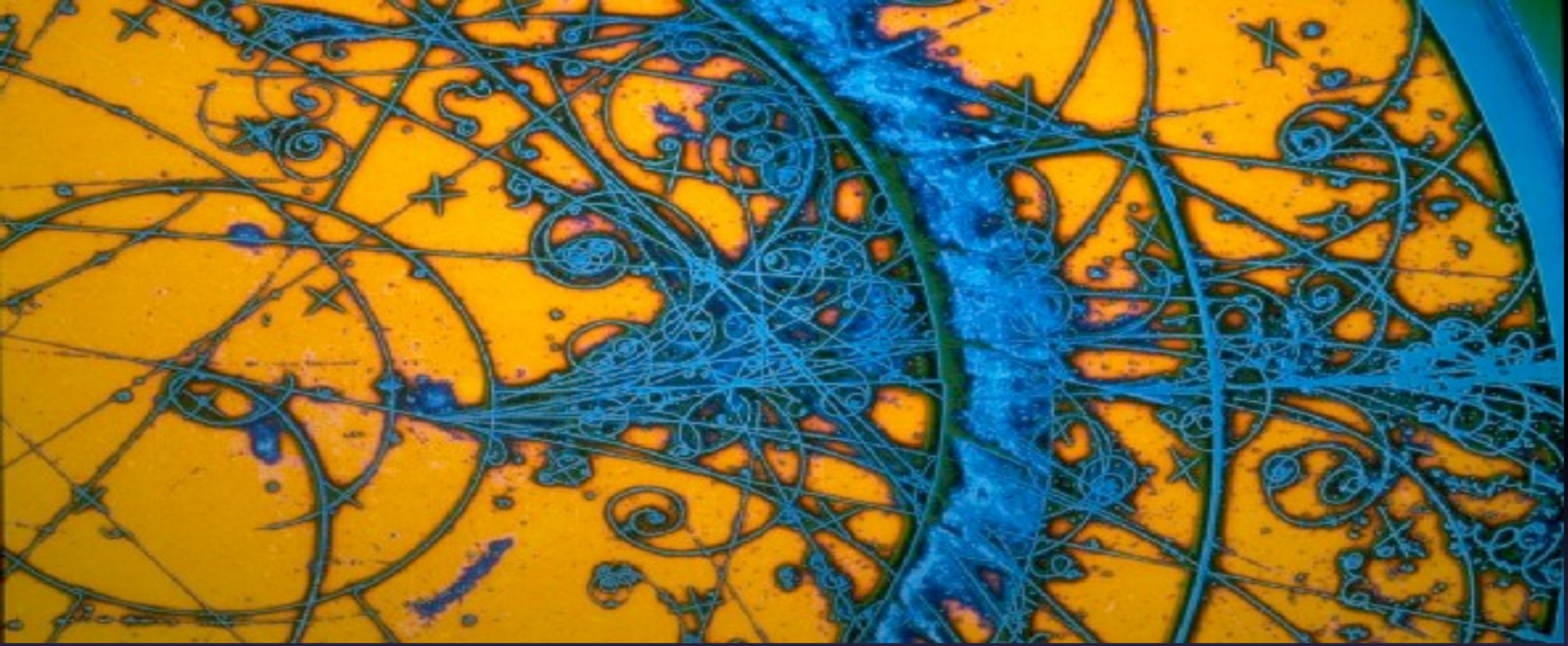


Detectors for Particle Physics

Interaction with Matter

D. Bortoletto

University of Oxford

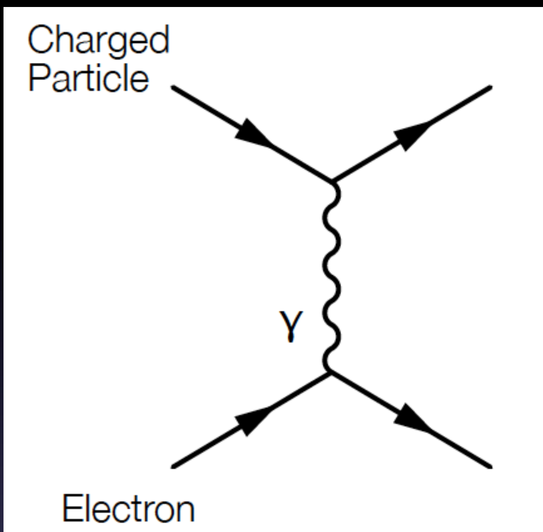
Detecting particles

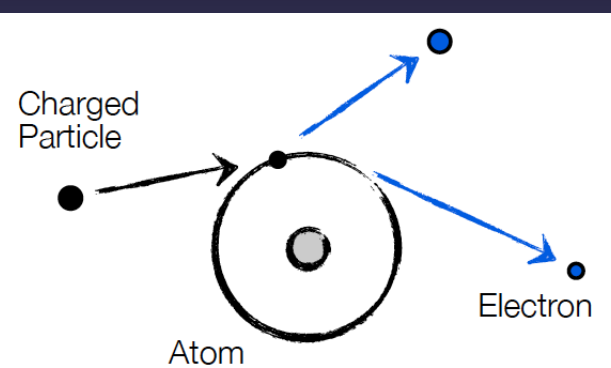
- Every effect of particles or radiation can be used as a working principle for a particle detector.

