Interferometry of rotating sources

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Outline

- Introduction
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- The exact model
- Differential HBT
- Results
- Summary

Introduction

- We study an exact rotating and expanding solution of the fluid dynamical model of heavy ion reactions, that take into account the rate of slowing down of the rotation due to the longitudinal and transverse expansion of the system.
- The parameters of the model are set on the basis of realistic 3+1D fluid dynamical calculation at TeV energies, where the rotation is enhanced by the build up of the Kelvin Helmholtz Instability in the flow.

Two particle correlation equations

$$C(p_1, p_2) = \frac{P_2(p_1, p_2)}{P_1(p_1)P_1(p_2)} \qquad P_1(p) = \int d^4x \; S(x, k)$$

$$P_2(p_1, p_2) = \int d^4x_1 d^4x_2 \, S(x_1, p_1) S(x_2, p_2) |\psi_{12}|^2$$

$$\psi_{12} = \frac{1}{\sqrt{2}} \left(e^{ip_1 \cdot x_1 + ip_2 \cdot x_2} + e^{ip_1 \cdot x_2 + ip_2 \cdot x_1} \right)$$

Center of mass momentum $$k = \frac{1}{2}(p_1 + p_2)$$

Relative momentum $$q = p_1 - p_2$$

$$C(k,q) = 1 + \frac{Re[J(k,q)J(k,-q)]}{\left|\int d^4x S(x,k)\right|^2}$$

$$J(k,q) = \int d^4x \; S(x,k) \exp\left[-\frac{q \cdot u(x)}{2T(x)}\right] \exp(iqx)$$

Jüttner distribution

$$f^J(x,p) = \frac{n(x)}{C_n} \exp\left(-\frac{p^\mu u_\mu(x)}{T(x)}\right)$$

Source function

$$S(x,k) = \frac{n(x)k^\mu \sigma_\mu}{C_n} \exp\left[-\frac{k^\mu u_\mu}{T(x)}\right]$$

Gaussian distribution

$$n(x) = n_s \exp\left(-\frac{x^2+y^2+z^2}{2R^2}\right)$$

Correlations for exact hydro model

Cylindrical symmetry, z is the long or beam axis, x is the direction of impact parameter and y determines the side direction

Density

$$N(r_\rho, r_y) = N_B \frac{C_n}{V} \exp(-r_\rho^2/2R^2) \exp(-r_y^2/2Y^2)$$

Scalar variables

$$s_\rho = r_\rho^2/R^2, \;\; s_\varphi = r_\varphi^2/S^2, \;\; s_y = r_y^2/Y^2$$

Density with scalar variables

$$N(s_\rho, s_y) = N_B \frac{C_n}{V} \exp(-s_\rho/2) \exp(-s_y/2)$$

$$\int_0^\infty \int_{-\infty}^\infty \int_0^{2\pi} r_\rho dr_\rho dr_y d\varphi = R^2 Y \int_0^1 \int_0^1 \int_0^{2\pi} \frac{ds_y ds_\rho d\varphi}{\sqrt{s_y}}$$

Equations for the correlation function

$$C(k,q) = 1 + \frac{Re[J(k,q)J(k,-q)]}{\left|\int d^4x S(x,k)\right|^2}$$

$$k_0 = \sqrt{\frac{2m_\pi}{\hbar c} + k^2}$$

$$q_0 = \frac{\boldsymbol{k} \cdot \boldsymbol{q}}{k_0}$$

$$J(k,q) \propto \int_0^1 \int_0^1 \int_0^{2\pi} w_s \gamma_s \left(k_0 + \boldsymbol{k} \cdot \boldsymbol{v_s}\right) \times$$
$$\exp\left[-\frac{\gamma_s}{T_s}((k_0 + q_0/2) - (\boldsymbol{k} + \boldsymbol{q}/2) \cdot \boldsymbol{v_s})\right] \times$$
$$\exp(i\boldsymbol{q} \cdot \boldsymbol{x}) e^{-s_\rho/2} e^{-s_y/2} \frac{ds_y ds_\rho d\varphi}{\sqrt{(s_y)}}$$

