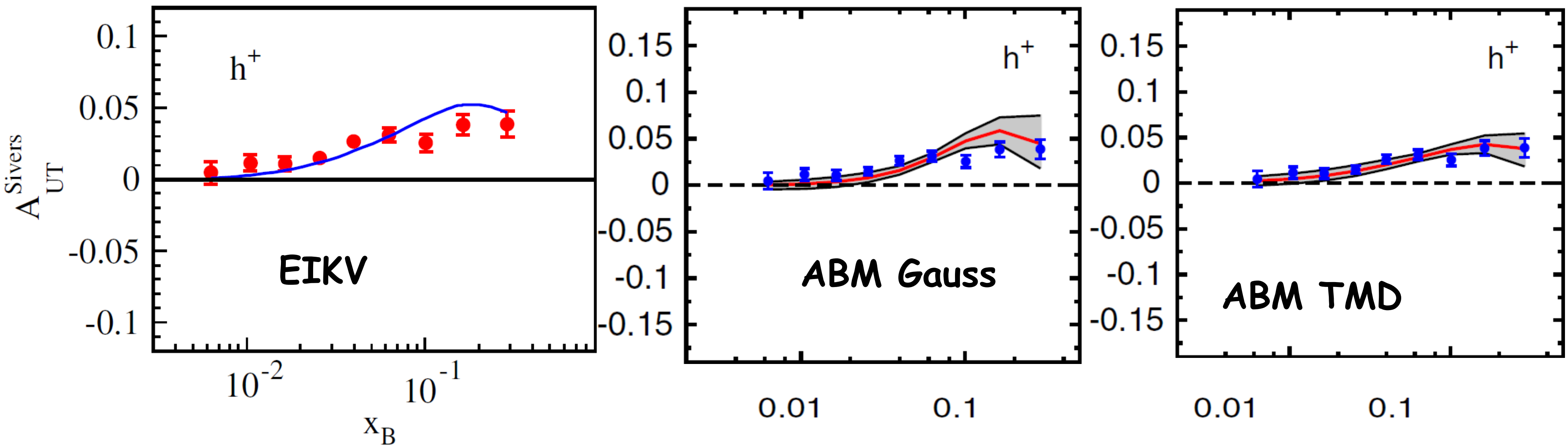


Conclusion II

- Different approaches can describe the Sivers asymmetry because the non pertubative behavior is probably dominant
- We do not have a full NLO and NLL fit of SIDIS data yet. We need to know well the twist 3 Qiu-Sterman functions and their (NLO) collinear evolution... which is not close
- Contrary to unpolarized PDF we do not know the collinear T_F Can we study T_F in SIDIS? (i.e. at large q_T, maybe at EIC?)

CSS/TMD evolution and Collins/Transversity

- Extraction of transversity and Collins functions using TMD evolution at NLL'

$$F_{UT} = -\frac{1}{2z_h^3}\int \frac{db\, b^2}{(2\pi)} J_1\!\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\rm PT}(Q,b_*)-S^{\rm (SIDIS)}_{\rm NP\,coll}(Q,b)} \times\ \delta C_{q\leftarrow i}\otimes h_1^i(x_B,\mu_b)\ \ \delta\hat{C}^{\rm (SIDIS)}_{j\leftarrow q}\otimes \hat{H}^{(3)}_{h/j}(z_h,\mu_b), (2)$$

$$Z_{uu}^{h_1h_2}(Q;P_{h\perp}) = \frac{1}{z_{h1}^2}\int_0^\infty \frac{db\, b}{(2\pi)} J_0(P_{h\perp}b/z_{h1})\ e^{-S_{\rm pert}(Q,b_*)-S_{\rm NP}^{e^+e^-}(Q,b)}\,\widetilde{Z}_{uu}^{h_1h_2}(b_*)\ ,$$

$$Z_{\rm collins}^{h_1h_2}(Q;P_{h\perp}) = \frac{1}{z_{h1}^2}\frac{1}{4z_{h1}z_{h2}}\int_0^\infty \frac{db\, b^3}{(2\pi)} J_2(P_{h\perp}b/z_{h1})\ e^{-S_{\rm pert}(Q,b_*)-S_{\rm NP\,collins}^{e^+e^-}(Q,b)}\,\widetilde{Z}_{\rm collins}^{h_1h_2}(b_*)$$

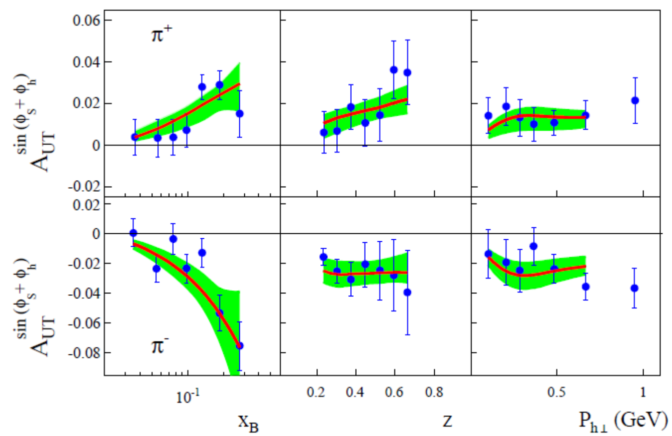
$$S_{\rm NP}^{e^+e^-}(Q,b) = g_2\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + \left(\frac{g_h}{z_{h1}^2}+\frac{g_h}{z_{h2}^2}\right)b^2\ ,$$

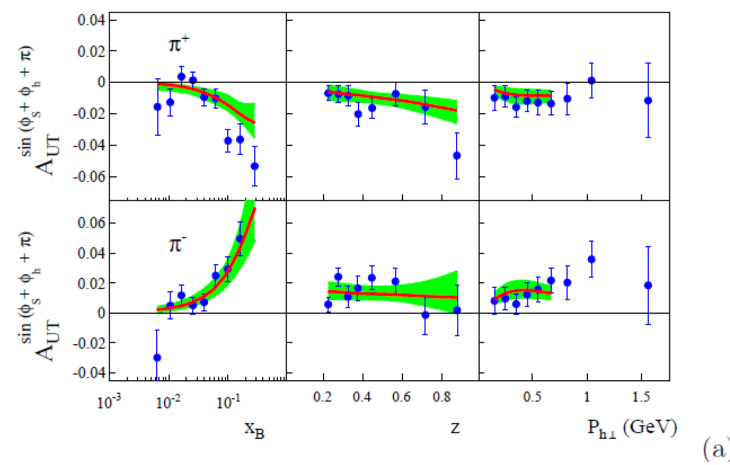
$$S_{\rm NP\,collins}^{e^+e^-}(Q,b) = g_2\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + \left(\frac{g_h-g_c}{z_{h1}^2}+\frac{g_h-g_c}{z_{h2}^2}\right)b^2$$

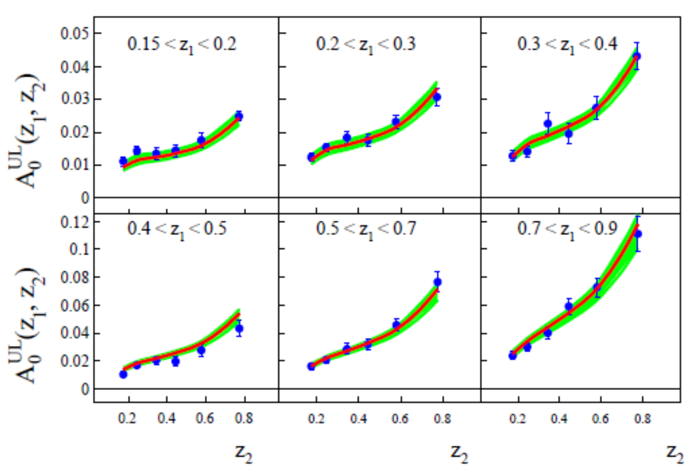
Kang, Prokudin, Sun and Yuan: Phys. Rev. D91 (2015) 071501 and ArXiv:1505.05589

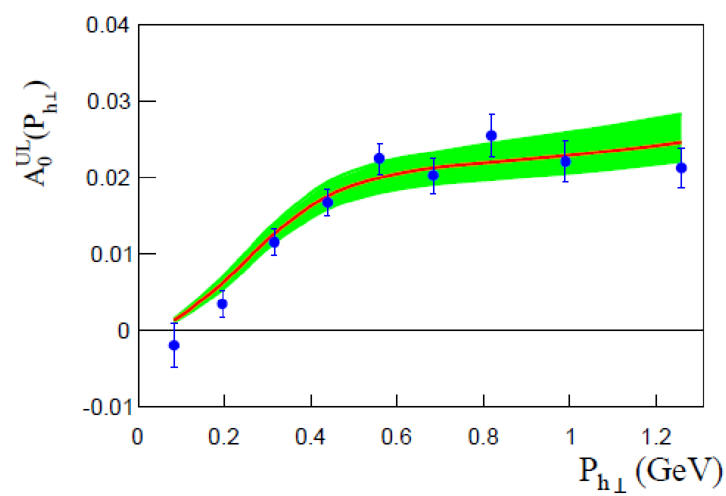
CSS/TMD evolution and Collins/Transversity

- Extraction of transversity and Collins functions using TMD evolution





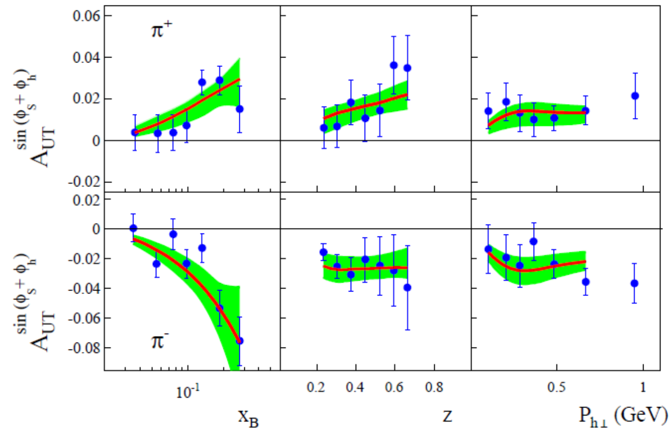


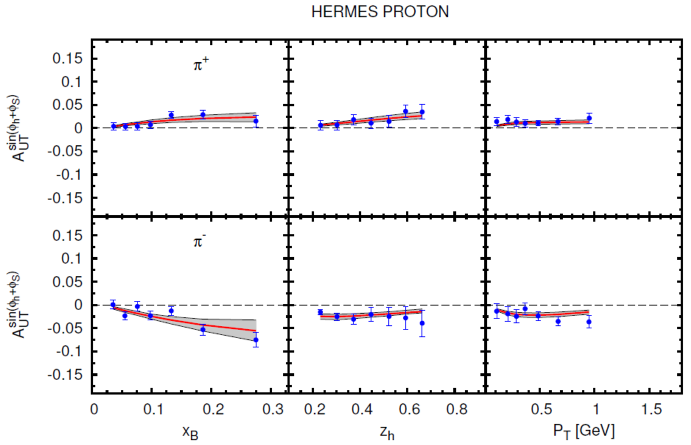


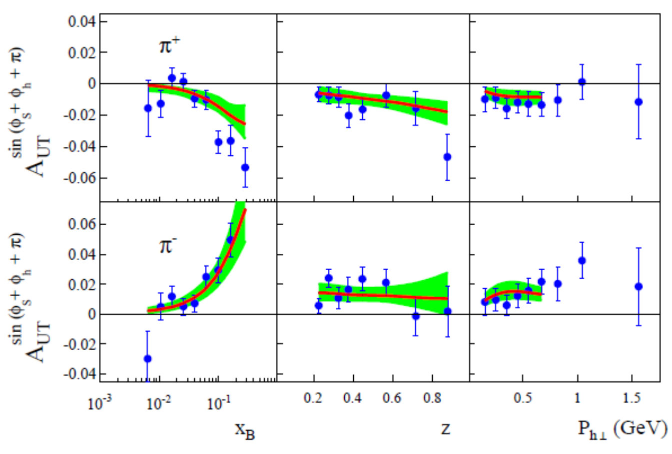
Kang, Prokudin, Sun and Yuan: Phys. Rev. D91 (2015) 071501 and ArXiv:1505.05589

CSS/TMD evolution and Collins/Transversity

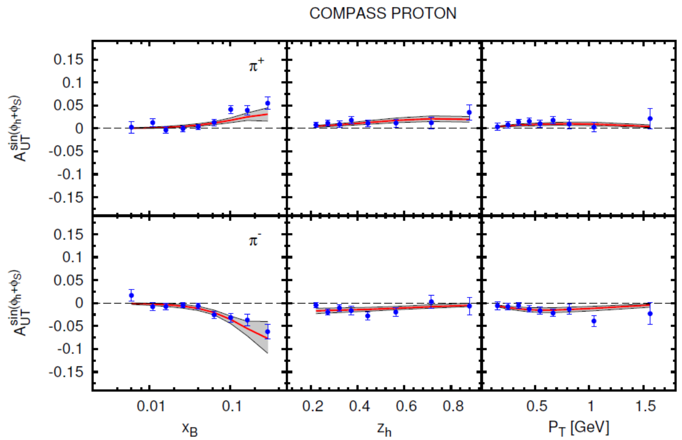
- TMD evolution
- Gaussians







(a)



CSS/TMD evolution and Collins/Transversity

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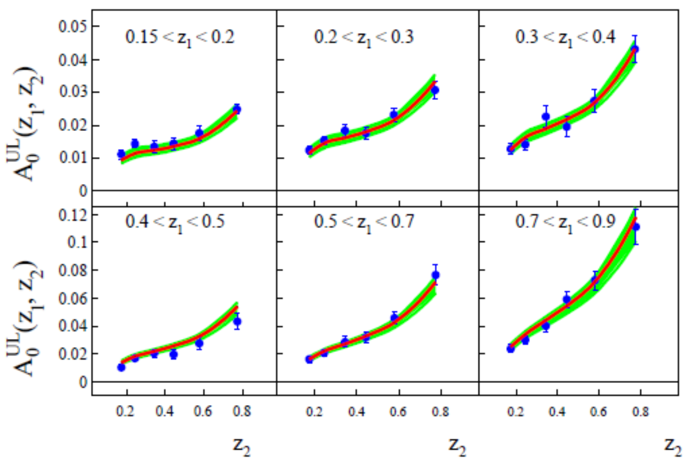
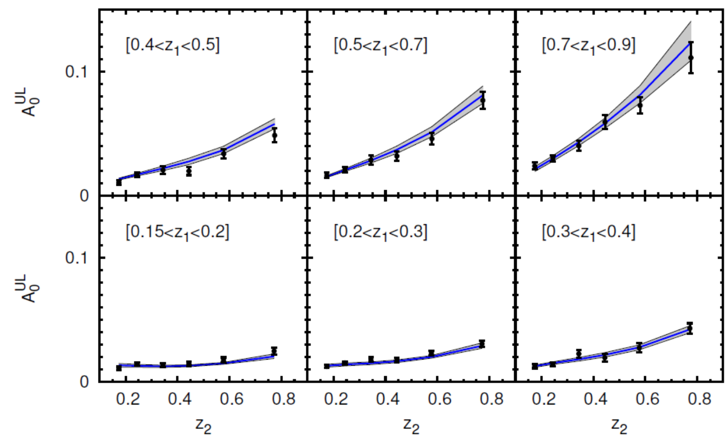
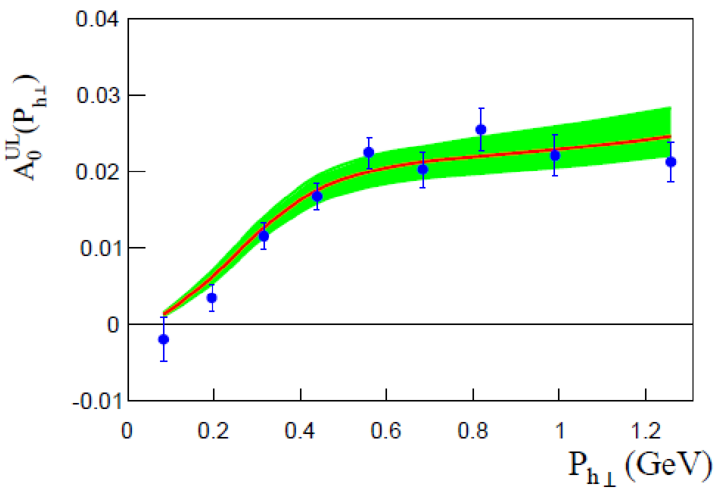
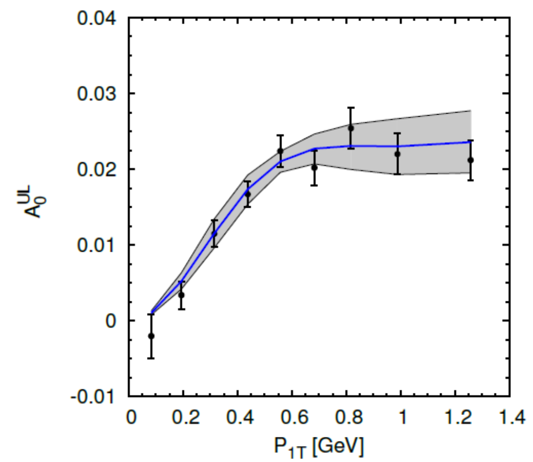




