Fluctuations of flow harmonics in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in the Glauber model *

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*based on arXiv:1510.08146

Outline

- In this talk we present predictions of the Glauber-like models of the initial stage of the heavy-ion reactions for **fluctuations of the flow harmonics**.

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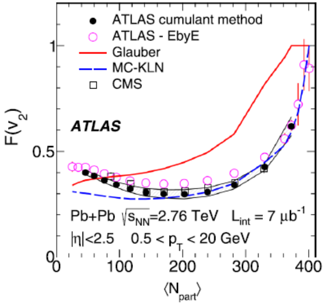
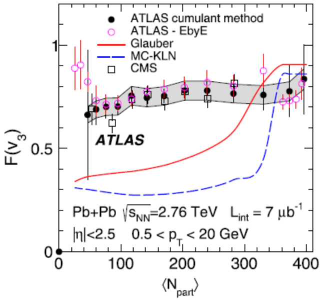
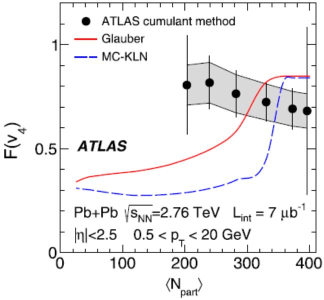
- The predictions are compared to the recent measurements of the fluctuations of the harmonic flow coefficients at the LHC done by **ALICE** (Phys. Rev. Lett. 105 (2010) 252302), **ATLAS** (Eur. Phys. J. C74 (2014) 3157, G. Aad et al., JHEP 1311 (2013) 183) and **CMS** (Phys. Rev. C89 (2014) 044906) experiments.







Figures taken from
G. Aad et al., Eur. Phys. J. C74 (2014) 3157

Glauber model details

We use GLISSANDO 2 to analyze two variants of the Glauber model with Monte Carlo simulations:

- The **mixed** model, amending wounded nucleons with an admixture of binary collisions in the proportion α. The successful fits to particle multiplicities (PHOBOS, B.B. Back et al., Phys. Rev. C70 (2004) 021902) give $\alpha = 0.145$ at $\sqrt{s_{NN}} = 200$ GeV. For the LHC energy of $\sqrt{s_{NN}} = 2.76$ TeV we take $\alpha = 0.15$.

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- Each source from the mixed model may deposit entropy with a certain distribution of strength. Therefore, we superpose the **Gamma distribution** over the distribution of sources and label this model. The choice of this distribution follow from the fact that when folded with the Poisson distribution for the production of the number of particles at freeze-out, it yields the popular negative binomial distribution.

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- In the analysis of the eccentricities we use value of $\nu = 0.9$ in the Gamma distribution

$$g(w,\nu) = \frac{w^{\nu-1}\nu^{\nu}\exp(-\nu w)}{\Gamma(\nu)}, \quad w \in [0,\infty).$$

which yields $\langle w \rangle = 1$ and $\sigma(w) = 1/\sqrt{\nu} = 1.054$.

The expression for the initial entropy distribution in the transverse plane is

$$s(\mathbf{x}_T) = \mathrm{const}\left(\frac{1-\alpha}{2}\sum_{i=1}^{N_\mathrm{w}} w_i g_i(\mathbf{x}_T) + \alpha \sum_{j=1}^{N_\mathrm{bin}} w_j g_j(\mathbf{x}_T)\right),$$

where

$$g_k(\mathbf{x}_T) = \exp\left(-\frac{(\mathbf{x}_T - \mathbf{x}_{T,k})^2}{2\sigma^2}\right)$$

describe the smearing of the sources (wounded nucleons or binary collisions) located at $\mathbf{x}_{T,k}$). The smearing parameter is $\sigma = 0.4$ fm (P. Bożek and W. Broniowski, Phys. Rev. C88 (2013) 014903). The center of the binary-collision source is at the mean of the location of the centers of the colliding nucleons.

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- The realistic nucleon-nucleon inelastic collision profile for the LHC energies is taken from M. Rybczyński and Z. Włodarczyk, J.Phys. G41 (2013) 015106.

