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The Standard Model of particle physics

Michael Krämer (RWTH Aachen University)

Bibliography

- I.J.R. Aitchison and A.J.G. Hey, "Gauge Theories in Particle Physics", IoP Publishing.
- R.K. Ellis, W.J. Stirling and B.R. Webber, "QCD And Collider Physics," Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 8 (1996) 1.
- D. E. Soper, Basics of QCD perturbation theory, arXiv:hep-ph/0011256.
- Lectures by Keith Ellis, Douglas Ross, Adrian Signer, Robert Thorne and Bryan Webber (thanks!).

The Standard Model Lagrangian is determined by symmetries

- space-time symmetry: global Poincaré-symmetry
- internal symmetries: local $SU(n)$ gauge symmetries

$$\mathcal{L}_{\rm SM} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\not{D}\psi \qquad \text{gauge sector}$$

$$+|DH|^2 - V(H) \qquad \text{EWSB sector}$$

$$+\psi_i\lambda_{ij}\psi_j H + \mathrm{h.c.} \qquad \text{flavour sector}$$

$$+N_i M_{ij} N_j \qquad \nu\text{-mass sector}$$

Outline

- QED and QCD as gauge theories
- QCD for the LHC
- Breaking gauge symmetries:

  the Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism

QCD as an SU(3) gauge theory

We start with the Dirac Lagrangian for a free quark q,

$$\mathcal{L} = \bar{q}\,(i\gamma^{\mu}\partial_{\mu} - m)\,q\,,$$

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Recall that $SU(3)$ is the group of special unitary transformations, i.e. the group of all 3×3 unitary matrices with determinant one. To specify an $SU(3)$ transformation, one needs $3^2 - 1 = 8$ real parameters, so we can write

$$e^{-i\omega^a T^a}$$

where the $\omega^a, a \in \{1, ..., 8\}$ are real parameters, and the T^a are called generators of the group. [If you are unfamiliar with the concept of a group generator, you can think of the T^a as traceless, hermitian 3×3 matrices.]

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$SU(3)$ is a non-abelian group, i.e. its generators and thus its elements do not commute,

$$[T^a, T^b] = if^{abc}\,T^c \neq 0 \quad \text{and} \quad e^{-i\omega_1^a T^a} e^{-i\omega_2^b T^b} \neq e^{-i\omega_2^b T^b} e^{-i\omega_1^a T^a}\,.$$

The $SU(3)$ transformations act on the quark fields, so q carries an index i, with $i \in \{1, ..., 3\}$:

$$q \to \left(e^{-i\omega^a T^a}\right) q \quad \text{or} \quad q_i \to \left(e^{-i\omega^a T^a}\right)_i^{\ j} q_j$$

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The $SU(3)$ invariant Lagrangian then becomes

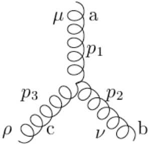
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} + \bar{q}^i \left(i\gamma^\mu D_\mu - m\right)_i^j q_j$$

with $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c$.

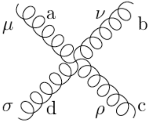
Because of the gauge symmetry, the gluon is massless.

The interactions of QCD follow from gauge invariance:

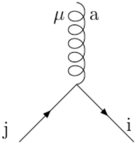
$$\mathcal{L}_{\text{interaction}} = g\, A^a_\mu\, \bar{q}\gamma^\mu T^a q - g\, f^{abc}(\partial_\mu A^a_\nu)A^{b\,\mu}A^{c\,\nu} - g^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{d\,\mu} A^{e\,\nu} \; :$$



$$-g\, f^{abc}\left(g_{\mu\nu}\,(p_1 - p_2)_\rho + g_{\nu\rho}\,(p_2 - p_3)_\mu + g_{\rho\mu}\,(p_3 - p_1)_\nu\right)$$



$$\begin{aligned}&-i\, g^2 f^{eab} f^{ecd}\,(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\\ &-i\, g^2 f^{eac} f^{ebd}\,(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho})\\ &-i\, g^2 f^{ead} f^{ebc}\,(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})\end{aligned}$$

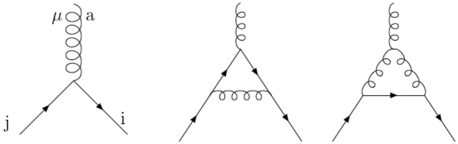


$$-i\, g\, \gamma^\mu\, (T^a)_{ij}$$

The QCD coupling

Consider a dimensionless physical observable R, e.g. the ratio of two cross sections, evaluated at some large energy scale Q. If $Q \gg m$, one can set $m \to 0$, and dimensional analysis implies that R should be independent of Q.

