Correlations and fluctuations of pions at the LHC

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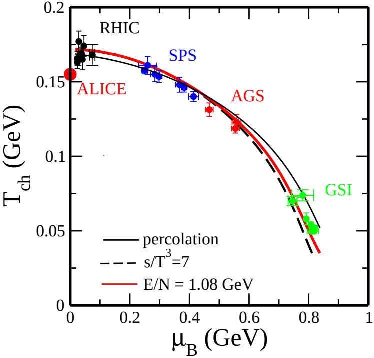
- Thermal model gives the freeze-out curve
- The prediction was too high for ratios to **pions**, especially proton to pion ratio
- The best fit of the **LHC** data still gives three standard deviations for protons
- The low-transverse-momentum **pion spectra** show up to 50% enhancement compared to hydrodynamic models
- The fit of the LHC data gives the parameters that **fall out** to the "wrong" side



Cleymans, Redlich, et. al., PRC (2006); EPJ (2015)

Possible explanations:

- hadronization and freeze-out in chemical **non-equilibrium** (Rafelski et al., PRC (2013))
- hadronic rescattering in the final stage (Becattini, Stock et al., PRL (2013))
- incomplete list of hadrons (Noronha-Hostler, Greiner, 1405.7298; NPA (2014))

Reasons for the non-equilibrium:

- super(over)cooling of the QGP (Shuryak, 1412.8393; Csorgo, Csernai, PLB (1994))
- gluon condensation in CGC (Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA (2012); Gelis, NPA (2014))

Cracow single freeze-out thermal model

The phase-space distribution of the primordial particles has the form:

$$f_i = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\gamma_i^{-1} \exp(\sqrt{p^2 + m_i^2}/T) \pm 1}, \quad \text{where} \quad \gamma_i = \gamma_q^{N_q^i + N_{\bar{q}}^i} \gamma_s^{N_s^i + N_{\bar{s}}^i} \exp\left(\frac{\mu_B B_i + \mu_S S_i}{T}\right),$$

and N_q^i, N_s^i are the numbers of light (u, d) and strange (s) quarks in the ith hadron. It includes all well established resonances from PDG. Resonance decay according to their branching ratios.

Single-freeze out model (Broniowski, Florkowski, PRL (2001))
Monte-Carlo implementations, THERMINATOR (Chojnacki, Kisiel, Florkowski, Broniowski, Comput. Phys. Commun. (2012))

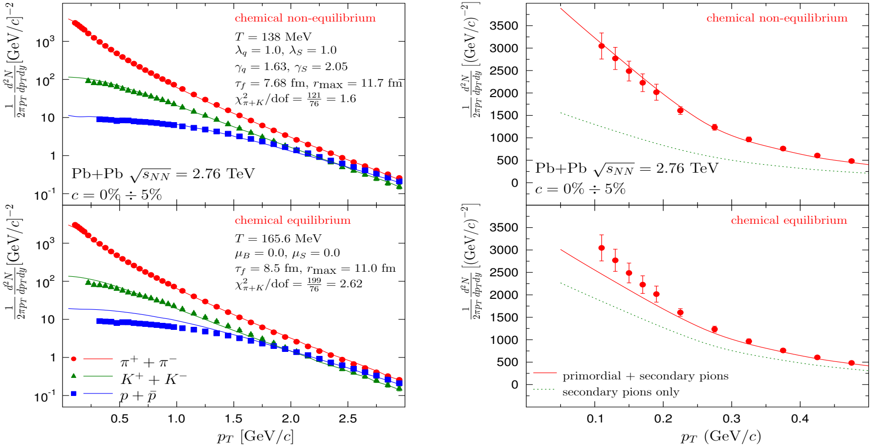
The spectra are calculated from the Cooper-Frye formula at the freeze-out **hyper surface**

$$\frac{dN}{dyd^2p_T} = \int d\Sigma_\mu p^\mu f(p \cdot u), \qquad t^2 = \tau_f^2 + x^2 + y^2 + z^2, \qquad x^2 + y^2 \le r_{\max}^2,$$

assuming the Hubble-like **flow**: $u^\mu = x^\mu / \tau_f$.

There is **only** one additional **parameter** in the model, because the product $\pi \tau_f r_{\max}^2$ is equal to the volume (per unit rapidity), while the ratio $r_{\max}/\tau_f$ determines the **slope** of the spectra.

