CERN Summer School 2015
Introduction to Accelerator Physics

Part III

by
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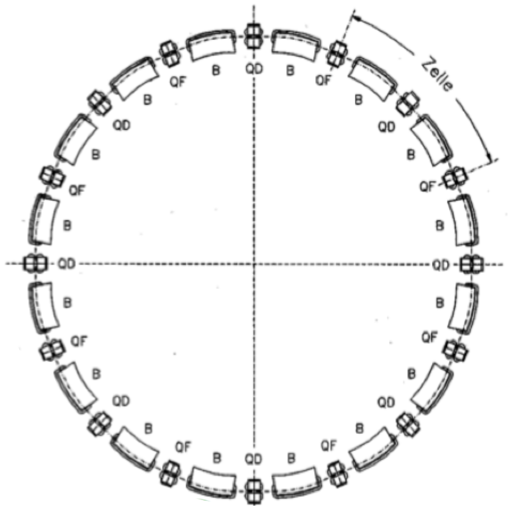
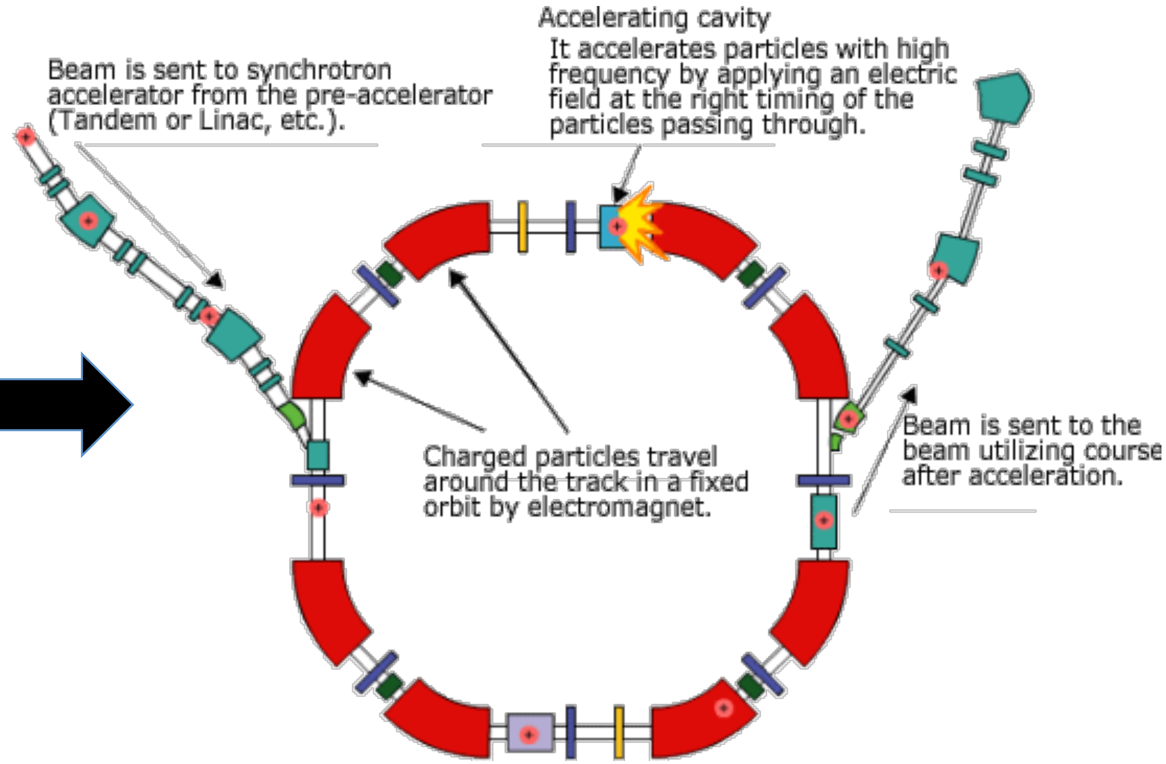
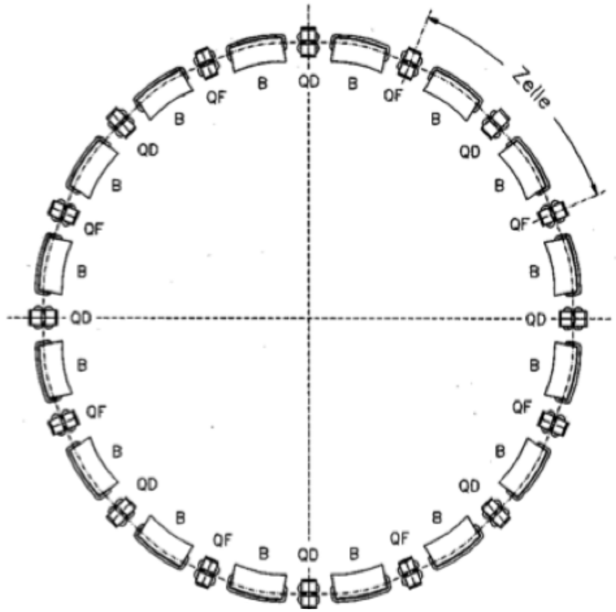
Today's Lecture

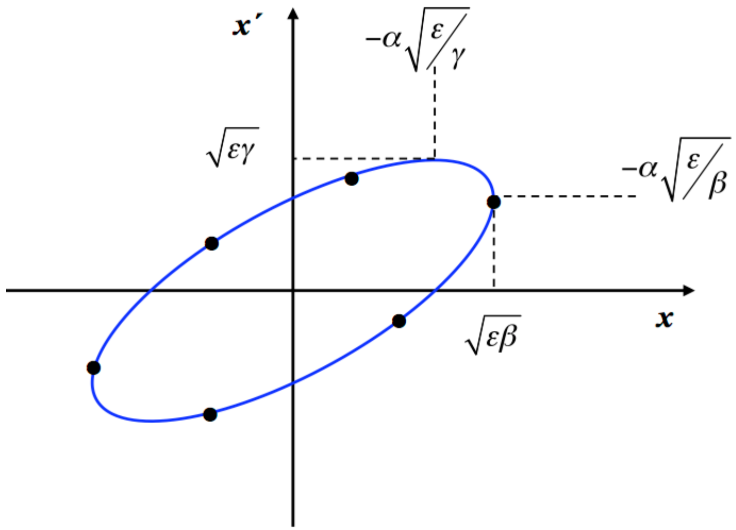




- Insertions
- Synchrotrons – acceleration – longitudinal motion

Reminder from last lecture





$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} \quad M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

Betatron oscillation $$x(s) = \sqrt{\epsilon}\sqrt{\beta(s)}\cos(\psi(s) + \phi)$$

Invariant of motion $$\epsilon = \gamma(s)x(s)^2 + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$

Insertions

Some other equipment needs to be fit into gaps in our regular structure

- RF cavities
- Injection, extraction equipment
- Experiments

Let’s have a look at very simply insertions:

DRIFTS – no focusing, no bending

The transfer matrix in a drift:

$$M_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Transformation of Twiss Parameters

Two positions in your ring: s_0, s

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

ε is constant

$$\varepsilon = \gamma_0 {x_0}^2 + 2\alpha_0 x_0 x_0' + \beta_0 {x_0'}^2$$
$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

Express x_0, x'_0 as function of x, x' using matrix M^{-1} and insert in equation for ε:

$$\varepsilon = \beta_0(m_{11}x' - m_{21}x)^2 + 2\alpha_0(m_{22}x - m_{12}x')(m_{11}x' - m_{21}x) + \gamma_0(m_{22}x - m_{12}x')^2$$

Sort via x, x' and compare coefficients...

Transformation of Twiss Parameters

The new parameters α, β, γ can be transformed through the lattice with the transfer matrix elements

...one can calculate the effect of the change of focusing on the optical parameters.

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{s_1} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21}m_{12} & m_{21} + m_{22}m_{11} & -m_{12}m_{22} \\ m_{12}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_{s_0}$$

Now back to the DRIFT...