Claus Grupen

Example of particle interactions

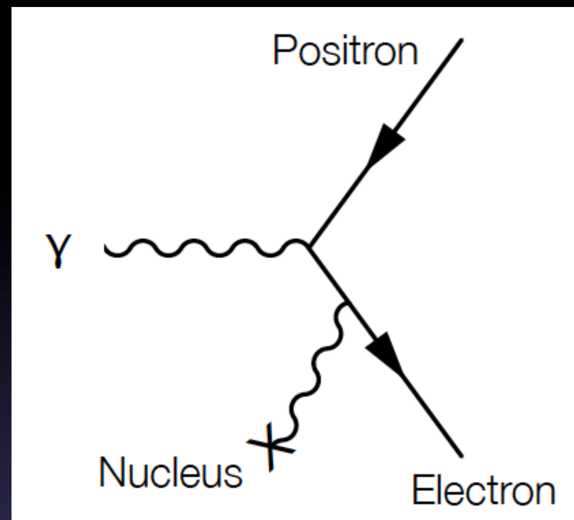
- Ionization

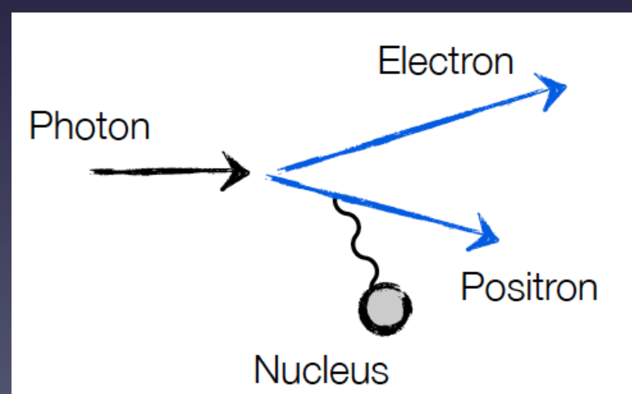




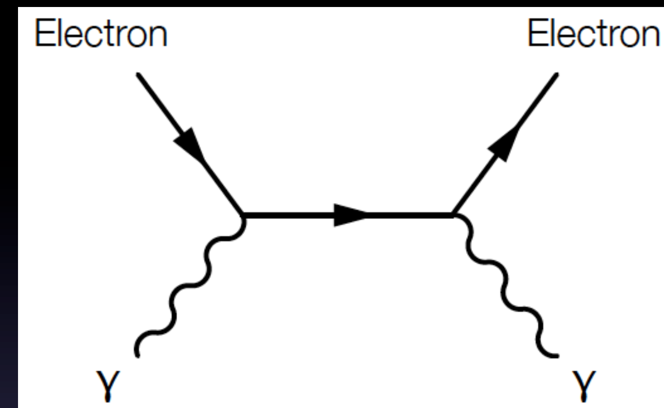
Delta-electrons

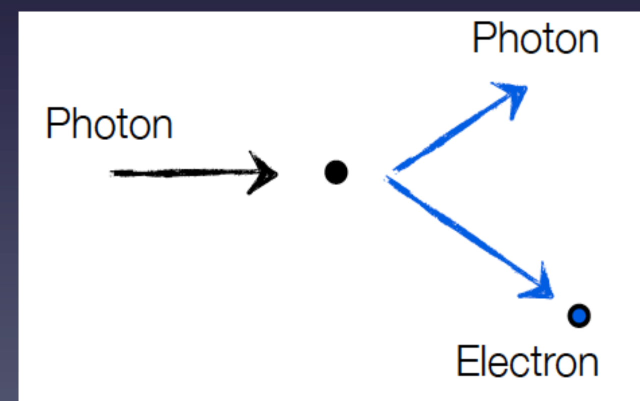
- Pair production



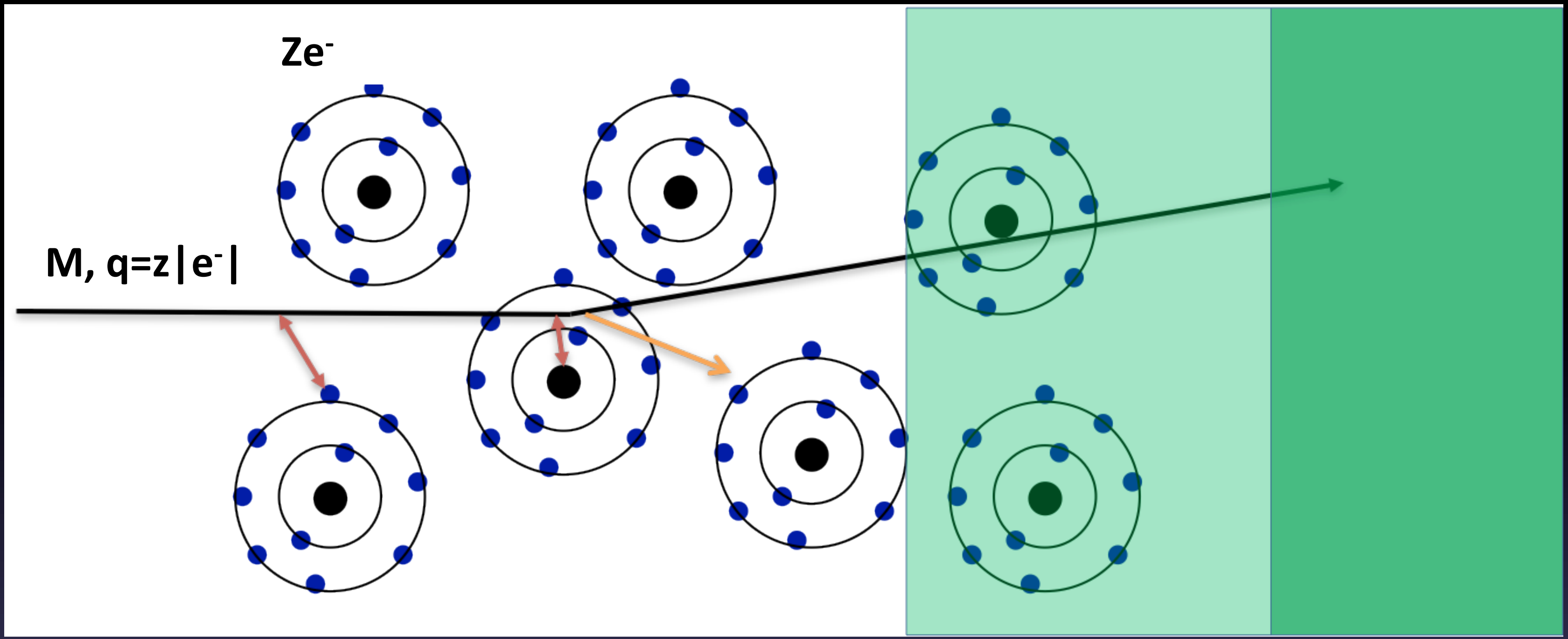


- Compton scattering





EM interaction of charged particles with matter



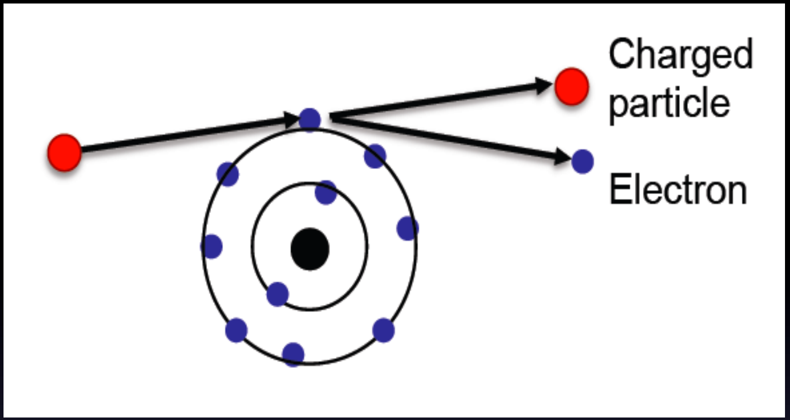
Interaction with the atomic electrons. Incoming particles lose energy and atoms are excited or ionized.

Interaction with the atomic nucleus. Particles are deflected and a Bremsstrahlung photon can be emitted.

If the particle's velocity is > the velocity of light in the medium → Cherenkov Radiation.
When a particle crosses the boundary between two media, there is a probability ≈1% to produce an X ray photon Transition radiation.

Energy Loss by Ionization

- Assume: $Mc^2 \gg m_e c^2$ (calculation for electrons and muons are more complex)
- Interaction is dominated by elastic collisions with electrons
  - The trajectory of the charged particle is unchanged after scattering
- Energy is transferred to the electrons



Energy loss (- sign)

Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Classical derivation in backup slides agrees with QM within a factor of 2

$\propto 1/\beta^2 \cdot \ln(\mathrm{const} \cdot \beta^2 \gamma^2)$

Energy loss by ionization

- The Bethe-Bloch equation for energy loss

Valid for heavy charged particles ($m_{incident} >> m_e$), e.g. proton, k, π, μ

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I^2} W_{max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

