TMDe 2015

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TRANSVERSE SPIN EFFECTS IN TWO HADRON ELECTROPRODUCTION

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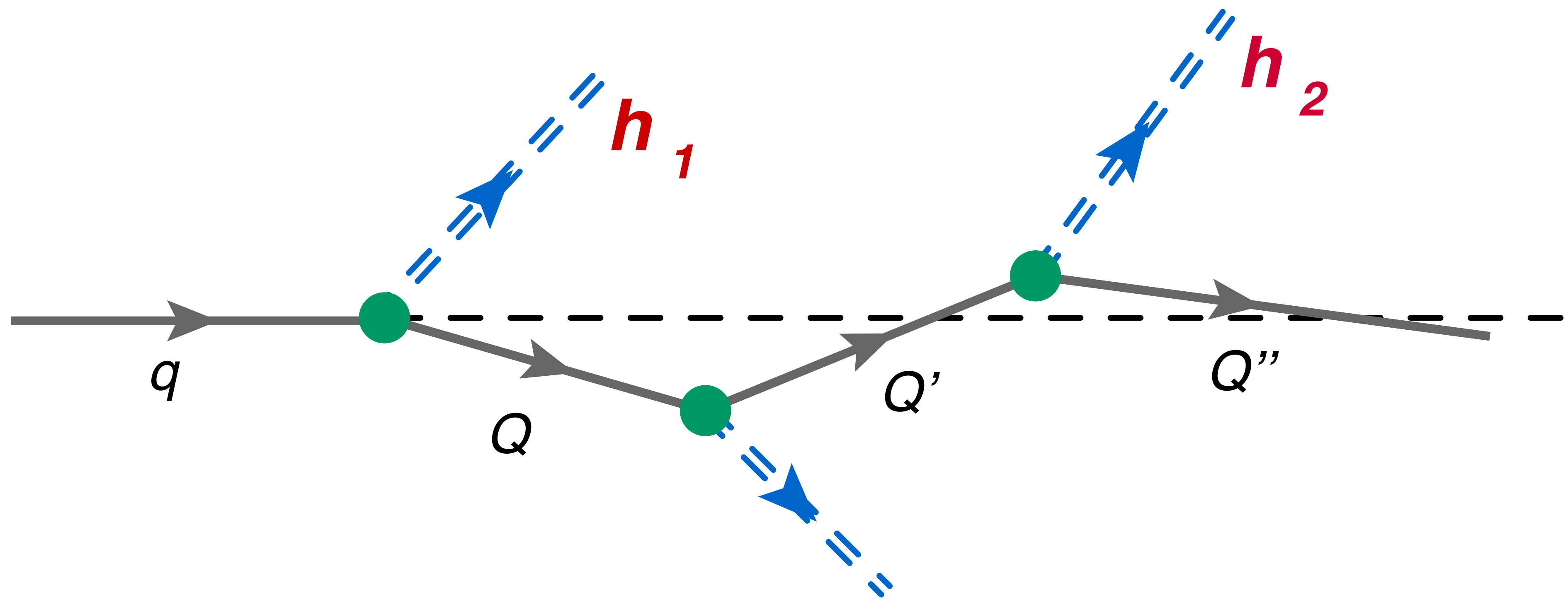
E.-C. Aschenauer, H. Avakian.

OUTLOOK

- ***Sivers Effect in Two Hadron SIDIS and Unpolarized Dihadron Fragmentation Functions.***

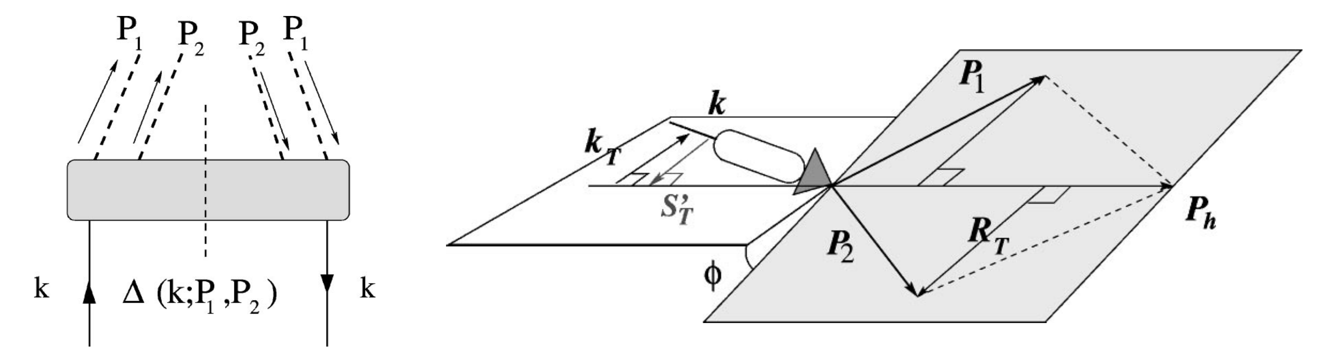
- ***Interference DiFF type modulations from Collins effect.***



TWO HADRON CORRELATIONS:
DIHADRON FRAGMENTATION FUNCTIONS

TWO-HADRON FRAGMENTATION

A. Bianconi, et al: PRD 62, 034008 (2000). M. Radici, et al: PRD 65, 074031 (2002).



- Kinematic Variables:

$$P_1=\left[\xi P_h^-, \frac{M_1^2+\vec{R}_T^2}{2\,\xi P_h^-}, \vec{R}_T\right], \qquad k=\left[\frac{P_h^-}{z}, z\frac{k^2+\vec{k}_T^2}{2P_h^-}, \vec{k}_T\right] \qquad z \equiv z_h = z_1+z_2$$

$$P_2=\left[(1-\xi)P_h^-, \frac{M_2^2+\vec{R}_T^2}{2(1-\xi)P_h^-}, -\vec{R}_T\right] \qquad \mathbf{R}=\frac{\mathbf{P}_1-\mathbf{P}_2}{2} \qquad \xi=\frac{z_1}{z_1+z_2}$$

- The relevant terms of the quark correlator at leading order for a **Transversely Polarized Quark:**

Unpolarized

$$\Delta^{[\gamma^-]} = D_1(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T)$$

Interference

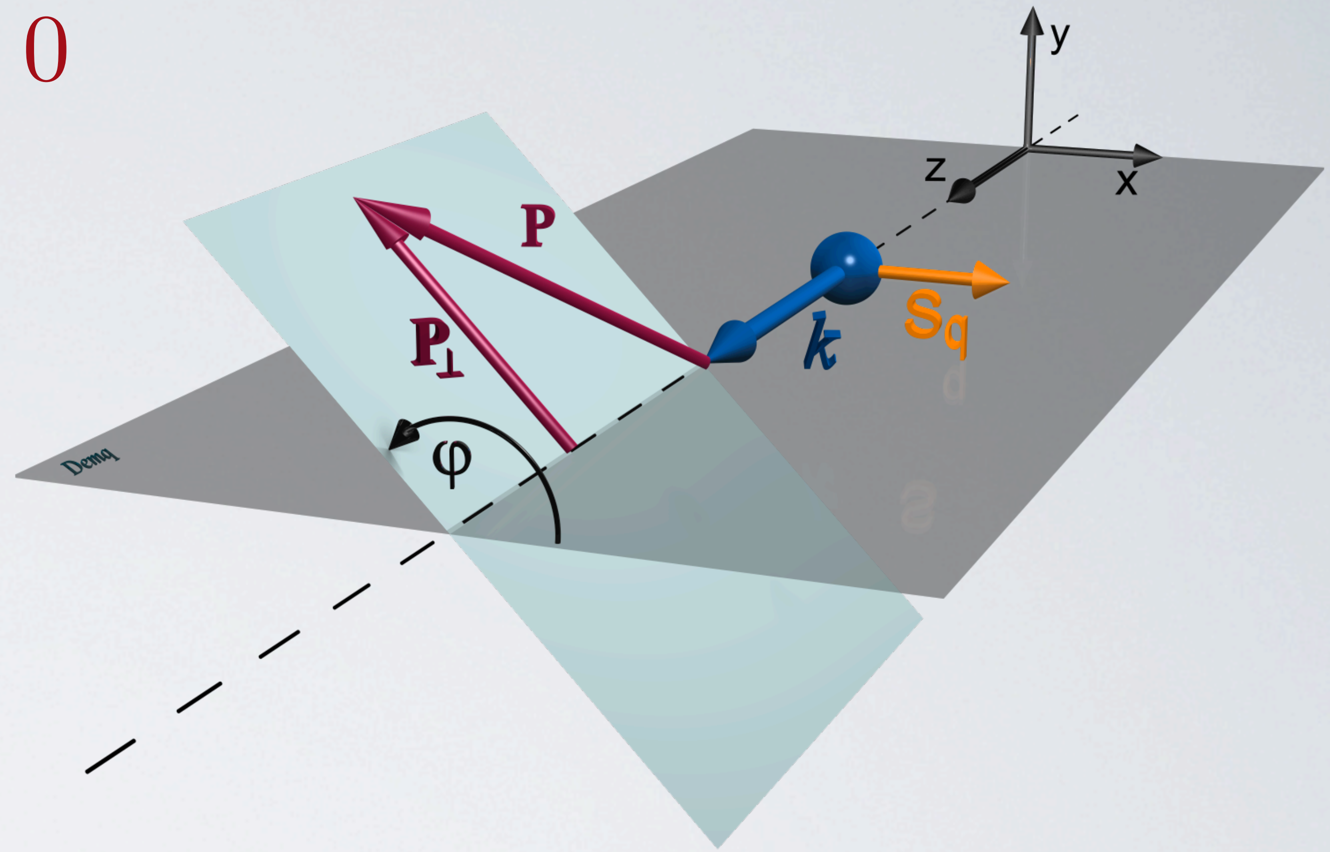
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1+M_2} H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1+M_2} H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T)$$

- **IFFS are Chiral-ODD: Need to be coupled with another chiral-odd quantity to be observed (e.g. transversity).**

TWO-HADRON FRAGMENTATION

✦**Transformation to frame** $\mathbf{k}_T = 0$

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$



$$\mathbf{P}_T = \mathbf{P}^{\perp}_{h_1} + \mathbf{P}^{\perp}_{h_2}$$
$$\mathbf{R} = (\mathbf{P}^{\perp}_{h_1} - \mathbf{P}^{\perp}_{h_2})/2$$

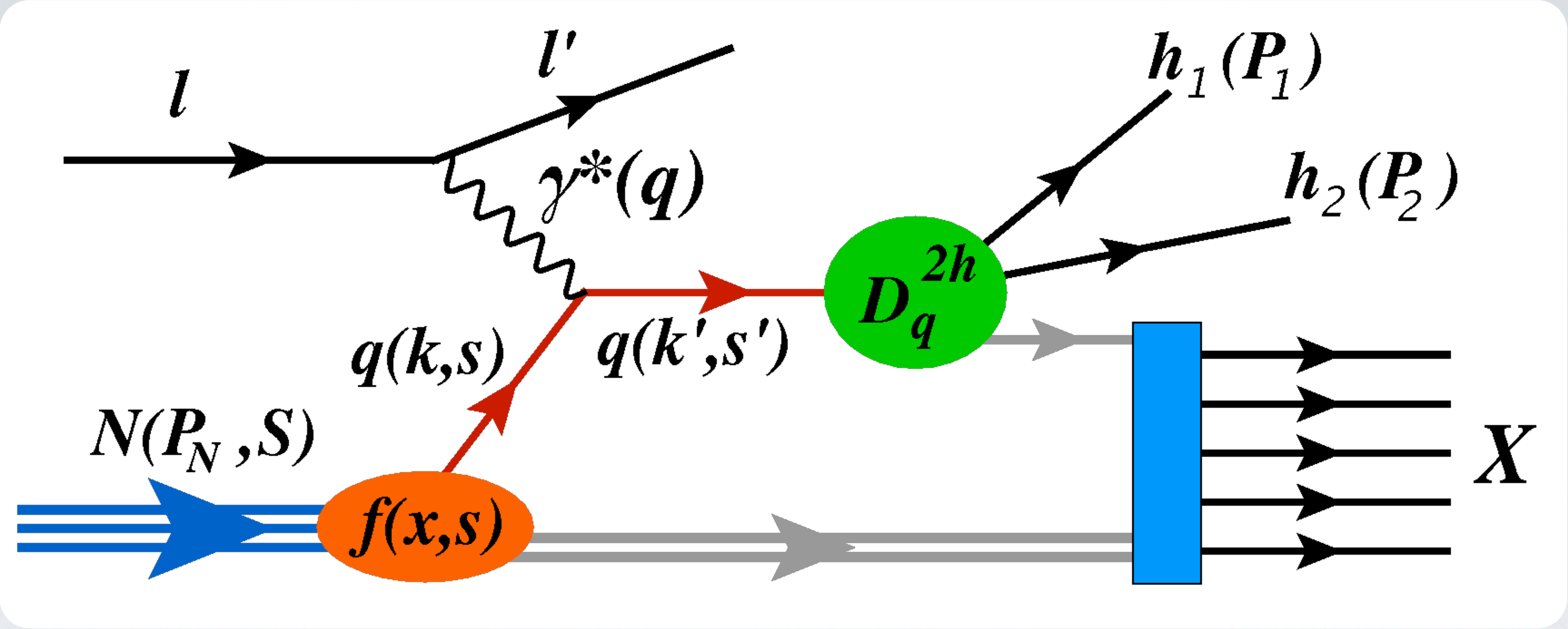
✦**Integrate over one or other momentum:**

$$D^{h_1 h_2}_{q^\uparrow}(\varphi_R) = D^{h_1 h_2}_{1,q} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D^{h_1 h_2}_{q^\uparrow}(\varphi_T) = D^{h_1 h_2}_{1,q} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

✦ **The IFF surviving after** $\mathbf{k}_T$ **integration is redefined as**

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$



Sivers Effect in Two Hadron SIDIS

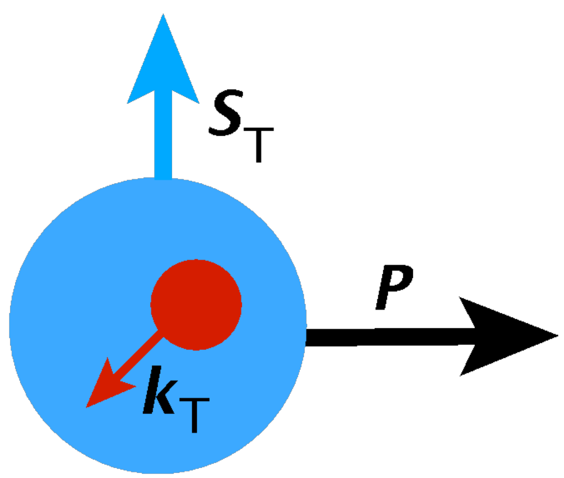
SIVERS PDF

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

D. Sivers, Phys.Rev. D41 (1990).

✦ Correlation of k_T and S_T

✦ Proposed by Dennis Sivers in 1990 to explain the single spin asymmetry in $pp^\uparrow \to \pi + X$.



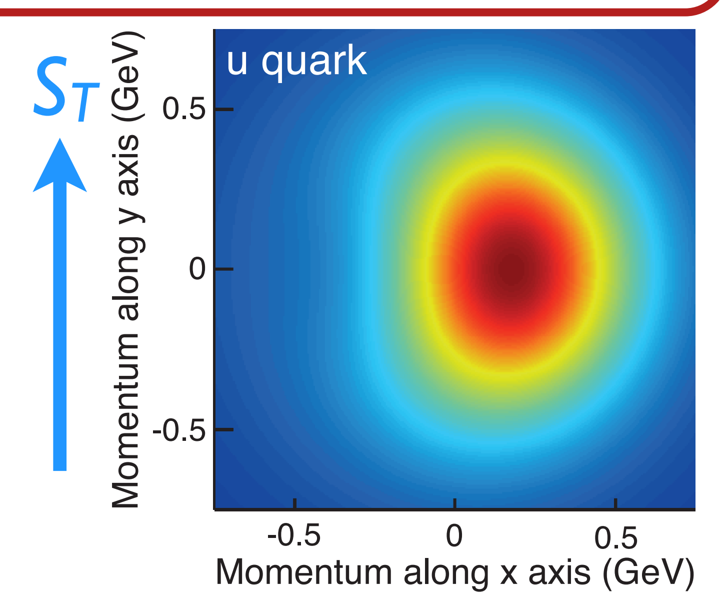
$$S_T k_T \sin(\varphi_k - \varphi_S)$$

$$f_\uparrow^q(x, \vec{k}_T) = f_1^q(x, k_T) + \frac{[\vec{S} \times \vec{k}_T]_3}{M} f_{1T}^{\perp q}(x, k_T)$$

✦ Naively *T-odd*, gauge-link should be included in the definition.

✦ Accessible in Polarized SIDIS, Drell-Yan.

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

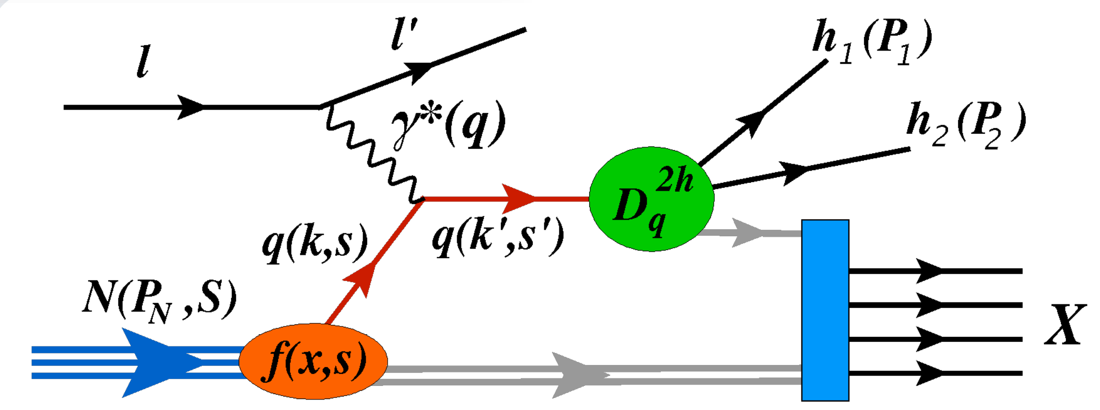


EIC White Paper, arXiv:1212.1701

TWO-HADRON SIDIS

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

- Correlations of quark's TM transferred to **two hadrons.**



$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{1T}\, d^2\boldsymbol{P}_{2T}} = C(x,Q^2)\,(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2\boldsymbol{k}_T\; f_1^q\; D_{1q}^{h_1 h_2} \qquad \sigma_S = \sum_q e_q^2 \int d^2\boldsymbol{k}_T \frac{[\boldsymbol{S}_T \times \boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}\; D_{1q}^{h_1,h_2}$$

- Unpolarized fully unintegrated dihadron Fragmentation Function

✦ **Single hadron** FF.

$$D_{1q}^{h}(z, P_\perp)$$

✦ **Dihadron** FF.

$$D_{1q}^{h_1,h_2}(z_1, z_2, P_{1\perp}, P_{2\perp}, \boldsymbol{P}_{1\perp}\cdot\boldsymbol{P}_{2\perp})$$

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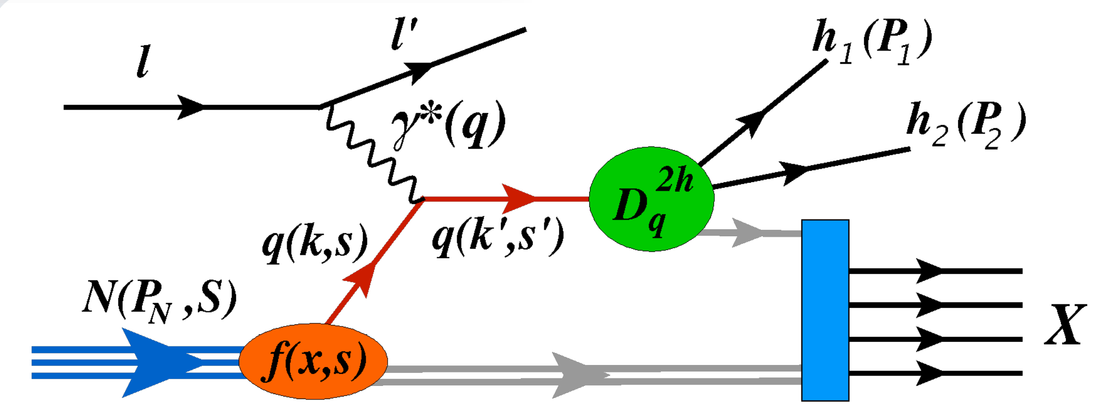


$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{1T}\, d^2\ \boldsymbol{P}_{2T}} = C(x,Q^2)\,(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2\boldsymbol{k}_T\ f_1^q\ D_{1q}^{h_1 h_2} \qquad \sigma_S = \sum_q e_q^2 \int d^2\boldsymbol{k}_T \frac{[\boldsymbol{S}_T \times \boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}\ D_{1q}^{h_1,h_2}$$

- Unpolarized fully unintegrated dihadron Fragmentation Function

✦ **Single hadron** FF.

$$D_{1q}^{h}(z, P_\perp)$$

✦ **Dihadron** FF.

$$D_{1q}^{h_1,h_2}(z_1, z_2, P_{1\perp}, P_{2\perp}, \boldsymbol{P}_{1\perp}\cdot\boldsymbol{P}_{2\perp})$$

two-hadron correlations

TWO-HADRON SIDIS

- Cross Section in terms of **Total and Relative Momenta**

$$\boldsymbol{P}_h = \boldsymbol{P}_1 + \boldsymbol{P}_2 \qquad \boldsymbol{R} = (\boldsymbol{P}_1 - \boldsymbol{P}_2)/2$$

- The Sivers term:

$$\sigma_S = S_T \left(\sigma_T \frac{P_{hT}}{M} \sin(\varphi_T - \varphi_S) + \sigma_R \frac{R_T}{M} \sin(\varphi_R - \varphi_S) \right)$$

$$\int d\varphi_R \, \sigma_S = S_T \left(\sigma_{T,0} \frac{P_{hT}}{M} + \sigma_{R,1} \frac{R}{2M} \right) \sin(\varphi_T - \varphi_S)$$

$$\int d\varphi_T \, \sigma_S = S_T \left(\sigma_{T,1} \frac{P_{hT}}{2M} + \sigma_{R,0} \frac{R}{M} \right) \sin(\varphi_R - \varphi_S)$$

- ***Non-vanishing σ_R is new! (shown explicitly in toy model)***

$$D_{1q}^{h_1,h_2}(z_1, z_2, P_{1\perp}, P_{2\perp}, \boldsymbol{P}_{1\perp} \cdot \boldsymbol{P}_{2\perp}) = D_{1q}^{h_1,h_2}(z_1, z_2) \frac{1}{\pi^2 \nu_1^2 \nu_2^2} e^{-P_{1\perp}^2/\nu_1^2 - P_{2\perp}^2/\nu_1^2} (1 + c\boldsymbol{P}_{1\perp} \cdot \boldsymbol{P}_{2\perp})$$

- ***Is this method useful?***

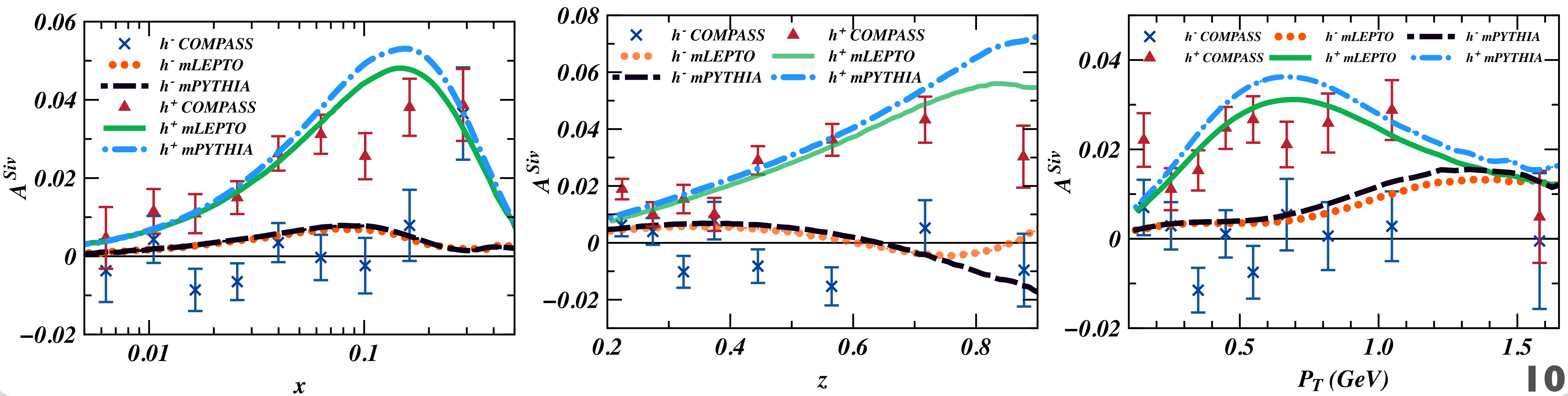
❖ Additional information on Sivers PDF: Flavor decomposition!
❖ Need the new (yet unknown) DiFFs!
❖ Is it feasible to measure these SSAs?

EVENT GENERATORS + SIVERS EFFECT

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

- Use ***PYTHIA 6.4*** (and ***LEPTO*** earlier) (*F77-yuk*).
- Incorporate dynamical hadronization mechanism: one, two,... hadron FFs.
- Sivers effect modulates quark TM's azimuthal angle: *relatively easy* to include in MC generators.
- Use Sivers PDF extraction from *Torino group*.
- Event generators allow to study *exp. kinematics effects*.

❖ ***Does it working?***

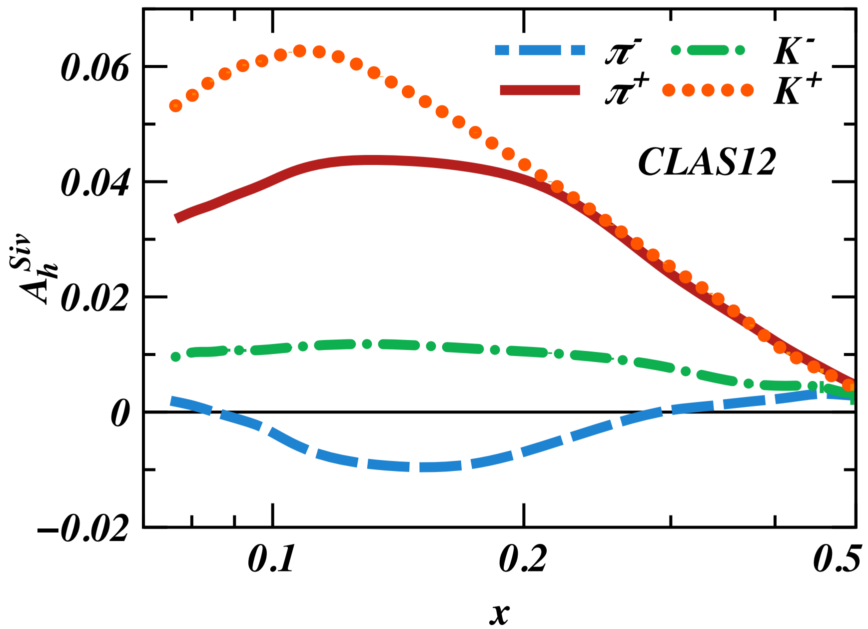


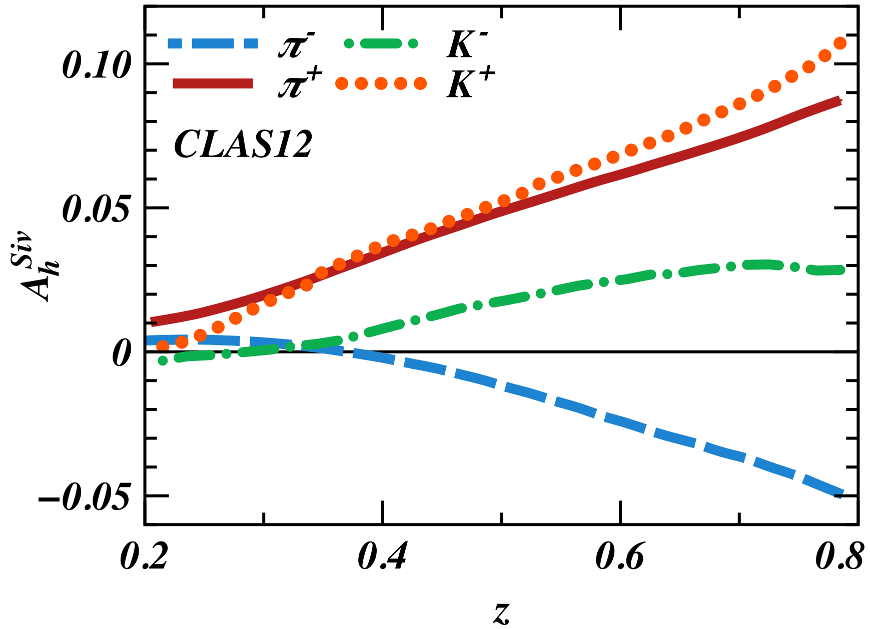
Sivers SSAs at CLAS12

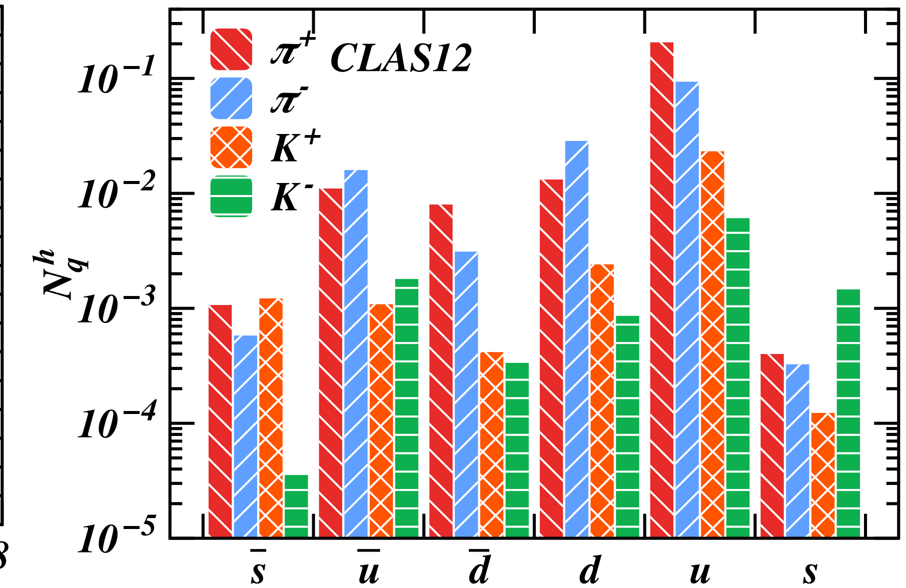
H.M et al., arXiv:1502.02669 (2015).