Single particle distribution

$$\int d^4x S(x,k) \propto \int w_s \gamma_s \left(k_0 + \boldsymbol{k} \cdot \boldsymbol{v_s}\right) \times$$
$$\exp\left[-\frac{\gamma_s}{T_s}(k_0 - \boldsymbol{k} \cdot \boldsymbol{v_s})\right] e^{-s_\rho/2} e^{-s_y/2} \frac{ds_y ds_\rho d\varphi}{\sqrt{(s_y)}}$$

Exact hydro model

L. P. Csernai, D. J. Wang and T. Csörgo''
PHYSICAL REVIEW C **90**, 024901 (2014)

Velocity

$$\boldsymbol{v}_s = \left(\dot{R}\sqrt{s_\rho}, R\omega\sqrt{s_\rho}, \dot{Y}\sqrt{s_y} \right)$$

or

$$\boldsymbol{v}_s = \Big(\dot{R}\sqrt{s_\rho}\ \sin(\varphi) + R\omega\sqrt{s_\rho}\ \cos(\varphi),$$
$$\dot{Y}\sqrt{s_y}, \dot{R}\sqrt{s_\rho}\ \cos(\varphi) - R\omega\sqrt{s_\rho}\ \sin(\varphi) \Big)$$

t (fm/c)	Y (fm)	$\dot{Y}$ (c)	ω (c/fm)	R (fm)	$\dot{R}$ (c)	φ (Rad)
0.	4.000	0.300	0.150	2.500	0.250	0.000
3.	5.258	0.503	0.059	3.970	0.646	0.307
8.	8.049	0.591	0.016	7.629	0.779	0.467

TABLE I. Time dependence of some characteristic parameters of the fluid dynamical calculation presented in Ref. [6]. R is the average transverse radius, Y is the longitudinal length of the participant system, φ is the angle of the rotation of the interior region of the system, around the y-axis, measured from the horizontal, beam (z) direction in the reaction, $[x, z]$ plane, $\dot{R}$, $\dot{Y}$ are the speeds of expansion in transverse and longitudinal directions, and ω is the angular velocity of the internal region of the matter during the collision.

$$C(k,q) = 1 + \frac{Re[J(k,q)J(k,-q)]}{\left|\int d^4x S(x,k)\right|^2}$$

Assume the temprature profile is flat

Differential Correlation Function (DCF)

$$\Delta C(k,q) \equiv C(k_+,q_{\rm out}) - C(k_-,q_{\rm out})$$

Correlation Function for exact model

$$C(k,q) = 1 + \frac{Re[J(k,q)J(k,-q)]}{\left|\int d^4x S(x,k)\right|^2}$$

t (fm/c)	Y (fm)	$\dot{Y}$ (c)	ω (c/fm)	R (fm)	$\dot{R}$ (c)	φ (Rad)
0.	4.000	0.300	0.150	2.500	0.250	0.000
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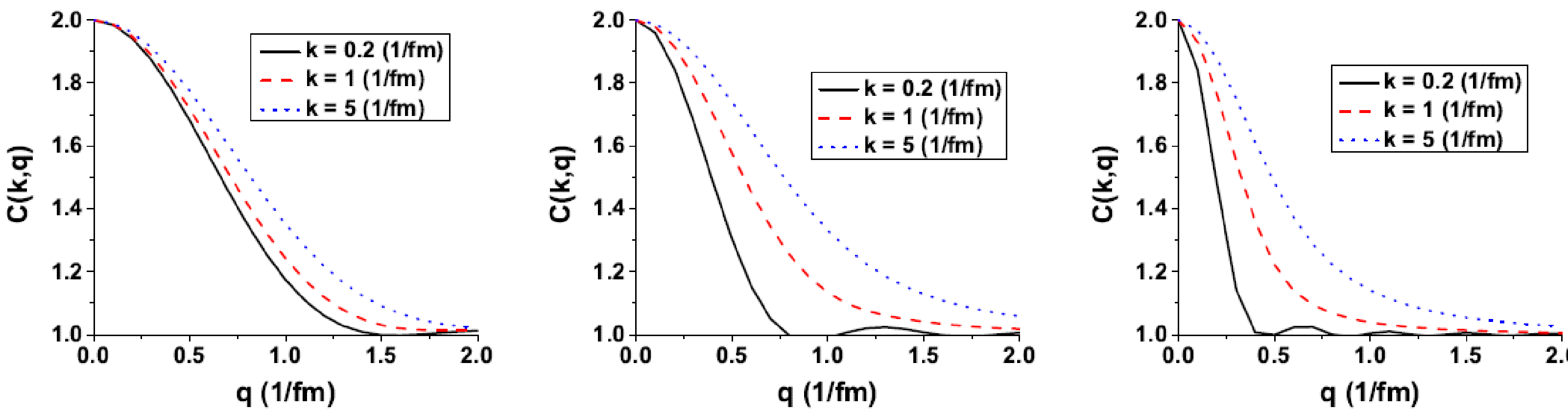