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- The total inelastic nucleon-nucleon cross section is equal to 64 mb for the analyzed Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

Fluctuations of elliptic and triangular flow

- Due to collectivity of the fireball evolution, the azimuthal anisotropy of hadrons produced in the final state reflects the initial spatial asymmetry of the fireball in the transverse plane, which is due to geometry and fluctuations event-by-event. The observed particle distributions are characterized by the harmonic flow coefficients v_n, defined as the Fourier coefficients of the expansion

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2\sum_{n=2}^{\infty} v_n \cos\left[n(\phi - \Psi_n)\right]\right].$$

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- The eccentricity coefficients ϵ_n parametrize the shape of the initial fireball, and are defined in a given event as

$$\epsilon_n e^{in\Phi_n} = \frac{\int dx_T s(\mathbf{x}_T)\rho^n e^{in\phi}}{\int dx_T s(\mathbf{x}_T)\rho^n},$$

  where ρ and ϕ are the polar coordinates corresponding to $\mathbf{x}_T$.

Fluctuations of elliptic and triangular flow

- It has been argued (e.g.: H. Niemi et al., Phys. Rev. C87 (2013) 054901) that to a good accuracy one has the proportionality (the "shape-flow" transmutation)

$$v_n = \kappa_n \epsilon_n, \quad n = 2, 3,$$

where the constants κ_n depend on features of the colliding system (centrality selection, mass numbers, collision energy) and the properties of the dynamics (viscosity of quark-gluon plasma, initial time of collective evolution, freeze-out conditions).

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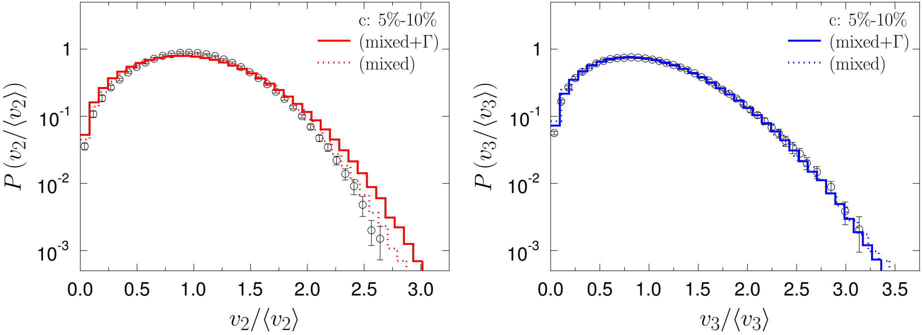
- Thus one obtains immediately the relation for the scaled (i.e., independent of the mean) quantities

$$\frac{v_n}{\langle v_n \rangle} = \frac{\epsilon_n}{\langle \epsilon_n \rangle}, \quad n = 2, 3,$$

where $\langle . \rangle$ denotes averaging over events in the given class.



Distributions of $\epsilon_n/\langle \epsilon_n \rangle$ for the model calculations, compared to the experimental distribution of $v_n/\langle v_n \rangle$ from the **ATLAS** collaboration, G. Aad et al., JHEP 1311 (2013) 183. Centrality 5 – 10%.