❖ ***Exploring the large x region.***

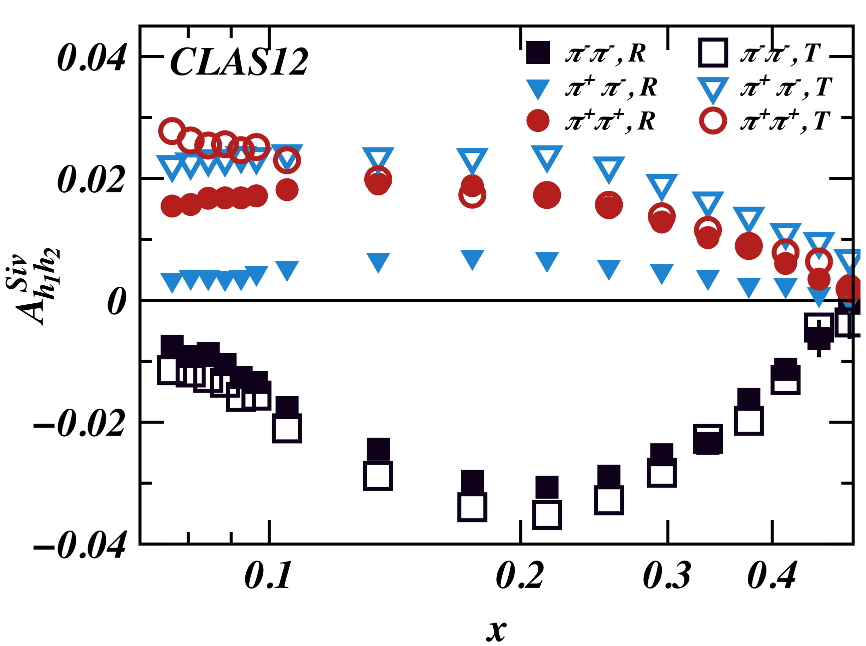
✦ **Single hadron** SSAs.

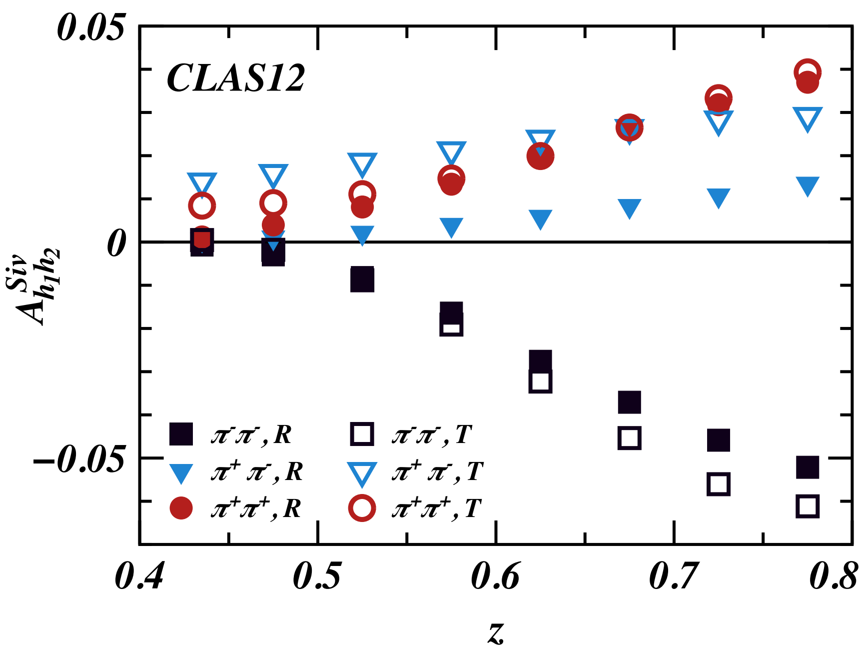


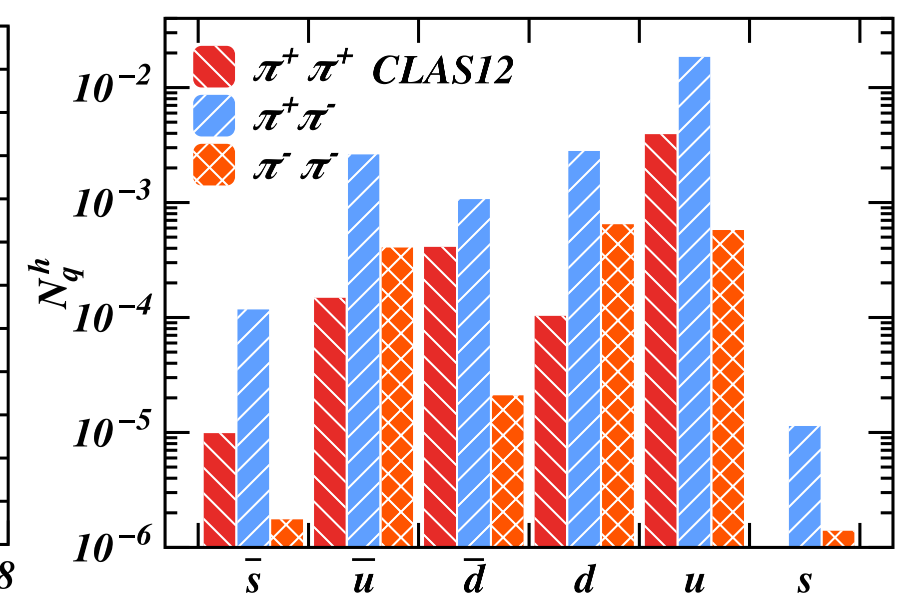




✦ **Dihadron** SSAs for pion pairs: identical pairs via z-ordering $z_1 \geq z_2$





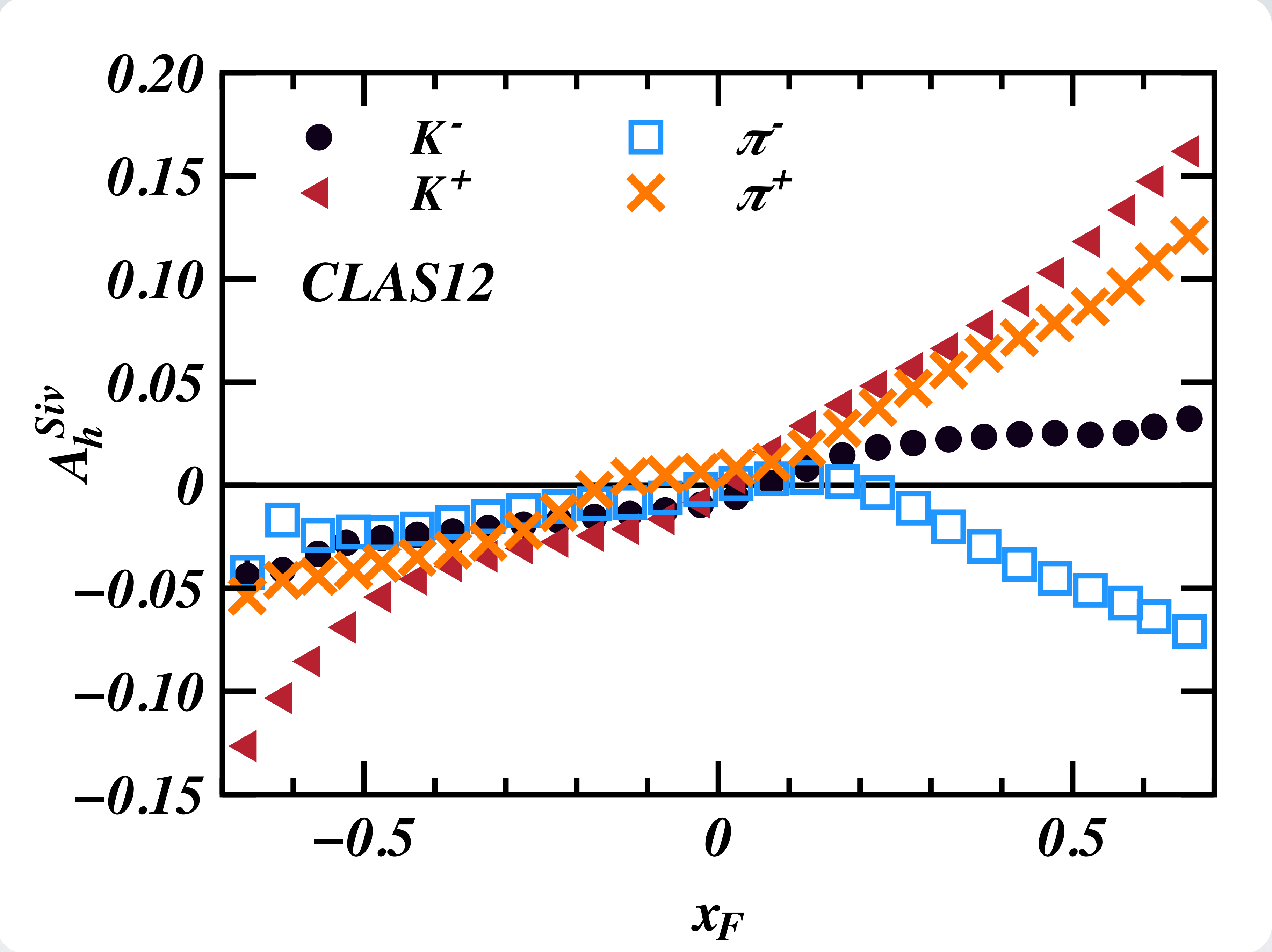


✦ Both Single and Dihadron SSAs are comparable in size!

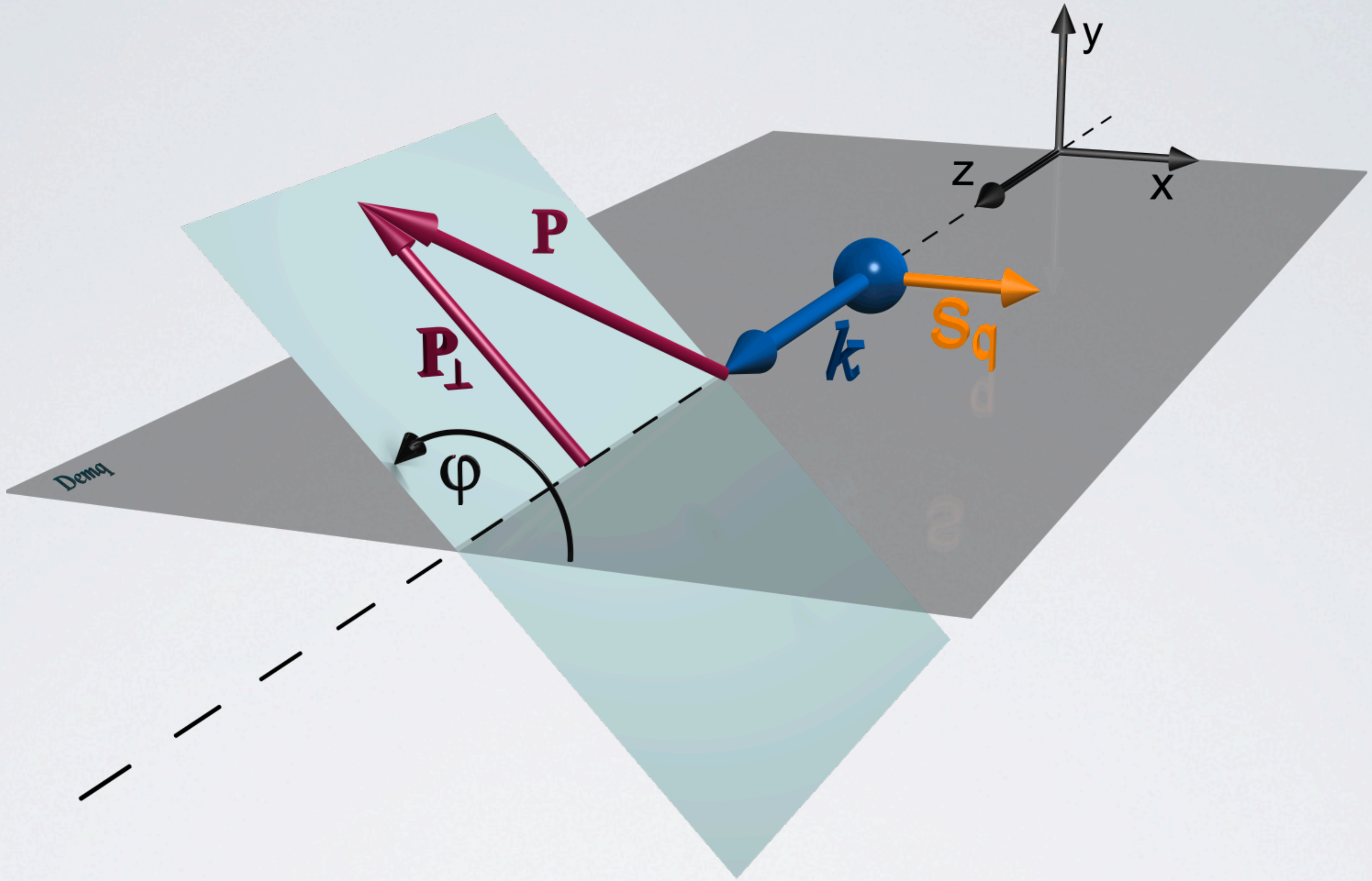
Sivers SSAs at CLAS12

H.M et al., arXiv:1502.02669 (2015), accepted in PRD.

- ***Explore Target Fragmentation Regions*** $x_F < 0$.



- Sivers SSA changes sign in some channels, fragmentation of nucleon remnant (recoil TM)!

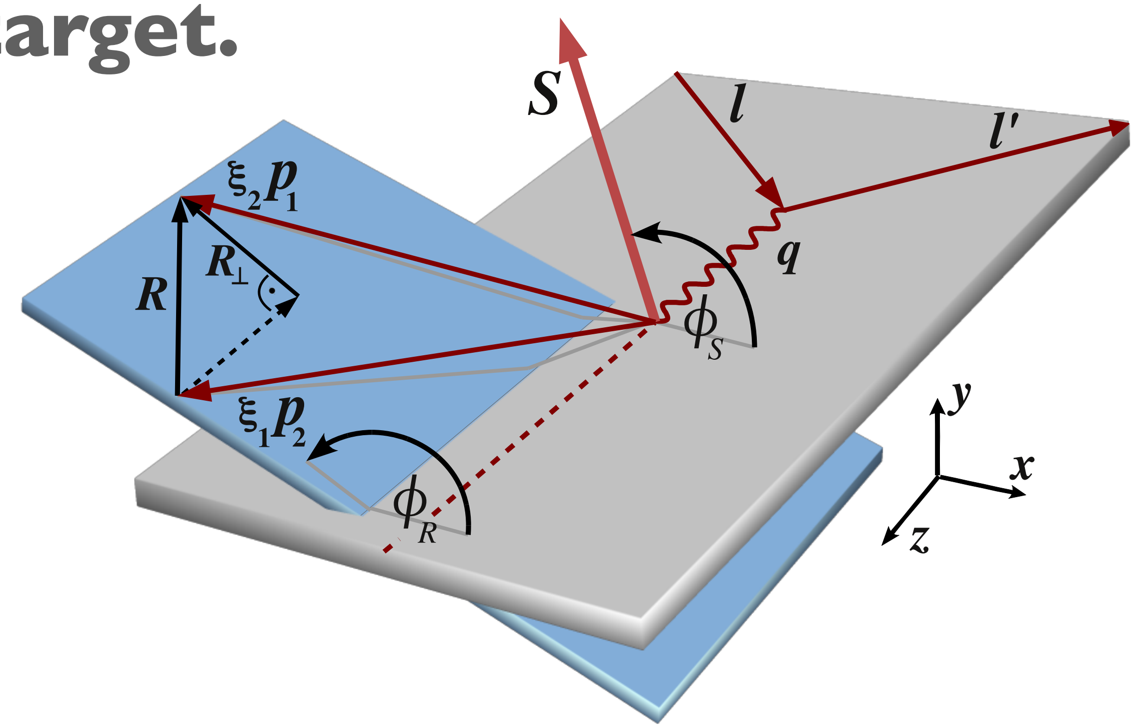


TRANSVERSELY POLARIZED QUARK FRAGMENTATION:
COLLINS EFFECT AND TWO-HADRON CORRELATIONS

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

✦**SIDIS with transversely polarized target.**



✦**Collins single spin asymmetry:**

$$A_{Coll} = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \, q \otimes D_1^{h/q}}$$

✦**Two hadron single spin asymmetry:**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

✦**Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

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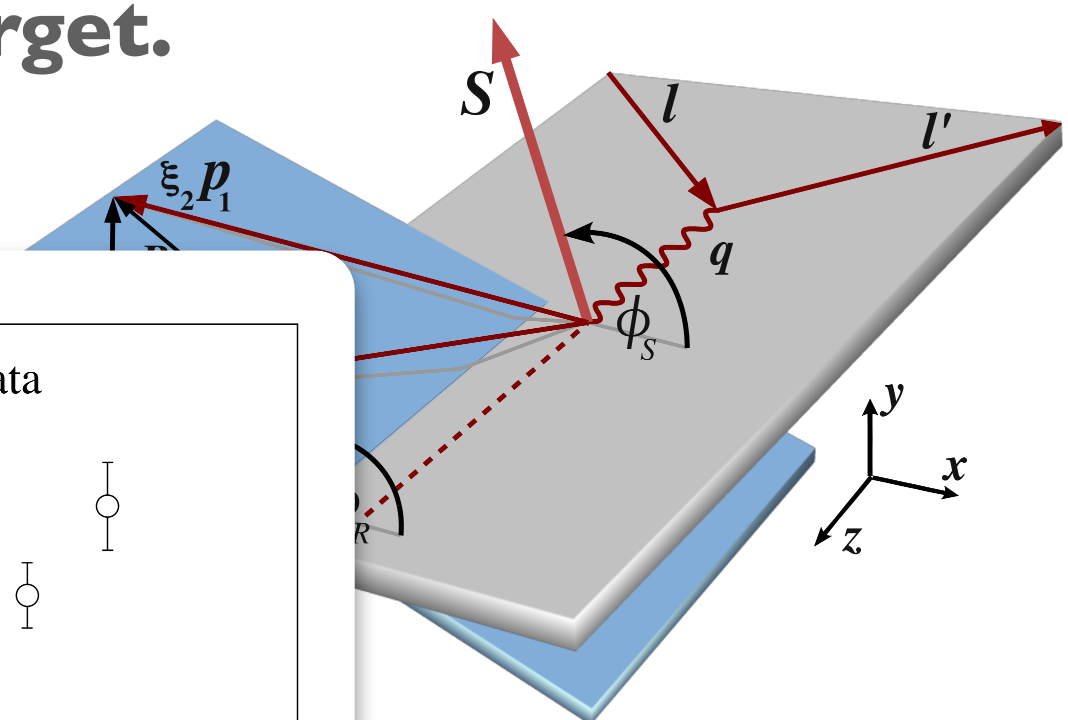
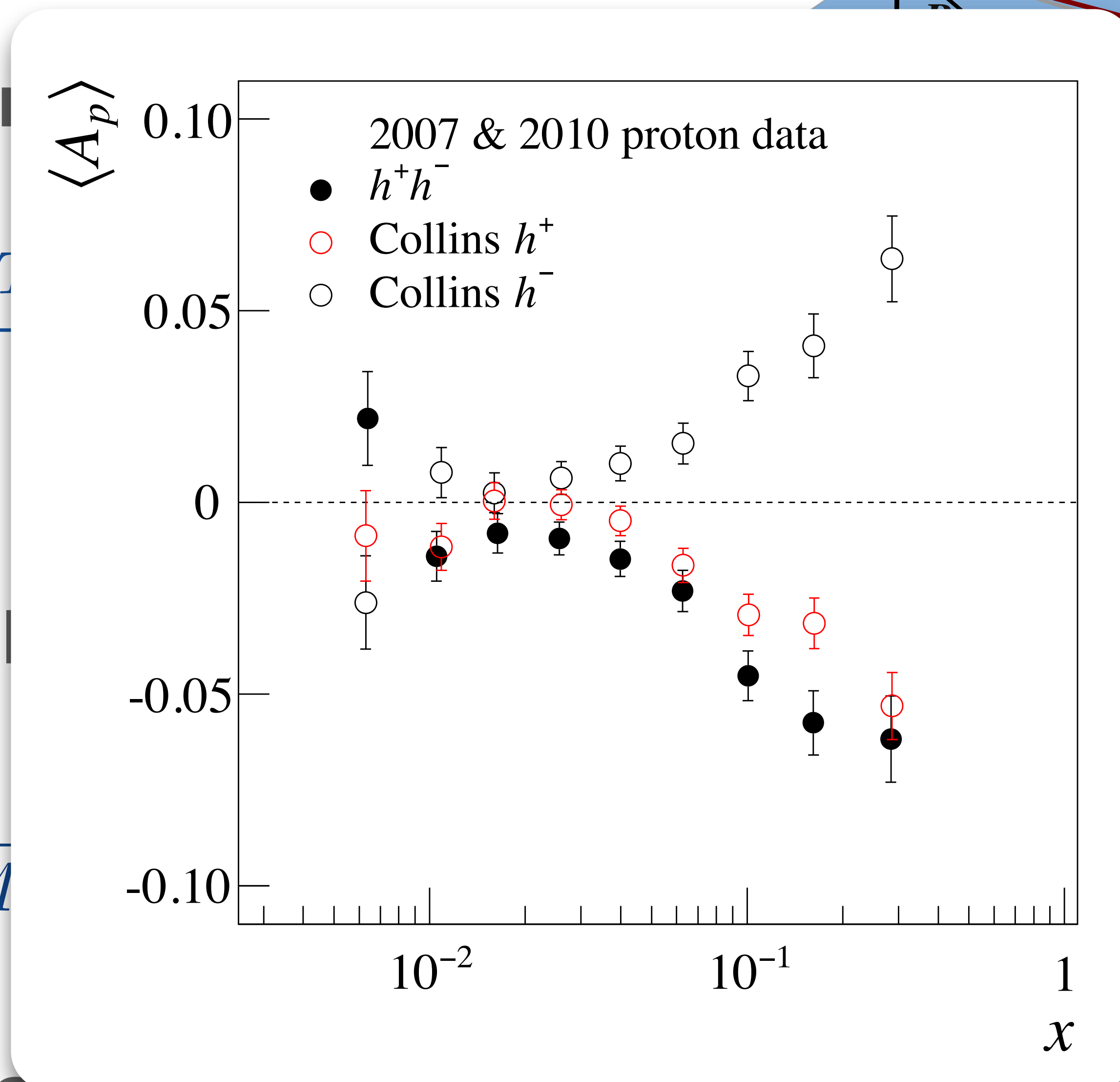


✦**Collins single spi**

$$A_{Coll} = \frac{\sum_q e_q^2\ \Delta_T}{\sum_q e_q^2}$$



✦**Two hadron singl**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1}{2M} \quad \frac{\cos\theta)}{\cos\theta)}$$

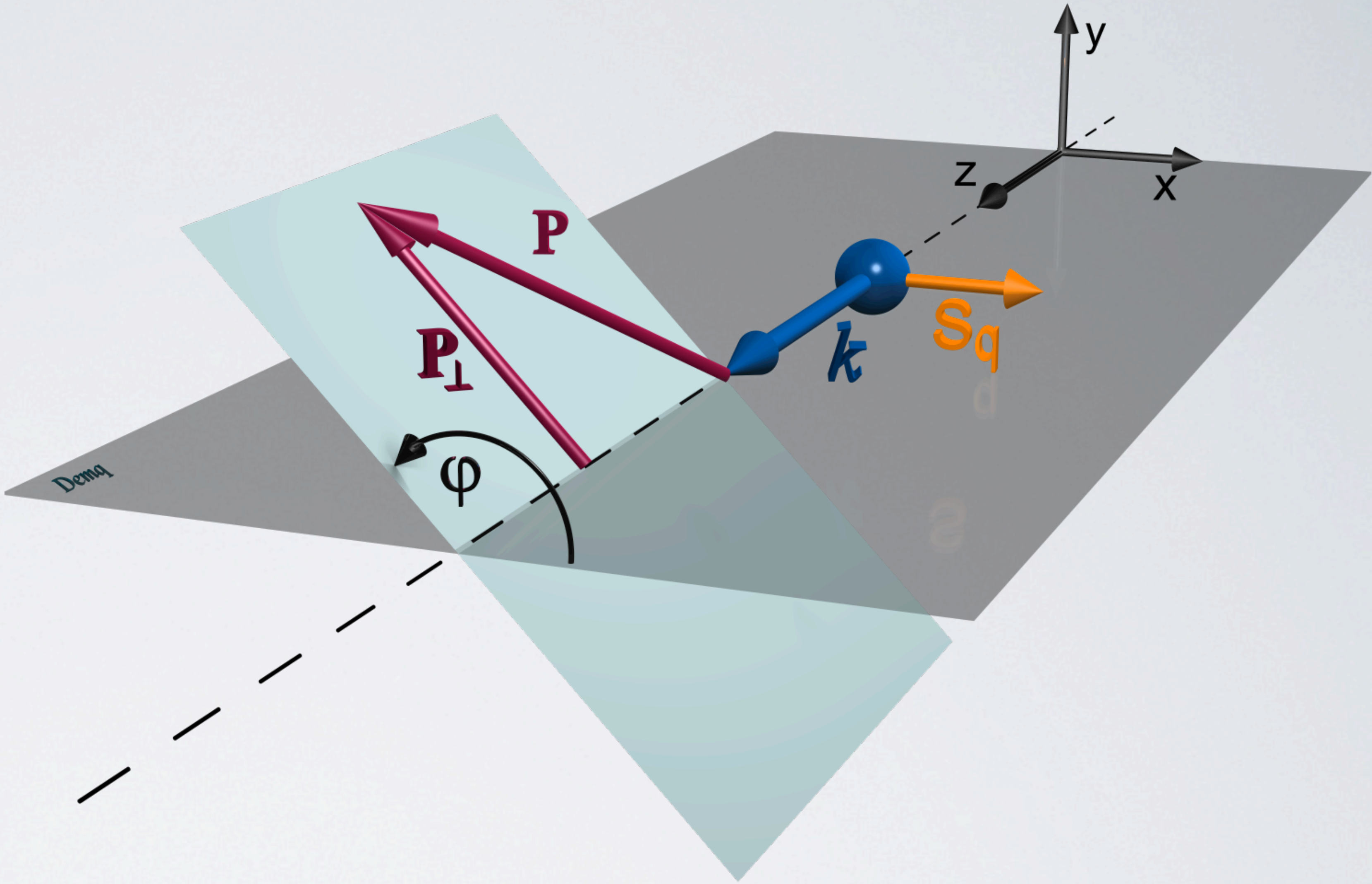
✦**Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2\boldsymbol{P}_1 - z_1\boldsymbol{P}_2}{z_1 + z_2}$$

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

$$D_{h/q^{\uparrow}}(z, P_{\perp}^{2}, \varphi) = D_{1}^{h/q}(z, P_{\perp}^{2}) - H_{1}^{\perp h/q}(z, P_{\perp}^{2}) \frac{P_{\perp} S_{q}}{z m_{h}} \sin(\varphi)$$

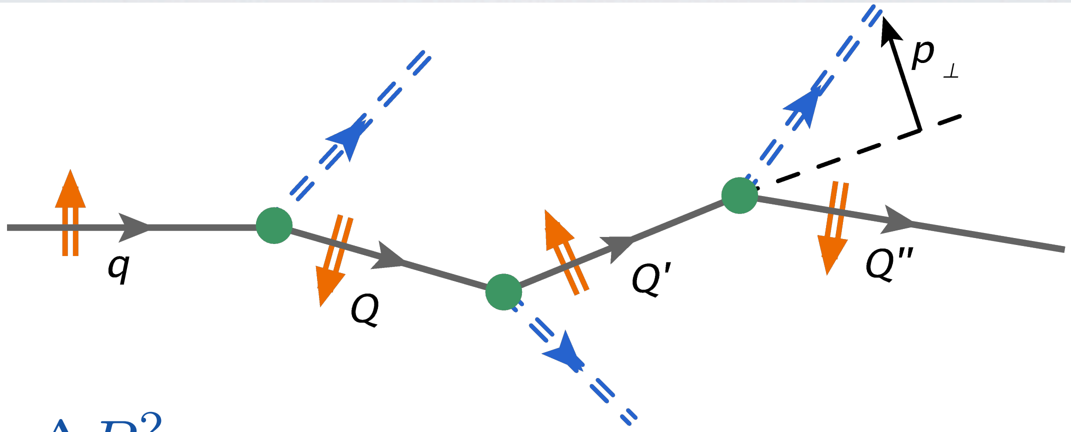
Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

COLLINS FRAGMENTATION FUNCTION FROM NJL-JET

H.M.,Bentz, Thomas, PRD.86:034025, 2012.

