TMDevolution: an overview

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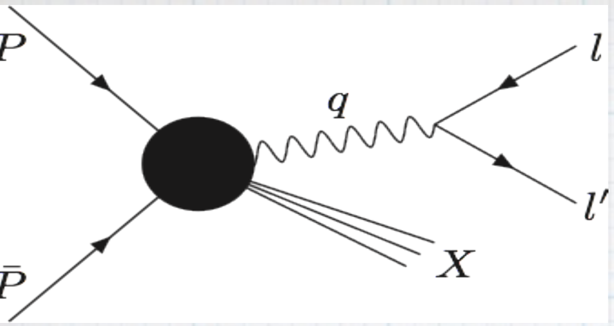


$$q^2 = Q^2 \gg q_T^2$$

A path for TMD extraction

Multi-differential cross-sections involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem are required.

Status: Currently only Drell-Yan (Photon, Vector Boson, Higgs..), SIDIS, ee->2h processes are known to admit a proper factorization theorem (Collins '11, Echevarría-Idilbi-S. '12):
Q=M>>max(qT,hadronization scale, e.g. 1 GeV)!

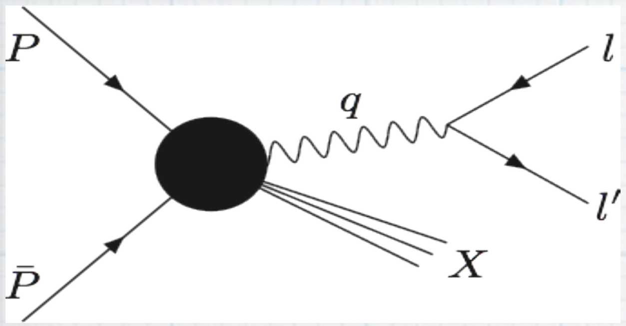
The evolution of TMDs allows compare experimental results at different M

Question: Can we check the UNIVERSALITY of TMDs?

In principle many different experiments can provide data; in practice..

DY: Tevatron data, LHC is starting now..
SIDIS:
1) Current experimental results only for Q=1-2 GeV!!! (Hermes, Compass). Is leading twist factorization still a good approximation? What would be the ideal photon momentum? Do we have to wait future colliders (EIC)?
2) Are fragmentation functions theoretically and experimentally sufficiently known?

DY, SIDIS, ee-> 2j, TMD's and energy scales



$$q^2 = Q^2 \gg q_T^2$$ Q=M=dilepton invariant mass

$q_T^2 \gg \Lambda_{QCD}^2$ $\Rightarrow$ $\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n;\mu^2)\, f_{\bar{n}}(x_{\bar{n}};\mu^2)$

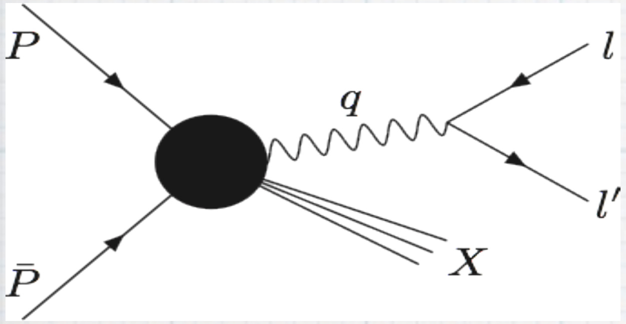
Example: Vector boson (Tevatron, LHC) and Higgs production at LHC (up to a certain precision, qT>5-10 GeV..),
Some DIS data from HERA

$q_T^2 \sim \Lambda_{QCD}^2$ $\Rightarrow$ $\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$

Example: DY Tevatron experiments (E288: Q=4-15GeV, qt<2 GeV)
no (usable) DIS data... waiting for EIC..

Issues: Can we understand Compass DY-DIS results in this formalism (Q=1-2 GeV)?
(Hermes, Compass, JLAB) $Q^2 \not\gg \Lambda_{QCD}^2 \quad q_T^2 \sim \Lambda_{QCD}^2$

TMD's factorization: principles and formalism



$$q^2 = Q^2 \gg q_T^2$$

Q=M=dilepton invariant mass

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$q_T^2 \gg \Lambda_{QCD}^2 \Rightarrow \tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n; \mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \mu^2)$

All coefficients are extracted matching effective field theories. During the matching the IR parts have to be regulated consistently above and below the matching scales

Practical issue: what's the best way to write the TMDs to recover the perturbative limit?

Important issue: The estimate of theoretical errors (convergence of QCD)

Evolution kernel for TMD's

TMD-PDF

$$\frac{d}{d\ln\zeta_F}\ln\tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_F,\mu^2) = -D(b_T;\mu^2)\,,$$

TMD-FF

$$\frac{d}{d\ln\zeta_D}\ln\tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_D,\mu^2) = -D(b_T;\mu^2)\,.$$

$$\frac{dD}{d\ln\mu} = \Gamma_{cusp}$$

$$\tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_{F,f},\mu_f^2) = \tilde{F}^{[\Gamma]}_{f/N}(x,\mathbf{b}_\perp,S;\zeta_{F,i},\mu_i^2)\,\tilde{R}\left(b_T;\zeta_{F,i},\mu_i^2,\zeta_{F,f},\mu_f^2\right)\,,$$

$$\tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_{D,f},\mu_f^2) = \tilde{D}^{[\Gamma]}_{h/f}(z,\mathbf{b}_\perp,S_h;\zeta_{D,i},\mu_i^2)\,\tilde{R}\left(b_T;\zeta_{D,i},\mu_i^2,\zeta_{D,f},\mu_f^2\right)\,,$$

$$\boxed{\tilde{R}(b;\zeta_i,\mu_i^2,\zeta_f,\mu_f^2) = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\,\gamma\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_f}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_f}{\zeta_i}\right)^{-D(b_T;\mu_i)}}\,,$$

We evolve from one M to another

Consistently the A.D. of the TMD is the opposite of the one of the hard coefficient

$$\gamma_H = -\gamma_F\left(\alpha_s(\mu),\ln\frac{\zeta_F}{\mu^2}\right) - \gamma_D\left(\alpha_s(\mu),\ln\frac{\zeta_D}{\mu^2}\right)$$

$$\gamma_{F,D}\left(\alpha_s(\mu),\ln\frac{\zeta_{F,D}}{\mu^2}\right) = -\Gamma_{\rm cusp}(\alpha_s(\mu))\ln\frac{\zeta_{F,D}}{\mu^2} - \gamma^V(\alpha_s(\mu))$$

D-resummation

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s) \qquad D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

LL NLL NNLL

$$\begin{aligned}
d_1(L_\perp) &= d_1^{(1)} L_\perp + d_1^{(0)} \\
d_2(L_\perp) &= d_2^{(2)} L_\perp^2 + d_2^{(1)} L_\perp + d_2^{(0)} \\
d_3(L_\perp) &= d_3^{(3)} L_\perp^3 + d_3^{(2)} L_\perp^2 + d_3^{(1)} L_\perp + d_3^{(0)} \\
d_4(L_\perp) &= d_4^{(4)} L_\perp^4 + d_4^{(3)} L_\perp^3 + d_4^{(2)} L_\perp^2 + d_4^{(1)} L_\perp + d_4^{(0)} \\
d_5(L_\perp) &= \ldots
\end{aligned}$$

$$D\left(b;Q_i\right) = D\left(b;\mu_b\right) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{\text{cusp}}; \qquad \mu_b = 2e^{-\gamma_E}/b$$

$$D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln(1-X)$$

$$\alpha_s(\mu_b) = \alpha_s(Q)/(1-X)$$

Landau pole

The perturbative expansion of the D is valid when logs are small

$$\mu \sim q_T \sim 1/b$$

Outside this region two strategies are proposed:

1. D-resummation (Becher, Neubert, Wilhelm; G. Echevarría, Idilbi, Schaefer, S.) $\mu = Q_0 + q_T$
2. Scale fixing (CSS: Collins, Rogers; Qiu, Zhang; BLNY; Boer, Sun, Yuan..) $\mu = 1/b; \quad \mu = 1/b^*$

Finally one gets to the pure non-perturbative part of D.

Is the NP part dominant?

If the answer is yes we are almost lost ..

Construction of unpolarized TMDPDFs

- Take the asymptotic limit (High Q, qT) of <u>each</u> TMDPDF

PDF

$$\tilde{F}_{q/N}(x,b_T;\zeta,\mu)=\left(\frac{\zeta}{\mu_b}\right)^{-D(b_T;\mu)}\sum_j\int_x^1\frac{dz}{z}\tilde{C}^{Q}_{q\leftarrow j}(x/z,b_T;\mu_b,\mu)f_{j/N}(x,\mu)M(x,b,\zeta)$$

OPE to PDF, valid ONLY for qT›› Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.