With the transfer matrix of the drift

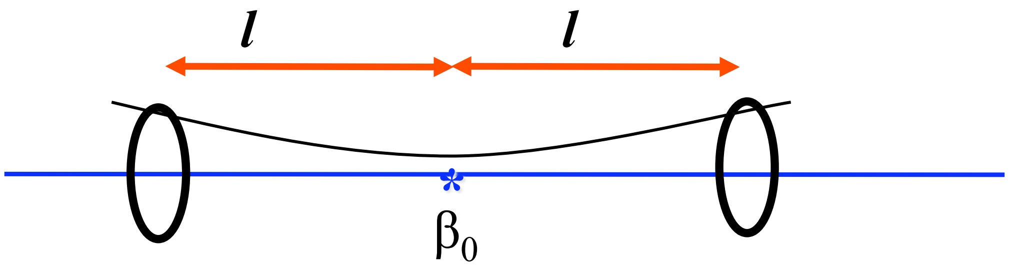
$$M_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

β, α and γ for any position s in the drift transform like

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$
$$\alpha(s) = \alpha_0 - \gamma_0 s$$
$$\gamma(s) = \gamma_0$$

Special drift: Minibeta insertion

Minibeta insertion is a symmetric drift space with a beta waist in the center of the insertion



On each side of the symmetry point a quadrupole doublet or triplet are used to generate the waist.
They are not part of the regular lattice.

E.g. collider experiments are located in minibeta insertions: smallest beam size possible for the colliding beam to increase probability of collisions.

How does the β at the quadrupoles depend on the β_0 (and l)?

At the location of the waist: $\alpha_0 = 0 \rightarrow \gamma_0 = \frac{1+\alpha_0^2}{\beta_0} = \frac{1}{\beta_0}$

Minibeta Insertion

We use this in

$$\beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$

Thus the beta function in the vicinity of the waist behaves like

$$\beta(s) = \beta_0 + \frac{s^2}{\beta_0}$$

- the smaller β_0, the larger the β at the end of the drift
- the larger the insertion, the larger the β at the end of the drift

Example of the LHC (design report values):

$\beta^* = \beta_0 =$ 55 cm

$\sigma^* =$ 16 μm

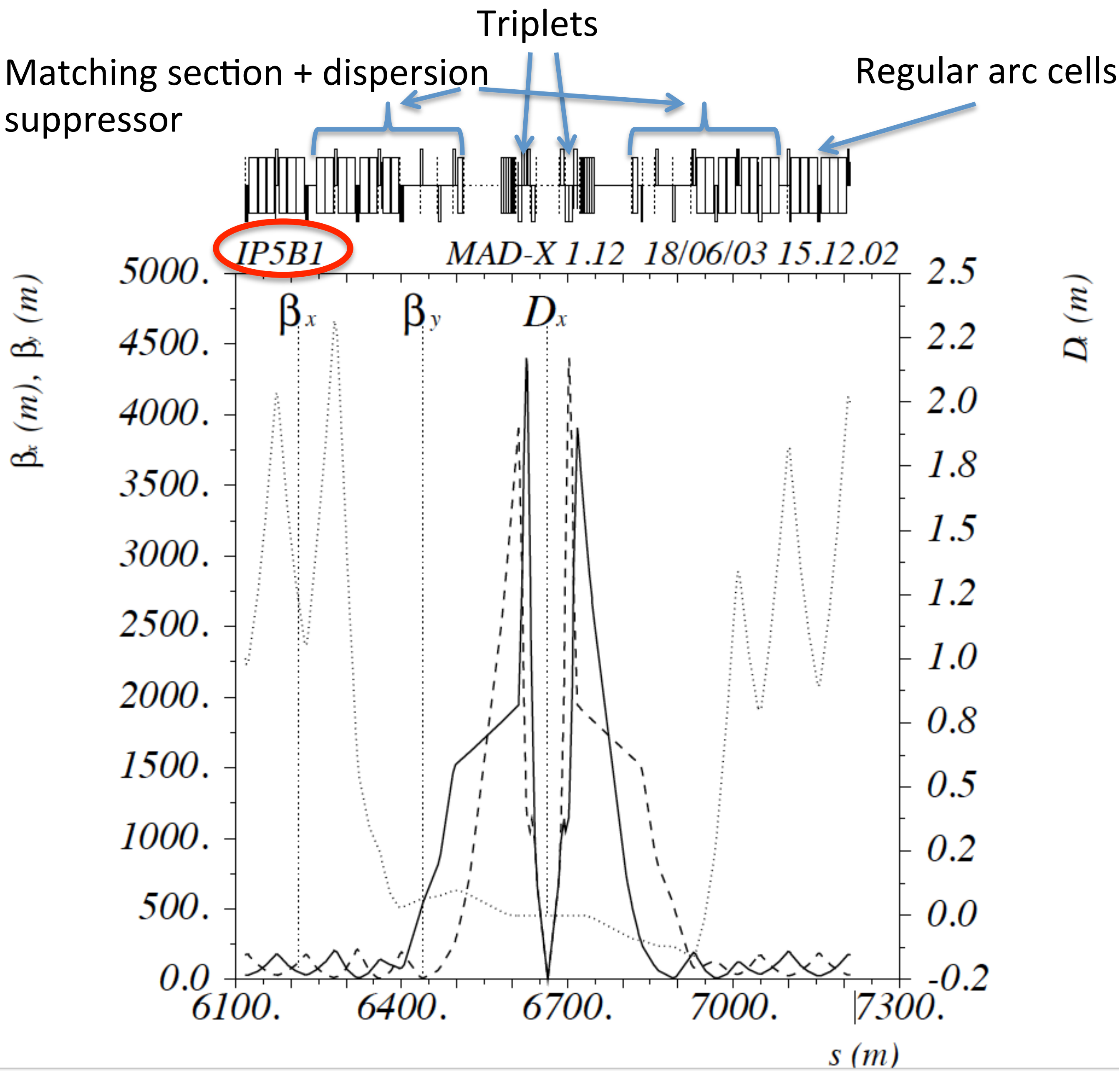
Maximum β in the triplet: 4.5 km

σ = 1.5 mm

Causes many issues:
Tighter tolerances on field errors

Smallest aperture in the machine.
Also, because of crossing angle.

