Evolution of Temperature Fluctuation in a Viscous Medium

(arXiv:1510.03154)

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Fluctuations: We encounter in everyday examples (temperature fluctuation in a room).

More Examples:

- Critical Opalescence: Density fluctuation near critical point
- Signature of temperature fluctuation in particle yield originated from high-energy collisions.
- CMB temperature fluctuation

Study of temperature fluctuation may be important for studying QCD phase transition [D K Mishra *et al.* JPG **42**, 105105(2015)]

and

for characterizing QGP [S Basu *et al.* arXiv:1405.3969 [nucl-ex], ibid. 1504.04502]

System with same temperature everywhere

System with different local temperatures

Hotspots

The Model: Medium consists of weakly interacting hot zones.

Within a certain pre-determined time slice Δt, the temperature of a certain hot zone does not change.

Within Δt, they are represented by a collection of canonical (say, no chemical potential) ensembles and their (inverse) temperature has a particular distribution.

The distribution determines the average inverse temperature β and its fluctuation $\Delta\beta$ at every hot zone.

With time, temperature distribution changes and so does $\Delta\beta$.

We assume particle distribution is affected by both β and $\Delta\beta$

$$\text{Ansatz:} \quad f = e^{-\beta(1+\Delta\beta)p} = f^{eq} + \Delta f$$

[S. Dodelson, Modern Cosmology]

Feed f inside the Boltzmann Transport Equation (BTE), which is the evolution equation for inhomogeneous, anisotropic distribution in presence of external force

Momentum space gradient

$$\frac{\partial f}{\partial t} + \vec{v}.\vec{\nabla} f + \vec{F}.\vec{\nabla}_p f = C[f]$$

Collision term (takes care of interaction due to which distribution may change)

External force

Inhomogeneity

We get the evolution equation for Δf and hence for $\Delta\beta$ assuming Relaxation Time Approximation (RTA)

$$C[f] = \frac{f - f^{eq}}{t_R}$$

Relaxation time (The time within which the distribution changes appreciably)

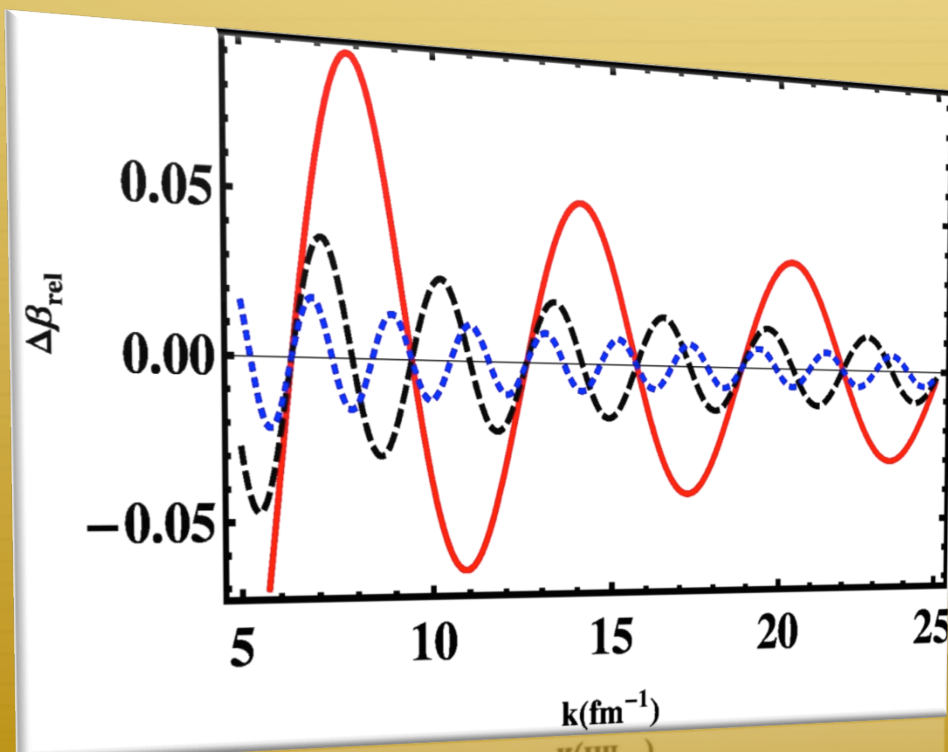
The Fourier Space (k-space) variation of *Δβ* (in a medium of almost massless particles):

$$\Delta\beta(\vec{k},\hat{p};t) = \Delta\beta(\vec{k},\hat{p};t^0)\, e^{-ik\mu(t-t^0)} e^{-\frac{t-t^0}{t_R}}$$

