Correlation measurement of particle strong interaction

R. Lednický @ JINR Dubna & IP ASCR Prague

- History
- QS correlations $\rightarrow$ femtoscopy with identical particles
- FSI correlations $\rightarrow$ femtoscopy with nonidentical particles
- Correlation study of strong interaction
- Summary

History of Correlation femtoscopy

measurement of space-time characteristics R, $c\tau \sim$ fm
of particle production using particle correlations

Fermi'34, GGLP'60, Dubna (GKPLL..'71-) …

β-decay: Coulomb FSI between $e^{\pm}$ and Nucleus in β-decay modifies the relative momentum ($\mathbf{k}$) distribution → Fermi (correlation) function

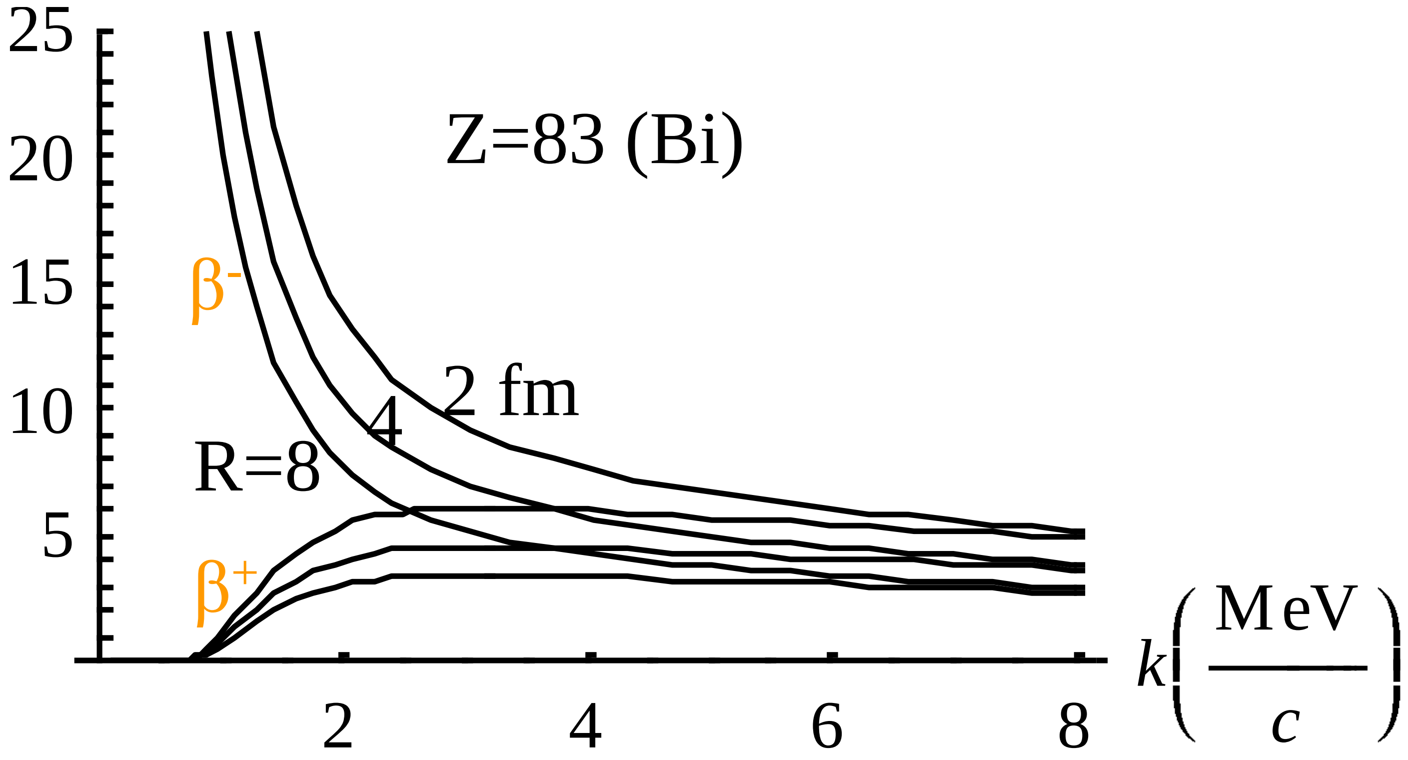
$$F(k,Z,R) = \langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$$

is sensitive to Nucleus radius R if charge $Z \gg 1$

$\psi_{-\mathbf{k}}(\mathbf{r})$ = electron – residual Nucleus WF ($\Delta t=0$)

Fermi function in β-decay

Fermi f-n $(k, Z, R) = \langle|\psi_{-k}(r)|^2\rangle \sim (kR)^{-(Z/137)^2}$



Modern correlation femtoscopy formulated by Kopylov & Podgoretsky

KP'71-75: **settled basics of correlation femtoscopy in > 20 papers** **(for non-interacting identical particles)**

- **proposed CF= N^{corr}/N^{uncorr} & mixing techniques to construct N^{uncorr} & two-body approximation to calculate theor. CF**
- **showed that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q***

smoothness approximation:

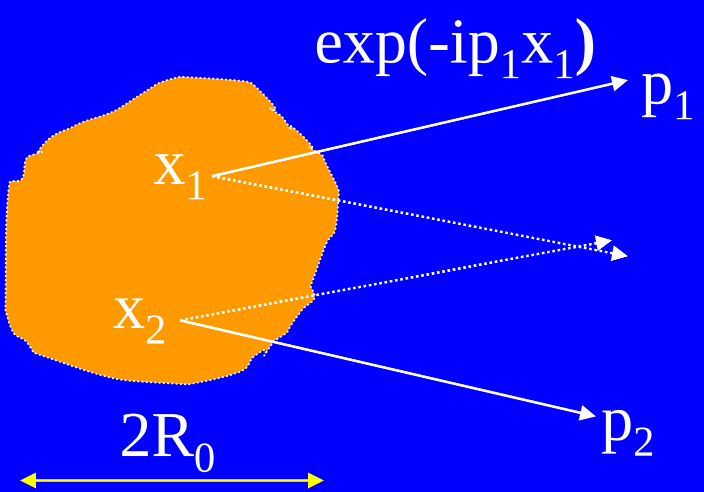
$$|\int d^4x_1 d^4x_2 \psi_{p_1p_2}(x_1,x_2)...|^2 \rightarrow \int d^4x_1 d^4x_2 |\psi_{p_1p_2}(x_1,x_2)|^2 ...$$

- **clarified role of space-time production characteristics: shape & time source picture from various q-projections**

QS symmetrization of production amplitude

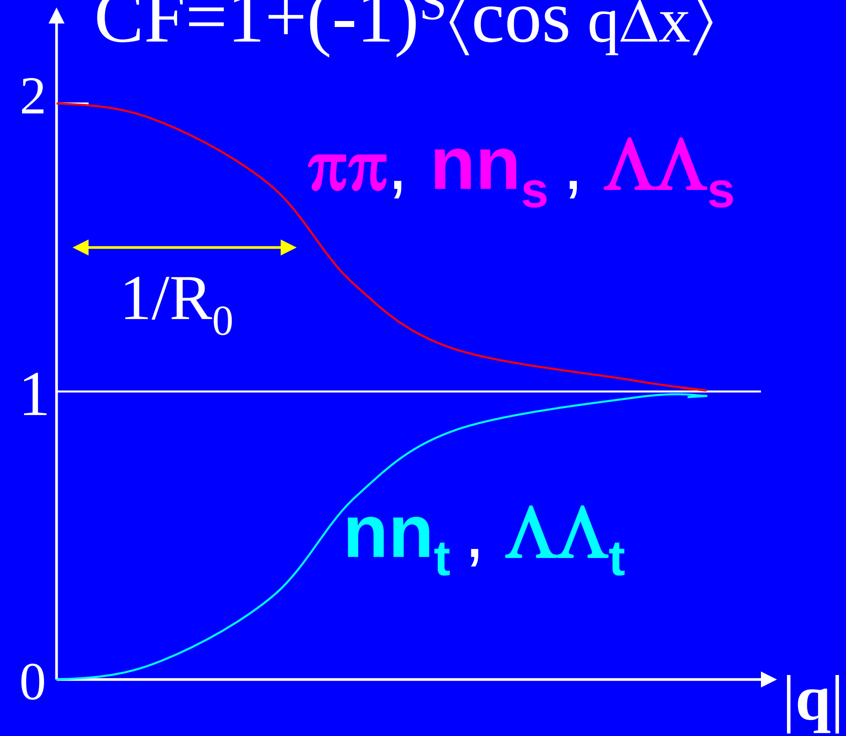
→ *momentum correlations of identical particles are sensitive to space-time structure of the source*

KP'71-75



total pair spin

$CF = 1 + (-1)^S \langle \cos q\Delta x \rangle$



$$q = p_1 - p_2 \xrightarrow{\textbf{PRF}} \{0, 2\mathbf{k^*}\}$$

$$\Delta x = x_1 - x_2 \rightarrow \{t^*, \mathbf{r^*}\}$$

$$CF \rightarrow \langle\, |\psi^{S(sym)}_{-k^*}(r^*)|^2 \,\rangle = \langle\, |\, [\, e^{-ik^*r^*} + (-1)^S e^{ik^*r^*}]/\sqrt{2}\, |^2 \,\rangle$$

! CF of noninteracting identical particles is independent of t* in PRF

KP model of single-particle emitters

Probability amplitude to observe a particle with 4-coordinate x from emitter A at x_A can depend on $x-x_A$ only and so can be written as:

$$\langle x|\psi_A\rangle = (2\pi)^{-4}\int d^4\kappa\, u_A(\kappa)\exp[i\kappa(x-x_A)].$$

