

# Description of multipole asymmetries with the Buda Lund hydrodynamical model

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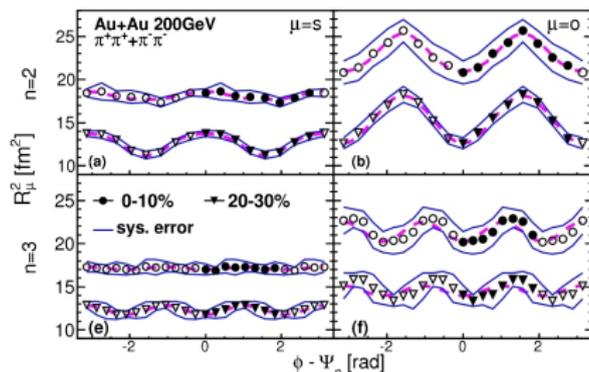
WPCF 2015, Warsaw

# Introduction

- QGP behaves like perfect fluid  $\rightarrow$  hydro description
- Finite number of nucleons  $\rightarrow$  generalized geometry is necessary
- Generalize the space-time and the velocity field distribution
- Higher order flows can be investigated
- HBT radii have  $\cos(n\phi)$  dependences in the respective reaction plane
- These can be studied experimentally:

Nucl.Phys. A904-905 (2013) 439c-442c

Phys.Rev.Lett. 112 (2014) 22, 222301



# The Buda-Lund model

Phys.Rev. C54 (1996) 1390 and Nucl.Phys. A742 (2004) 80-94

- Hydro-model:  $S(x, p) = \frac{g}{(2\pi)^3} \frac{p^\nu d^4 \Sigma_\nu(x)}{B(x, p) + s_q}$  where  
 $B(x, p) = \exp \left[ \frac{p^\nu u_\nu(x) - \mu(x)}{T(x)} \right]$  is the Boltzmann phase-space distribution and the  $p^\nu d^4 \Sigma_\nu(x) = p^\nu u_\nu \delta(\tau - \tau_0) d^4 x$
- Spatial elliptical asymmetry is ensured by the scaling variable

$$s = \frac{r_x^2}{2X^2} + \frac{r_y^2}{2Y^2} + \frac{r_z^2}{2Z^2}$$

- The asymmetry in the velocity field is also elliptical

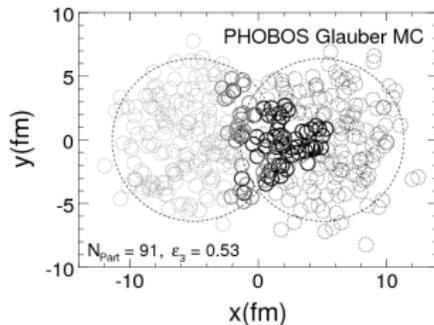
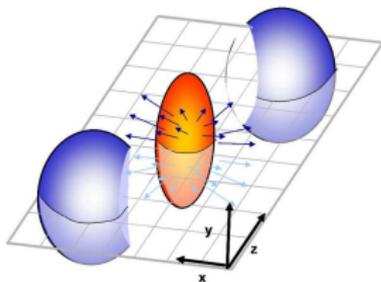
$$u_\mu = \left( \gamma, r_x \frac{\dot{X}}{X}, r_y \frac{\dot{Y}}{Y}, r_z \frac{\dot{Z}}{Z} \right)$$

# Generalization of the model I.

- The spatial asymmetry is described by the scaling variable
- General  $n$ -pole spatial asymmetry (elliptical case:  $n = 2$ ):

$$s = \frac{r^2}{2R^2} \left( 1 + \sum_n \epsilon_n \cos(n(\phi - \Psi_n)) \right) + \frac{r_z^2}{2Z^2}$$

- $\Psi_n$  is the angle of the  $n$ -th order reaction plane



## Generalization of the model II.

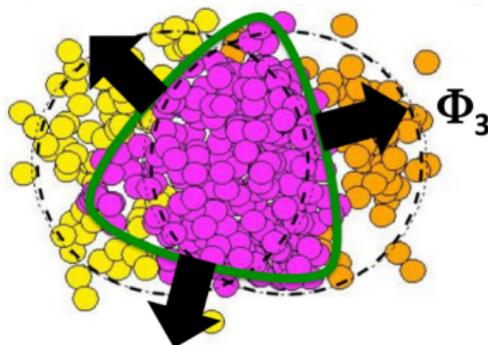
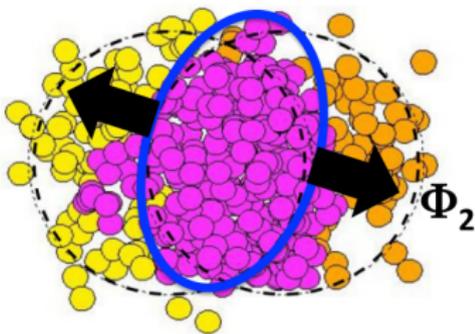
- Derive the velocity field from a potential:  $u_\mu = \gamma(1, \partial_x \Phi, \partial_y \Phi, \partial_z \Phi)$
- General  $n$ -pole asymmetrical potential (elliptical case:  $n = 2$ ):

$$\Phi = Hr^2 \left( 1 + \sum_n \chi_n \cos(n(\phi - \Psi_n)) \right) + H_z r_z^2$$