Status: This formula predicts that one TMDPDF matches onto a sum of PDFs. Currently all analysis of low energy data have fully exploited this up to first order

$$\tilde{C}^{Q}_{q\leftarrow q}=\mathcal{O}(\alpha_s^0)$$
$$\tilde{C}^{Q}_{q\leftarrow g}=\mathcal{O}(\alpha_s^1)$$
$$\tilde{C}^{Q}_{q\leftarrow \bar{q}}=\mathcal{O}(\alpha_s^2)$$
$$\tilde{C}^{Q}_{q\leftarrow q'}=\mathcal{O}(\alpha_s^2)$$

2-loop matching of PDFs deduced from the calculation of the cross section [Firenze (Catani et al.), Zurich (Gehrmann. et al)]. No direct application of the TMD formalism.

Construction of unpolarized TMDPDFs

Scales and Theoretical errors:

- Take the asymptotic limit (High Q, qT) of each TMDPDF

$$\tilde{F}_{q/N}(x, b_T; \boxed{\zeta, \mu}) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \tilde{C}^{Q}_{q\leftarrow j}(x/z, b_T; \mu_b, \mu) f_{j/N}(x, \mu) M(x, b, \zeta)$$

"Mu-b Scale"

$Q^2 \gg q_T^2 \gg \Lambda^2_{QCD}$ Perturbative regime: 3 scales $\zeta,\ \mu,\ \mu_b$

de Florian, Catani, Ferrera, Grazzini, ..
Chiu, Jain, Neill, Rothstein, Vaidya,..

$Q^2 \gg q_T^2 \sim \Lambda^2_{QCD}$ TMD regime: 2 scales ζ, μ

Then $\mu_b = 2/(e^{2\gamma} b)$
defines the TMD scheme/model

Construction of unpolarized TMDs

- Take the asymptotic limit (High Q, qT) of <u>each</u> TMDPDF

2-loop matching of PDFs:
Florence (Catani et al.), Zurich (Gehrmann. et al)

$$\tilde{F}_{q/N}(x, b_T; \zeta, \mu) = \left(\frac{\zeta}{\mu_b}\right)^{-D(b_T;\mu)} \sum_j \int_x^1 \frac{dz}{z} \tilde{C}^{Q}_{q\leftarrow j}(x/z, b_T; \mu_b, \mu) f_{j/N}(x, \mu) M(x, b, \zeta)$$

- Exponentiation of part of the coefficient and complete resummation of the logs in the exponent
(Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)

$$\tilde{C}_{q\leftarrow j}(x, \vec{b}_\perp, \mu) \equiv \exp(h_\Gamma - h_\gamma)\hat{C}_{q\leftarrow j}(x, \vec{b}_\perp, \mu)$$

$$\frac{dh_\Gamma}{d\ln\mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d\ln\mu} = \gamma^V$$

$$h_\Gamma^R(b, \mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma^F_{cusp}(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)} .$$