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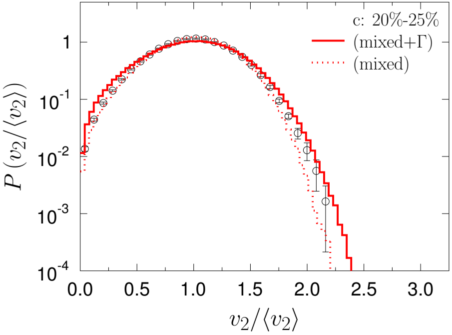
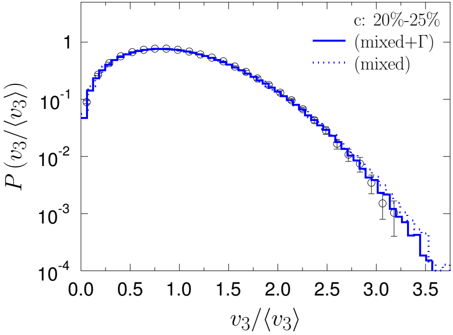
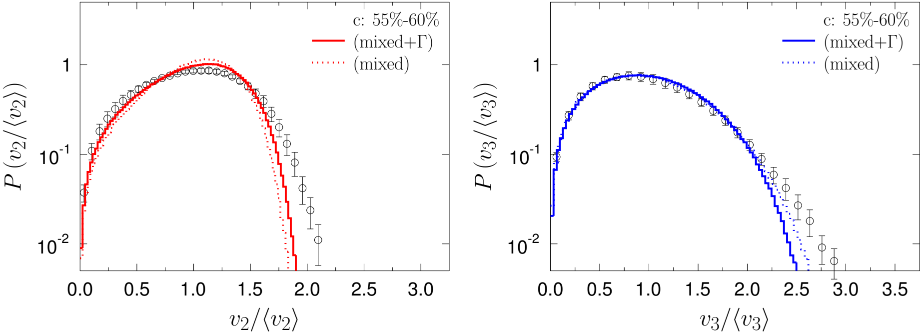




Distributions of $\epsilon_n/\langle\epsilon_n\rangle$ for the model calculations, compared to the experimental distribution of $v_n/\langle v_n\rangle$ from the **ATLAS** collaboration, G. Aad et al., JHEP 1311 (2013) 183. Centrality 20 – 25%.



Distributions of $\epsilon_n/\langle\epsilon_n\rangle$ for the model calculations, compared to the experimental distribution of $v_n/\langle v_n\rangle$ from the **ATLAS** collaboration, G. Aad et al., JHEP 1311 (2013) 183. Centrality 55 – 60%.

- Next, we explore the two-particle and four-particle cumulant moments, defined as

$$\epsilon_n\{2\} = \langle \epsilon_2^2 \rangle^{1/2},$$
$$\epsilon_n\{4\} = 2\left(\langle \epsilon_n^2 \rangle^2 - \langle \epsilon_n^4 \rangle\right)^{1/4}.$$

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- More specifically, we take the scaled event-by-event standard deviation, $\sigma(\epsilon_n)/\langle \epsilon_n \rangle$, and the $F_n(\epsilon_n)$ moments defined as

$$F(\epsilon_n) = \sqrt{\frac{\epsilon_n\{2\}^2 - \epsilon_n\{4\}^2}{\epsilon_n\{2\}^2 + \epsilon_n\{4\}^2}}.$$

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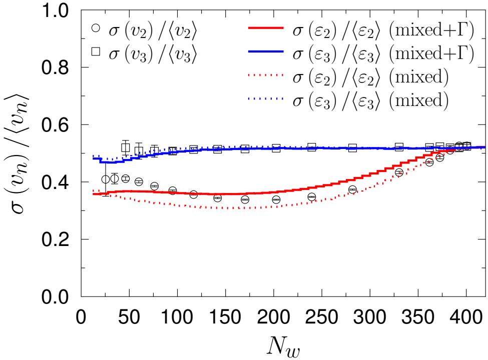
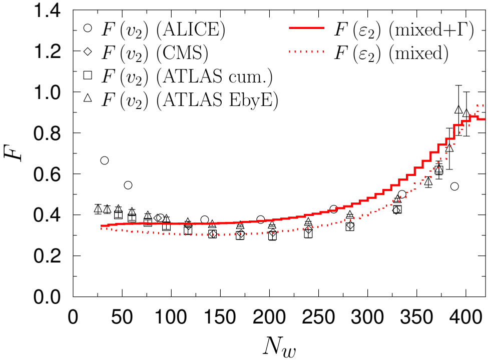
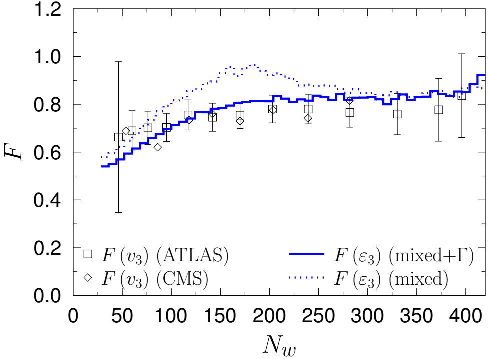
- These measures are analogously defined for the flow coefficients v_n.