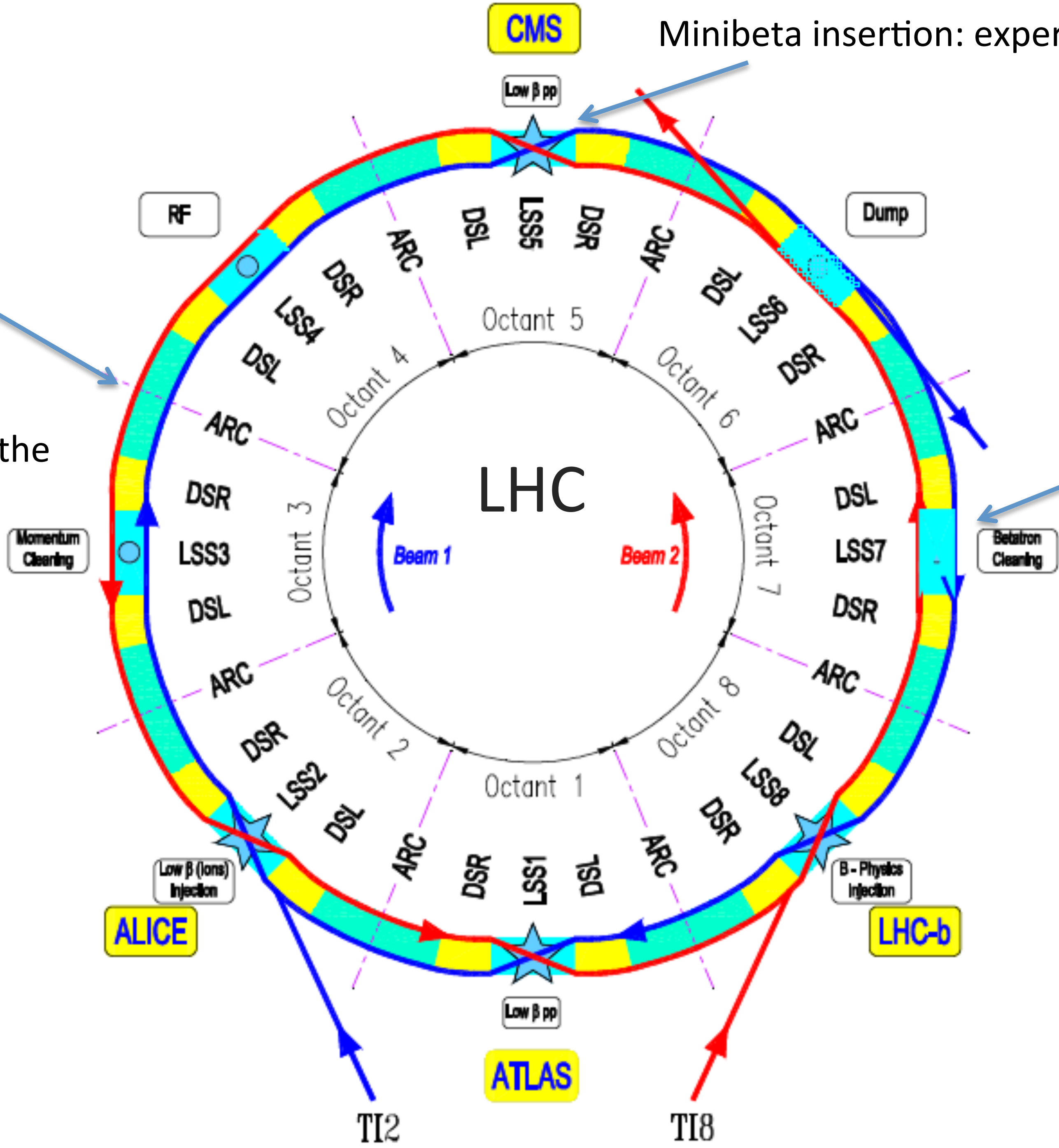
Minibeta insertion – Example LHC



Minibeta insertion: experiment insertion

LHC arc:
Regular FODO
structure

All dipoles,
all QF and QD the
same strength



Insertions:
Long straight
sections

Focusing
adjusted for
particular
optics
requirements

LHC superconducting cavities

8 cavities per beam in 2 cryo-modules per beam.
Can deliver 2 MV per cavity. Accelerating field 5 MV/m
RF frequency: 400 MHz.

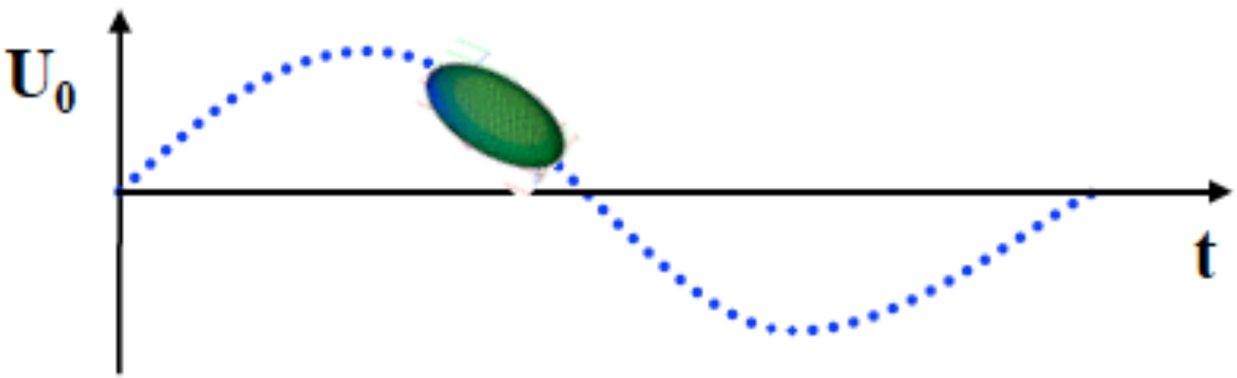
Acceleration

Using RF acceleration: multiple application of the same accelerating voltage.

Brilliant idea to gain higher energies

...but accelerating voltage is changing with time while particles are going through the RF system.

➔ Longitudinal dynamics



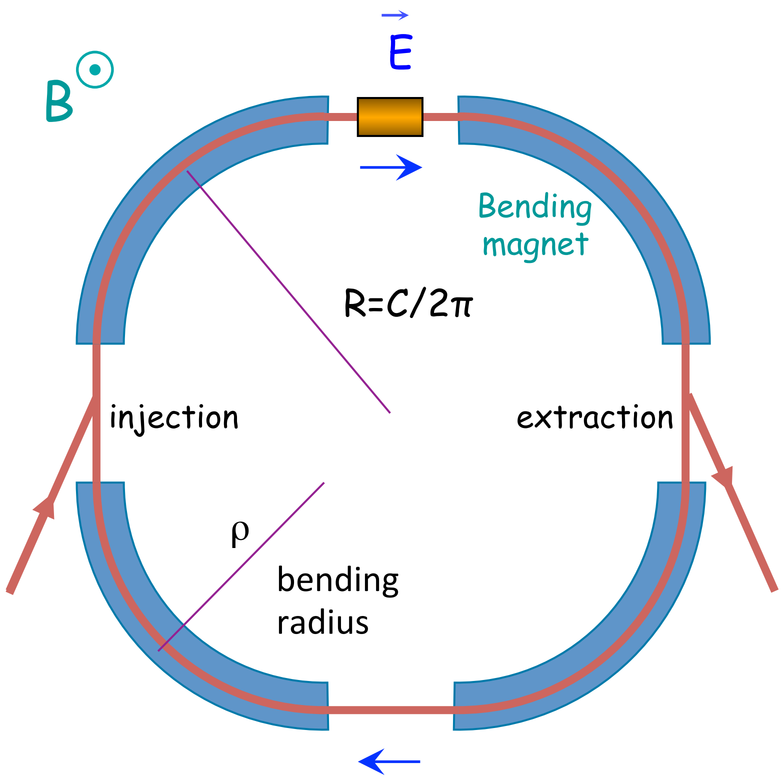
Not all particles arrive at the same time.

Not all particles will receive the same energy gain.

Not all particles will have the same energy.

Acceleration in a Synchrotron

Synchrotron: there is a synchronous RF phase of the RF field for which the energy gain fits the increase of the magnetic field



Energy gain per turn $eV \sin\phi = eV \sin\omega_{RF}t$

Reference particle, synchronous particle $\phi = \phi_s = const$

RF synchronism: the RF frequency must be locked to the revolution frequency $\omega_{RF} = h\omega_{rev}$

h...harmonic number

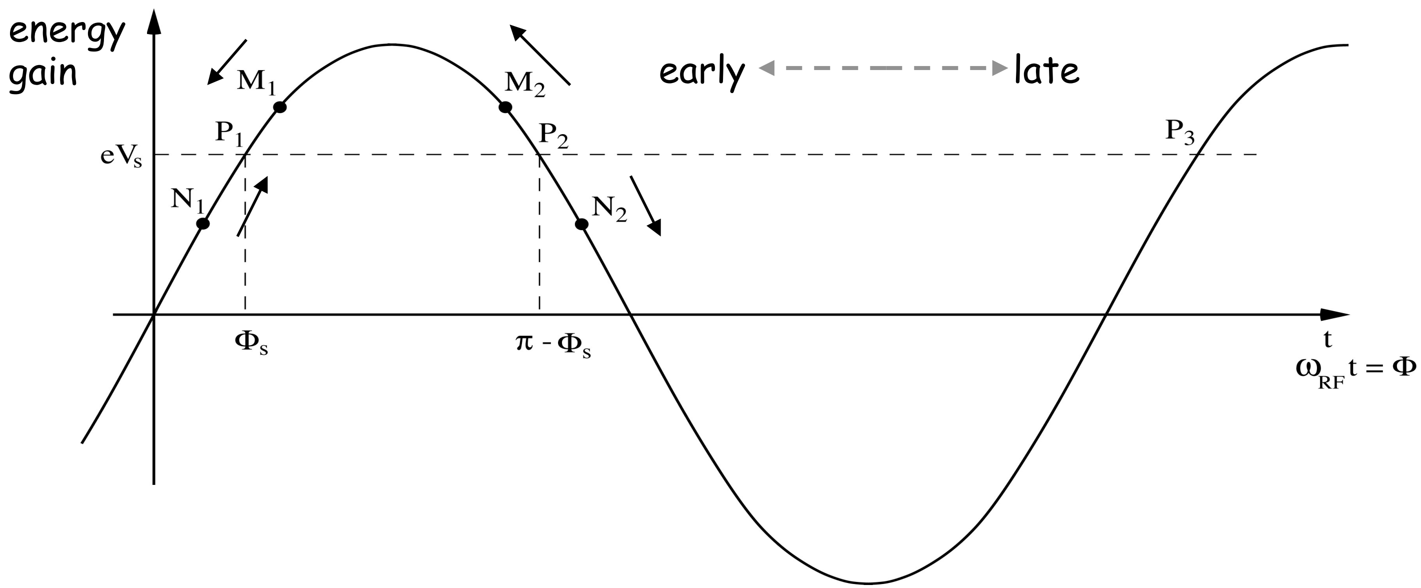
constant orbit, bending radius

variable magnetic field

Principle of Phase Stability

Assume the situation where energy increase is transferred into a velocity increase

Particles P_1, P_2 have the synchronous phase.