Transfering to 4-momentum representation: $\langle p|x\rangle = \exp(-ipx)$ $\Rightarrow$

$$\langle p|\psi_A\rangle = \int d^4x\, \langle p|x\rangle\langle x|\psi_A\rangle = u_A(p)\exp(-ipx_A)$$

and probability amplitude to observe two spin-0 bosons:

$$T_{AB}^{\mathrm{sym}}(p_1,p_2) = [\langle p_1|\psi_A\rangle\langle p_2|\psi_B\rangle + \langle p_2|\psi_A\rangle\langle p_1|\psi_B\rangle]/\sqrt{2}.$$

Corresponding **momentum correlation function**:

$\Delta x = x_A - x_B$

$$R(p_1,p_2) = 1 + \frac{\Re\sum_{AB} u_A(p_1)u_B(p_2)u_A^*(p_2)u_B^*(p_1)\exp(-iq\Delta x)}{\sum_{AB}|u_A(p_1)u_B(p_2)|^2} \doteq 1 + \langle\cos(q\Delta x)\rangle$$

if $u_A(p_1) \approx u_A(p_2)$: "smoothness assumption"

Assumptions to derive KP formula

$$\boxed{CF - 1 \propto \langle \cos q\Delta x \rangle}$$

- **two-particle approximation (small freeze-out PS density f)**
  ~ OK, $\langle f \rangle \ll 1$? low p_t

- **smoothness approximation: $R_{emitter} \ll R_{source} \Leftrightarrow \langle|\Delta p|\rangle \gg \langle|q|\rangle_{peak}$**
  ~ OK in HIC, $R_{source}^2 \gg 0.1\ \mathrm{fm}^2 \approx p_t^2$-slope of direct particles

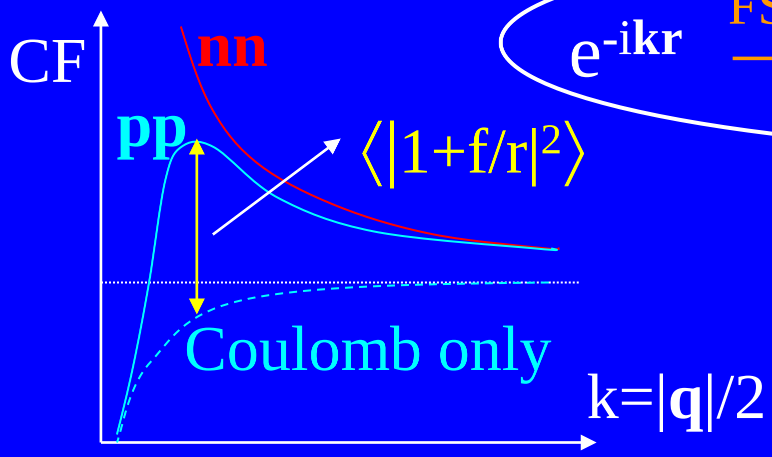
- **neglect of FSI**
  OK for photons, ~ OK for pions up to Coulomb repulsion

- **incoherent or independent emission**
  2π and 3π CF data approx. consistent with KP formulae:
  $CF_3(123) = 1+|F(12)|^2+|F(23)|^2+|F(31)|^2+2\mathrm{Re}[F(12)F(23)F(31)]$
  $CF_2(12) = 1+|F(12)|^2, \quad F(q)| = \langle e^{iqx} \rangle$

Final State Interaction

Similar to Coulomb distortion of β-decay Fermi'34: $\langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$

Migdal, Watson, Sakharov, … Koonin, GKW, LL, ...



$$e^{-i\mathbf{kr}} \xrightarrow{\text{FSI}} \psi_{-\mathbf{k}}(\mathbf{r}) \xrightarrow[\text{Coulomb}]{\text{s-wave} \atop \text{strong FSI}} [\, e^{-i\mathbf{kr}} + \overbrace{f(k)}^{f_c\sqrt{A_c}\cdot(G_0+iF_0)}e^{ikr}/r \,]$$

$$e^{i\delta_c}\underbrace{\sqrt{A_c}}_{\text{Point-like Coulomb factor}} \qquad F = 1 + \frac{\mathbf{kr}+kr}{k\underbrace{a}_{\text{Bohr radius}}} + \ldots$$

⇒ FSI is sensitive to source size r and scattering amplitude f

It **complicates CF analysis** but makes possible

→ **Femtoscopy with nonidentical particles** πK, πp, .. **& Coalescence** **deuterons**, ..

→ **Study "exotic" scattering** ππ, πK, KK, πΛ, pΛ, ΛΛ, ..

→ **Study relative space-time asymmetries** delays, flow

"Fermi-like" CF formula

$$\boxed{CF = \langle\, |\psi_{-k^*}(r^*)|^2 \,\rangle}$$

Koonin'77: nonrelativistic & unpolarized protons

RL, Lyuboshitz'82: generalization to relativistic & polarized & nonidentical particles & estimated the effect of nonequal times

Assumptions:

- same as for KP formula in case of pure QS &
- equal time approximation in PRF

RL, Lyuboshitz'82 → eq. time conditions: $|t^*| \ll m_{1,2} r^{*2}$

$|k^* t^*| \ll m_{1,2} r^*$

OK (usually, to several % even for pions) fig.

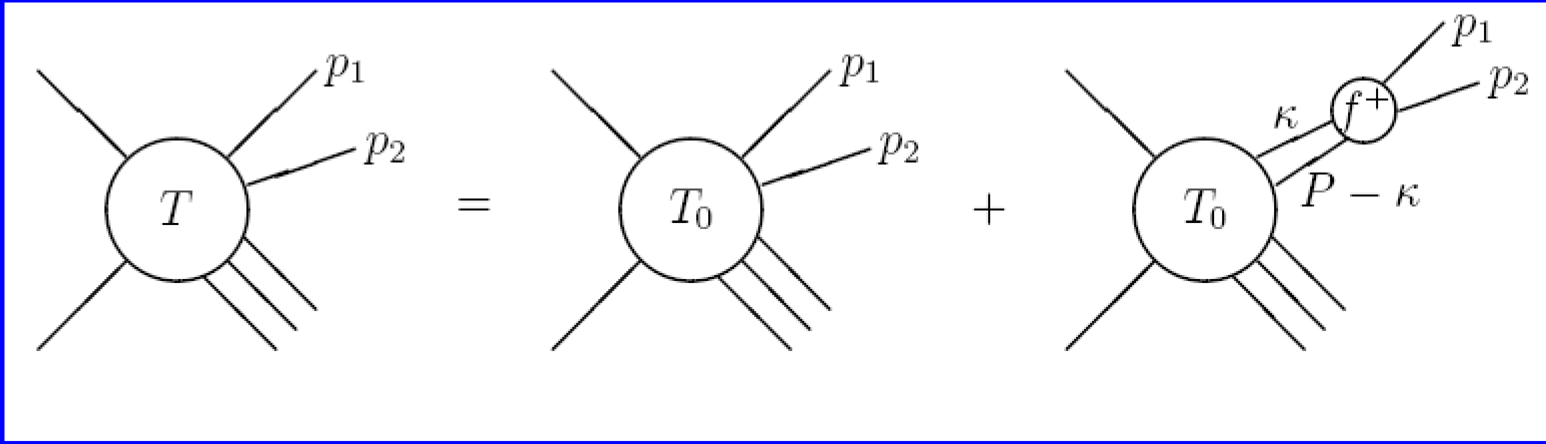
- $t_{FSI} = d\delta/dE \gg t_{prod}$

t_{FSI}(s-wave) $= \mu f_0/k^* \;\rightarrow\; |k^*| = \frac{1}{2}|q^*| \ll$ hundreds MeV/c ≈ typical momentum transfer in production

RL, Lyuboshitz ..'98

& account for coupled channels within the same isomultiplet only: $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, $\pi^- p \leftrightarrow \pi^0 n$, $K^+K^- \leftrightarrow K^0\bar{K}^0$, ...

BS-amplitude Ψ



$$T(p_1,p_2;\alpha) = T_0(p_1,p_2;\alpha) + \Delta T(p_1,p_2;\alpha)$$

$$\Delta T(p_1,p_2;\alpha) = \frac{\mathrm{i}\sqrt{P^2}}{2\pi^3}\int \mathrm{d}^4\kappa \frac{T_0(\kappa, P-\kappa;\alpha) f^{S*}(p_1,p_2;\kappa,P-\kappa)}{(\kappa^2-{m_1}^2-\mathrm{i}0)[(P-\kappa)^2-{m_2}^2-\mathrm{i}0]}$$

Inserting KP amplitude $T_0(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)exp(-ip_1x_A-ip_2x_B)$ in ΔT and taking the amplitudes $u_A(\kappa)$ and $u_B(P-\kappa)$ out of the integral *at* $\kappa \approx p_1$ and $P-\kappa \approx p_2$ (again "smoothness assumption") $\Rightarrow$

Product of plane waves $\rightarrow$ BS-amplitude Ψ :