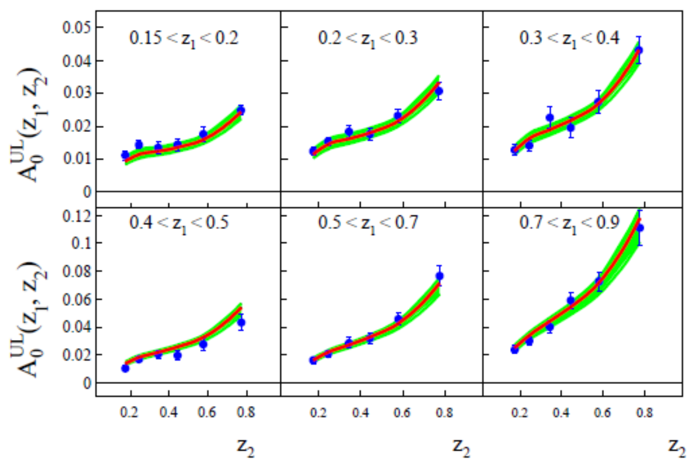


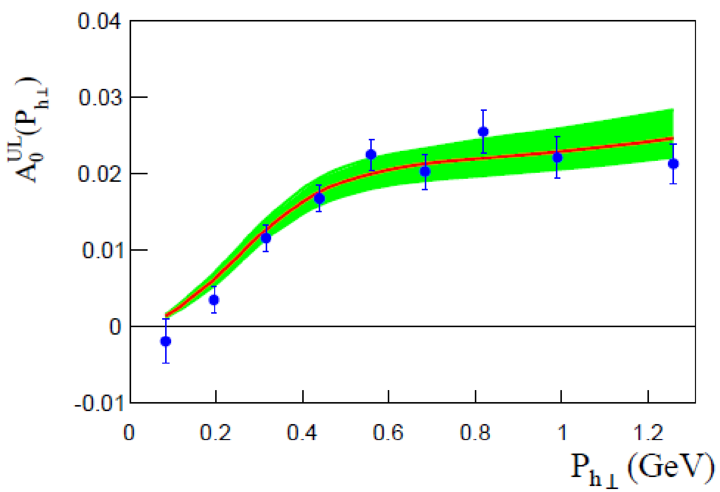


Kang, Prokudin, Sun and Yuan: Phys. Rev. D91 (2015) 071501 and ArXiv:1505.05589

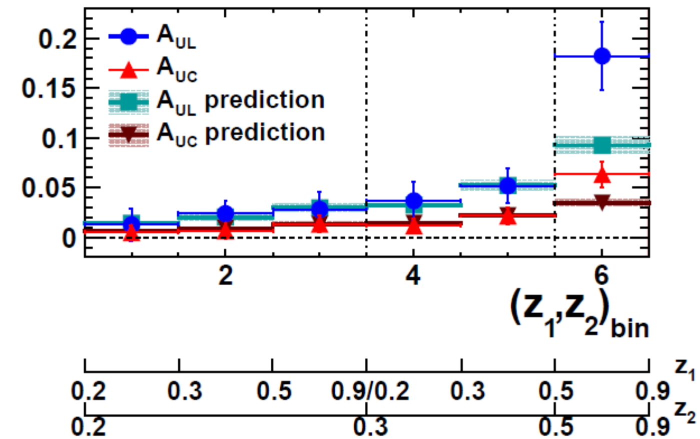
CSS/TMD evolution and Collins/Transversity

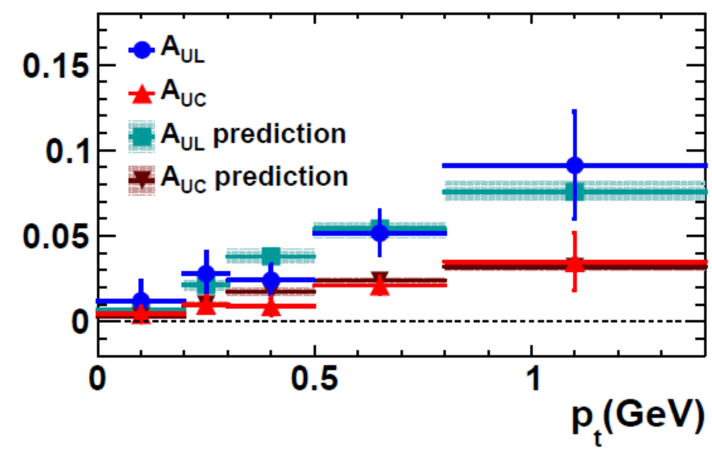
- TMD evolution





- TMD evolution effects at BES??

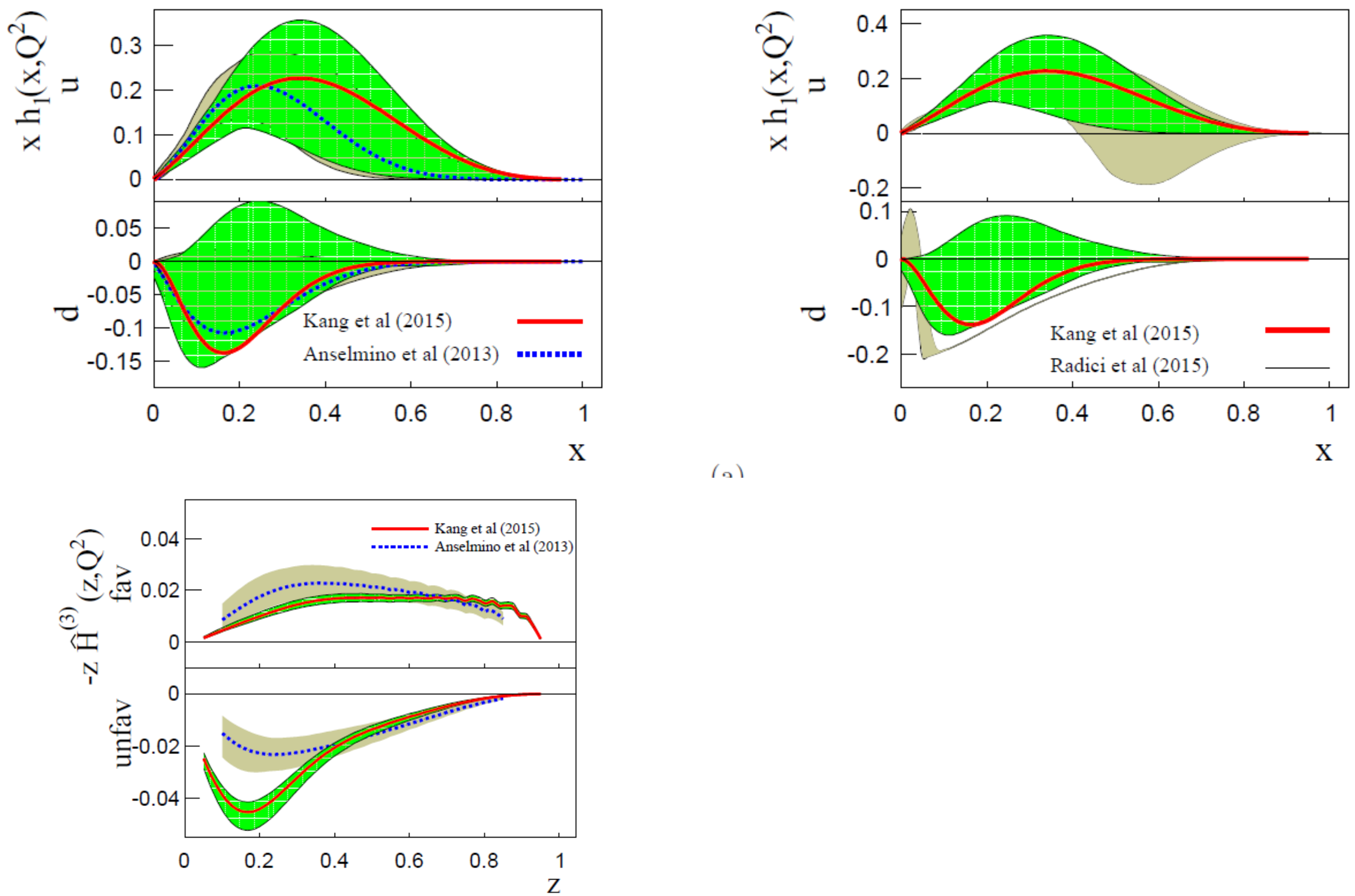




Kang, Prokudin, Sun and Yuan: ArXiv:1505.05589 and BES: 1507.06824

CSS/TMD evolution and Collins/Transversity

- Extraction of transversity and Collins functions using TMD evolution

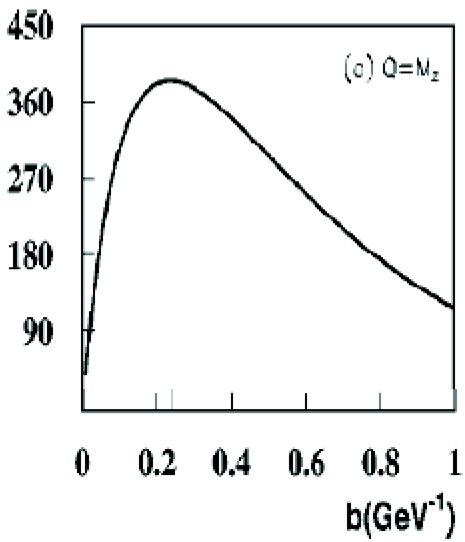


Kang, Prokudin, Sun and Yuan: ArXiv:1505.05589 and BES: 1507.06824

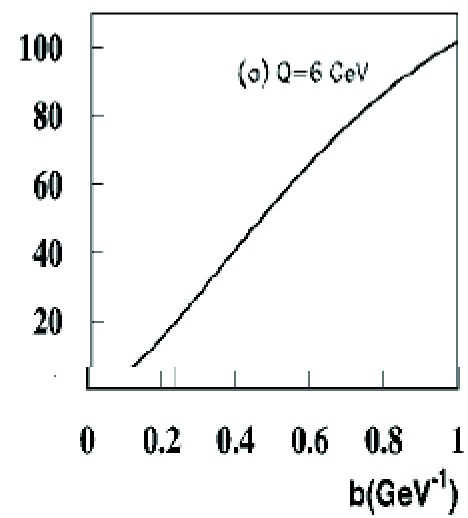
Backup Slides

Integrand of the FT in SIDIS

Integrand of the FT in DY



$\sqrt{s} = 1.8 \text{ TeV}$

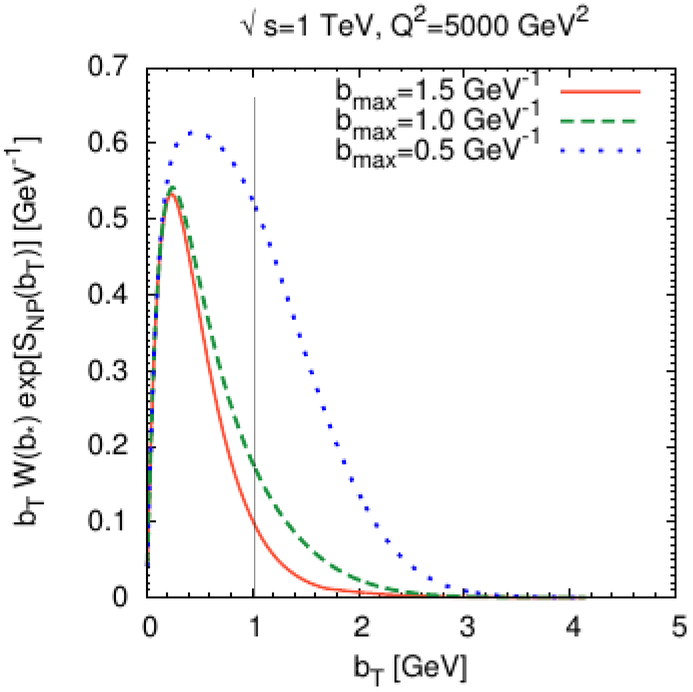


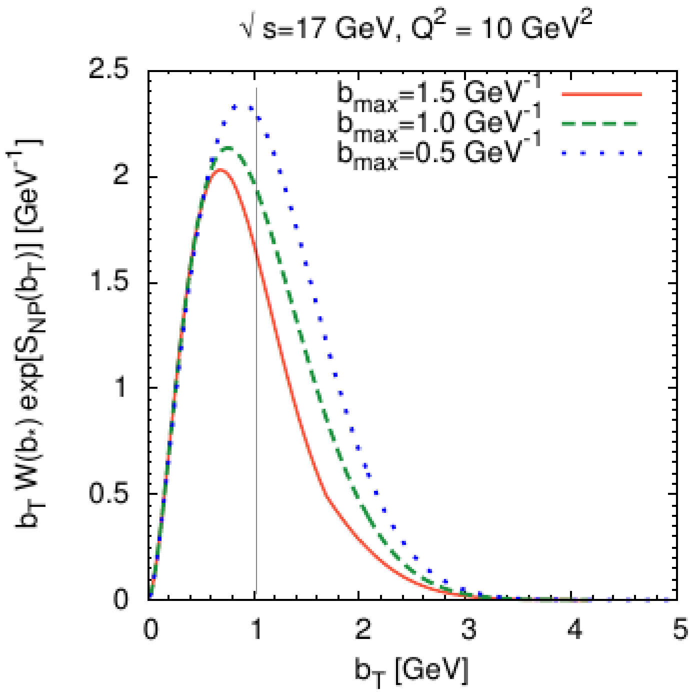
$\sqrt{s} = 27.4 \text{GeV}$

$$\frac{1}{2\pi}\int_0^\infty db\, b\, J_0(Q_T b)\, e^{S(b,Q)} \sum_{ij} \sigma_{ij\to V}(Q)\, \tilde{W}_{ij}(b, \frac{c}{b}, x_A, x_B)$$

Qiu and Zhang, Phys. Rev. D63 (2001) 114011

Integrand of the FT in SIDIS





$$W^{\rm NLL} = \pi\sigma_0^{\rm DIS}\int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{\rm SIDIS}(x,z,b_*,Q)\exp\left[S_{\rm NP}(x,z,b_T,Q)\right]$$

Boglione et al, JHEP 02 (2015) 095

Theoretical uncertainties in pQCD

- Perturbative, fixed order, calculations are affected by theoretical uncertainties due, for instance, to the choice of the factorization scale.
  The cross section depends on logs like:

$$\ln(Q/\mu_F)$$

Theoretical uncertainties in CSS

Theoretical uncertainties in pQCD

- To "optimize" the expansion the factorization scale is set to be equal to the hard scale:

$$\ln(Q/\mu_F) \longrightarrow \mu_F = Q$$

- The theoretical error is built changing the value of the factorization scale. Usually:

$$Q/2 < \mu_F < 2Q$$

Theoretical uncertainties in pQCD

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- The theoretical error is built changing the value of the factorization scale. Usually:

$$Q/2 < \mu_F < 2Q$$

Theoretical uncertainties in resummation

- Similarly, in resummation several scales appear. For instance, using the standard CSS nomenclature we have:

$$C_1/b_T \qquad\qquad C_2Q \qquad\qquad C_3/b_T$$

- Studying the theoretical uncertainties in resummation is important because it gives us a measure of how much we know of the perturbative part of the cross section and correspondingly how much we have to model.

- This is particularly important for low energy SIDIS data that, contrary to Drell-Yan data, are difficult to describe with resummation.

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
$$+ Y(x_1, x_2, q_T, C_4 Q)$$

Y-factor (matching function)

Perturbative resummed part of the cross section

Non-perturbative function

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
$$+ Y(x_1, x_2, q_T, C_4 Q)$$

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2 / b_{max}^2}}$$

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
$$+Y(x_1, x_2, q_T, C_4Q)$$

$$W_j(x_1, x_2, Q, C_1/b_*, C_2Q, C_3/b_*) = \sum_{i,k} \exp\left[S(b_*, C_1/b_*, C_2Q)\right] \left[C_{ji} \otimes f_i\right] \left[C_{\bar{j}k} \otimes f_k\right]$$

Perturbative Sudakov

Convolutions of PDFs and Wilson Coefficients

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
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$$S(b_T, Q, C_1, C_2) = - \int_{C_1^2/b_T^2}^{C_2^2 Q^2} \frac{d\kappa^2}{\kappa^2} \left[A(\alpha_s(\kappa), C_1) \ln\left(\frac{C_2^2 Q^2}{\kappa^2}\right) + B(\alpha_s(\kappa), C_1, C_2)\right]$$

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i\boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
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Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2dydq_T^2} = \sigma_0\left\{\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,C_1/b_*,C_2Q,C_3/b_*)F_{NP}(x,b_T,Q)\right\}$$
$$+Y(x_1,x_2,q_T,C_4Q)$$

$$W_j(x_1,x_2,Q,C_1/b_*,C_2Q,C_3/b_*) = \sum_{i,k}\exp\left[S(b_*,C_1/b_*,C_2Q)\right]\left[C_{ji}\otimes f_i\right]\left[C_{\bar{j}k}\otimes f_k\right]$$

$$S(b_T,Q,C_1,C_2) = -\int_{C_1^2/b_T^2}^{C_2^2Q^2}\frac{d\kappa^2}{\kappa^2}\left[A(\alpha_s(\kappa),C_1)\ln\left(\frac{C_2^2Q^2}{\kappa^2}\right)+B(\alpha_s(\kappa),C_1,C_2)\right]$$

Sudakov hard scale

Sudakov soft scale

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i\boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
$$+ Y(x_1, x_2, q_T, C_4 Q)$$

$$W_j(x_1, x_2, Q, C_1/b_*, C_2 Q, C_3/b_*) = \sum_{i,k} \exp\left[S(b_*, C_1/b_*, C_2 Q)\right] \left[C_{ji} \otimes f_i\right] \left[C_{\bar{j}k} \otimes f_k\right]$$

$$S(b_T, Q, C_1, C_2) = -\int_{C_1^2/b_T^2}^{C_2^2 Q^2} \frac{d\kappa^2}{\kappa^2} \left[A(\alpha_s(\kappa), C_1) \ln\left(\frac{C_2^2 Q^2}{\kappa^2}\right) + B(\alpha_s(\kappa), C_1, C_2) \right]$$

Cross section with scales in the CSS formalism

- Drell-Yan cross section

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$$[C_{ji} \otimes f_i] = \int_x^1 \frac{dz}{z} C_{ji}(z, b, \mu = \frac{C_3}{b}, C_1, C_2) f_i(x/z, \mu = \frac{C_3}{b})$$

Cross section with scales in the CSS formalism

- Drell-Yan cross section

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \left\{ \int \frac{d^2 \boldsymbol{b}_T e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T}}{(2\pi)^2} \sum_j e_j^2 W_j(x_1, x_2, C_1/b_*, C_2 Q, C_3/b_*) F_{NP}(x, b_T, Q) \right\}$$
$$+ Y(x_1, x_2, q_T, C_4 Q)$$

$$W_j(x_1, x_2, Q, C_1/b_*, C_2 Q, C_3/b_*) = \sum_{i,k} \exp\left[S(b_*, C_1/b_*, C_2 Q)\right] \left[C_{ji} \otimes f_i\right] \left[C_{\bar{j}k} \otimes f_k\right]$$

$$[C_{ji} \otimes f_i] = \int_x^1 \frac{dz}{z} C_{ji}(z, b, \mu = \frac{C_3}{b}, C_1, C_2) f_i(x/z, \mu = \frac{C_3}{b})$$

Cross section with scales in the CSS formalism

- Some examples of logs:

$$A^{(2)}(C_1) = 2C_F\left[\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_G - \frac{5}{9}N_f - \beta_0 \ln(b_0/C_1)\right] \qquad B^{(1)}(C_1, C_2) = -C_F[3 + 4\ln(C_2 b_0/C_1)]$$

$$C_{jg}^{(1)}(z, b, \mu) = 2\,T_F\left\{[z(1-z)] - \ln(\mu b/b_0)[z^2 + (1-z)^2]\right\}$$

$$\mu = \frac{C_3}{b}$$

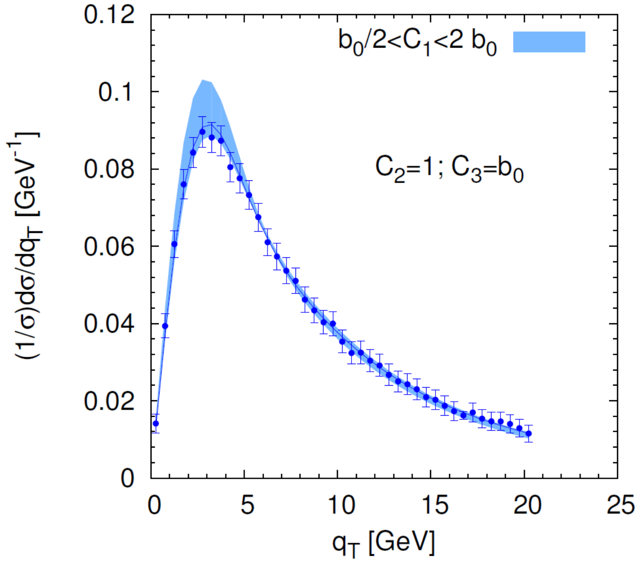
$$b_0 = 2\exp(-\gamma_E)$$

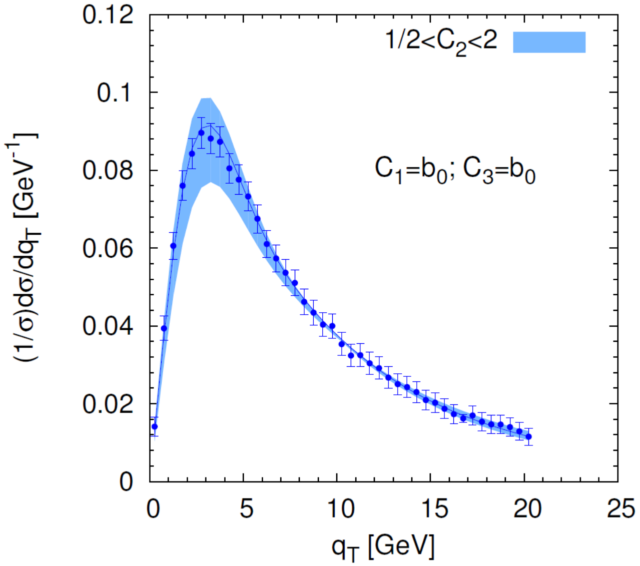
...The exact values [of the scales] can be chosen to "optimize" the perturbation expansion, that is, to keep higher-order correction moderately small. We have left this possibility open by including the constant C_1 and C_2...