This is not true in quantum field theory. Quantum fluctuations change the value of the effective coupling:



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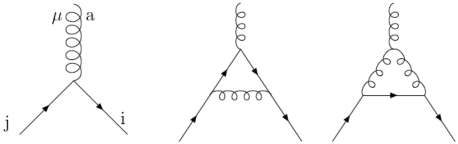

The calculation of R as a perturbation series in the coupling $\alpha_s \equiv g/4\pi$ requires renormalization to remove ultraviolet contributions. This introduces a second mass scale μ – the point at which the UV contributions are subtracted. Thus

$$R = R(Q^2/\mu^2, \alpha_s(\mu^2))\,.$$

However, a physical observable must not depend on the scale μ, i.e.

$$\mu^2 \frac{d}{d\mu^2} R(Q^2/\mu^2, \alpha_s(\mu^2)) = \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right] R = 0 \,.$$

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Introducing

$$t = \ln\left(\frac{Q^2}{\mu^2}\right), \quad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

we have

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The β-function has a perturbative expansion and can be extracted from an explicit calculation of higher-order loop-corrections to propagators and vertices.

The running of the coupling at one-loop is thus determined from

$$\frac{\partial \alpha_s(Q^2)}{\partial t} = \beta(\alpha_s(Q^2)) \quad \text{and} \quad \beta(\alpha_s) = -b\alpha_s^2$$

which yields

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\, b \ln(Q^2/\mu^2)} \quad \text{with} \quad b = \frac{33 - 2n_f}{12\pi}\,.$$

For $n_f \leq 16$ the QCD coupling decreases with increasing Q^2. This is the famous property of asymptotic freedom.

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Note that in QED one finds $b = -1/3$ so that the QED coupling

$$\alpha_{\rm QED}(Q^2) = \frac{\alpha_{\rm QED}(\mu^2)}{1 - \frac{\alpha_{\rm QED}(\mu^2)}{3\pi} \ln(Q^2/\mu^2)}$$

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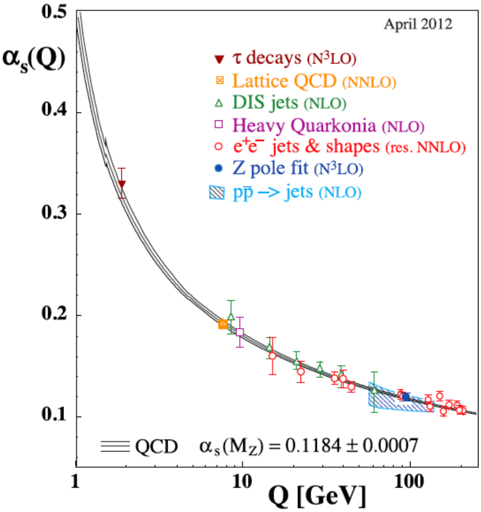
The running QCD coupling

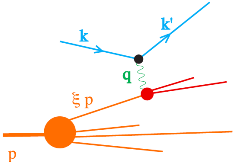


non-perturbative $\longleftrightarrow$ perturbative

Deeply inelastic scattering

Consider the scattering of a high-energy charged lepton off a proton target.

In the parton model we imagine the proton, or any other hadron, to be made of point-like constituents, the partons. The photon scatters from a point-like quark with fraction ξ of the proton's momentum.



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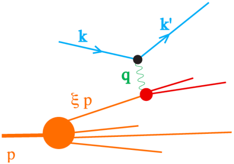
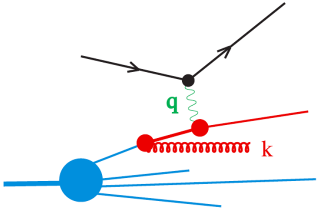


The parton model leads to an intuitive formula that relates the lepton-hadron cross section to the cross section for the electron-parton scattering:

$$\frac{d\sigma^{(lh)}}{dxdQ^2} = \sum_a \int_0^1 d\xi \, f_{a/h}(\xi) \, \frac{d\sigma^{(la)}}{dxdQ^2},$$

where $d\sigma^{(lh)}$ is the inclusive cross section for lepton-nucleon scattering, while $d\sigma^{(la)}$ is the parton-electron cross section, with the parton's momentum given by ξp, ξ between zero and one, and $f_{a/h}(\xi)$ is a parton distribution function.

In QCD, beyond the lowest order approximation, we have to include diagrams like



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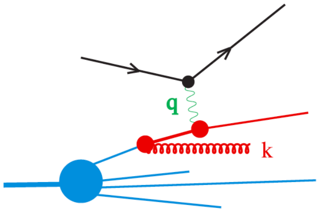

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The limit $k_T \to 0$ corresponds to a long-range part of QCD which is not calculable in perturbation theory. However, there is a factorisation theorem which states that the long-range contributions can be absorbed in the parton distribution functions.