- $u^\mu \partial_\mu s = 0$  is satisfied:
  - in  $\mathcal{O}(\epsilon_n)$  and  $\mathcal{O}(\chi_n)$  if  $\dot{\epsilon}_n = -2H\chi_n$
  - different  $n$  does not coupling
- From Euler-equation:
  - dynamics of the  $\chi_n$ s:  $\dot{\chi}_n = \chi_{n,0} \left( \frac{\tau}{\tau_0} \right)$
  - the initial perturbations are constants during the evolution in first order
- There is multipole solution: Phys.Rev.C90,054911 (2014) based on Heavylon Phys.A21:73-84,2004

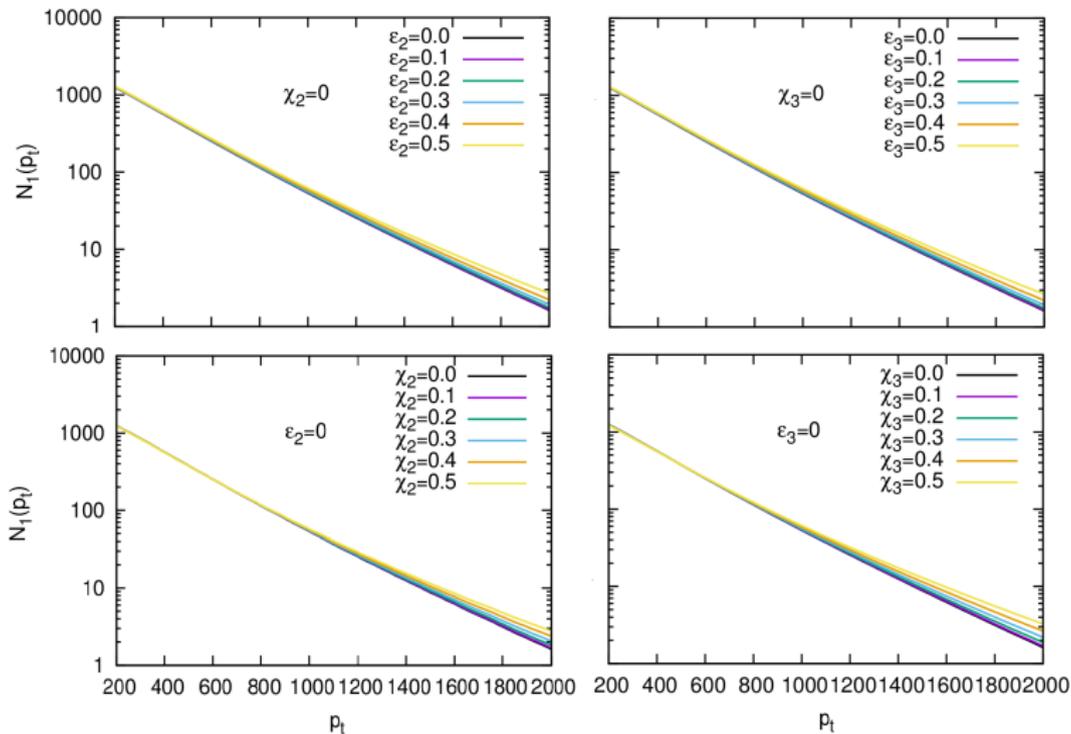
# Observables at freeze-out

- Invariant transverse momentum distribution, flows, azimuthally sensitive HBT radii
- All asymmetries are investigated in their respective reaction plane
- Rotate the system to the second / third order plane and average on the angle of the third / second order plane
- The proper parameters can be set to zero

