# Precision determination of the top mass

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**I.**: **Basic facts about the top quark**

**II**: **Basic facts about the mass in general**

**III**: **Implications on the precision for the top mass**

# Why do we care about precision on the top mass?

- Obviously, the value of the top mass affects the measured top cross sections.
- Affects searches for new physics with top background, BSM decays into tops, etc.
- Top mass close to the electro-weak breaking scale, impact on precision physics of the Higgs sector.
  If there is new physics associated with electro-weak symmetry breaking top physics is a place to look for.
- If the Standard Model is assumed to be valid to very high scales, the stability of the electro-weak vacuum depends crucially on the precise numerical value of $m_t$.

# Basic facts about top

The essential numbers:

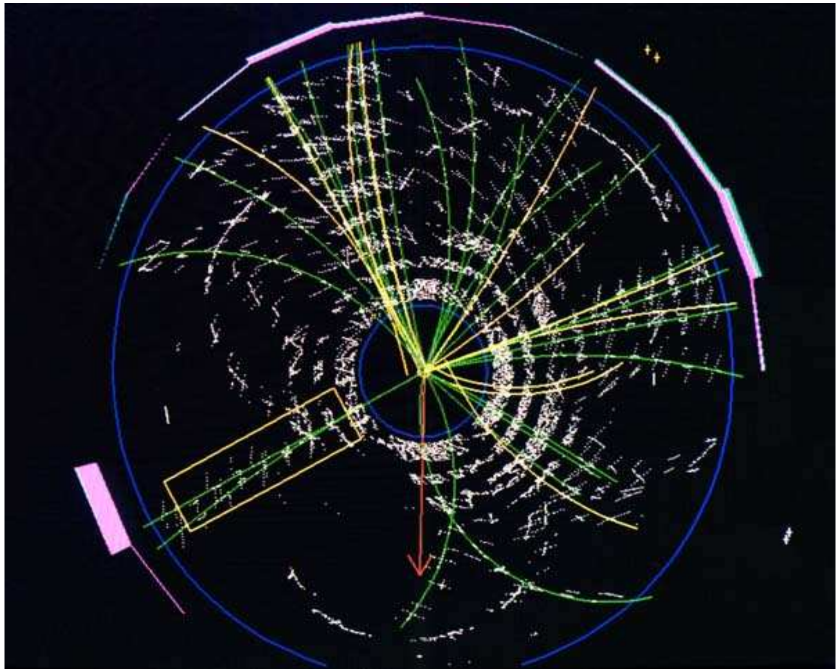
Mass:

$$m_t = 173.21 \pm 0.51 \pm 0.71 \text{ GeV}$$

Width:

$$\Gamma = 2.0 \pm 0.5 \text{ GeV}$$

Discovered at the Tevatron in 1995



A $t\bar{t}$ event from CDF.

# Basic facts about top

The top quark is special:

+ The large top mass sets a hard scale.

+ Lifetime shorter than characteristic hadronization time scale.

$\Rightarrow$ Top physics is (mainly) described by perturbative QCD.

But, of course as any quark of the $2^{\text{nd}}$ or $3^{\text{rd}}$ generation:

- The top quark is a colour-charged particle.

- The top quark is not a stable particle.

$\Rightarrow$ There is no asymptotic free top state,
non-perturbative effects (might) enter here through the back door.

# Why are there theory talks on the top mass?

Up to now top quark produced only at hadron colliders.

Up to now there is no “theory-free” experimental determination of the top mass.

Experimental measurements rely on theoretical input through template method / matrix element method.

The error on the top mass is approaching $O(\Lambda_{\mathrm{QCD}})$.

Can we in principle improve the error below $O(\Lambda_{\mathrm{QCD}})$?

We would like to be able to reduce the theory error systematically by calculating higher-order corrections.