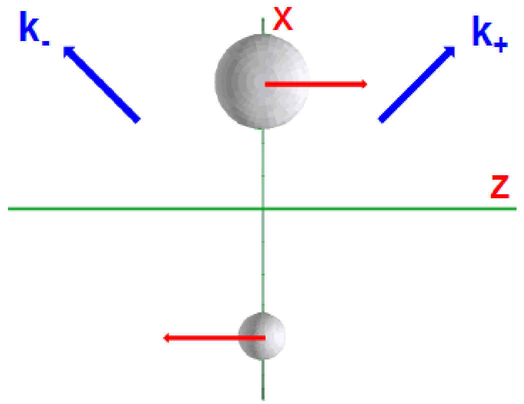
Correlation Function $C(k, q_{out})$ for the exact hydro model as function of $q = q_{out}$, with
(a) R = 2.500 fm, $\dot{R}$ = 0.250 c, Y = 4.000 fm, $\dot{Y}$ = 0.300 fm, ω = 0.150 c/fm,at t = 0.0 fm/c. (left figure)
(b) R = 3.970 fm, $\dot{R}$ = 0.646 c, Y = 5.258 fm, $\dot{Y}$ = 0.503 fm, ω = 0.059 c/fm, at t = 3.0 fm/c. (middle figure)
(c) R = 7.629 fm, $\dot{R}$ = 0.779 c, Y = 8.049 fm, $\dot{Y}$ = 0.591 fm, ω = 0.016 c/fm, at t = 8.0 fm/c. (right figure)
The solid black line is for $k = 0.2\, fm^{-1}$, the dashed red line is for $k = 1\, fm^{-1}$ and the dotted blue line is for $k = 5\, fm^{-1}$.

Differential HBT method

$$\hat{\mathbf{k}}_{\pm} = (a, 0, \pm b)\mathrm{fm}^{-1}, \;\; k_x = a|\mathbf{k}|, \;\; k_z = \pm b|\mathbf{k}|$$

$$a^2 + b^2 = 1$$

$$\hat{\mathbf{q}}_{out} = (a, 0, \pm b), \;\; q_x = a|\mathbf{q}|, \;\; q_z = \pm b|\mathbf{q}|$$
$$\hat{\mathbf{q}}_{side} = (0, 1, 0), \quad q_y = |\mathbf{q}|$$
$$\hat{\mathbf{q}}_{long} = (\mp b, 0, a), \quad q_x = \mp b|\mathbf{q}|, \;\; q_z = a|\mathbf{q}|$$



Simple example with 2 sources only (right figure):

$$C(k_{(\pm)}, q_{out}) = 1 + \exp(-R^2q^2)\,\frac{(1+\epsilon^2)\cosh\left(\frac{2\gamma k_z v_z}{T_s}\right) + 2\epsilon\sinh\left(\frac{2\gamma k_z v_z}{T_s}\right) + (1-\epsilon^2)\cosh\left(\frac{\gamma q_z v_z}{T_s}\right)\cos\left(q_x d_x\right)}{(1+\epsilon^2)\cosh\left(\frac{2\gamma k_z v_z}{T_s}\right) + 2\epsilon\sinh\left(\frac{2\gamma k_z v_z}{T_s}\right) + (1-\epsilon^2)}$$