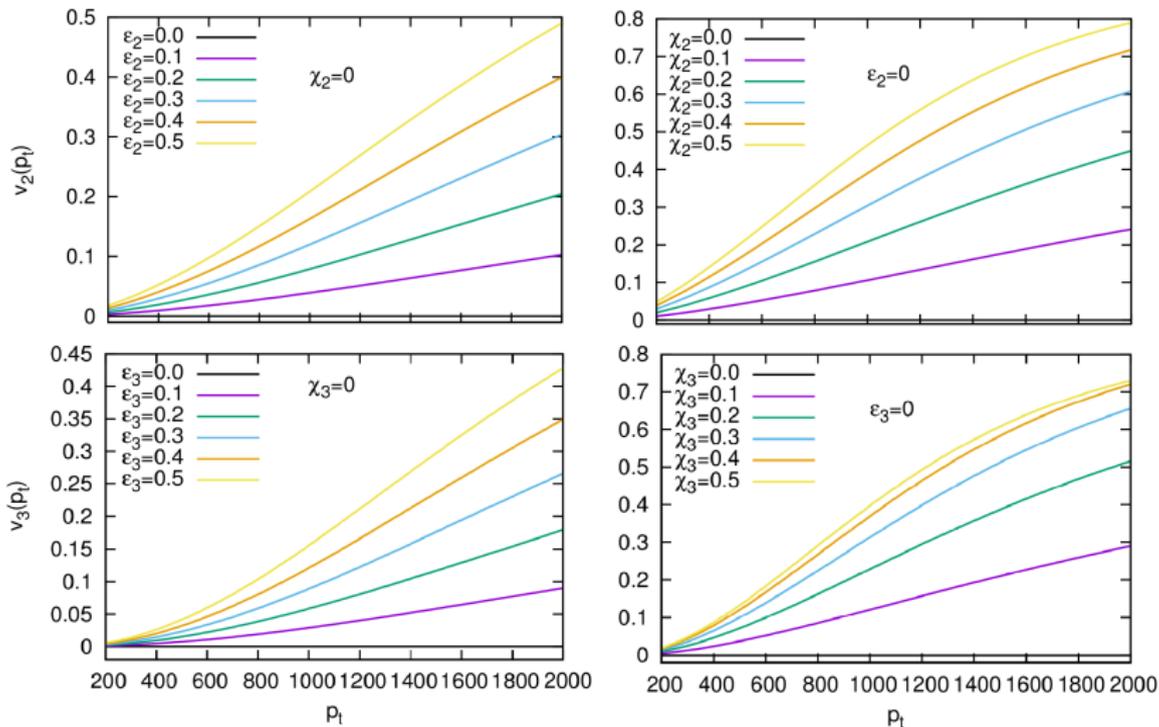


# Invariant momentum distribution

Significant change could be at high  $p_t$ , the log slope is not affected strongly

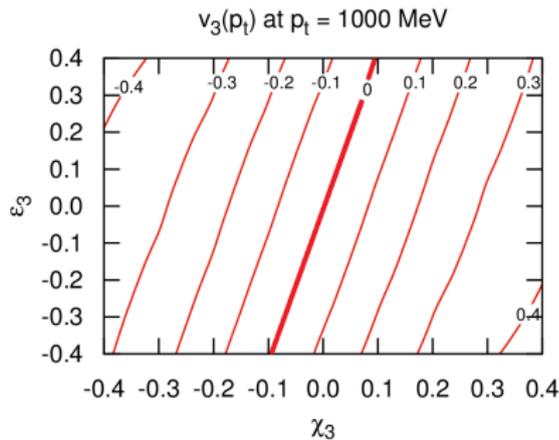
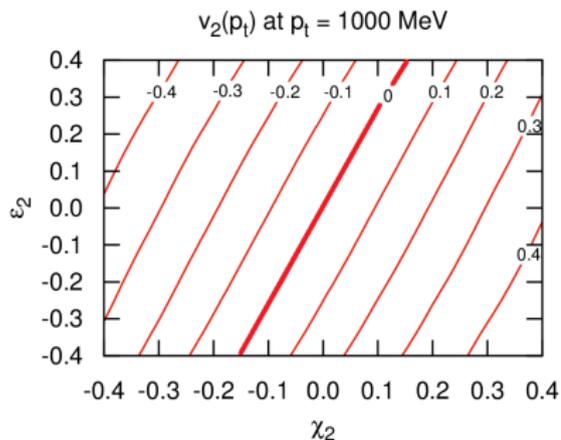


Elliptic and triangular flows are affected by their own asymmetry parameters



# Mixing of parameters

- The parameters affect the flows together
- The generalization of velocity field is necessary



- Calculate in the *out – side – long* system

$$R_{\text{out}}^2 = \langle r_{\text{out}}^2 \rangle - \langle r_{\text{out}} \rangle^2 \text{ and } R_{\text{side}}^2 = \langle r_{\text{side}}^2 \rangle - \langle r_{\text{side}} \rangle^2$$

where  $r_{\text{out}} = r \cos(\phi - \alpha) - \beta_t t$  and  $r_{\text{side}} = r \sin(\phi - \alpha)$

→ C. J. Plumberg, C. Shen, U. W. Heinz Phys.Rev. C88 (2013) 044914

- There can be higher order parts

→ B. Tomášik and U. A. Wiedemann, in *QGP3*, pp. 715–777.

- We use the following parameterization in

- elliptical case:

$$R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},2}^2 \cos(2\alpha) + R_{\text{out},4}^2 \cos(4\alpha) + R_{\text{out},6}^2 \cos(6\alpha)$$

- triangular case:

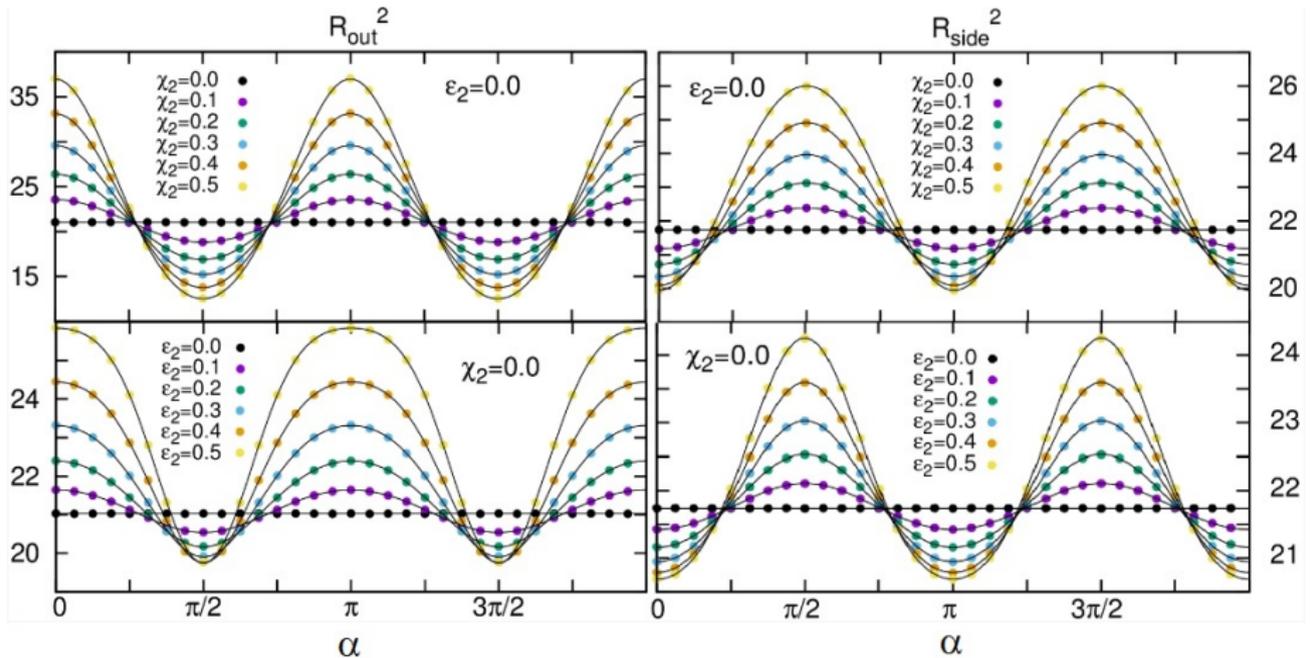
$$R_{\text{out}}^2 = R_{\text{out},0}^2 + R_{\text{out},3}^2 \cos(3\alpha) + R_{\text{out},6}^2 \cos(6\alpha) + R_{\text{out},9}^2 \cos(9\alpha)$$

- Similar to the  $R_{\text{side}}^2$

# Results of the parametrization – Second order case

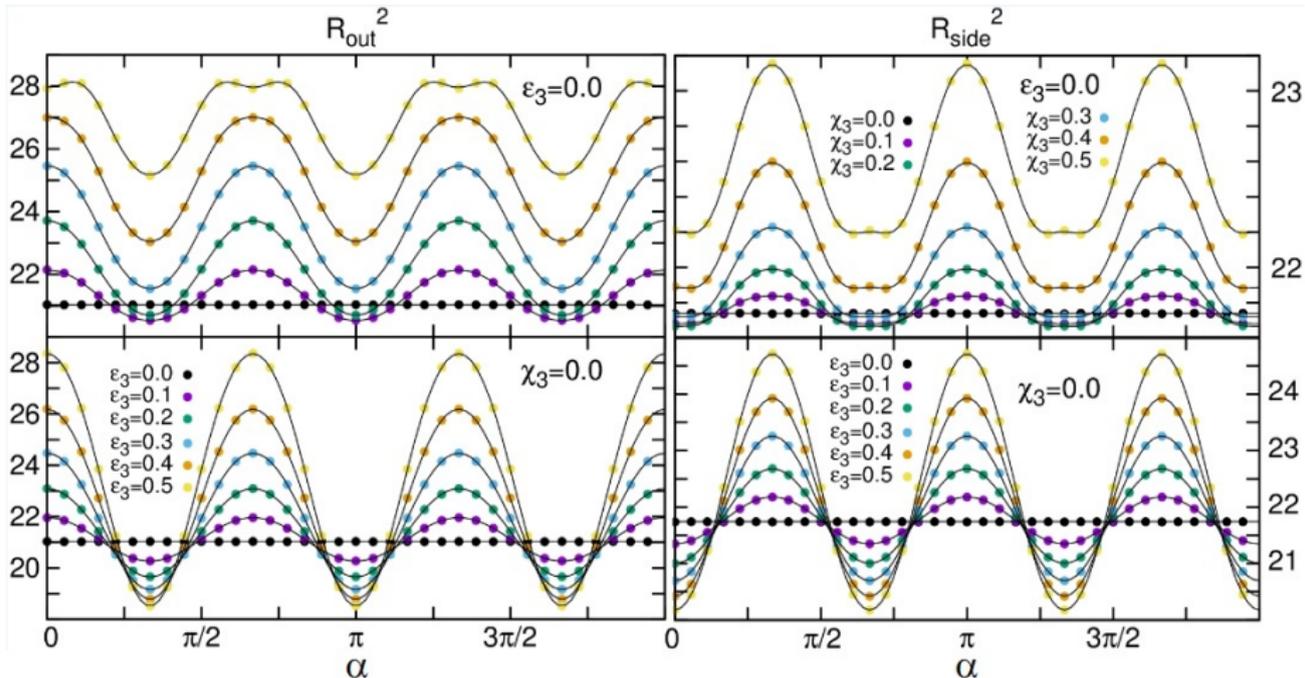
This case already have investigated: Eur.Phys.J.A37:111-119,2008