M_1 & N_1 will move towards P_1 => stable

M_2 & N_2 will go away from P_2 => unstable (and finally be lost)

Frequency change during the energy ramp

The energy ramp is the phase in the operational cycle of a synchrotron where the energy and hence the B fields change.

The RF frequency also has to change:

$$\frac{f_{RF}(t)}{h} = \frac{v(t)}{2\pi R_s} = \frac{1}{2\pi}\frac{qc^2}{E_s(t)}\frac{\rho}{R_s}B(t)$$

with

$$p(t) = qB(t)\rho$$

$$E = mc^2$$

And with:

$$E^2 = (m_0c^2)^2 + p^2c^2$$

$$\frac{f_{RF}(t)}{h} = \frac{c}{2\pi R_s}\frac{B(t)}{\sqrt{B(t)^2 + \left(\frac{1}{c\rho}\frac{m_0c^2}{q}\right)^2}}$$

For large B(t) (i.e. v ≈ c):

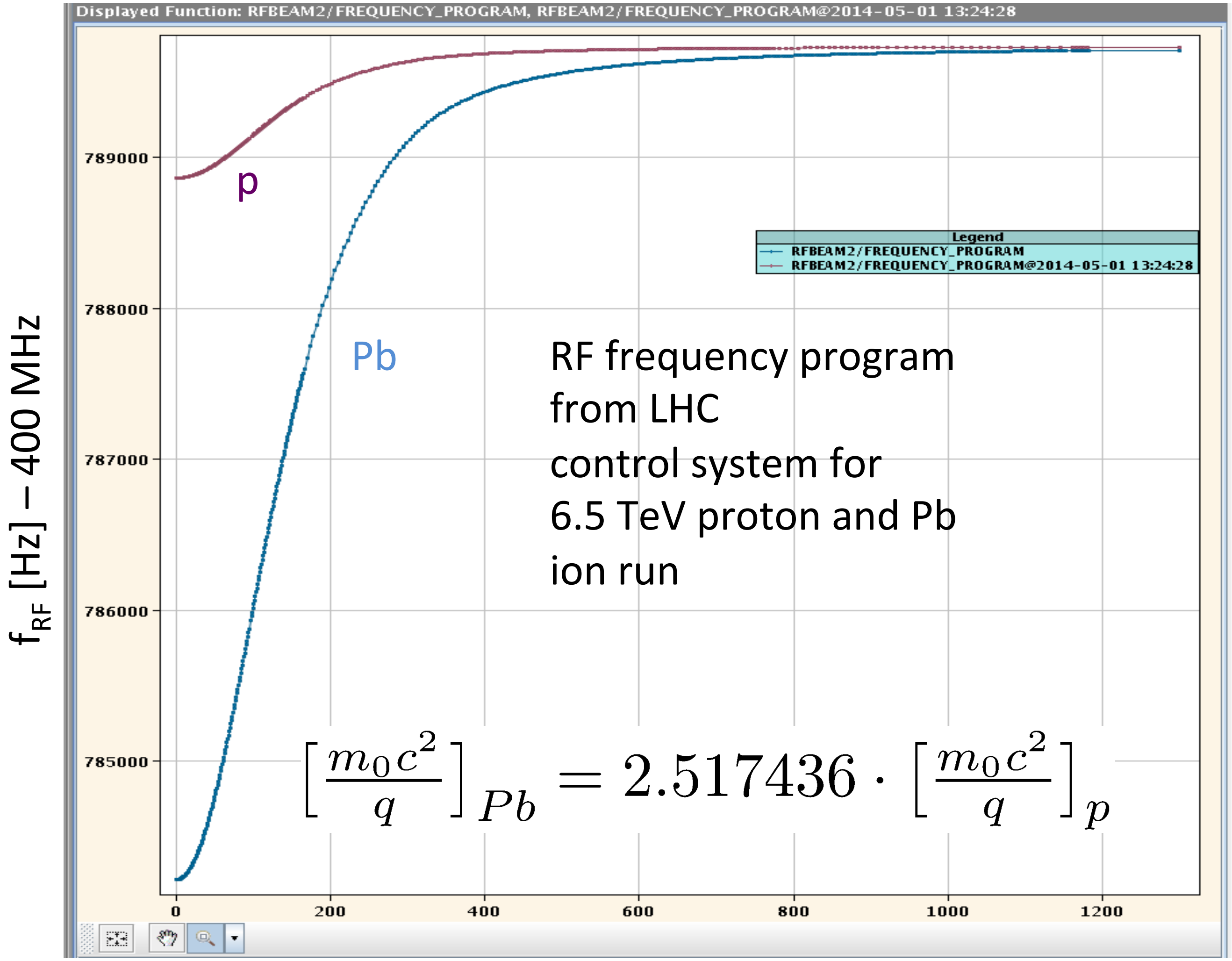
$$f_{RF} \rightarrow \frac{ch}{2\pi R_s}$$

Frequency ramp: LHC p^+ versus Pb^{82+}

The LHC can accelerate protons and heavier ions.

In the past: runs with p^+ and Pb^{82+}

For the ramp of lead ions larger frequency swing



RF voltage and phase during ramp

Energy gain per turn: $\Delta E = eV \sin \phi_s$

with $E^2 = E_0^2 + p^2 c^2 \rightarrow \Delta E = v \Delta p$

and $v = \frac{2\pi R}{T_{turn}}$

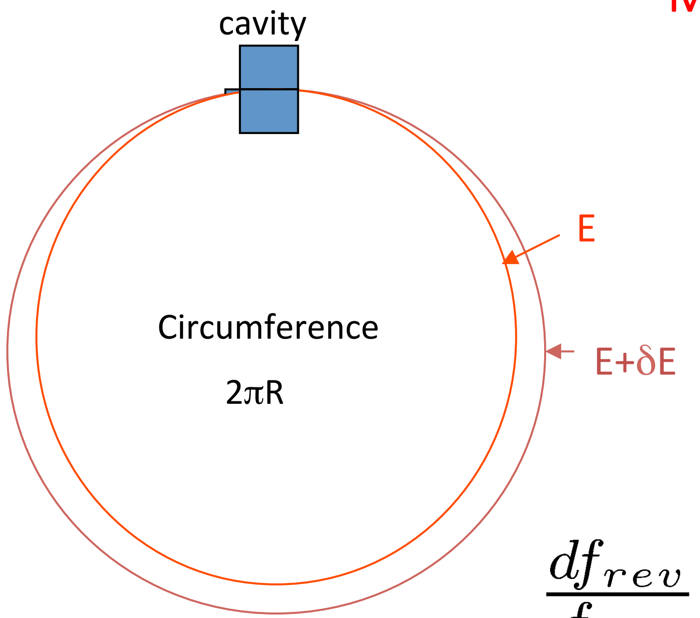
$$2\pi R \frac{dp}{dt} = q \cdot V \cdot \sin \phi_s$$

Stable phase and total supplied RF voltage change during acceleration.

The ramp in the LHC is slow. Takes > 15 minutes. Total energy gain per turn is only about 500 keV (ϕ close to 180°).

Some definitions

If a particle is slightly shifted in momentum, it will run on a different orbit with a different length.



Momentum compaction α: $\alpha = \frac{dL/L}{dp/p}$

The particle will also have a different velocity and hence a different revolution frequency: the slippage factor

$$\eta = \frac{df_{rev}/f_{rev}}{dp/p} = \frac{1}{\gamma^2} - \alpha$$

without prove

$$\frac{df_{rev}}{f_{rev}} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p}$$