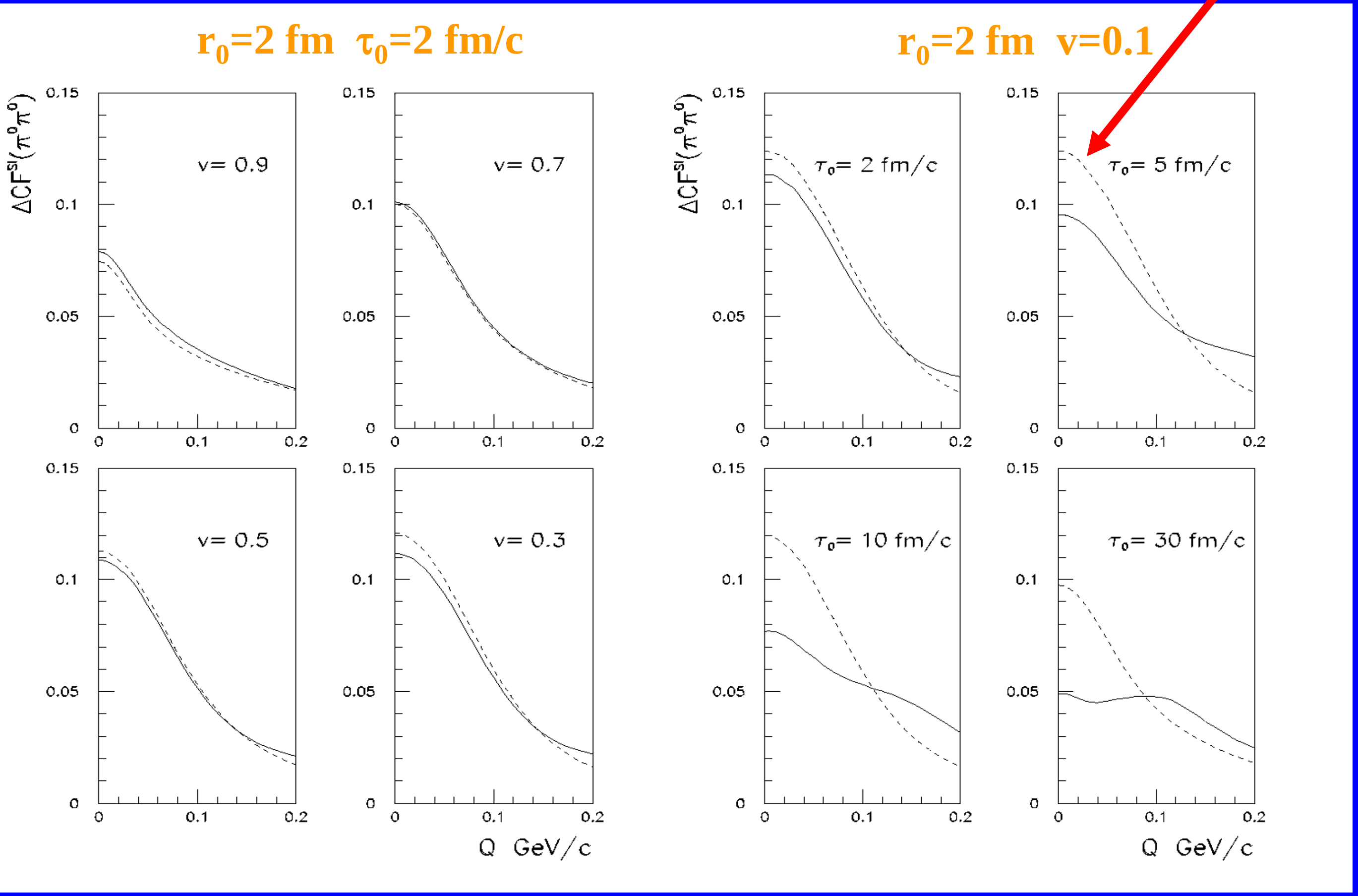
$$T(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\,\Psi p_1 p_2(x_A, x_B)$$

Effect of nonequal times in pair cms

RL, Lyuboshitz SJNP 35 (82) 770; RL nucl-th/0501065 $\Psi^{S(+)}_{p_1,p_2}(x_1,x_2) \rightarrow e^{iPX}\psi^{S}_{-\mathbf{k}^*}(\mathbf{r}^*)$

Applicability condition of equal-time approximation: $|t^*| \ll m_{1,2}r^{*2}$

$|k^*t^*| \ll m_{1,2}r^*$



$\downarrow$

OK for heavy particles & small k*

→ OK within 5% even for pions if $\Delta\tau=\tau_0 \sim r_0$ or lower

Note

- Formally (FSI) correlations in beta decay and multiparticle production are determined by the same (Fermi) function $\langle|\psi_{-\mathbf{k}}(\mathbf{x})|^2\rangle$

- But it appears for different reasons in
beta decay: a weak **r**-dependence of $\psi_{-\mathbf{k}}(\mathbf{r})$ within the nucleus volume + point like emission + equal times
and in
multiparticle production in usual events of HIC:
a small space-time extent of the emitters compared to their separation + sufficiently small phase space density + a small effect of nonequal times in usual conditions

Using spherical wave in the outer region ($r>\varepsilon$) & inner region ($r<\varepsilon$) correction $\rightarrow$ **analytical dependence on scatt. amplitudes f_L and source radius r_0** LL'81

Inner region: $W(r) \rightarrow W(0)$ & integral relation (single channel and no Coulomb) with the phase shifts δ_L and momentum derivative δ_L' :

$$\int d^3\mathbf{r}[|\psi_{-k}(\mathbf{r})|^2 - 1] = (2\pi/k^3)\sum_L(2L+1)\{k\delta_L' - \tfrac{1}{2}[\sin 2(k\varepsilon+\delta_L) - \sin 2(k\varepsilon)] + ..\}$$

$\Rightarrow$ FSI contribution to the CF of nonidentical particles, assuming Gaussian source function $W(r)=\exp(-r^2/4r_0^2)/(2\sqrt{\pi}\, r_0)^3$:

$$\boxed{\text{for } kr_0 \ll 1:\ \ \Delta CF^{FSI} = \tfrac{1}{2}|f_0/r_0|^2[1-d_0/(2r_0\sqrt{\pi})] + 2f_0/(r_0\sqrt{\pi}) \sim r_0^{-1} \text{ or } r_0^{-2}}$$

f_0 and d_0 are the s-wave scatt. length and eff. radius entering in the ($L=0$) amplitude $f_L(k) = \sin\delta_L \exp(i\delta_L)/k \approx (1/f_L + \tfrac{1}{2}d_L k^2 - ik)^{-1}$

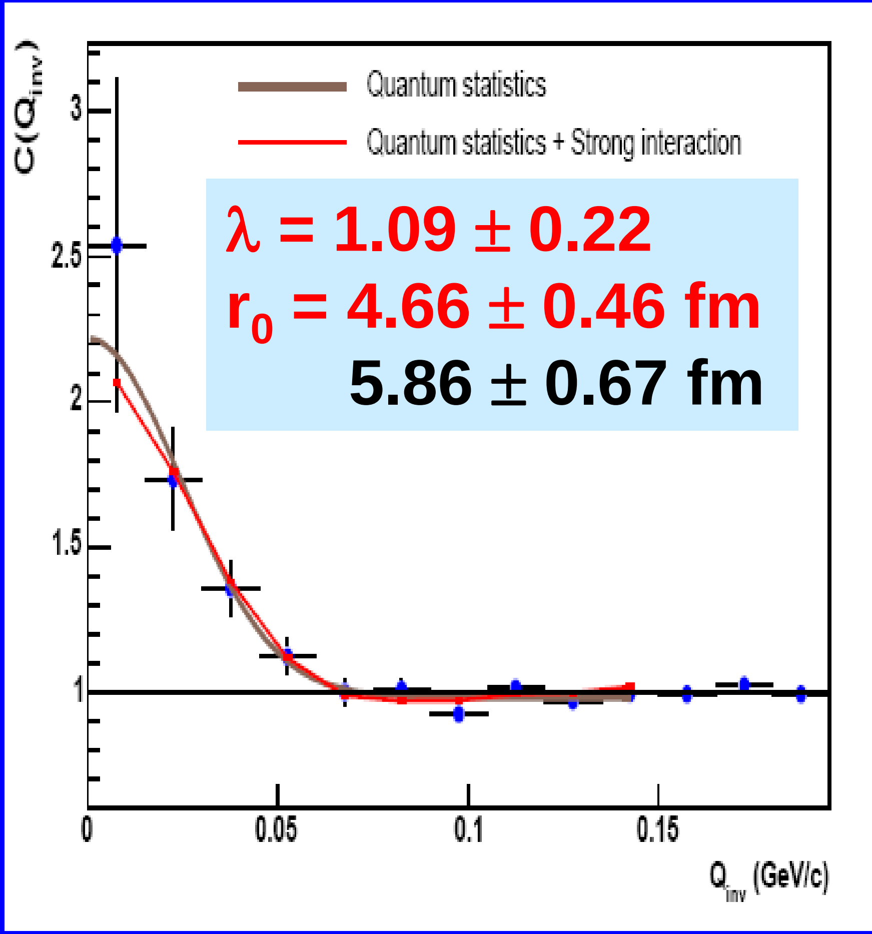
$$\boxed{\text{for } kr_0 \gg 1:\ \ \Delta CF^{FSI} = (2\pi/k^2)W(0)\sum_L(2L+1)\delta_L' \sim r_0^{-3}}$$

