Theoretical Concepts in Particle Physics (3)

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Yesterday...

- Correction
  - Q: the implication of $x_1 \to -3x_2$ and $x_2 \to -x_1/3$?
  - A: $q_1 = x_1 - 3x_2,\ q_2 = x_2 + 3x_1$ and we have $V(q_1)$
  - We can also have $x_1 x_2$
- Free fields are set of harmonic oscillators
- Particles are excitations of these oscillators

More on QFT

What about masses?

- A "free" Lagrangian gives massless particle

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 \Rightarrow \omega = k \quad (\text{or } E = P)$$

- We can add "potential" terms (without derivatives)

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + m^2 \phi^2$$

- Here m is the mass of the particle. Still free particle
- (HW) Show that m is a mass of the particle by showing that $\omega^2 = k^2 + m^2$

What about other temrs?

- How do we choose what terms to add to $\mathcal{L}$?
- Must be invariant under the symmetries
- We keep some leading terms (usually, up to ϕ^4)
- Lets add $\lambda\phi^4$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

- We get the non-linear wave equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial t^2} = m^2\phi + \lambda\phi^3$$

- We do not know how to solve it

What about fermions?

- We saw how massless and massive bosons are related to SHO
- The spin of the particle is related to the polarization of the classical field
  - Scalar field : spin zero paricles
  - vector field : spin one paricles
- We can construct a fermion SHO

$$[a, a^\dagger] = 1 \quad \to \quad \{b, b^\dagger\} = 1$$

- No classical analogue since $b^2 = 0$
- We can then think of fermionic fields. They can generate only one particle in a given state

A short summary

- Particles are excitations of fields
- The fundeumental Lagrangian is giving in term of fields
- Our aim is to find $\mathcal{L}$
- We can only solve the linear case, that is, the equivalent of the SHO
- What can we do with higher order temrs?

Perturbation theory

Perturbation theory

$$H = H_0 + H_1 \qquad H_1 \ll H_0$$

- In many cases pertubation theory is a mathematical tool
- There are cases, however, that PT is a better way to describe the physics
- Many times we prefer to work with EV of H (why?)
- Yet, at times it is better to work with EV of H_0 (why?)

1st and 2nd order PT

$$H = H_0 + H_1 \qquad H_1 \ll H_0$$

- In first order we care only about the states with the same energy

$$\langle f|H_1|i\rangle \qquad E_f = E_i$$

- 2nd order pertubation theory probe the whole spectrum

$$\sum_n \frac{\langle f|H_1|n\rangle\langle n|H_1|i\rangle}{E_n - E_f} \qquad E_f = E_i \qquad E_n \neq E_i$$

PT for 2 SHOs

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- Classically α moves energy between the two modes
- How it goes in QM?
- Recall the Fermi golden rule

$$P \propto |\mathcal{A}|^2 \times \text{P.S.} \qquad \mathcal{A} \sim \langle f|\alpha x^2 y|i\rangle$$

- The relevant thing to calculate is the transition amplitude, $\mathcal{A}$.

Transitions

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- Recall

$$x \sim a_x + a_x^\dagger \qquad y \sim a_y + a_y^\dagger$$

- When $\mathcal{A}$ is non-zero?

$$\mathcal{A} \sim \langle f | \alpha x^2 y | i \rangle$$

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- Recall

$$x \sim a_x + a_x^\dagger \qquad y \sim a_y + a_y^\dagger$$

- When $\mathcal{A}$ is non-zero?

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- Since $H_1 \sim x^2 y$ we see that $\Delta n_y = \pm 1$ and $\Delta n_x = 0, \pm 2$
- What could you say if the perturbation was $x^2 y^3$?

PT for 2 SHOs again

$$V(x,y) = \frac{kx^2}{2} + \frac{4ky^2}{2} + \alpha x^2 y$$

- Consider $|i\rangle = |0,1\rangle$
- Since $\omega_y = 2\omega_x$ only $f = |2,0\rangle$ is allowed by energy conservation and by the perturbation

$$\mathcal{A} \sim \langle 2,0|\alpha x^2 y|0,1\rangle \sim \alpha\langle 2,0|(a_x+a_x^\dagger)(a_x+a_x^\dagger)(a_y+a_y^\dagger)|0,1\rangle$$