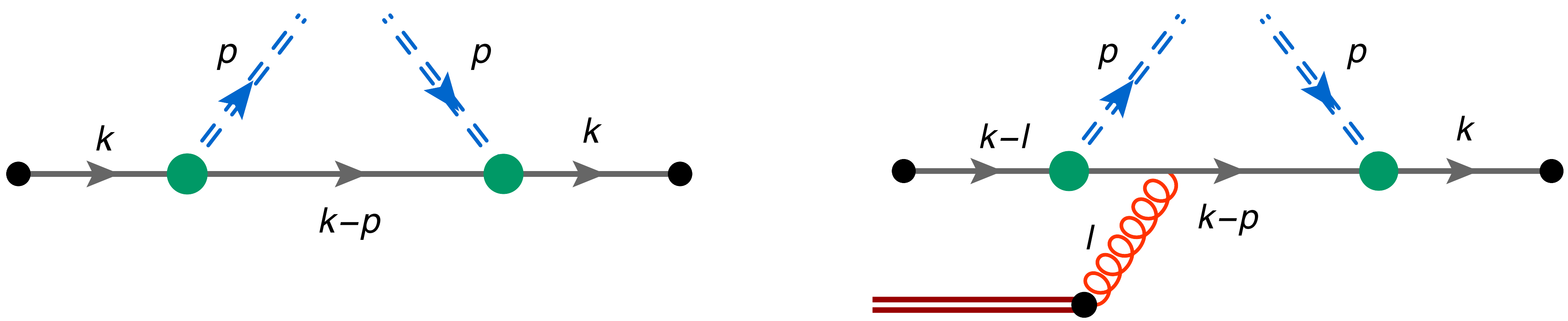
- **Extend the NJL-jet Model to Include the Quark's Spins.**



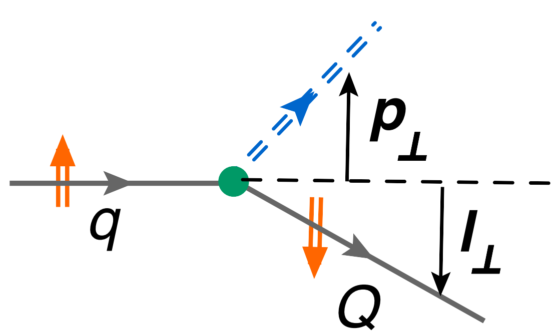
$$D_{h/q^{\uparrow}}(z, P_{\perp}^2, \varphi)\, \Delta z\, \frac{\Delta P_{\perp}^2}{2}\, \Delta\varphi = \left\langle N_{q^{\uparrow}}^h(z, z+\Delta z; P_{\perp}^2, P_{\perp}^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

- **Model Calculated Elementary Collins Function as Input**

A. Bacchetta et. al., PLB659, 234 (2008).



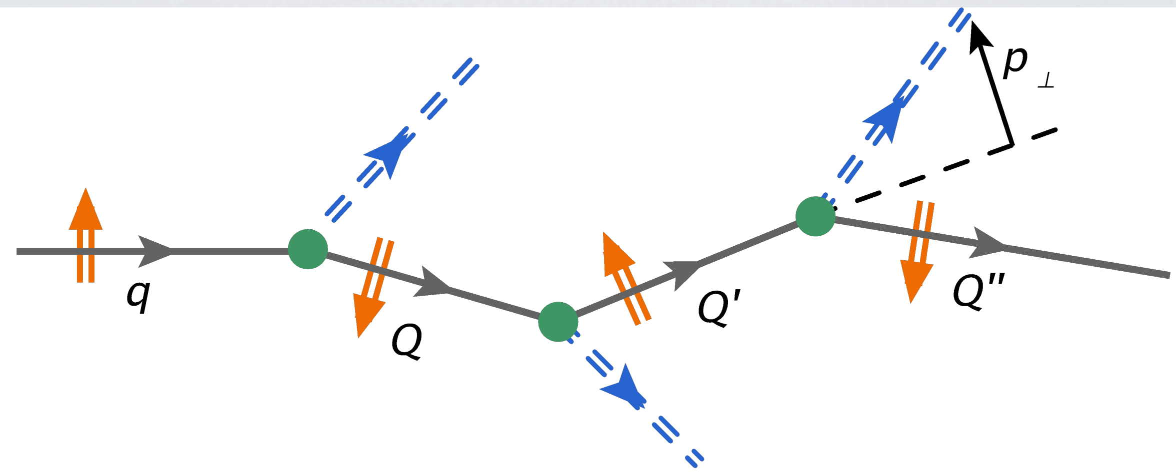
- ***Spin flip probability:*** $\mathcal{P}_{SF}$



POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- **Use the NJL-jet Model including Collins effect (Mk 2) to study DiFFs.**



$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle.$$

- ***Choose a constant Spin flip probability:*** $\mathcal{P}_{SF}$

- ***Simple model to start with:***

Only pions and extreme ansatz for the Collins term in elementary function.

$$d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)$$

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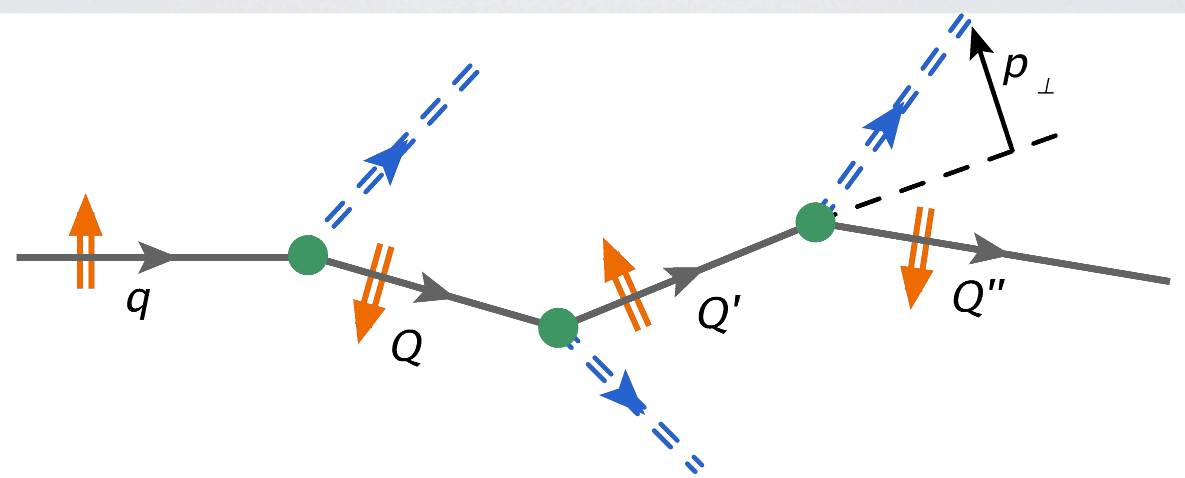

$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle.$$

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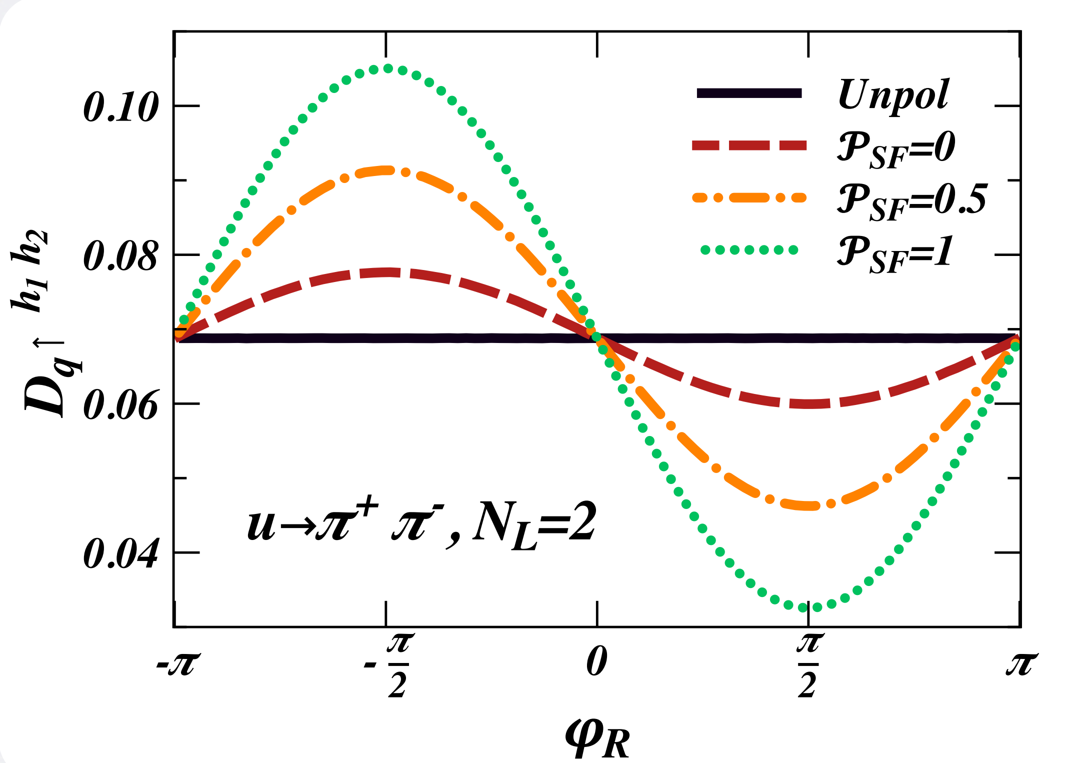
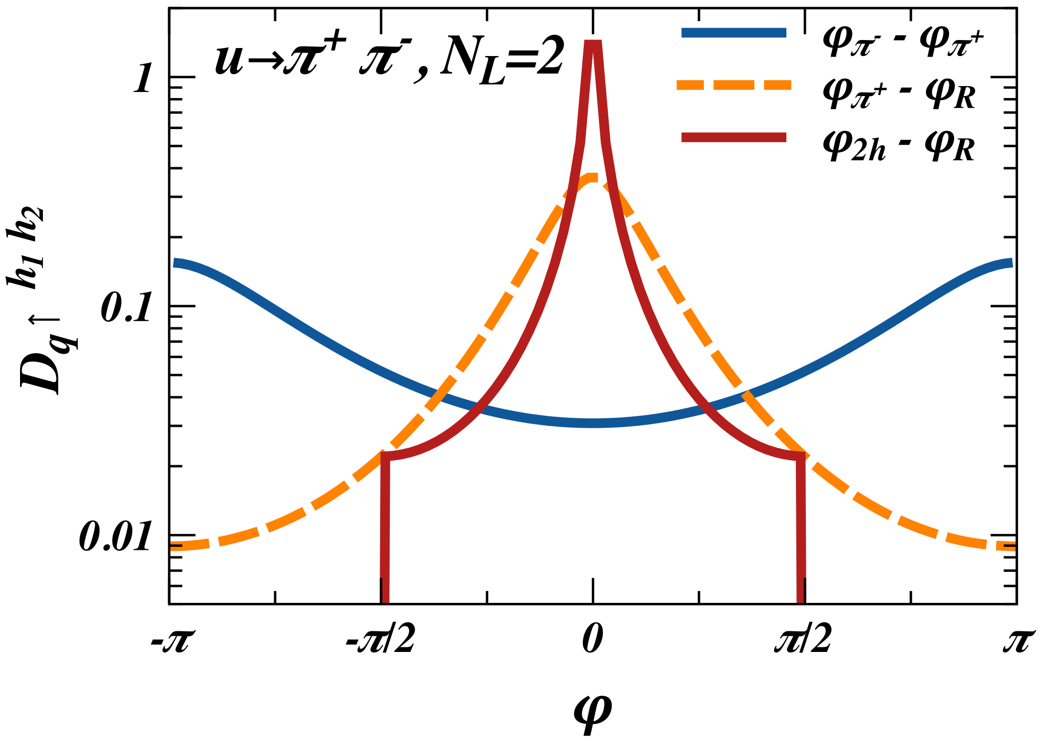


ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Quark-Jet



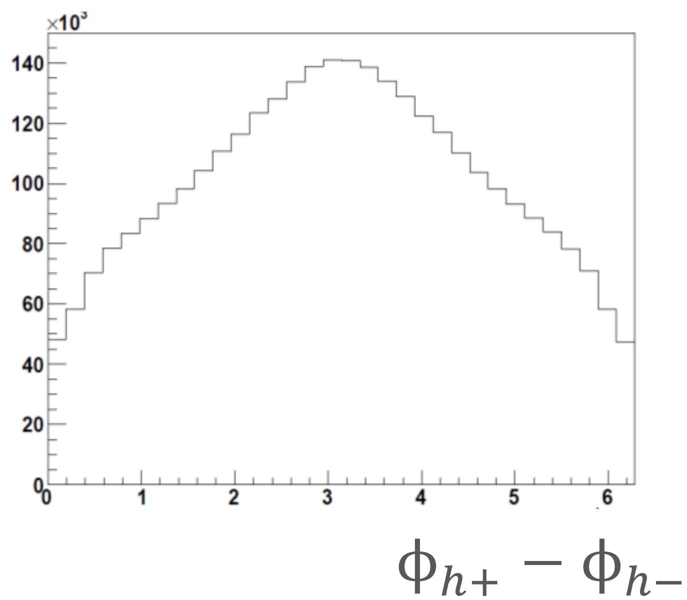
- We define:

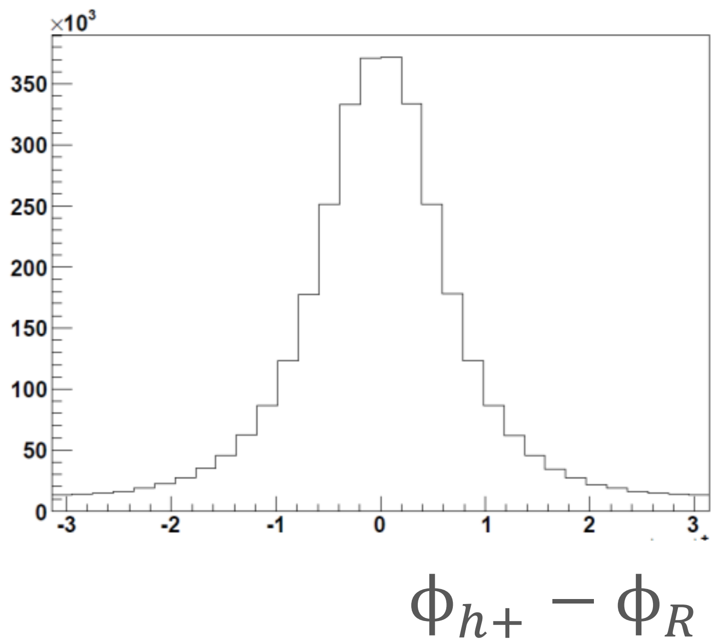
$$\mathbf{P}_{2h} = \frac{\widehat{\mathbf{P}}_1 - \widehat{\mathbf{P}}_2}{2}$$

- No Spin Dependence Included!

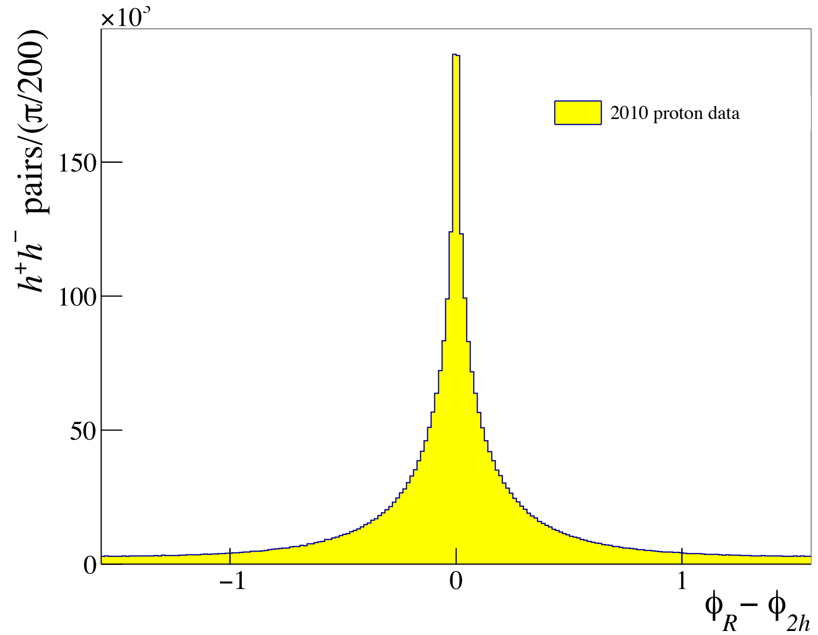
COMPASS Results

F. Bradamante - COMO 2013.



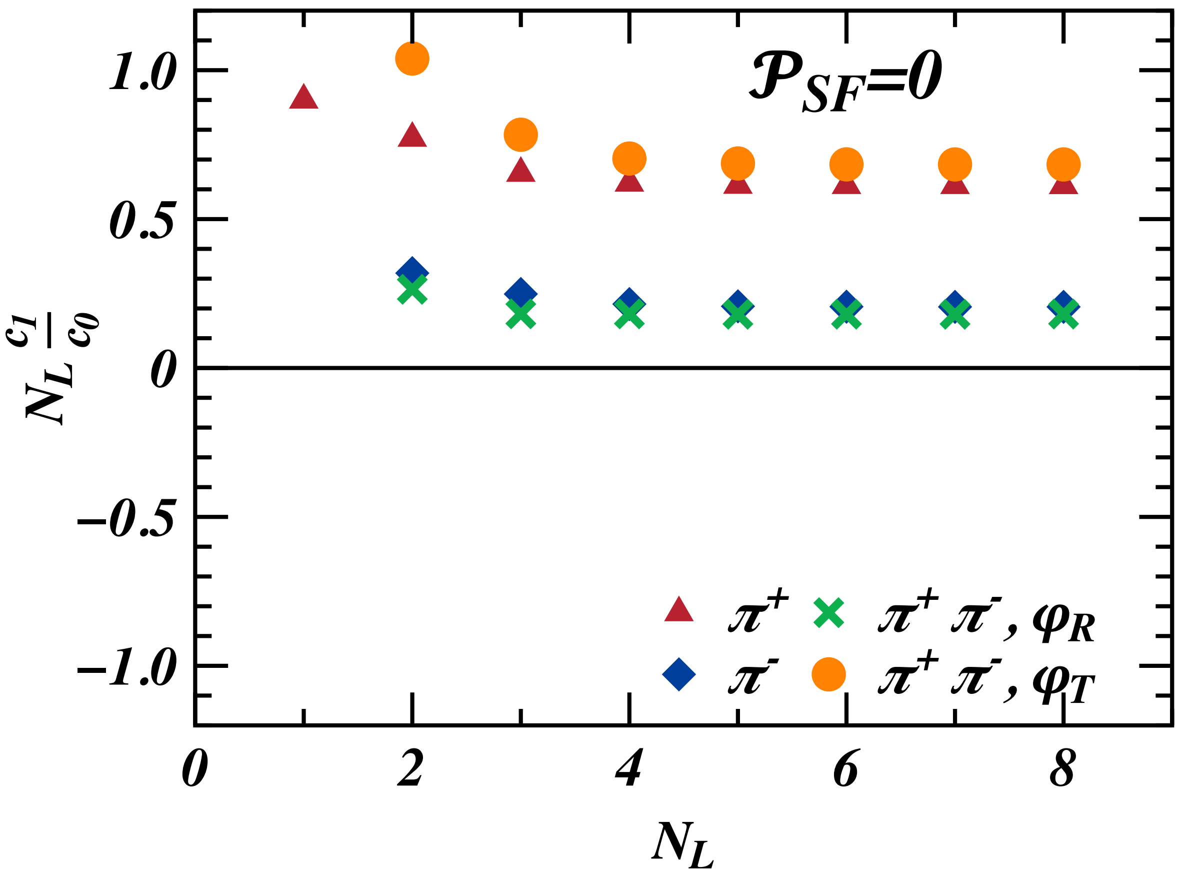


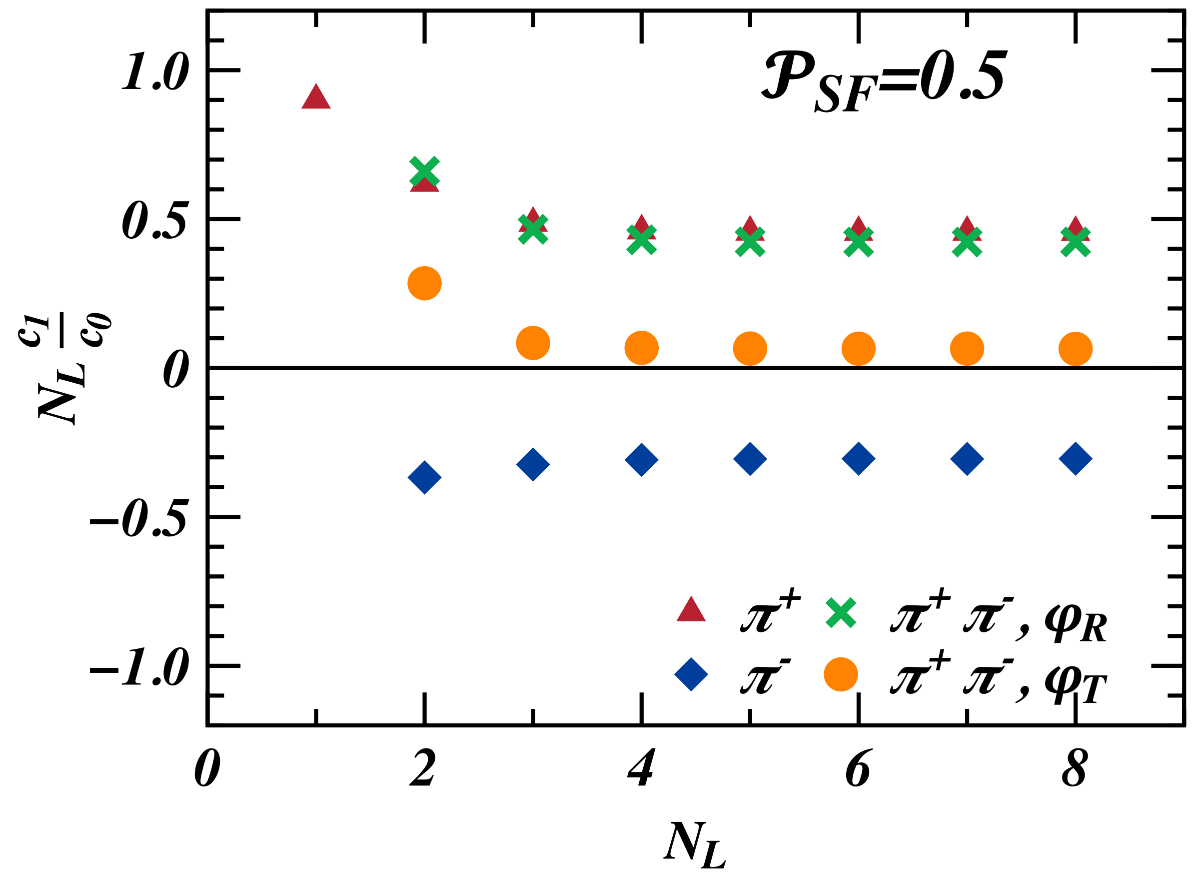
PLB736, 124-131 (2014).

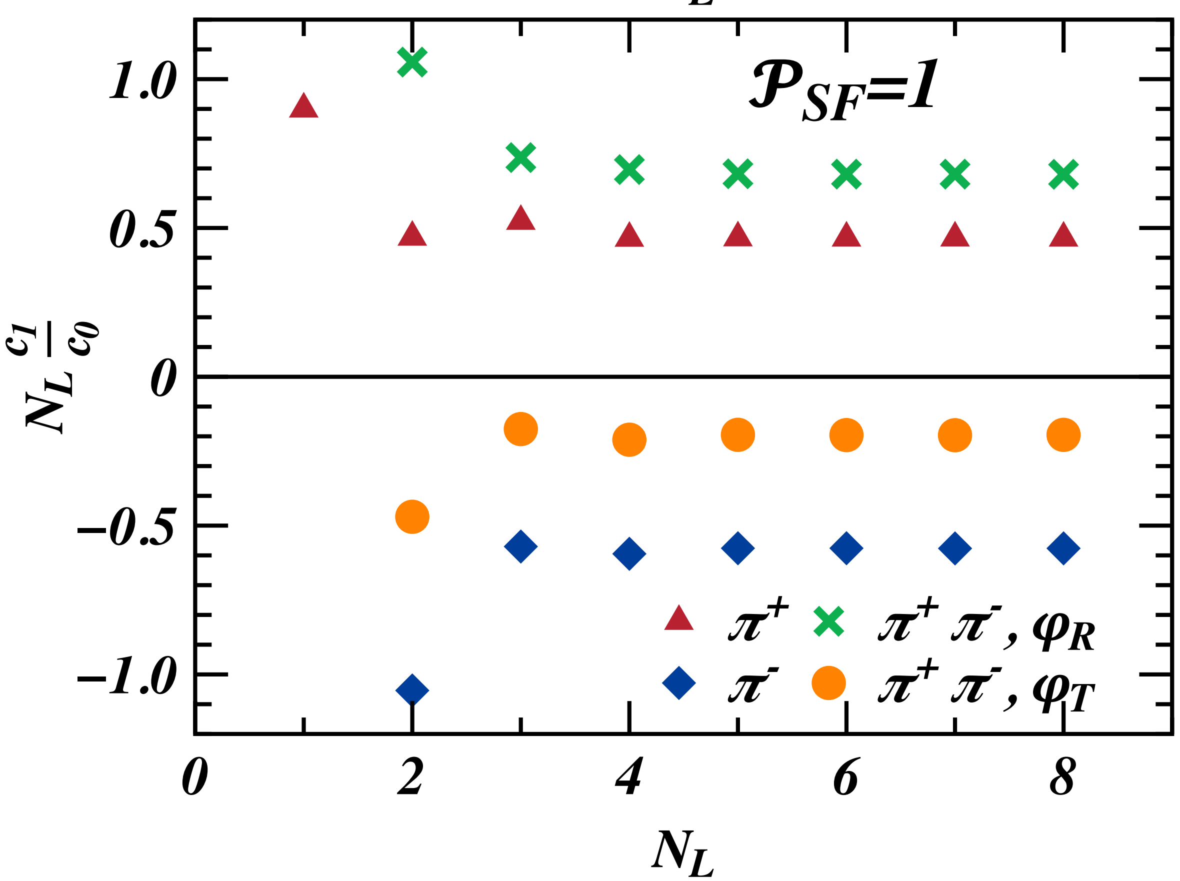


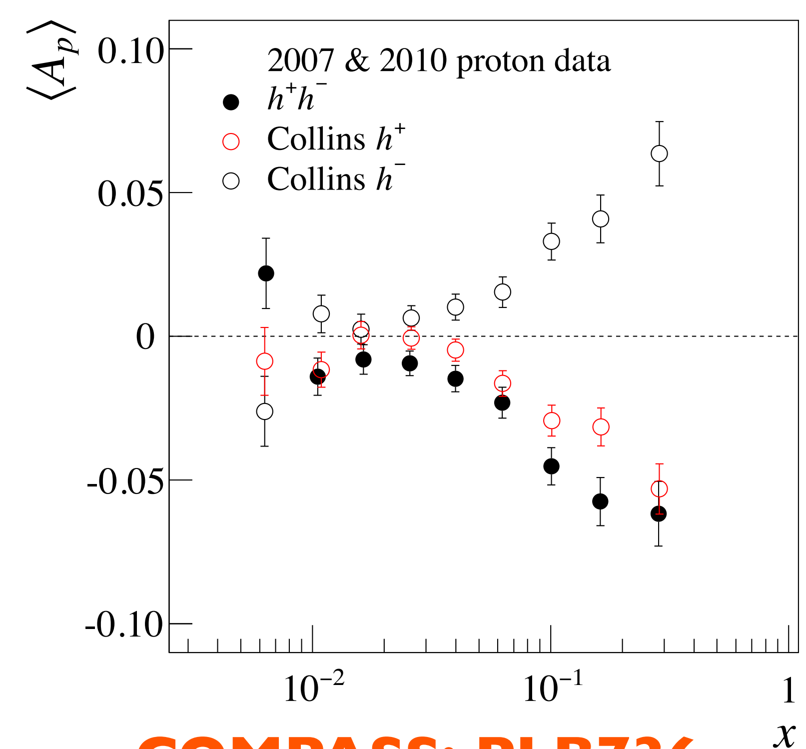
INTEGRATED ANALYZING POWERS

$z_{1,2} > 0.2, z > 0.2$







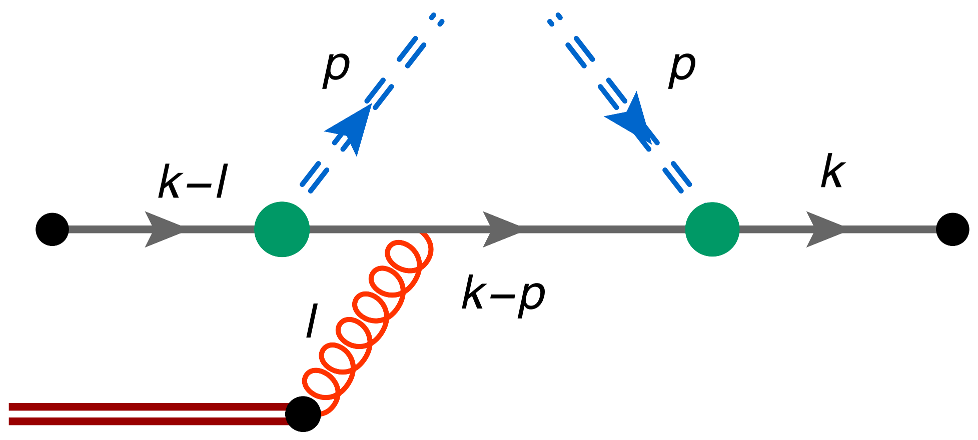


COMPASS: PLB736, 124-131 (2014).