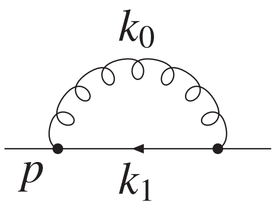
# Basic facts about a fermion mass

Theorists like Lagrangians:

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}_{\text{bare}}\left(i\not{D} - m_{\text{bare}}\right)\Psi_{\text{bare}}$$

Beyond leading-order in perturbation theory: The (one-loop) self-energy:

$$-i\Sigma =$$

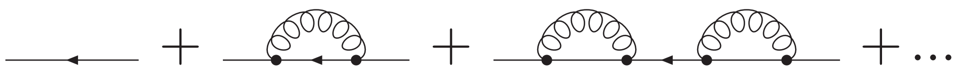



$$= \frac{g^2 C_F}{\mu^{D-4}} \int \frac{d^D k}{(2\pi)^D}\, i\gamma_\rho \frac{i}{\not{k}_1 - m_{\text{bare}}}\, i\gamma^\rho \frac{(-i)}{k_0^2} = -i\left(A\not{p} + B m_{\text{bare}}\right)$$

- In four space-time dimensions this integral is divergent.
- $D = 4 - 2\varepsilon$ is a regulator, divergences show up as poles $1/\varepsilon$.
- $A$ and $B$ depend on $p^2$, $m_{\text{bare}}^2$ and $\mu^2$.

# Basic facts about a fermion mass

Resummed self-energy insertions:



$$= \frac{i}{\not{p} - m_{\rm bare} - \Sigma}$$

$$= \frac{i(1+A)}{\not{p} - (1+A+B)m_{\rm bare}} + O\left(\alpha_s^2\right)$$

Renormalisation:

$$\psi_{\rm bare} = \sqrt{Z_2}\,\psi_{\rm renorm}$$

$$m_{\rm bare} = Z_m\, m_{\rm renorm}$$

All renormalisation schemes entail:

- Wave function renormalisation: Absorb UV-divergences of $(1+A)$ in the numerator.
- Mass renormalisation: Absorb UV-divergences of $(1+A+B)$.

## The $\overline{\mathrm{MS}}$-scheme

Absorb only the parts proportional to $\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)$ and nothing else into $Z_m$:

$$Z_m = 1 - (A+B)_{\mathrm{div}}$$

The propagator is then

$$\frac{i}{\not{p} - m_{\overline{\mathrm{MS}}} - (A+B)_{\mathrm{fin}} m_{\overline{\mathrm{MS}}}}$$

- $m_{\overline{\mathrm{MS}}}$ depends on the scale $\mu$: Running mass.
- Presence of $(A+B)_{\mathrm{fin}} m_{\overline{\mathrm{MS}}}$: The propagator does not have a pole at $m_{\overline{\mathrm{MS}}}$, matrix elements do not factor at $p^2 = m^2_{\overline{\mathrm{MS}}}$.
- $(A+B)_{\mathrm{fin}}$ depends on $p^2$: Propagator does not yield Breit-Wigner shape.

# The $\overline{\mathrm{MS}}$-scheme

$m_{\overline{\mathrm{MS}}}$ is an example of a short-distance mass.

Can extract $m_{\overline{\mathrm{MS}}}$ from an infrared safe observable for a process like $pp \to l\bar{\nu}jjb\bar{b}$ at high energies by comparing

$$\sigma_{\mathrm{exp}} \quad \text{with} \quad \sigma_{\mathrm{theo}}(m_{\overline{\mathrm{MS}}})$$

Moch, Langenfeld, Uwer, '09;

Czakon, Fiedler, Mitov, '13;

Dowling, Moch, '13

Can also use $pp \to t\bar{t} + \text{jet}$.

Dittmaier, Uwer, S.W., '07,

Melnikov, Schulze, '10

# The on-shell-scheme

Define $Z_m$ such that the propagator has a pole at $m_{\text{pole}}$.

The propagator is then by definition

$$\frac{i}{\not{p} - m_{\text{pole}}}$$

+ $m_{\text{pole}}$ is complex, includes the width.

+ Matrix elements factor at $p^2 = m^2_{\text{pole}}$.

