Radiative corrections to QED processes

F. Piccinini

INFN Pavia

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and
Radiative Corrections

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material from talk of C.M. Carloni Calame at FCC meeting in Pisa 2015

QED processes: candles for e^+e^- collider luminosity

- Luminosity is a machine (process independent) parameter entering every experimental cross-section

$$\frac{N_{obs}}{\mathcal{L}} = \sigma \quad \rightarrow \quad \mathcal{L} = \frac{N_{obs}^X}{\sigma_{theory}^X}$$

- in order to minimize $\delta\mathcal{L}$, the reference process X has to have large statistics, be well calculable theoretically, be cleanly detectable (small systematics)
- the best choice are QED processes, in particular Bhabha
  - ⋆ at small angles at LEP
    - huge statistics, by far dominated by photon t-channel contribution (only QED, no "Z contamination")
  - ⋆ at large angles at flavour factories
    - no need of dedicated detectors, also here dominated by t-channel photon exchange
- to reduce the theoretical error on σ all the relevant radiative corrections (RC) must be included

One example: neutrino counting

$$R^0_{\rm inv} = \frac{\Gamma_{\rm inv}}{\Gamma_{ll}} = \sqrt{\frac{12\pi R^0_l}{\sigma^0_{\rm had} m_Z^2}} - R^0_l - (3+\delta_\tau)$$

- assuming lepton universality

$$\left(R^0_{\rm inv}\right)_{\rm exp} = N_\nu \left(\frac{\Gamma_{\nu\bar\nu}}{\Gamma_{ll}}\right)_{\rm SM}$$

- from LEP Z-peak measurements

$$\begin{aligned} N_\nu &= 2.9840 \pm 0.0082 \\ \delta N_\nu &\simeq 10.5 \frac{\delta n_{\rm had}}{n_{\rm had}} \oplus 3.0 \frac{\delta n_{\rm lept}}{n_{\rm lept}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}} \\ \frac{\delta \mathcal{L}}{\mathcal{L}} &= 0.061\% \Longrightarrow \delta N_\nu = 0.0046 \end{aligned}$$

ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

- δN_ν severely afftected by luminosity uncertainty (theory dominated at LEP)

perturbative side: QED Radiative Corrections

- Large part of RC in Bhabha is due to photonic corrections, driven at $\mathcal{O}(\alpha)$ by $\alpha\times$ the collinear log $L = \log\left(\frac{st}{um_e^2}\right) - 1$

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

  - $\star$ $L \simeq \log(s/m_e^2) - 1 \simeq 14$ at flavour factories (large angle)
  - $\star$ $L \simeq \log(-t/m_e^2) - 1 \simeq 16$ at LEP1 or FCC-ee-Z (small angle)
  - $\star$ $L \simeq \log(-t/m_e^2) - 1 \simeq 19$ at FCC-ee-t

- a $\sim 18\%$ increase of QED RC impact from LEP1/FCC-ee-Z to FCC-ee-t is naively expected

Monte Carlo Tools for Luminometry

- The calculations must be implemented into MC event generators
  - $\rightarrow$ any complex event selection criteria can be accounted for
  - $\rightarrow$ realistic simulation of the process

A. Arbuzov *et al.*, Phys. Lett. B **383** (1996) 238 [hep-ph/9605239]
S. Jadach *et al.*, Physics at LEP2, vol. 2 [hep-ph/9602393]
S. Actis *et al.*, Eur. Phys. J. C **66** (2010) 585 [arXiv:0912.0749]

- MCs for LEP
  - ⋆ **BHLUMI** - Jadach et al.
  - ⋆ **SABSPV** - Cacciari et al.
  - ⋆ **BHAGEN95** - Remiddi et al.
  - ⋆ **NLLBHA** - Arbuzov et al.
- MCs for flavour factories
  - ⋆ **BabaYaga** (**3.5** and **NLO**) - C.M. Carloni Calame et al.
  - ⋆ **BHWIDE** - Jadach et al.
  - ⋆ **MCGPJ** - Arbuzov et al.