IMPROVED MODEL FOR COLLINS EFFECT

- ✦**Use the *spectator model* for Collins function.**

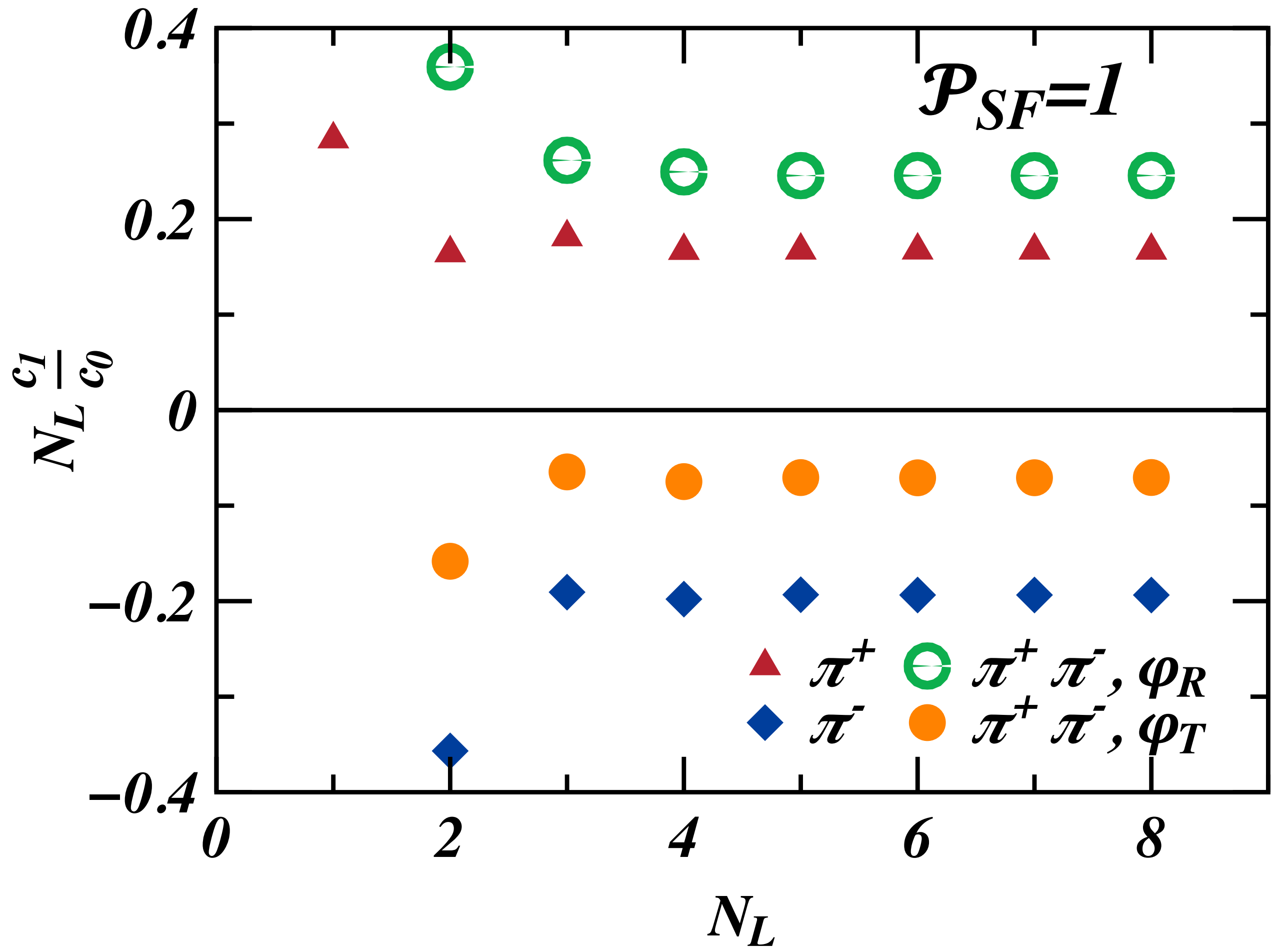
$$H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$



- ✦**Include both *pion and kaon* channels.**

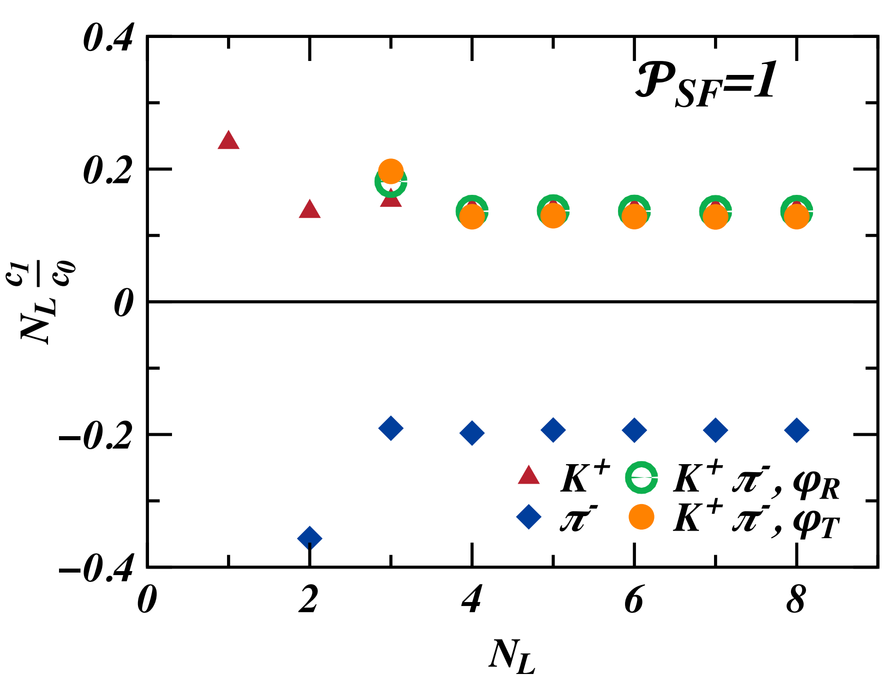
IMPROVED MODEL RESULTS

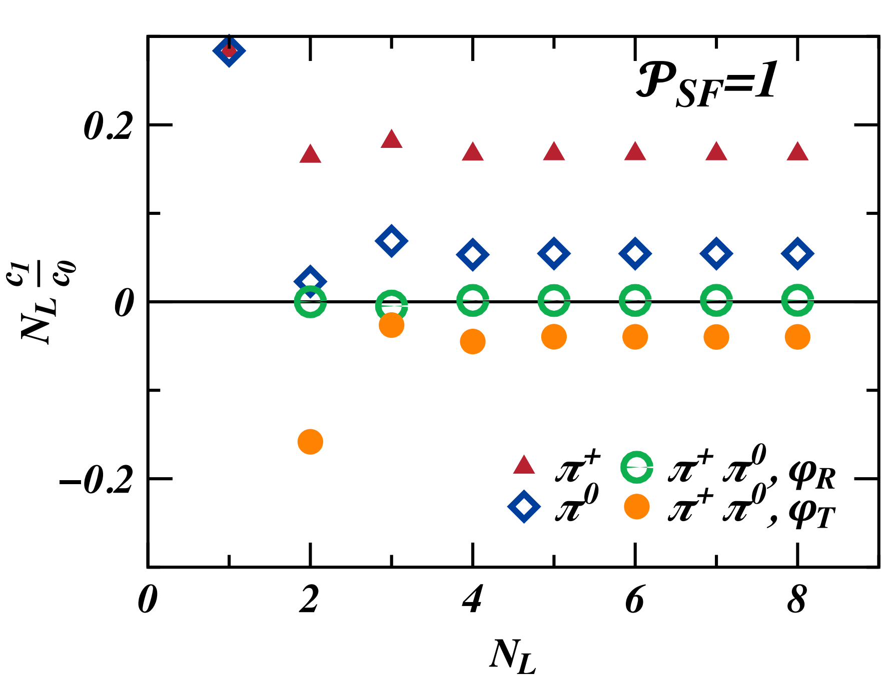
- ✦ Qualitatively the ***same*** picture as in the Toy-model calculations.

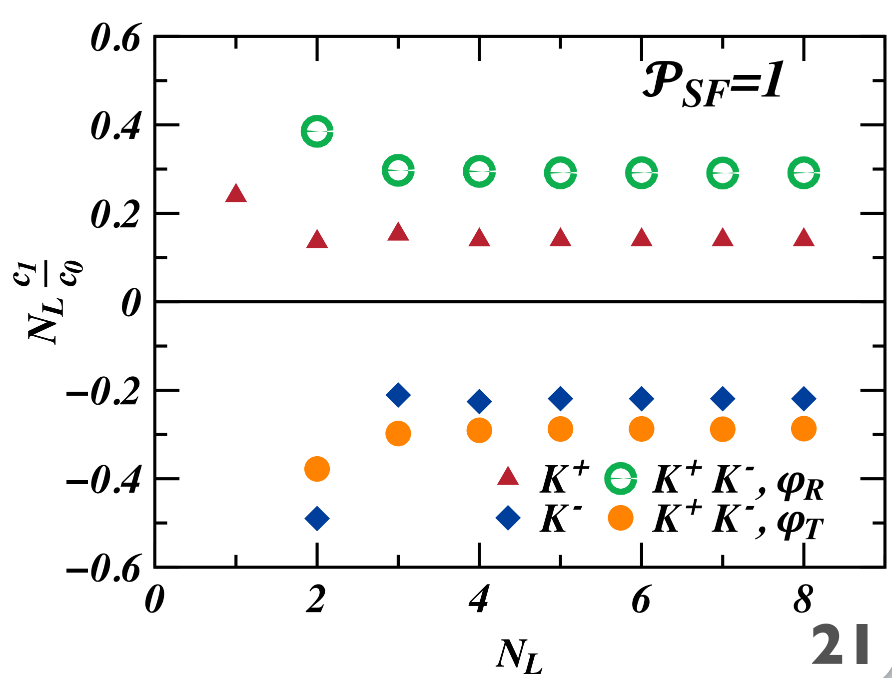


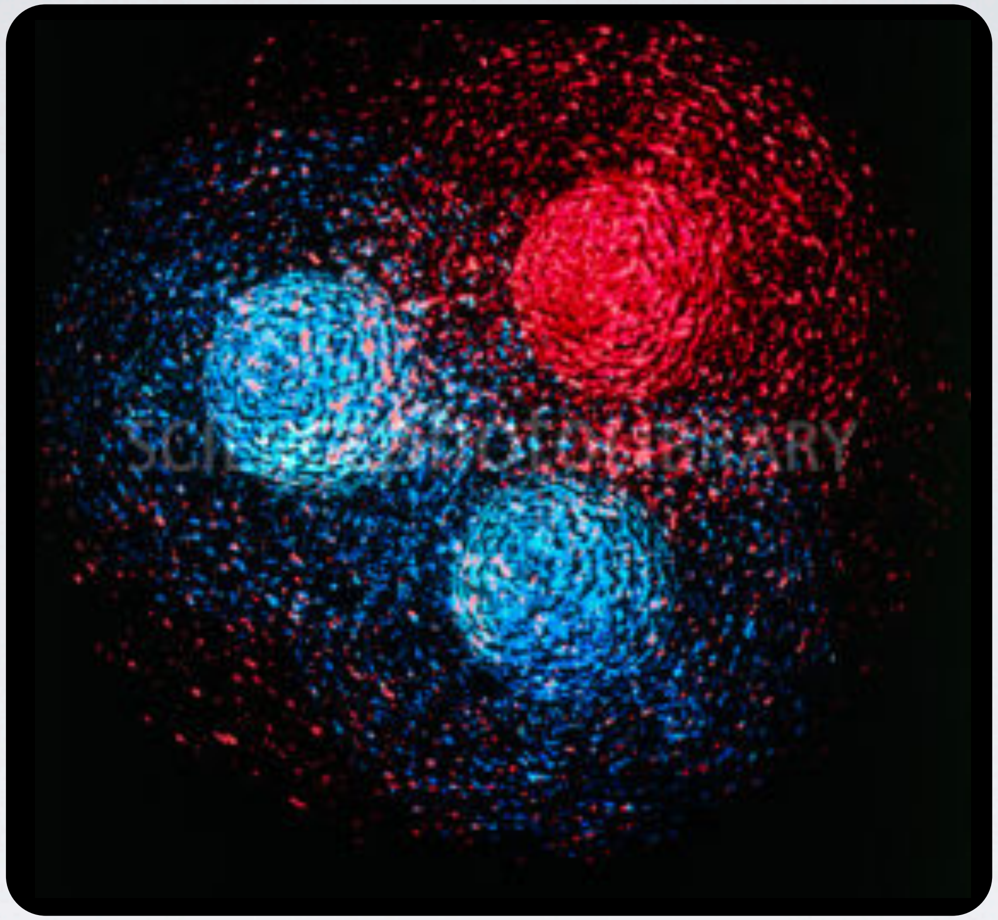
- ✦ ***Consistent*** with COMPASS results.

- ✦ Predictions for various hadron pairs.





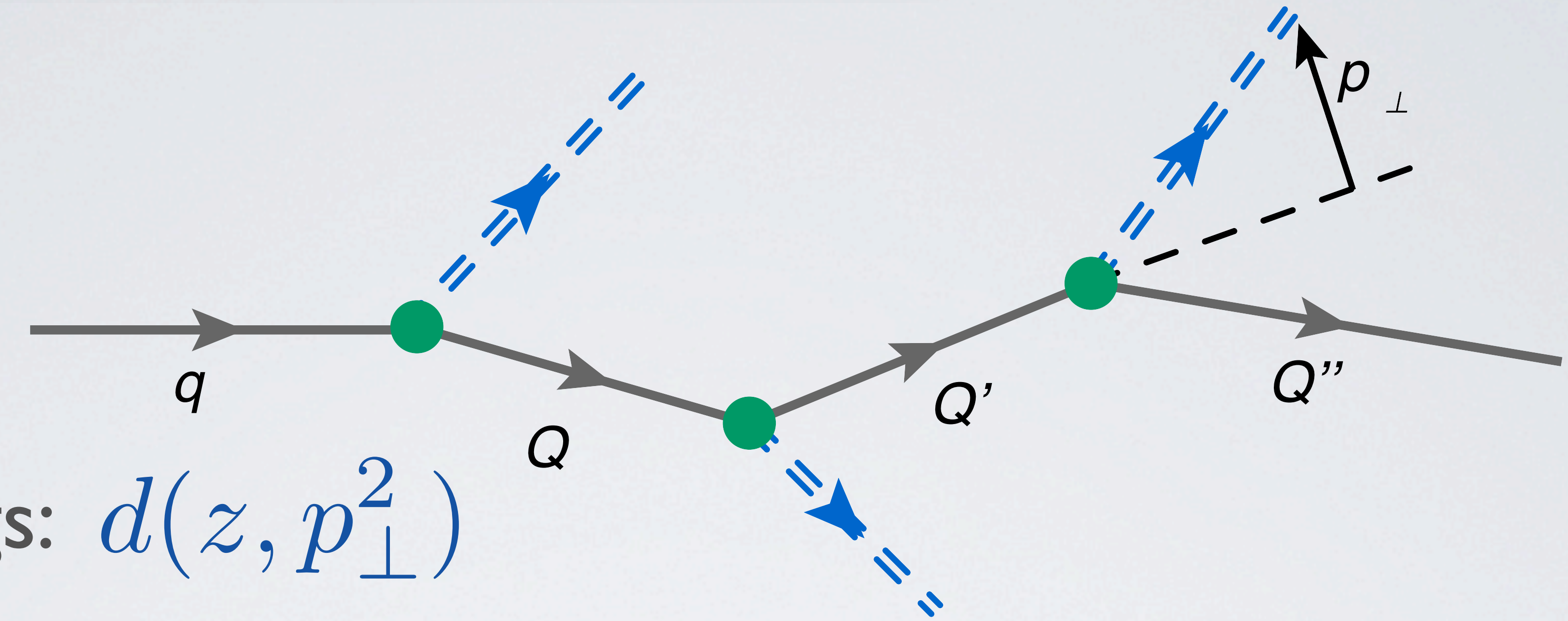




TRANSVERSE MOMENTUM DEPENDENCE

TMD FRAGMENTATION FUNCTIONS

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

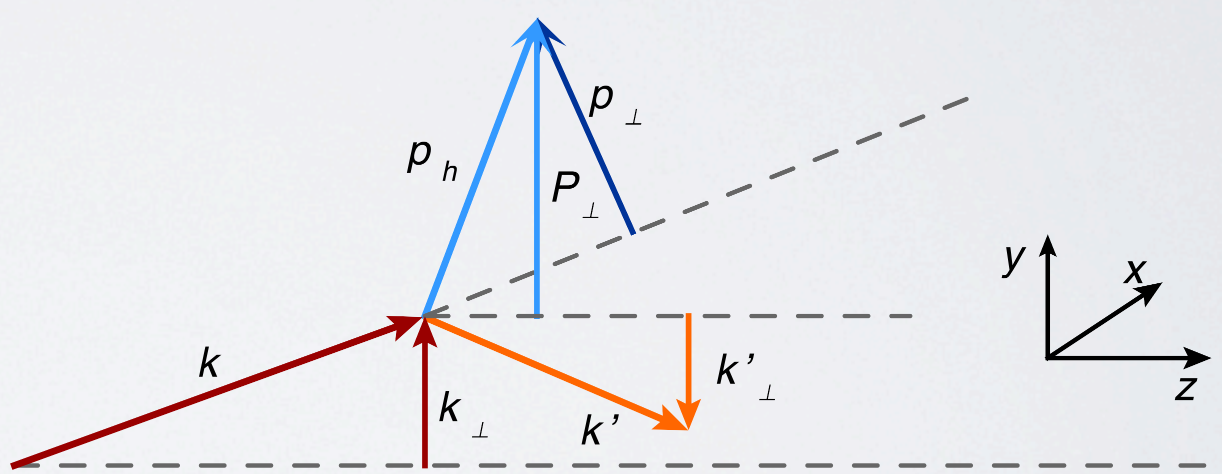


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp$$

$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



- Calculate the Number Density

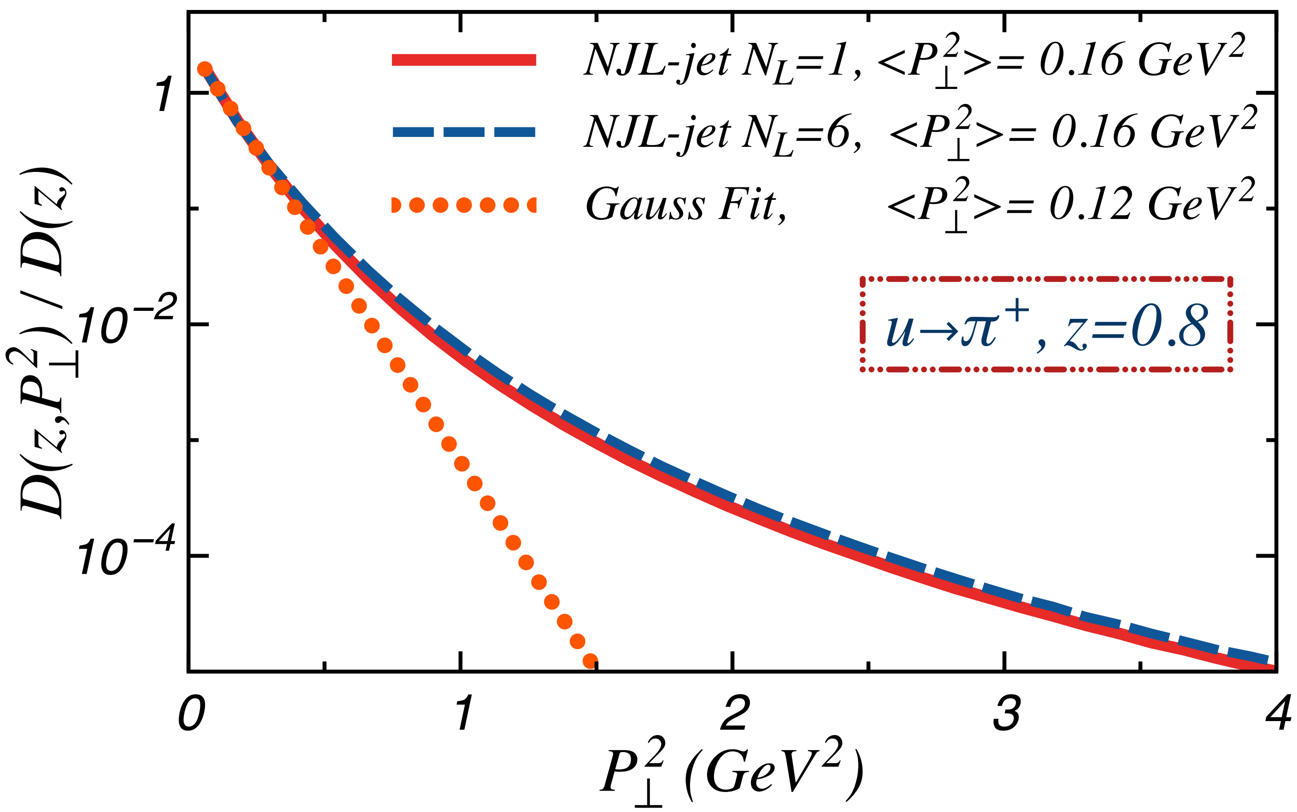
$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2+\Delta P_\perp^2)}{N_{Sims}}.$$

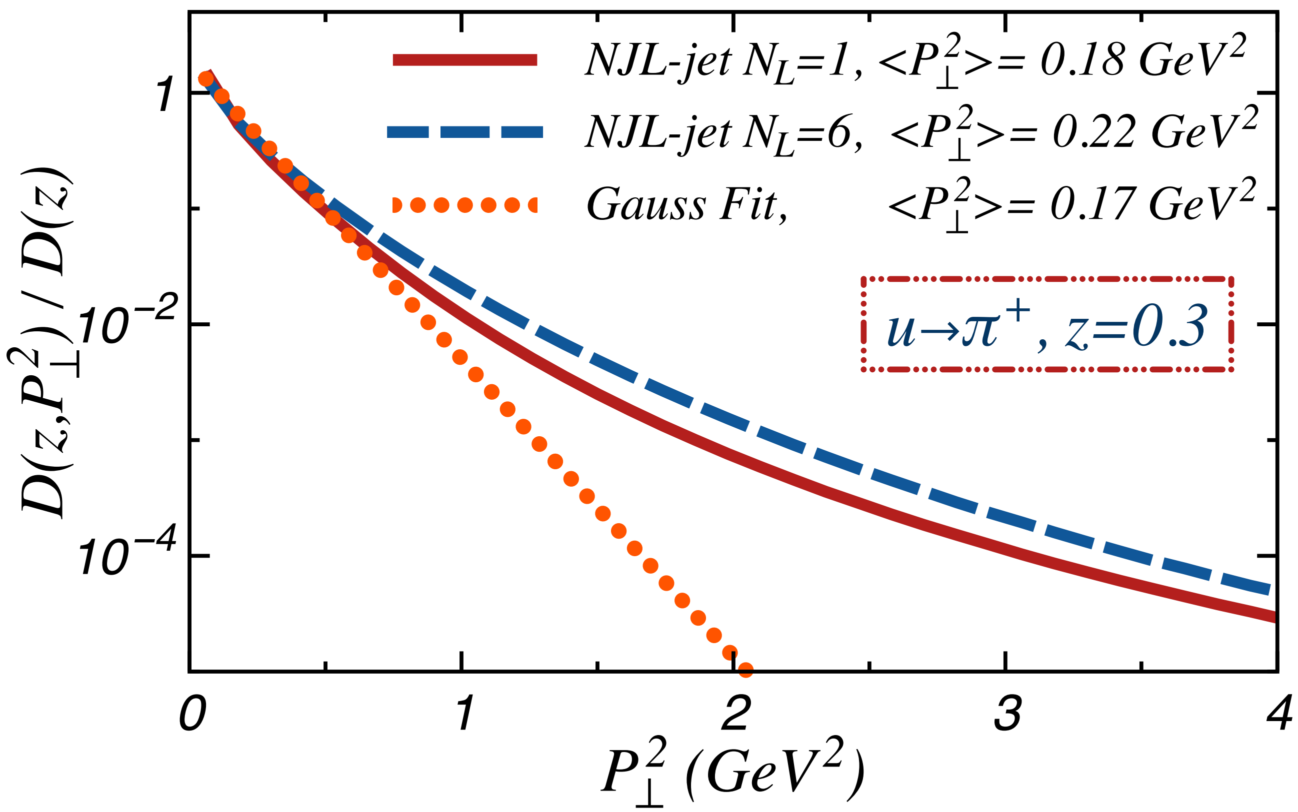
COMPARISON WITH GAUSSIAN ANSATZ

- TMD Dependence of the splitting function.

$$d_q^h(z,p_\perp^2) \sim \frac{p_\perp^2 + [(z-1)M_1 + M_2]^2}{[p_\perp^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_h^2]^2} \frac{1}{[1+(M_{12}^2/\Lambda_{12}^2)^2]^2}$$

- TMD Dependence of the full fragmentation function.





- Gaussian ansatz: $D(z,P_\perp^2) = D(z)e^{-P_\perp^2/\langle P_\perp^2\rangle}/\pi\langle P_\perp^2\rangle$

✓ Multiple hadron emissions *broaden* the TM dependence, more significant at *small z*.