Mainly  $\cos(2\phi)$  behavior but higher order oscillations are also present



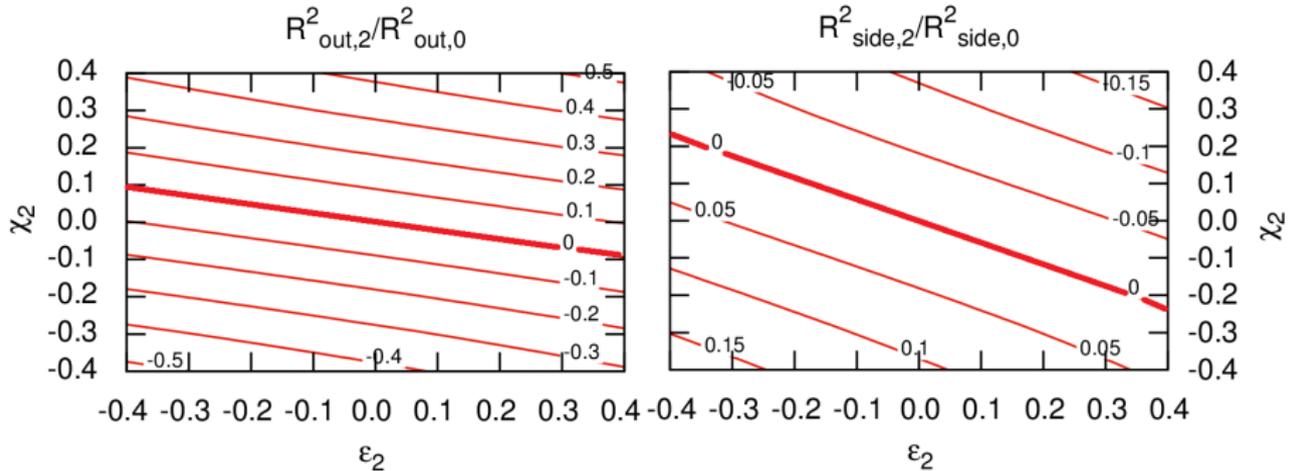
# Results of the parametrization – Third order case

Mainly  $\cos(3\phi)$  behavior but higher order oscillations are also present



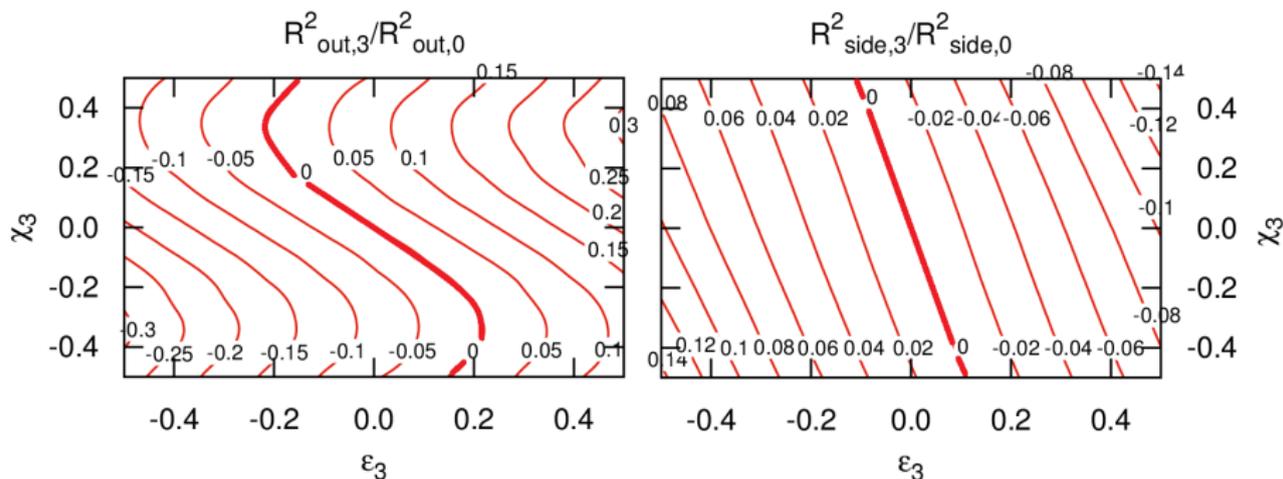
# Mixing of the parameters

The dependence of the amplitudes of the  $R_{\text{out}}^2$  and  $R_{\text{side}}^2$  in the second order case



# Mixing of the parameters

The dependence of the amplitudes of the  $R_{\text{out}}^2$  and  $R_{\text{side}}^2$  in the third order case