$$\gamma_{tr} = \frac{1}{\sqrt{\alpha}}$$

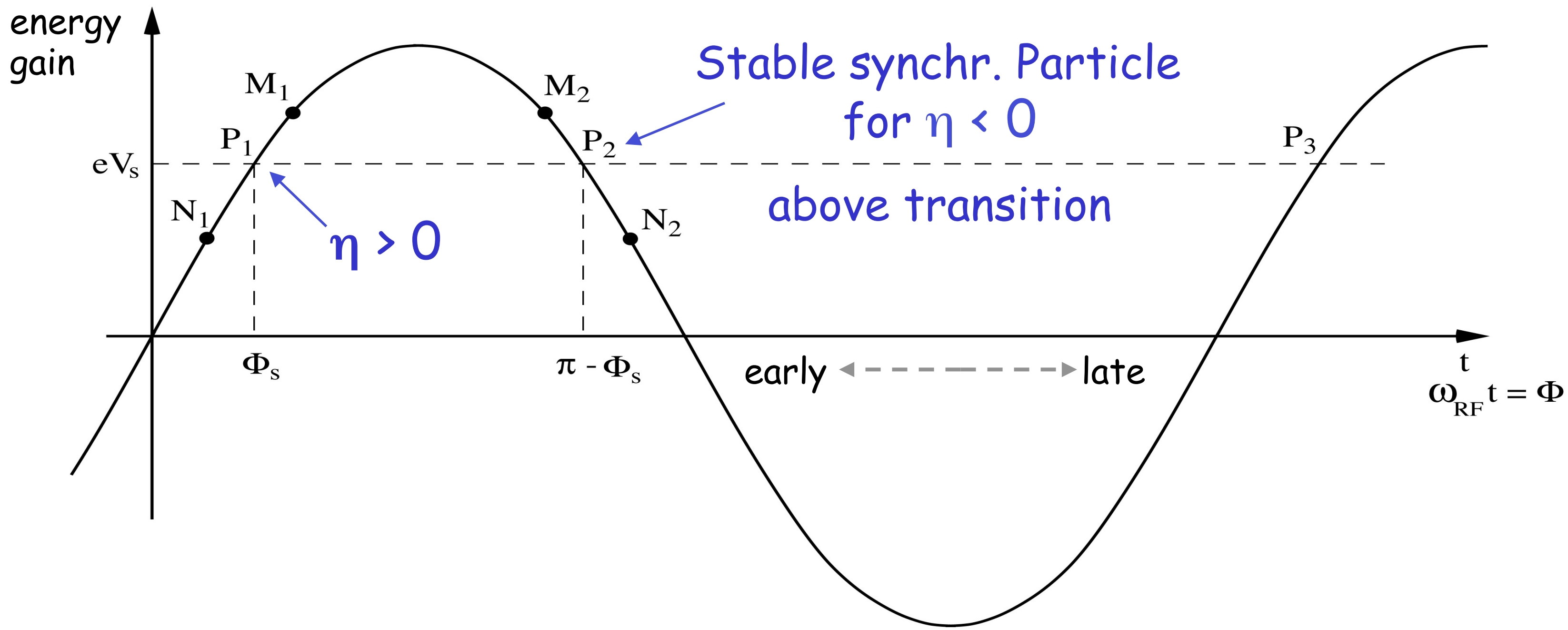
Momentum compaction defines transition energy.

Phase Stability in a Synchrotron

$$\frac{df_{rev}}{f_{rev}} = \eta \frac{dp}{p} = \left(\frac{1}{\gamma^2} - \alpha\right)\frac{dp}{p}$$

Below transition ($\eta > 0$): higher momentum, higher f_{rev}

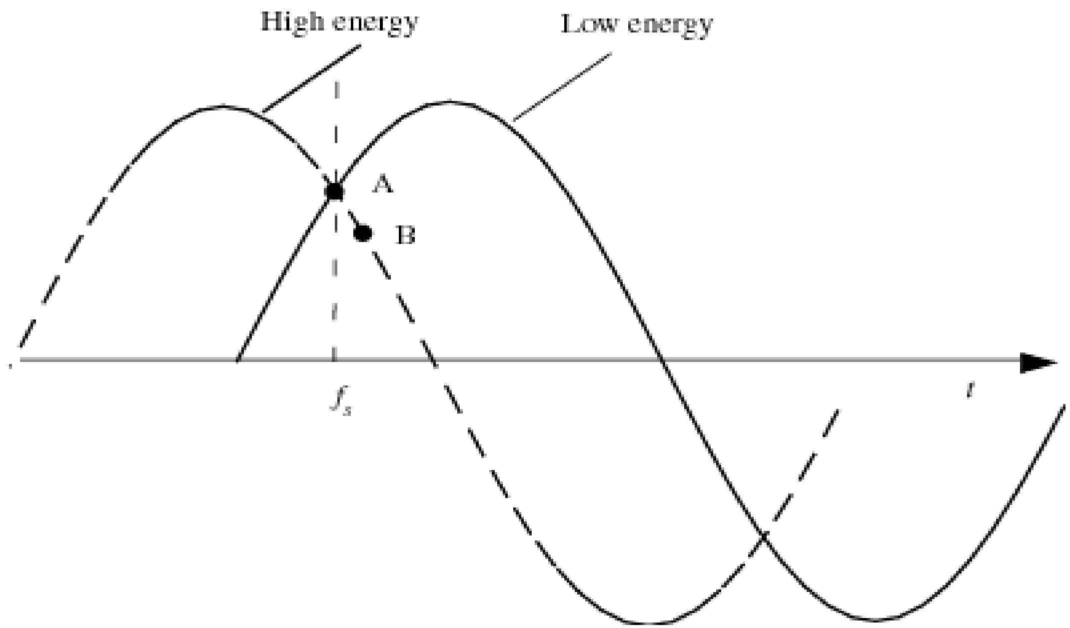
Above transition ($\eta < 0$): higher momentum, lower f_{rev}



Courtesy F. Tecker for drawings

Crossing Transition during Acceleration

Crossing transition during acceleration makes the previously stable synchronous phase unstable. The RF system needs to make a rapid change of the RF phase, a ‘phase jump’



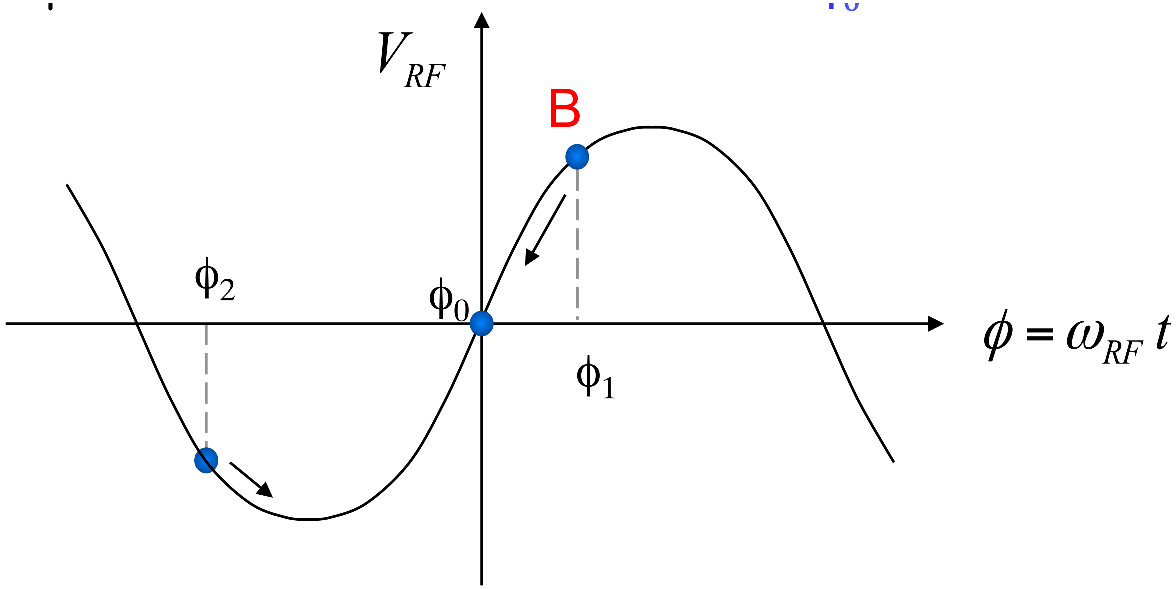
In the LHC transition energy γ_{tr} = 53 GeV. Injection energy is 450 GeV. The LHC is always above transition.

Synchrotron Oscillations

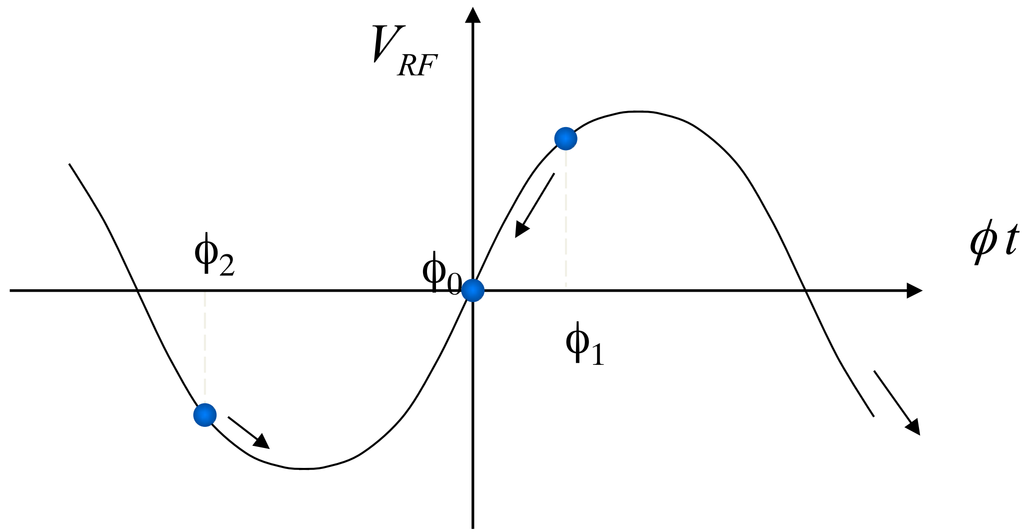
Like in the transverse plane the particles are performing an oscillation in longitudinal space.