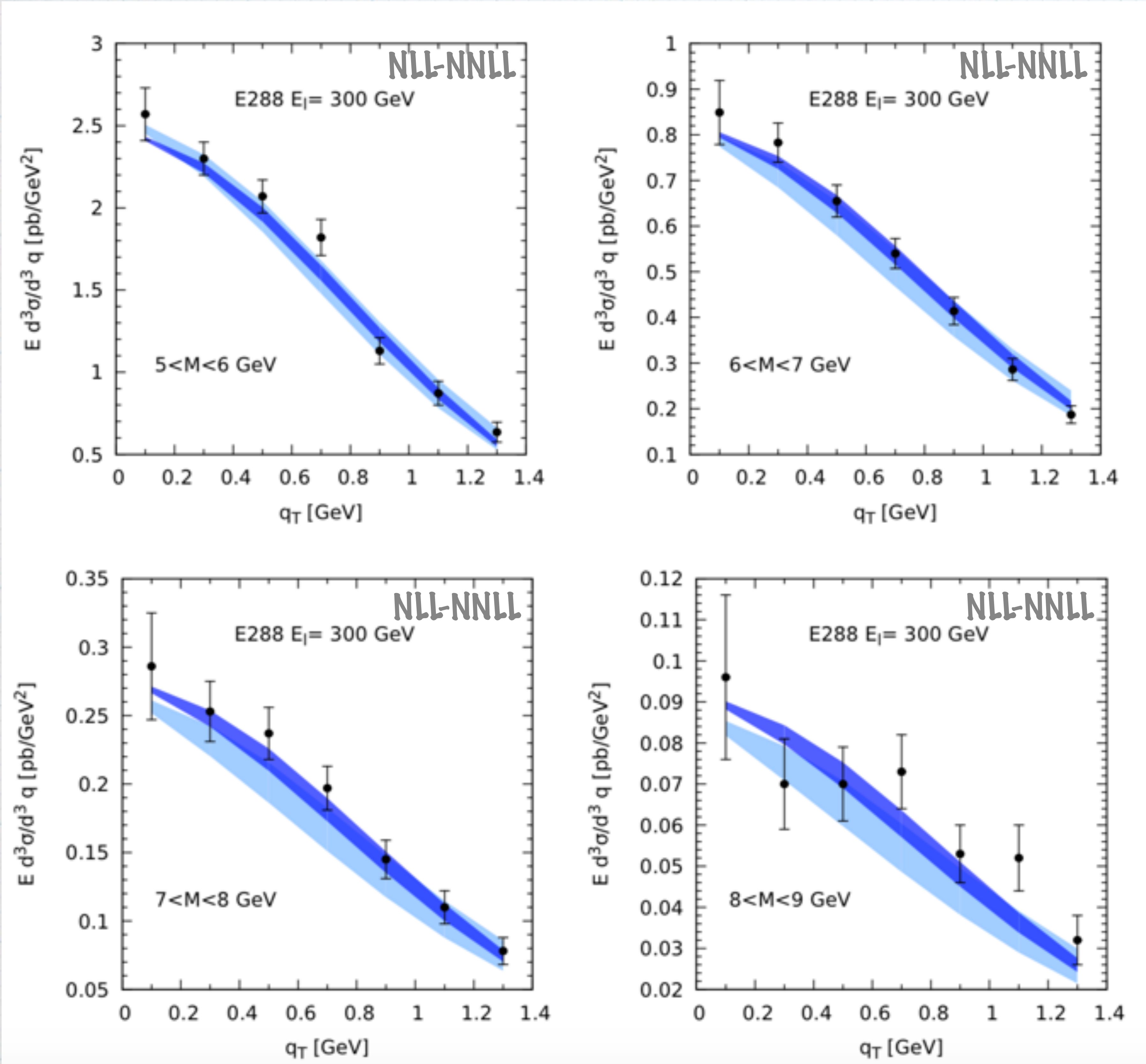
Same resummation as for the D

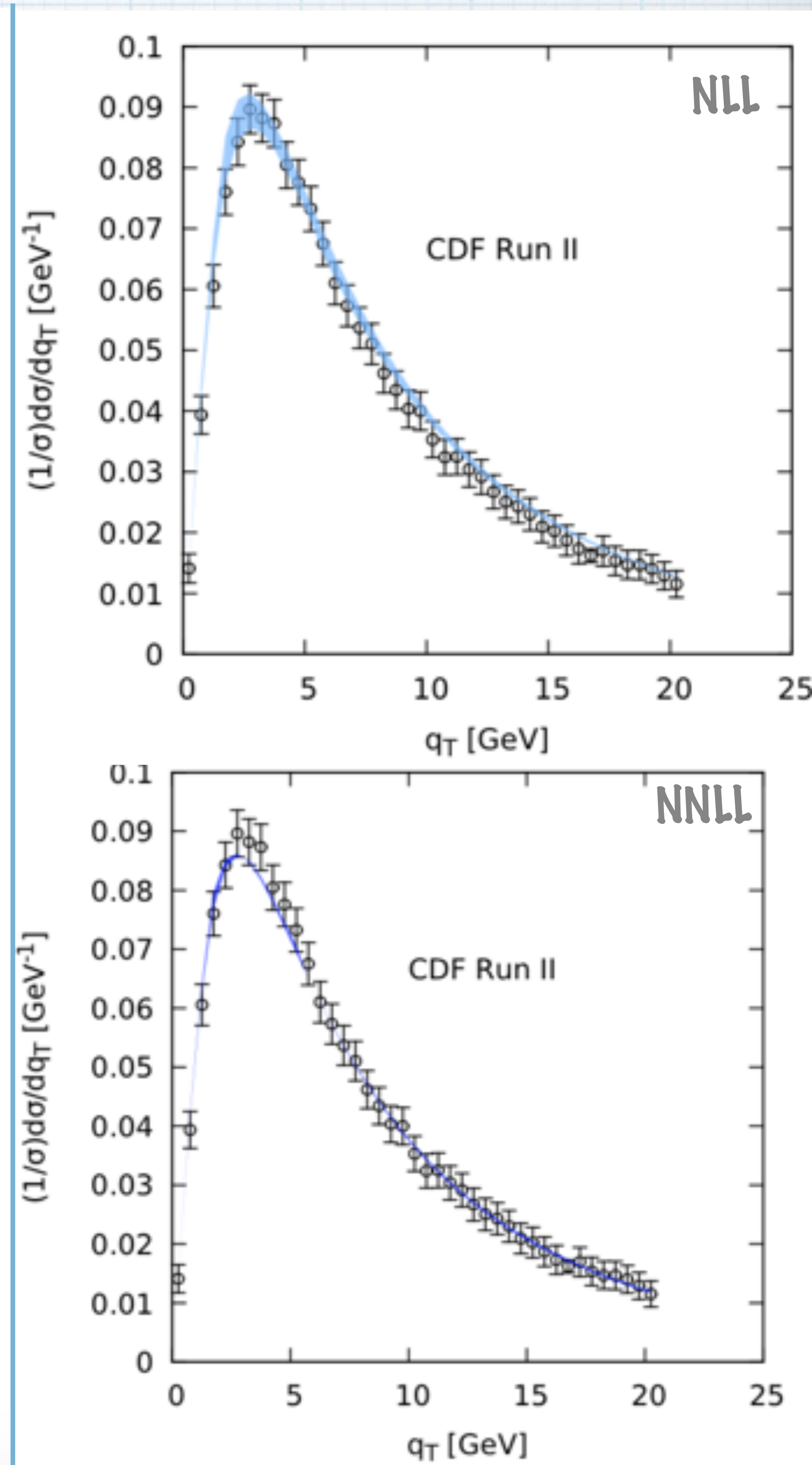
finally write a(1/b) in terms of a(mu) and fix mu=Qi.

Logs are minimized with the choice
mu=Qi=Q0+qT

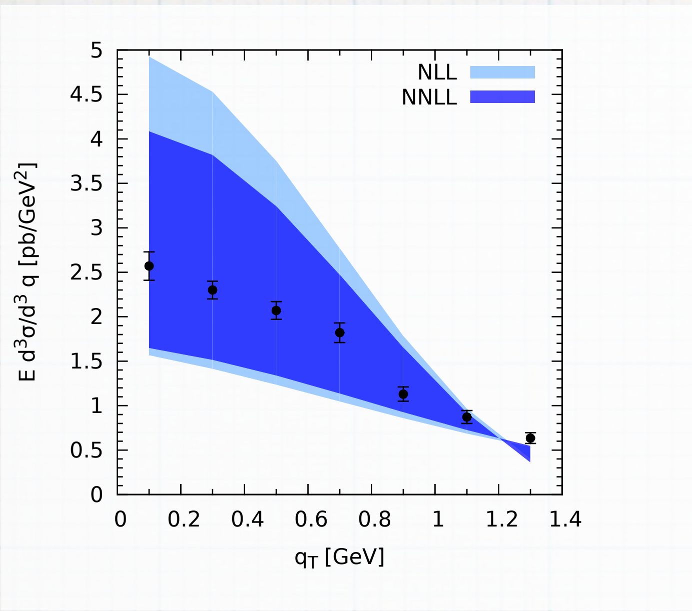
Scale dependence: ζ, μ bands on DY data

Figures from: D' Alesio, Echevarría,Melis, S.





Mu-b error on E288 data



The matching on the PDF can
be understood only as a model:

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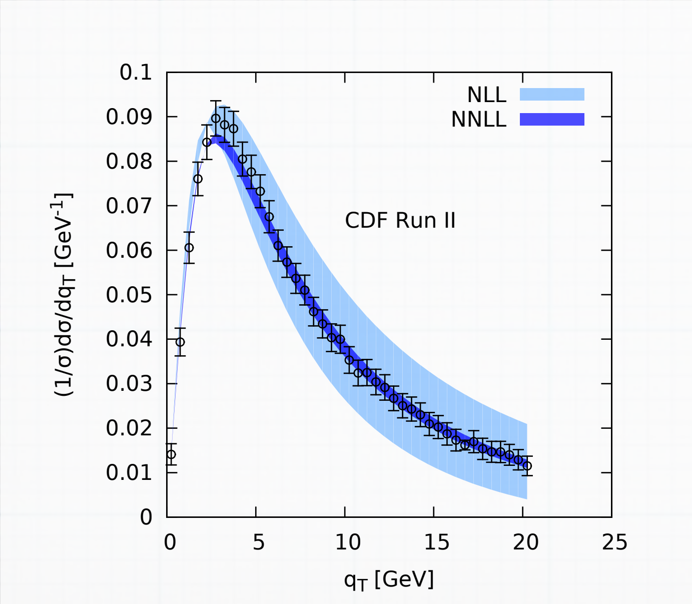
the rapidity scale error is too big
to allow a perturbative treatment.
Higher twist? NNLL′/NNLO? See talk of S. Melis

Mu-b error on Z boson data

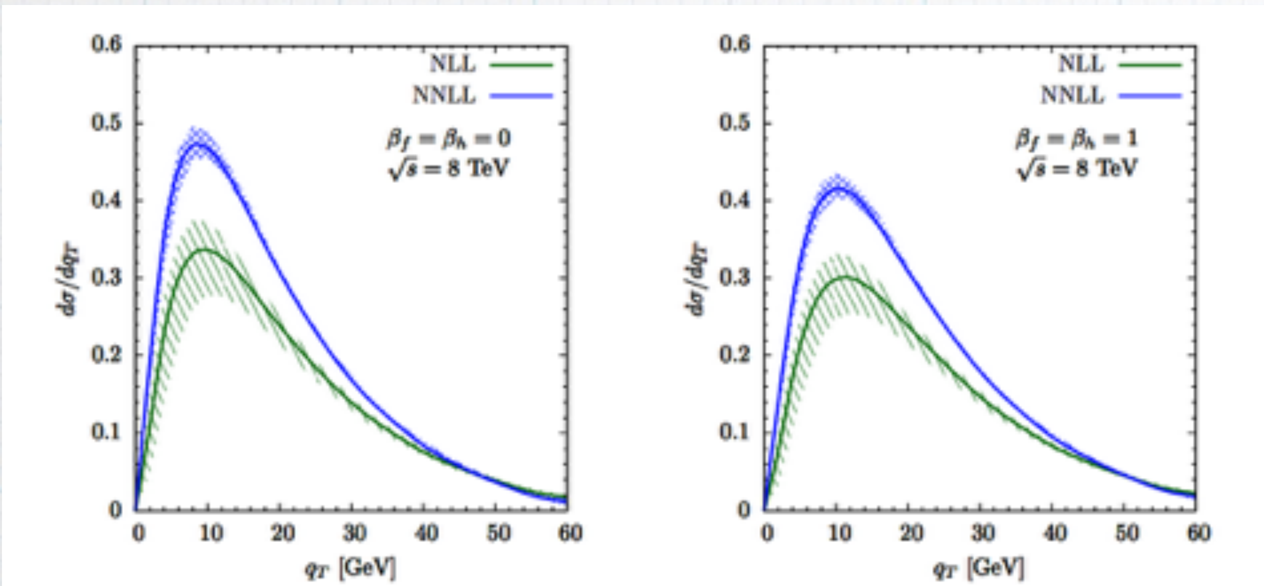


For these data the matching is
perturbatively under control at NNLL

Figures done including the model of D′ Alesio, Echevarría,Melis,S.