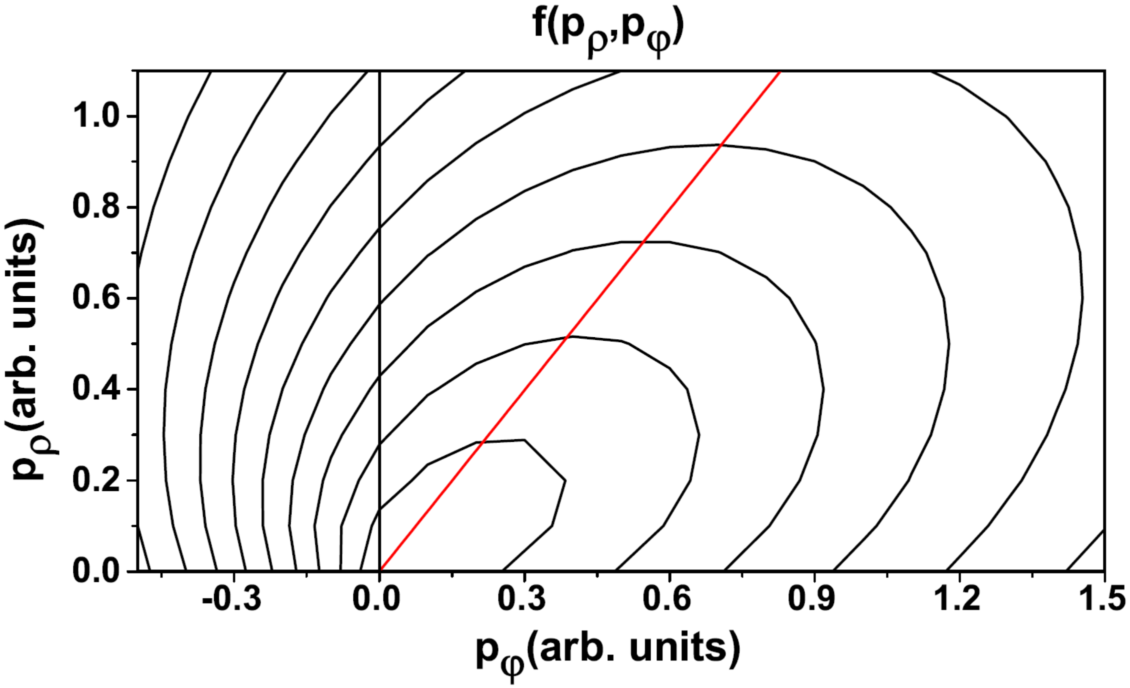
$$C(k_{(\pm)}, q_{side}) = 1 + \exp(-R^2q^2)\ ,$$

$$C(k_{(\pm)}, q_{long}) = 1 + \exp(-R^2q^2)\,\frac{(1+\epsilon^2)\cosh\left(\frac{2\gamma k_z v_z}{T_s}\right) + 2\epsilon\sinh\left(\frac{2\gamma k_z v_z}{T_s}\right) + (1-\epsilon^2)\cosh\left(\frac{\gamma q_z v_z}{T_s}\right)\cos\left(q_x d_x\right)}{(1+\epsilon^2)\cosh\left(\frac{2\gamma k_z v_z}{T_s}\right) + 2\epsilon\sinh\left(\frac{2\gamma k_z v_z}{T_s}\right) + (1-\epsilon^2)}$$

$$\Delta C(k,q) \equiv C(k_+, q_{\mathrm{out}}) - C(k_-, q_{\mathrm{out}})$$

- If the rotation or radial expansion is removed the differential correlation function would vanish in the exact hydro model.
- Although our spatial source configuration is azimuthally symmetric, our phasespace configuration is not.