Particles keep oscillating around the stable synchronous particle varying phase and dp/p.

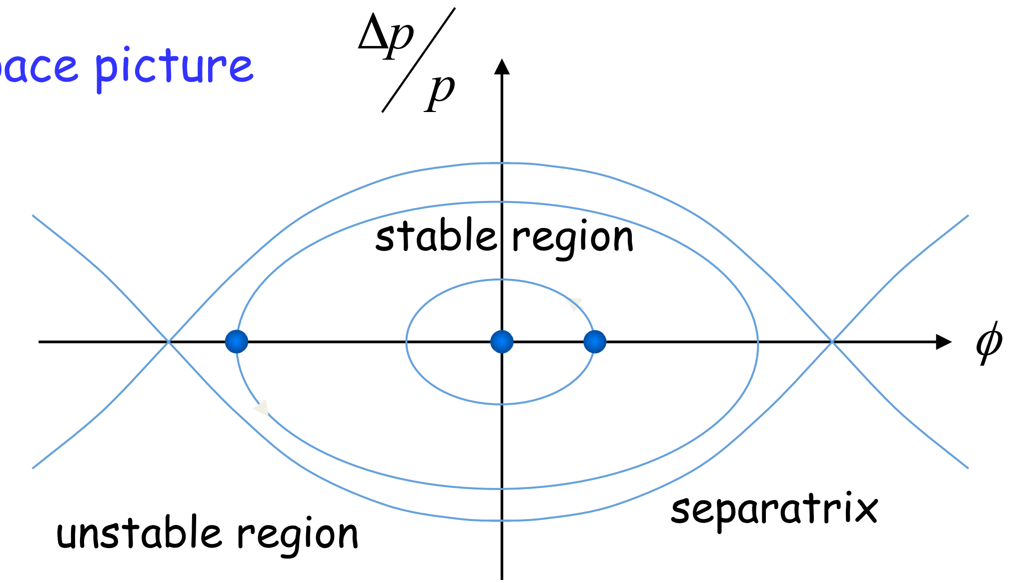
Assume: no acceleration, B = const, below transition $\eta > 0$
Stable phase = 0. B will oscillate around ϕ_0.



Synchrotron Oscillations – No acceleration

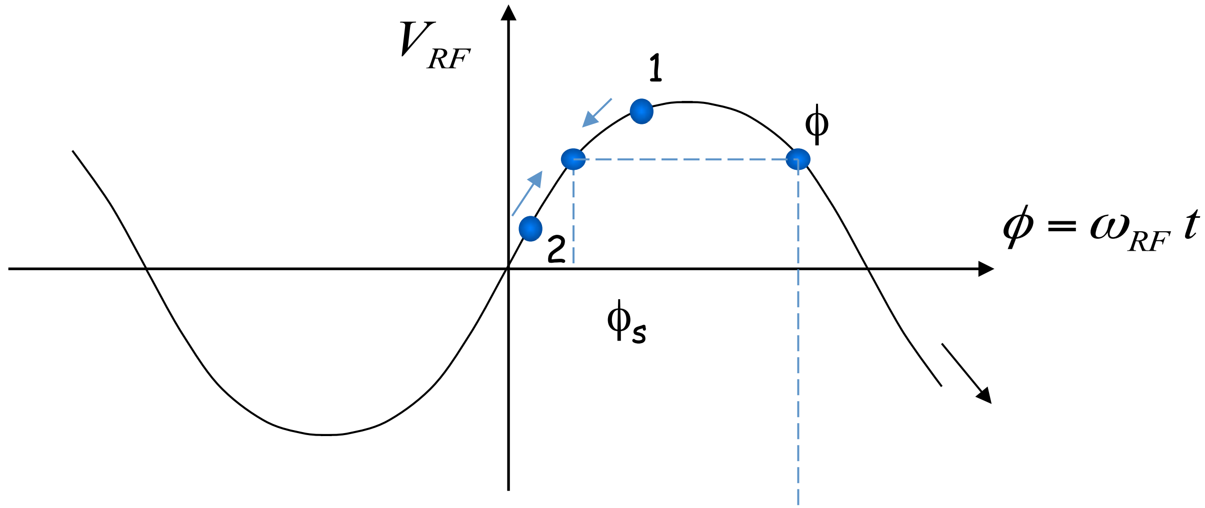


Phase space picture



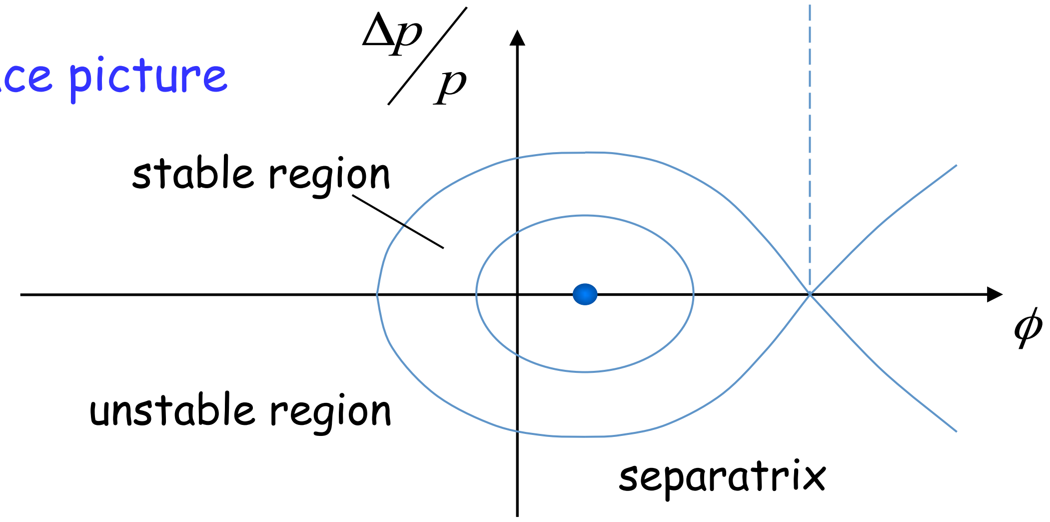
Synchrotron Oscillations – with Acceleration