# Conclusions

- Generalization of  $u^\mu$  and  $s$  with  $u^\mu \partial_\mu s = 0$  kept valid
- The dynamics of the parameters can be obtained
- Absolute value of the azimuthal HBT radii depend on asymmetries
- Higher order oscillation can be observed in HBT radii
- The spatial and velocity field anisotropies both influence the  $v_n$  coefficient and the HBT radii
- The asymmetry parameters can be disentangle from the flows and the amplitudes

# Thank you for your attention!

And let me invite you to the 15th Zimanyi School in Budapest  
<http://zimanyischool.kfki.hu/15/>

## ZIMÁNYI SCHOOL '15



Arnold Gross: Lexicon

15. Zimányi

**WINTER SCHOOL ON  
HEAVY ION PHYSICS**

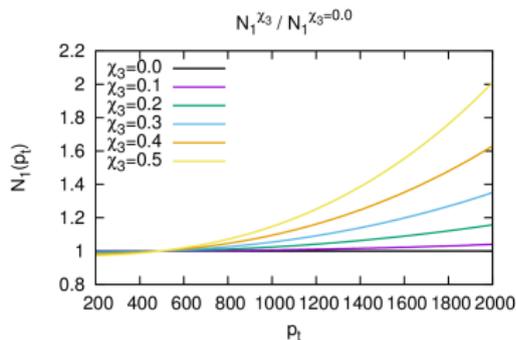
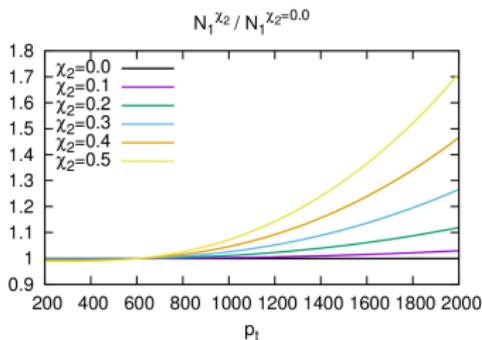
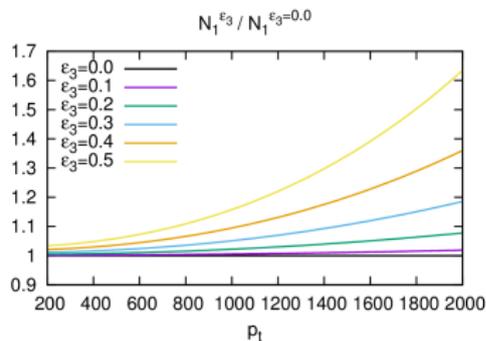
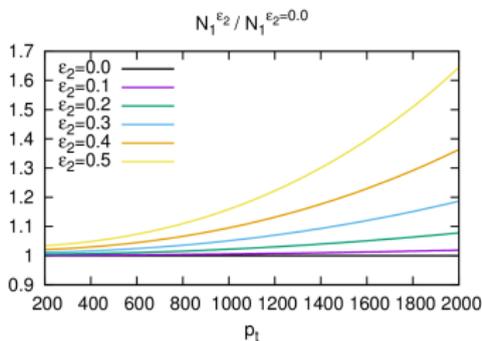
**Dec. 7. - Dec. 11.,  
Budapest, Hungary**



József Zimányi (1931 - 2006)