- a_y in y annihilates the y "particle" and $(a_x^\dagger)^2$ in x^2 creates two x "particles"
- It is a decay of a particle y into two x particles with width $\Gamma \propto \alpha^2$ and thus $\tau = 1/\Gamma$

Even More PT

$$H_1 = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- Calculate $y \to 3x$ using 2nd order PT

$$\mathcal{A} \sim \langle 3,0,0|\mathcal{O}|0,1,0\rangle \qquad \mathcal{O} \sim \sum \frac{\langle 3,0,0|V'|n\rangle\langle n|V'|0,1,0\rangle}{E_n - E_{0,1,0}}$$

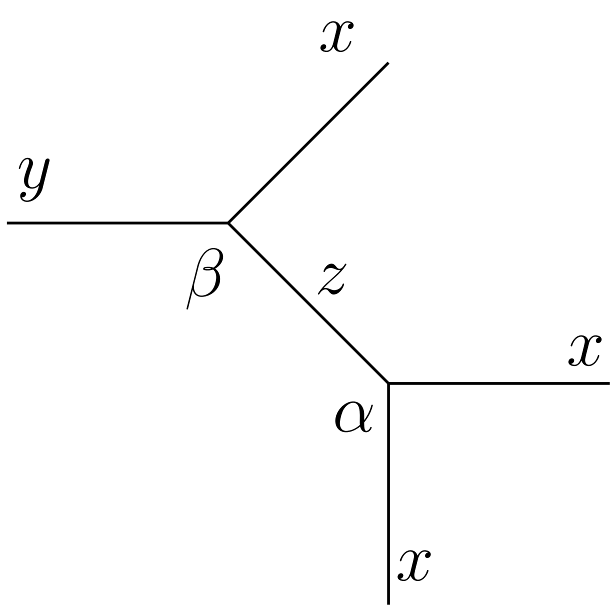
- Which intermediate states? $|1,0,1\rangle$ and $|2,1,1\rangle$
  - $\mathcal{A}_1 = |0,1,0\rangle \xrightarrow{\beta} |1,0,1\rangle \xrightarrow{\alpha} |3,0,0\rangle$
  - $\mathcal{A}_2 = |0,1,0\rangle \xrightarrow{\alpha} |2,1,1\rangle \xrightarrow{\beta} |3,0,0\rangle$
- The total amplitude is then

$$\mathcal{A} \propto \alpha\beta\left(\frac{\#}{\Delta E_1} + \frac{\#}{\Delta E_2}\right) \propto \alpha\beta\left(\frac{\#}{8} + \frac{\#}{12}\right)$$

Closer look

$$V' = \alpha x^2 z + \beta xyz \qquad \omega_z = 10,\ \omega_y = 3,\ \omega_x = 1$$

- We look at $\mathcal{A} = |0,1,0\rangle \xrightarrow{\beta} |1,0,1\rangle \xrightarrow{\alpha} |3,0,0\rangle$



$$\mathcal{A} \propto \frac{\alpha\beta}{\Delta E_z}$$

Feynman diagrams

Using PT for fields

- For SHOs we have $x_i \sim a_i + a_i^\dagger$
- For fields we then have

$$\phi \sim \int \left[a(k) + a^\dagger(k)\right] dk$$

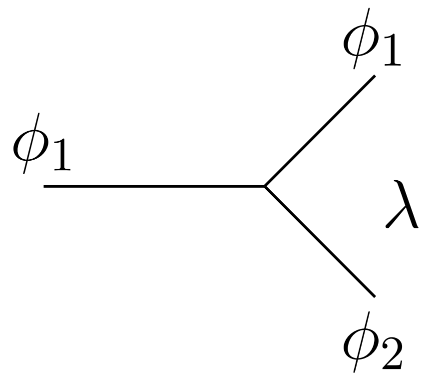
Quantum field $=$ creation and annihilation operators

Feynman diagrams

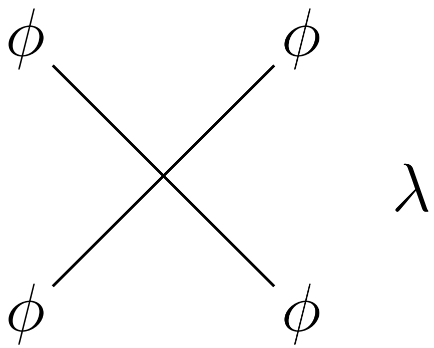
- A graphical way to do perturbation theory with fields
- Unlike SHOs before, a particle can have any energy as long as $E \geq m$
- Operators with 3 or more fields generate transitions between states. They give decays and scatterings
- Decay rates and scattering cross sections are calculated using the Golden Rule
- Amplitudes are calculated from $\mathcal{L}$
- We generate graphs where lines are particles and vertices are interactions