$2\pi N_a r_e^2 m_e c^2$ = 0.1535 MeV cm^2/g

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a \beta^2 \gamma^2\right)$$

Fundamental constants

r_e=classical radius of electron

m_e=mass of electron

N_a=Avogadro's number

c =speed of light

$$r_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2}$$

Absorber medium

I = mean ionization potential

Z = atomic number of absorber

A = atomic weight of absorber

ρ = density of absorber

δ = density correction

C = shell correction

Incident particle

z = charge of incident particle

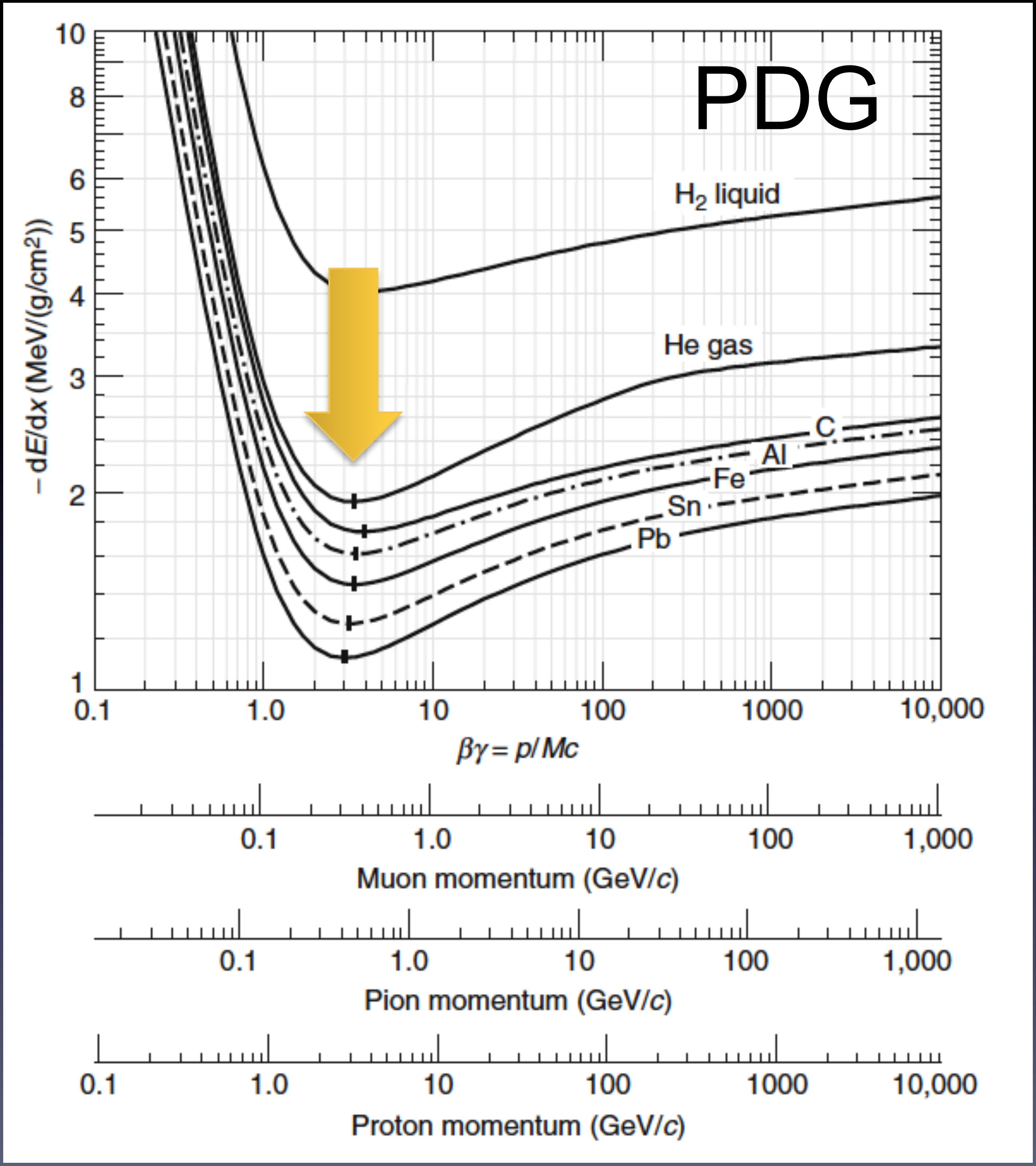
β = v/c of incident particle

γ = $(1-\beta^2)^{-1/2}$

W_{max} = max. energy transfer in one collision

The Bethe-Bloch Formula

- Common features:
  - fast growth, as $1/\beta^2$, at low energy
  - wide minimum in the range $3 \leq \beta\gamma \leq 4$,
  - slow increase at high $\beta\gamma$.
- A particle with d**E**/d**x** near the minimum is a minimum-ionizing particle or mip.
- The mip's ionization losses for all materials except hydrogen are in the range 1-2 MeV/(g/cm^2)
  - increasing from large to low Z of the absorber.

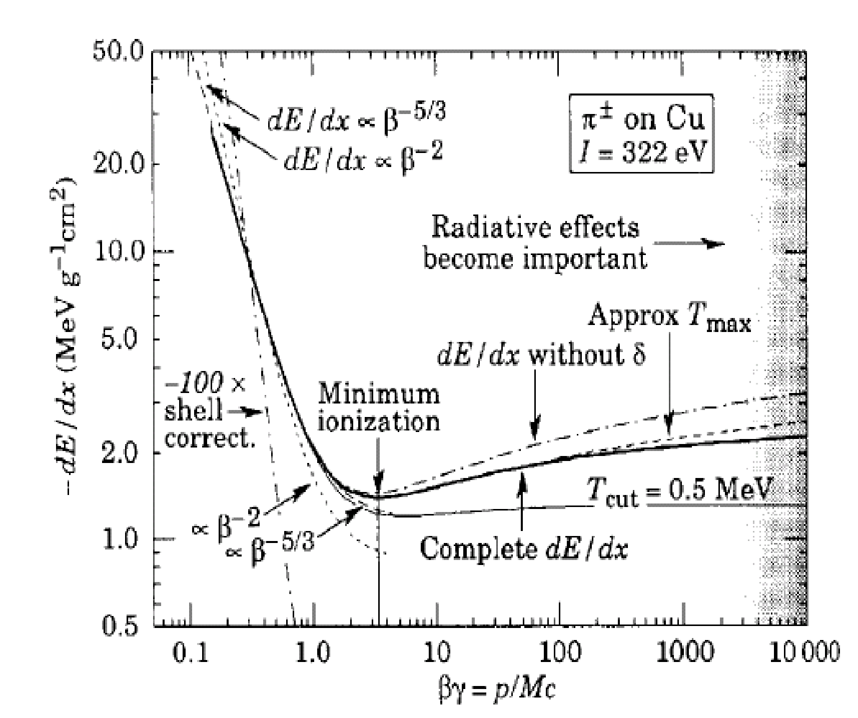


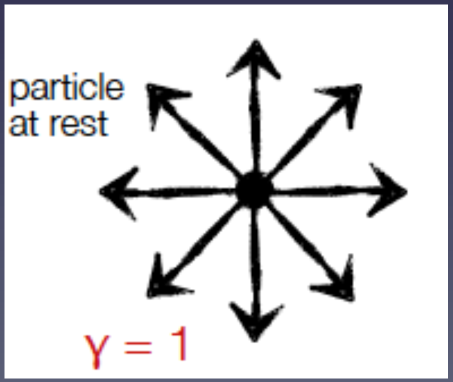
Understanding Bethe-Bloch

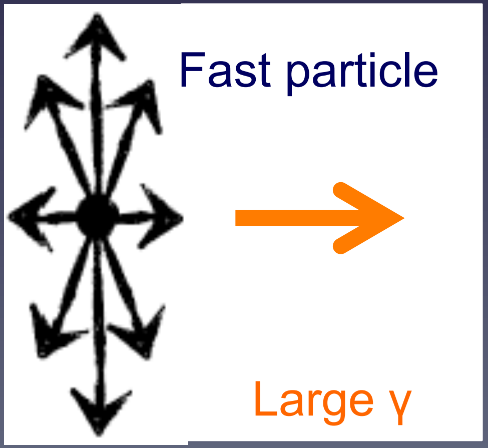
- dE/dx falls like $1/\beta^2$ [exact dependence $\beta^{-5/3}$]
  - Classical physics: slower particles "feel" the electric force from the atomic electron more

$$\Delta p_{\perp} = \int F_{\perp} dt = \int F_{\perp} \frac{dt}{dx} dx = \int F_{\perp} \frac{dx}{v}$$

- Relativistic rise as $\beta\gamma>4$
  - Transversal electric field increases due to Lorentz boost



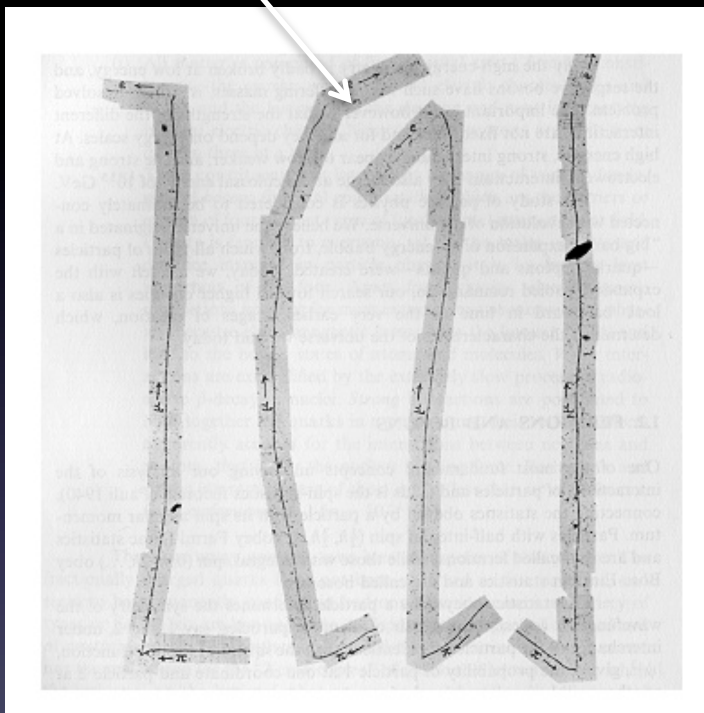




- Shell corrections
  - if particle v ≈ orbital velocity of electrons, i.e. $\beta c \sim v_e$. Assumption that electron is at rest breaks down → capture process is possible .
- Density effects due to medium polarization (shielding) increases at high γ