+ Propagator corresponds to a Breit-Wigner shape.

- The pole mass is not a short distance mass.

## Non-perturbative sensitivity related to the pole mass

The pole mass is ambigous by an amount $O(\Lambda_{\mathrm{QCD}})$:

- In the on-shell scheme, the renormalisation constant $Z_m$ contains contributions from all momentum scales, not just the ultraviolet region.
- In higher orders, subsets of diagrams are dominated by the IR-region.
- Therefore, the full perturbative series can only be summed up to an (infrared) renormalon ambiguity.
- The renormalon ambiguity is of $O(\Lambda_{\mathrm{QCD}})$.

Bigi, Shifman, Uraltsev, Vainshtein, '94, Beneke, '94, Smith, Willenbrock, '96

## Conversion between the pole mass and the $\overline{\mathrm{MS}}$-mass

In perturbation theory one has with $\bar{m} = m_{\overline{\mathrm{MS}}}(\mu = m_{\overline{\mathrm{MS}}})$

$$m_{\mathrm{pole}} = \bar{m} \times \left[1 + c_1 \frac{\alpha_s(\bar{m})}{\pi} + c_2 \left(\frac{\alpha_s(\bar{m})}{\pi}\right)^2 + c_3 \left(\frac{\alpha_s(\bar{m})}{\pi}\right)^3 + c_4 \left(\frac{\alpha_s(\bar{m})}{\pi}\right)^4 + \ldots\right]$$

Melnikov, van Ritbergen, '99; Chetyrkin, Steinhauser, '99; Marquard, A. Smirnov, V. Smirnov, Steinhauser, '15

Numerically for the top quark:

$$m_{\mathrm{pole}} = \bar{m} \times [1 + 0.046 + 0.010 + 0.003 + 0.001 + \ldots]$$

The conversion formula is again only an asymptotic series and has an renormalon ambiguity as well.

# Crude estimates of the ambiguity

From the truncation of the conversion formula between $m_{\text{pole}}$ and $\bar{m}$:

$$\delta m_{\text{pole}} \quad \approx \quad \mathcal{O}(\,200\,\text{MeV}\,)$$

From the estimate of the renormalon:

$$\delta m_{\text{pole}} \quad \approx \quad \mathcal{O}(\,270\,\text{MeV}\,)$$

What about determining the non-perturbative effects by comparing two different non-perturbative models?

Engineer A: $13^2 = 172$ (sic)

Engineer B: $13^2 = 174$ (sic)

This does not imply $13^2 = 173 \pm 1$ (sic)

# Measuring the peak position

Can one translate a measurement of the peak position into a theoretical well defined short-distance top mass?

Remark: Experimentalists can measure many things to high precision (average number of pions in $pp$ collisions, etc.), the question is if and how a quantity can be related to a quantity depending only on short-distance physics.

Let's split up this question:

- Which scales are involved?
- How to define a short-distance mass at a given scale?
- How to translate the measurement?

## The involved scales

In order to avoid large logarithms:

- Describe physics at a particular scale $\mu$ by an appropriate effective theory.
- Evolution operators sum up large logarithms.

From a study of $e^+e^- \to t\bar{t}$:

| Scale | Matrix elements | Effective theory | Affects | Remarks |
|---|---|---|---|---|
| $Q \ldots m_t$ | hard function | QCD | norm of the distribution | depends on $m_t$ |
| $m_t \ldots \Gamma_t$ | jet function | SCET | shape and position | depends on $m_t$ |
| $\Gamma_t \ldots \Lambda_{QCD}$ | soft function | top-HQET | shape and position | independent of $m_t$ |

$\Rightarrow$ Need a short-distance mass definition for scales down to $\Gamma_t$.

Fleming, Hoang, Mantry, Stewart, '07

# The MSR mass

Short-distance mass: any mass definition not affected by a renormalon ambiguity.

Idea for construction: Remove contributions giving rise to this ambiguity (known from bottomium, potential subtracted mass).