$\gamma < \gamma_{tr}$



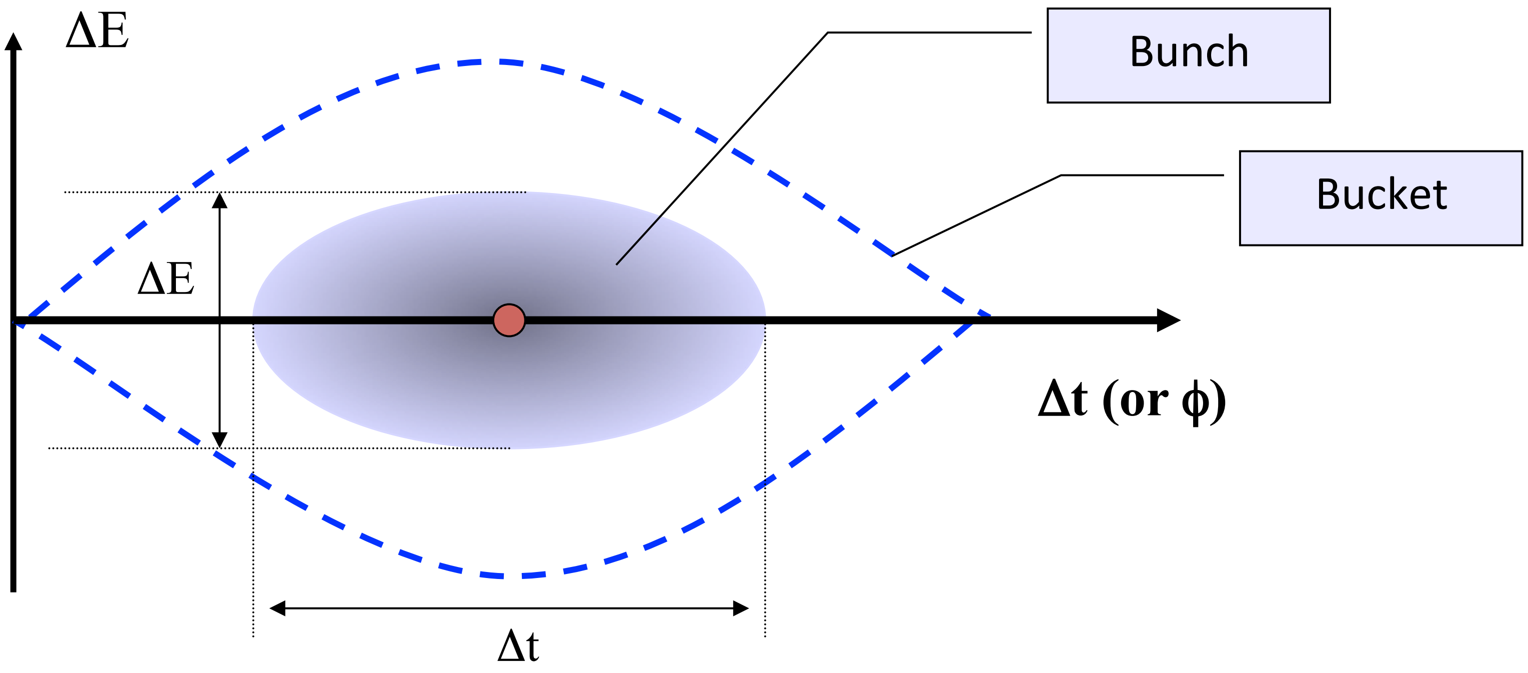
$$\phi_s < \phi < \pi - \phi_s$$

Phase space picture



Bucket & Bunch

The bunches of the beam fill usually a part of the bucket area.

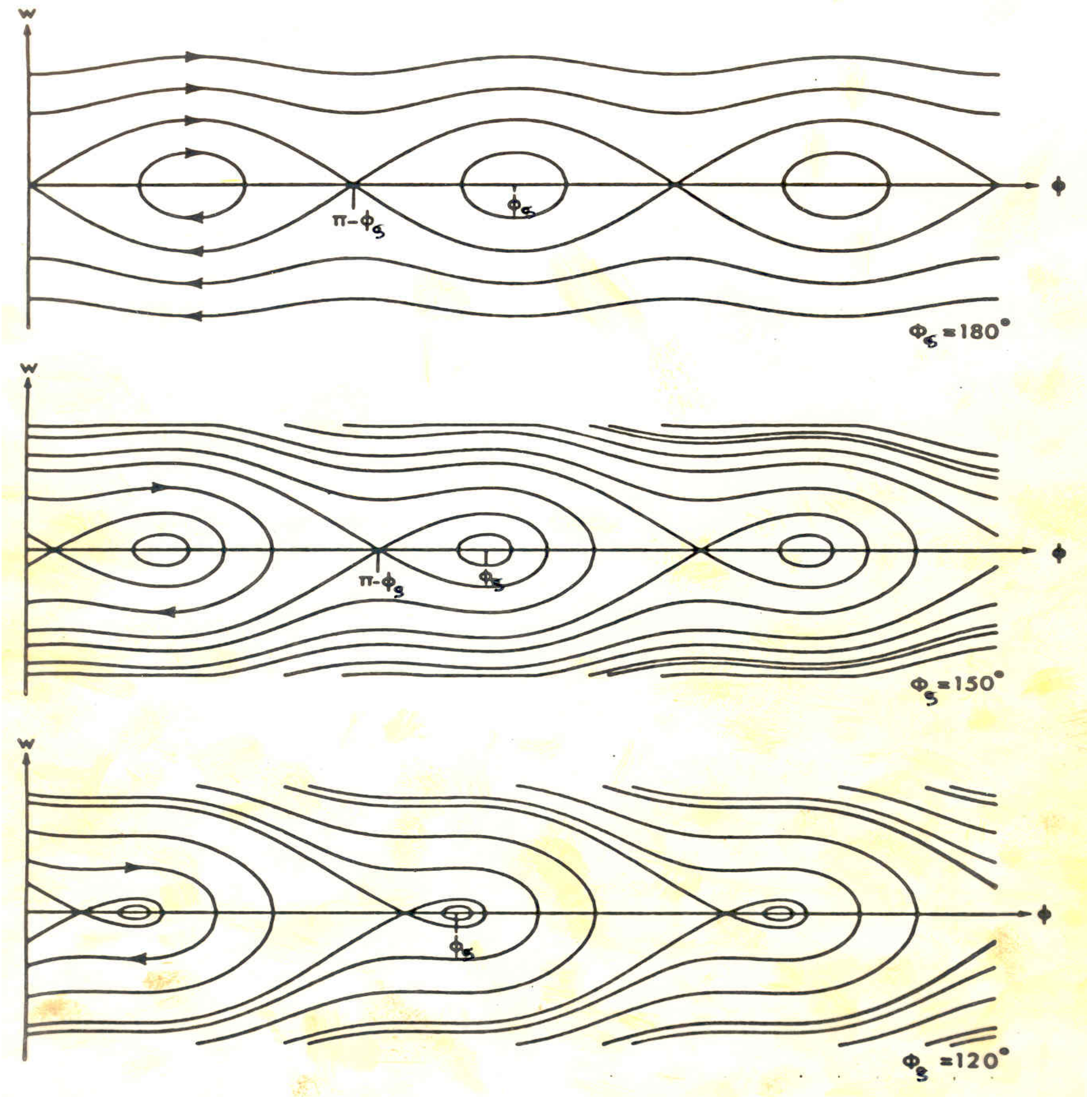


Bucket area = **longitudinal Acceptance** [eVs]

Bunch area = **longitudinal beam emittance** = **$\pi.\Delta E.\Delta t/4$ [eVs]**

The ratio between these two is called filling factor.

RF Acceptance versus Synchronous Phase



RF acceptance plays an important role for capture at injection and the stored beam lifetime

The areas of stable motion (closed trajectories) are called "BUCKET".

As the synchronous phase gets closer to 90° the buckets gets smaller.

The number of circulating buckets is equal to "h".

The phase extension of the bucket is maximum for ϕ_s =180° (or 0°) which correspond to no acceleration . The RF acceptance increases with the RF voltage.

Synchrotron motion

Synchronous particle: (p_s,ϕ_s)

Another particle P: (p,ϕ)

$$\Delta\phi = \phi - \phi_s$$

P will also have a different revolution frequency

$$\frac{d\Delta\phi}{dt} = -2\pi h \Delta f_{rev} \qquad \frac{d^2\Delta\phi}{dt^2} = -2\pi h \frac{d\Delta f_{rev}}{dt}$$

When crossing the cavity, the momentum increase will be different for the two particles:

$$2\pi R \frac{dp_s}{dt} = q \cdot V \cdot \sin\phi_s$$

$$2\pi R \frac{dp}{dt} = q \cdot V \cdot \sin\phi$$

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

Synchrotron motion

Remember:

$$\eta = \frac{df_{rev}/f_{rev}}{dp/p} = \frac{\Delta f_{rev}/f_{rev}}{\Delta p/p_s}$$

Use η in:

$$\frac{d^2\Delta\phi}{dt^2} = -2\pi h \frac{d\Delta f_{rev}}{dt} = -\frac{2\pi\eta h f_{rev}}{p_s}\frac{d\Delta p}{dt}$$

With:

$$2\pi R \frac{d\Delta p}{dt} = q \cdot V \cdot \sin\phi - q \cdot V \cdot \sin\phi_s$$

We get a second-order non-linear differential equation describing the synchrotron motion:

$$\frac{d^2\Delta\phi}{dt^2} + \frac{\eta \cdot f_{RF}}{R \cdot p_s} q \cdot V (\sin\phi - \sin\phi_s) = 0$$

Synchrotron motion

For small amplitude oscillations, where we have small phase deviations from the synchronous particle:

And: $$\sin\phi - \sin\phi_s = \sin(\phi_s + \Delta\phi) - \sin\phi_s \cong \cos\phi_s \Delta\phi$$

We can linearize the equation from above

$$\frac{d^2\Delta\phi}{dt^2} + \left[\frac{\eta f_{RF}\cos\phi_s}{Rp_s} qV\right]\Delta\phi = 0$$

$$\frac{d^2\Delta\phi}{dt^2} + \Omega_s^2\Delta\phi = 0$$

$$\Omega_s = \sqrt{\frac{\eta f_{RF}\cos\phi_s}{Rp_s} qV}$$

An **undamped resonanotor** with **resonant frequency Ω_s** called the synchrotron frequency.