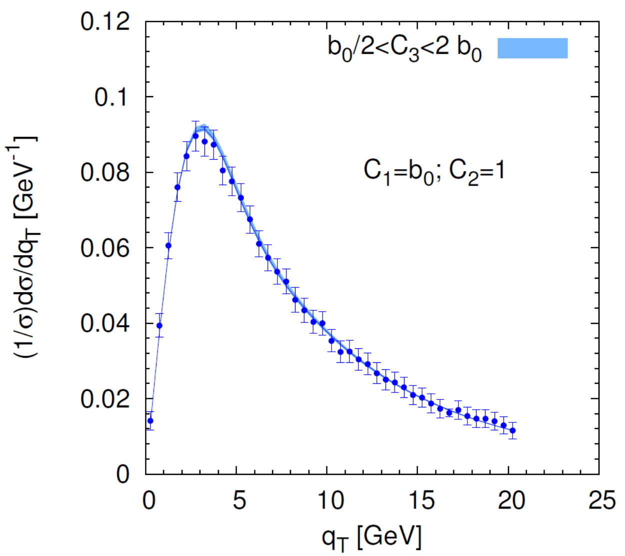
Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

- Standard choice: $C_1 = C_3 = b_0,\ C_2 = C_4 = 1$

Drell-Yan

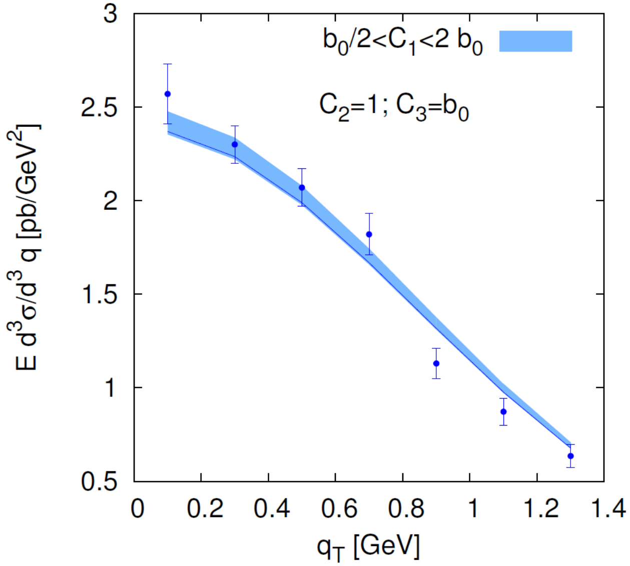


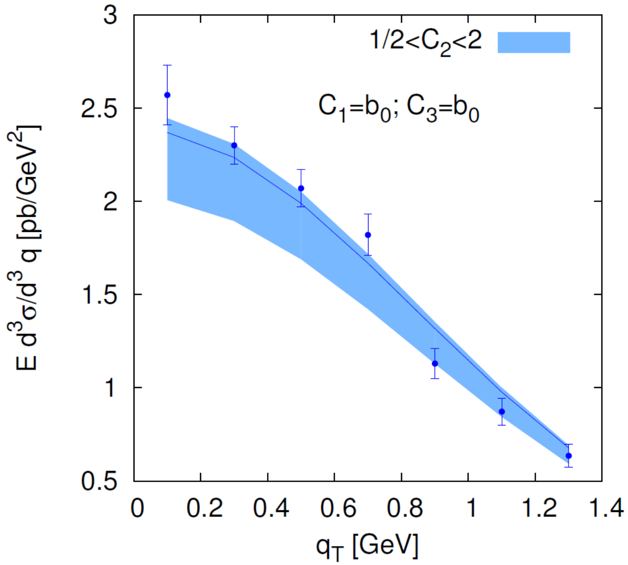


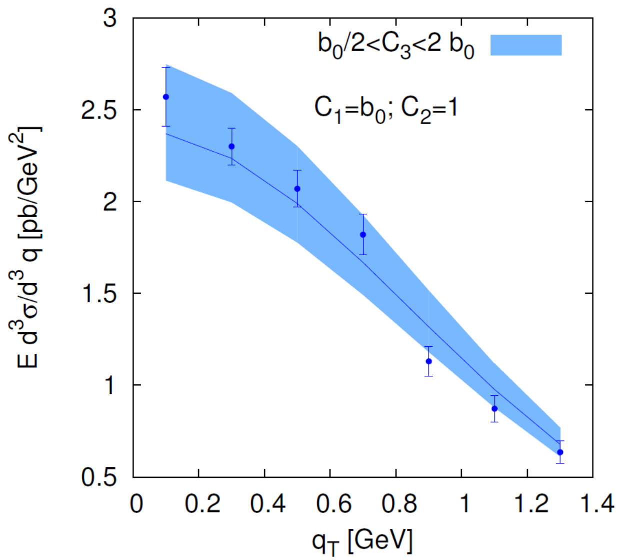


- CDF Run II (High energy exp)
- NLL BLNY parametrization, $b_{max}=0.5\ GeV^{-1}$

Drell-Yan



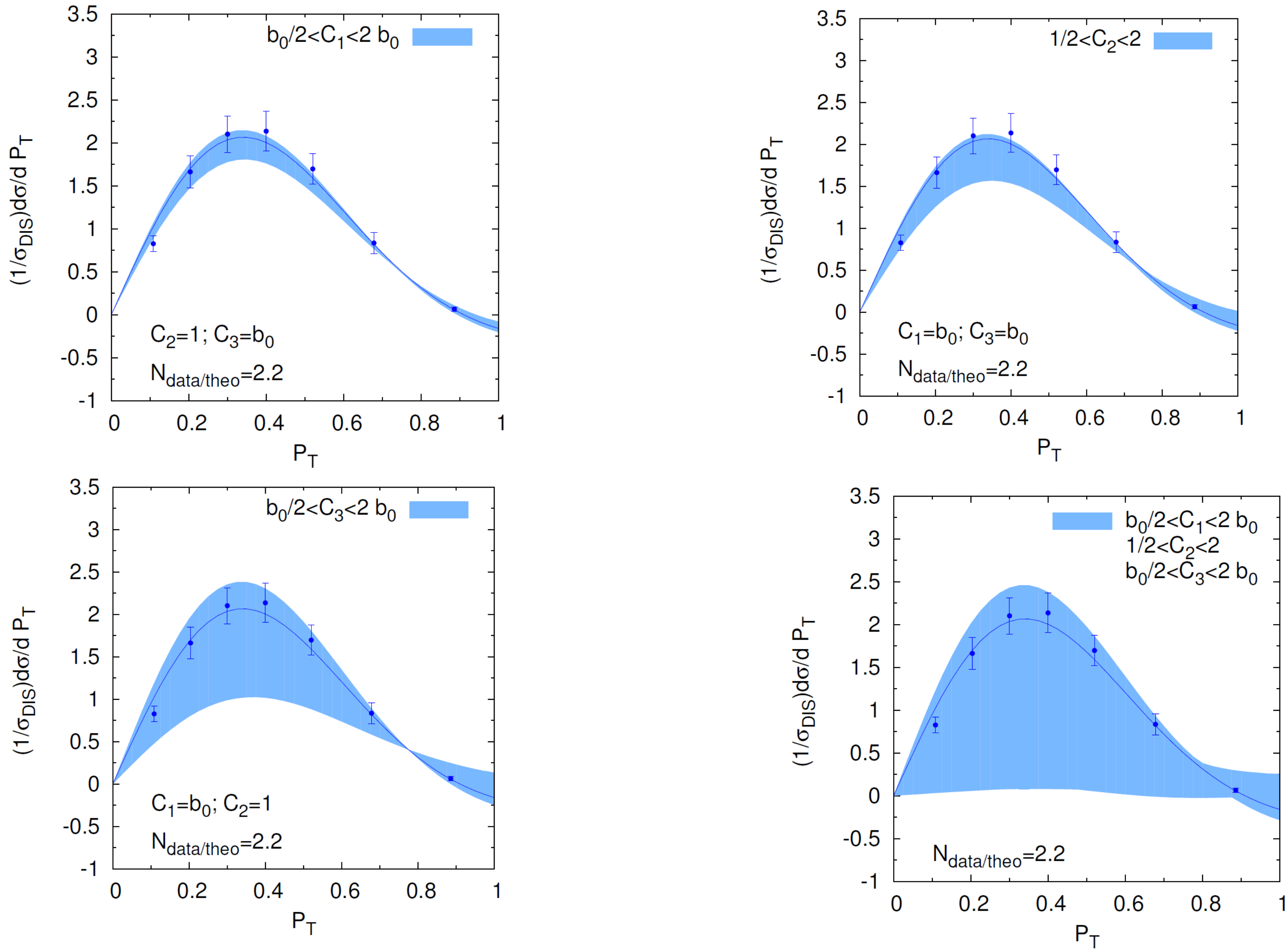




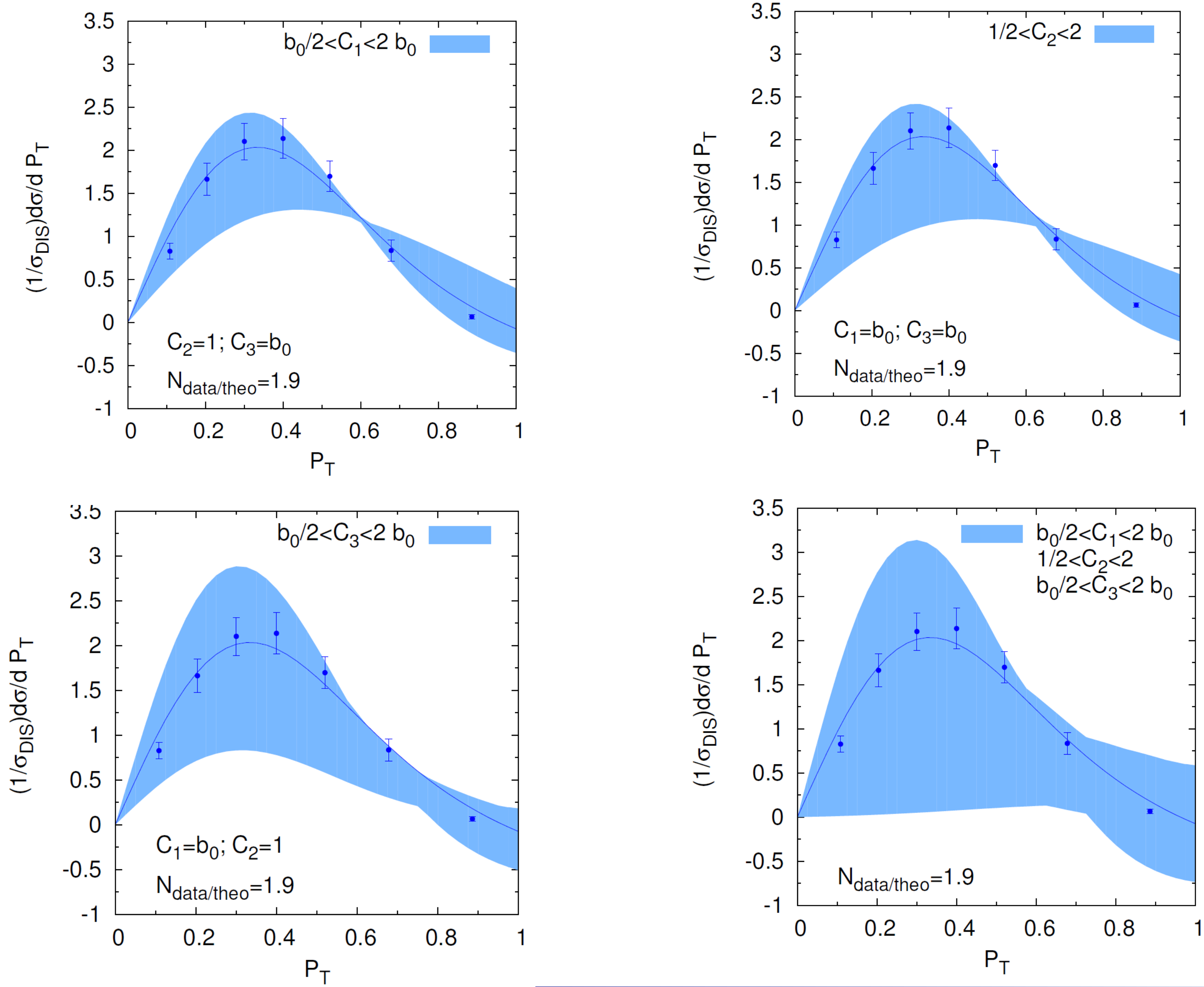
- E288 $\sqrt{s}$=23.8 GeV (Low energy exp)
- Q=5 GeV

- NLL BLNY parametrization, $b_{max}=0.5\ GeV^{-1}$

HERMES b_{max}=0.5 GeV^{-1}



HERMES KN (b_{max}=1.5 GeV^{-1})



Yuan-Sun phenomenolgy

- Then Anselmino et al like parametrization for the Sivers function at the scale of HERMES

$$\tilde{F}^{\alpha}_{\rm sivers}(Q_0, b) = \frac{ib^{\alpha}_{\perp}M}{2}\sum_q e_q^2 \Delta f_q^{\rm sivers}(x) D_q(z) e^{-(g_0-g_s)b^2 - g_h b^2/z_h^2}$$

$$\Delta f_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}} f_q(x)$$

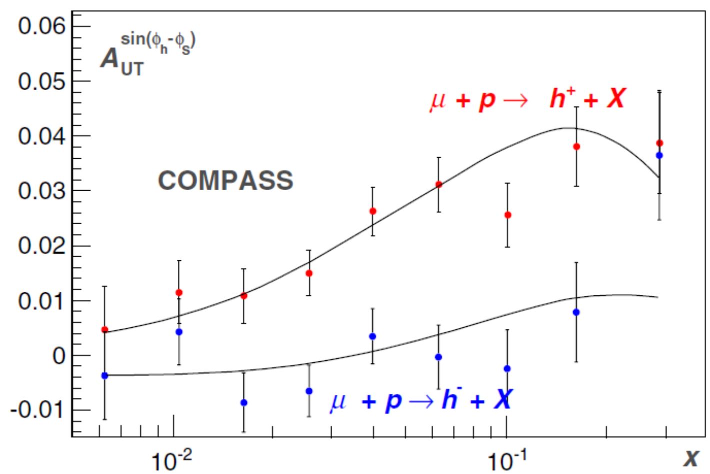
Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

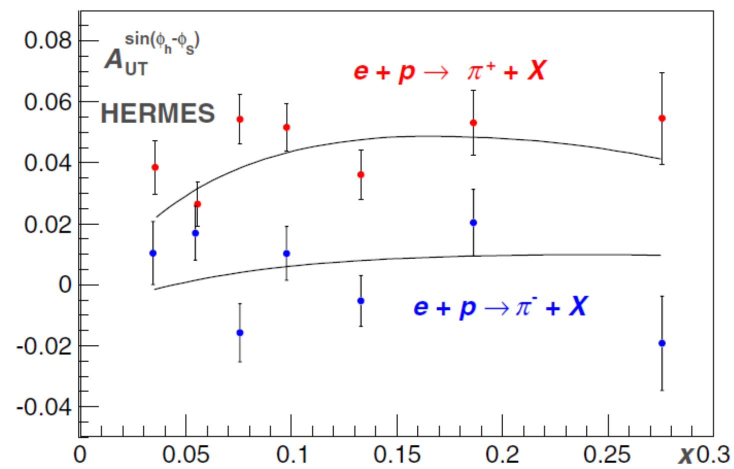
Yuan-Sun phenomenolgy

TABLE I. Parameters $\{a_i^0\}$ describing our optimum Δf_i in Eq. (5) at the input scale $Q^2 = 2.4$ GeV.

flavor i	N_i	α_i	β_i	g_s (GeV^2)
u	0.13 ± 0.023	0.81 ± 0.16	4.0 ± 1.2	0.062 ± 0.005
d	-0.27 ± 0.12	1.41 ± 0.28	4.0 ± 1.2	0.062 ± 0.005
s	0.07 ± 0.06	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
$\bar{u}$	-0.07 ± 0.05	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005
$\bar{d}$	-0.19 ± 0.12	0.58 ± 0.39	4.0 ± 1.2	0.062 ± 0.005