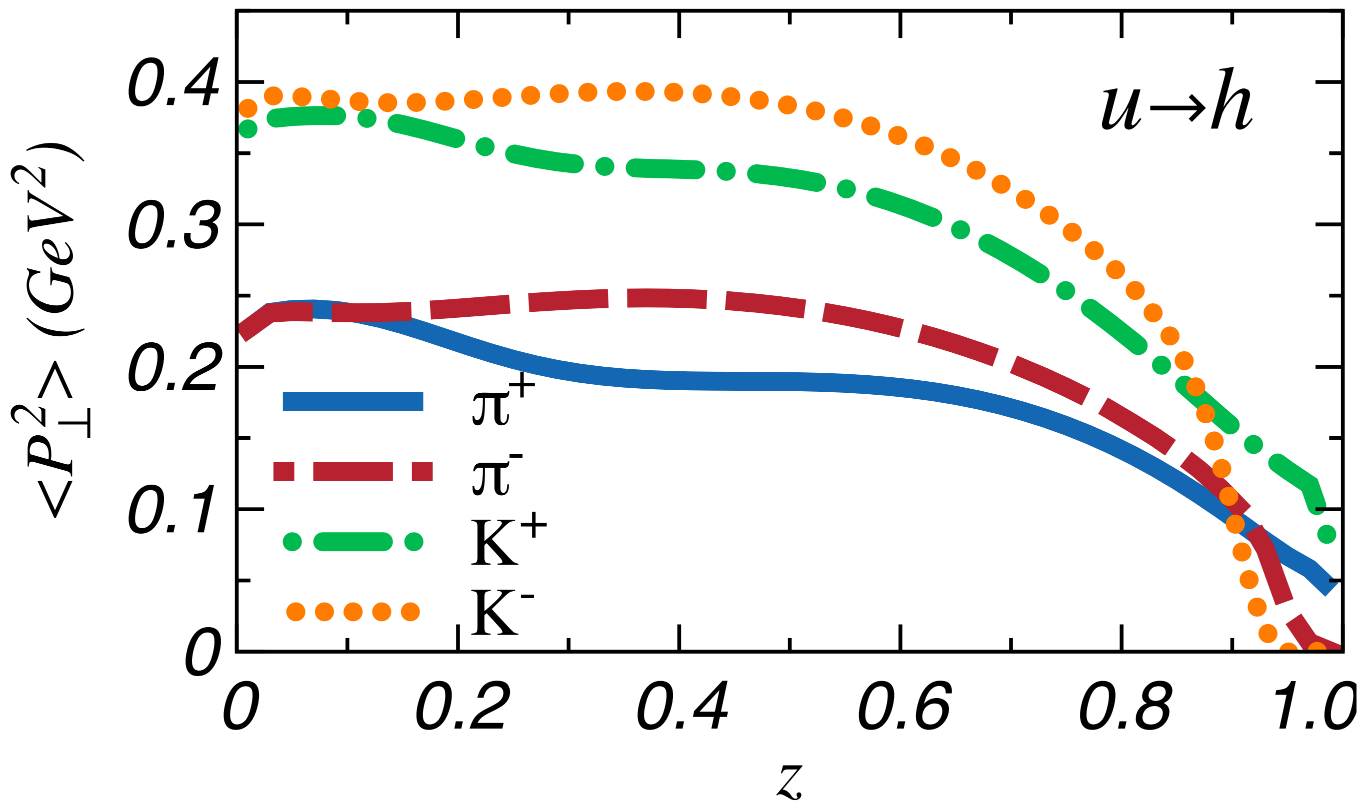
AVERAGE TRANSVERSE MOMENTA VS Z

FRAGMENTATION

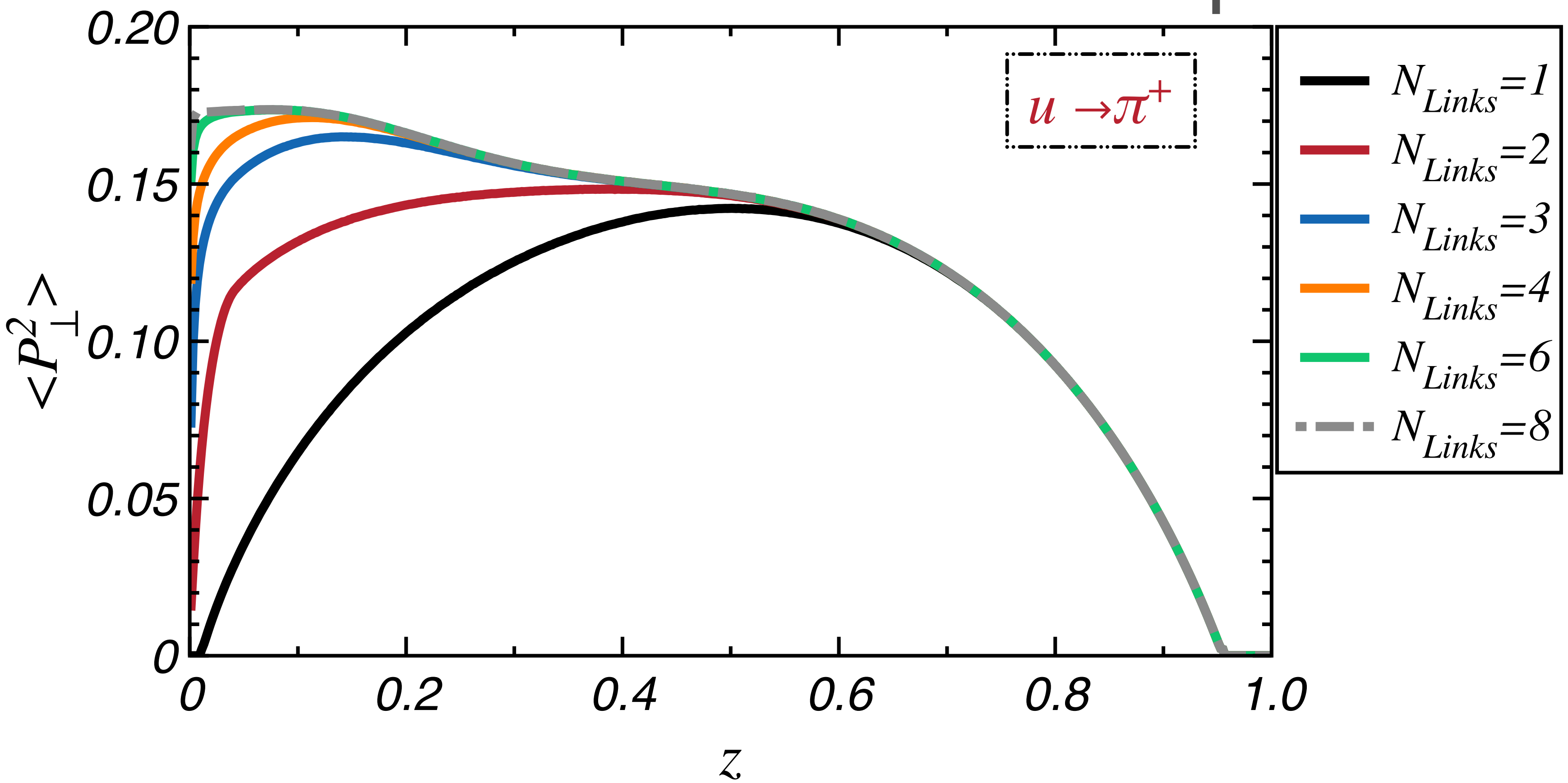
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data:

A. Signori, et al: JHEP 1311, 194 (2013)



✓Multiple hadron emissions: *broaden* the TM dependence at *low z*!



CONCLUSIONS

- ***Two-Hadron SIDIS*** will provide information for mapping the TM and flavor dependencies of Sivers and Transversity PDFs.
- We need unintegrated Dihadron Fragmentation Functions.
- Measurements of ***IFFs*** for both ***relative*** and ***total*** TM will be crucial for understanding the hadronization process.
- The modified full Event Generators *incorporating Sivers effect* (*mPYTHIA*): a useful tool for phenomenological studies.
- *mPYTHIA* predictions show *a great potential* for *measuring* two-hadron SIDIS SSAs at *CLAS12* and *EIC*.

BACKUP SLIDES

WHITE PAPER FOR NSAC-LRP

✦White paper on extracting DiFFs at BELLE II.

https://www.phy.anl.gov/nsac-lrp/Whitepapers/StudyOfFragmentationFunctionsInElectronPositronAnnihilation.pdf

TWO-HADRON SIDIS

- Cross Section in terms of **Total and Relative Momenta**

$$\boldsymbol{P}_h = \boldsymbol{P}_1 + \boldsymbol{P}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{P}_1 - \boldsymbol{P}_2)$$

- The Sivers term:

$$\sigma_S = S_T \left(\sigma_T \frac{P_{hT}}{M} \sin(\varphi_T - \varphi_S) + \sigma_R \frac{R_T}{M} \sin(\varphi_R - \varphi_S) \right)$$

$$\int d\varphi_R \; \sigma_S = S_T \left(\sigma_{T,0} \frac{P_{hT}}{M} + \sigma_{R,1} \frac{R}{2M} \right) \sin(\varphi_T - \varphi_S)$$

$$\int d\varphi_T \; \sigma_S = S_T \left(\sigma_{T,1} \frac{P_{hT}}{2M} + \sigma_{R,0} \frac{R}{M} \right) \sin(\varphi_R - \varphi_S)$$

✦**Non-vanishing σ_R is new!** **Contradiction with earlier results** Bianconi: PRD62, 034008 (2000) **? No:** Kotzinian: EPJConf. 85 02026 (2015)

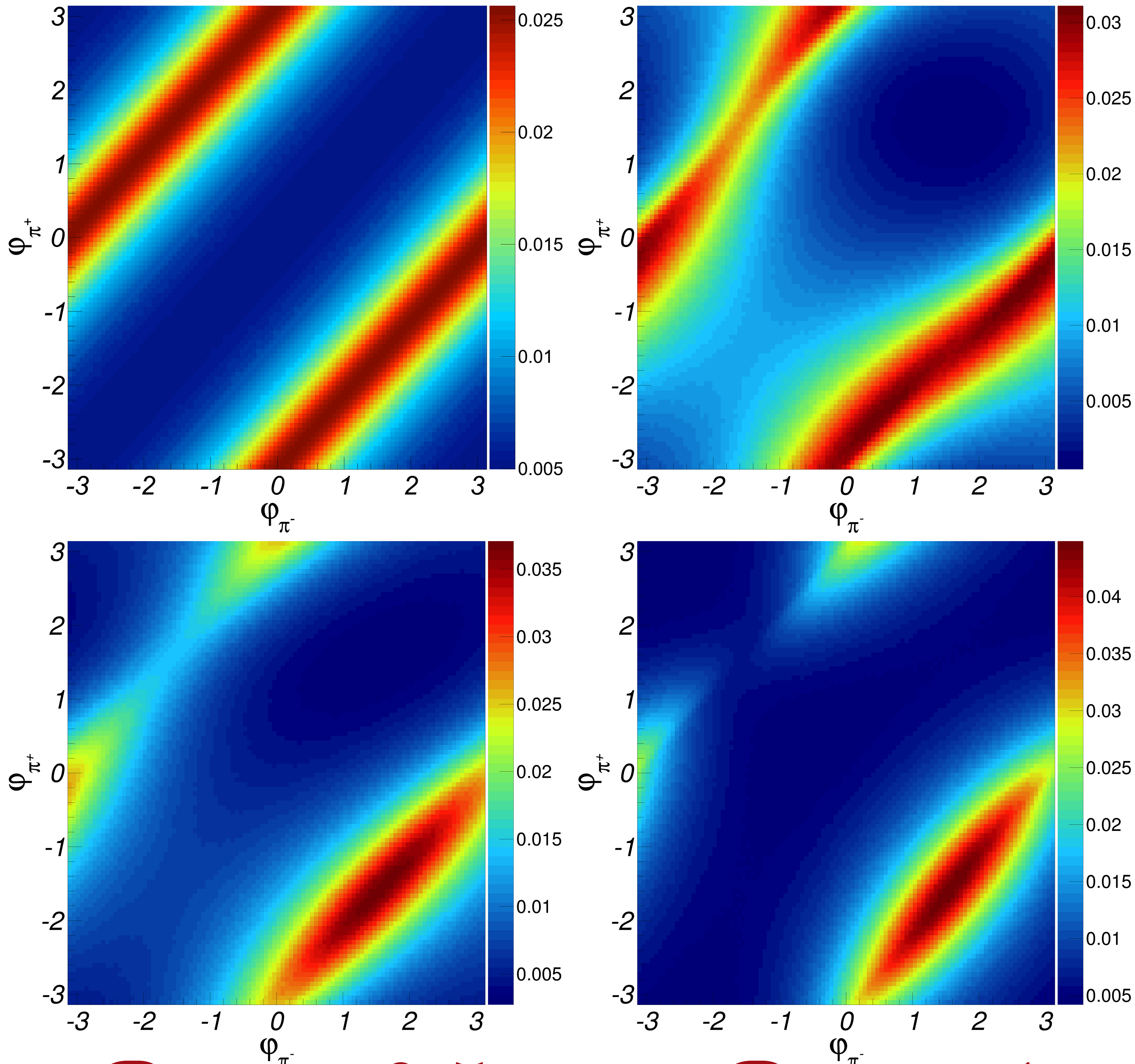
$$\boldsymbol{R}^P \equiv \boldsymbol{R} - (\boldsymbol{R} \cdot \hat{\boldsymbol{P}}_h)\hat{\boldsymbol{P}}_h \qquad \boldsymbol{R}^P \simeq \xi_2 \boldsymbol{P}_1 - \xi_1 \boldsymbol{P}_2$$

$$\boldsymbol{R}^P_T \simeq \xi_2 \boldsymbol{P}_{1\perp} - \xi_1 \boldsymbol{P}_{2\perp} \qquad \xi_i \equiv z_i/(z_1 + z_2)$$

No k_T dependence at LO! No contradiction, different R !

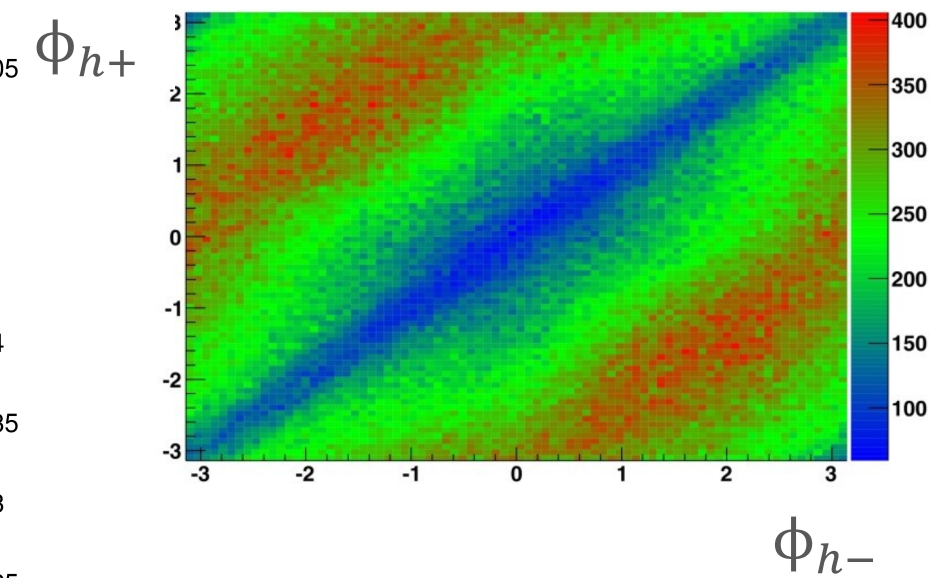
ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Unpolarized $\qquad \mathcal{P}_{SF} = 0$



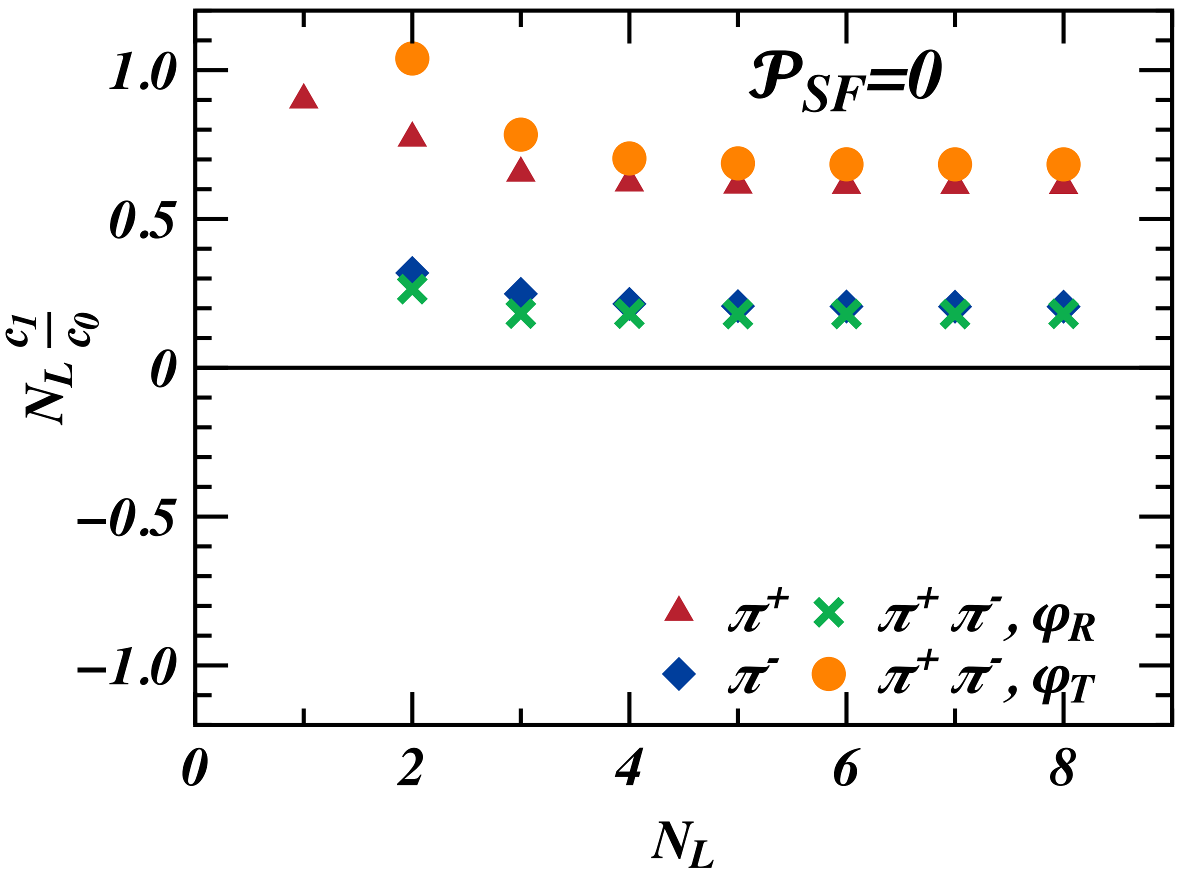
$\mathcal{P}_{SF} = 0.5 \qquad \mathcal{P}_{SF} = 1$

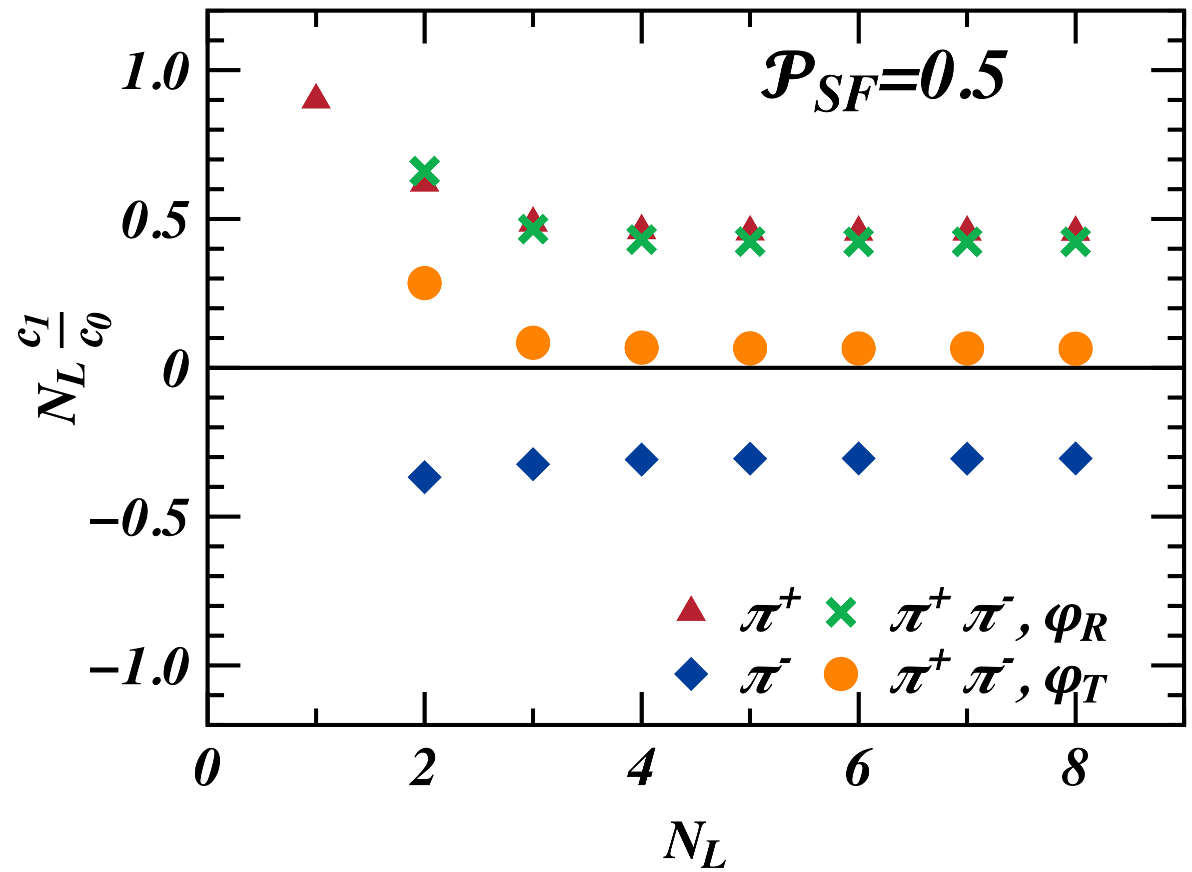
COMPASS Preliminary:
F. Bradamante - COMO 2013.

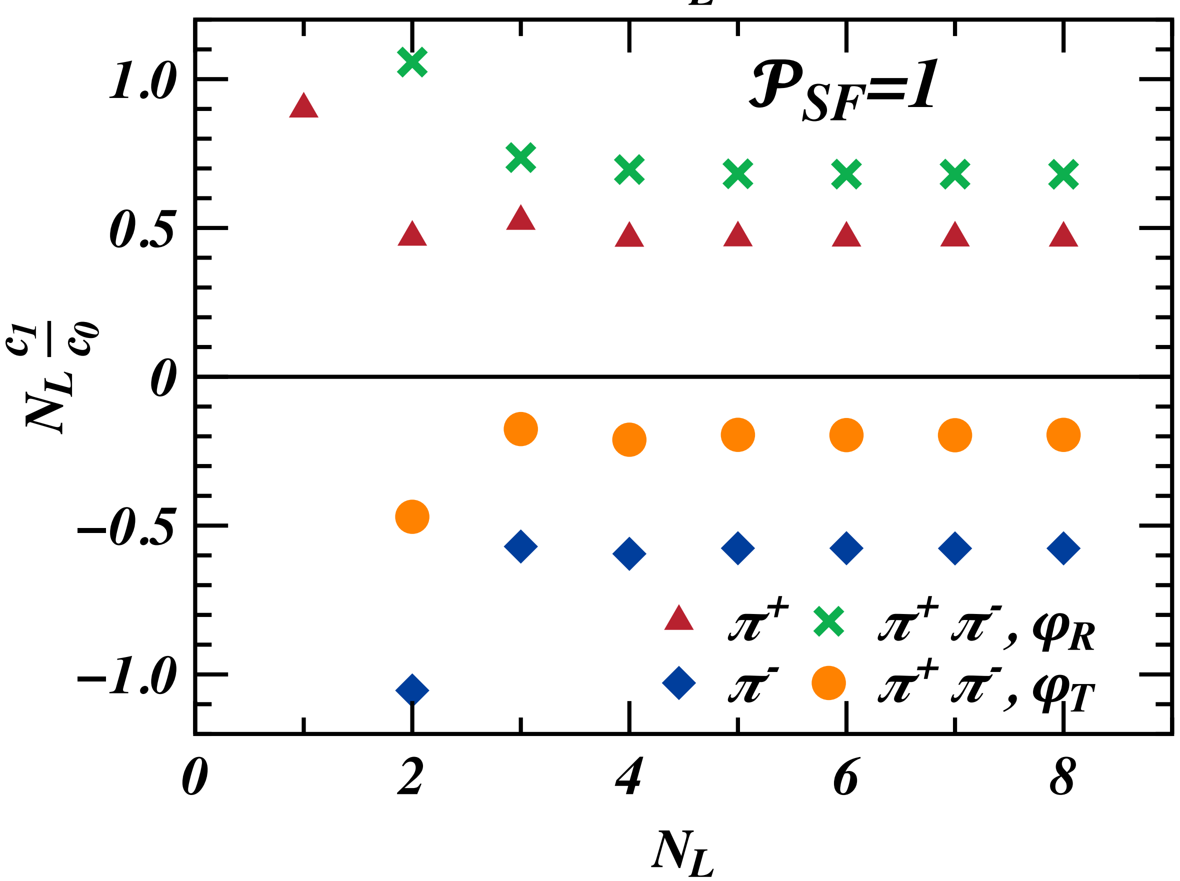


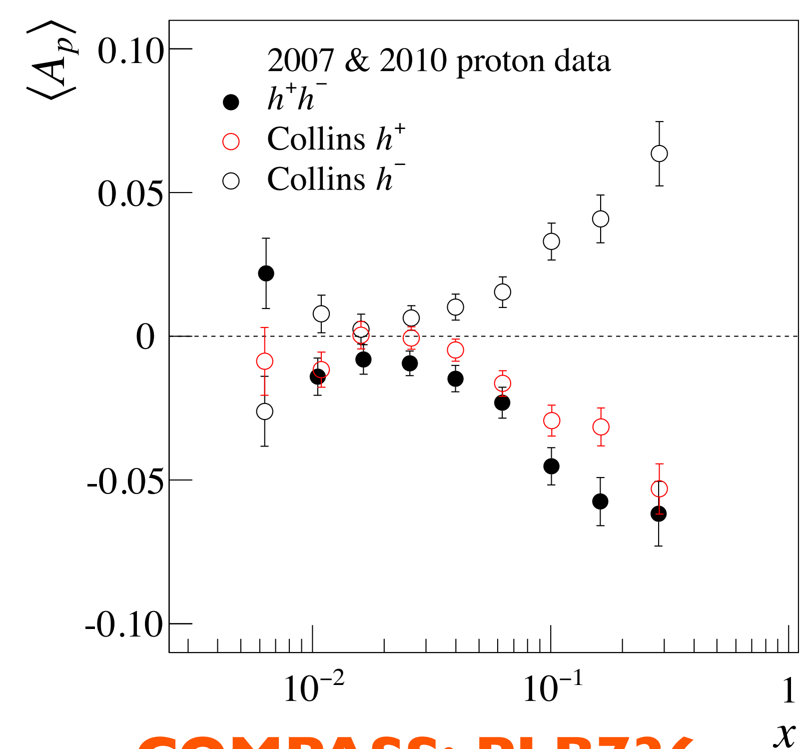
INTEGRATED ANALYZING POWERS

$z_{1,2} > 0.2, z > 0.2$









COMPASS: PLB736, 124-131 (2014).

INTEGRATED ANALYZING POWERS

Single Hadron

$$N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \, q \otimes D_1^{h/q}}$$

DiHadron

$$N_{h^+h^-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)$$

$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

$$D_{q^\uparrow} = c_0 - \sin(\phi) c_1$$

124-131 (2014).

Single Hadron

$$N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)$$

$$\phi_C = \phi_h - \phi_{S'} = \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \, q \otimes D_1^{h/q}}$$

DiHadron

$$N_{h+h-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)$$

$$\phi_{RS} = \phi_R - \phi_{S'} = \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

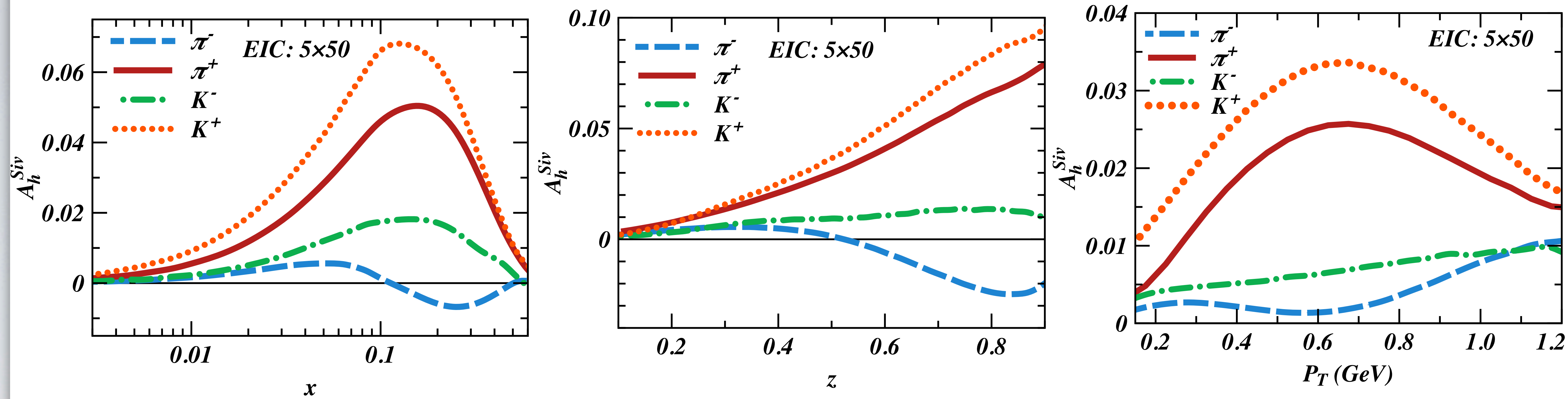
$$D_{q\uparrow} = c_0 - \sin(\phi) c_1$$

124-131 (2014).