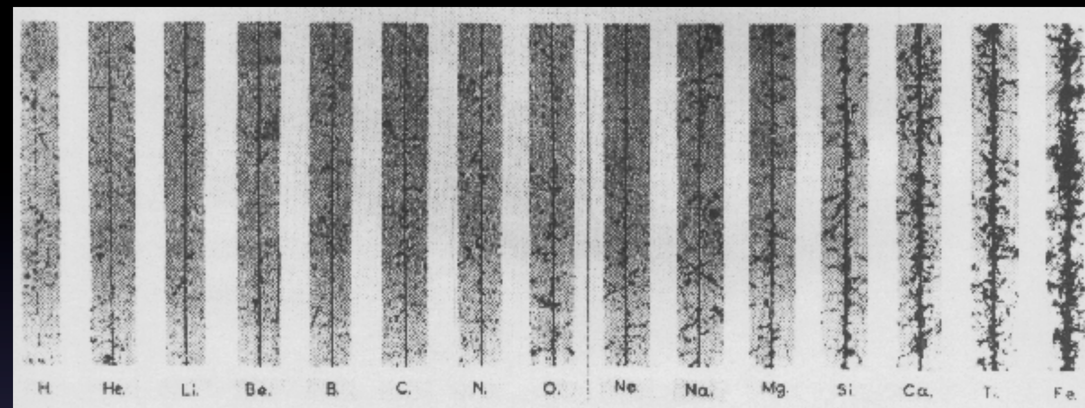
Understanding Bethe-Bloch

Small energy loss
→ Fast Particle



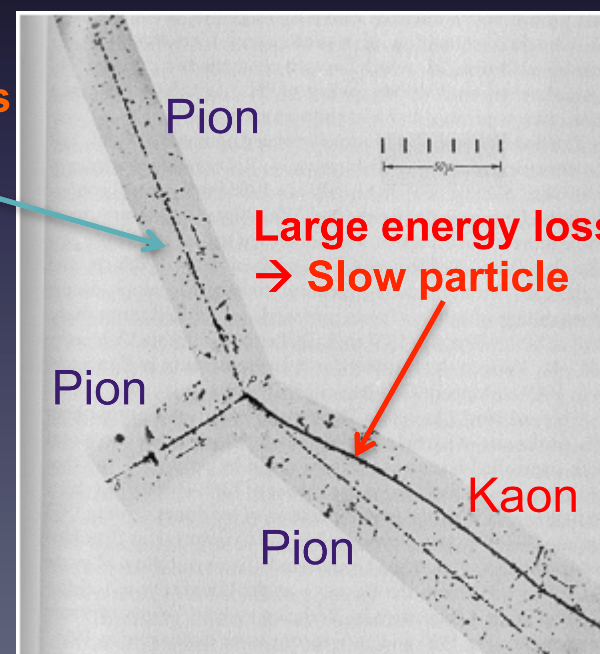
Discovery of muon and pion

Cosmic rays: dE/dx≈z^2



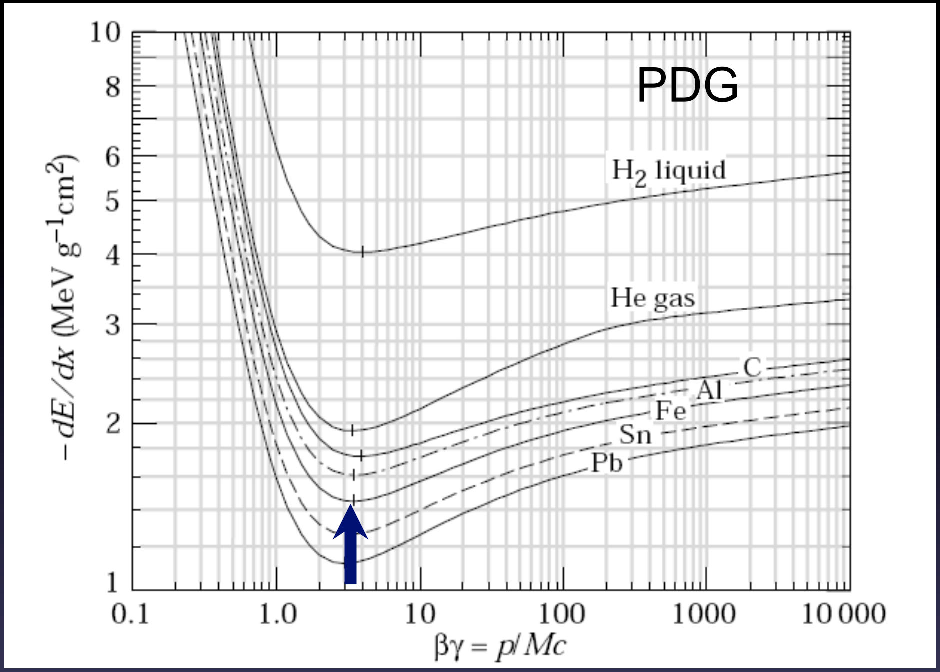
Small energy loss
→ Fast particle

Large energy loss
→ Slow particle



Bethe-Bloch: Order of magnitude

- For Z ≈ 0.5 A
  - 1/ρ dE/dx ≈ 1.4 MeV cm ^{2}/g for βγ ≈ 3
- Can a 1 GeV muon traverse 1 m of iron ?
  - Iron: Thickness = 100 cm; ρ = 7.87 g/cm ^{3}
  - dE ≈ 1.4 MeV cm ^{2}/g × 100 cm ×7.87g/cm ^{3}= 1102 MeV
  - This is only an average value
- dE/dx must be taken in consideration when you are designing an experiment

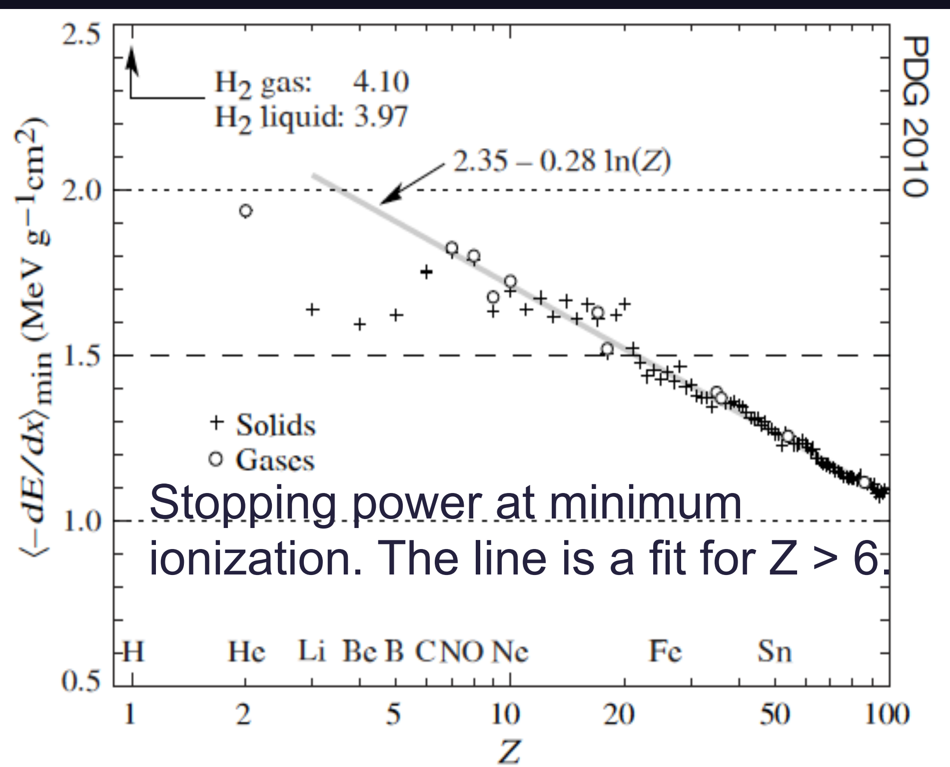


This number must be multiplied with ρ [g/cm ^{3}] of the Material → dE/dx [MeV/cm]

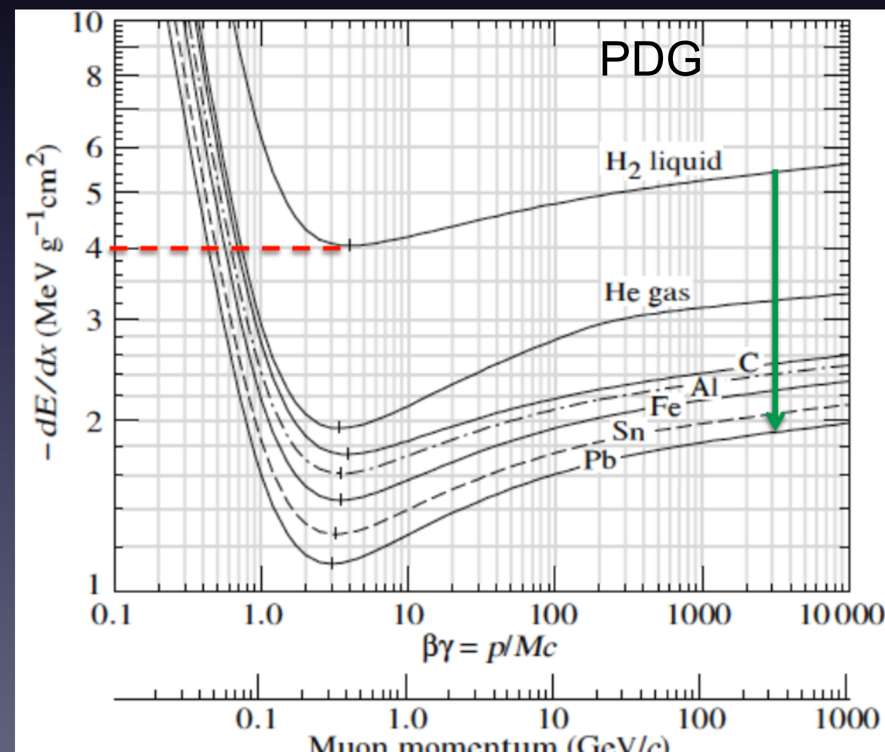
Bethe-Bloch dependence on Z/A

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2\beta^2\gamma^2}{I^2}W_{max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z}\right]$$

- Minimum ionization ≈ 1 - 2 MeV/g cm^{-2}. For H_2: 4 MeV/g cm^{-2}
- Linear decrease as a function of Z of the absorber



Stopping power at minimum ionization. The line is a fit for Z > 6.