$$\delta_L' = [(2L+1)/2k]\sin(2\delta_L) - (d_L/k^{2L})\sin^2\delta_L$$

FSI effect on CF of neutral kaons

Lyuboshitz-Podgoretsky'79:
K_sK_s from $K\bar{K}$ also show BE enhancement

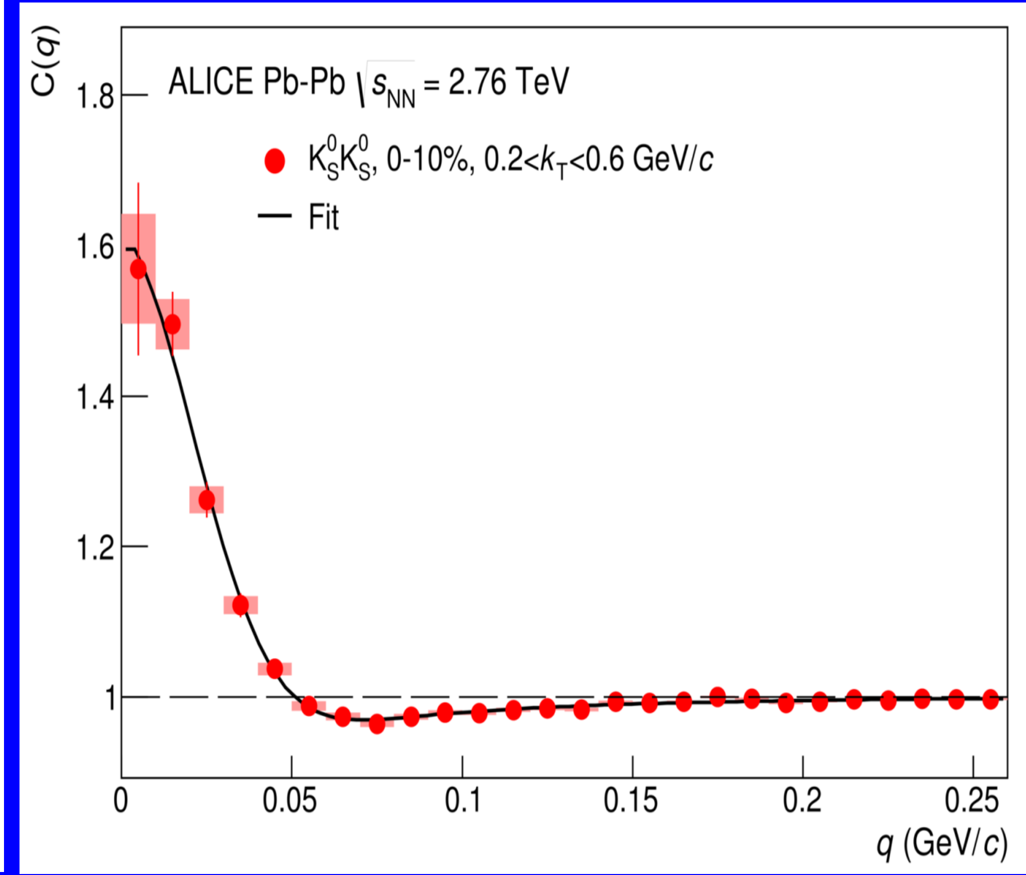
STAR data on CF(K_sK_s)



Goal: no Coulomb. But R may go up by ~1 fm if neglecting FSI in $K\bar{K}$ (~50% K_sK_s)↔ $f_0(980)$ & $a_0(980)$

RL-Lyuboshitz'82

ALICE data on CF(K_sK_s)



Even stronger effect of KK-bar FSI on K_sK_s correlations in pp-collisions at LHC

ALICE: PLB 717 (2012) 151

e.g. for $k_t < 0.85$ GeV/c, N_{ch}=1-11 the neglect of FSI increases λ by ~100% and R_{inv} by ~40%

$\lambda = 0.64 \pm 0.07 \quad \rightarrow \quad 1.36 \pm 0.15 \;\; > 1$!

$R_{inv} = 0.96 \pm 0.04 \quad \rightarrow \quad 1.35 \pm 0.07$ fm

ArXiv.org:1506.07884

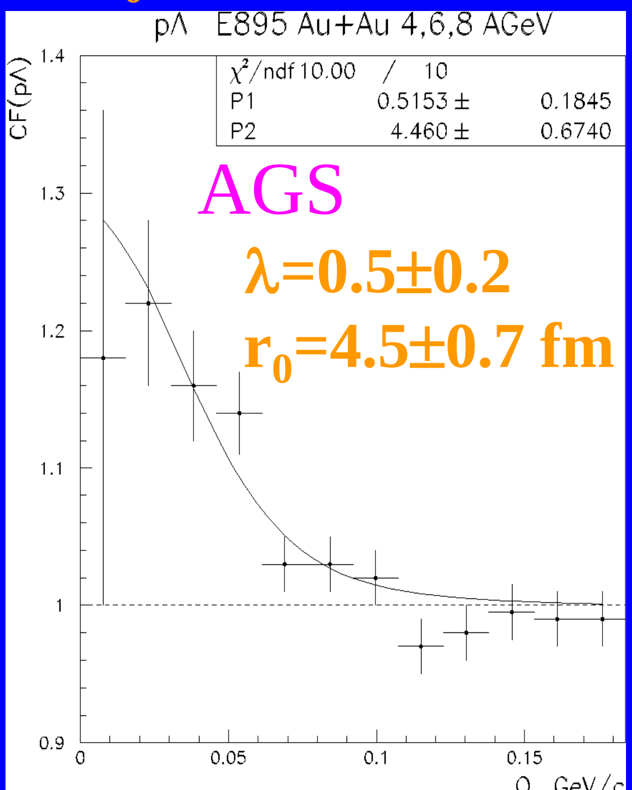
Correlation femtoscopy with nonid. particles

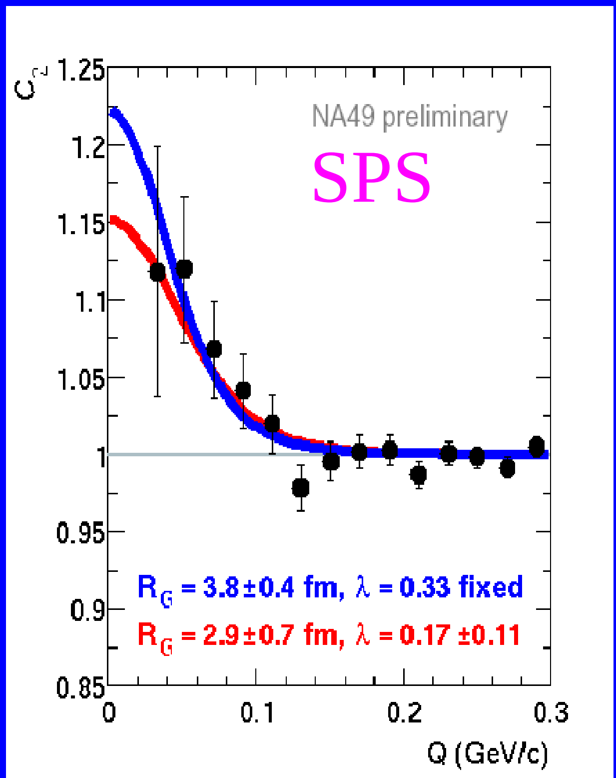
pΛ CFs at AGS & SPS & STAR

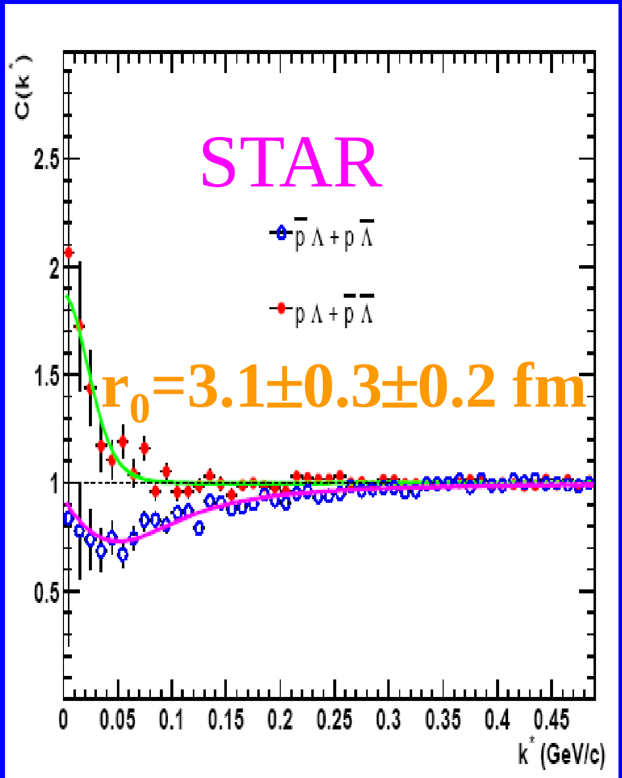
Goal: No Coulomb suppression as in pp CF &
Wang-Pratt'99 Stronger sensitivity to r_0

	singlet	triplet
Scattering lengths, fm:	2.31	1.78
Effective radii, fm:	3.04	3.22

Fit using RL-Lyuboshitz'82 with
λ consistent with estimated impurity
r_0~ 3-4 fm consistent with the radius from pp CF & m_t scaling







Pair purity problem for pΛ CF @ STAR

Particle	Identification	Fraction Primary
p	$76 \pm 7\%$	$52 \pm 4\%$
$\overline{p}$	$74 \pm 7\%$	$48 \pm 4\%$
Λ	$86 \pm 6\%$	$45 \pm 4\%$
$\overline{\Lambda}$	$86 \pm 6\%$	$45 \pm 4\%$

$\Rightarrow$ **PairPurity ~ 15%**

Assuming no correlation for misidentified particles and particles from weak decays

$\rightarrow$ $$C^{corr}_{measured}(k^*) = \frac{C_{measured}(k^*) - 1}{\text{PairPurity}} + 1$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(Qr_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Qr_0) \right],$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$.