Monte Carlos building blocks

In order to achieve a $\mathcal{O}(0.1\%)$ accuracy, the most precise event generators implement

- Leading Log QED RC up to all orders (QED PS, YFS, SF,...)
- exact QED NLO corrections (matched with LL)
- relevant weak corrections (at LEP energies)
- photon vacuum polarization corrections (see later)
- part of QED NNLO corrections
- fully exclusive event generation

Besides technical reliability, at present the main sources of theoretical error in MCs are

- ⋆ partial inclusion of QED NNLO: the error starts at $\mathcal{O}(\alpha^2)$
  $\rightarrow$ this can be improved in a "short" time
- ⋆ vacuum polarization: hadronic contribution depends on low energy data
  $\rightarrow$ upcoming measurements are essential

Theoretical error on SABS at LEP1

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- the theoretical accuracy has been achieved with a hard work of many groups and comparing independent calculations/codes

...and at flavour factories from Eur. Phys. J. C **66** (2010) 585

Main conclusion of the Luminosity Section of the WG Report

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

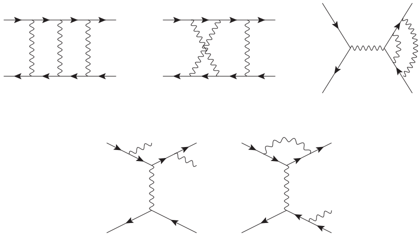
Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ [in progress]	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$	0.03	0.016	0.03
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07	0.06	0.06

- For the experiments in proximity of the ψ/Υ's resonances, the accuracy slightly deteriorates

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP/SLC

NNLO Bhabha calculations

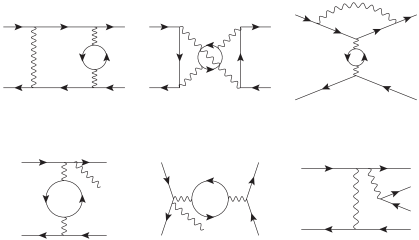
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

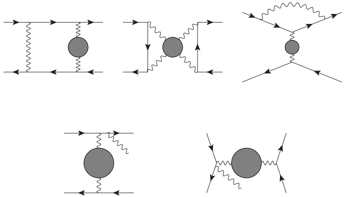
S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26



- Heavy fermion and hadronic loops

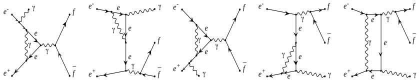
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419



Theoretical accuracy of MC's, comparisons with NNLO

- ⋆ The NNLO QED RC to Bhabha have been calculated but not available into EGs yet
- the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) (though large NNLO RC already included by expon. and by $\mathcal{O}(\alpha)$ LL $\times$ finite-NLO)
- e.g., **BabaYaga** formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

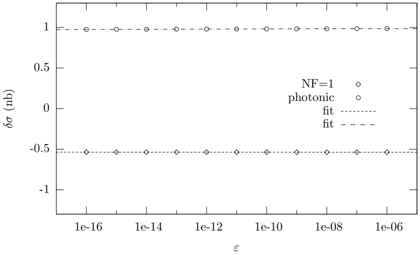
  - $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2) \longrightarrow$ compared with the corresponding available NNLO QED calculation
  - $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ estimated relying on existing (partial) results
  - $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

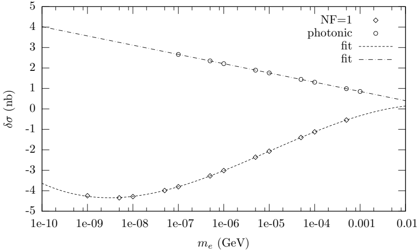
Comparison with (a subset of) NNLO

G. Balossini et al., NPB758 (2006) 227

Comparison of $\sigma^{\alpha^2}_{\rm SV}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)





★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma^{\alpha^2}_{\rm SV}(\text{Penin}) - \sigma^{\alpha^2}_{\rm SV}(\text{BabaYaga@NLO}) \; < \; 0.02\% \times \sigma_0$$

Resummation beyond α^2

- $\star$ with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

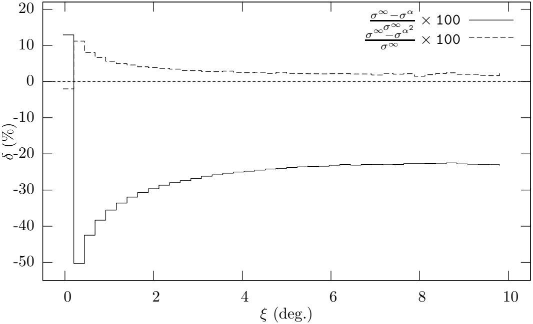


Figure : Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution at DAΦNE

- $\star$ resummation beyond α^2 still important

Vacuum Polarization

- The running of the EM constant α is an important effect
- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\qquad \Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\text{top}}(q^2) + \Delta\alpha^{(5)}_{\text{had}}(q^2)$
- $\Delta\alpha^{(5)}_{\text{had}}$ is an intrinsically non-perturbative contribution. It can be calculated from $e^+e^- \to hadrons$ data using dispersion relations