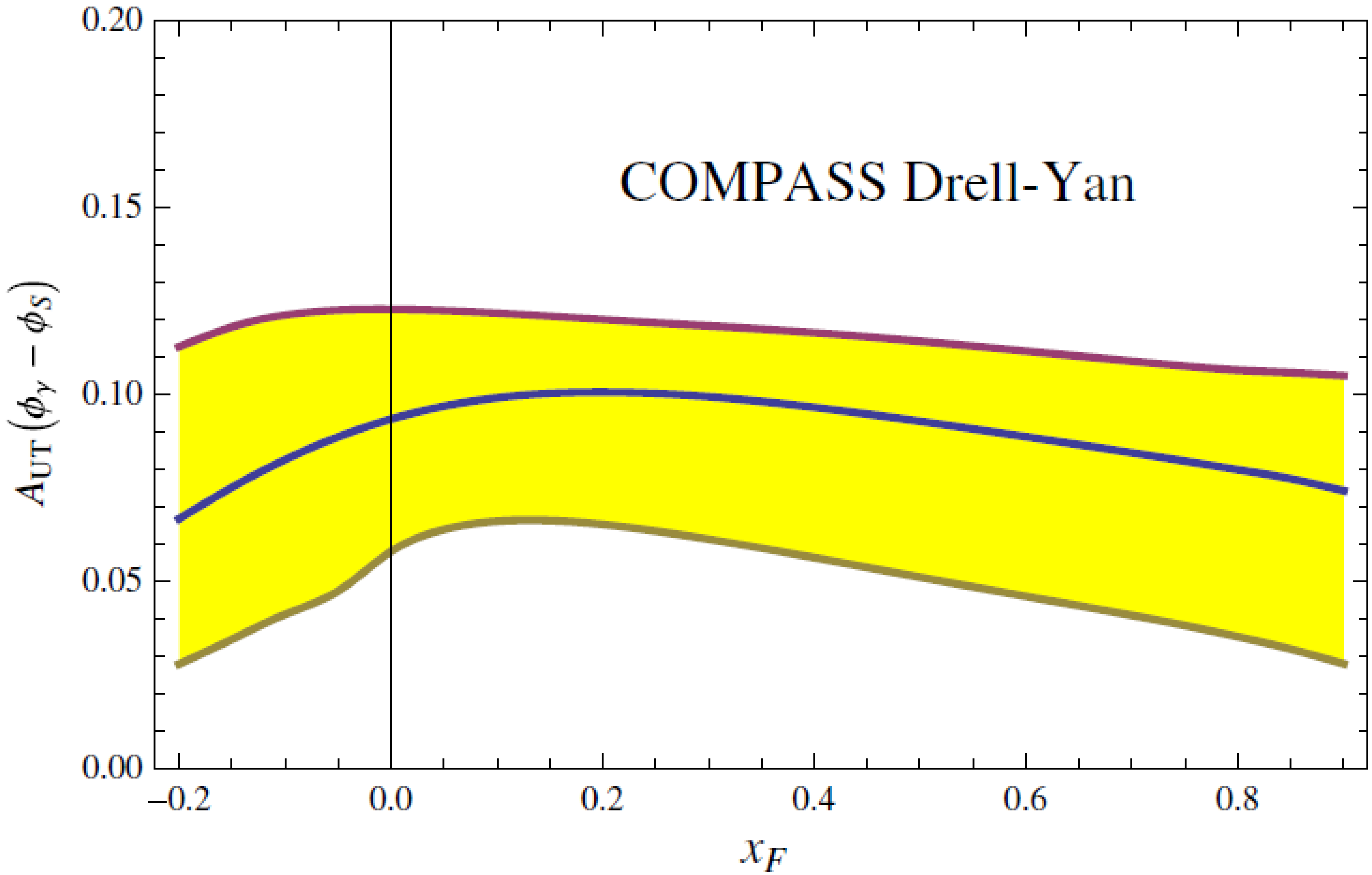
$$\chi^2/\bar{\mathrm{d}}.\mathrm{o.f} = 1.08$$





Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

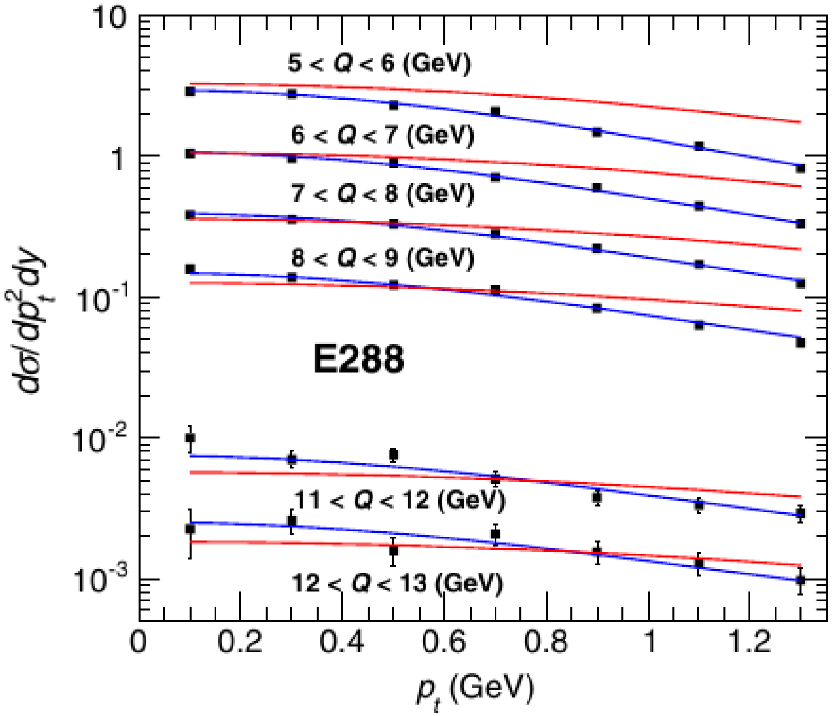
Yuan-Sun phenomenolgy



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Yuan-Sun

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$



This formulation maximize the
Non perturbative input
Maybe not suitable for DY...

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Echevarria-Idilbi-Kang-Vitev phenomenology

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Restart from the TMD evolution in the CSS-like version

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b)$$
$$\exp\left\{\int_{\mu_b}^{Q} \frac{d\kappa}{\kappa}\gamma_F(\kappa; 1) - \ln\left(\frac{Q}{\kappa}\right)\gamma_K(\kappa)\right\}$$
$$\exp\left\{-g_P(x, b_T) - g_K(b_T)\ln\left(\frac{Q}{Q_0}\right)\right\}$$

- Make some approximation to simply life

$$\tilde{C}_{ji}(z, \alpha(\mu)) = \delta_{ij}\delta(1-z)$$ At LO; PDF at LO

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Simple parametrizations for the non-perturbative part:

$$F_{NP}(b_T, Q)^{\rm pdf} = \exp\left[-b_T^2\left(g_1^{\rm pdf} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

$$F_{NP}(b_T, Q)^{\rm ff} = \exp\left[-b_T^2\left(g_1^{\rm ff} + \frac{g_2}{2}\ln(Q/Q_0)\right)\right]$$

- Choose Q_0^2=2.4 GeV^2 as reference scale. We know that simple gaussian models describe well SIDIS data...

$$g_1^{\rm pdf} = \frac{\langle k_\perp^2\rangle_{Q_0}}{4}, \qquad g_1^{\rm ff} = \frac{\langle p_T^2\rangle_{Q_0}}{4z^2}$$

$$\langle k_\perp^2\rangle_{Q_0} = 0.25 - 0.44\ {\rm GeV}^2, \qquad \langle p_T^2\rangle_{Q_0} = 0.16 - 0.20\ {\rm GeV}^2$$

- We know that DY data can be described using:

$$b_{\rm max} = 1.5\ {\rm GeV}^{-1} \quad g_2 = 0.184 \pm 0.018\ {\rm GeV}^2$$

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

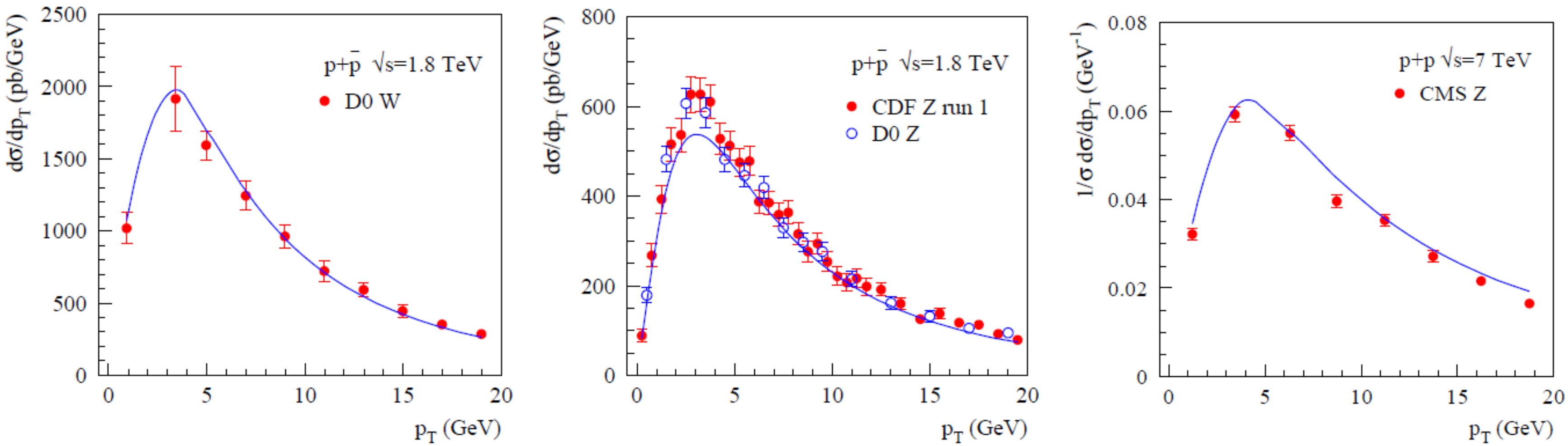
- Try to find reasonable parameters to describe data and see what happens...

$$\langle k_\perp^2 \rangle_{Q_0} = 0.38 \text{ GeV}^2, \qquad \langle p_T^2 \rangle_{Q_0} = 0.19 \text{ GeV}^2, \qquad g_2 = 0.16 \text{ GeV}^2$$

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Z and W-Boson Production

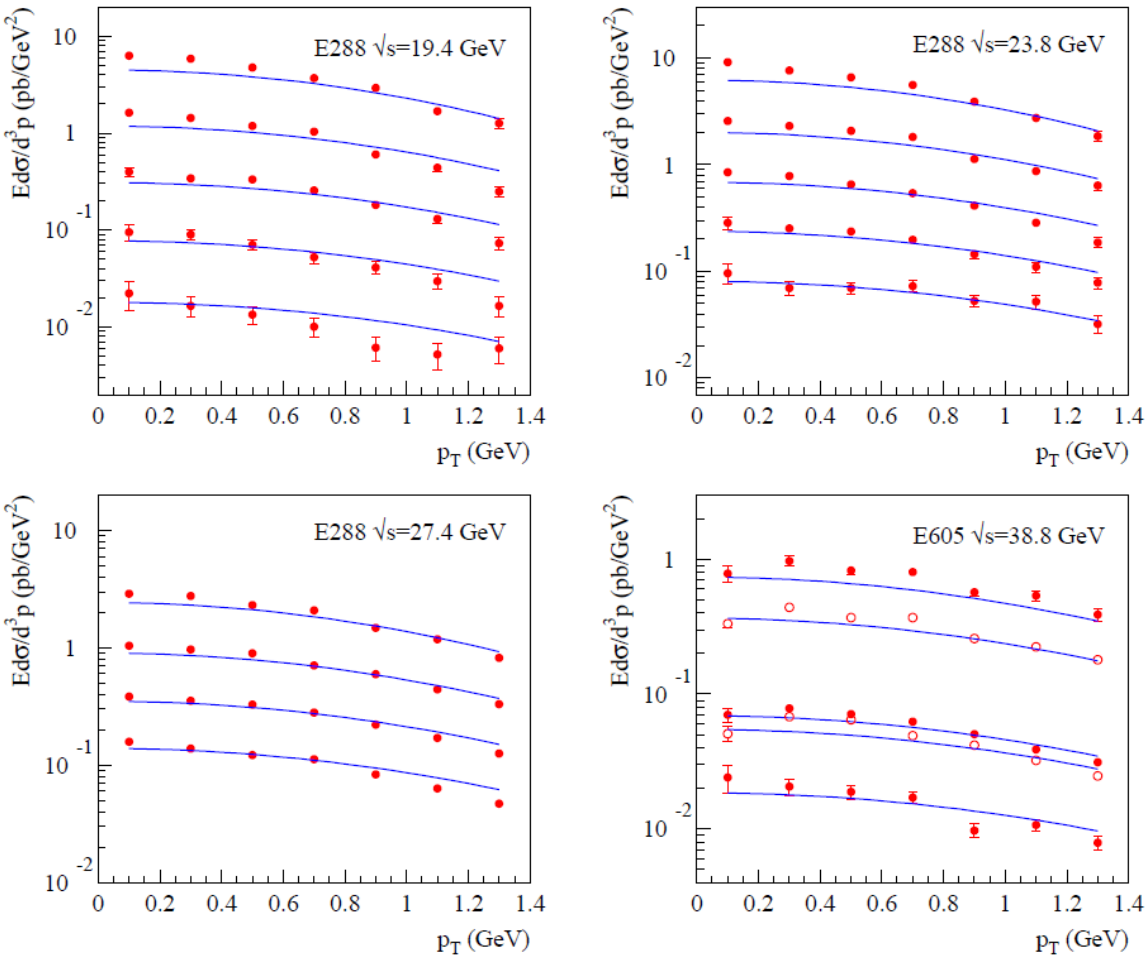


MSTW2008 PDF

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Low energy Drell-Yan

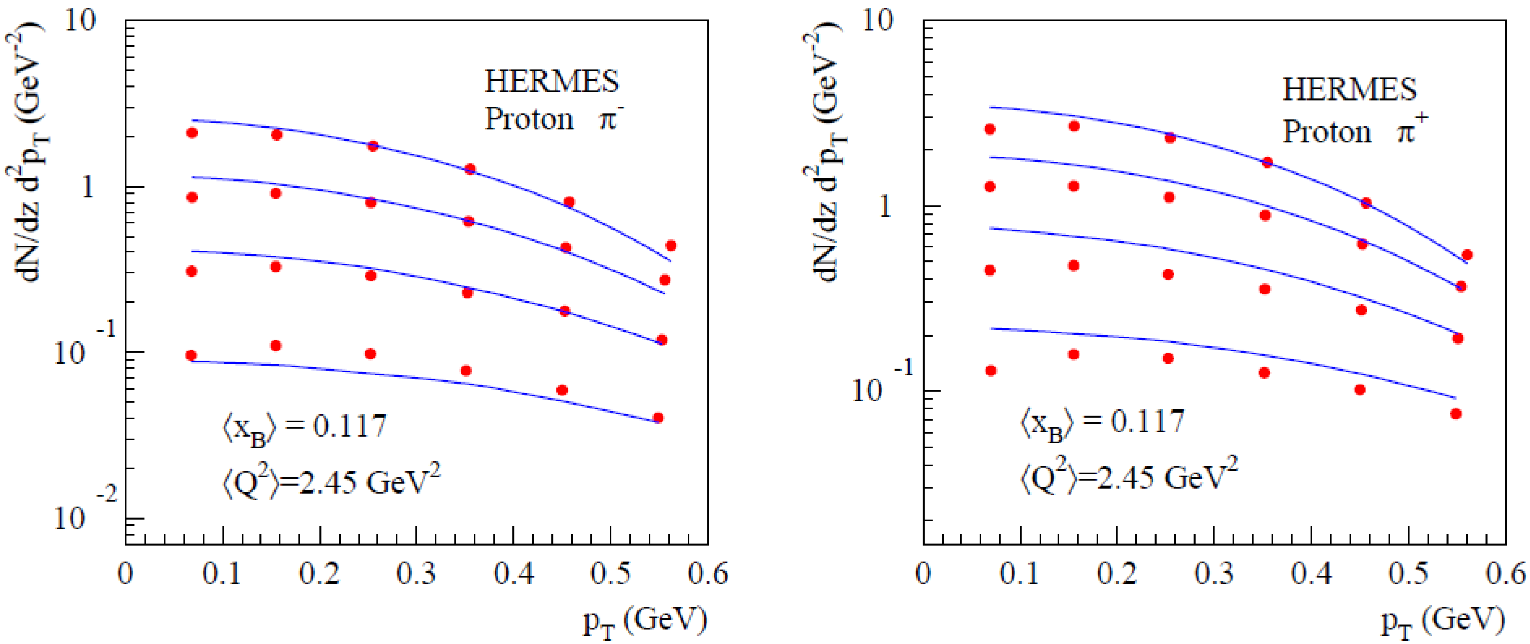


EKS98 Cu PDF

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

HERMES SIDIS data

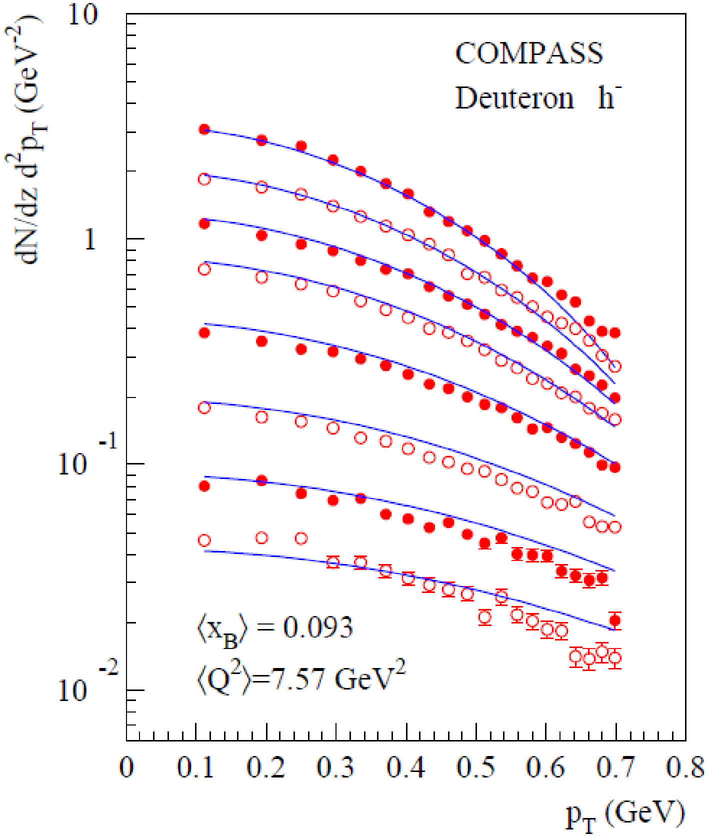


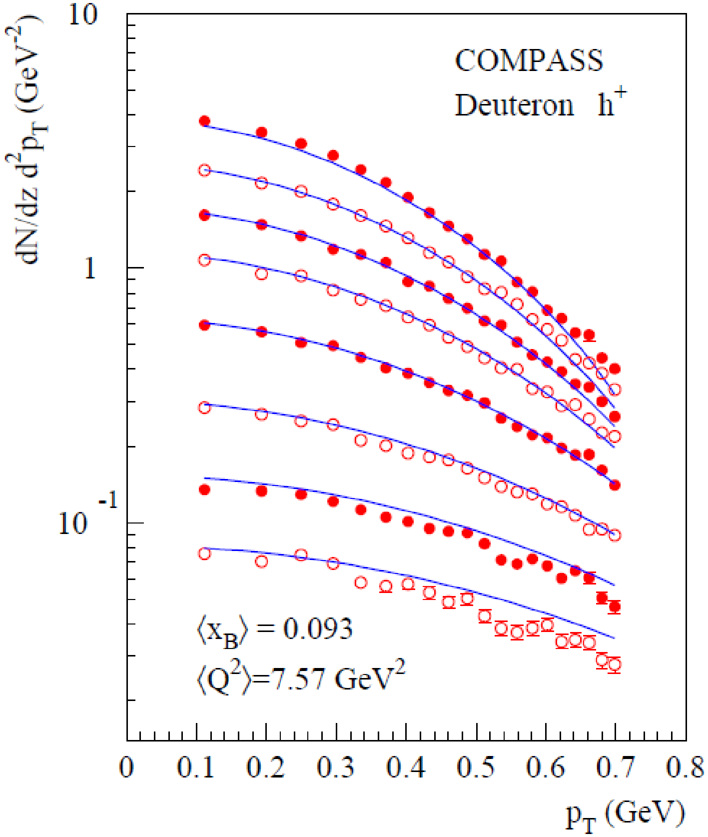
MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