Provided average (inverse) temperature varies slowly with time

After averaging over $\mu = \hat{k}.\hat{p}$

$$\Delta\beta_{rel}(\vec{k},t) = e^{-\frac{t-t^0}{t_R}} \frac{\sin k(t-t^0)}{k(t-t^0)}$$



$(t-t^0)$ values for $t_R = 3$ fm

1 fm

2 fm

3 fm

Observations:

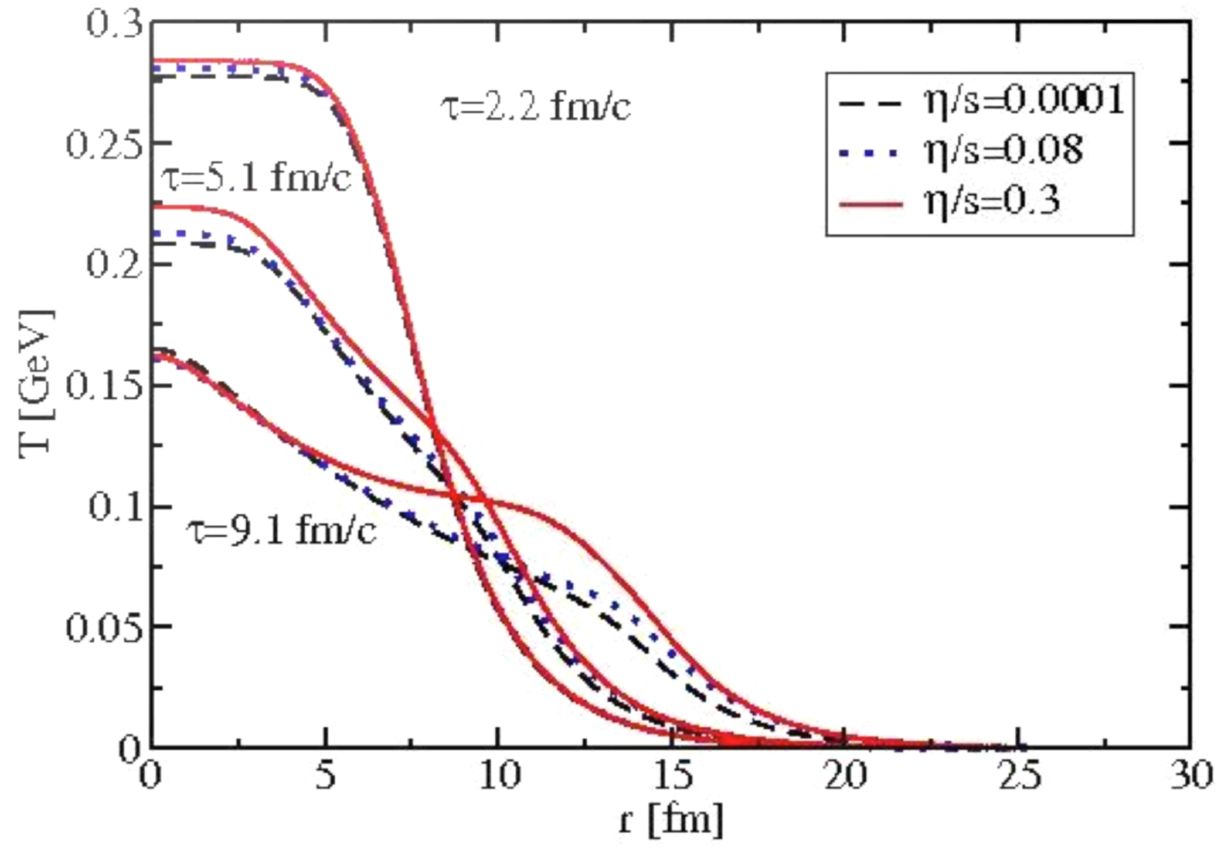
- The amplitude of $\Delta\beta_{rel}$ dies down with time and is more towards the smaller k, i.e. towards large radius of the system.
- Analysis driven by BTE is limited by the constraint over the observation time

$$(t - t^0) << t_R$$

- Given the constraint, $\Delta\beta_{rel}$ is independent of t_R

Any generalized analysis possible which will be able to avoid the constraint ?