$\leftarrow$ Fit using RL-Lyuboshitz'82 (for np)

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1}$$

Pairs	Fractions (%)
p_{prim}-Λ_{prim}	15
p_{Λ}-Λ_{prim}	10
p_{Σ^+}-Λ_{prim}	3
p_{prim}-Λ_{Σ^0}	11
p_{Λ}-Λ_{Σ^0}	7
p_{Σ^+}-Λ_{Σ^0}	2
p_{prim}-Λ_{Ξ}	9
p_{Λ}-Λ_{Ξ}	5
p_{Σ^+}-Λ_{Ξ}	2

$\leftarrow$ but, there can be residual correlations for particles from weak decays requiring knowledge of $\Lambda\Lambda$, pΣ, $\Lambda\Sigma$, $\Sigma\Sigma$, pΞ, $\Lambda\Xi$, $\Sigma\Xi$ correlations

Correlation study of strong interaction

$\pi^+\pi^-$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ s-wave scattering parameters from NA49 and STAR

Fits using RL-Lyuboshitz'82

$\bar{p}\Lambda$: STAR data accounting for residual correlations
- Kisiel et al, PRC 89 (2014) : assuming a universal $\mathrm{Im}f_0$
- Shapoval et al PRC 92 (2015): Gauss. parametr. of res. CF

$\mathrm{Re}f_0 \approx 0.5$ fm, $\mathrm{Im}f_0 \approx 1$ fm, $r_0 \approx 3$ fm

$\Lambda\Lambda$: NA49: $|f_0(\Lambda\Lambda)| \ll f_0(NN) \sim 20$ fm

STAR, PRL 114 (2015): $f_0(\Lambda\Lambda) \approx -1$ fm, $d_0(\Lambda\Lambda) \approx 8$ fm

$\pi^+\pi^-$: NA49 vs RQMD with SI scale: $f_0 \rightarrow$ sisca f_0 (=0.232fm)

sisca = 0.6±0.1 compare with

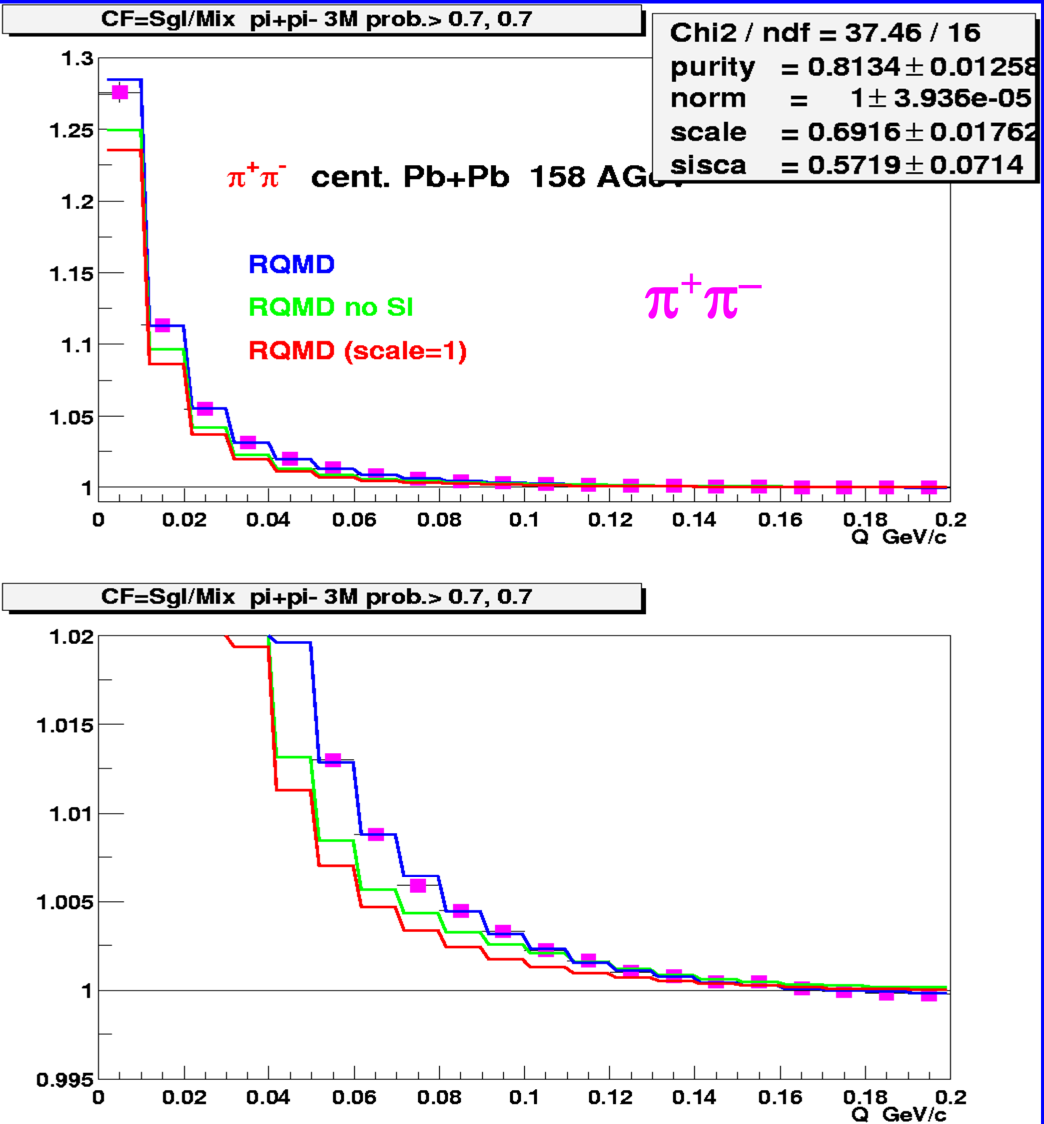
~0.8 from SχPT & BNL data E765 $K \rightarrow e\nu\pi\pi$

Here a suppression can be due to eq. time approx.

$\bar{p}\bar{p}$: STAR, Nature (2015): f_0 and d_0 coincide with table pp-values

Correlation study of particle interaction

CF=Norm [Purity RQMD(r* → Scale•r*)+1-Purity]



$\pi^+\pi^-$ scattering length
f_0 from NA49 CF

Fit CF($\pi^+\pi^-$) by RQMD
with SI scale:
$f_0 \to$ sisca f_0^{input}
f_0^{input} = 0.232 fm

sisca = 0.6±0.1
Compare with
~0.8 from SχPT
& BNL E765
$K \to e\nu\pi\pi$

Correlation study of strong interaction

$\Lambda\Lambda$ scattering lengths f_0 from STAR correlation data

Fit using RL-Lyuboshitz (82): **$\lambda \approx 0.18$, $r_0 \approx 3$ fm, $a_{res} \approx -0.04$, $r_{res} \approx 0.4$ fm**

$f_0 \approx -1$ fm, $d_0 \approx 8$ fm $\Rightarrow$ no s-wave resonance

$$CF=1+\lambda[\Delta CF^{FSI}+\Sigma_S \rho_S (-1)^S \exp(-r_0^2 Q^2)]$$

$$+\ a_{res} \exp(-r_{res}^2 Q^2)$$

$\rho_0 = \frac{1}{4}(1-P^2)$ $\rho_1 = \frac{1}{4}(3+P^2)$ P=Polar.=0

$$\Delta CF^{FSI} = 2\rho_0[\tfrac{1}{2}|f^0(k)/r_0|^2(1-d_0^0/(2r_0\sqrt{\pi}))$$

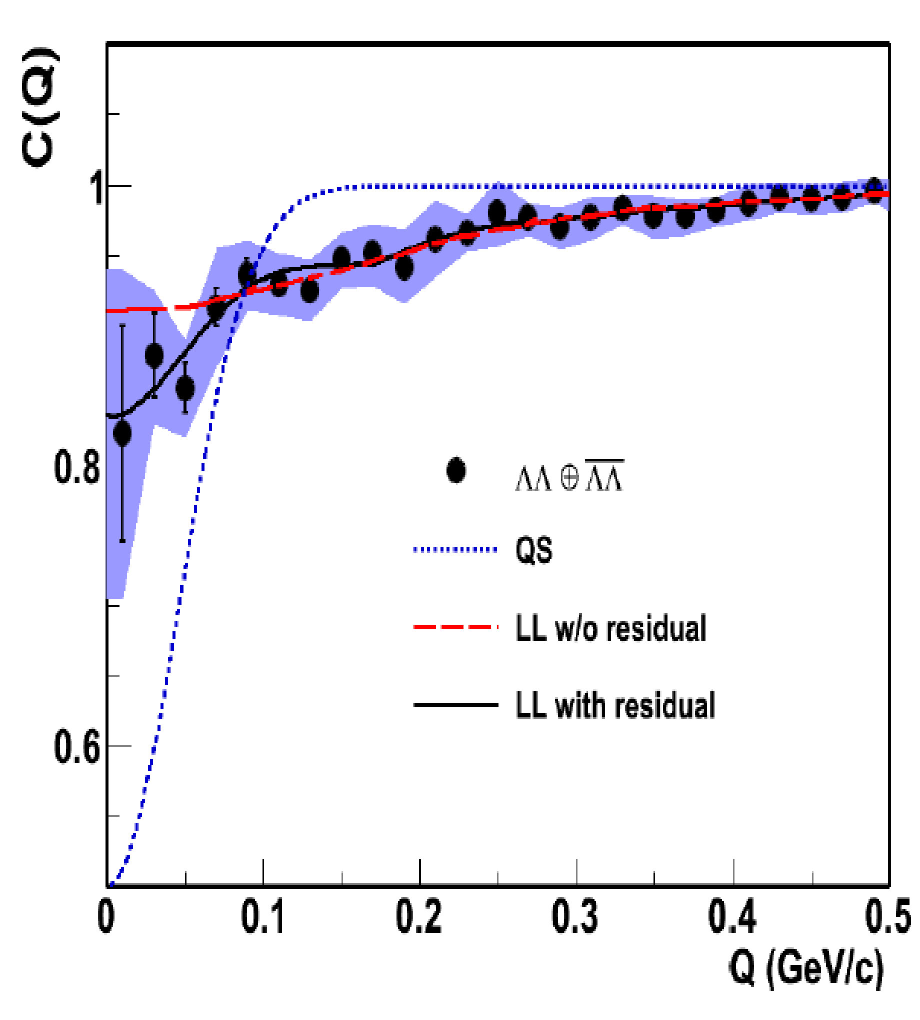
$$+2\mathrm{Re}(f^0(k)/(r_0\sqrt{\pi}))F_1(r_0 Q)$$