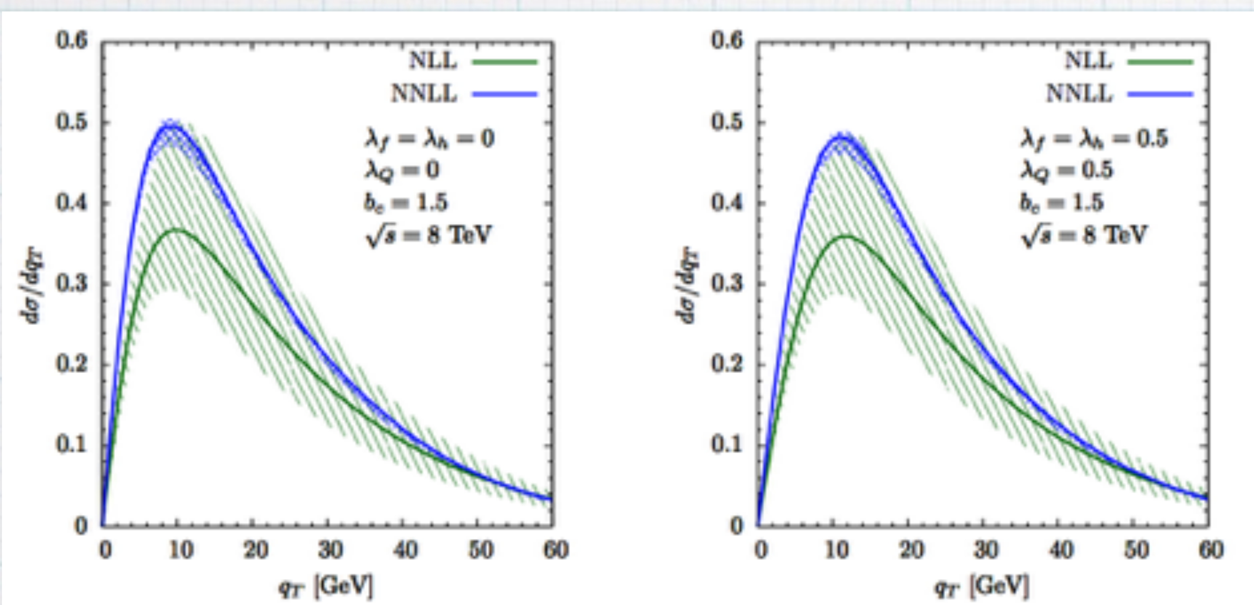
The Higgs case

Figures from: Echevarría, Kasemets, Mulders,Pisano
See also: Neill, Rothstein,Vaidya;
Becher, Neubert, Wilhelm;
Bozzi,Catani, de Florian, Grazzini...



Exponential model and $\mu = Q_0 + q_T$

Biggest error from mu-b scale



Gaussian model and $\mu_b = 1/\hat{b}_T$ where $\hat{b}_T = b_c\sqrt{1 - \exp[-b_T^2/b_c^2]}$

From NLL to NNLL.... to NNLL´/NNLO

Higher perturbative orders allow to improve the convergence significantly!

Improvement NLL->NNLL

MSTW08

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	1.10	1.48
E288 200	35	1.53	2.60
E288 300	35	1.50	1.12
E288 400	49	2.07	1.79
R209	6	0.16	0.25
CDF Run I	32	0.74	1.31
D0 Run I	16	0.43	1.44
CDF Run II	41	0.30	0.62
D0 Run II	9	0.61	2.40

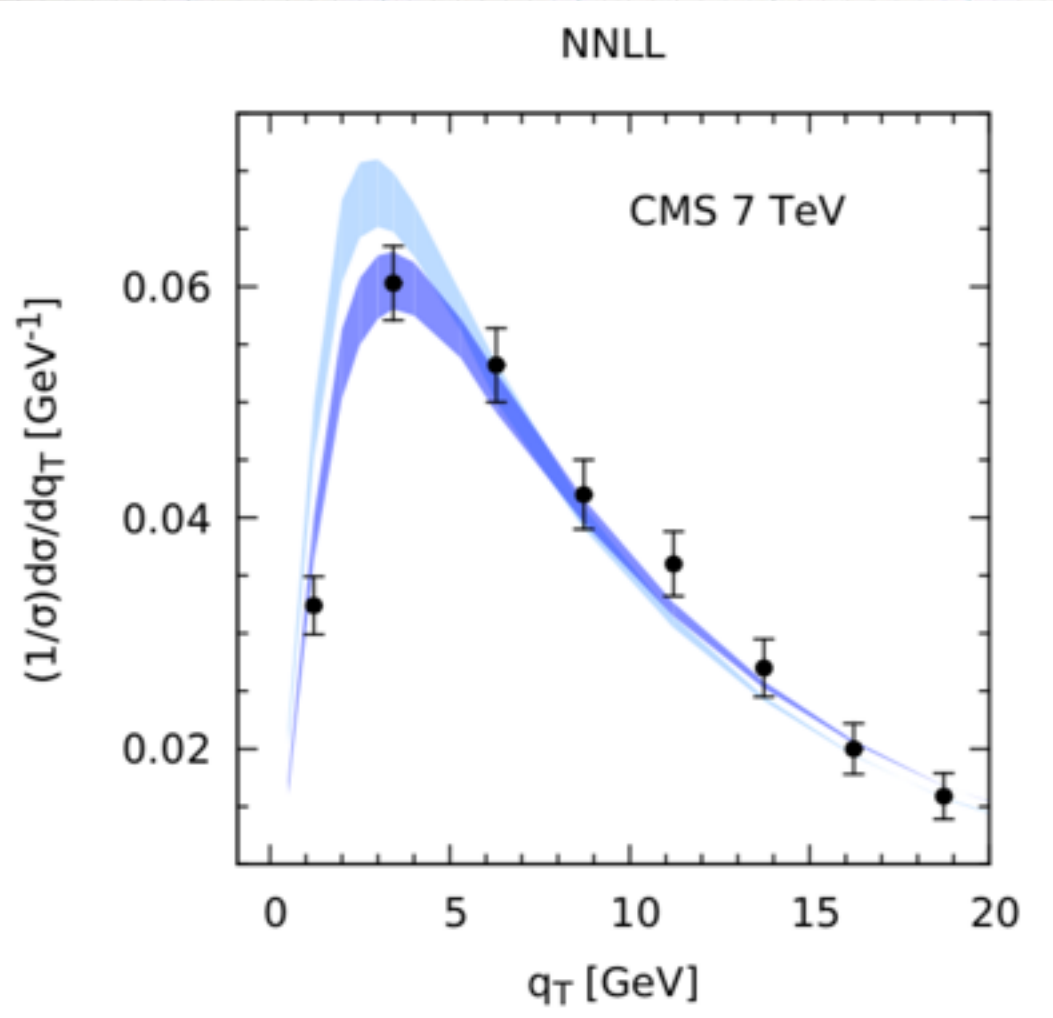
NLL	223 points	χ^2/d.o.f. $= 1.51$
	$\lambda_1 = 0.26^{+0.05_{\rm th}}_{-0.02_{\rm th}} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{\rm th} \pm 0.03_{\rm stat}$ GeV^2
	$N_{\rm E288} = 0.9^{+0.2_{\rm th}}_{-0.1_{\rm th}} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.3 \pm 0.01_{\rm th} \pm 0.2_{\rm stat}$
NNLL	223 points	χ^2/d.o.f. $= 1.12$
	$\lambda_1 = 0.33 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{\rm th} \pm 0.03_{\rm stat}$ GeV^2
	$N_{\rm E288} = 0.85 \pm 0.01_{\rm th} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.5 \pm 0.01_{\rm th} \pm 0.2_{\rm stat}$

CTEQ10

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	0.96	1.79
E288 200	35	1.58	2.61
E288 300	35	1.09	1.10
E288 400	49	1.17	2.43
R209	6	0.20	0.35
CDF Run I	32	0.83	1.55
D0 Run I	16	0.48	1.79
CDF Run II	41	0.38	0.79
D0 Run II	9	1.036	3.28

NLL	223 points	$\chi^2/\mathrm{dof} = 1.79$
	$\lambda_1 = 0.28 \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.14 \pm 0.04_{\rm stat}$ GeV^2
	$N_{\rm E288} = 1.02 \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.4 \pm 0.2_{\rm stat}$
NNLL	223 points	$\chi^2/\mathrm{dof} = 0.96$
	$\lambda_1 = 0.32 \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.12 \pm 0.03_{\rm stat}$ GeV^2
	$N_{\rm E288} = 0.99 \pm 0.05_{\rm stat}$	$N_{\rm R209} = 1.6 \pm 0.3_{\rm stat}$