Phase space distribution



- The schematic phase space distribution of the rotating and expanding source in the momentum space.

$$\int_0^{2\pi} \exp\left(k\sqrt{s_\rho}\ \left(a[\dot{R}\sin(\varphi) + R\omega\cos(\varphi)] + c[\dot{R}\cos(\varphi) - R\omega\sin(\varphi)]\right)\right) d\varphi$$

$$\neq \int_0^{2\pi} \exp\left(k\sqrt{s_\rho}\ \left(a[\dot{R}\sin(\varphi) + R\omega\cos(\varphi)] - c[\dot{R}\cos(\varphi) - R\omega\sin(\varphi)]\right)\right) d\varphi$$

$$\Rightarrow \Delta C \neq 0$$

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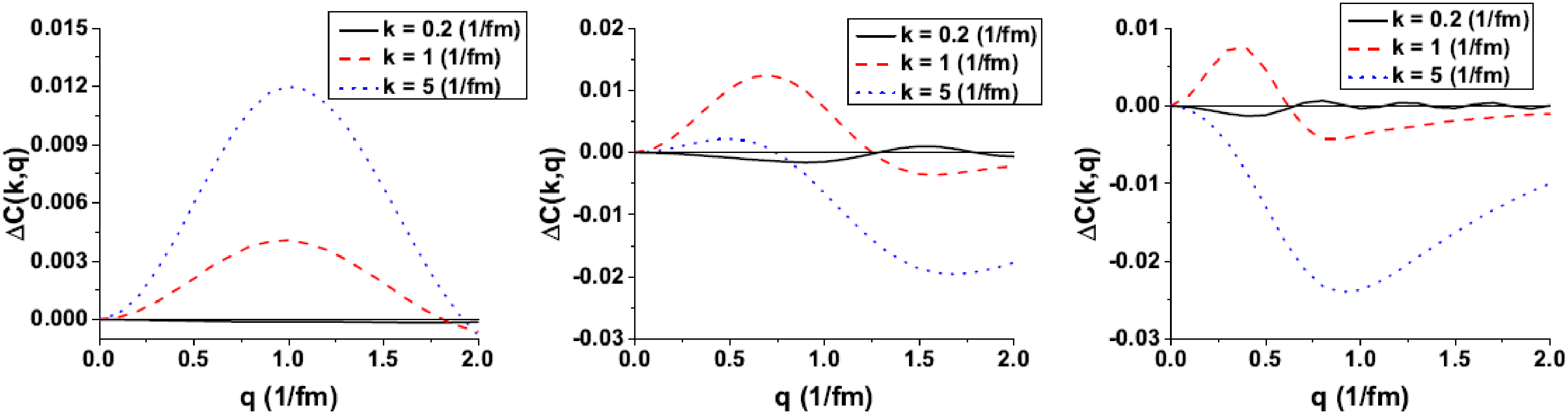
- The momentum of the expansion increases with the radius just as the momentum arising from the rotation. So, higher radial flow momenta correspond to higher rotation momenta as indicated by the red line.
- At constant p_ρ the distribution peaks at the momentum p_ϕ indicated by the red line; the Jüttner distribution, is not symmetric, it is elongated towards higher momenta.
- The resulting thermally smeared distribution is indicated by the contour lines.

Differential CF for the exact model

$$\Delta C(k,q) \equiv C(k_+,q_{\text{out}}) - C(k_-,q_{\text{out}})$$



Differential Correlation Function for the exact hydro model as function of $q = q_{out}$, with
(a) R = 2.500 fm, $\dot{R}$ = 0.250 c, Y = 4.000 fm, $\dot{Y}$ = 0.300 fm, ω = 0.150 c/fm at t = 0.0 fm/c. (left figure)
(b) R = 3.970 fm, $\dot{R}$ = 0.646 c, Y = 5.258 fm, $\dot{Y}$ = 0.503 fm, ω = 0.059 c/fm at t = 3.0 fm/c. (middle figure)
(c) R = 7.629 fm, $\dot{R}$ = 0.779 c, Y = 8.049 fm, $\dot{Y}$ = 0.591 fm, ω = 0.016 c/fm at t = 8.0 fm/c. (right figure)
Where the solid black line is for $k = 0.2\, fm^{-1}$, the dashed red line is for $k = 1\, fm^{-1}$ and the dotted blue line is for $k = 5\, fm^{-1}$.
In (a) the solid black line is close to the axis.