(some...) COMPASS SIDIS data





MSTW2008 PDF and DSS

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Ready for Sivers! Again a CSS-like version approximated at LO

$$F_{UT}^{\sin(\phi_h-\phi_s)} = \frac{1}{4\pi}\int_0^\infty db\, b^2 J_1(P_{h\perp}b/z_h)\sum_q e_q^2\, T_{q,F}(x_B, x_B, c/b_*) D_{h/q}(z_h, c/b_*)$$
$$\times \exp\left\{-\int_{c^2/b_*^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left(g_1^{\rm ff}+g_1^{\rm sivers}+g_2\ln\frac{Q}{Q_0}\right)\right\}$$

- The Qiu-Sterman function treated at LO as a Sivers function.

$$T_{q,F}(x,x,\mu) = N_q\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}x^{\alpha_q}(1-x)^{\beta_q}f_{q/A}(x,\mu)$$

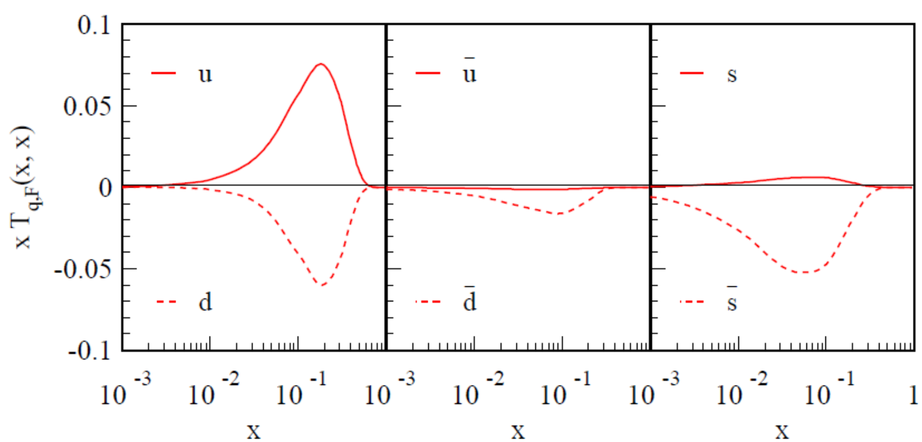
Using an Anselmino-like parametrization.

Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

- Fit of HERMES, COMPASS and JLAB data

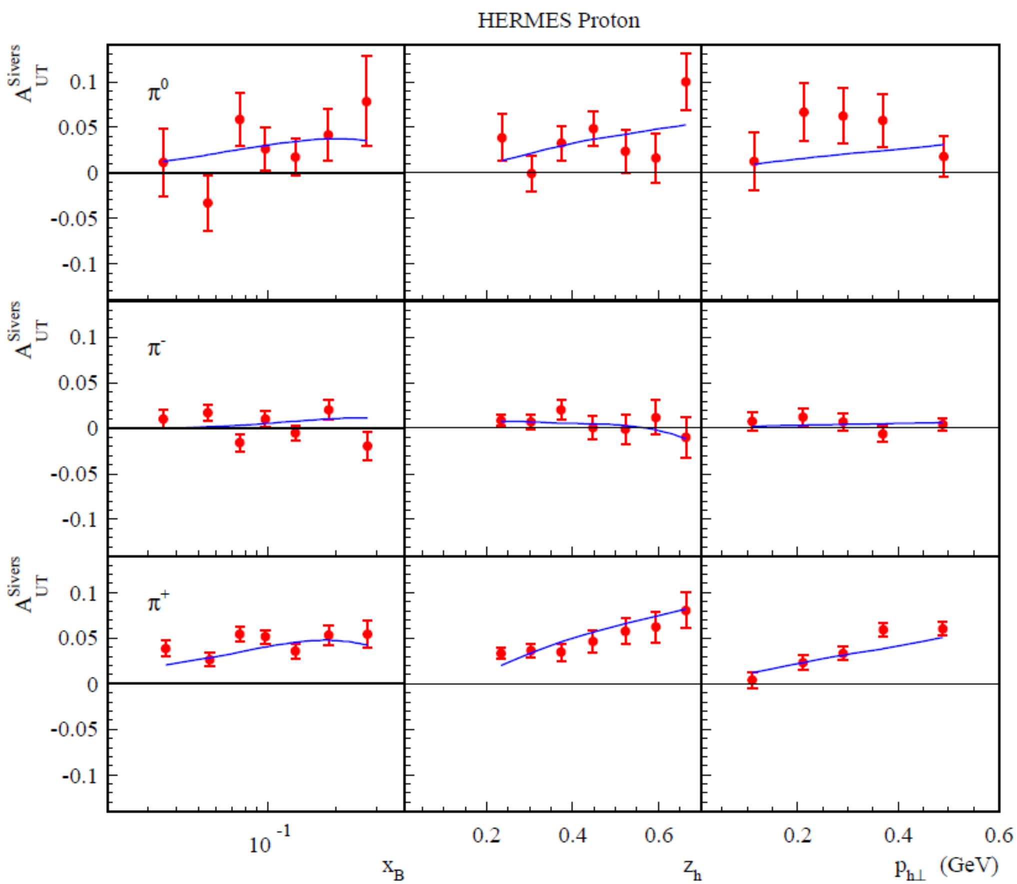
$\chi^2/d.o.f. = 1.3$	
$\alpha_u = 1.051^{+0.192}_{-0.180}$	$\alpha_d = 1.552^{+0.303}_{-0.275}$
$\alpha_{\text{sea}} = 0.851^{+0.307}_{-0.305}$	$\beta = 4.857^{+1.534}_{-1.395}$
$N_u = 0.106^{+0.011}_{-0.009}$	$N_d = -0.163^{+0.039}_{-0.046}$
$N_{\bar{u}} = -0.012^{+0.018}_{-0.020}$	$N_{\bar{d}} = -0.105^{+0.043}_{-0.060}$
$N_s = 0.103^{+0.548}_{-0.604}$	$N_{\bar{s}} = -1.000 \pm 1.757$
$\langle k_{s\perp}^2 \rangle = 0.282^{+0.073}_{-0.066}$ GeV^2	



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

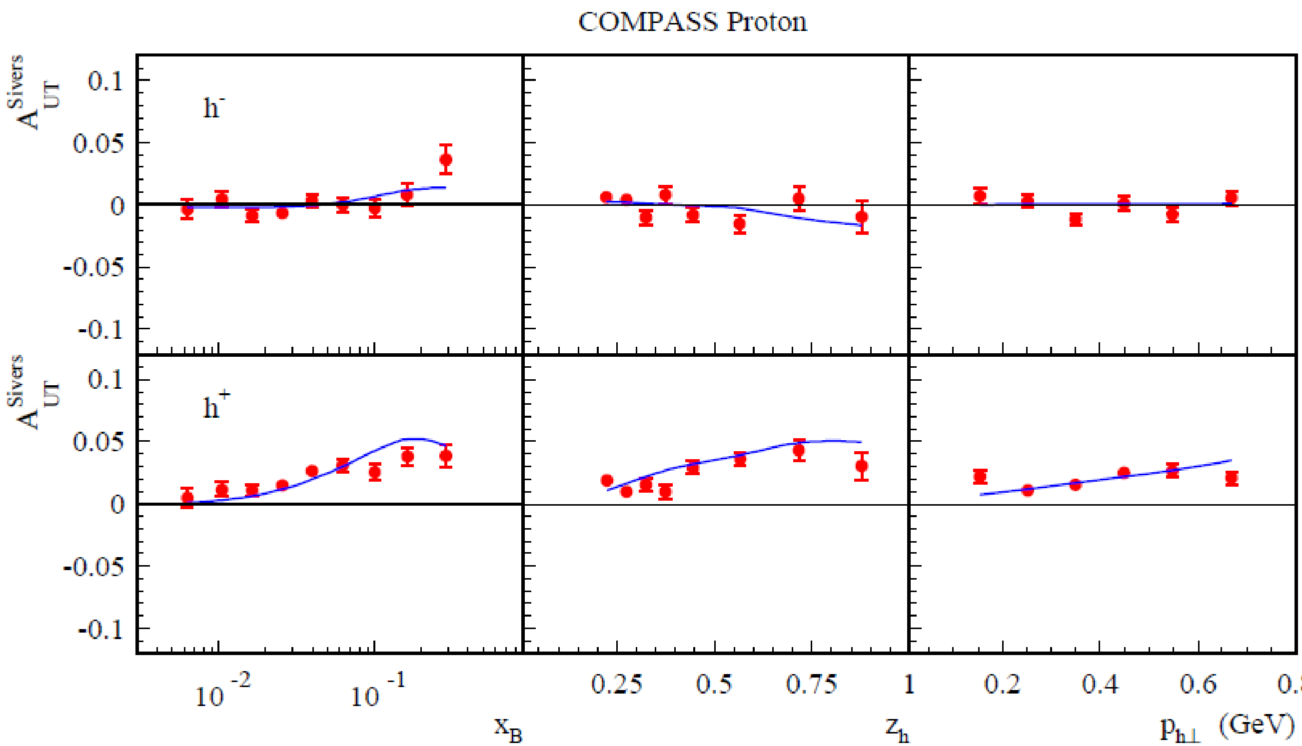
HERMES SIDIS data



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

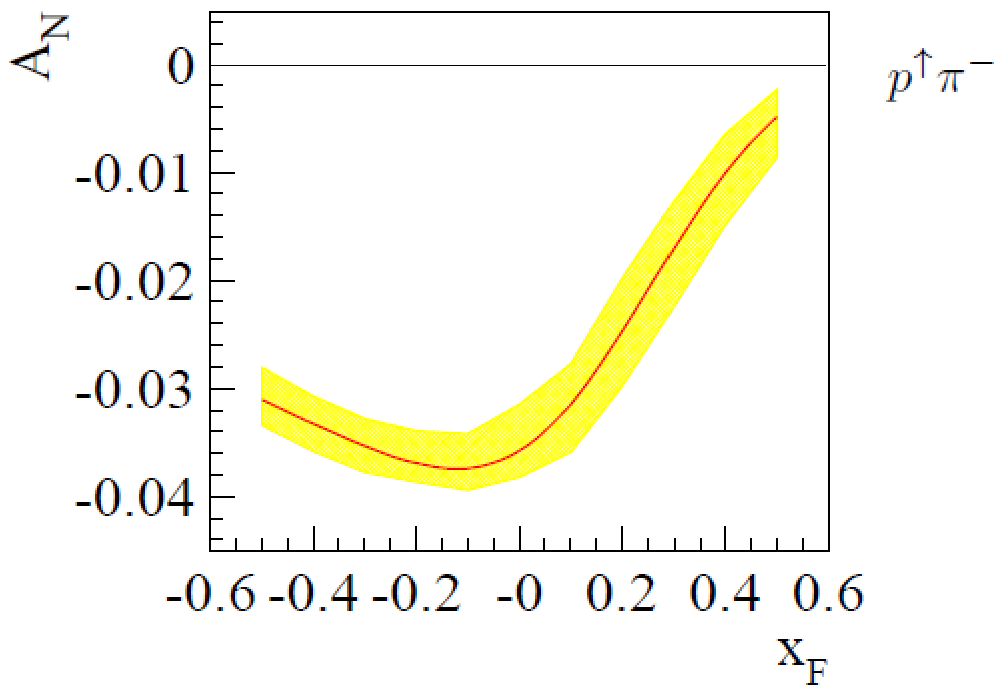
COMPASS SIDIS data



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

EIKV phenomenology

Predicition for COMPASS Drell-Yan



Echevarria, Idilbi, Kang, Vitev Arxiv: 1401.5078

Extraction of transversity & Collins functions

- Azimuthal asymmetry in polarized SIDIS

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, \boldsymbol{k}_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

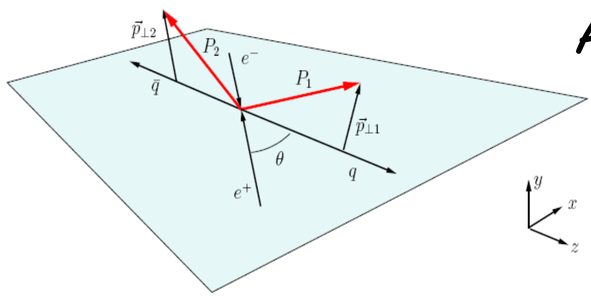
Transversity — $h_{1q}(x, \boldsymbol{k}_\perp)$

Collins function — $\Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2 \frac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\, \sin(\phi+\phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

Extraction of transversity & Collins functions

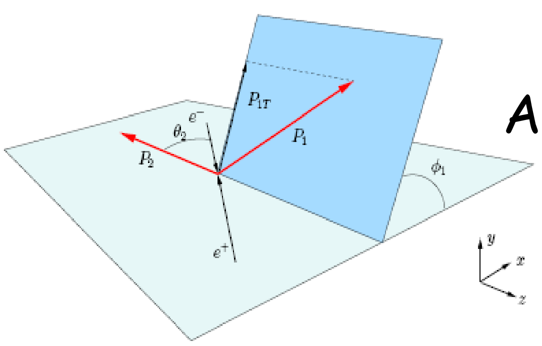
- $e^+e^- \to h_1\, h_2\, X$ BELLE Data



A_{12} asymmetry → **Thrust axis method**

$$A(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{8}\frac{\sin^2\theta}{1+\cos^2\theta}\cos(\varphi_1+\varphi_2)\,\frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$



A_0 asymmetry → **Hadronic plane method**

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi}\frac{z_1\, z_2}{z_1^2+z_2^2}\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\cos(2\,\phi_1)\,\frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Extraction of transversity & Collins functions

- To avoid acceptance effects the BELLE Collaboration considered ratio of different combinations of hadron pairs:

$$\frac{\text{Unlike-sign } (\pi^+\pi^- + \pi^-\pi^+)}{\text{Like-sign } (\pi^+\pi^+ + \pi^-\pi^-)} \rightarrow A^{UL} \text{ asymmetry}$$

$$\frac{\text{Unlike-sign } (\pi^+\pi^- + \pi^-\pi^+)}{\text{Charged } (\pi^+\pi^+ + \pi^-\pi^- + \pi^+\pi^- + \pi^-\pi^+)} \rightarrow A^{UC} \text{ asymmetry}$$

- A_{12}^{UL} A_{12}^{UC} A_0^{UL} A_0^{UC}

Parametrizations

➢Gaussian parametrization of the unpolarized PDF & FF:

- $$f_{q/p}(x, k_\perp) \; = \; f_{q/p}(x) \, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

- $$D_{h/q}(z, p_\perp) \; = \; D_{h/q}(z) \, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

[*] $\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2 \qquad \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Parametrizations

➢Parametrization of Transversity function:

✎ $$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x) + \Delta q(x)\right]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

Unpolarized PDF

Helicity PDF

$$\mathcal{N}_q^T(x) = N_q^T\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

N_q^T, α, β free parameters

Parametrizations

➢Parametrization of the Collins function:

✎ $$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$
- $h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}$

Unpolarized FF

N_q^C, γ, δ, M_h free parameters

✔Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) \le 2\, D_{\pi/q}(z,k_\perp)$$

✔Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)	DGLAP evolution
$\chi^2_{\mathrm{tot}} = 255.8$	$\chi^2_{\mathrm{tot}} = 315.6$
$\chi^2_{\mathrm{d.o.f}} = 1.02$	$\chi^2_{\mathrm{d.o.f}} = 1.26$