This will involve apart from the UV-renormalisation scale $\mu$ a second scale $R$.

The $\overline{\mathrm{MS}}$-mass is a short-distance mass, and $R = \bar{m}$ in this case.

The $\mathrm{MSR}$-mass (read: $\bar{m}$ substituted by $R$) is the two-scale generalisation with a UV-scale $\mu$ and an IR-scale $R$, such that

$$m_{\mathrm{MSR}}(R=0) = m_{\mathrm{pole}}, \qquad m_{\mathrm{MSR}}(R=\bar{m}) = \bar{m}.$$

Hoang, Jain, Scimemi, Stewart, '08

# An analogy

Jet cross section: Jets defined by

- an infrared-safe jet algorithm (SISCone, $k_t$-algorithm, anti-$k_t$-algorithm, etc.)
- parameters associated to this algorithm ($R$, $f$, $n_{\rm pass}$, $y_{\rm cut}$, etc.)

Top mass: Mass defined by

- a short-distance renormalisation scheme ($\overline{\rm MS}$-scheme, MSR-scheme, etc.)
- parameters associated to this scheme ($\mu$, $R$, etc.)

# Translating the measurement

Theory sneaks in though template method / matrix element method.

Analogy of factorisation:

| | |
|---|---|
| Effective theory: | Hard function / jet function / soft function |
| Monte Carlo: | Hard matrix element / parton shower / hadronisation |

Parton shower has a lower cut-off.

$\Rightarrow$ Monte Carlo mass is something like a short-distance mass.

Translation for Pythia:

$$m_{\text{Pythia}} = m_{\text{MSR}}\,(R = 1...9\ \text{GeV})$$

This introduces an uncertainty of the order of $1\ \text{GeV}$ on the translation from the Monte Carlo mass to a theoretically well defined short-distance mass.

Hoang, Stewart, ’08

# Work to do for hadron colliders

- Work out in detail factorisation and short-distance mass in $pp$-collisions.
  - Coloured initial states.
  - Jets instead of hemisphere masses.
- Compare in detail MC mass with a well defined short-distance mass.
  - Establish that shower cut-off effectively implements some short-distance mass.
  - Improve translation from MC mass to well-defined short-distance mass.
- Consider practical issues:
  - Can we write a dedicated event generator, based on a well-defined short distance mass?

    Proposals (not specific to top) to go from SCET to exclusive event generators

    Bauer and Schwartz, '06; Bauer, Tackmann and Thaler, '08

# Measuring the top mass at an $e^+e^-$-collider

At an $e^+e^-$-machine: extract top mass from threshold scan around $350\ \mathrm{GeV}$.

| Effective theory | Relevant modes | Scale |
|---|---|---|
| QCD | hard | $m_t$ |
| NRQCD | soft | $m_t v$ |
| potential NRQCD | ultra-soft | $m_t v^2$ |

Appropriate short-distance mass definitions:

- $1\mathrm{S}$ mass (A. Hoang, Z. Ligeti, A. Manohar, T. Teubner)
- potential subtracted mass (M. Beneke)

Theory prediction at N$^3$LO in potential NRQCD

C. Anzai, M. Beneke, Y. Kiyo, B. Kniehl, P. Marquard, A. Penin, J. Piclum, T. Rauh, K. Schuller, D. Seidel, A. Smirnov, V. Smirnov, M. Steinhauser, Y. Sumino

Precision on top mass expected to be below $100\ \mathrm{MeV}$.

# Summary

- The value of the top mass is essential for many precision measurements.
- Want to have a well defined short-distance mass.
- Assume that today we would have both a hadron collider and an $e^+e^-$-collider available:
  - Current state-of-the-art: a threshold scan at an $e^+e^-$-machine will give the most precise measurement.
- Assume a hadron collider today and an $e^+e^-$-collider in the future:
  - Theoretical uncertainties for a hadron collider will go down, threshold scan at an $e^+e^-$-machine will probably stay the most precise option.