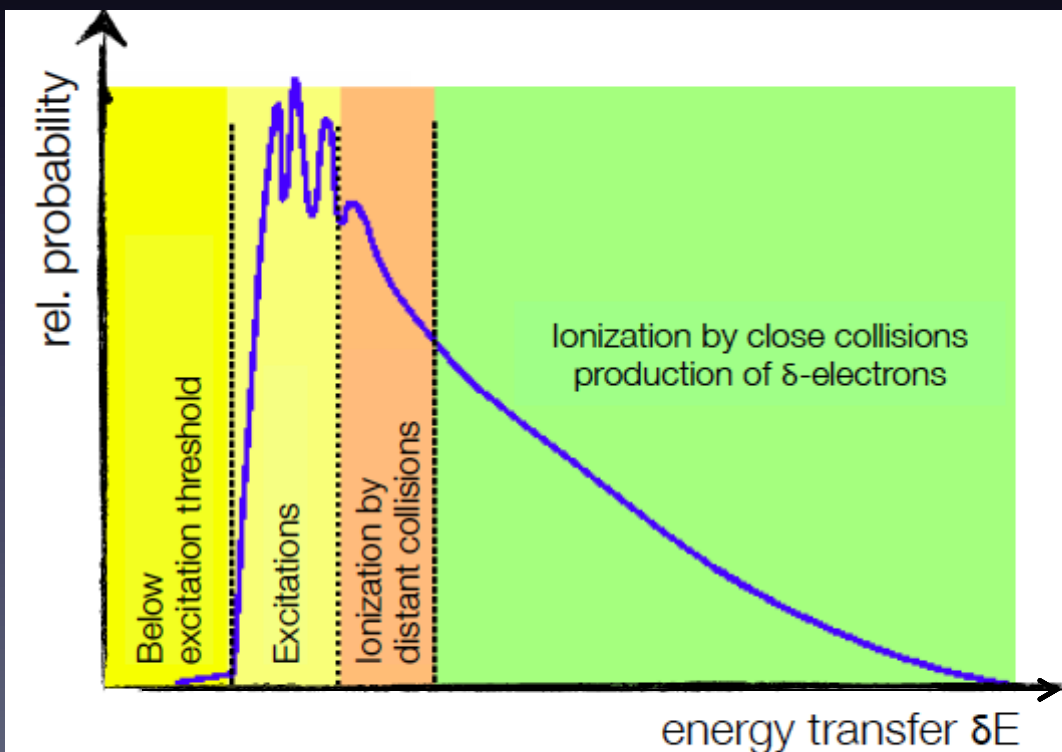


dE/dx Fluctuations

- The statistical nature of the ionizing process results in large fluctuations of energy loss (Δ) in absorbers which are thin compared with the particle range.

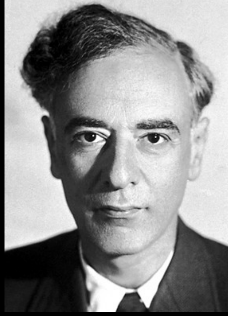
$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N= number of collisions
δE=energy loss in a single collision



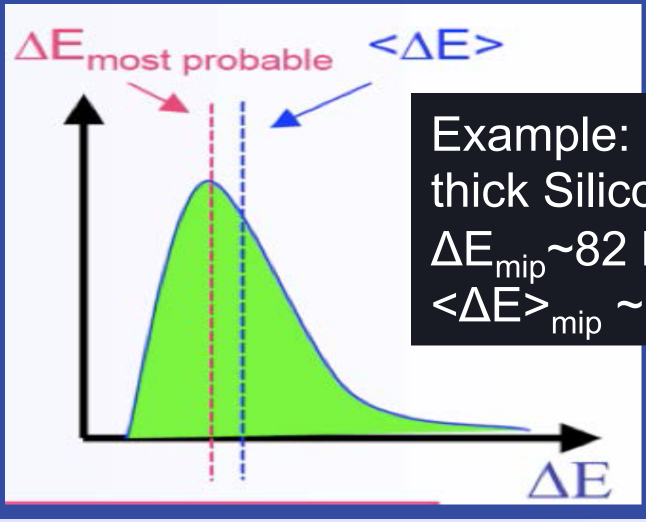
- Ionization loss is distributed statistically
- Small probability to have very high energy delta-rays (or knock-on electrons)

dE/dx Fluctuations

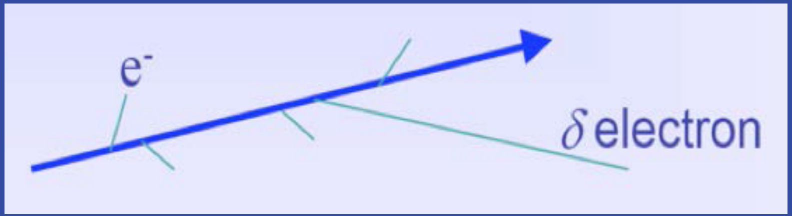


- A real detector (limited granularity) cannot measure <dE/dx>
  - It measures the energy ΔE deposited in layers of finite thickness Δx
  - Repeated measurements are needed

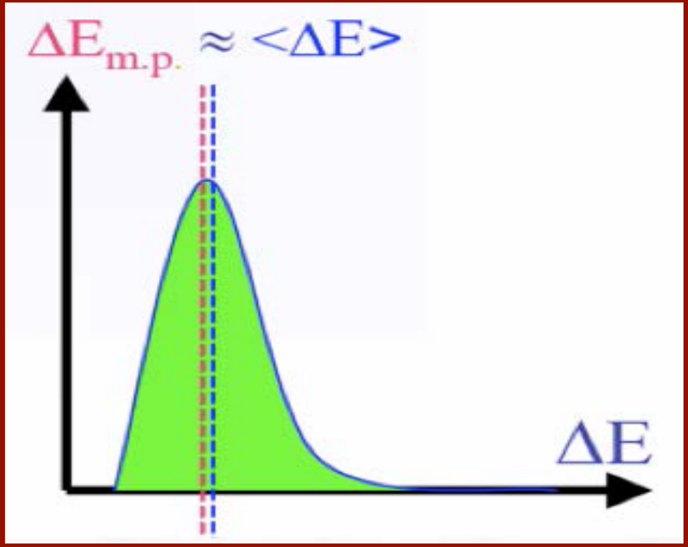
- Thin layers or low density materials: dE/dx has large fluctuations towards high losses (Landau)

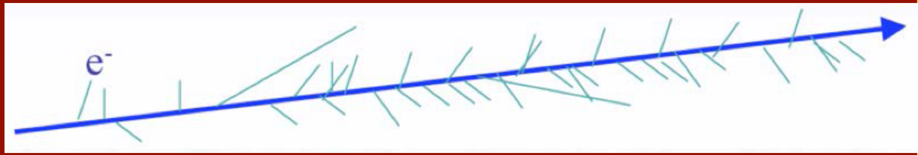


Example: 300 μm thick Silicon
ΔE_{mip}~82 keV,
$<\Delta E>_{mip}$ ~115 keV



- Thick layers and high density materials: the dE/dx is a more Gaussian-like (many collisions