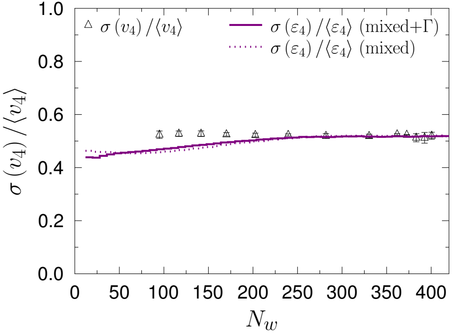
The scaled event-by-event standard deviation for the eccentricities, $\sigma(\epsilon_n)/\langle\epsilon_n\rangle$, and for the harmonic flow coefficients $\sigma(v_n)/\langle v_n\rangle$, plotted as functions of the number of wounded nucleons N_w. The dashed (solid) lines correspond to our simulation in the mixed (mixed+Γ). The data come from the **ATLAS** collaboration.

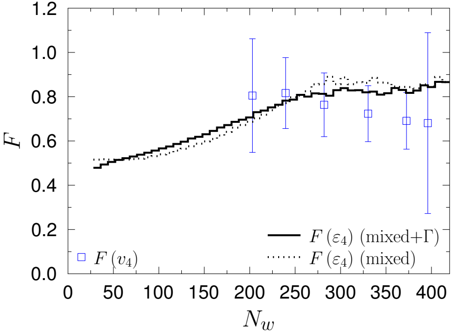


The relative elliptic flow event-by-event fluctuations measure $F(v_2)$, plotted as a function of the number of wounded nucleons N_w. Result of our simulation with the mixed model is displayed with the full line, whereas the dashed line shows the outcome of the mixed+Γ model. The points show the data from **ATLAS**, **ALICE**, and **CMS** experiments.



The relative elliptic flow event-by-event fluctuations measure $F(v_3)$, plotted as a function of the number of wounded nucleons N_w. Result of our simulation with the mixed model is displayed with the full line, whereas the dashed line shows the outcome of the mixed+Γ model. The data from **ATLAS** and **CMS** collaborations are shown with the points.





The measures $\sigma(\epsilon_4)/\langle\epsilon_4\rangle$ and $F(v_4)$, compared to the **ATLAS** data for the corresponding quantities for v_4.

Conclusions

- Glauber model works within expected accuracy for the flow measures $\sigma(\epsilon_n)/\langle\epsilon_n\rangle$ and $F(v_n)$, for $n = 2, 3$, but also for $n = 4$.

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- Our results for the investigated measures do not depend strongly on the details of the Glauber model (overlaid distribution, correlations in the nuclear distributions, wounding profile, etc.), hence are robust for the investigation of flow fluctuations.

Running the simulations

To reproduce the results of the simulations presented here, or to extend them to other physical cases, the user should download the package GLISSANDO 2 ver. 2.9 from the web page:

http://www.ujk.edu.pl/homepages/mryb/GLISSANDO/

and after unpacking execute (on UNIX systems) the following commands:

```
make
./glissando2 input/mixed_gamma.dat output/mixed_gamma_01.root
root -b -l -q -x "macro/eps_fluct.C(\"output/mixed_gamma\",1)"
```

More statistics can be accumulated by running, for instance

```
./glissando2 input/mixed_gamma.dat output/mixed_gamma_02.root
...
./glissando2 input/mixed_gamma.dat output/mixed_gamma_10.root
root -b -l -q -x "macro/eps_fluct.C(\"output/mixed_gamma\",10)"
```

The plots are placed in the output directory. The present code has been checked with ROOT ver. 5.34.