Yes, to start with, one of the ways is to get the temperature profile of the medium at different stages (i.e. proper time τ) of evolution.



R. Baier P. and Romatschke, EPJC 51, 677(2007)

The temperature profile obtained from the theoretical analysis for a viscous QGP medium (created in a central HIC) evolving hydrodynamically can be described by the following function:

$$\beta_M(r;t) = \beta_0(t)\left[e^{a(t)\left(\frac{r}{r_0}-1\right)}+1\right]$$

With the tabulated details:

$\tau(fm/c)$	$\beta_0(GeV^{-1})$	a	$r_0(fm)$
2.2	3.45	5.99	7.96
5.1	4.55	3.42	8.41
9.1	5.56	1.91	8.71

From this, we can generate $\{\beta_{Mi}\}$, a sequence of radially varying inverse temperature values at different stages.

$$\{\beta_{Mi}\}_\tau \equiv \{\beta_{M0}, \beta_{M1}, \beta_{M2}, \beta_{M3}, \ldots\ldots, \beta_{Mn}\}_\tau$$

$$\langle \beta_M \rangle = \frac{\beta_{M0} + \beta_{M1} + \beta_{M2} + \beta_{M3} + \ldots\ldots + \beta_{Mn}}{n}$$

$$\langle \beta_M^2 \rangle = \frac{\beta_{M0}^2 + \beta_{M1}^2 + \beta_{M2}^2 + \beta_{M3}^2 + \ldots\ldots + \beta_{Mn}^2}{n}$$

So, for a given $\{\beta_{Mi}\}$, we can define

- An average $\langle \beta_M \rangle$

- Also, a fluctuation on top of it:

$$\Delta\beta(r;t) = \beta_M(r;t) - \langle \beta_M \rangle$$

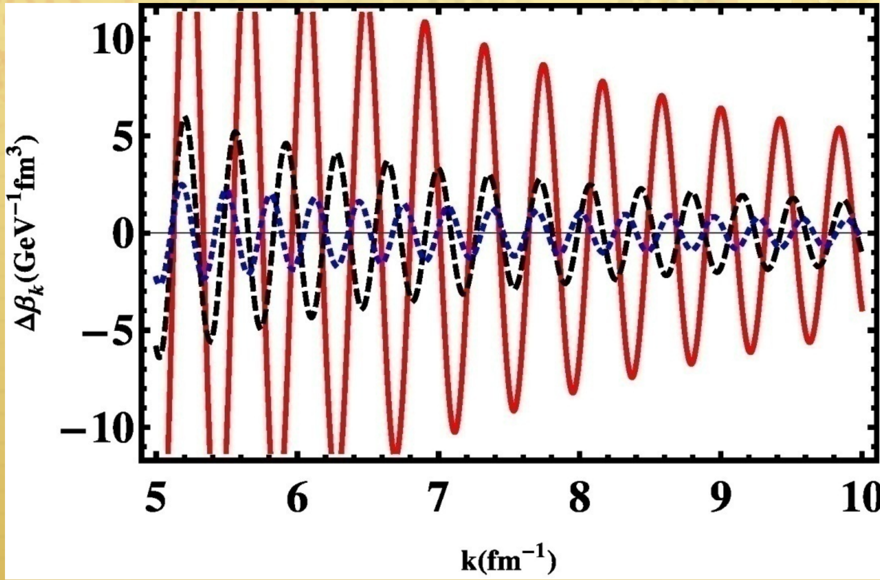
$$= \beta_0(t) e^{a(t)\left(\frac{r}{r_0}-1\right)} + \delta\beta(t)$$

with $\delta\beta(t) = \beta_0(t) - \langle \beta_M \rangle$

The Fourier Space (k-space) variation of inverse temperature fluctuation:

$$\Delta\beta(k,t) = \frac{2\beta_0(t)}{(2\pi)^2 k}\int_0^R e^{a(t)\left(\frac{r}{r_0}-1\right)} r \sin kr \, dr + \delta\beta(t)\delta(\vec{k})$$

where, $\delta(\vec{k})$ is the Dirac delta function.