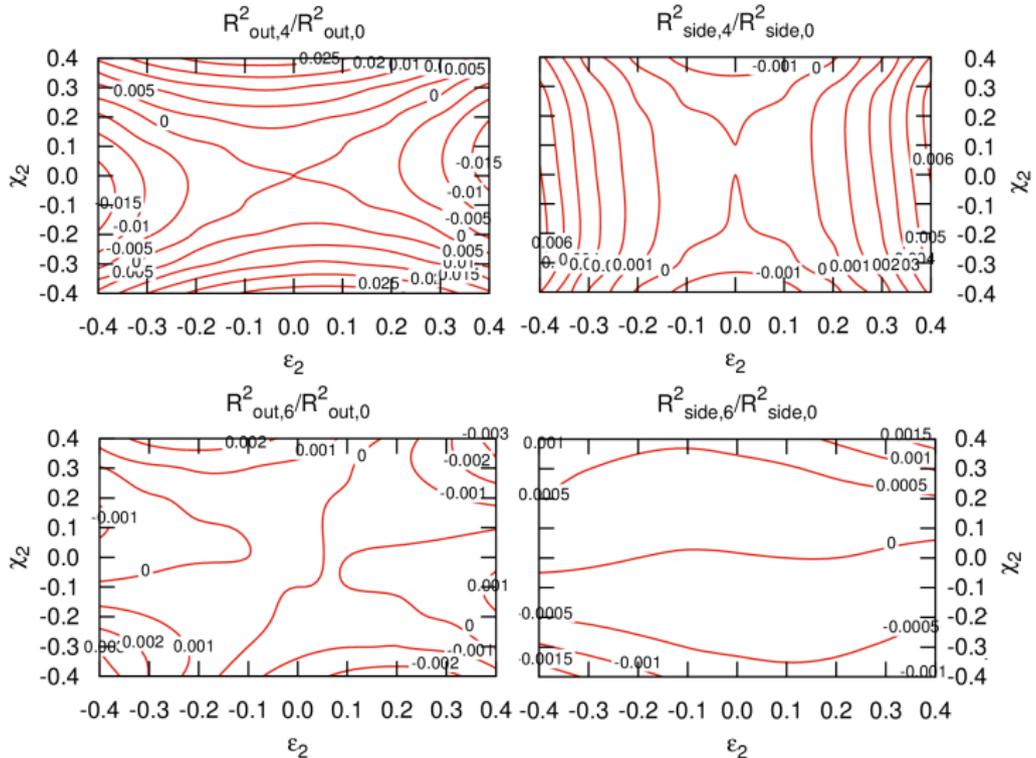
# Backup slides – About the spectra

Plot the  $N_1$  with non zero coefficient divide by  $N_1$  with zero coefficient



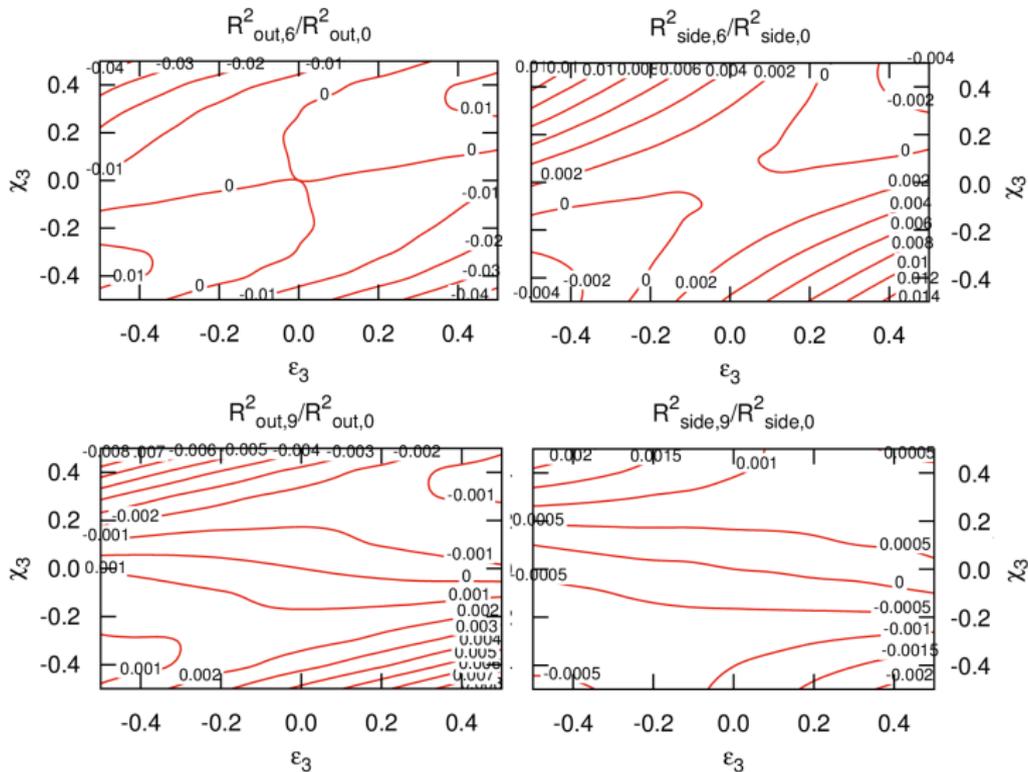
# Backup slides – Higher order amplitudes

Second order:



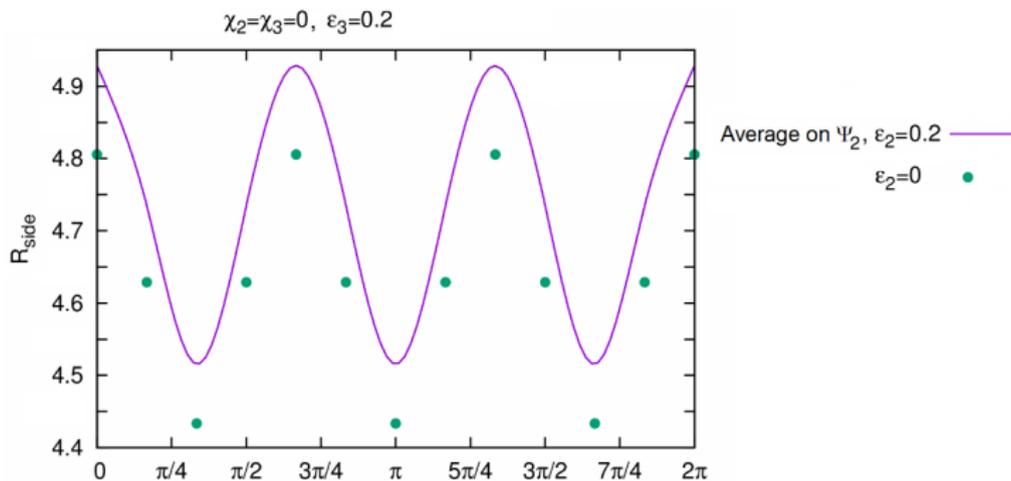
# Backup slides – Higher order amplitudes