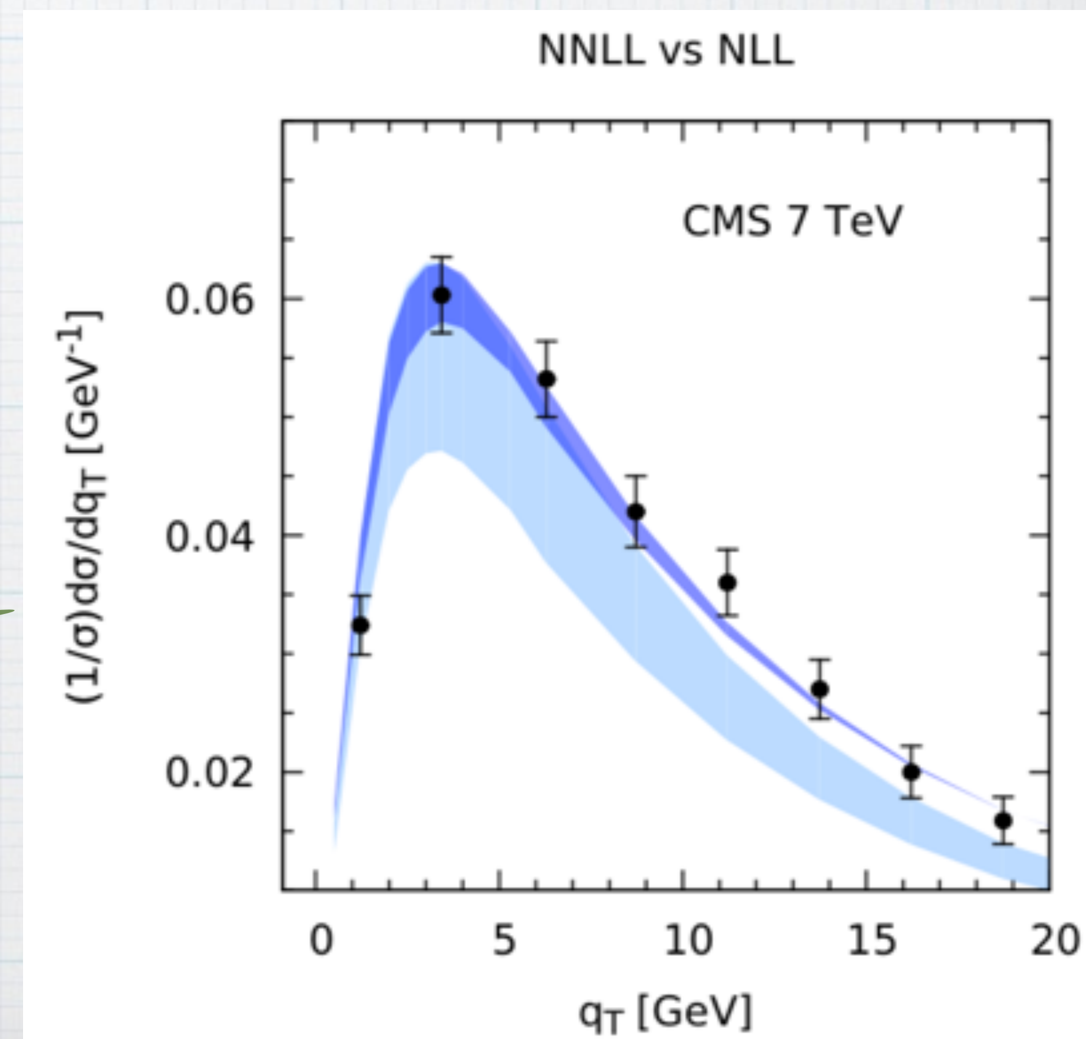
Predictions for CMS



Pure-perturbative vs complete TMDs at NNLL

Very large bins!! (not shown)

NLL vs NNLL for complete TMDs: scale dependence



CMS goes at smaller values of Bjorken x than TeVatron: broader bands (See Belinsky et al. for small x resummation)

Work in progress

with U. D' Alesio, M. Echevarría, S. Melis

Data analysis/fits:

Full inclusion of two loop results (NNLL/NNLO)

Scale dependences

Improved non-perturbative inputs for weak boson productions

LHC results

TMDFF at NNLO

with M.G. Echevarría, A. Vladimirov. arXiv:1509.XXXX

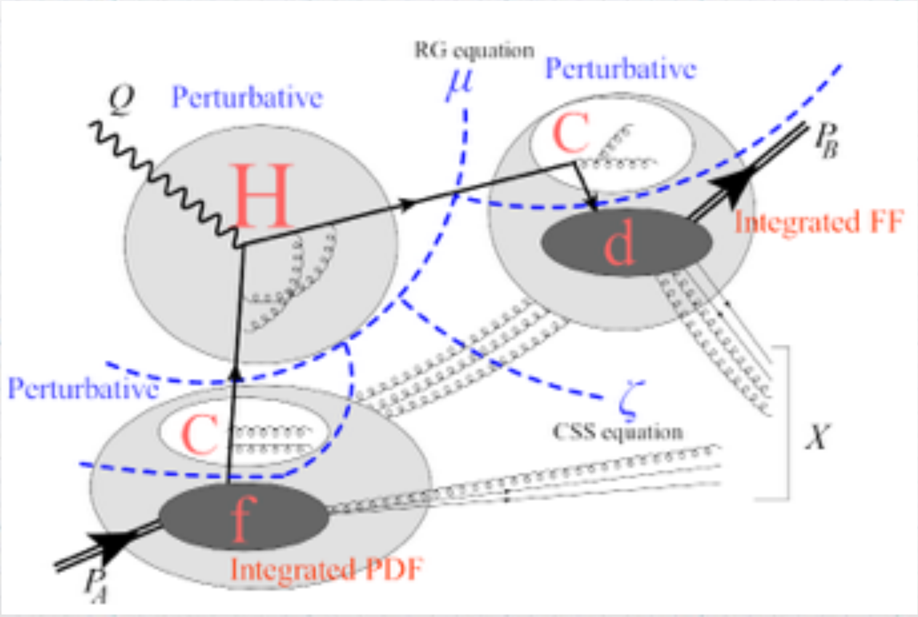
TMDFF at NNLO

TMD formalism never been directly tested at 2-loops:

All higher perturbative coefficients deduced from calculations of the product of 2 TMD's

We need (a regulator which allows) to calculate:

* The Universal Soft function (to be used for all spin dependent TMDs)
* The matching of the naive TMD's onto the collinear functions. We provide the matching the unpolarized TMDFF onto FF.



TMD structures in SIDIS (EIS formulation)

$$W = H(Q/\mu) \int \frac{db}{(2\pi)^2} \Delta^{(0)}_{A/f}(z_A, b; \mu, \delta^+) \Phi^{(0)}_{f'/B}(z_B, b; \mu, \delta^-) S(b; \delta^+, \delta^-)$$

* Each matrix element is zero-bin subtracted (suffix (0))

$$\Delta^{(0)}_{A/f}(z_A, b; \mu, \delta^+) = \Delta^{naive}_{f'/B}(z_B, b; \mu, \delta^-)/S(b; \delta^+, \delta^-)$$

* Rapidity divergences regulated by deltas

$$S(b_T) = \frac{1}{N_c} \langle 0 | [-\infty_n, b_T, \infty_{\bar{n}}][\infty_{\bar{n}}, 0, -\infty_n] | 0 \rangle, \quad [\gamma] \sim P \exp\left(-ig \int_\gamma A_\gamma\right)$$

The soft factor contains only rapidity/collinear divergences

TMD structures in SIDIS (EIS formulation)

$$W = H(Q/\mu)\int \frac{db}{(2\pi)^2} D_{A/f}(z_A, b; \mu, \zeta_A) F_{f'/B}(z_B, b; \mu, \zeta_B)$$

* Each TMD is

$$D_{A/f}(z_A, b; \mu, \zeta_A) = \Delta^{naive}_{f'/B}(z_B, b; \mu, \delta^+)/\sqrt{S(b; \delta_+^2 \alpha)}$$

* Rapidity divergences canceled within one TMD

TMDFF structures (EIS formulation)

Motivations and goals

- We would like to check the cancelation of divergences individually for every TMD
  - We need expression for soft factor
  - We need expression for naive collinear TMD
- The expression for TMD FF is under interest
  - It is novel part of information, which cannot be get from [Gehrmann,et al] (since they restrict they-self to space-like separators only)
  - It is needed for N^3LO analysis of TMD FF. So TMD FF and TMD PDF would be considered on equal footing.