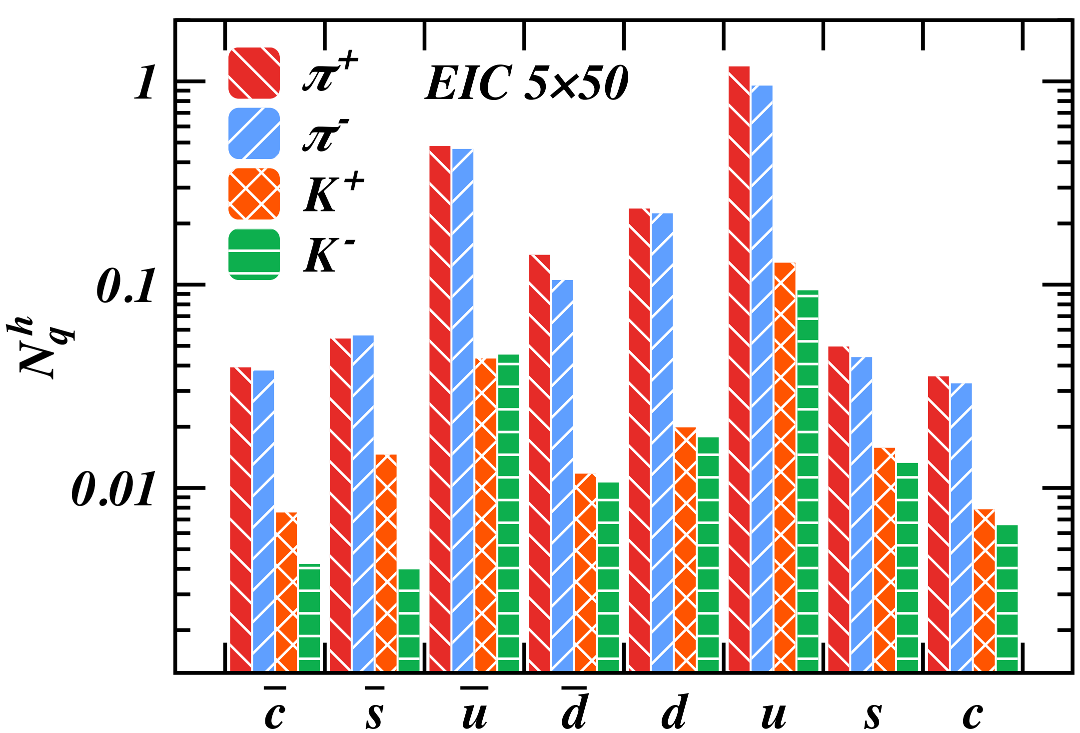
mPYTHIA RESULTS FOR EIC: ONE H

H.M et al., arXiv:1502.02669 (2015).

✦ SSAs for charged pions and kaons from ***proton*** target - low x region.



✦ **Average** number of hadrons by struck quark flavor.

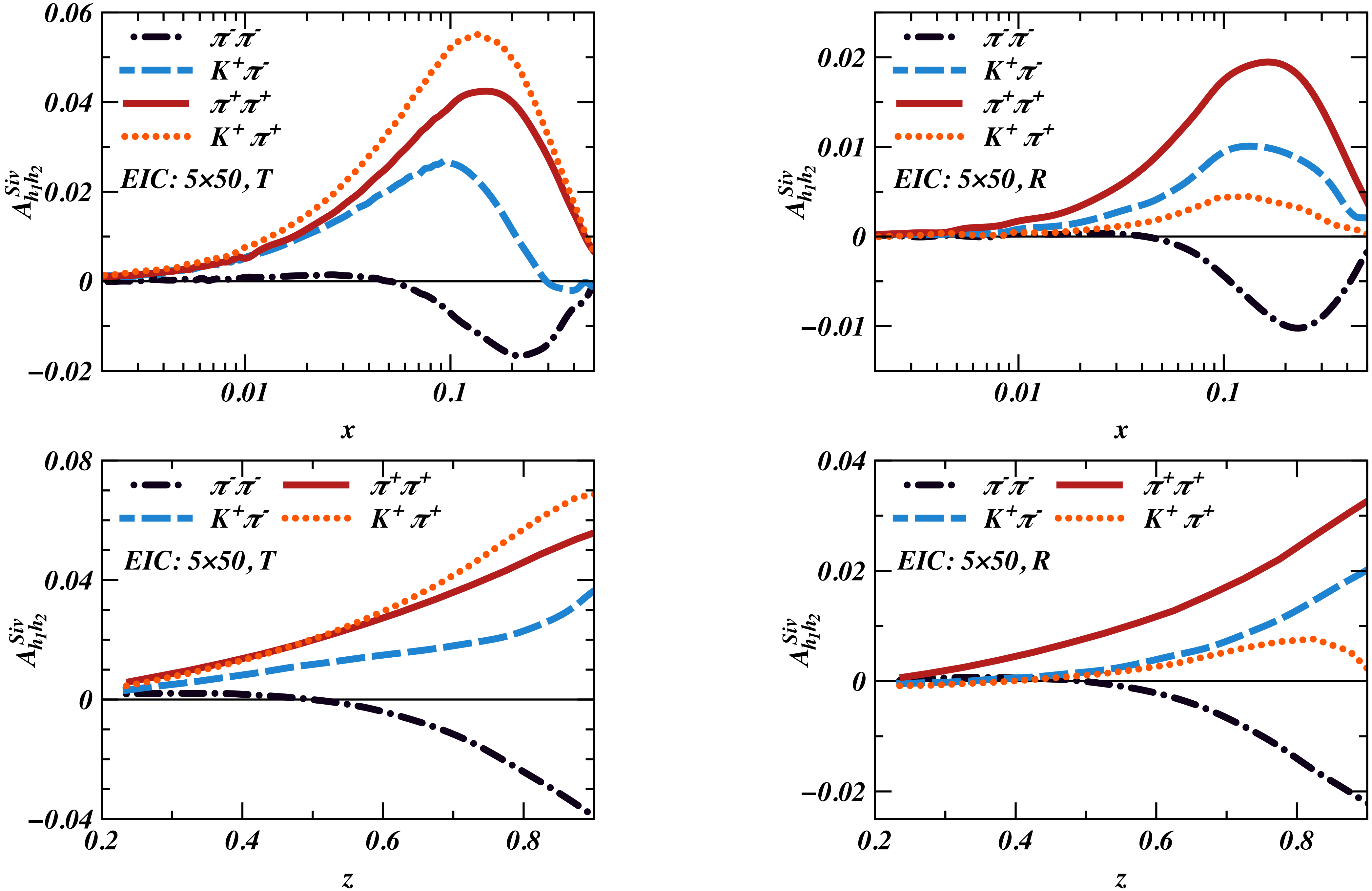


✦ π^+ multiplicities larger than K^+, but kaon SSAs are larger. Up quark dominates the multiplicities.

Dihadron Sivers SSAs for EIC

H.M et al., arXiv:1502.02669 (2015).

✦ Identical pairs via z-ordering: $z_1 \geq z_2$ (so $\sigma_R \neq 0$)



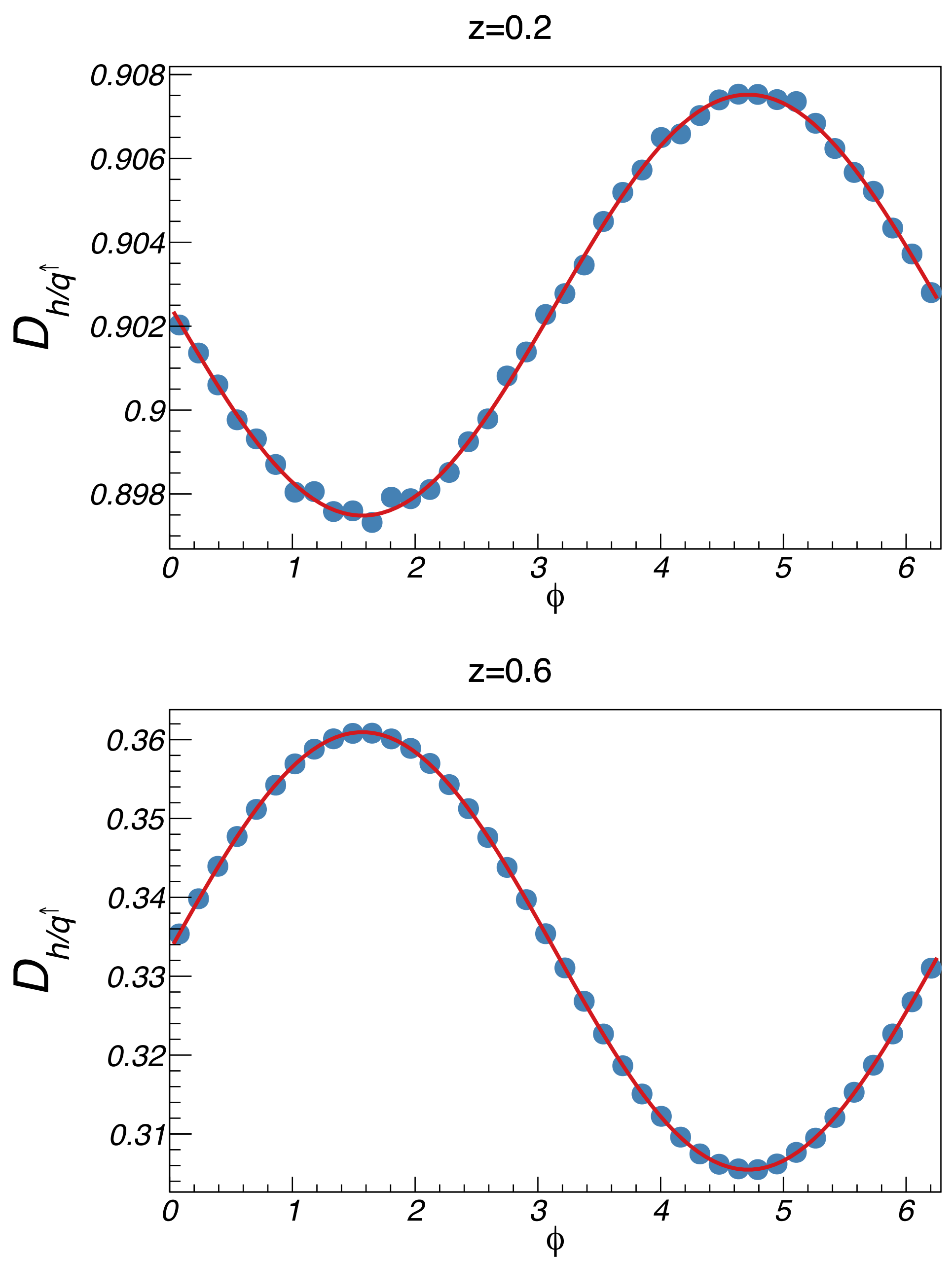
• Dihadron SSAs are *comparable* to single hadron ones!

(the one- and two-hadron FFs should mostly cancel in the ratios)

INTEGRATED POLARIZED FRAGMENTATIONS

- Integrate Polarized Fragmentations over $P_\perp^2$

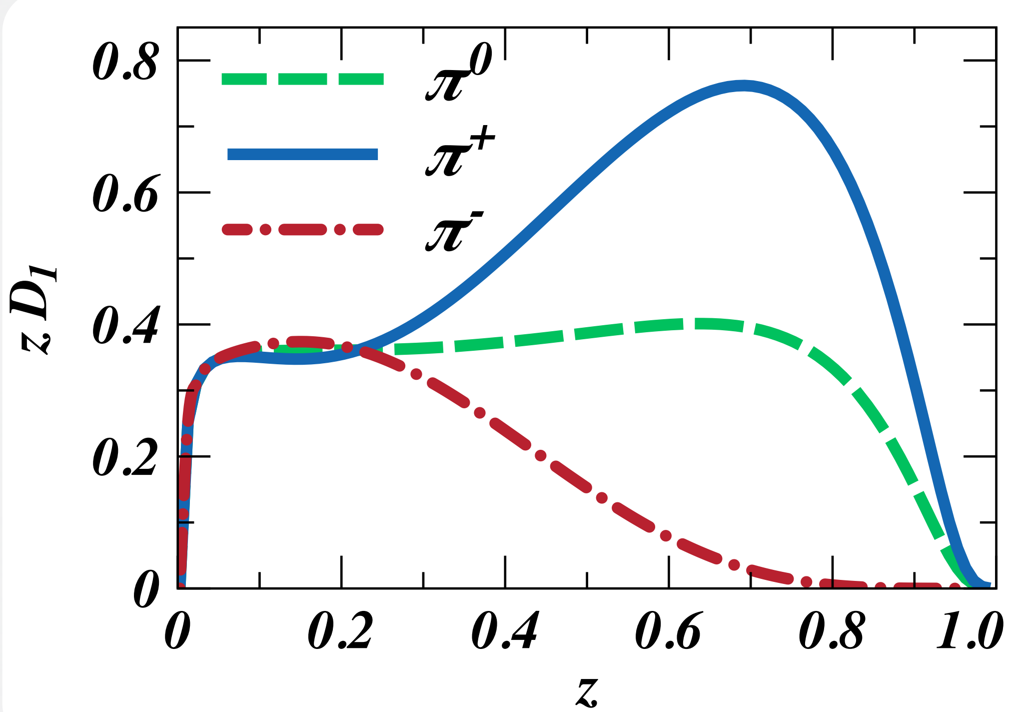
$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2 \; D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z) \; - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$



$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2 \; D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $F(c_0,c_1) = c_0 - c_1 \sin(\varphi)$



COLLINS EFFECT - MK2

MK2 Model Assumptions:

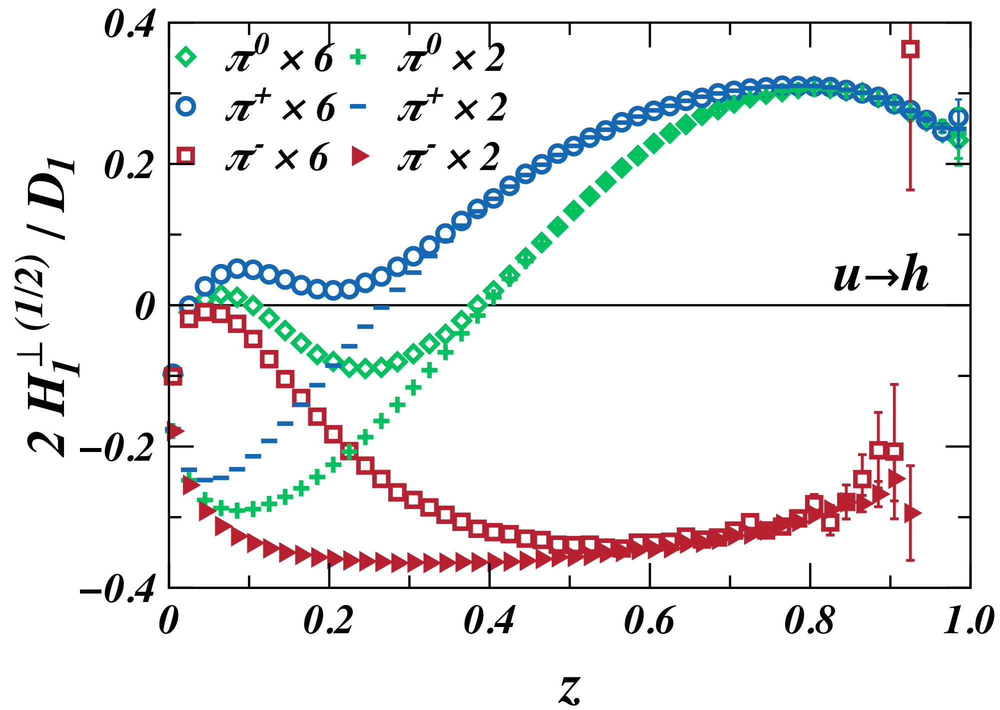
H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

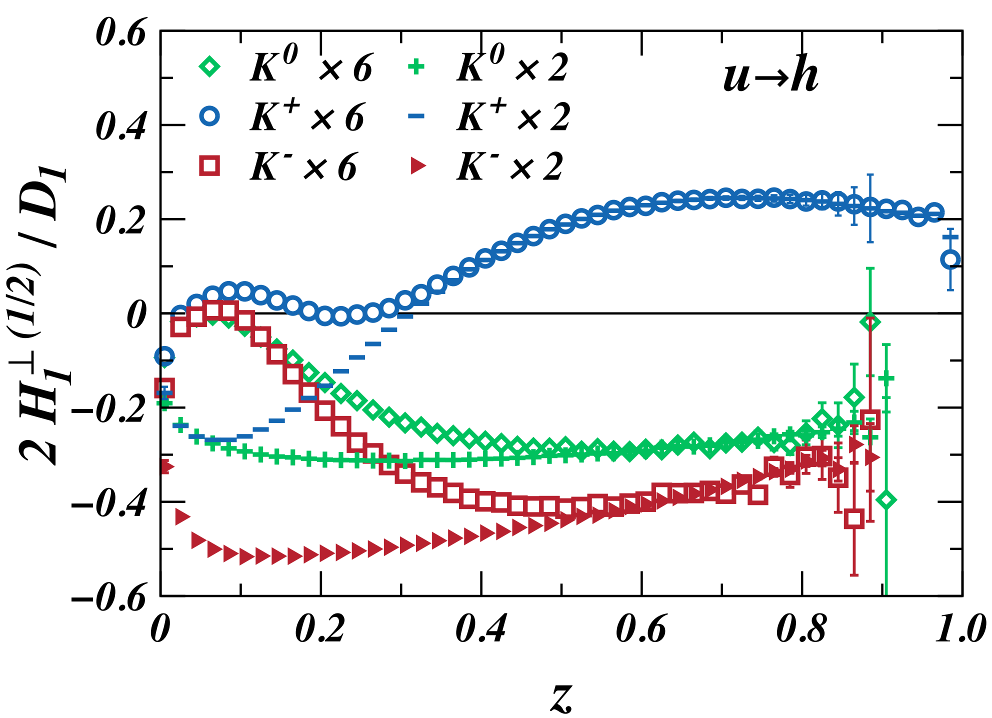
1. Allow for Collins Effect only in a SINGLE emission vertex - N_L^{-1} scaling of the resulting Collins function.
2. Use constant values for $\mathcal{P}_{SF}$

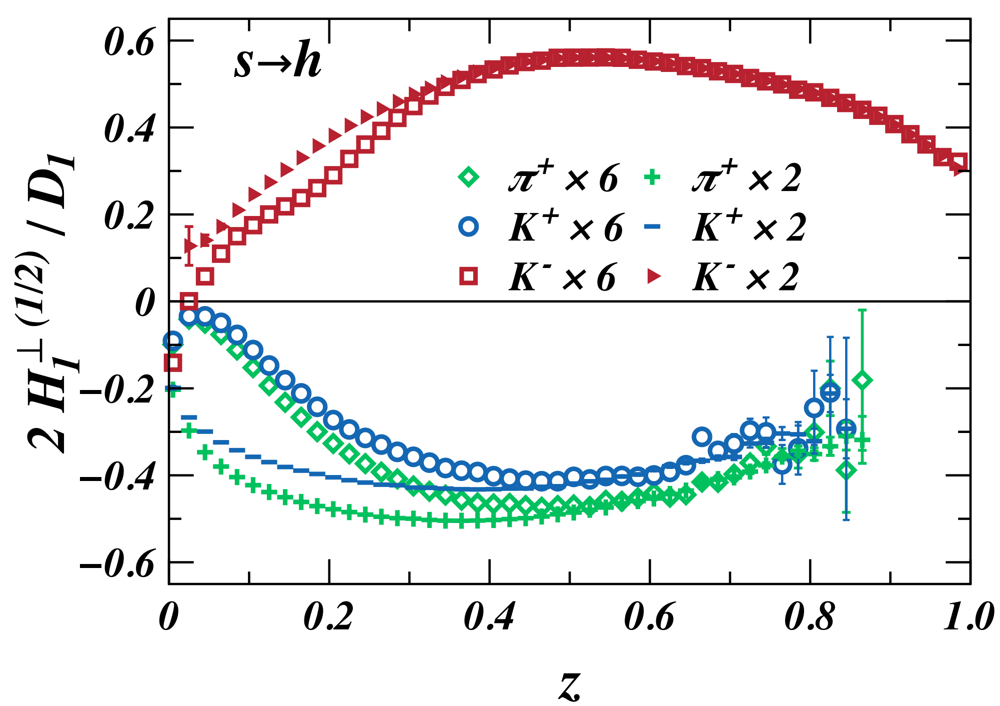
$$\mathcal{P}_{SF} = 1$$

✦ The results for N_L=2 and N_L=6, scaled up by a factor N_L.

$$F(c_0, c_1) = c_0 - c_1 \sin(\varphi)$$







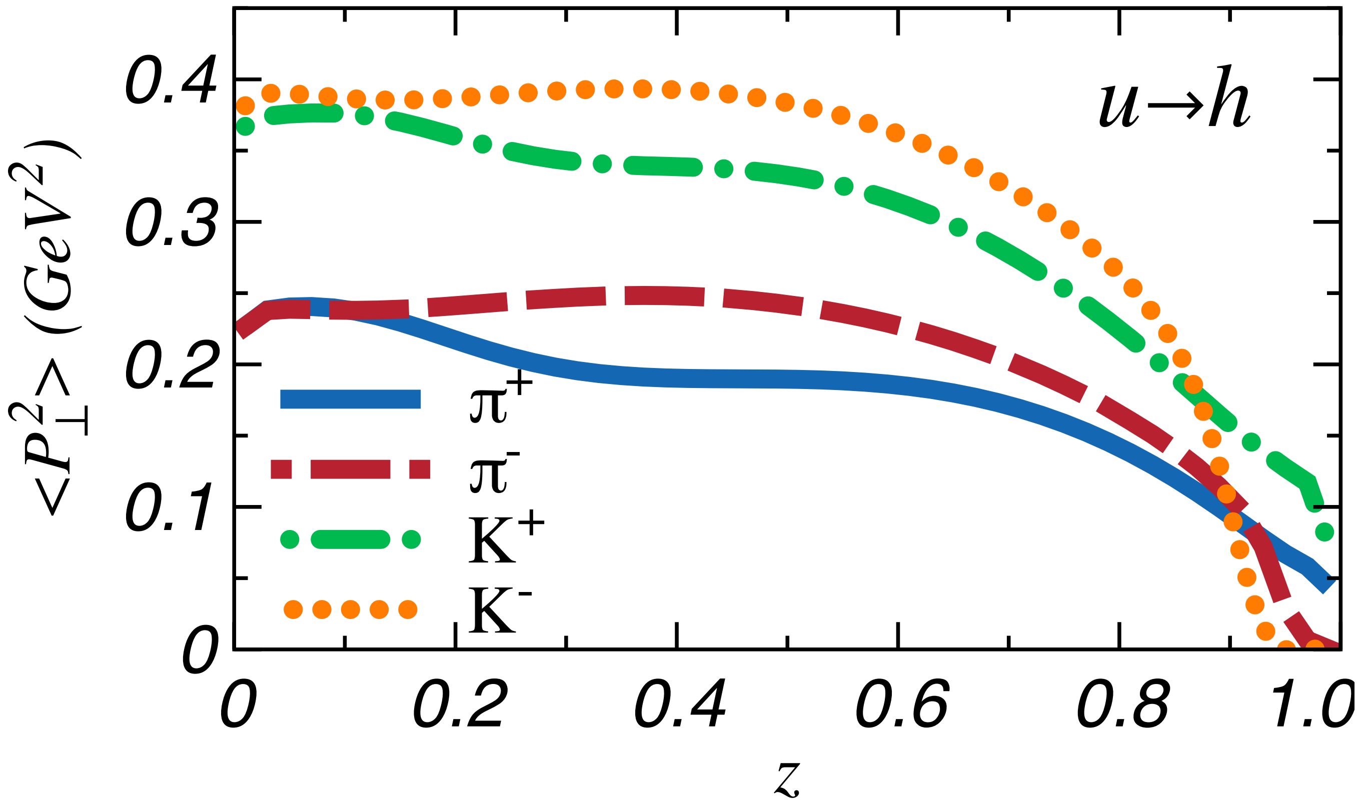
AVERAGE TRANSVERSE MOMENTA VS Z

FRAGMENTATION

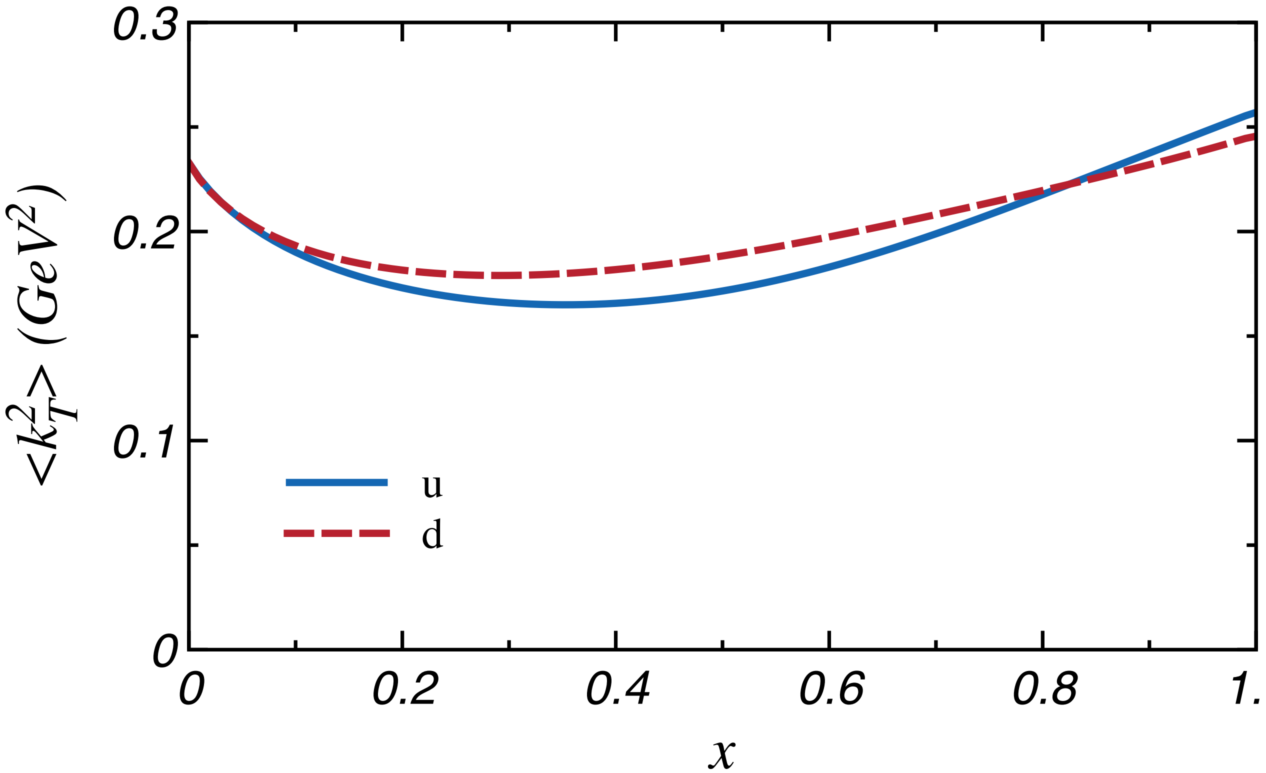
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data:

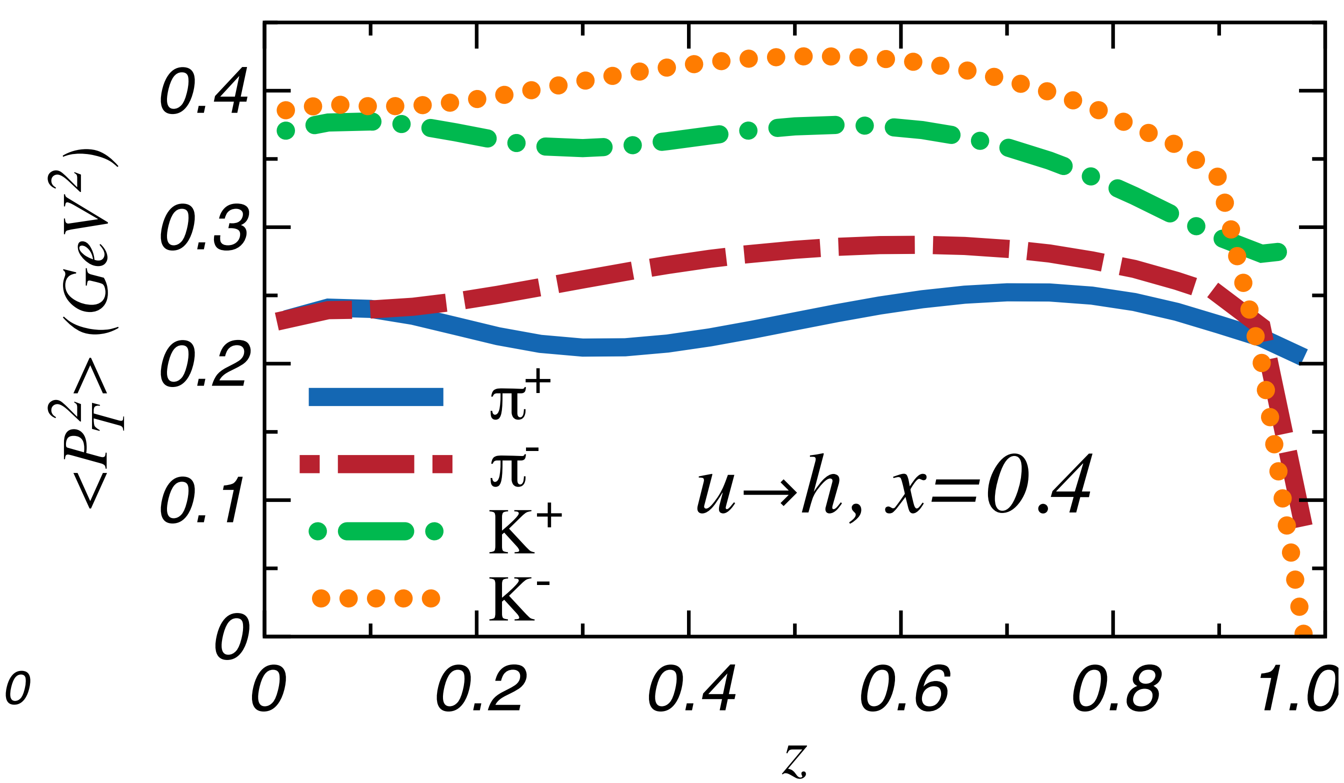
A. Signori, et al: JHEP 1311, 194 (2013)



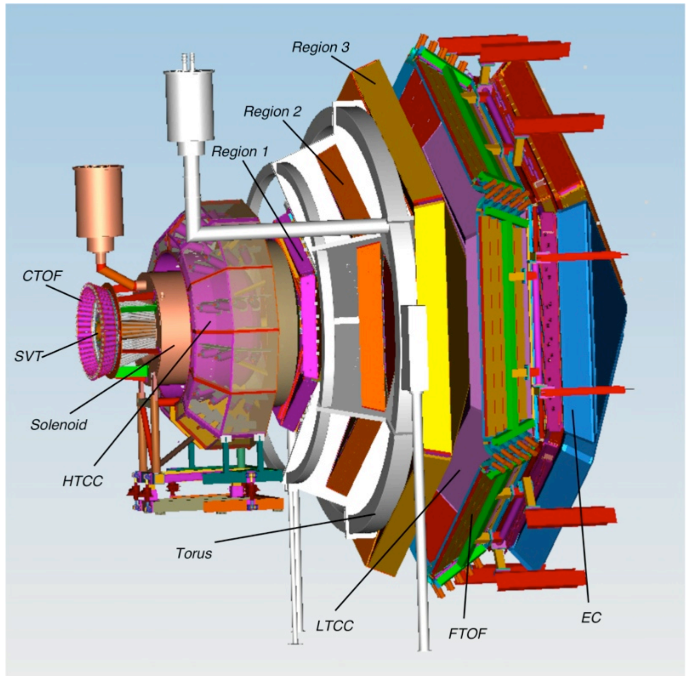
PDF



SIDIS



CLAS12 @ JLAB 12GeV



- Upcoming SIDIS experiment, 1H and 2H
- 11 GeV electron off polarized proton target.
- Access to large x region of nucleon structure.
- We use mPYTHIA for SIDIS predictions.