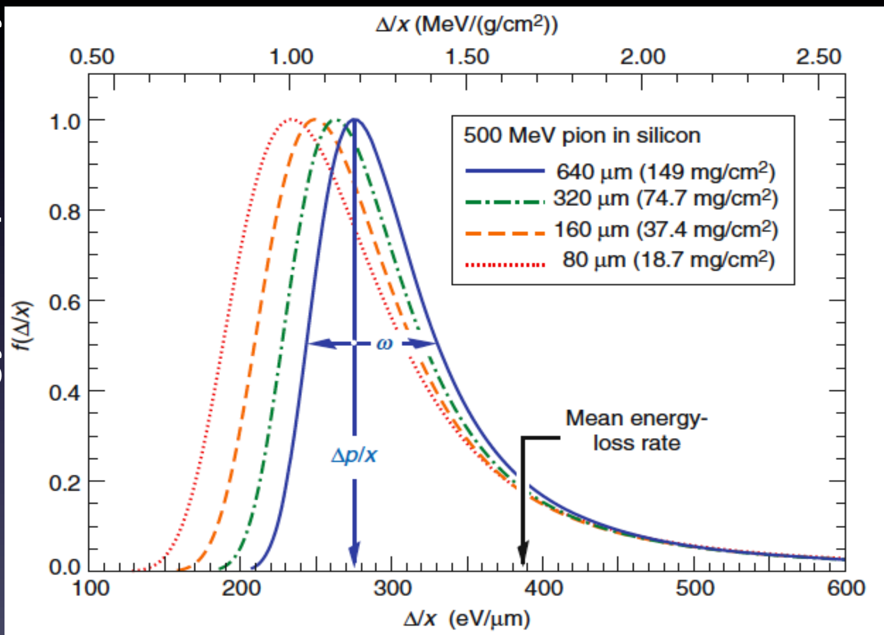


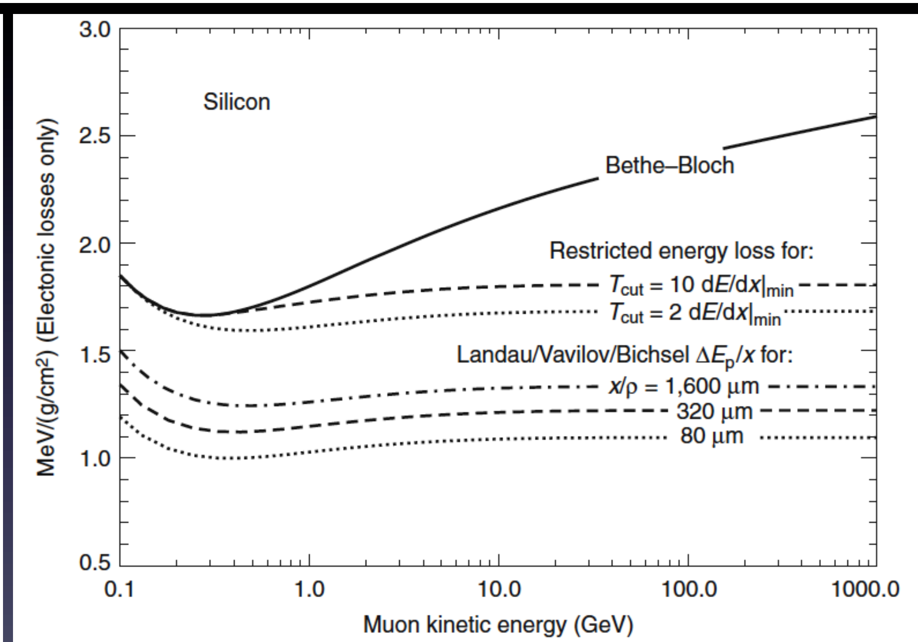
Landau Distribution

- For thin (not too thin) absorbers the Landau distribution offers a good approximation of the energy loss (Gaussian-like + tail due to high energy delta-rays which might leave the detector)

Landau distribution- Most Probable Value (MPV) dE/dx ≠ average dE/dx

Normalized energy loss probability





- An approximation of the Landau distribution:

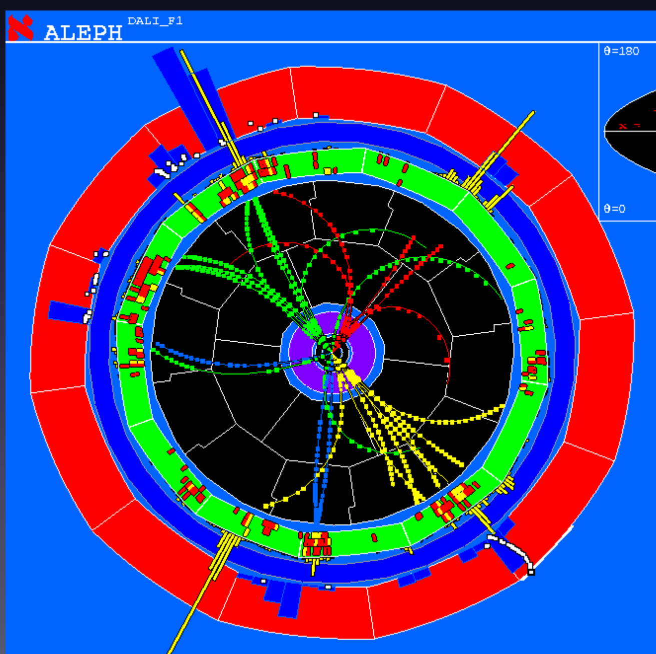
$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\lambda + e^{-\lambda}\right)\right]$$

$$\lambda = \frac{\Delta E - \Delta E^{MP}}{\xi}$$

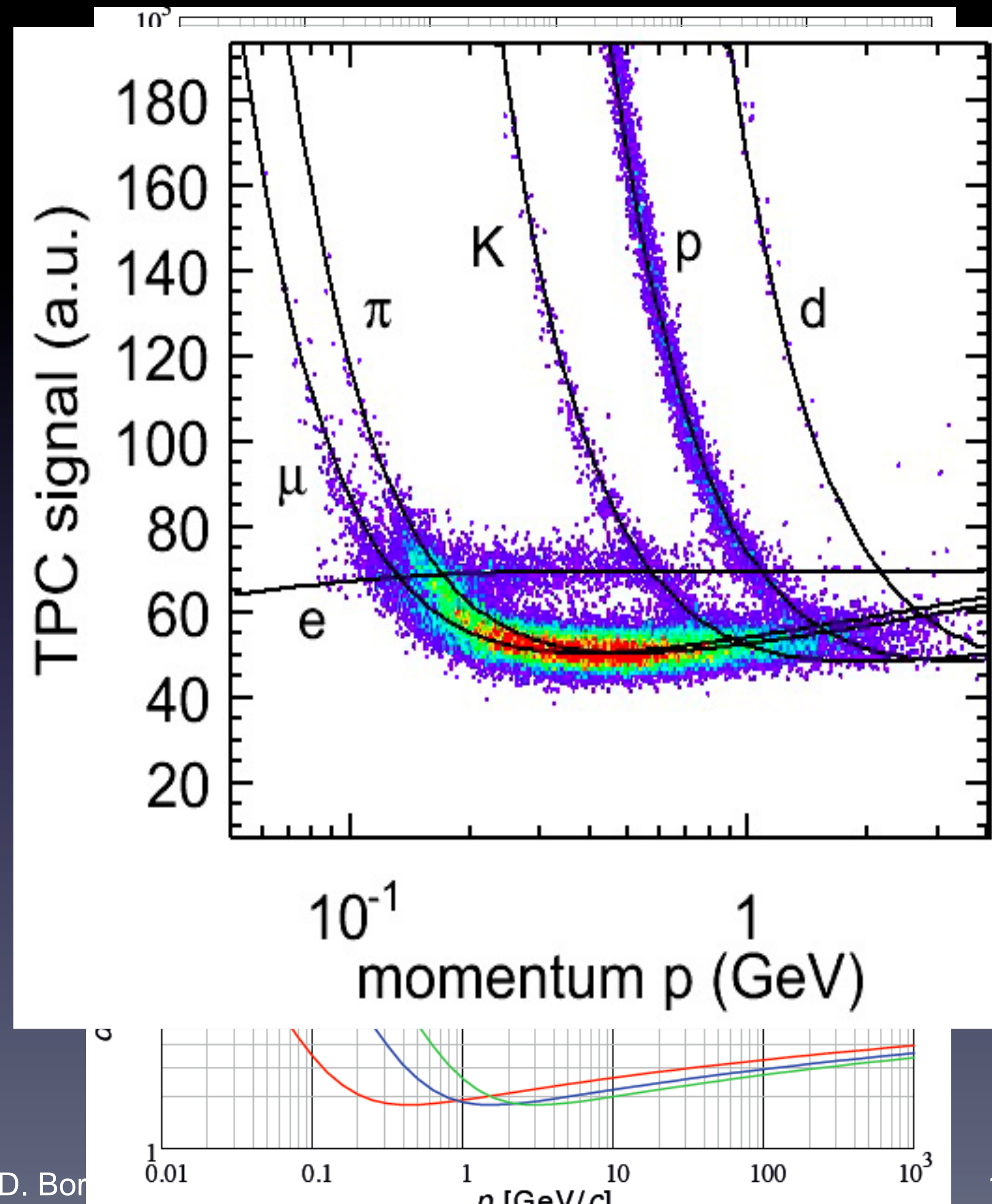
ξ Is material dependent

dE/dx and particle ID

- dE/dx is a function of $\beta\gamma = P/Mc$ and it is independent of M.
- By measuring P and the energy loss independently → Particle ID in certain momentum regions