Correlation function for different angular velocities and directions

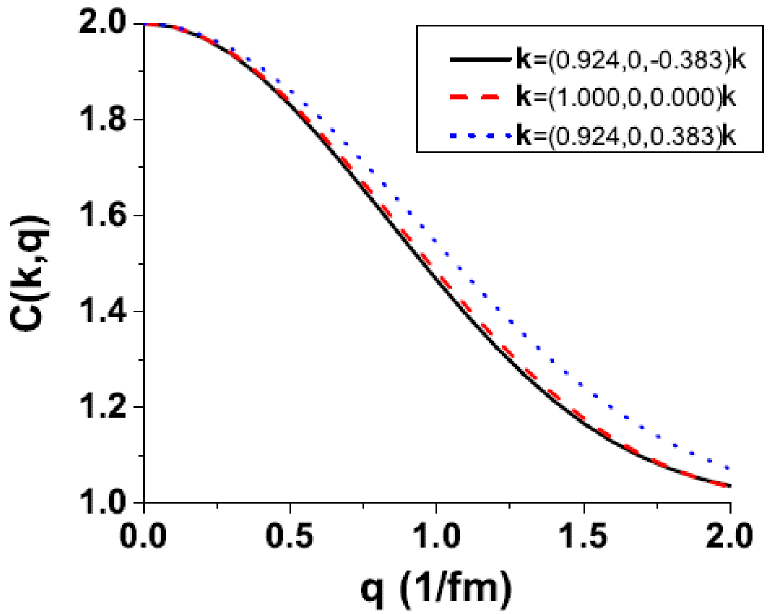


FIG. 1. Correlation Function $C(k, q)$ for the exact hydro model. R = 2.50 fm, $\dot{R}$ = 0.25 c, Y = 4.00 fm, $\dot{Y}$ = 0.30 fm, ω = 0.30 c/fm, at t = 0.0 fm/c with k = 5 (1/fm). The solid black line is for measuring the correlation function at $\boldsymbol{k}^- = (0.924, 0, -0.383)k$, the dashed red line is for $\boldsymbol{k} = (1, 0, 0)k$ and the dotted blue line is for $\boldsymbol{k}^+ = (0.924, 0, 0.383)k$.