- Include the kinematical cuts on $x, Q^2, W, \theta_{e'}, \theta_h, M_{Mis}, z, ...$

$$0.075 \leq x \leq 0.532$$

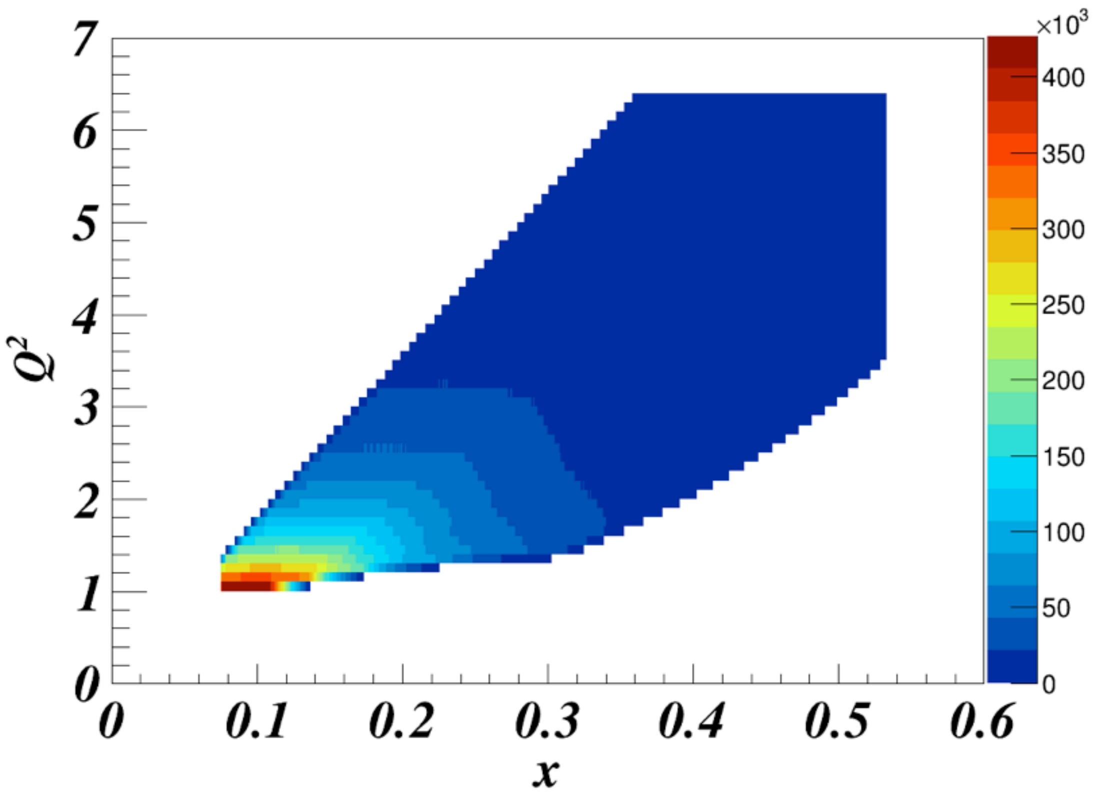
$$1 \text{ GeV} \leq Q^2 \leq 6.3 \text{ GeV}$$

$$W \geq 2 \text{ GeV}$$

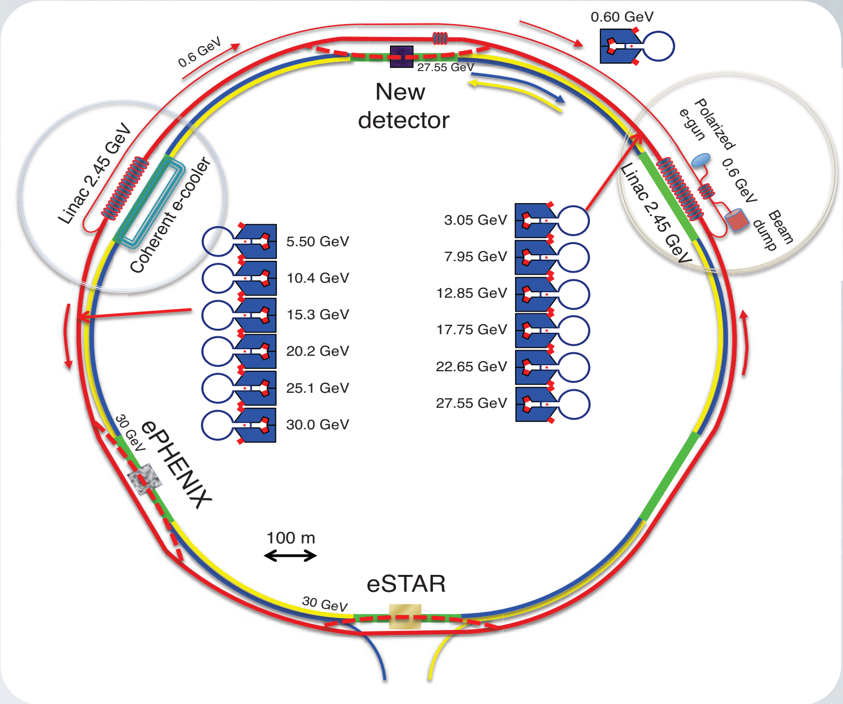
$$MMis_{(ep)-(e'hX)} \geq 1.5 \text{ GeV}$$

$$MMis_{(ep)-(e'h_1h_2X)} \geq 1.5 \text{ GeV}$$

DIS kinematics

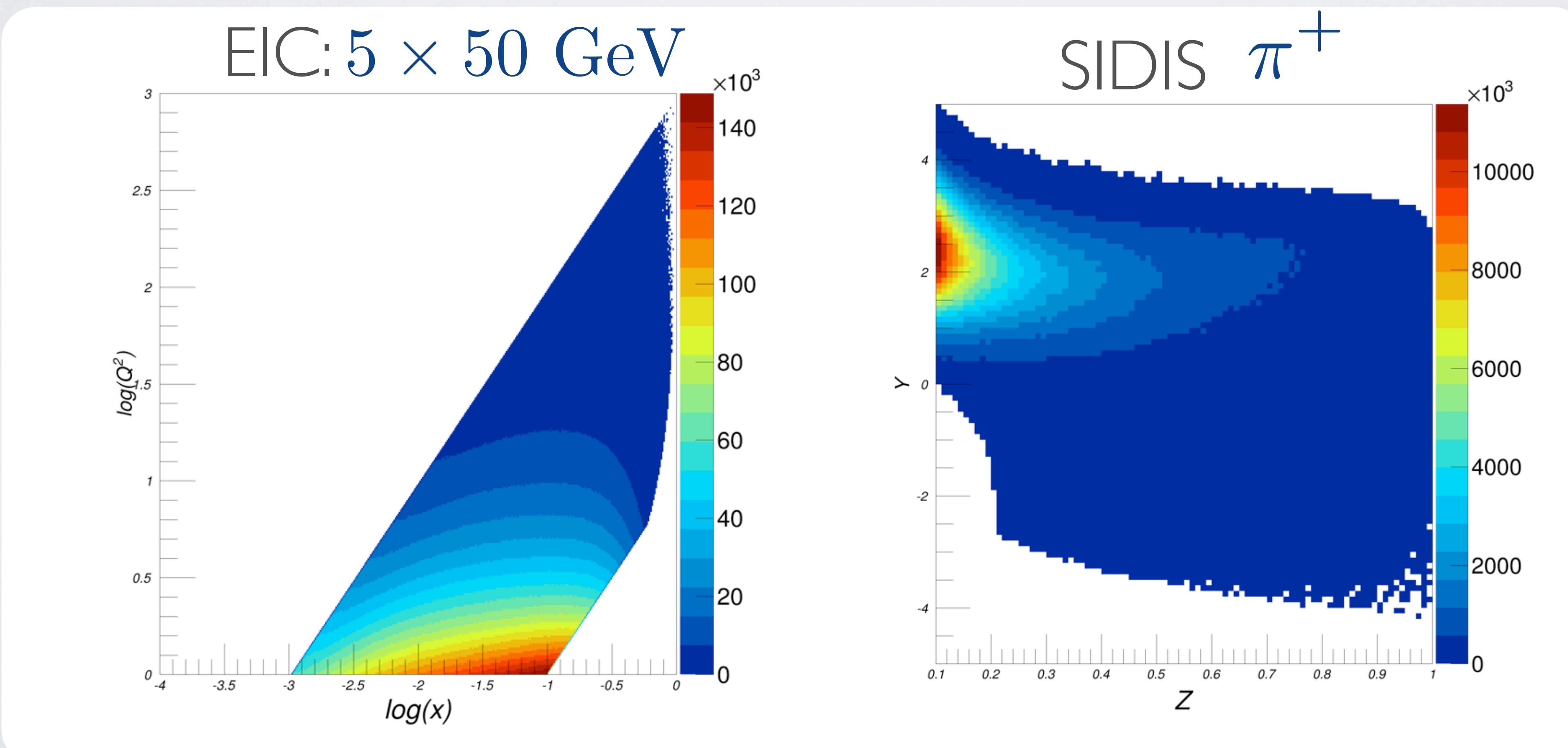


EIC: eRHIC



White Paper -- Accardi et. al. : 1212.1701(2012).

- EIC using RHIC + electron ring.
- Various proposed beam momenta: $l_e \times P_N$
- We use mPYTHIA for SIDIS predictions.



- **Sivers Single Spin Asymmetry:**

$$\langle \sin(\phi - \phi_S) \rangle^h_{UT} \equiv \frac{\int d\phi_h d\phi_S \sin(\phi_h - \phi_S)[d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)]}{\int d\phi_h d\phi_S [d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)]}$$

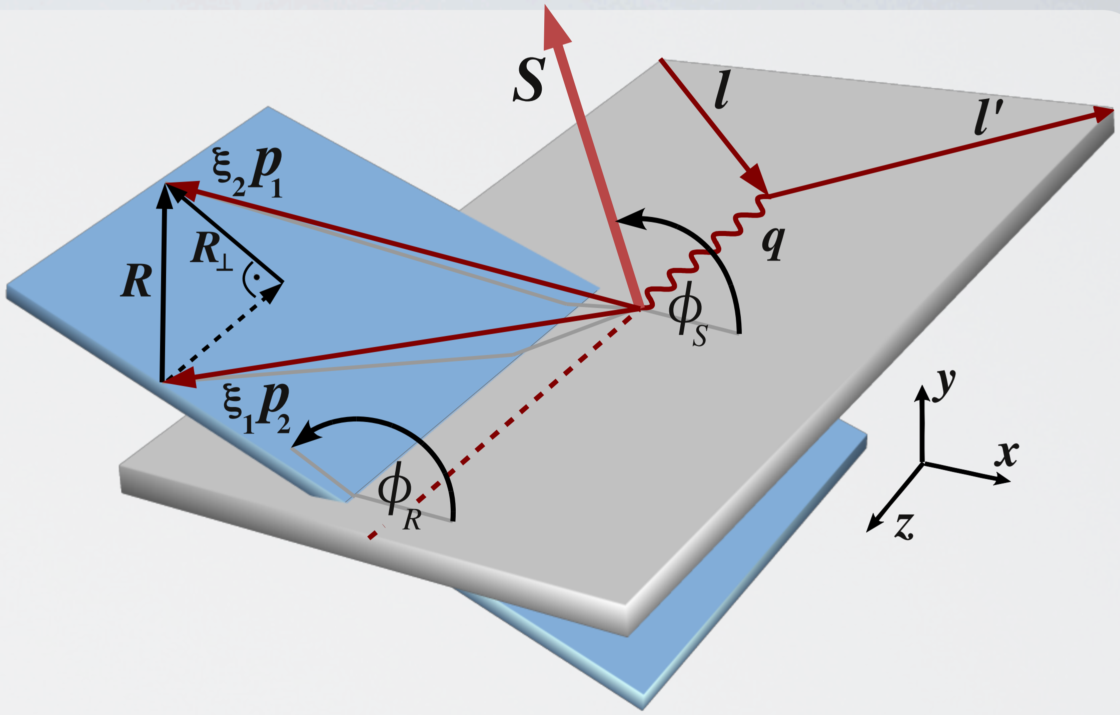
$$A^P_{Siv} \equiv 2\langle \sin(\phi - \phi_S) \rangle^h_{UT}$$

$$\langle \sin(\phi - \phi_S) \rangle^h_{UT} \sim \frac{\mathcal{C}[f^{\perp,q}_{1T} \;\; D^{h/q}_1]}{\mathcal{C}[f^q_1 \; D^{h/q}_1]}$$

ACCESS TO TRANSVERSITY PDF FROM DiFF

M. Radici, et al: PRD 65, 074031 (2002).

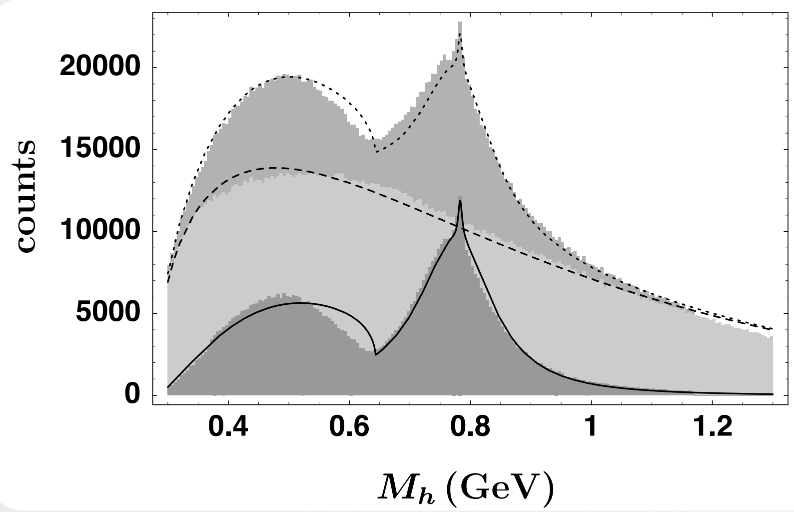
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!

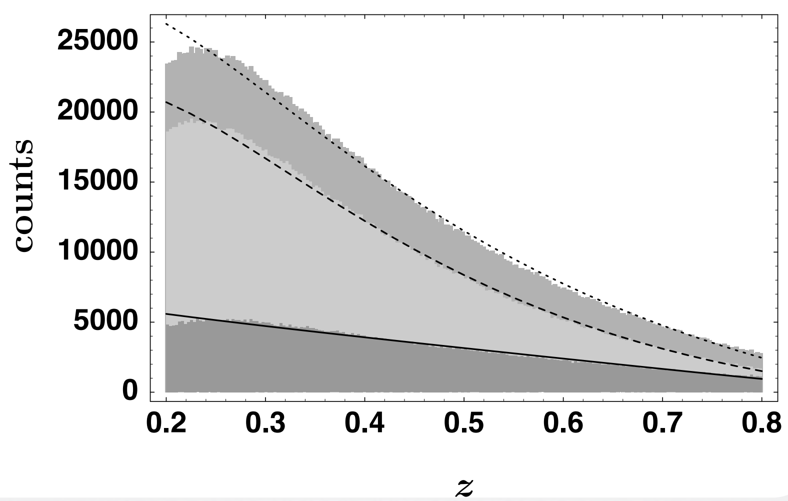


$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2\ h_1^q(x)/x\ H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2\ f_1^q(x)/x\ D_1^q(z, M_h^2)}$$

- Empirical Model for D_1^q has been fitted to PYTHIA simulations.

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



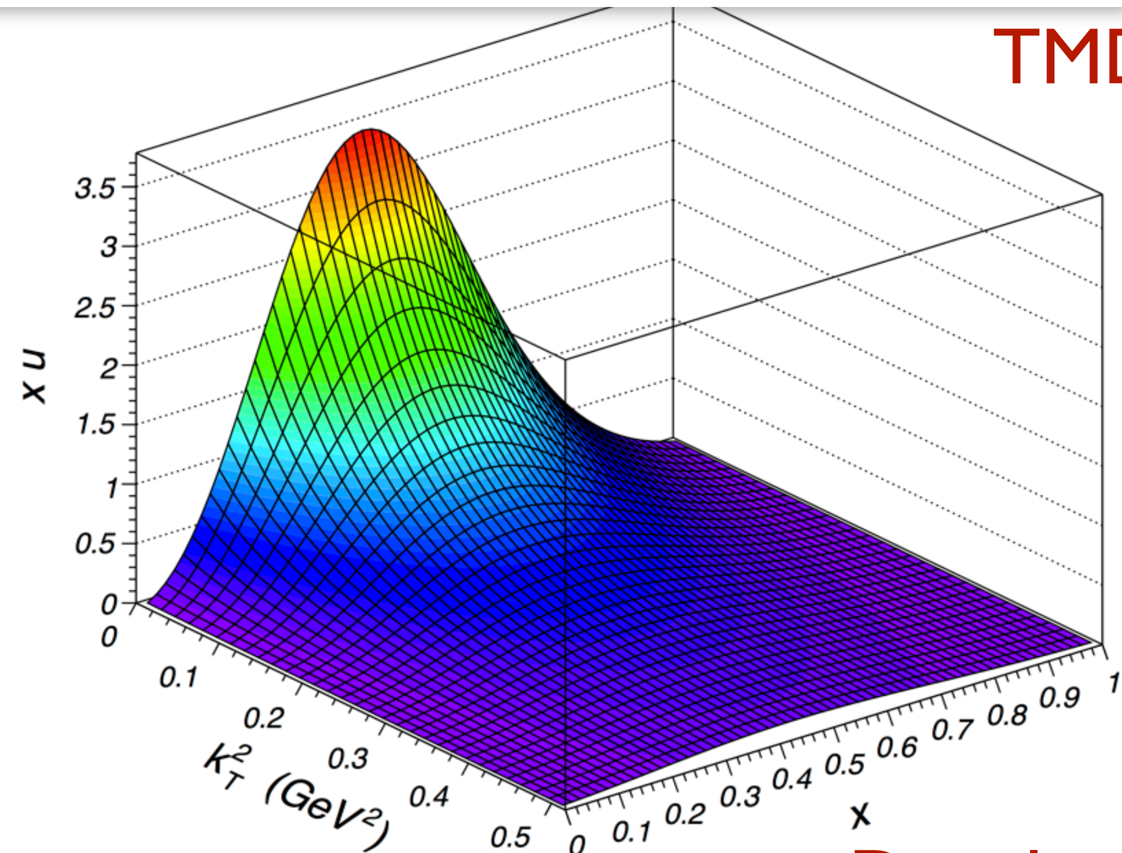


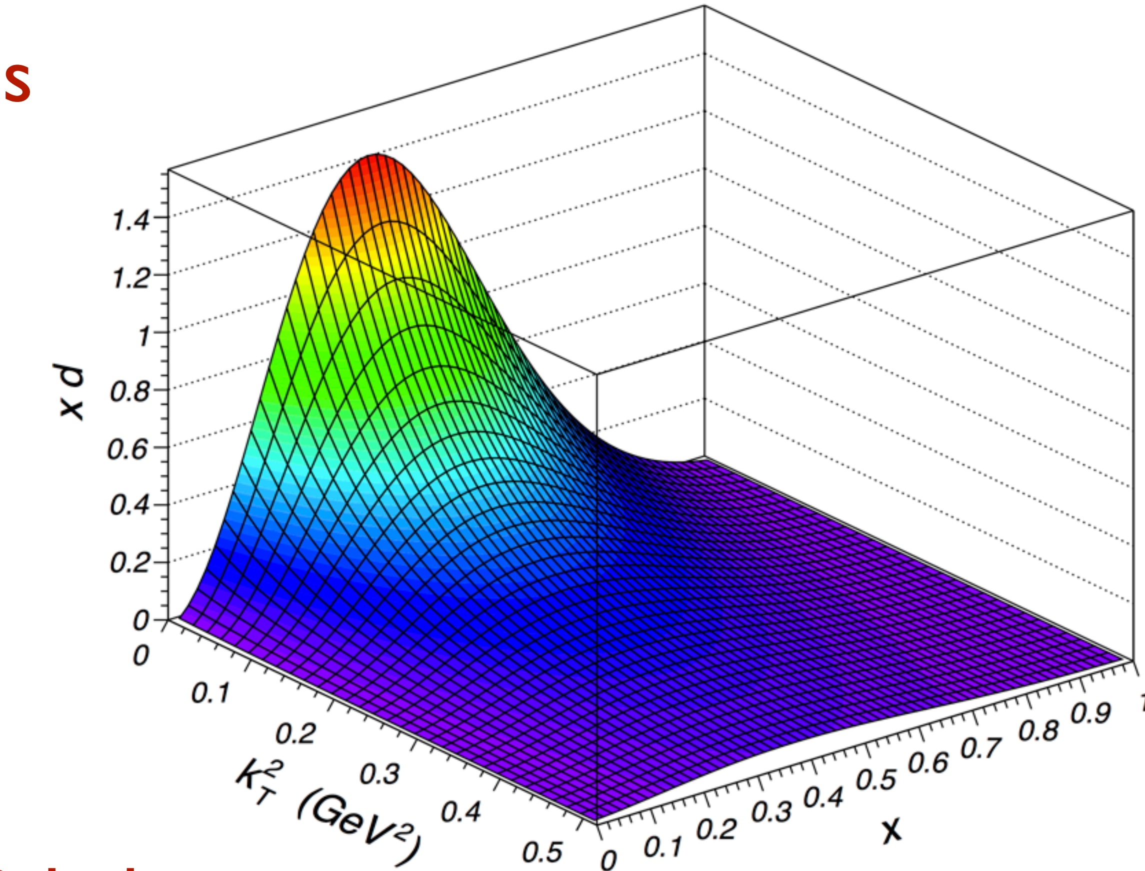
Experiments:
BELLE,
HERMES,
COMPASS.