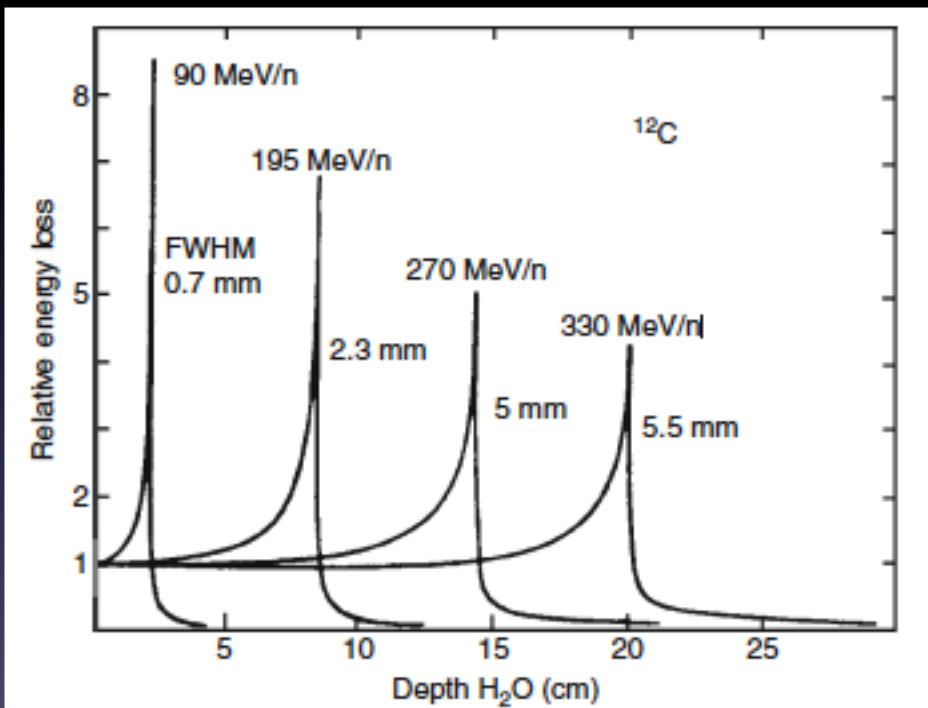
$$P_T\left[GeV/c\right] = 0.3B[T]\rho[m]$$

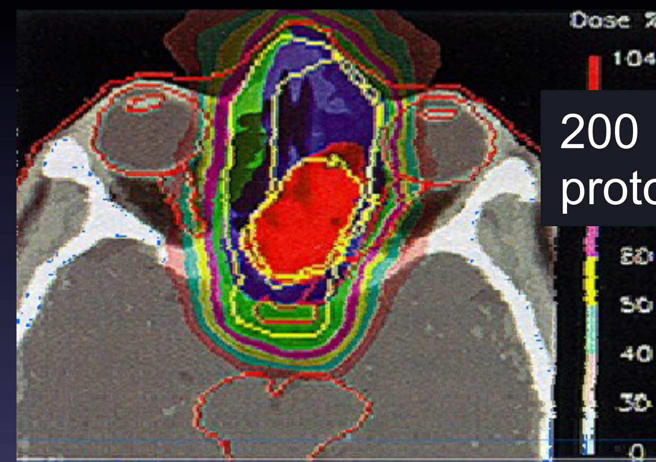


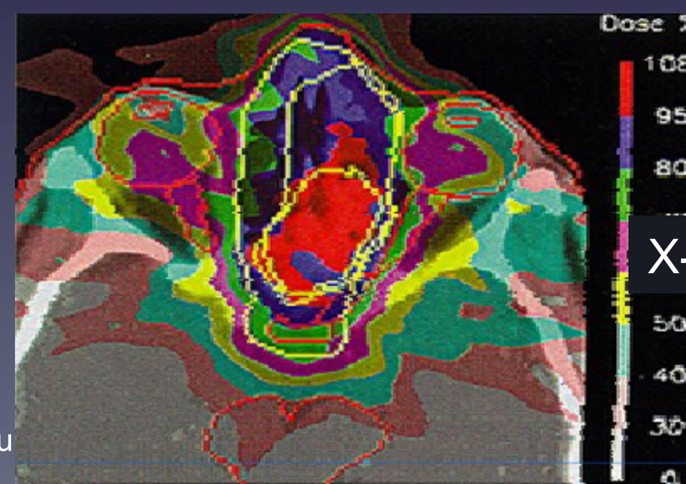
Energy loss at small momenta

- If the energy of the particle falls below $\beta\gamma=3$ the energy loss rises as $1/\beta^2$ → Particles deposit most of their energy at the end of their track → Bragg peak

Hadron therapy: Protons 200 MeV 1 nA
Carbon ions 4800 MeV 0.1 nA







Critical for radiation therapy

Range of particles in matter

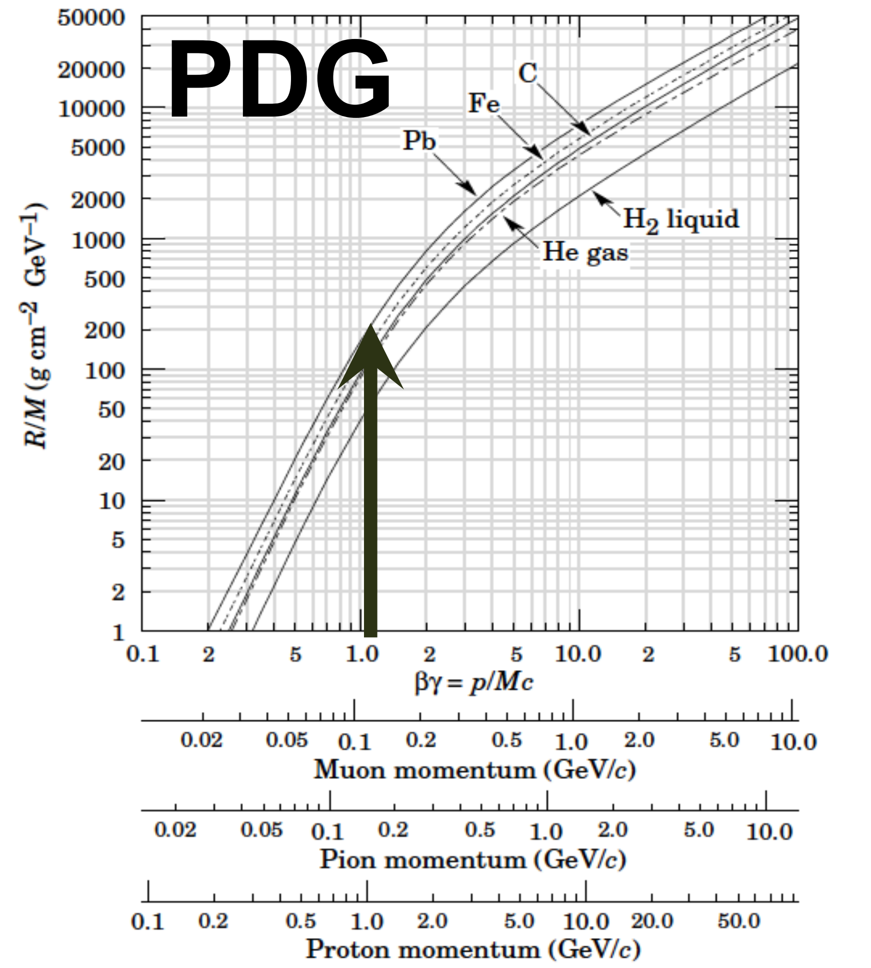
- A particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at a distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{z^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho R(\beta_0\gamma_0)}{Mc^2} = \frac{1}{z^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

- R/M is ≈ independent of the material
- R is a useful concept only for low-energy hadrons ($R < \lambda_I$ =the nuclear interaction length)