Periodic motion is stable if $\eta.\cos(\phi_s) > 0$:

$$\gamma \leq \gamma_{tr} \Rightarrow \eta \geq 0 \Rightarrow \cos\phi_s \geq 0 \Rightarrow \phi_s \in [0, \pi/2]$$ Acceleration below transition

$$\gamma \geq \gamma_{tr} \Rightarrow \eta \leq 0 \Rightarrow \cos\phi_s \leq 0 \Rightarrow \phi_s \in [\pi/2, \pi]$$ Acceleration above transition

Back to transverse motion

We have heard that not all particles have exactly the same momentum.

In fact a bunch of particles contains a distribution of dp/p.

We have neglected this so far.

The typical momentum spread is in the order of $dp/p < 1.0 \times 10^{-3}$.

What does this do to the transverse motion of the particles?

Without going through the derivation: ➔ inhomogeneous differential equation

$$x'' + x(\frac{1}{\rho^2} - k) = 0$$

$$y'' + ky = 0$$

➔

$$x'' + x(\frac{1}{\rho^2} - k) = \frac{\Delta p}{p_0}\frac{1}{\rho}$$

$$y'' + ky = 0$$

Dispersion

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p_0}\frac{1}{\rho}$$

General solution:

$$x(s) = x_h(s) + x_i(s)$$

$$x_h''(s) + K(s) \cdot x_h(s) = 0 \qquad x_i''(s) + K(s) \cdot x_i(s) = \frac{1}{\rho} \cdot \frac{\Delta p}{p}$$

Define dispersion as:

$$D(s) = \frac{x_i(s)}{\Delta p/p}$$

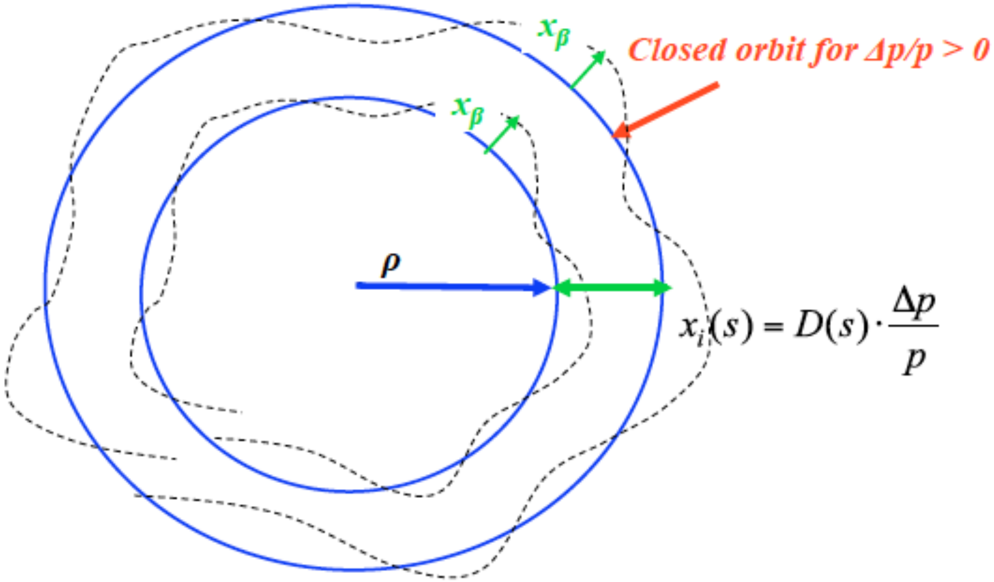
Dispersion is the trajectory an ideal particle would have with $\Delta p/p = 1$.

The trajectory of any particle is the sum of X_β (s) plus dispersion × momentum offset.

D(s) is just another trajectory and will therefore be subject to the focusing properties of the lattice.

Dispersion

Imagine homogenous dipole field all around the ring.



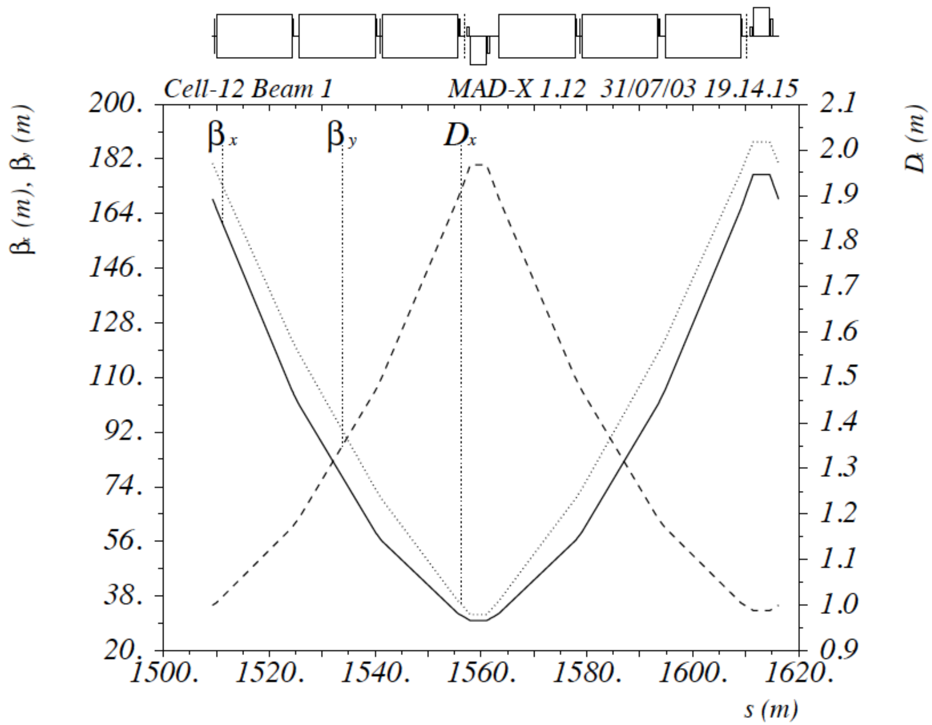
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} \Rightarrow \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}_{s_1}$$

Also has effect on beam size: momentum spread of beam $\Delta p/p$

$$\sigma = \sqrt{\beta\varepsilon} \quad \Rightarrow \quad \sigma = \sqrt{\beta\varepsilon + D^2 (\frac{\Delta p}{p})^2}$$

Dispersion

Dispersion is created by dipole magnets and then focused by quadrupole magnets…



ξ: The contribution to the dispersion from dipoles between s_0 and s_1.

How does dispersion transform:

$$D(s_1) = m_{11_{s_0 \to s_1}} D(s_0) + m_{12_{s_0 \to s_1}} D'(s_0) + \xi$$

With $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$

Dispersion

What is the contribution of a dipole magnet to the dispersion?

The contribution to dispersion from s_0 to s_1 can be calculated with

$$D(s_1) = m_{12}(s_1) \int_{s_0}^{s_1} \frac{1}{\rho} m_{11}(\tilde{s}) d\tilde{s} - m_{11}(s_1) \int_{s_0}^{s_1} \frac{1}{\rho} m_{12}(\tilde{s}) d\tilde{s}$$

In quadrupoles and drifts: $1/\rho = 0$ ➔ no dispersion created.

In dipoles

$$M = \begin{pmatrix} \cos\frac{l}{\rho} & \rho \sin\frac{l}{\rho} \\ -\frac{1}{\rho}\sin\frac{l}{\rho} & \cos\frac{l}{\rho} \end{pmatrix} \quad ➔ \quad \begin{aligned} D(s) &= \xi = \rho \cdot (1 - \cos\frac{l}{\rho}) \\ D'(s) &= \sin\frac{l}{\rho} \end{aligned}$$

Dispersion

As mentioned earlier: dispersion contributes to beam size

$$\sigma = \sqrt{\beta\varepsilon + D^2(\frac{\Delta p}{p})^2}$$

In the LHC: average dispersion $\overline{D} = 1.5m$, $\Delta p/p = 1.1 \times 10^{-4}$

Beam size at experiment: $\sigma^*_\beta = 16\mu m$

including dispersion $\sigma^*_{\beta+D} = 165\mu m$!!!

At the experiments: keep beam size as small as possible

No contribution from dispersion ➔ "dispersion suppressor" cells

Dispersion suppressor: special cells in lattice to cancel/reduce dispersion in an insertion.

Minibeta insertion – Example LHC