$$-\ \mathrm{Im}(f^0(k)/r_0)F_2(r_0 Q)]$$

$$f^S(k)=(1/f_0^S+\tfrac{1}{2}d_0^S k^2 - ik)^{-1} \quad k=Q/2$$

$$F_1(z)=\int_0^z dx\, \exp(x^2-z^2)/z$$

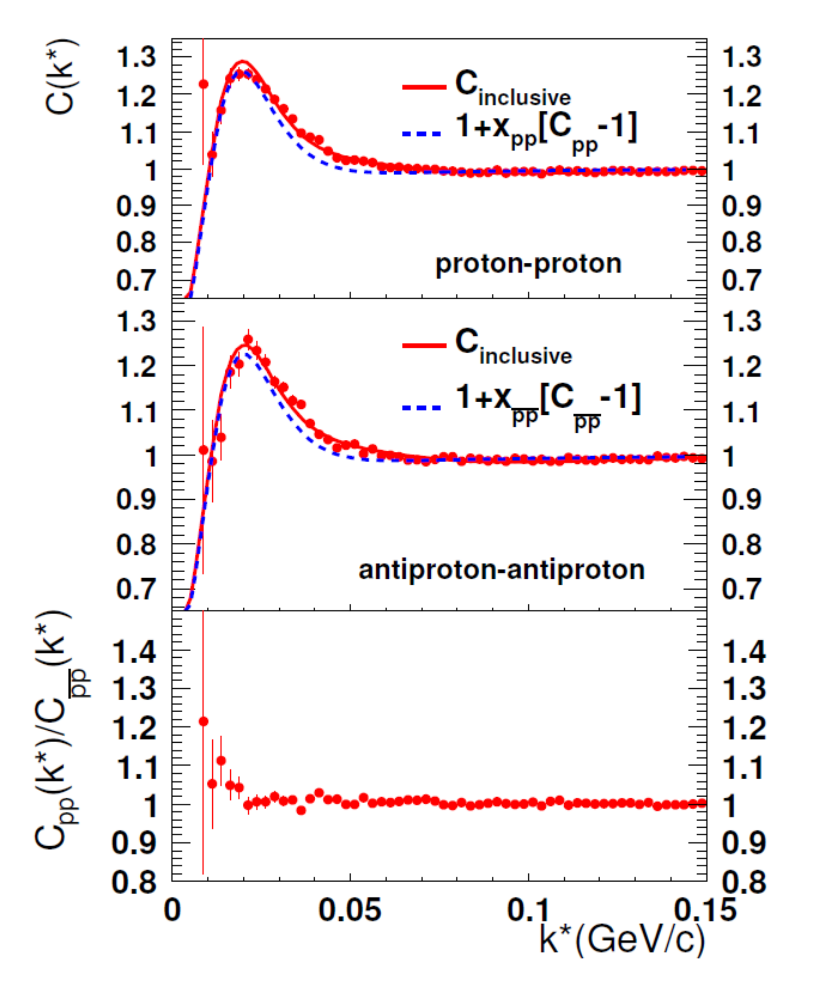
$$F_2(z)=[1-\exp(-z^2)]/z$$



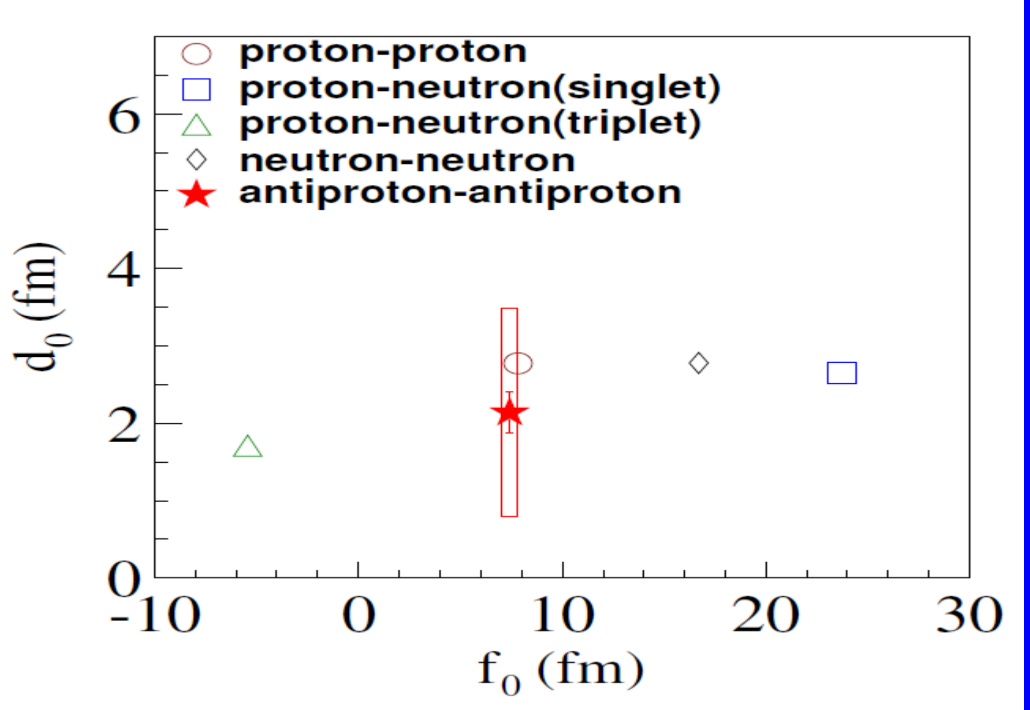
Correlation study of strong interaction

$\bar{p}\bar{p}$ s-wave scattering parameters from STAR correlation data

$$C_{\text{inclusive}}(k^*) = 1 + x_{pp}[C_{pp}(k^*;R_{pp}) - 1] + x_{p\Lambda}[\tilde{C}_{p\Lambda}(k^*;R_{p\Lambda}) - 1] + x_{\Lambda\Lambda}[\tilde{C}_{\Lambda\Lambda}(k^*) - 1]$$

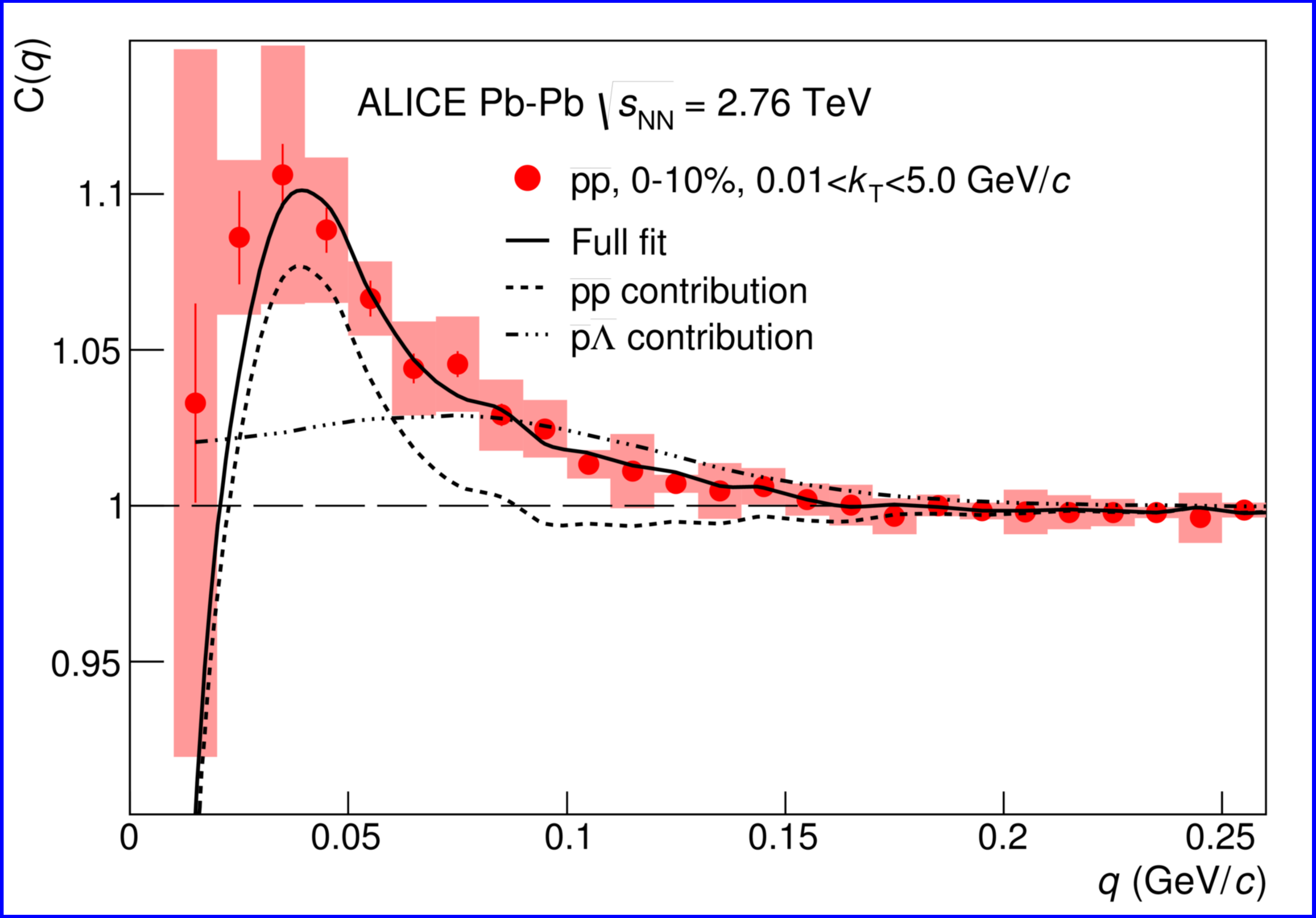


	DCA	x_{pp}	$x_{p\Lambda}$	$x_{\Lambda\Lambda}$
proton-pronton	$2cm$	0.45	0.375	0.077
proton-proton	$1cm$	0.51	0.335	0.055
pbar-pbar	$2cm$	0.42	0.385	0.092
pbar-pbar	$1cm$	0.485	0.35	0.063