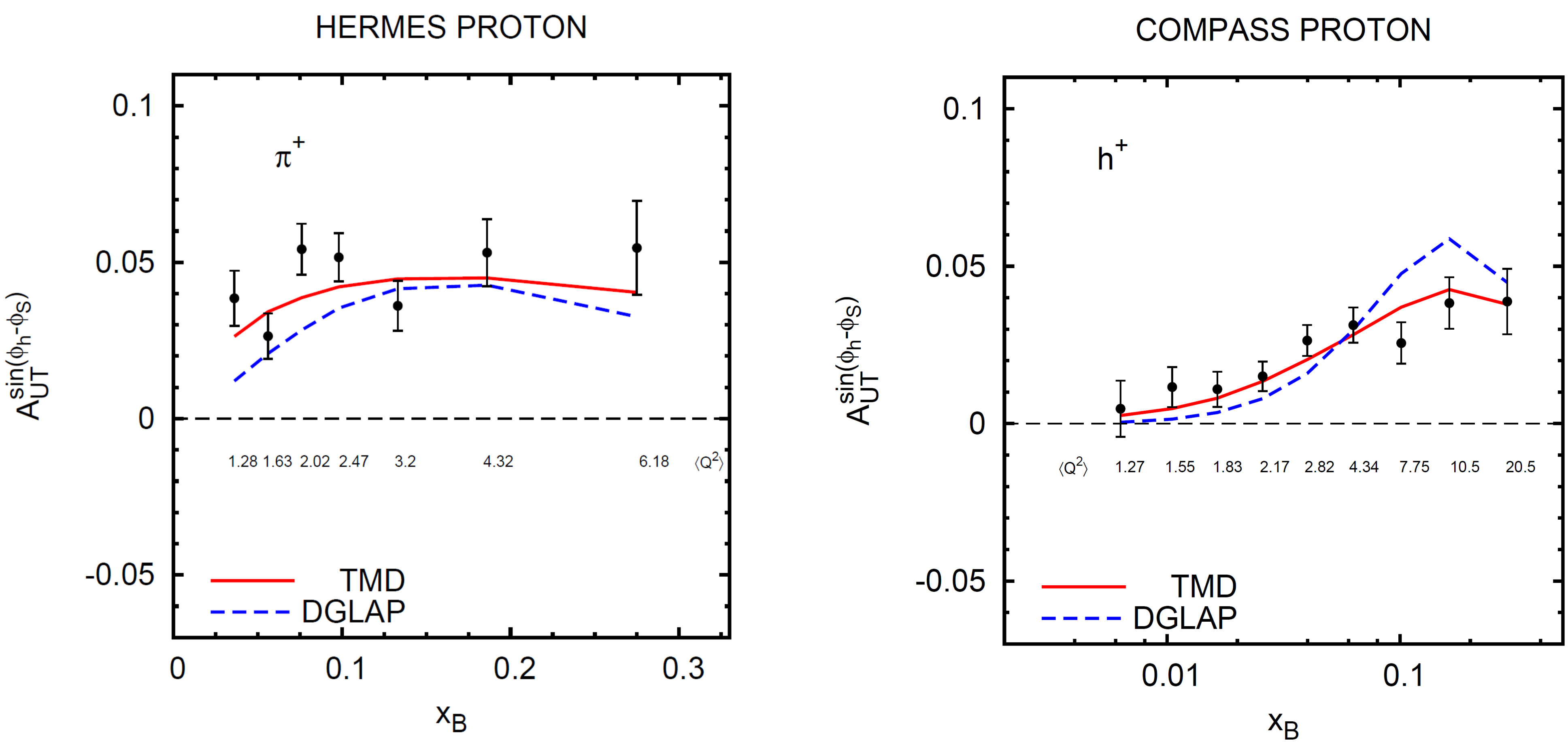
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

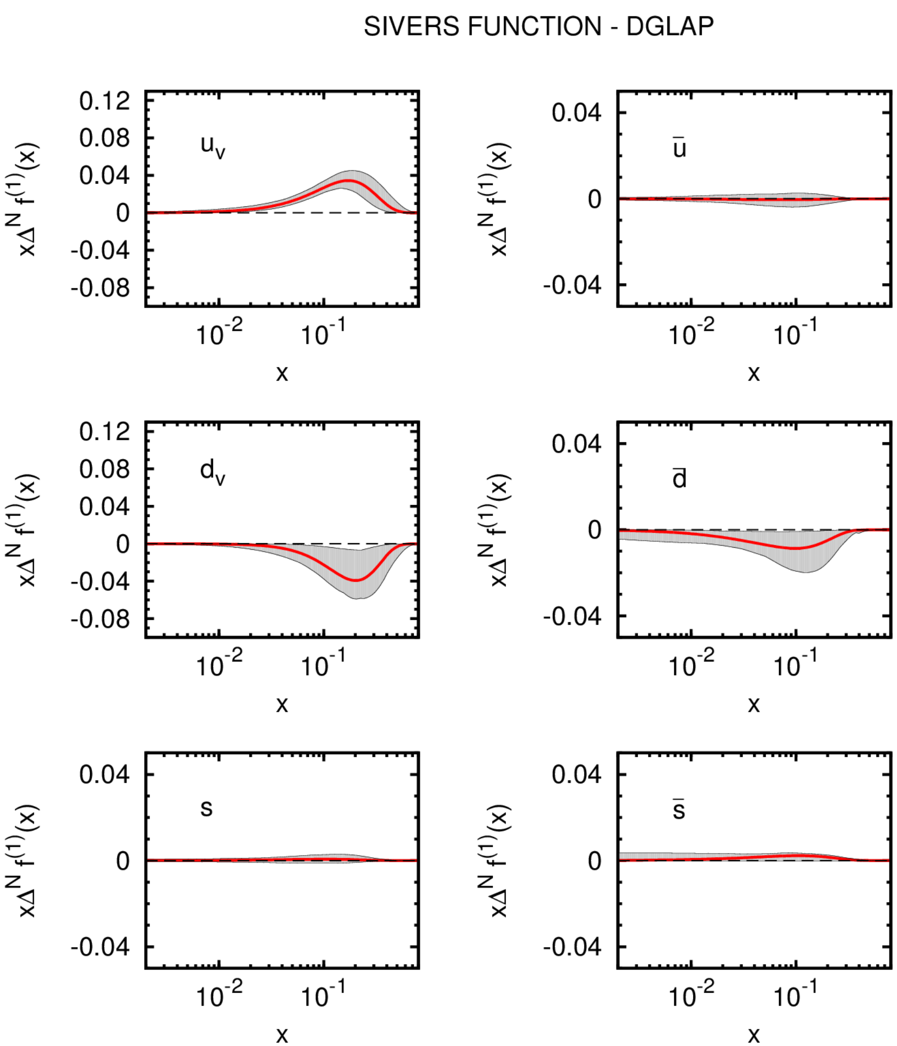
11 free parameters, 261 points

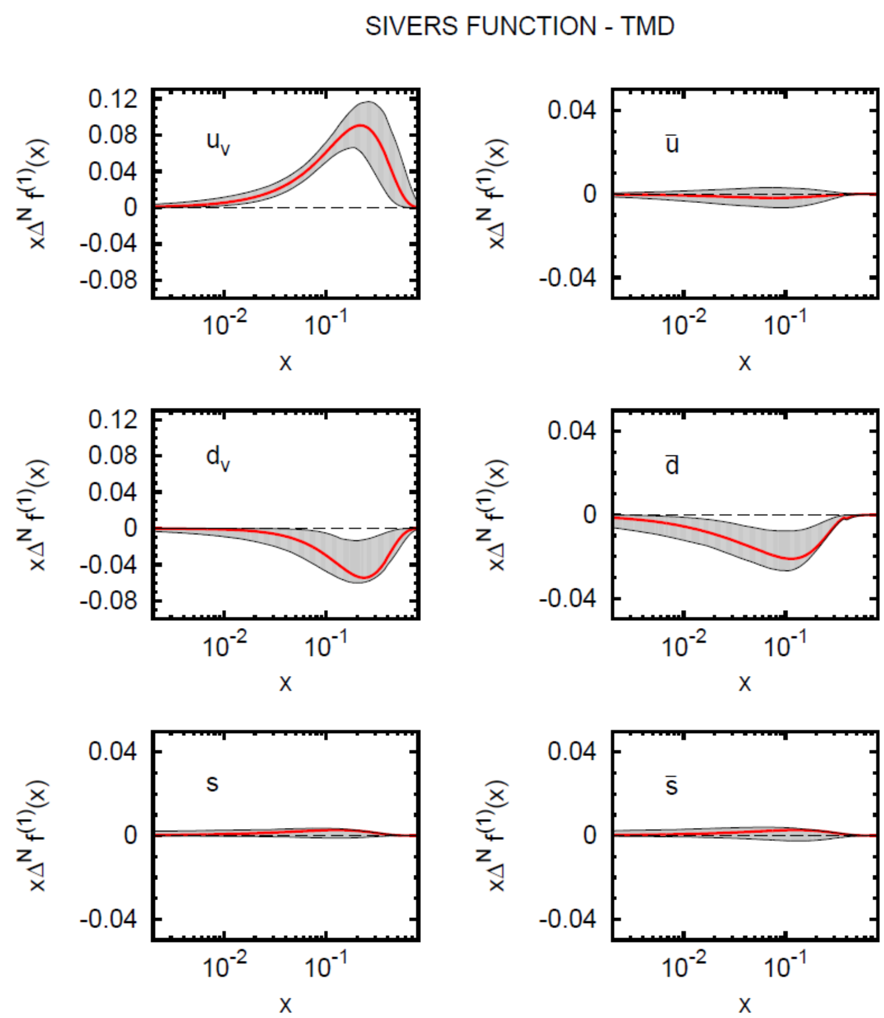
	TMD Evolution (Exact)		DGLAP Evolution
	$\chi^2_{tot} = 255.8$		$\chi^2_{tot} = 315.6$
	$\chi^2_{d.o.f} = 1.02$		$\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$	7 points	$\chi^2_x = 27.5$
	$\chi^2_z = 4.3$		$\chi^2_z = 8.6$
	$\chi^2_{P_T} = 9.1$		$\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$	9 points	$\chi^2_x = 29.2$
	$\chi^2_z = 17.8$		$\chi^2_z = 16.6$
	$\chi^2_{P_T} = 12.4$		$\chi^2_{P_T} = 11.8$

Fit of HERMES and COMPASS SIDIS data

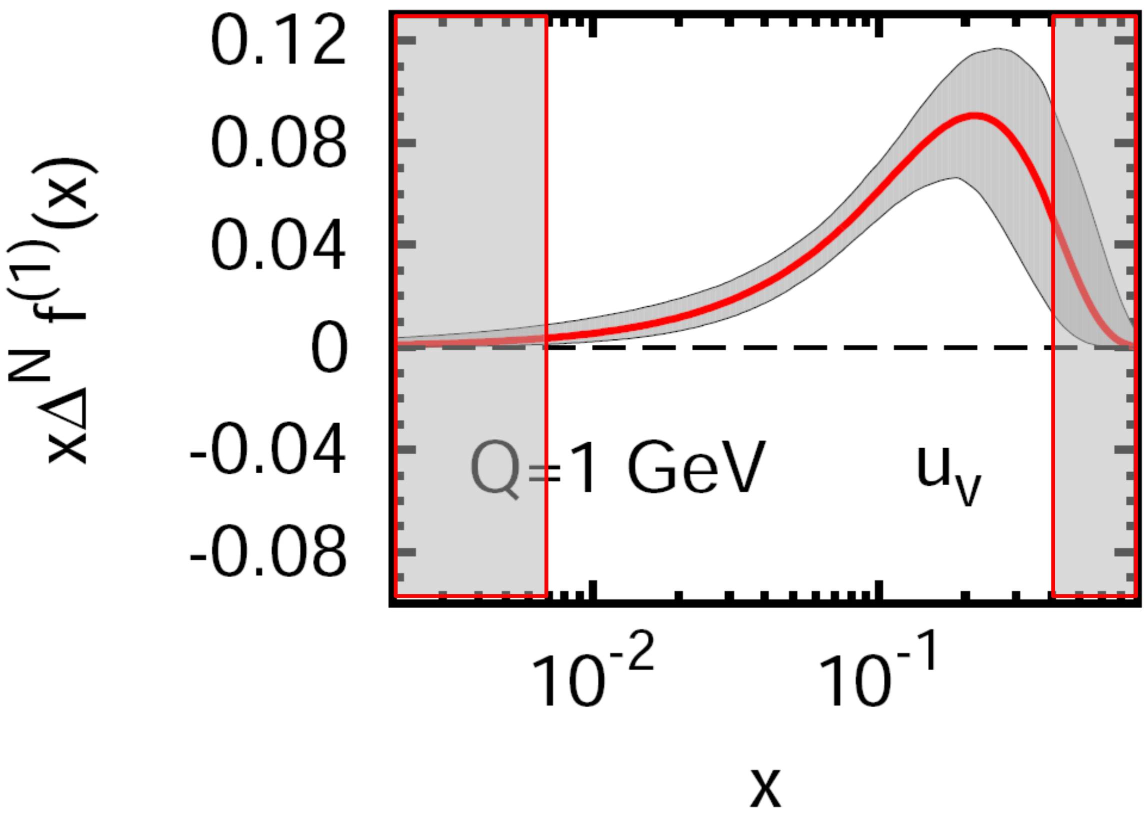


Sivers functions





Sivers functions



Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) = 2\mathcal{N}_q(x) h(k_\perp) \widehat{f}_{q/p}(x, k_\perp; Q)$$
$$= 2\mathcal{N}_q(x) f_{q/p}(x; Q) \sqrt{2e}\,\frac{k_\perp}{M_1}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF (DGLAP) → $f_{q/p}(x; Q)$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

Collins TMD evolution of the Sivers function (PRD85,2012)

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (44)$$

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}^{\text{Sivers}}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b) \times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ -g^{\text{Sivers}}_{f/P}(x, b_T) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}. \quad (47)$$

CSS formalism

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part

Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

Pdfs convoluted with the Wilson Coefficients

$$[C_{ji}\otimes f_i](x,\mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z,\alpha_s(\mu)) f_i(x/z,\mu)$$

$$C_{ji}(z,\alpha(\mu)) \quad = \quad \delta_{ij}\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n C_{ij}^{(n)}(z)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2 dy dq_T^2} = \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part　　　　Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

Sudakov factor

$$S_j(b_T,Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$A_j(\alpha(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_j^{(n)}$$

$$B_j(\alpha(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_j^{(n)}$$

Leading Log (LL)	:	$A^{(1)}$;
Next to LL (NLL)	:	$A^{(2)}, B^{(1)}, C^{(1)}$;
Next to NLL (NNLL)	:	$A^{(3)}, B^{(2)}, C^{(2)}$;
Fixed order $\alpha_s(FXO)$	:	$A^{(1)}, B^{(1)}, C^{(1)}$;

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

CSS formalism

Evolution equations:

$$\frac{\partial W_j(x_1, x_2, b_T, Q)}{\partial \ln(Q^2)} = \Big[K(b_T\mu) + G(Q/\mu)\Big] W_j(x_1, x_2, b_T, Q)$$

$$\frac{dK(b_T\mu, \alpha_s(\mu)}{d\mu} = -\gamma_K(\alpha_s(\mu))$$

$$\frac{dG(Q/\mu, \alpha_s(\mu)}{d\mu} = +\gamma_K(\alpha_s(\mu))$$

$$\frac{\partial W_j(x_1, x_2, b_T, Q)}{\partial \ln(Q^2)} = \left\{ -\int_{C_1^2/b_T^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left[A_j(\alpha_s(\kappa)) \ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa)) \right] \right\} W_j(x_1, x_2, b_T, Q)$$

$\rightarrow S_j(b_T, Q)$

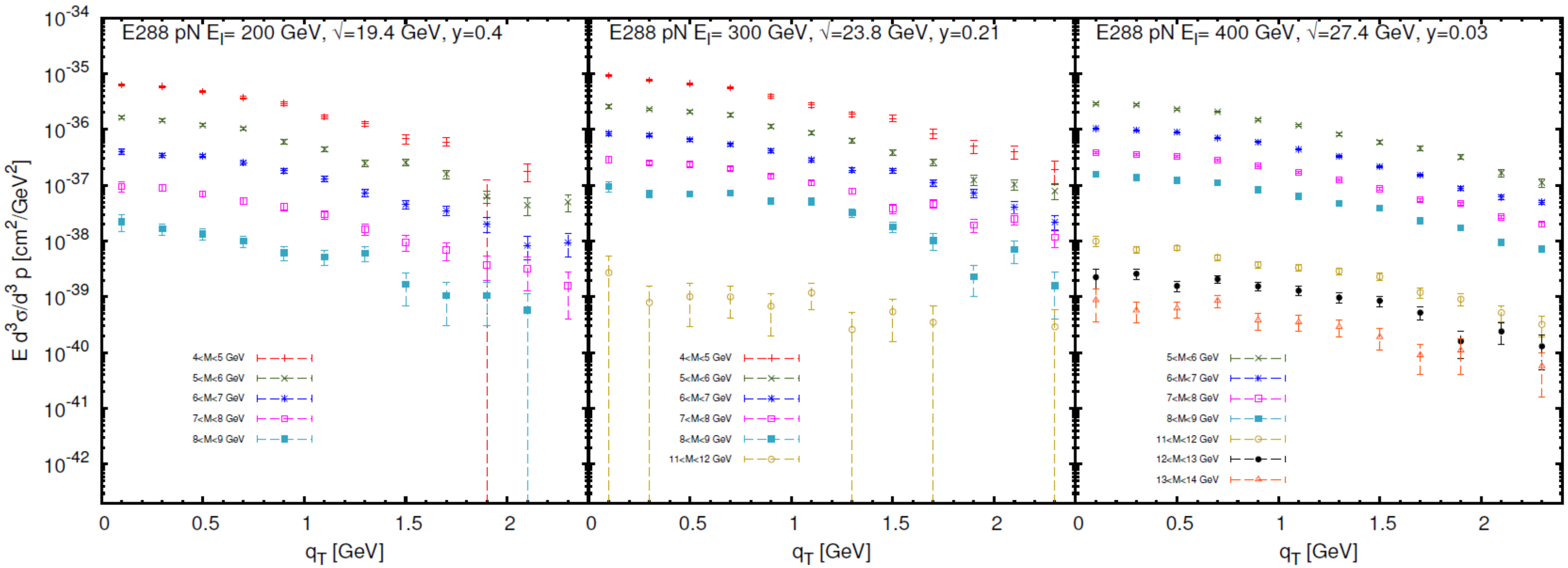
$$W_j(x_1, x_2, b_T, Q) = \exp\left[S_j(b_T, Q)\right] W_j(x_1, x_2, b_T, C_1/b_T)$$

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

Drell-Yan phenomenology

➢ Low energy data

	E288 200	E288 300	E288 400	E605	R209
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	800 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p Cu	p p
Q range	4-9 GeV	4-9; 11-12 GeV	5-9; 11-14 GeV	4-9; 10.5-18 GeV	5-8; 11-25 GeV
Other kin. var	y=0.4	y=0.21	y=0.03	$-0.1 < x_F < 0.2$	
Observable	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$d\sigma/dq_T^2$



➢ The P_T distribution seems to be Gaussian....