The fits to the ratios of hadron abundances (Rafelski et al., PRC (2013)) yield γ_q which is very close to the critical pion chemical potential: $\mu_\pi = 2T \ln \gamma_q \simeq 134$ MeV $\simeq m_{\pi^0} \simeq 134.98$ MeV



The **spectra** favor the **non-equilibrium** model. It may suggest that a substantial part of π^0 **mesons form** the **condensate** (V.B., Florkowski, Rybczynski, PRC (2014))

Multiplicity fluctuations of any order can be calculated using the definition of susceptibility:

$$\chi_n = \left.\frac{\partial^n(\mathcal{P}/T^4)}{\partial(\mu/T)^n}\right|_T$$

where $\mathcal{P}$ is pressure, μ-chemical potential, T-temperature.

The characteristic observables are the

$\omega = \sigma^2/\langle N\rangle$ - **scaled variance**: variance over the mean,

$S \cdot \sigma$ - normalized **skewness**

$\kappa \cdot \sigma^2$ - normalized **kurtosis**

They are related to the susceptibilities and central moments:

$$\omega = \frac{\chi_2}{\chi_1} = \frac{\mu_2}{\langle N\rangle} \qquad S \cdot \sigma = \frac{\chi_3}{\chi_2} = \frac{\mu_3}{\mu_2} \qquad \kappa \cdot \sigma^2 = \frac{\chi_4}{\chi_2} = \frac{\mu_4}{\mu_2} - 3\,\mu_2$$

where $\mu_n = \langle (N - \langle N\rangle)^n\rangle = \sum_N (N - \langle N\rangle)^n \cdot P(N)$ are central moments of the $P(N)$ multiplicity distribution.

Fluctuations of primary pions

In the pion gas one can make **analytical calculations**

$$\langle N\rangle = \sum_p \langle n_p\rangle \qquad\qquad \langle n_p\rangle = \frac{1}{\exp\left[\left(\sqrt{p^2+m_i^2} - \mu_\pi\right)/T\right]-1}$$

$$\omega = \frac{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}{\sum_p\langle n_p\rangle}$$

$$S\cdot\sigma = \frac{\sum_p\left(\langle n_p\rangle + 3\langle n_p\rangle^2 + 2\langle n_p\rangle^3\right)}{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}$$

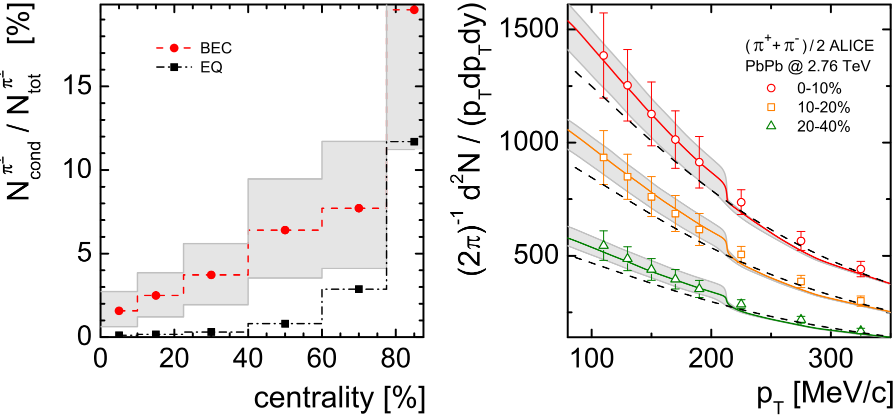
$$\kappa\cdot\sigma^2 = \frac{\sum_p\left(\langle n_p\rangle + 7\langle n_p\rangle^2 + 12\langle n_p\rangle^3 + 6\langle n_p\rangle^4\right)}{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}$$

The standard replacement of the sum by the integral over the momentum levels $\sum_p \to V/(2\pi)^3\int d^3p$ leads to the divergence of all $\int d^3p\langle n_p\rangle^m$ with $m \geq 2$ and infinite variance, skewness and kurtosis at BEC.

There are no divergences in **finite volume**, that can be taken into account by keeping the **zero momentum level** of the sum (V.B., Gorenstein, PRC (2008), V.B. EPJ (2015)):

$$\langle N\rangle = \langle n_0\rangle + \frac{V}{(2\pi)^3}\int_0^\infty \langle n_p\rangle\, d^3p = N_{cond} + N_{norm}$$

The p_T distribution is $\frac{dN}{dy d\phi_p \, p_T dp_T} = \frac{N_{cond}}{V} \frac{\tau_f^3}{m^2} \theta \left(r_{max} - p_T \tau_f / m \right)$ (V.B., Florkowski, PRC (2015)):



Condensate rate and p_T spectrum for charged pions. The grey area show the **10%** deviation from the best fit.