Correlation study of strong interaction

$\bar{p}\bar{p}$ & pp ALICE correlation data



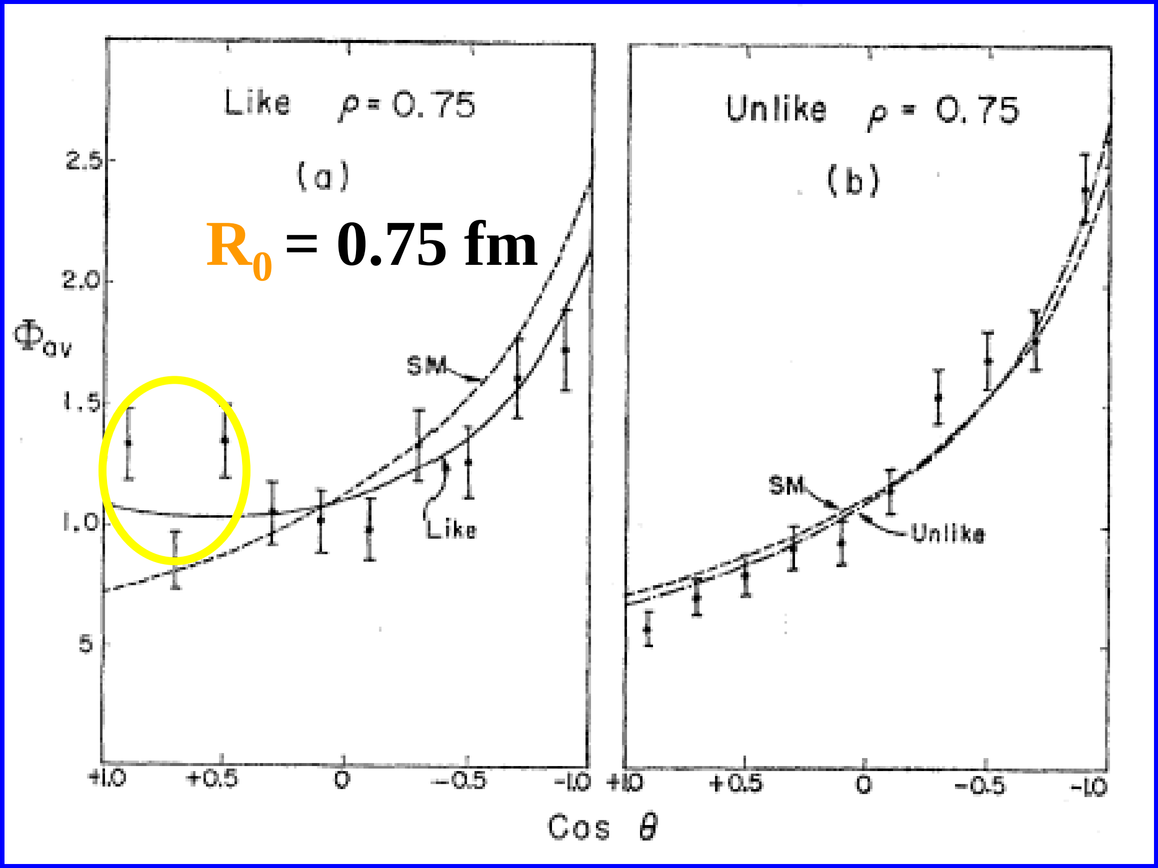
Summary

- Assumptions behind femtoscopy theory in HIC OK at $k \to 0$.
- Wealth of data on correlations of various particles ($\pi^{\pm}$,$K^{\pm 0}$,$p^{\pm}$,Λ,Ξ) is available & gives unique space-time info on production characteristics thanks to the effects of QS and FSI.
- Info on two-particle s-wave strong interaction:
  $\pi\pi$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ scattering amplitudes
  from HIC at SPS and RHIC
  (on a way to solving the problem of residual correlations).
  A good perspective: high statistics RHIC & LHC data.

2xGoldhaber, Lee & Pais

GGLP'60: enhanced $\pi^+\pi^+$, $\pi^-\pi^-$ vs $\pi^+\pi^-$ at small opening angles – interpreted as BE enhancement depending on fireball radius R_0

$\bar{p}\, p \rightarrow 2\pi^+\, 2\pi^-\, n\pi^0$



Femtoscopy through Emission function $G(p,x)$

One particle:

$$E\, d^3N/d^3p = \sum_\alpha |T_\alpha(p)|^2 = \int d^4x\, d^4x'\, \exp[-i\, p(x-x')]\, \sum_\alpha T_\alpha(x) T_\alpha^*(x')$$

$$= \int d^4x\, G(p,x) \qquad x,x' \rightarrow\ x=\tfrac{1}{2}(x+x'),\ \varepsilon = x-x'$$

$G(p,x)$ = partial Fourier transform of space-time density matrix $\sum_\alpha T_\alpha(x) T_\alpha^*(x')$

Two id. nonint. pions:

$$E_1E_2 d^6N/d^3p_1 d^3p_2 = \int d^4x_1 d^4x_2\, [G(p_1,x_1;p_2,x_2) + G(p,x_1;p,x_2)\cos(q\Delta x)]$$

$$p = \tfrac{1}{2}(p_1+p_2) \quad q = p_1-p_2 \quad \Delta x = x_1-x_2$$

$$\mathrm{Corr}(p_1,p_2) = \int d^4x_1 d^4x_2 G(p,x_1;p,x_2)\cos(q\Delta x) \,/\, \int d^4x_1 d^4x_2 G(p_1,x_1;p_2,x_2)$$

$$\approx \langle \cos(q\Delta x) \rangle$$

if $G(p_1,x_1;p_2,x_2) \approx G(p,x_1;p,x_2)$: "smoothness assumption"

Smoothness approximation: $r_A \ll r_0$ $(q \ll \Delta p)$

$$W(p_1,p_2) = \sum_\alpha \left|\int d^4x_1 d^4x_2\, \psi_{\mathbf{p1p2}}(x_1,x_2)\, T(x_1,x_2;\alpha)\right|^2$$

$$= \sum_\alpha \int d^4x_1 d^4x_1' d^4x_2 d^4x_2'$$

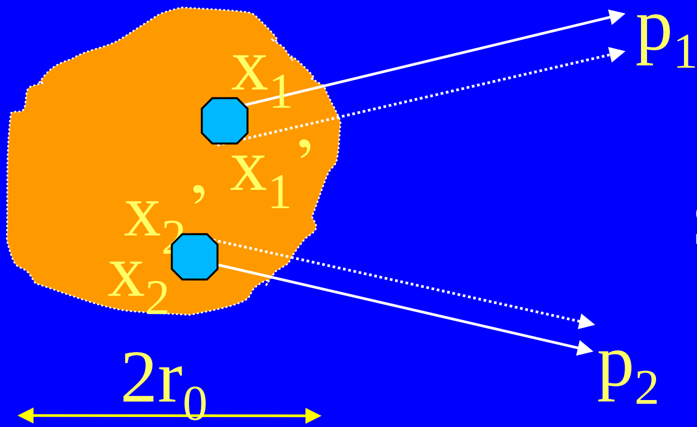
$$\psi_{p1p2}(x_1,x_2)\psi_{p1p2}^*(x_1',x_2')$$

$$T(x_1,x_2;\alpha)T^*(x_1',x_2';\alpha)$$

$$\approx \int d^4x_1 d^4x_2\, G(x_1,p_1;x_2,p_2)\, |\psi_{\mathbf{p1p2}}(x_1,x_2)|^2$$

r_0 - Source radius

r_A - Emitter radius



Source function $G(x_1,p_1;x_2,p_2) =$

$$\sum_\alpha \int d^4\varepsilon_1 d^4\varepsilon_2 \exp(ip_1\varepsilon_1 + ip_2\varepsilon_2)\,.$$

$$T(x_1+\tfrac{1}{2}\varepsilon_1, x_2+\tfrac{1}{2}\varepsilon_2;\alpha)T^*(x_1-\tfrac{1}{2}\varepsilon_1, x_2-\tfrac{1}{2}\varepsilon_2;\alpha)$$

For non-interacting identical spin-0 particles – exact result ($p=\tfrac{1}{2}(p_1+p_2)$):

$$W(p_1,p_2) = \int d^4x_1 d^4x_2\, [G(x_1,p_1;x_2,p_2) + G(x_1,p;x_2,p)\cos(q\Delta x)]$$

approx. result: $\approx \int d^4x_1 d^4x_2\, G(x_1,p_1;x_2,p_2)\,[1+\cos(q\Delta x)]$

$$= \int d^4x_1 d^4x_2\, G(x_1,p_1;x_2,p_2)\, |\psi_{\mathbf{p1p2}}(x_1,x_2)|^2$$