Examples of vertices

$$\mathcal{L} = \lambda \phi_1^2 \phi_2 :$$



$$\mathcal{L} = \lambda \phi^4 :$$



Calculations

- We usually care about $1 \to n$ or $2 \to n$ processes
- We need to make sure we have energy conservation
- External (Internal) particles are called on(off)–shell
  - On-shell: $E^2 = p^2 + m^2$
  - Off-shell: $E^2 \neq p^2 + m^2$
- $\mathcal{A} =$ the product of all the vertices and internal lines
- Each internal line with q^μ gives suppression

$$\frac{1}{m^2 - q^2}$$

- There are many more rules to get all the factors right

Examples of amplitudes

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$$\Gamma(Z \to XY)$$

- Energy conservation condition
- Draw the diagram and estimate the amplitude

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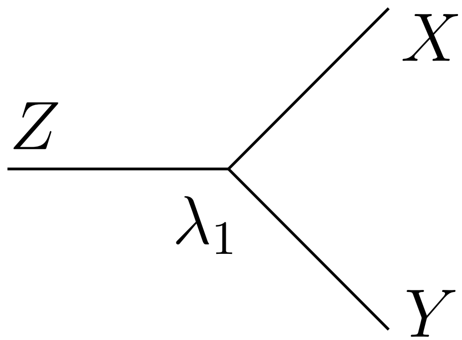
- Energy conservation condition $m_Z > m_X + m_Y$
- Draw the diagram and estimate the amplitude



$$\mathcal{A} \propto \lambda_1$$

Examples of amplitudes (2)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$\Gamma(Y \to 3X)$

- Energy conservation condition
- Draw the diagram and estimate the amplitude

Examples of amplitudes (2)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

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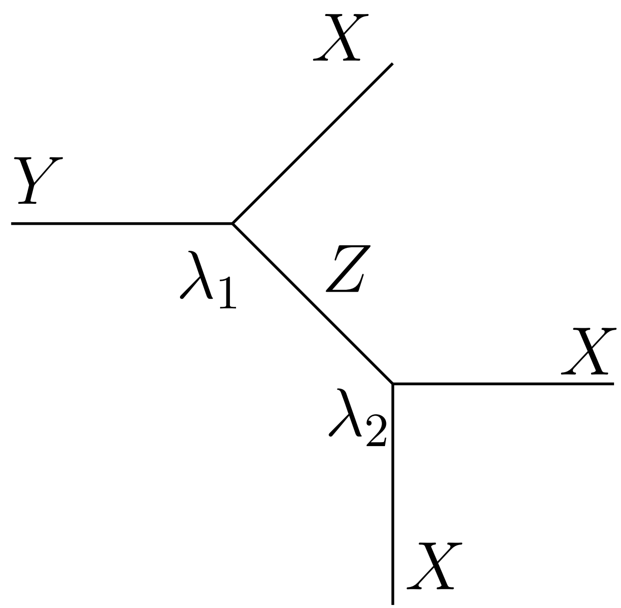
- Energy conservation condition $m_Y > 3m_X$
- Draw the diagram and estimate the amplitude



$$\mathcal{A} \propto \lambda_1 \lambda_2 \times \frac{1}{\Delta E_Z^2}$$

$$= \lambda_1 \lambda_2 \times \frac{1}{m_Z^2 - q^2}$$

Examples of amplitudes (HW)

$$\mathcal{L} = \lambda_1 XYZ + \lambda_2 X^2 Z$$

$\sigma(XX \to XY)$

- Energy conservation condition
- Draw the diagram and estimate the amplitude

Some summary

- Quadratic terms describe free fields. Free particles cannot be created nor decay
- We use perturbation theory were terms with 3 or more fields in $\mathcal{L}$ are consider small
- These terms can generate and destroy particles and give dynamics
- Feynman diagrams are a tool to calculate transition amplitudes
- Many more details are needed to get calculation done
- Once calculations and experiments to check them are done, we can test our theory