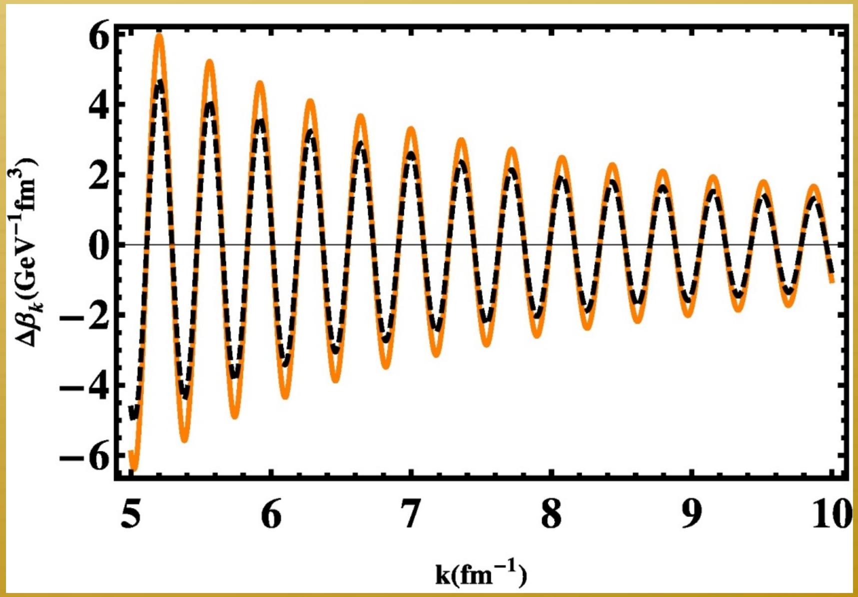
2.2 fm

5.1 fm

9.1 fm

$\eta / s = 0.08$

$\eta / s = 0.30$



- And, a relative variance for the collection $\{\beta_{M\,i}\}$:

$$\frac{\left\langle \beta_M{}^2 \right\rangle - \left\langle \beta_M \right\rangle^2}{\left\langle \beta_M \right\rangle^2} = \frac{\sigma_\beta{}^2}{\beta^2} = \Re_\beta$$

Recall:

$$\frac{<(\frac{1}{T})^2> - <(\frac{1}{T})>^2}{<(\frac{1}{T})>^2} = q - 1$$

G. Wilk and Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770

τ	$\Re_\beta$
2.2	0.047
5.1	0.011
9.1	0.002

$\frac{\eta}{s} = 0.08$

$\frac{\eta}{s}$	$\Re_\beta$
0.08	0.012
0.30	0.011

$\tau = 5.1\ fm$

Let us assume that the system produced by central HIC freezes-out by 9.1 fm and compare theoretically obtained $\Re_\beta$ value at the boundary with the similar experimentally observed (q-1) parameter **[G. Wilk and Z. Wlodarczyk PRL 84, 2770(2000)]** for hadron spectra at $\sqrt{s_{NN}} = 200$ GeV within 0-10% centrality **[Z. Tang *et al.* PRC 79, 051901(R)(2009)]**.

$\Re_\beta^{Theory}$	**(q-1)**
0.013	0.018 ± 0.005

Cosmological connections:

- Temperature fluctuation of our universe can be explained by the modified Boltzmann-Gibbs formula with (q-1) value 0.045 ± 0.005 *[A. Bernui et al. PLA 356, 426(2006)]*

- Study on the similarity between the surface of the last scattering for CMB radiation and the freeze-out surface in RHICE

- Similarity with HIC experimental results: needs review

Summary and Conclusion:

- Time evolution of (inverse) temperature fluctuation
- With time, amplitude of inverse temperature fluctuation decreases
- With distance amplitude of inverse temperature fluctuation increases
- Relative fluctuation at the boundary is comparable with the experimental value under similar conditions.

- More studies required on the evolution of temperature fluctuation with more generalized scenarios.

- Exploring possible connections with experiment in a more detailed manner.

Thank you