- The inclusion of several more levels would lead to finer steps
- The data on multiplicities and spectra are compatible with **5%** of the **condensate**

Estimate of the resonance decay contribution

For two uncorrelated multiplicity distributions $P_1(N_1)$ and $P_2(N_2)$:

$$\langle N \rangle = \langle N_1 \rangle + \langle N_2 \rangle = \sum_{N_1} N_1 \; P_1(N_1) + \sum_{N_1} N_1 \; P_1(N_1)$$

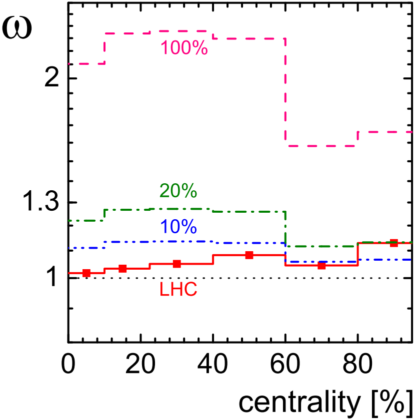
$$\omega = \omega_1 \frac{\langle N_1 \rangle}{\langle N \rangle} + \omega_2 \frac{\langle N_2 \rangle}{\langle N \rangle}$$

Similarly for kurtosis and skewness:

$$S \cdot \sigma = S_1 \cdot \sigma_1 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + S_2 \cdot \sigma_2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle}$$

$$\kappa \cdot \sigma^2 = \kappa_1 \cdot \sigma_1^2 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + \kappa_2 \cdot \sigma_2^2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle}$$

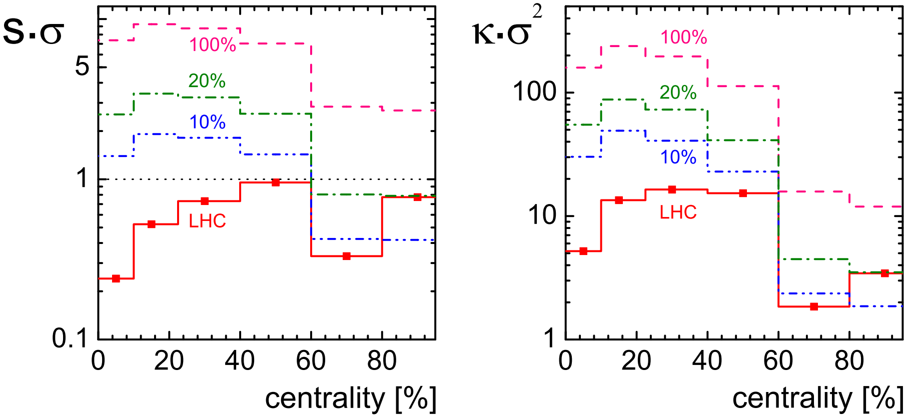
For Poisson distribution $\omega_i = 1$. Experimentally seen fluctuations are $\omega_i \lesssim 2$.



Scaled variance for different condensate rate $\langle N_1 \rangle / \langle N \rangle$, assuming $\omega_2 = 1$ for resonances.

One can increase the condensate rate in the most central collisions parameters up to **10%** making the cut $p_T < p_T^{BEC}$. **ALICE** collaboration claims that coherent fraction in charged pion emission may reach **23%** for $p_T < 300$ MeV (PRC 89, 024911 (2014)).

For Poisson distribution $S \cdot \sigma = \kappa \cdot \sigma^2 = 1$, while for Gauss distribution $S \cdot \sigma = \kappa \cdot \sigma^2 = 0$.



Skewness and kurtosis for different condensate rate assuming Poisson distribution for resonances.

- Skewness is small and positive at the LHC
- **Kurtosis** is **large** even for the current data
- The increase of the condensate rate to **10%** could lead to a **detectable signal**

Conclusions

- The intriguing possibility of Bose-Einstein condensation of pions at the LHC is examined with the use of higher order moments of the multiplicity distribution
- The scaled variance, skewness and kurtosis are calculated for the pion system
- The effects of resonance decays are estimated
- The obtained results show the possibility to see a **significant increase** of the **kurtosis** for the case of BEC in the measured data