NJL: NUCLEON PDFS - TMD RESULTS

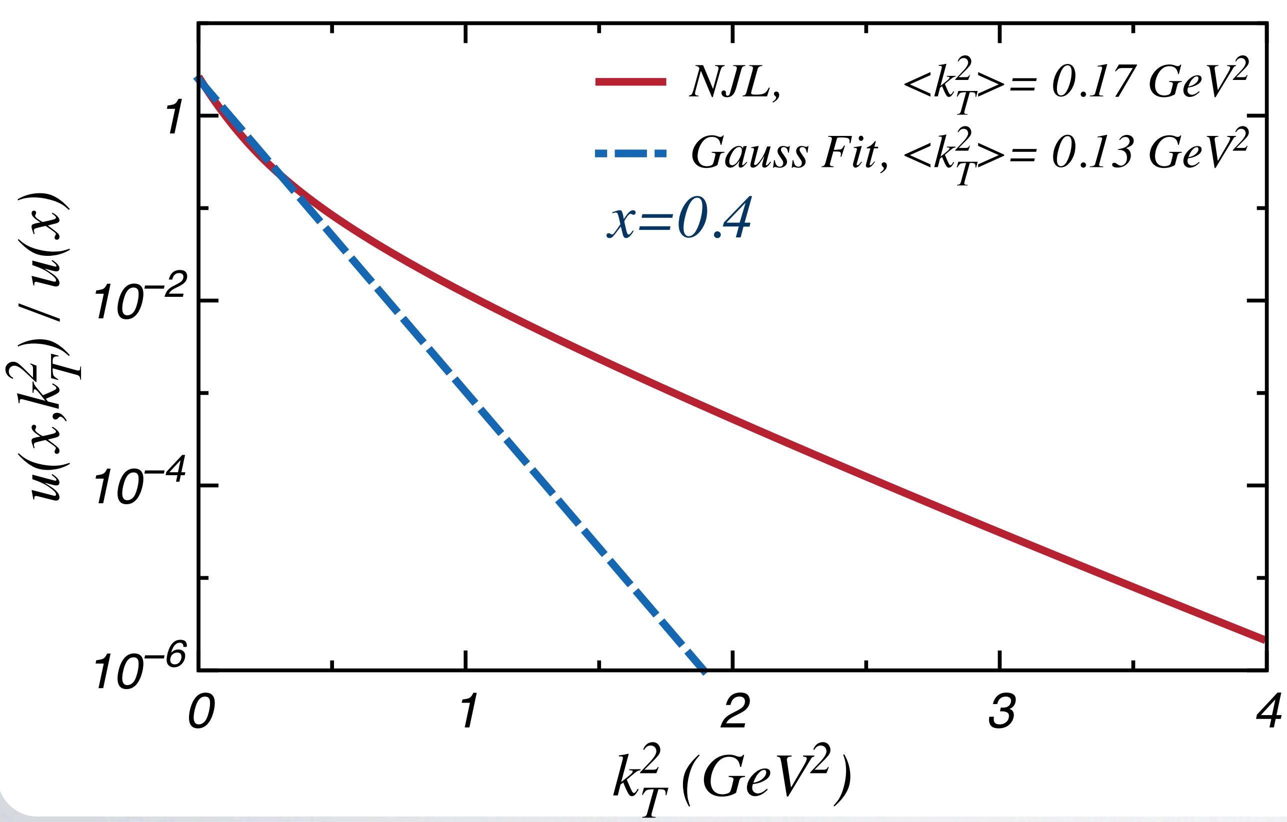
H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

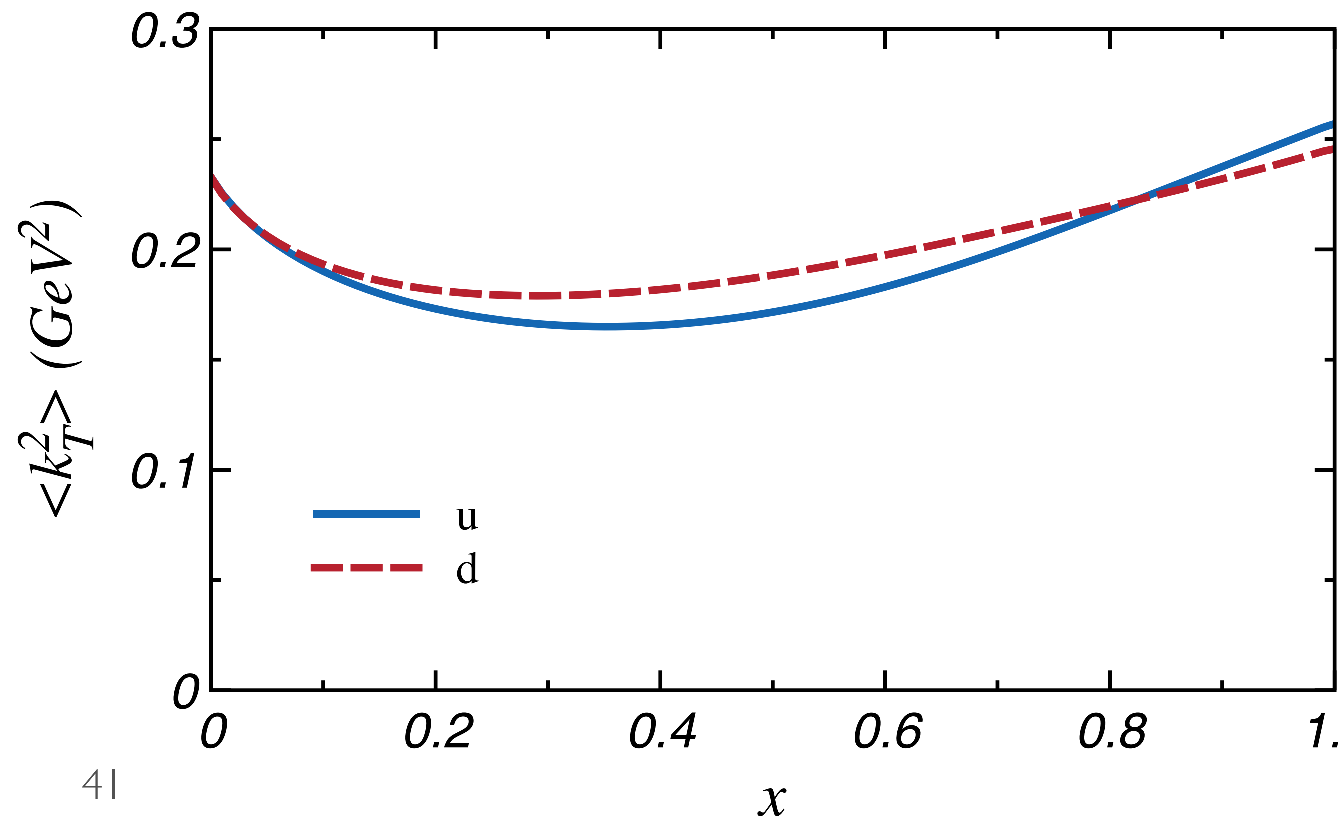
TMD PDFs





Details of TMD behavior





GAUSSIAN ANSATZ

- Need to calculate convolution PDFs and FFs:

$$F_{UU} = \sum_q e_q^2 \; f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\to lq} \otimes D_q^h(z, P_\perp^2, Q^2)$$

- Using Gaussian Ansatz For TM dependences of PDFs and FFs:

$$f_1^q(x, k_T^2) = f_1^q(x) \frac{e^{-k_T^2/\langle k_{T,q}^2\rangle}}{\pi\langle k_{T,q}^2\rangle} \qquad D_q^h(z, P_\perp^2) = D(z)_q^h \frac{e^{-P_\perp^2/\langle P_\perp^{2,q\to h}\rangle}}{\pi\langle P_\perp^{2,q\to h}\rangle}$$

- Only involved collinear PDFs and FFs.

$$F_{UU} = \sum_q e_q^2 \; f_1^q(x, Q^2) D_q^h(z, Q^2) \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle(z) = \langle P_\perp^2\rangle + z^2\langle k_T^2\rangle$$

$$\langle k_T^2\rangle \equiv \frac{\int d^2\mathbf{k_T} \; k_T^2 f(x, k_T^2)}{\int d^2\mathbf{k_T} \; f(x, k_T^2)} \qquad \langle P_\perp^2\rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$$

EMPIRICAL EXTRACTIONS OF SIVERS PDF

M. Anselmino et. al.: PRD 72, 094007 (2005). PRD 86, 014028 (2012).

- *Sivers SSAs from SIDIS*
- Use LO expression for factorized cross-section.
- Parametrize PDFs and FFs.
- Use Gaussian TMD dependence.
- Also *TMD evolution* in 2012.

$$A^h_{Siv} \equiv 2\frac{\int d\varphi_S d\varphi_h\,(\sigma^h_\uparrow - \sigma^h_\downarrow)\sin(\varphi_h - \varphi_S)}{\int d\varphi_S d\varphi_h\,(\sigma^h_\uparrow + \sigma^h_\downarrow)}.$$

$$A^h_{Siv} \sim \mathcal{C}[k_T f^{\perp q}_{1T}\ D_1]/\mathcal{C}[f^q_1\ D^{h/q}_1]$$

$$f^q_1(x,k_T) = f_q(x)\frac{1}{\pi\mu^2}e^{-k_T^2/\mu^2}$$

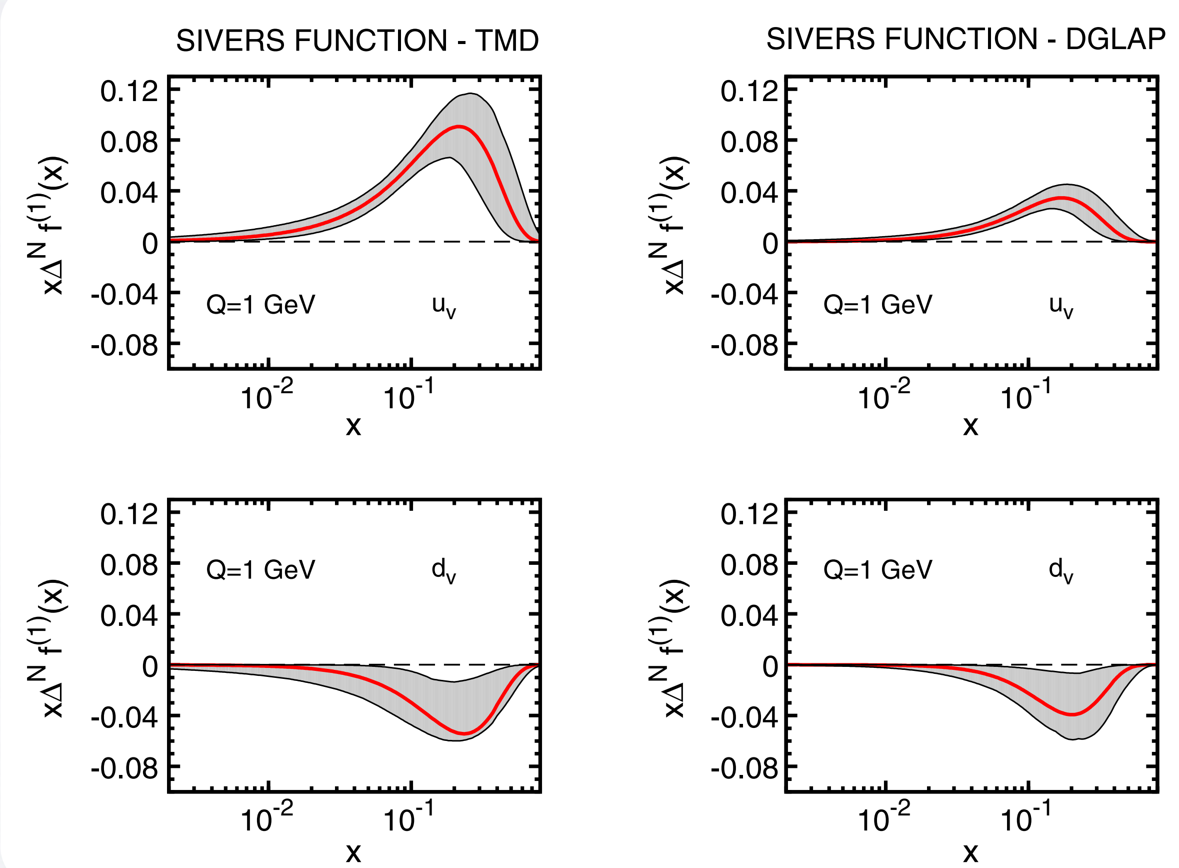
$$\Delta^N f_{q/p^\uparrow}(x,k_T) = \mathcal{N}_q(x)h(k_T)f^q_1(x,k_T)$$

$$\Delta^N f_{q/p^\uparrow} \equiv -\frac{2k_T}{M}f^{\perp q}_{1T}$$

- ***Fits to HERMES and COMPASS:***

▸ ***Current Data can only afford:***

- ***Large*** uncertainties, esp. for sea.
- ***Approximations:*** TM and flavor dependence of FF, etc.



EXTRACTIONS WITH TMD EVOLUTION

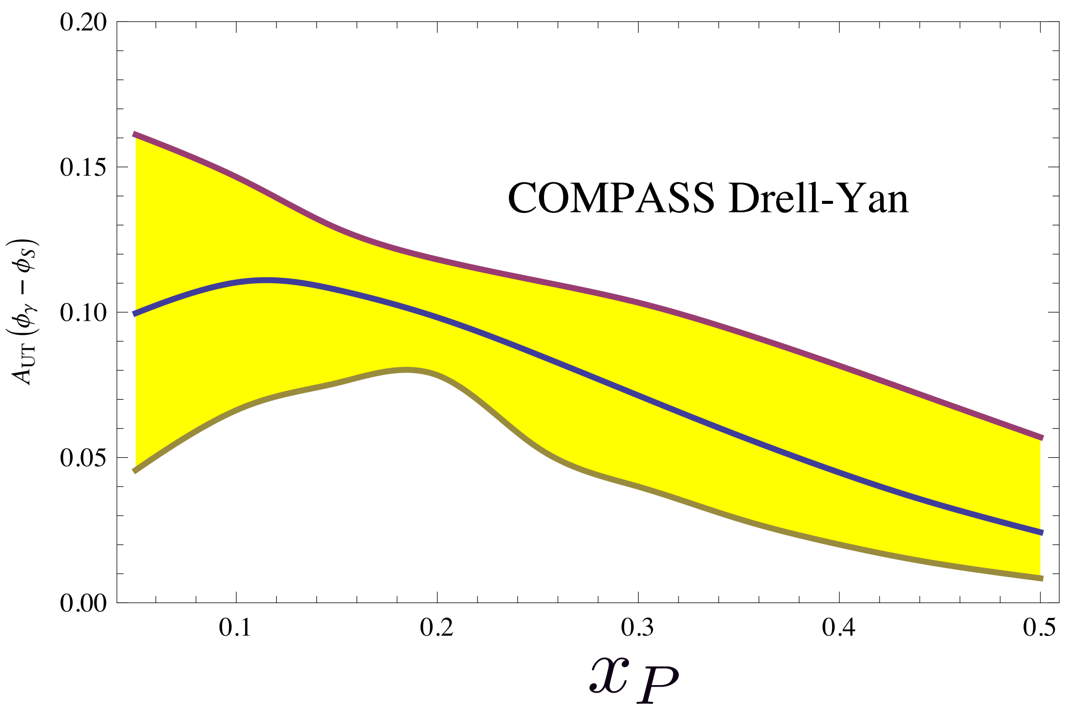
Sun, Yuan, PRD88 (2013), 114012

✦ Sun-Yuan prescription for TMD evolution.

✦ Gaussian TM dependence of NP TMD dependence at initial scale.

✦ Fit HERMES & COMPASS multiplicities and Sivers SSAs.

✦ Predict Sivers SSA and W production in COMPASS DY and PP.



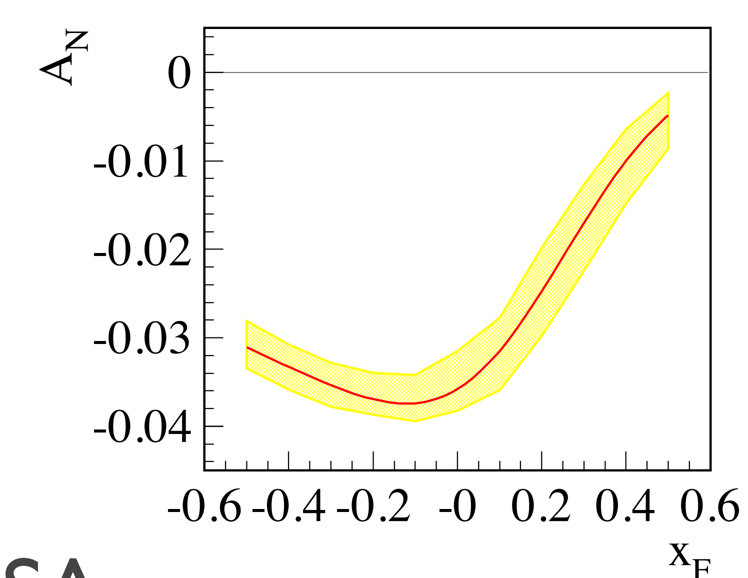
Echevarria et al.: PRD.89 074013, (2014)

✦ Find non-perturbative Sudakov factor that describes W, Z production in $P\bar{P}$ at Fermilab +HERMS & COMPAS.

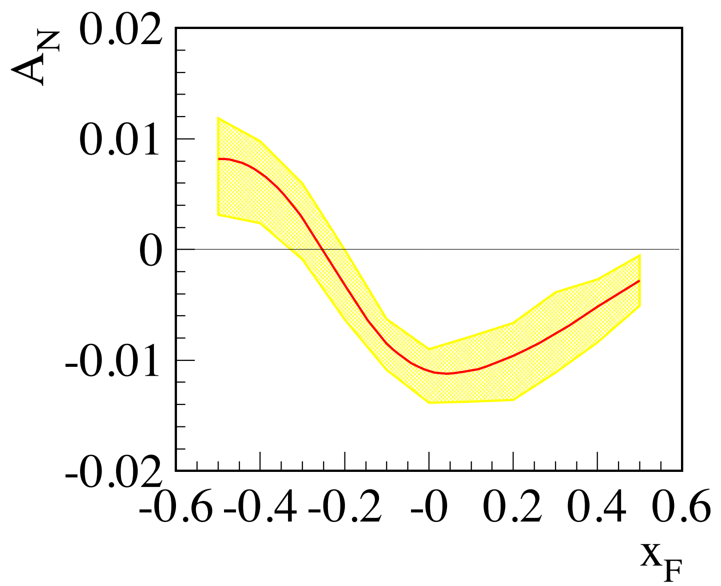
✦ Use it to fit Sivers SSA at HERMES, COMPASS, JLAB.

✦ Predict Sivers Effect for DY SSA.

COMPASS



RHIC



EXTRACTIONS WITH TMD EVOLUTION

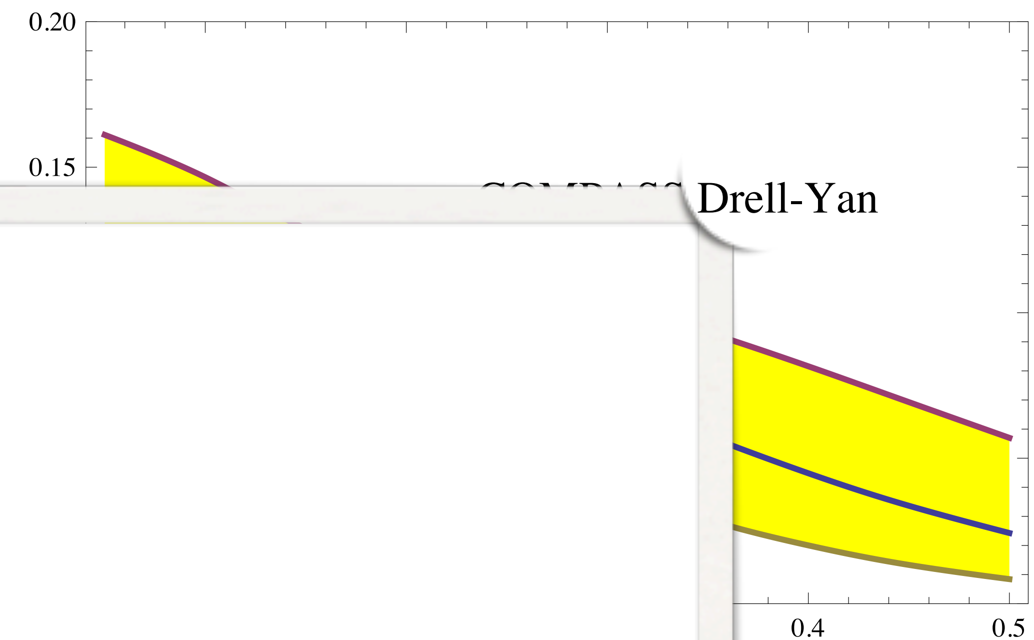
Sun, Yuan, PRD88 (2013), 114012

✦ Sun-Yuan prescription for TMD evolution.

✦ Gaussian TM dependence of NP TMD dependen

✦ Fit HER and Sivers

✦ Predict COMPAS



Approximations:

- ✦*Gaussian TM ansatz.*
- ✦***Flavor dependence.***
- ✦*TMD Evolution prescription.*
- ✦*TM and flavor dependence of FF.*

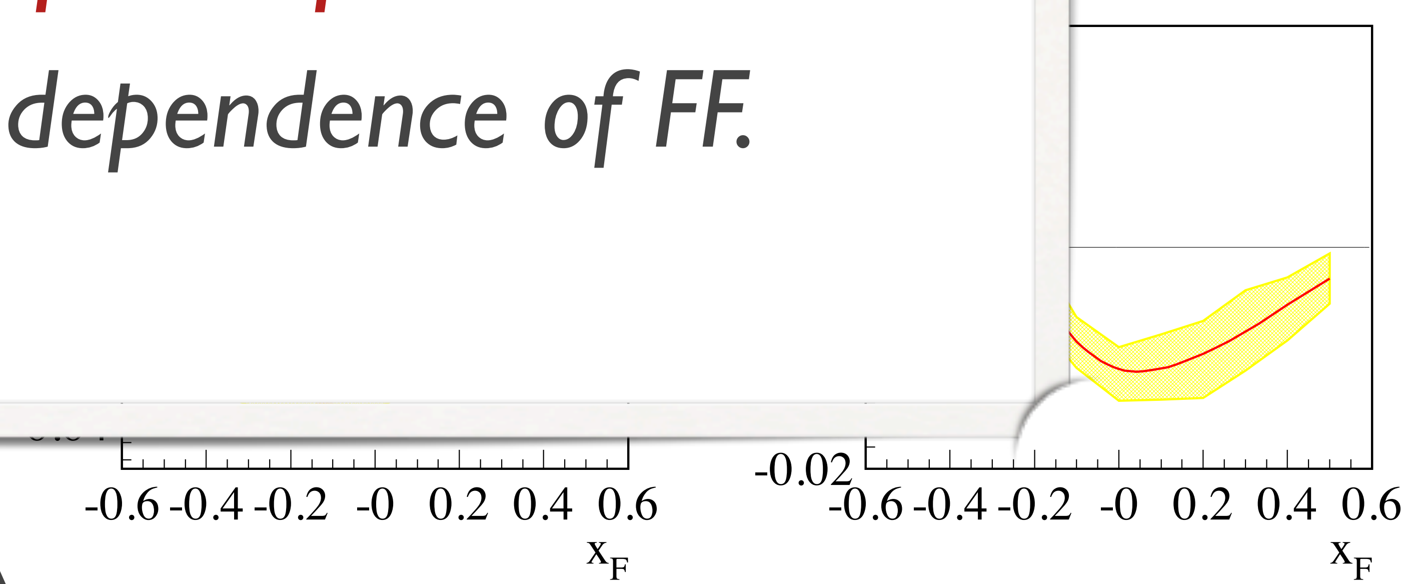
Echevarria e

✦ Find no factor that productio +HERMS &

✦ Use it t HERMES, COMPASS, JLAB.

✦ Predict Sivers Effect for DY SSA.

c



LO APPROXIMATION FOR SSA

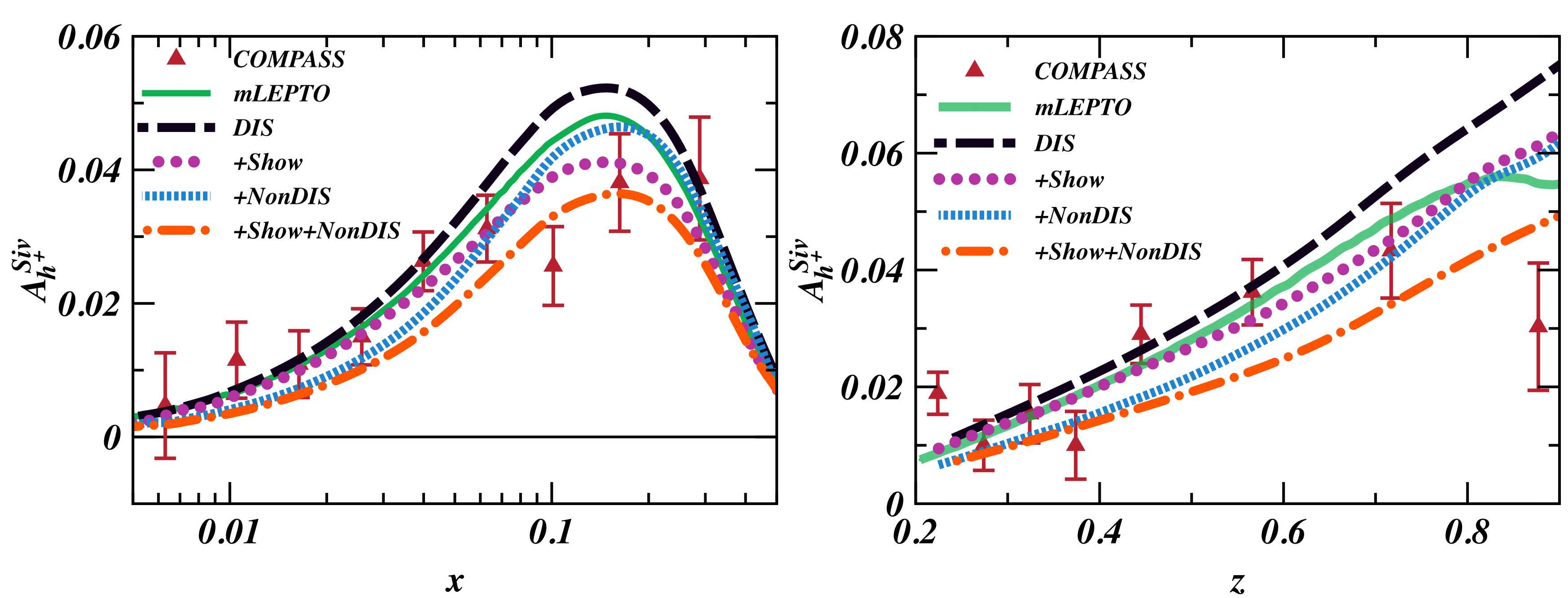
- Fits for *Sivers PDF* from HERMES and COMPASS data utilize *LO DIS-only* expressions for *SSAs*.

M. Anselmino et. al: PRD 86, 014028 (2012).

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \Delta^N \hat{f}_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \hat{f}_{q/p}(x,k_\perp,Q)\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)}.$$

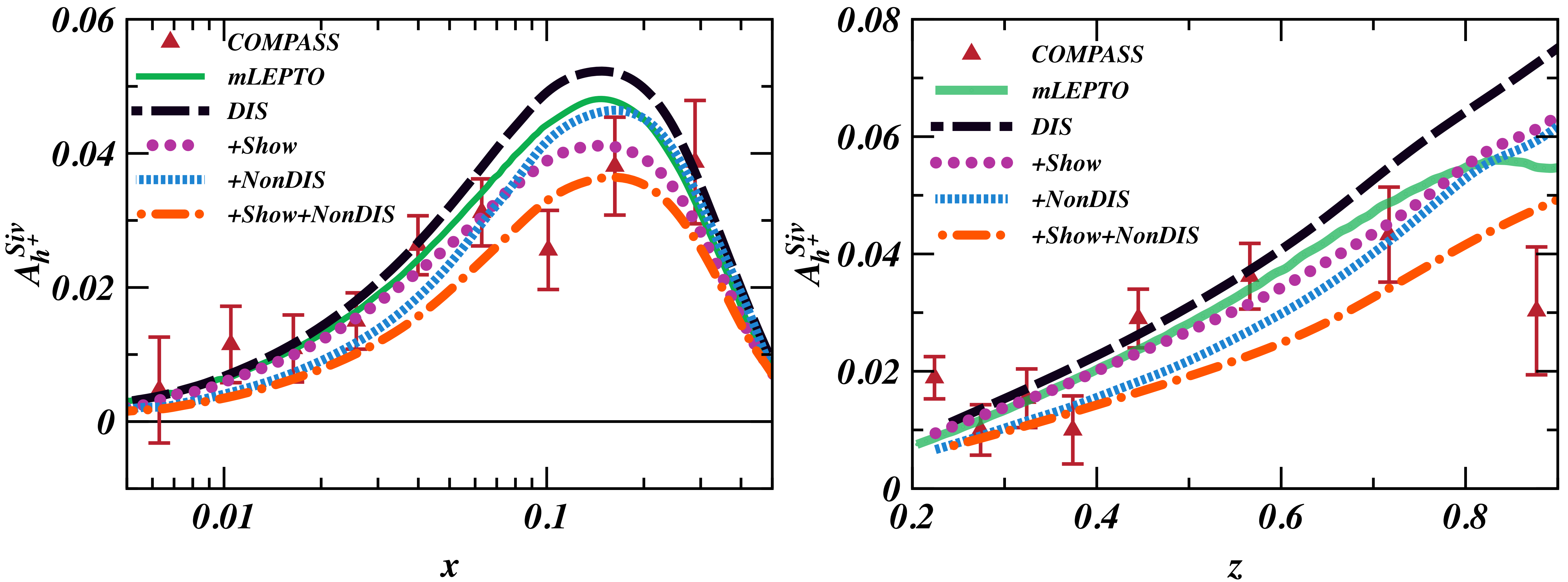
- **Is this justified at COMPASS energies?**
- ***Test using mPYTHIA: turn on non-DIS effects (VMD, GVMD, "direct") and parton showering (QCD+QED).***

H.M et al., arXiv:1502.02669 (2015).



LO APPROXIMATION FOR SSA

H.M et al., arXiv:1502.02669 (2015).

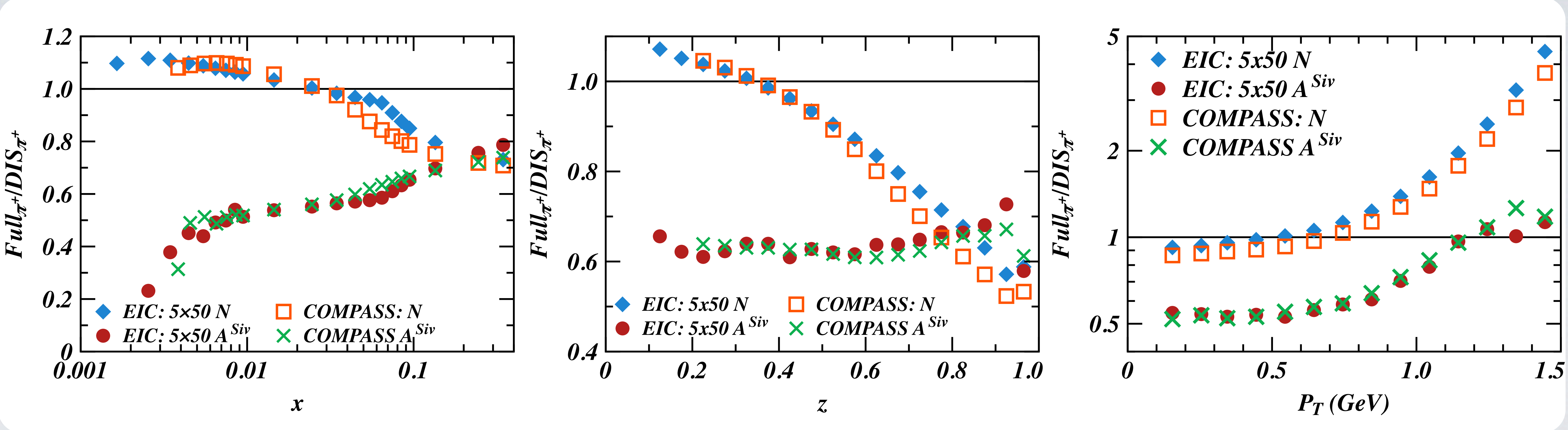


- **Significant** effects, but still agrees with data**!**
- Current Sivers PDF extractions *may* be underestimated.
- Note: no *model-independent* way to exclude non-DIS effects.

Can We Still Use These Parametrizations?

H.M et al., arXiv:1502.02669 (2015).

- ***How reliable are our SSA predictions for other experiments?***
- ***Construct Ratios of Full (non-DIS + showers) to LO DIS results for multiplicities and Sivers SSAs at COMPASS and EIC.***



- The Ratios are *very close* between COMPASS and EIC:
- We can *reliably estimate* SSAs if we use only *LO DIS* terms with the *current parametrization* of Sivers PDFs.