$$D_{q/f}(z_A, b_T; \zeta, \mu) = \sum_j \int_{z_A}^1 \frac{dz}{z^{3-2\epsilon}} d_{q/j}(z, \mu) C_{j/f}\left(\frac{z_A}{z}, b_T; \zeta, \mu\right) + \mathcal{O}(b_T),$$

$$C_{j/f}(z, b_T; \zeta, \mu) = \underbrace{C^{[0]}_{j/f}(z, b_T; \zeta, \mu)}_{\delta_{jf}\delta(z-1)} + \overbrace{a_s}^{\frac{g^2}{(4\pi)^2}} \underbrace{C^{[1]}_{j/f}(z, b_T; \zeta, \mu)}_{\substack{\text{[Collins,textbook]} \\ \text{[EIS,1402.0869]}}} + a_s^2 \underbrace{C^{[2]}_{j/f}(z, b_T; \zeta, \mu)}_{\text{desired}} + \ldots$$

TMDFF expansion

Small-b_T factorization

$$D(z, b_T) = C(z, b_T) \otimes \frac{d(z)}{z^{3-2\epsilon}},$$

$$C^{[0]} = \delta(1-z), \quad d^{[0]}(z) = \delta(1-z), \quad D^{[1]}(z, b_T) = \delta(1-z)$$

The order-by-order perturbative definition of matching coefficient:

$$C^{[1]}_{j/f} = D^{[1]}_{j/f} - \frac{d^{[1]}_{j/f}}{z^{3-2\epsilon}}$$

$$C^{[2]}_{j/f} = D^{[2]}_{j/f} - \sum_x D^{[1]}_{j/x} \otimes \frac{d^{[1]}_{x/f}}{z^{3-2\epsilon}} - \frac{d^{[2]}_{j/f}}{z^{3-2\epsilon}}$$

- The main difficulty is to calculate $D^{[2]}_{j/f}$

TMDFF expansion

1-loop

$$D_{q/q}^{[1]} = \tilde{\Delta}_{q/q}^{[1]} - \tilde{S}_{+}^{[1]} - Z_2^{[1]} + Z_D^{[1]}$$

$\sim \delta(1-z)$

2-loops

$$\tilde{D}_{i/f}^{[2]} = \underbrace{\overbrace{\tilde{\Delta}_{i/f}^{[2]} - \tilde{S}_{+}^{[1]}\tilde{\Delta}_{i/f}^{[1]}}^{\text{cros.rap.div.free}} - \tilde{S}_{+}^{[2]}\delta_{if} + \frac{3\tilde{S}_{+}^{[1]}\tilde{S}_{+}^{[1]}}{2}\delta_{if}}_{\text{rap.div.free}}$$

$$+ \left(Z_D^{[1]} - Z_2^{[1]}\right)\left(\tilde{\Delta}_{i/f}^{[1]} - \frac{\tilde{S}_{+}^{[1]}\delta_{if}}{2}\right)$$

$$+ \left(Z_D^{[2]} - Z_2^{[2]} - Z_2^{[1]}Z_D^{[1]} + Z_2^{[1]}Z_2^{[1]}\right)\delta_{if} \ .$$

Pure UV

Regularization

Regularizations

- Massless quarks
- On-shell incoming partons

dimensional regularization $d = 4 - 2\epsilon$
- UV divergences
- Other IR divergences (mass-divergences) $(\lambda, \lambda, \lambda)$
- Collinear divergences $(\lambda^2, 1, \lambda)$

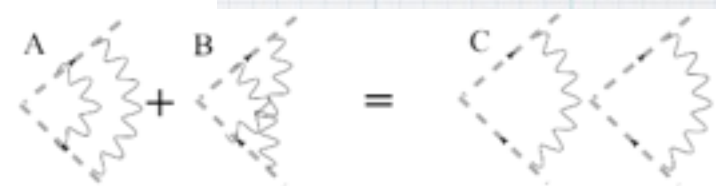
"δ-regularization"
- Collinear divergences $(\lambda^2, 1, \lambda)$
- Rapidity divergences $(\lambda, \lambda^{-1}, 1)$

δ-regularization

In original EIS approach the rapidity divergences were regularized as

$$\frac{1}{k^{\pm} + i0} \longrightarrow \frac{1}{k^{\pm} + i\delta^{\pm}}, \qquad \delta^{\pm} \to +0.$$

At two-loop such regularization violates exponentiation, and may result to non-cancelation of divergences.



$$= \frac{1}{(p^+ + i\delta)(p^+ + k^+ + 2i\delta)(p^+ + k^+ + l^+ + 3i\delta)}$$

δ-regularization preserving exponentiation

The regularization should be implemented on the level of operator

$$P\exp\left[-ig\int_0^{\infty} d\sigma A_{\pm}(\sigma n)\right] \longrightarrow P\exp\left[-ig\int_0^{\infty} d\sigma A_{\pm}(\sigma n) e^{-\delta^{\pm}|\sigma|}\right]$$

Then exponentiation is exact

$$\text{Diag}_A + \text{Diag}_B = \frac{\text{Diag}_C^2}{2}$$

2-loop structure

Soft function is linear in the rapidity regulator

Counting powers of $\ln\delta$

- Logarithm of soft-factor must be proportional to single $\ln(\delta^+\delta^-)$, otherwise definition of individual TMDs impossible.

$$S = \exp\left[A\ln(\delta^+\delta^-) + B\right] = 1 + \underbrace{S^{[1]}}_{C_F\ln\delta} + \underbrace{S^{[2]}}_{\cancel{C_F^2\ln^2\delta} + C_F C_A\ln\delta + C_F N_F\ln\delta}$$

- $\Delta^{[1]} \sim C_F\left[(...)_+ + \delta(\bar{z})(\ln\delta + ..)\right]$

2-loop structure

$C_F C_A$ and $C_F N_f$ part

$$D^{[2]} = \Delta^{[2]} - \cancel{\frac{S^{[1]}\Delta^{[1]}}{2}} - \frac{S^{[2]}\Delta^{[0]}}{2} + \cancel{\frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8}} + \ldots.$$

- Structure of $\Delta^{[2]} \sim C_A$ and $\sim N_f$ part should be

$$(\text{free of } \ln\delta)_+ + \delta(1-z)\,(\text{linear in } \ln\delta)$$

- Rather straightforward cancelation, can be traced diagram-by-diagram.

C_F^2 part

$$D^{[2]} = \Delta^{[2]} - \overbrace{\frac{S^{[1]}\Delta^{[1]}}{2}}^{\ln\delta \times (..)_+ + \delta(\bar{z})(\ln^2\delta + ..)} - \underbrace{\frac{S^{[2]}\Delta^{[0]}}{2} + \frac{3S^{[1]}S^{[1]}\Delta^{[0]}}{8}}_{\delta(\bar{z})(\ln^2\delta + ..)} + \ldots.$$

- Structure of $\Delta^{[2]} \sim C_F^2$ part should be

$$(\ln\delta + ..)_+ + \delta(1-z)\left(\ln^2\delta + \ln\delta + ..\right)$$

- Complected cancelation between higher ϵ terms of products of one-loop expressions.

2-loop structure: cancellation of divergences

CF^2

$\ln^4\delta$ and $\ln^3\delta$ cancel in the sum of $\Delta^{[2]}$

$\ln^2\delta$ and $\ln\delta$ cancel in the combination

$$\left(\Delta^{[2]} - \frac{1}{2}S^{[1]}\Delta^{[1]}\right)|_{+,\,C_F}$$

$$\left(\Delta^{[2]} - \frac{1}{2}S^{[1]}\Delta^{[1]} + \frac{1}{8}S^{[1]}S^{[1]}\Delta^{[0]}\right)|_{\delta(1-z),\,C_F}$$

higher orders of εexpansion (ε^n) cancel

All Nf divergences cancel

CF CA

$\Delta^{[2]}_{+,C_A}$ is free of $\ln\delta$

$\ln\delta$ in $\Delta^{[2]}_{\delta(1-z),C_A}$ is canceled in the combination with two loop soft factor

2-loop structure: recursion relations

The most general structure at order "n" which respects RGE

$$\tilde{C}_{if}^{[n]} = \sum_{l=0}^{n} \sum_{k=l}^{2n} \tilde{C}_{if}^{(n;k,l)} \boldsymbol{L}_{\mu}^{k-l} \lambda_{\zeta}^{l}$$

$$\mathbf{L}_\mu = \ln \frac{\mu^2 b^2 e^{2\gamma}}{4}; \ \lambda_\zeta = \ln \frac{\mu^2}{Q^2}$$

and one can find (and check) recursion relations

$$(k-l+1)\tilde{C}_{jf}^{(n;k+1,l)} = \sum_{r=1}^{n} \frac{\Gamma_{cusp}^{[r]}}{2} \tilde{C}_{jf}^{(n-r;k-1,l-1)} - \frac{\gamma_{Vjf}^{[r]} + 2(n-r)\beta^{[r]}}{2} \tilde{C}_{jf}^{(n-r;k,l)} - \frac{\mathcal{P}_{j/h}^{[r]}}{z^2} \otimes \tilde{C}_{jf}^{(n-r;k,l)} - \sum_{m=0}^{r} d^{(r;m)} \tilde{C}_{jf}^{(n-r;k-m,l)}.$$