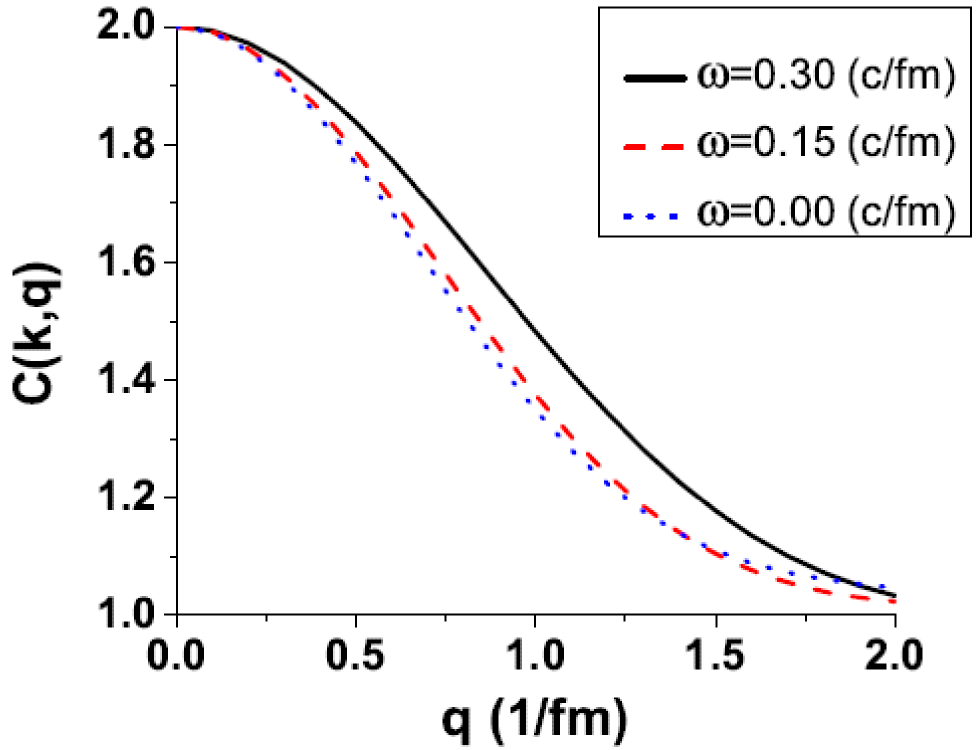


FIG. 2. Correlation Function $C(k,q)$ for the exact hydro model. R = 2.50 fm, $\dot{R}$ = 0.25 c, Y = 4.00 fm, $\dot{Y}$ = 0.30 fm at t = 0.0 fm/c with k = 5 (1/fm). The solid black line is for $\omega = 0.30$ c/fm, the dashed red line is for $\omega = 0.15$ c/fm and the dotted blue line is $\omega = 0.00$ c/fm.

Results from combining the effects for increasing angular velocity and the use of differential HBT and determining the size:

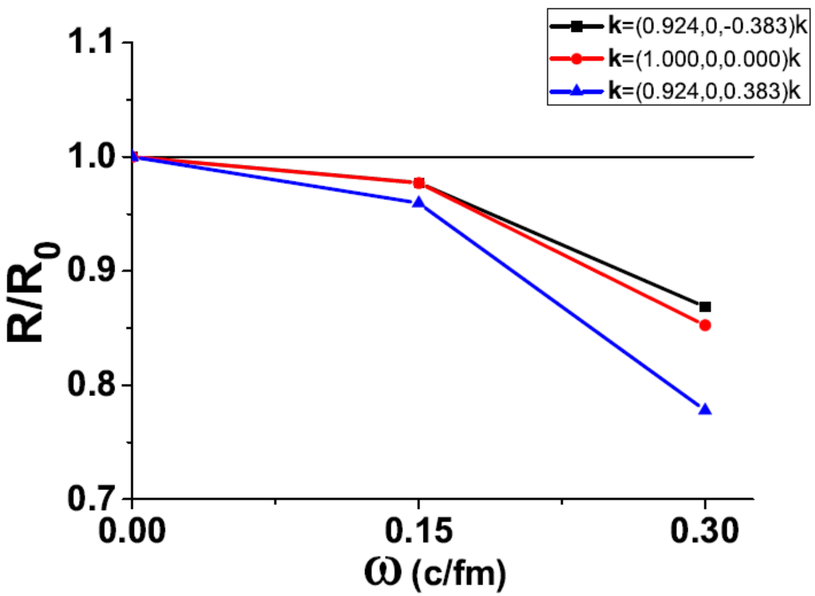


FIG. 3. Ratio of radius from the fit for the correlation function in Fig. 1 and 2 for different directions as a function of ω, the black line is for $\boldsymbol{k}^- = (0.924, 0, -0.383)k$, the red line is for $\boldsymbol{k} = (1, 0, 0)k$ and the blue line is for $\boldsymbol{k}^+ = (0.924, 0, 0.383)k$. R_0 is the observed radius of the system without rotation.

The detector at $k^+ = (0.924, 0, 0.383)k$ shows a smaller measured radius of for the exact hydro model while the radius showed at $k^- = (0.924, 0, -0.383)k$ is larger. This is also dependent on expansion velocity, temperature and size of the system. The axial size, Y, is not affected by the rotation.

Thus the model results show that rotation influences the HBT evaluation similarly like the expansion.

arXiv:1508.04017

Summary

- Different values of the angular velocity will change the measured size of the system
- It will also create smaller and larger values for the correlation function when measuring at different directions.
- The rotation of the cylindrically symmetric system will be observed as an asymmetric object.

Thank you for your attention.