Drell-Yan phenomenology

- Are data distributed as a Gaussian? Do data scale as 1/M2+DGLAP+KIN

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$

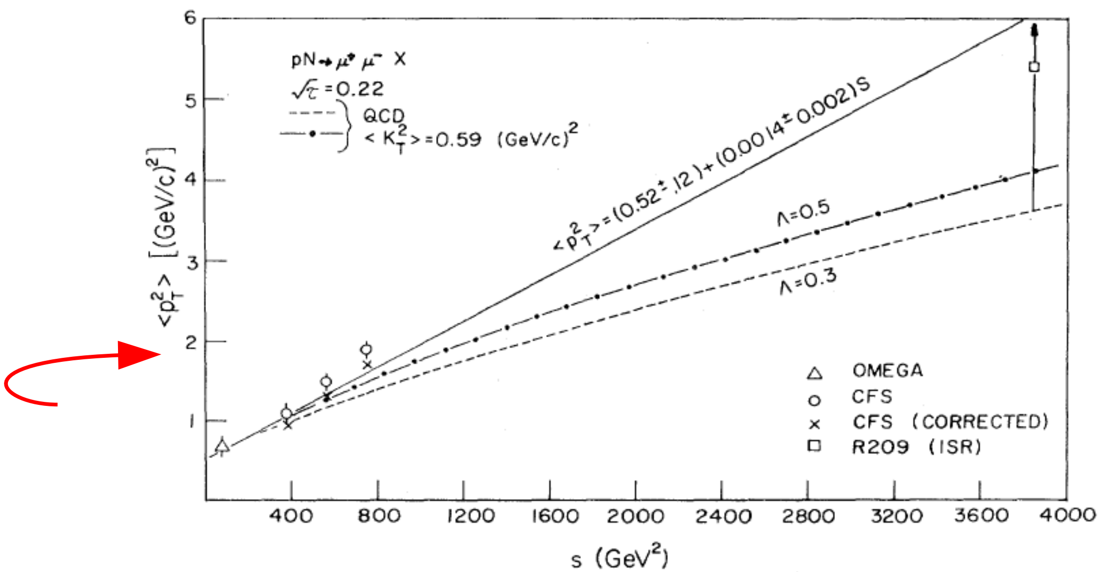


FIG. 3. $\langle p_T{}^2\rangle$ vs s for dimuons produced in p-nucleon interactions. The solid curve is the linear fit to the data. The dashed and dot-dash curves are the predictions of first-order QCD using the Altarelli *et al.* prescription for different values of Λ.

$$\langle K_\perp^2\rangle = \alpha_s(Q^2)\sqrt{S}\, f(\tau, \alpha_s(Q^2)) + \dots$$

Cox and Malhotra,Phys. Rev. D29(1984)

TMD evolution

TMD in the b space:

$$\tilde{F}(x, b_T, Q, \zeta_F) = \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b)$$

$$\exp\left\{ \ln\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\kappa}\right) \gamma_K(\kappa) \right\}$$

$$\exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\}$$

Related to the evolution in the cut off parameter of the TMD:

$$\frac{\partial \ln \tilde{F}(x, \mathbf{b}_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\mathbf{b}_T; \mu)$$

However.... at first order in the strong coupling constant:

$$\tilde{K}(\mu, b_T) = -\frac{\alpha_s(\mu)}{\pi} \ln(\mu^2 b_T^2 / C_1^2) \quad \text{if } \mu_b = C_1/b_* \quad \tilde{K}(b_*, \mu_b) = 0$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

TMD evolution

TMD in the b space:

$$\tilde{F}(x, b_T, Q, \zeta_F) = \sum_j \int_x^1 \frac{dy}{y} \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) f_j(y, \mu_b)$$

$$\exp\left\{ \ln\left(\frac{\sqrt{\zeta_F}}{\mu_b}\right) \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \gamma_F(\kappa; 1) - \ln\left(\frac{\sqrt{\zeta_F}}{\kappa}\right) \gamma_K(\kappa) \right\}$$

$$\exp\left\{ -g_P(x, b_T) - g_K(b_T) \ln\left(\frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}}\right) \right\}$$

Second part of the part of the Sudakov form factor, notice that depends on ζ_F

at order α_s:

$$\gamma_F(\mu; \zeta_F/\mu^2) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln(\frac{\zeta_F}{\mu^2}) \right)$$

$$\gamma_K(\mu) = 2C_F \frac{\alpha_s(\mu)}{\pi}$$

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

TMD evolution

Collins suggest that: $\zeta_F = Q^2$ $\quad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}$ $\quad \mu_b = C_1/b_*$

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \exp[S_{RAC}(b_*, Q^2)] F_{NP}(x, b_T, Q)$$

It can be show easily that at first order in the strong coupling constant:

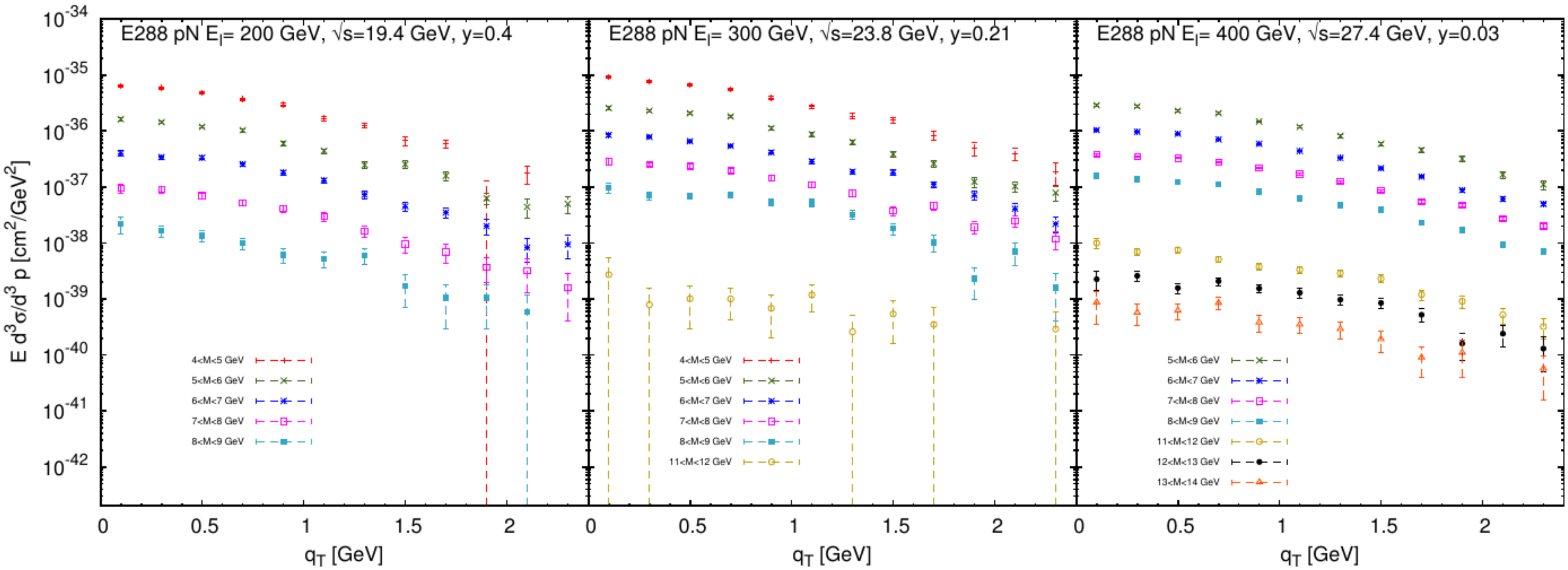
$$S_{RAC}(b_T, Q^2) = C_F \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \frac{\alpha_s(\kappa)}{\pi} \left[\frac{3}{2} - \ln\left(\frac{Q^2}{\kappa^2} \right) \right] \equiv \frac{1}{2} S_{CSS}(b_T, Q^2)$$

TMD evolution is more general than CSS which is a particular case of the TMD one
Previous studies performed with CSS are still valid!

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

Drell-Yan phenomenology

➢Low energy data example: FERMILAB E288 at 3 different energies



➢The P_T distributions seem to be Gaussian....

TMD evolution modelling
Rogers & Aybat

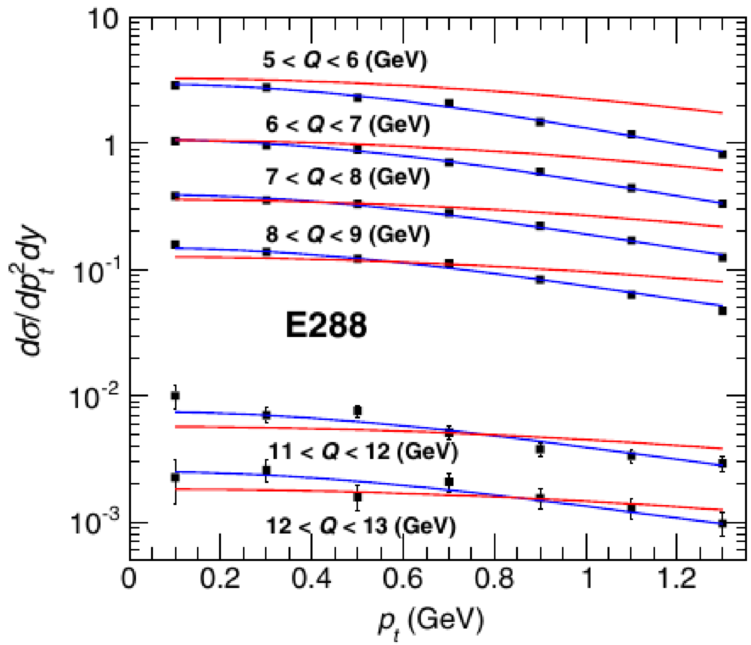
$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\tilde{F}(x,b_T,Q_0,Q_0^2) = f(x,Q_0)\exp\left[-\frac{\langle k_\perp^2\rangle}{4}b_T^2\right]$$

$$g_K(b_T) = \frac{1}{2}g_2 b_T^2$$ g_2 from DY

Average transverse momentum from SIDIS (HERMES)

Red line, prediction based on the above formula with the parameter as in Rogers,Aybat 2011



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
Yuan-Sun phenomenology

- Yuan-Sun explanation: the Sudakov form factor must be modified taking into account that low energy data are almost in a non perturbative region.

$$S_{\mathrm{Sud}} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

$$\tilde{F}_{UU}(Q;b) = e^{-S_{\mathrm{sud}}(Q,Q_0,b)} \tilde{F}_{UU}(Q_0;b),$$

$$\tilde{F}_{UU}(Q_0,b) = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

- Notice that there is not any b* and therefore any b_{max}.

See for a interesting discussion Section VII of Aidala, Field, Gamberg, Rogers, Phys.Rev. D89 (2014) 094002

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.

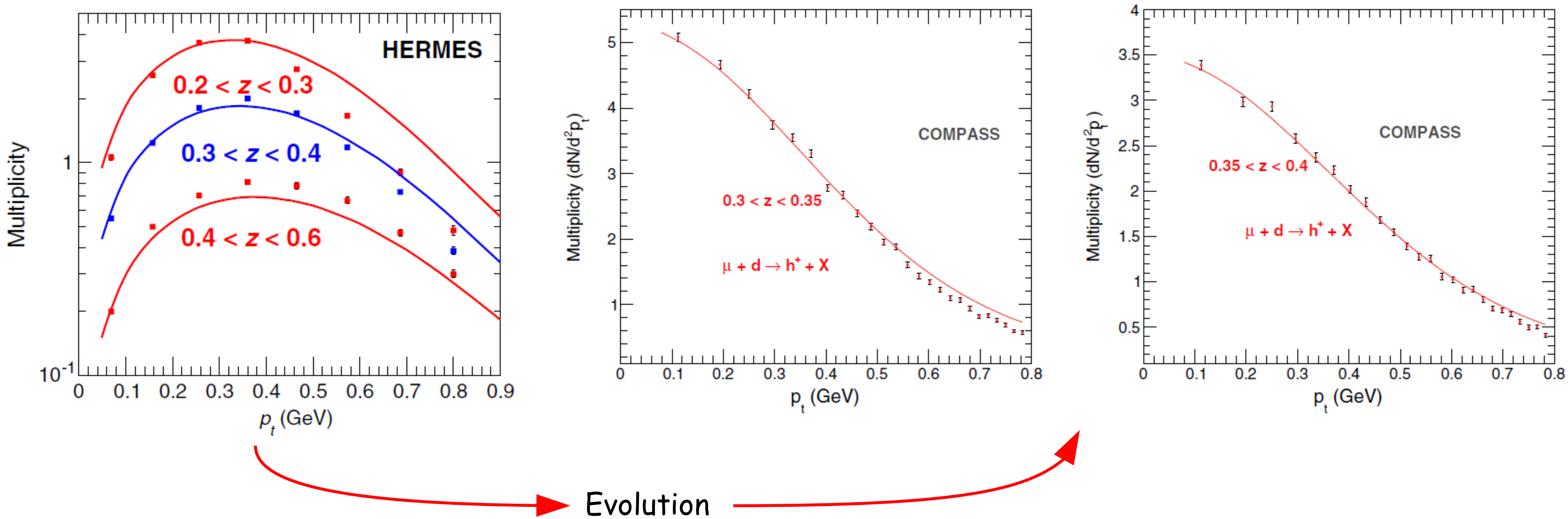
$$\tilde{F}_{UU}(Q_0, b) \quad = \sum_q e_q^2 f_q(x_B, \mu = Q_0) D_q(z_h, \mu = Q_0) e^{-g_0 b^2 - g_h b^2 / z_h^2}$$

$$\tilde{W}_{UU}(Q_0, b) = \sum_q e_q^2 f_q(x, \mu = Q_0) f_{\bar{q}}(x', \mu = Q_0) e^{-g_0 b^2 - g_0 b^2};$$

Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
Yuan-Sun phenomenology

- Gaussian parametrization for the PDF and the fragmentation function at the scale of HERMES.
- Parameters g_0 and g_h as in Schweitzer et al, Phys. ReV. D81,094019 (2010)



Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Alternative TMD evolution
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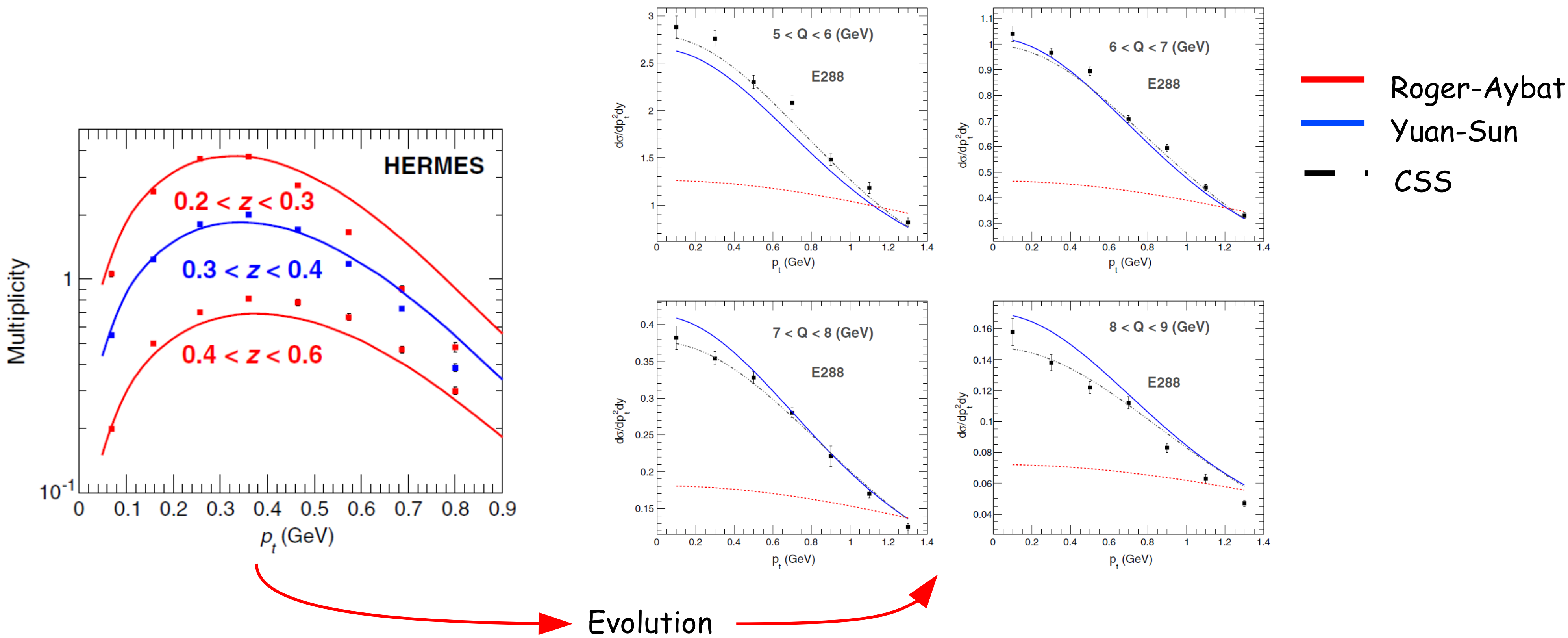


Sun and Yuan, Phys. Rev. D88, 034016 (2013), Phys. Rev. D88, 114012 (2013)

Drell-Yan phenomenology

- Simple phenomenological ansatz

$$f_{q/p}(x, k_\perp) = f(x)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

→ Factorization of longitudinal and transverse degrees of freedom; Gaussian distribution of transverse momentum

In this way the distribution in P_T is just a guassian! →

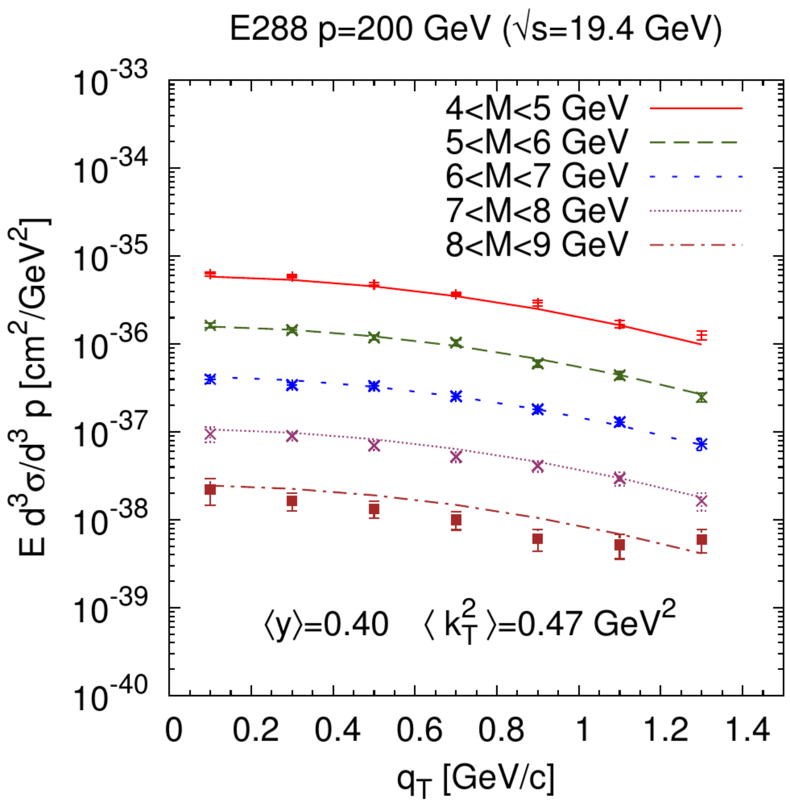
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2}\sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$

- Where for pp or pN scattering we just have: $\langle P_T^2\rangle = 2\langle k_\perp^2\rangle$

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1)\bar{f}_{q/h_2}(x_2)\frac{\exp(-P_T^2/\langle P_T^2\rangle)}{\pi\langle P_T^2\rangle}$$



Nice!