Separating short- and long-distance physics requires the introduction of a factorisation scale μ_F.

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation

The parton distribution function can be defined in terms of quark and gluon field operators. They are universal, i.e. independent of the particular hard scattering process. Pdfs could, in principle, be calculated using lattice QCD, but currently they are determined from experiment.

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The dependence of the parton distribution function on the renormalisation scale μ_F is determined by the DGLAP equation:

$$\frac{d}{d\ln\mu_F} f_{a/h}(x,\mu_F) = \sum_b \int_x^1 \frac{d\xi}{\xi}\ P_{ab}(x/\xi, \alpha_s(\mu_F))\ f_{b/h}(\xi,\mu_F).$$

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The splitting function P_{ab} has a perturbative expansion

$$P_{ab}(x/\xi, \alpha_s(\mu_F)) = P_{ab}^{(1)}(x/\xi) \; \frac{\alpha_s(\mu_F)}{\pi} + P_{ab}^{(2)}(x/\xi) \; \left(\frac{\alpha_s(\mu_F)}{\pi}\right)^2 + \cdots .$$

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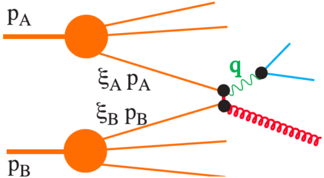
The DGLAP-equation is one of the most important equations in perturbative QCD.

Hadron-hadron collisions

In hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



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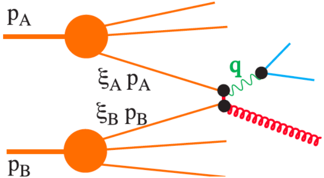
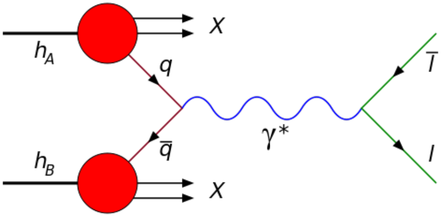


The cross section for a hard scattering process initiated by two hadrons with momenta p_A and p_B takes a factored form similar to that found for deeply inelastic scattering

$$d\sigma(p_A,p_B) = \sum_{a,b}\int d\xi_A d\xi_B\ f_{a/A}(\xi_A,\mu_F)\ f_{b/B}(\xi_B,\mu_F) \times d\hat{\sigma}_{ab}(\xi_A p_A,\xi_B p_B,\mu_F).$$

Hadron-hadron collisions

Historically, the most convincing evidence that the quark-parton-model provides the correct framework for high-energy processes in general came from its success in describing the Drell-Yan process (1971).



This was the birth of quantitative hadron collider phenomenology.

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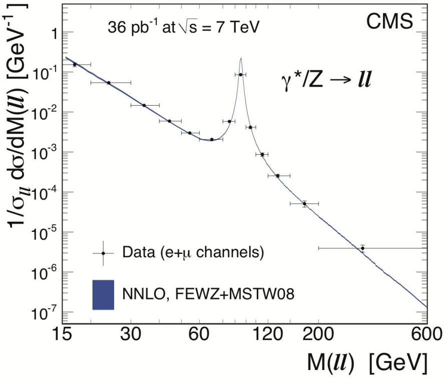

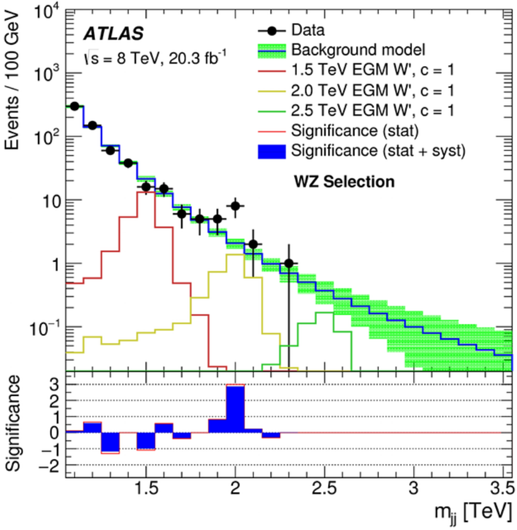
ATLAS
√s = 8 TeV, 20.3 fb-1
Data
Background model
1.5 TeV EGM W', c = 1
2.0 TeV EGM W', c = 1
2.5 TeV EGM W', c = 1
Significance (stat)
Significance (stat + syst)
WZ Selection
Events / 100 GeV
Significance
m_jj [TeV]
1.5
2
2.5
3
3.5