Sample of the result

$$\tilde{C}^{[2]}_{N_f} = \frac{4}{3z}\left(\frac{1+z^2}{1-z}\right)_+ \mathbf{L}_\mu{}^2 + \frac{8}{9z}\left(\frac{8-6z+8z^2-3(1+z^2)\ln z}{1-z}\right)_+ \mathbf{L}_\mu$$

$$+\delta(1-z)\left[\frac{8}{9}\mathbf{L}_\mu{}^3 - \frac{4}{3}\mathbf{L}_\mu{}^2\lambda_\zeta + \frac{2}{9}\mathbf{L}_\mu{}^2 - \frac{40}{9}\mathbf{L}_\mu\lambda_\zeta - \frac{1}{3}(26-4\pi^2)\mathbf{L}_\mu - \frac{112}{27}\lambda_\zeta\right] + \tilde{C}^{(2,0,0)}$$

$$C^{(2,0,0)}_{N_F} = \left[\left(\frac{2}{3}\ln^2 z - \frac{20}{3}\ln z + \frac{112}{27}\right)p(z) - \frac{16}{3}\bar{z}\ln z - \frac{4}{3}\bar{z}\right]_+ + \delta(\bar{z})\left(-\frac{2717}{162} + \frac{25\pi^2}{9} + \frac{52}{9}\zeta_3\right).$$

Conclusions

- The correct measurement of non-perturbative effects in transverse momentum dependent observables requires the use of TMDs on very different energy spectrum
- Golden energy range for TMDs, Q>2-3 GeV, qT<<Q. LHC, ee collider (Belle, Bes) and EIC can provide a huge development of the field
- The evolution of TMD's should be used at highest available order to control the perturbative series (NNLL only achieved in a limited set of TMDs)
- The control of perturbative error (2-3 scales) is fundamental to understand the nature of non-perturbative effects
- The universality of TMDs requires the computation of TMDFF with the same degree of precision of TMDPDF:
- We have completed the calculation of the universal soft factor and the matching of the unpolarized non-singlet quark TMDFF onto FFs at NNLO using the EIS formulation: The result has passed all consistences check… to be full released soon
- The soft function can be used for the evaluation of the matching of all TMDs

Thanks!!!

Back up slides

Experimental Data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

Z, run I:

Becher, Neubert, Wilhelm 2011:
ad hoc model for these data at low qT

Catani et al. 2009: Minimal Subtraction

Z, run I and low energy data
BLNY-RESBOS: model for everything

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	y=0.4	y=0.21	y=0.03	
Observable	$Ed^3\sigma/d^3\boldsymbol{p}$	$Ed^3\sigma/d^3\boldsymbol{p}$	$Ed^3\sigma/d^3\boldsymbol{p}$	$d\sigma/dq_T^2$

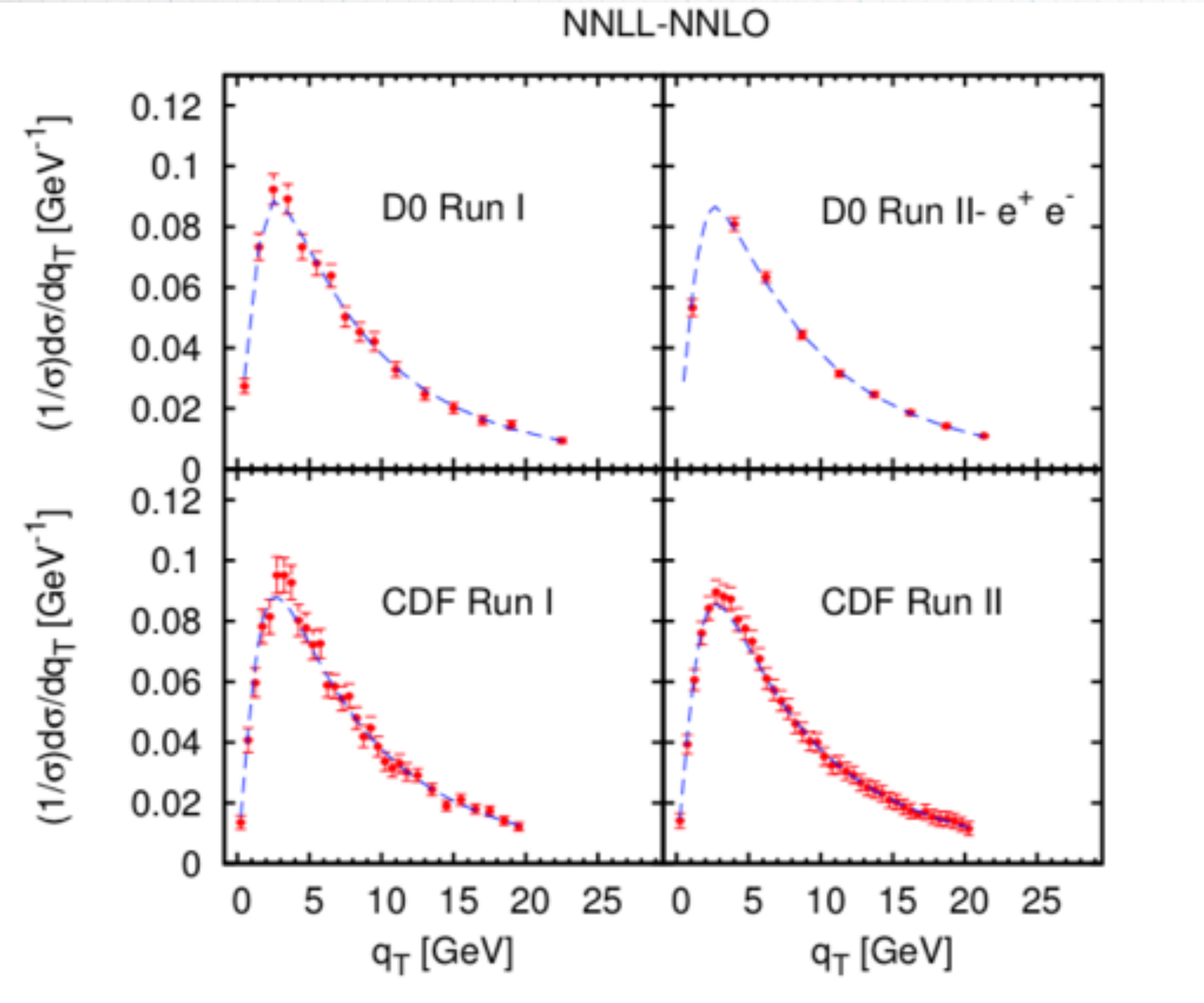
Expected to be insensitive to Landau pole region
Factorization hypothesis hold

Opportunity for ATLAS/CMS: unexplored measurement of DY

$$\frac{d\sigma}{dm_{\ell\ell}\,dq_T\,dy}$$

with $10 \text{ GeV} \simeq m_{\ell\ell} \simeq 70 \text{ GeV}$

Results at NNLL: Z production



Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs exponential) and (poorly) sensitive just to λ_1.
In order to fix it we need the global fit