Third order:



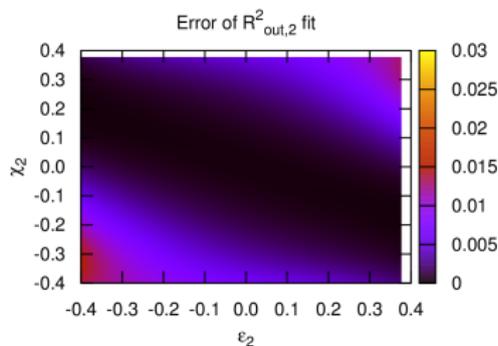
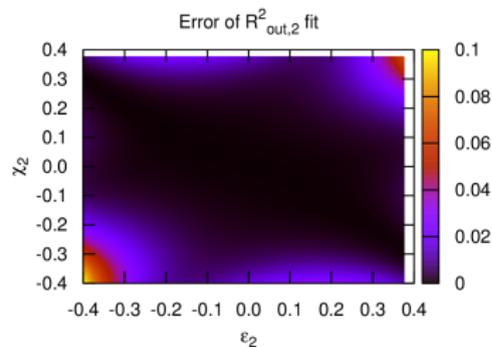
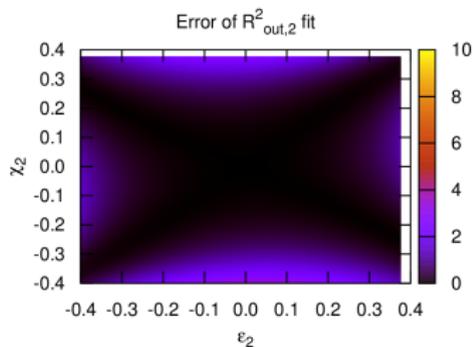
# Backup slides – Averaging

## Averaging vs. set-to-zero



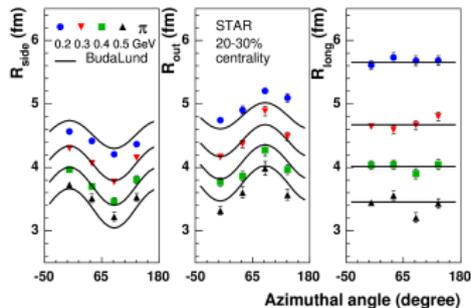
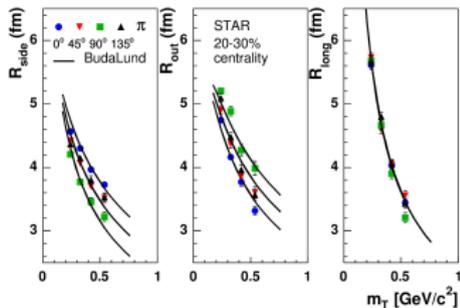
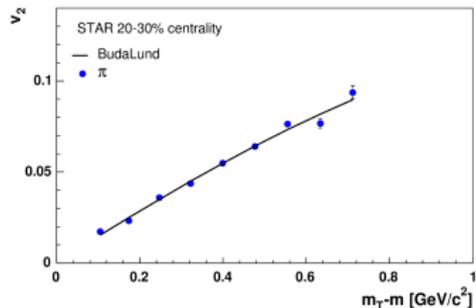
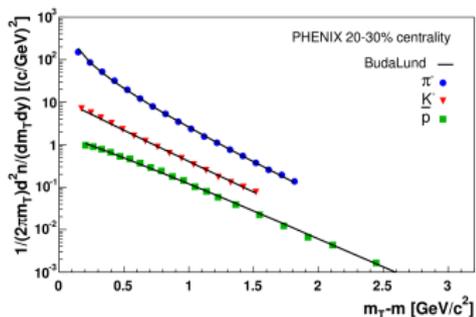
# Backup slides – Square of residuals

An example: square of residuals of  $R_{out}^2$  with different parametrizations



# Earlier results

Fits with elliptical Buda Lund model: Eur.Phys.J. A47 (2011) 58-66



## Value of the parameters

Meaning	Sign	Value
Mass of the particle	$m$	140 MeV
Freeze-out time	$\tau_0$	7 fm/c
Freeze-out temperature	$T_0$	170 MeV
Temperature-asymmetry parameter	$a^2$	0.3
Spatial slope parameter	$b$	-0.1
Transverse size of the source	$R$	10 fm
Longitudinal size of the source	$Z$	15 fm
Velocity-space transverse size	$H$	10 fm/c
Velocity-space longitudinal size	$H_z$	16 fm/c
Elliptical spatial asymmetry parameter	$\epsilon_2$	0.0
Triangular spatial asymmetry parameter	$\epsilon_3$	0.0
Elliptical velocity-field asymmetry parameter	$\chi_2$	0.0
Triangular velocity-field asymmetry parameter	$\chi_3$	0.0

Usually one anisotropy parameter is varied, and the others are kept zero