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = -\frac{q^2\alpha}{3\pi}\left[P\!\!\!\!\int_{4m_\pi^2}^{E^2_{cut}} \frac{R^{data}_{had}(s)}{s(s-q^2)}ds + P\!\!\!\!\int_{E^2_{cut}}^{\infty} \frac{R^{pQCD}_{had}(s)}{s(s-q^2)}ds \right]$$

- it is affected by an error, due to low energy data on $\sigma_{had}(s)$. In turn, it reflects on Bhabha predictions
- an historical perspective on the evolution of the error
  - $\Delta\alpha(M_Z^2) = 0.0280 \pm 0.0007 \Longrightarrow \alpha^{-1}(M_Z^2) = 128.89 \pm 0.09$ H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398
  - $\Delta\alpha(M_Z^2) = 0.02750 \pm 0.00033$ H. Burkhardt and B. Pietrzyk, Phys. Rev. D84 (2011) 037502
  - $\Delta\alpha(M_Z^2) = 0.027498 \pm 0.000135 [0.027510 \pm 0.000218]$ F. Jegerlehner, arXiv:1107.4683
  - $\Delta\alpha(M_Z^2) = 0.02757 \pm 0.0001$ Davier, Hoecker, Malaescu, Zhang, arXiv:1010.4180
  - $\Delta\alpha(M_Z^2) = 0.027626 \pm 0.000138$ T. Teubner et al., Nucl. Phys. Proc. Suppl. 225 (2012) 282

BabaYaga for Bhabha at FCC-ee, examples

- example with **BabaYaga** run at FCC-ee, in the small angle regime
- e.g. $3^\circ < \theta_- < 6^\circ$, $174^\circ < \theta_+ < 177^\circ$, $E_- E_+ / E_b^2 > z_{min} = 0.8$

	FCC-ee-Z 91 GeV	FCC-ee-W 160 GeV	FCC-ee-H 240 GeV	FCC-ee-t 350 GeV
σ_0^{VP} (nb)	36.0030	11.8062	5.28998	2.50709
+ Z	$+0.064\%$	-0.062%	-0.044%	-0.030%
+ QED NLO	-17.41%	-18.73%	-19.57%	-20.35%
+ QED h.o.	$+0.54\%$	$+0.66\%$	$+0.71\%$	$+0.77\%$
VP	$+5.17\%$	$+6.27\%$	$+7.14\%$	$+7.99\%$
$\delta\Delta\alpha_h$ [1]	$\pm0.021\%$	$\pm0.027\%$	$\pm0.030\%$	$\pm0.032\%$

⋆ Once full NNLO is merged into MCs, VP error will become the dominant one if the accuracy has to be pushed up to $\mathcal{O}(0.01\%)$

[1] **This uncertainty is by far dominated by the error on $\alpha(t)$**

$e^+e^- \to \gamma\gamma$

- $e^+e^- \to \gamma\gamma$ could be used to cross-check independently $\mathcal{L}$ measurements
  - ⋆ At present, its theoretical accuracy is similar to Bhabha (NLO + h.o.)
  - ⋆ Advantages: no Z exchange diagrams (at LO), no photon VP corrections (up to NNLO)
  - ⋆ Disadvantages: lower x-section, less clean signal (?)
  - ⋆ Efficiency in detecting $\gamma\gamma$ events?
- e.g. at least 2 γ's with $10^\circ < \theta_\gamma < 170^\circ$, $E_1 E_2 / E_b^2 > z_{min} = 0.8$

	FCC-ee-Z	FCC-ee-W	FCC-ee-H	FCC-ee-t
σ_0 (pb)	60.962	19.785	8.793	4.135
+ QED NLO	-8.61%	-9.06%	-9.40%	-9.71%
+ QED h.o.	-0.37%	-0.41%	-0.42%	-0.44%

⋆ What is a realistic experimental error estimate for $\gamma\gamma$@FCC-ee?

Summary

- On the theory side, precise luminosity determination at FCC-ee can greatly benefit from past experience, LEP and flavour factories
- Tools to reach a theoretical accuracy at less than 0.1%, with small angle Bhabhas, are already on the market
- There is room for improvements, to reach $\mathcal{O}(0.01\%)$ accuracy
  - new data at low energies for hadronic contribution to VP
  - sistematically include QED NNLO into event generators
  - include in MCs exact $\mathcal{O}(\alpha)$ weak corrections to full Bhabha
- QED exponentiation still needed to achieve high accuracy
- Can $e^+e^- \to \gamma\gamma$ be a cross-reference process?