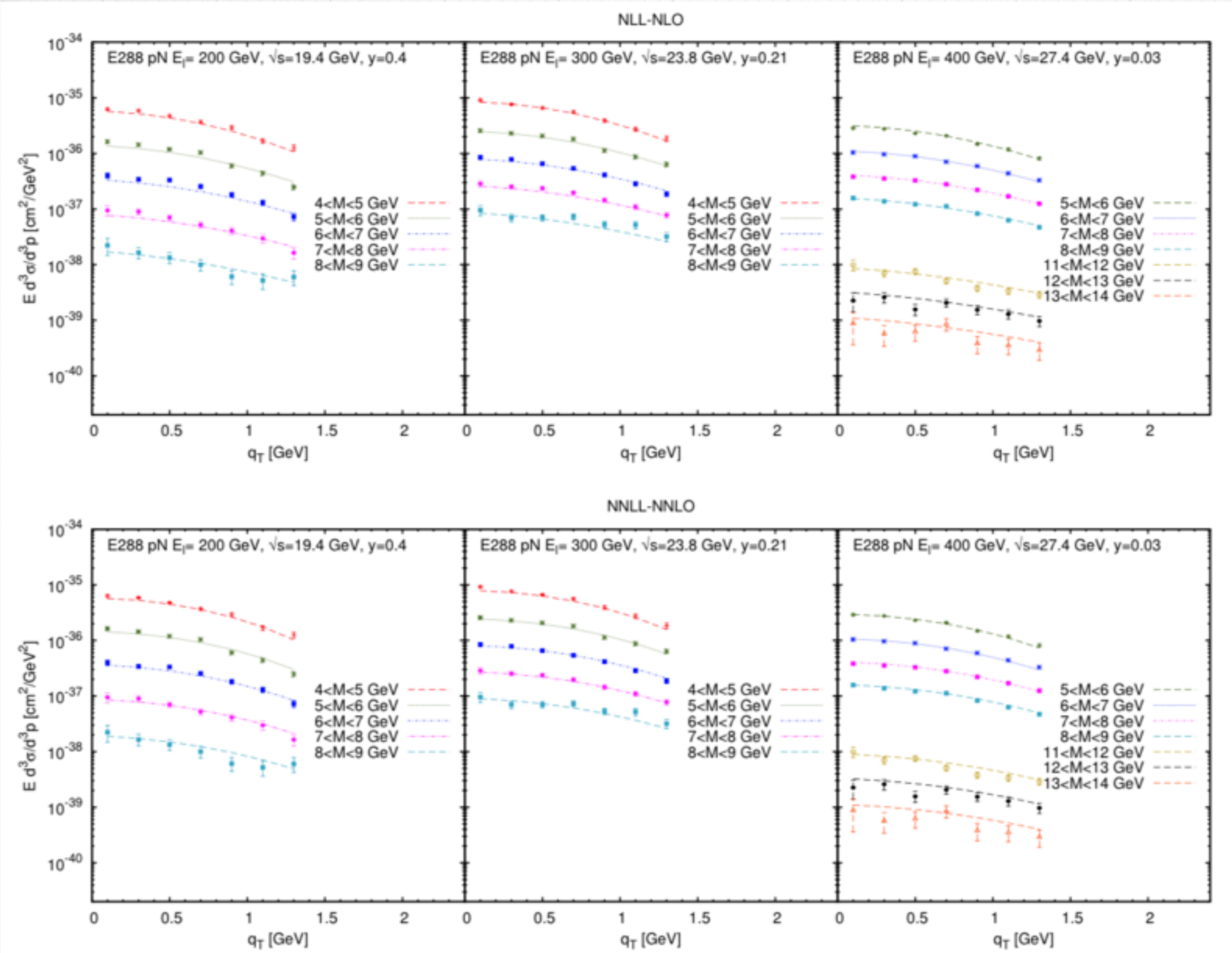
DYNNLO: Catani, Grazzini '07, Catani, Cieri, Ferrera, de Florian, Grazzini '09

Data: $$\frac{1}{\sigma_{exp}}\left(\frac{d\sigma}{dq_T}\right)_{exp}$$

Theory: $$\frac{1}{\sigma_{th}}\left(\frac{d\sigma}{dq_T}\right)_{th}$$

Message:
One cannot fix the NP part of TMD's just looking at Z-boson production:
Extrapolating parameters from Z to W may not be accurate enough.

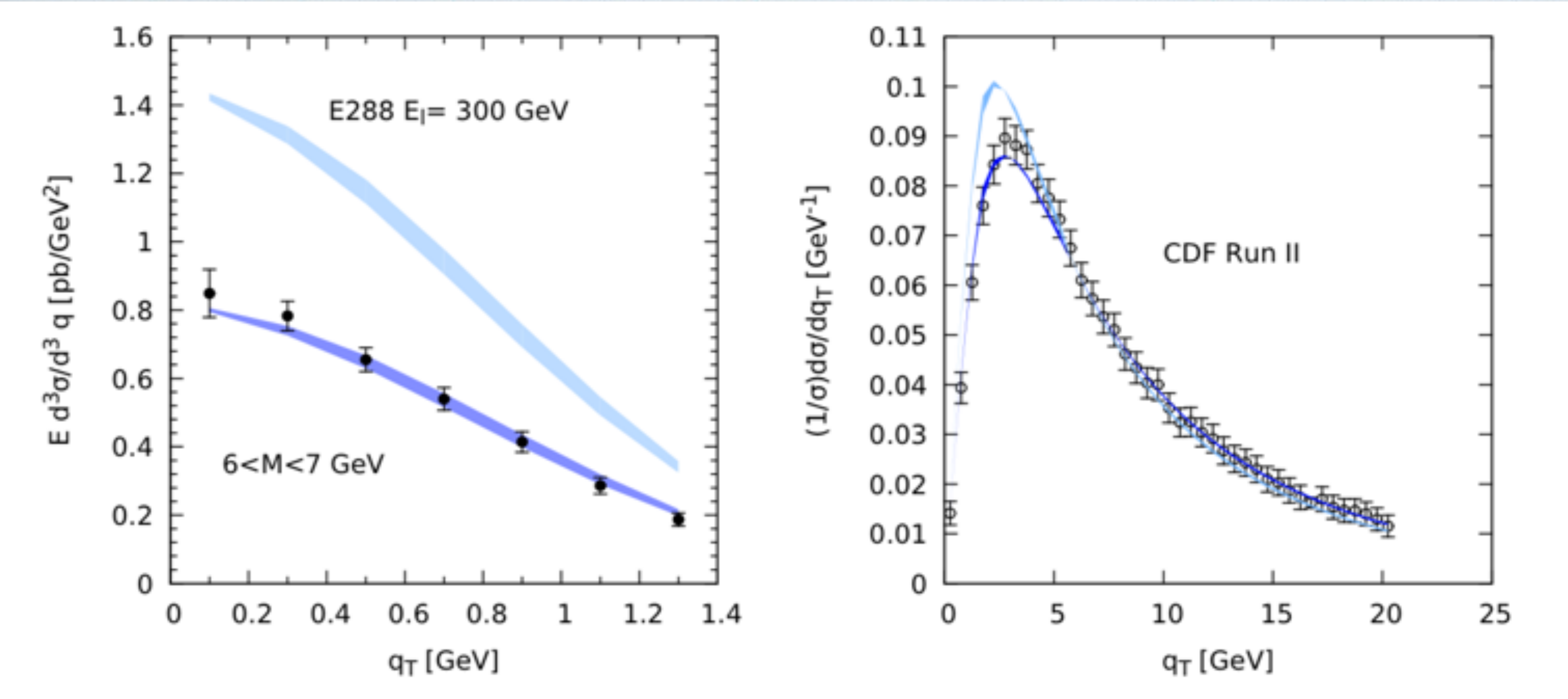
Results at NNLL



Exp. Normalization
NE288, NR209
deduced from the fit.

Total: 4 parameters

Model dependence



Non-perturbative inputs necessary for the peak region in Z-production: **Consistency between DY and Z data**

Theoretical arguments suggest also a non-perturbative Q-dependence of the evolution kernel (check RESBOS).
We test

$$M_q(x,b,Q) = \exp[-\lambda_1 b]\,(1+\lambda_2 b^2+\ldots)\left(\frac{Q^2}{Q_0^2}\right)^{-\lambda_3 b^2/2}$$

Model dependence

$Q_0 = 2.0$ GeV $+ q_T$		NNLL	NLL
λ_1		$0.29 \pm 0.04_{\text{stat}}$ GeV	$0.27 \pm 0.06_{\text{stat}}$ GeV
λ_2		$0.170 \pm 0.003_{\text{stat}}$ GeV^2	$0.19 \pm 0.06_{\text{stat}}$ GeV^2
λ_3		$0.030 \pm 0.01_{\text{stat}}$ GeV^2	$0.02 \pm 0.01_{\text{stat}}$ GeV^2
N_{E288}		$0.93 \pm 0.01_{\text{stat}}$	$0.98 \pm 0.06_{\text{stat}}$
N_{R209}		$1.5 \pm 0.1_{\text{stat}}$	$1.3 \pm 0.2_{\text{stat}}$
χ^2		180.1	375.2
	points	χ^2/points	χ^2/points
	223	0.81	1.68
	points	χ^2/dof	χ^2/dof
	223	0.83	1.72
E288 200	35	1.35	2.28
E288 300	35	0.98	1.22
E288 400	49	1.05	2.33
R209	6	0.27	0.40
CDF Run I	32	0.70	1.50
D0 Run I	16	0.41	1.77
CDF Run II	41	0.25	0.76
D0 Run II	9	0.82	3.2

No significative improvement:
1- Resummation in the evolution kernel greatly reduce TMD model dependence
2- The bulk of non-perturbative QCD corrections is scale independent

CTEQ10