Further information: the M^2 dependence is described by model and it is given by the interplay between $1/M^2$ born cross section +DGLAP+Kinematics

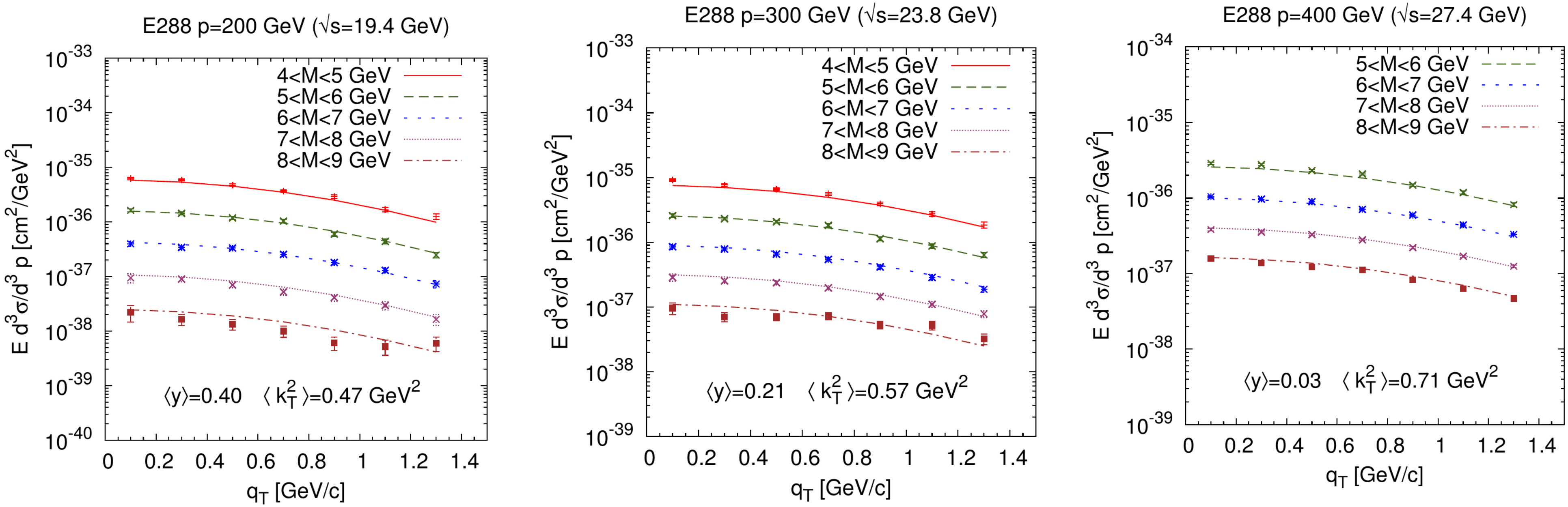
Is the width of the gaussian a measure of transverse momentum?

The model does not answer directly to this question.

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

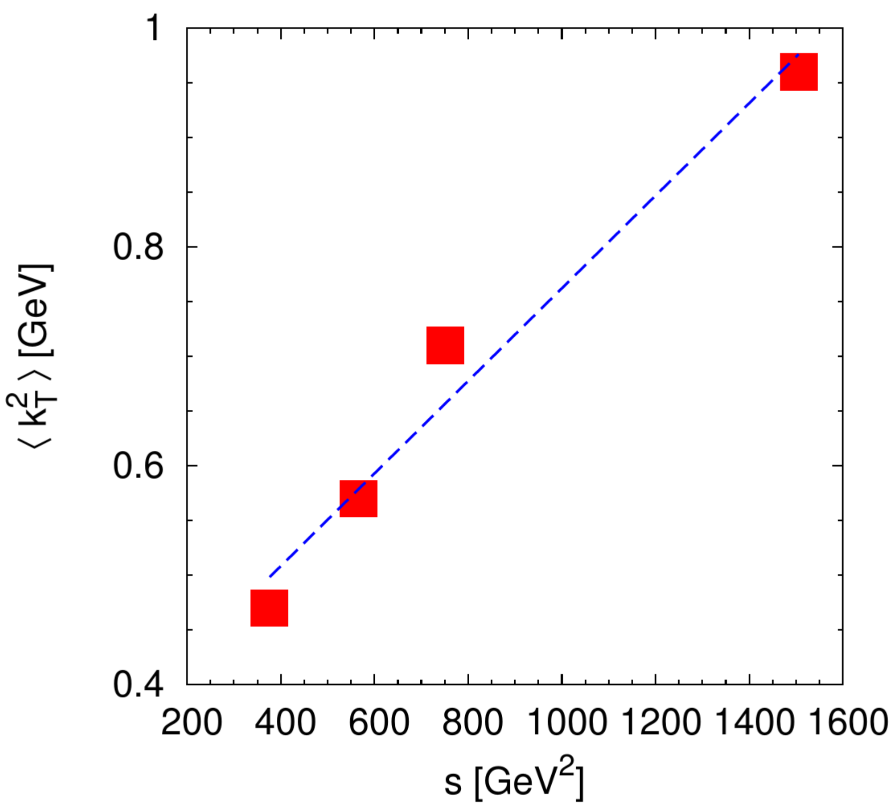


- Each data set is gaussian but with a different width

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



- QCD prediction?

$$\langle K_\perp^2 \rangle^{pert.} = \alpha_s(Q^2)\, s\, f(\tau, \alpha_s(Q^2)) + \dots$$

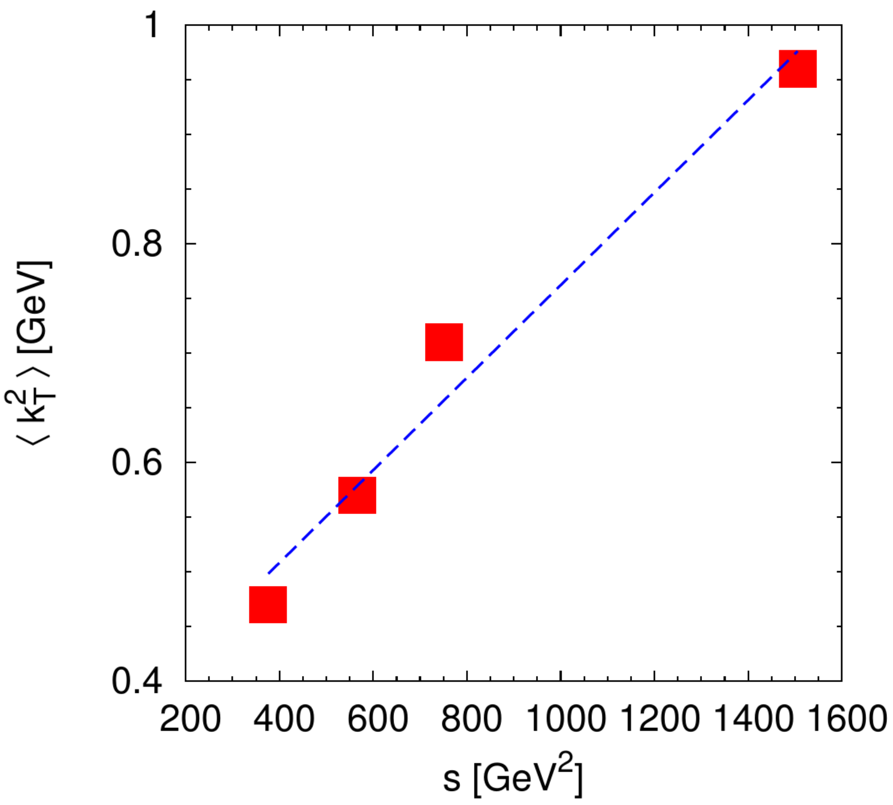
- Altarelli, Parisi and Petronzio
  Phys.Lett. B76 (1978) 351

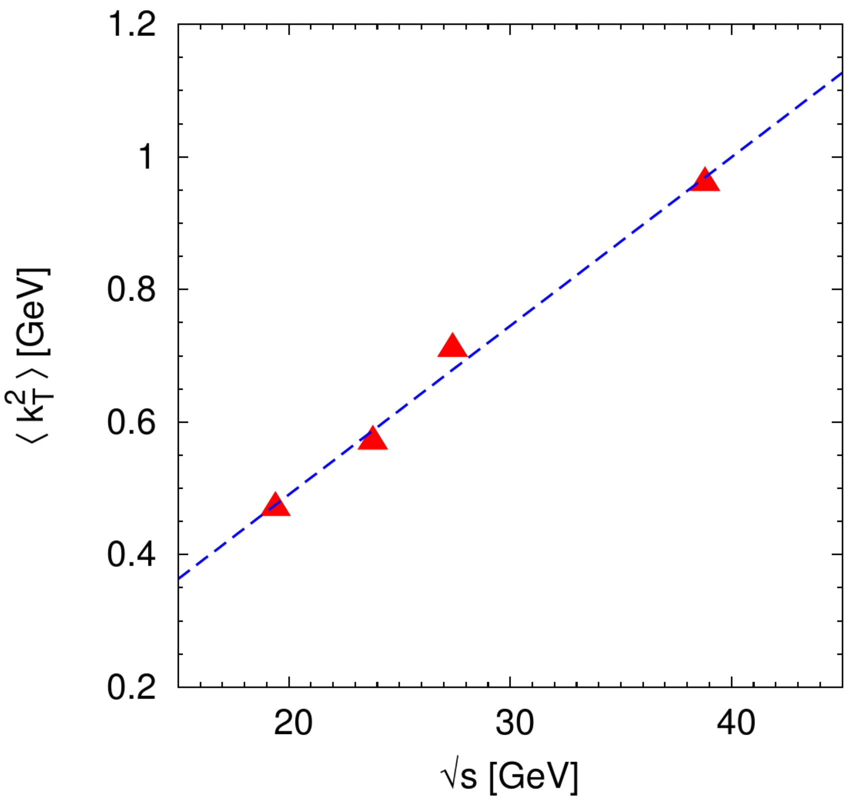
See, for SIDIS, also
Schweitzer, Metz, Teckentrup
Phys.Rev. D81 (2010) 094019

Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$

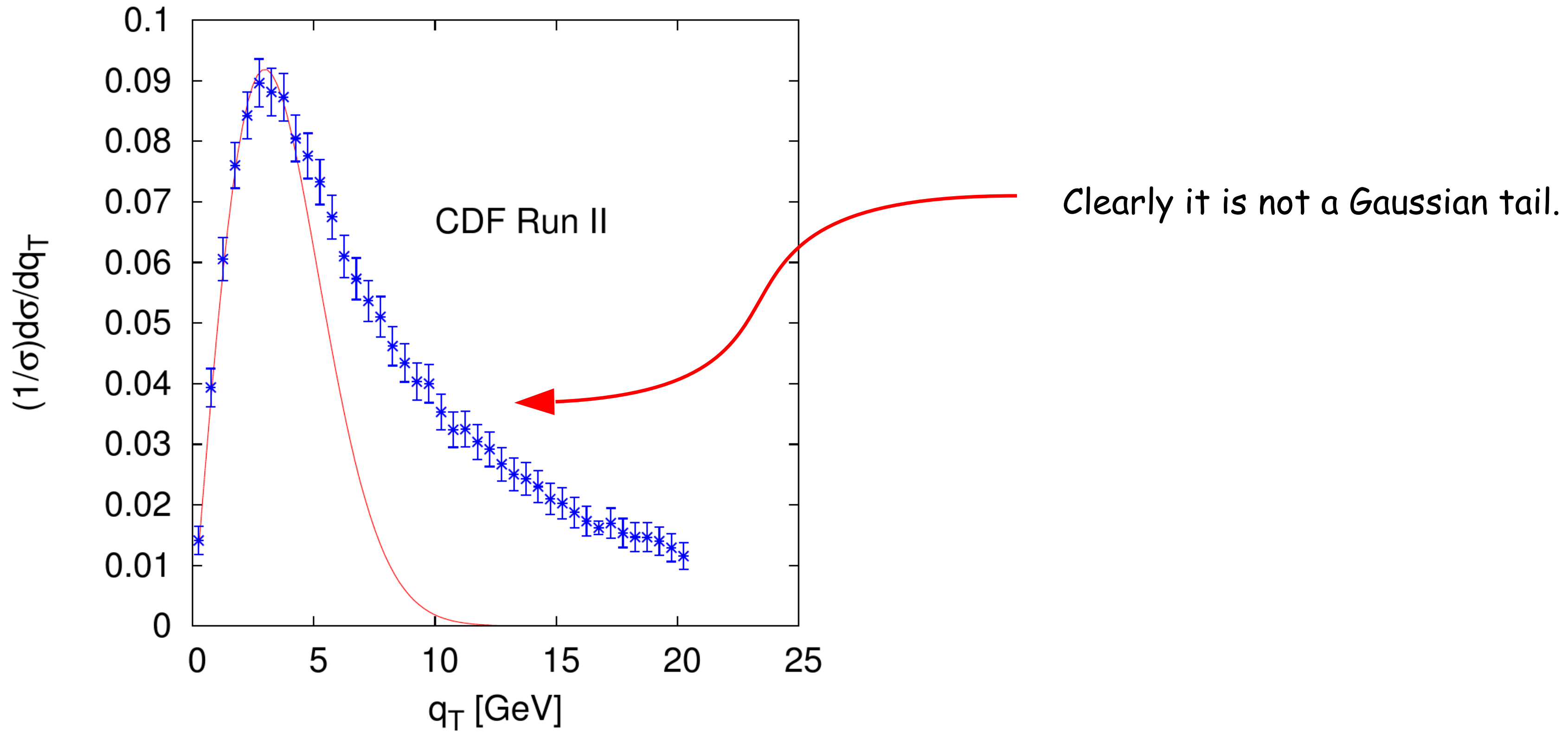




Drell-Yan phenomenology

- Are data gaussian distributed?

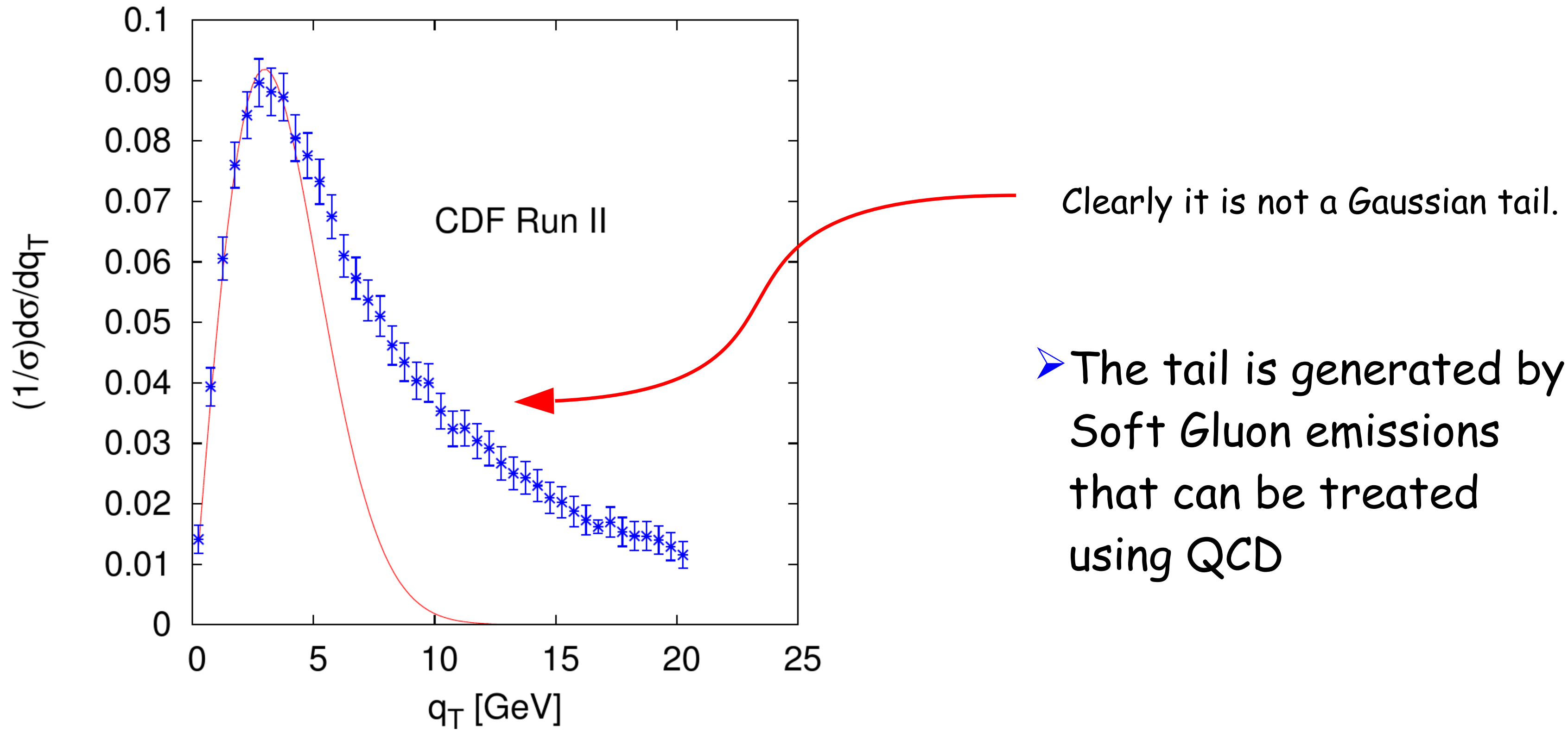
$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



Drell-Yan phenomenology

- Are data gaussian distributed?

$$\frac{d\sigma}{dP_T^2} \propto \frac{\alpha_{em}}{M^2} \sum_q f_{q/h_1}(x_1) \bar{f}_{q/h_2}(x_2) \frac{\exp(-P_T^2/\langle P_T^2 \rangle)}{\pi \langle P_T^2 \rangle}$$



- The tail is generated by Soft Gluon emissions that can be treated using QCD

$$\frac{\partial T_{q,F}(x,x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu) + \frac{N_c}{2}\left[\frac{(1+z)T_{q,F}(\xi,x,\mu)-(1+z^2)T_{q,F}(\xi,\xi,\mu)}{1-z} + T_{\Delta q,F}(x,\xi,\mu)\right]\Big\}, \quad (3)$$

where $z = x/\xi$ and $P_{qq}(z)$ is the splitting kernel for unpolarized quark distribution function given by

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right], \quad (4)$$

and the quark-gluon correlation function $T_{\Delta q,F}(x_1,x_2,\mu)$ is given by [18]

$$T_{\Delta q,F}(x_1,x_2) = \int \frac{dy_1^- dy_2^-}{4\pi}\, e^{ix_1P^+y_1^-}\, e^{i(x_2-x_1)P^+y_2^-}\,\langle P,s_T|\bar{\psi}_q(0)\,\gamma^+\gamma^5\left[i\,s_T^\alpha\,F_\alpha{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle. \quad (5)$$

ArXiv:1205.1019