Phase space density from CFs and spectra

$$\bar{f}(p_t) \equiv \frac{\int d^3r\, f(p,r)\cdot f(p,r)}{\int d^3r\, f(p,r)}$$

Bertsch'94

$$\sim \frac{1}{2\pi m_\pi}\frac{dN}{p_t dp_t dy}\int d^3Q_{\mathrm{inv}}\, C(p_t, Q_{\mathrm{inv}})$$

$$f_{\max}(p_t, r) \approx 2\sqrt{2}\,\bar{f}(p_t)$$

Lisa ..'05

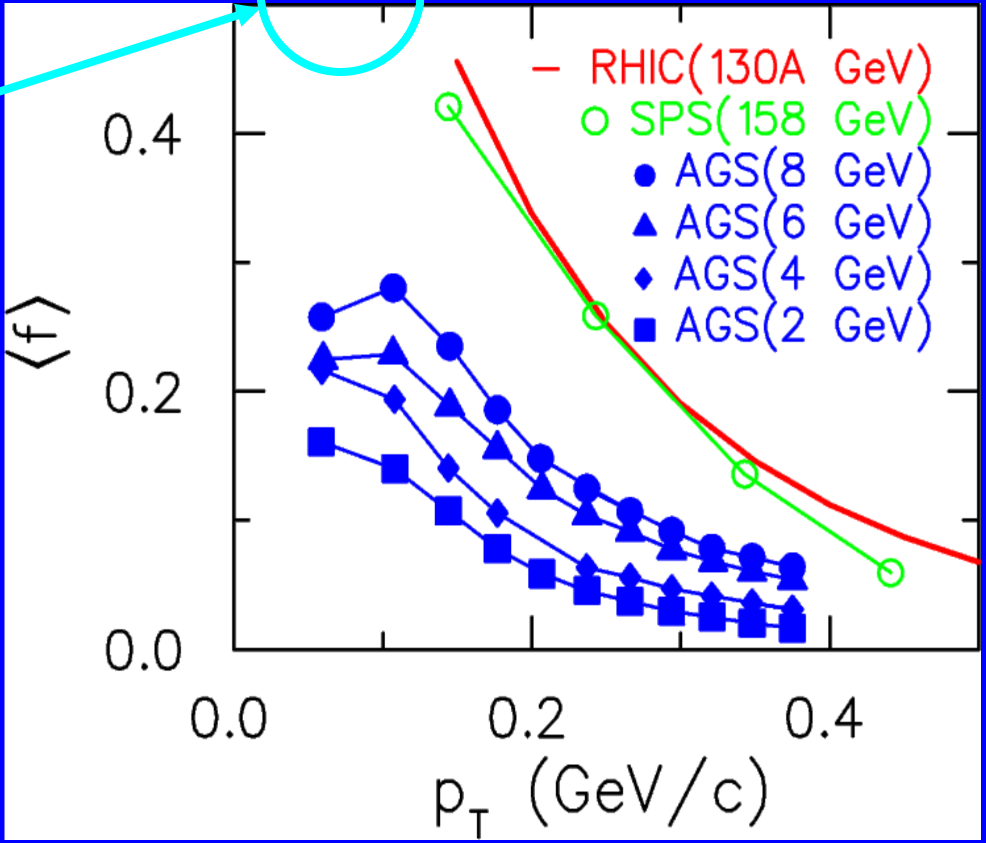
$\langle f \rangle$ rises up to SPS

May be high phase space density at low p_t ?

↓

? Pion condensate or laser

? Multiboson effects on CFs spectra & multiplicities

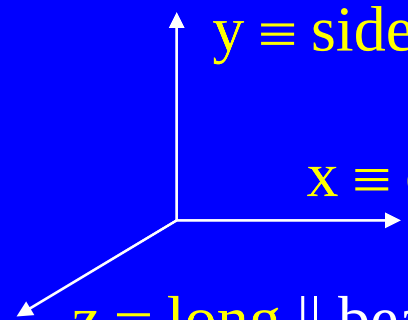


"General" parameterization at $|\mathbf{q}| \to 0$

Particles on mass shell & azimuthal symmetry $\Rightarrow$ 5 variables:

$\mathbf{q} = \{q_x, q_y, q_z\} \equiv \{q_{out}, q_{side}, q_{long}\}$, pair velocity $\mathbf{v} = \{v_x, 0, v_z\}$

$q_0 = \mathbf{q}\mathbf{p}/p_0 \equiv \mathbf{q}\mathbf{v} = q_x v_x + q_z v_z$



Grassberger'77
RL'78

$$\langle \cos q\Delta x\rangle = 1 - \tfrac{1}{2}\langle (q\Delta x)^2\rangle + .. \approx \exp(-R_x^2 q_x^2 - R_y^2 q_y^2 - R_z^2 q_z^2 - 2R_{xz}^2 q_x q_z)$$

Interferometry or correlation radii:

$$R_x^2 = \tfrac{1}{2}\langle (\Delta x - v_x \Delta t)^2 \rangle,\ R_y^2 = \tfrac{1}{2}\langle (\Delta y)^2 \rangle,\ R_z^2 = \tfrac{1}{2}\langle (\Delta z - v_z \Delta t)^2 \rangle$$

Podgoretsky'83, Bertsch, Pratt'95; so called out-side-long parameterization

Csorgo, Pratt'91: $\boxed{\text{LCMS } v_z = 0}$

cross-sections. Let two protons be emitted independently with equal momenta $\boldsymbol{p}$ from space-time points $(\boldsymbol{r}_1 t_1)$, $(\boldsymbol{r}_2 t_2)$, where $t_2 \geqslant t_1$. The joint probability of observing protons with momenta $\boldsymbol{p}_1, \boldsymbol{p}_2$ (both approximately equal to $\boldsymbol{p}$) is then given by the square of the overlap between the single-particle wave packets centered at $\boldsymbol{r}_1' = \boldsymbol{r}_1 + \boldsymbol{V}(t_2 - t_1)$, $\boldsymbol{r}_2$ and the final state wavefunction. Here, $\boldsymbol{V} = \boldsymbol{p}/m = (\boldsymbol{p}_1 + \boldsymbol{p}_2)/2m$ is the laboratory velocity of the p-p center-of-mass. After similarly considering the case $t_1 > t_2$, the double differential two-proton inclusive cross-section may be approximated as

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2} = \int_{-\infty}^{\infty} \mathrm{d}t_1\,\mathrm{d}t_2 \int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\, D(\boldsymbol{r}_1 t_1, \boldsymbol{p}) D(\boldsymbol{r}_2 t_2, \boldsymbol{p})$$

$$\times \left\{\tfrac{1}{4}\left|{}^1\Psi_{\boldsymbol{p}_1\boldsymbol{p}_2}(\boldsymbol{r}_1', \boldsymbol{r}_2)\right|^2 + \tfrac{3}{4}\left|{}^3\Psi_{\boldsymbol{p}_1\boldsymbol{p}_2}(\boldsymbol{r}_1', \boldsymbol{r}_2)\right|^2\right\}. \quad (1)$$

The singlet and triplet p-p scattering wavefunctions for protons of momenta $\boldsymbol{p}_1, \boldsymbol{p}_2$ are respectively ${}^1\Psi$ and ${}^3\Psi$. They are respectively symmetric and anti-symmetric under the interchange of their spatial or momentum arguments and satisfy the two-body p-p Schroedinger equation containing nuclear and coulomb potentials.

S. Koonin 1977

Intuitive generalization of KP formula for non-interacting protons to interacting non-relativistic protons (in the laboratory):

1st proton at t_2: $r_1' = r_1 + V(t_2 - t_1)$

2nd proton at t_2: r_2

FSI determined by spatial separation $r_1' - r_2$ at t_2= emission time of later particle

Problems:

- not applicable to lighter particles (pions) emitted with relativistic velocities
- generally, Ψ is the B-S amp., explicitly depending on time separation

Gyulassy, Kaufmann, Wilson 1979

Plane wave $\xrightarrow{\text{FSI}}$ Bethe-Salpeter amplitude

$$\exp(-ip_1x_1-ip_2x_2) \rightarrow \psi_{p1p2}(x_1,x_2)$$

In pair CMS, only relative quantities are relevant: $q=\{0,2\mathbf{k}\}$, $\Delta x=\{t,\mathbf{r}\}$

$$\exp(ikr) \rightarrow \psi_q(t,r)$$

at $t = 0$, the reduced B-S ampl. coincides with the usual WF:

$$\psi_q(t=0,r) = [\psi_{-k}(r)]^*$$

Note: in beta-decay $A \rightarrow A' + e + \nu$

$t(A')-t(e) = 0$ in the A rest frame $\approx t$ in A'e-pair CMS

Lednicky, Lyuboshitz 1981

- Eq. time approximation $t=0$ is valid on condition $|t| \ll \mu r^2$

Usually OK to several % even for pions

- Smoothness approx. applied also to non-id. particles

NA49 central Pb+Pb 158 AGeV vs RQMD: FSI theory OK

Long tails in RQMD: $\langle r^* \rangle$ = **21** fm for r^* < **50** fm

29 fm for r^* < **500** fm

Fit $\boxed{\text{CF=Norm [Purity RQMD}(r^* \rightarrow \text{Scale} \bullet r^*)\text{+1-Purity]}}$

⇒ **RQMD overestimates r^* by 10-20% at SPS** cf **~ OK at AGS**

worse at RHIC