1GeV p in Pb   ρ(Pb)= 11.34 g/cm^3

R/M(Pb)=200 g cm^{-2} GeV^{-1}

R=(200/11.34) cm ≈ 20 cm

Range of particles in matter

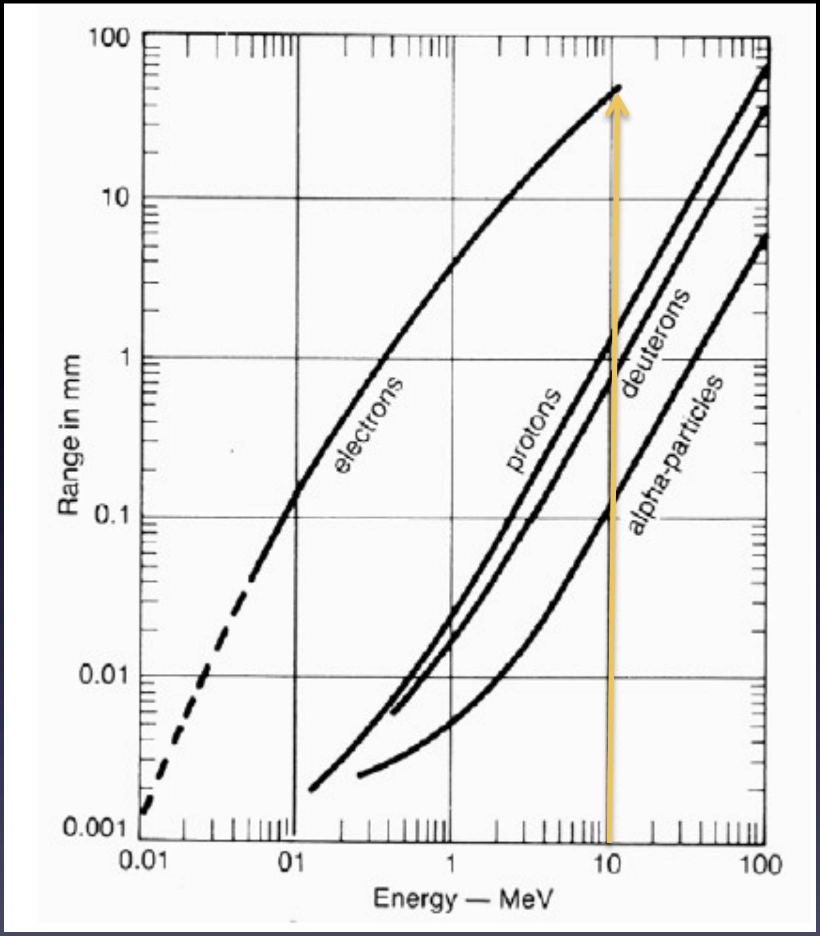
- A particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at a distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{z^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho R(\beta_0\gamma_0)}{Mc^2} = \frac{1}{z^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

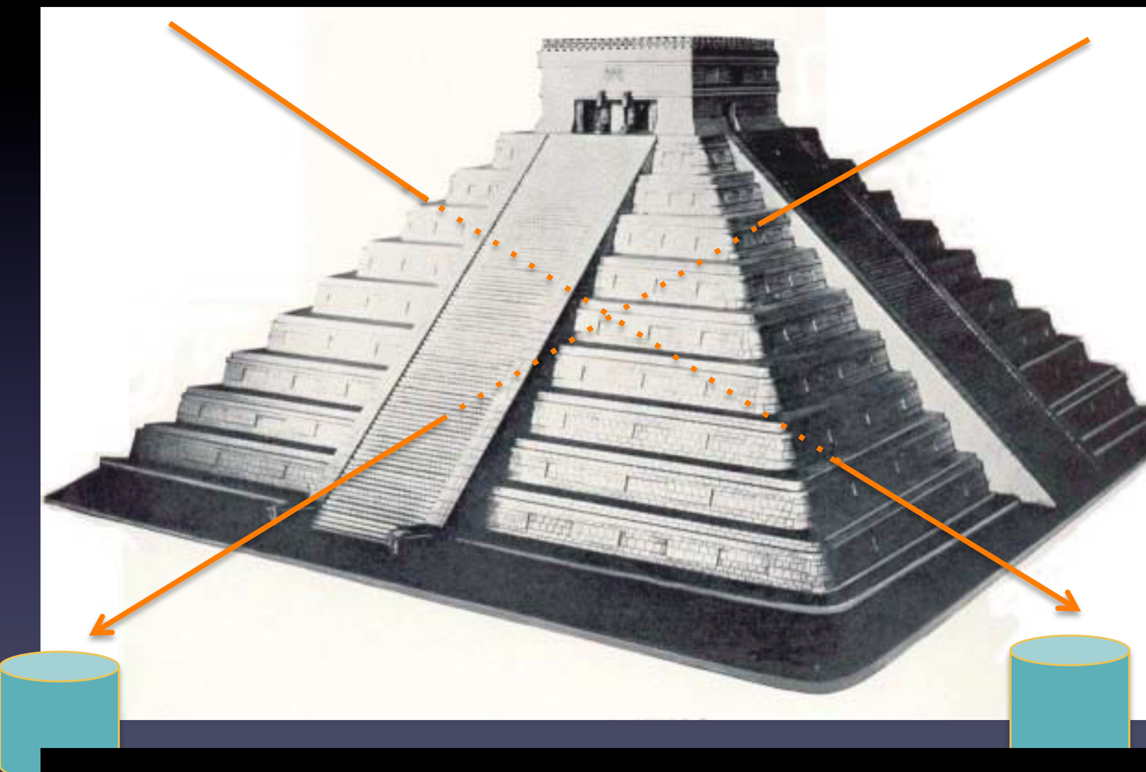
- R/M is ≈ independent of the material
- R is a useful concept only for low-energy hadrons ($R < \lambda_I$ =the nuclear interaction length)

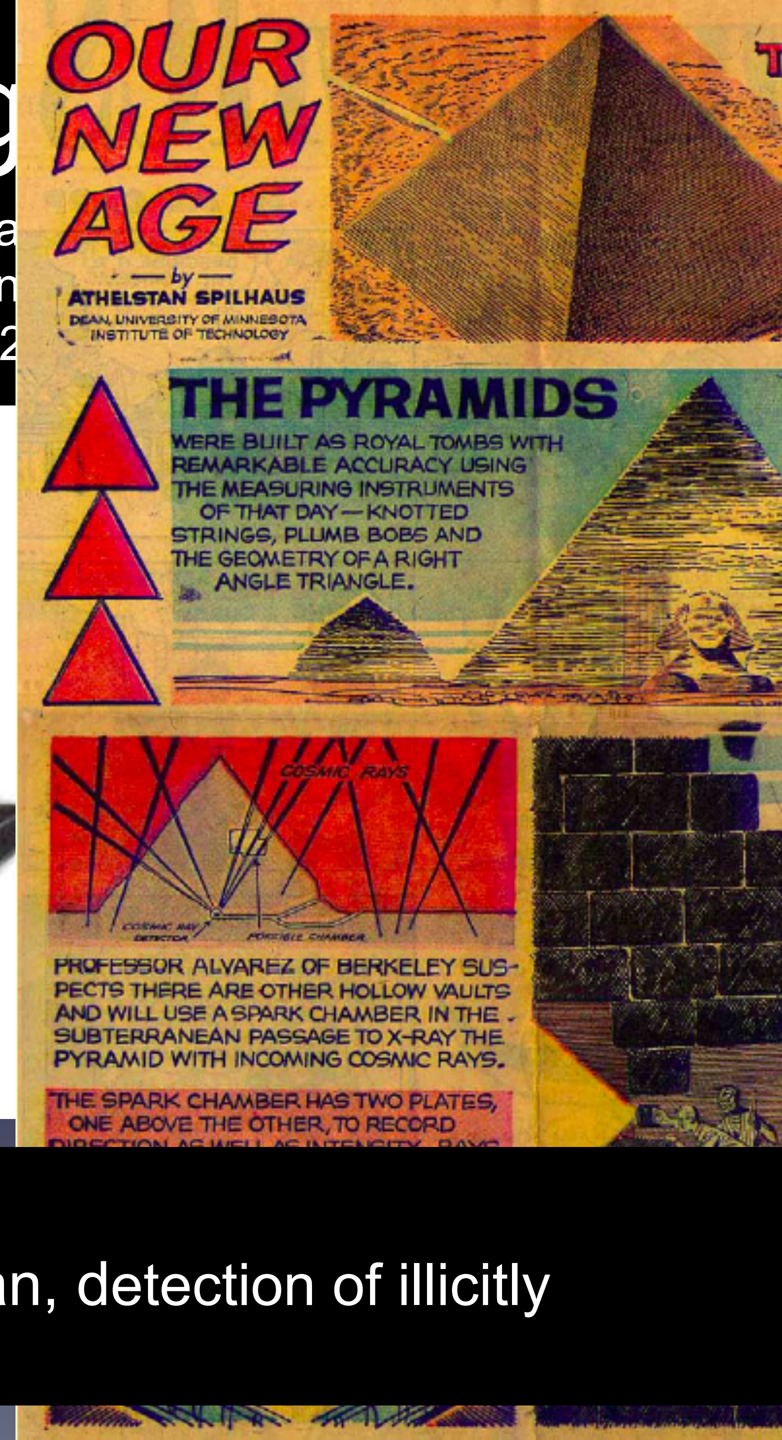


Mean free path in plastic scintillator for various charged particle

Muon Tomog

- L. Alvarez in the 60s used the measurement of a
  cosmic ray muons to look for hidden chambers in
  Pyramid → Muon Tomography (Science 167, 832

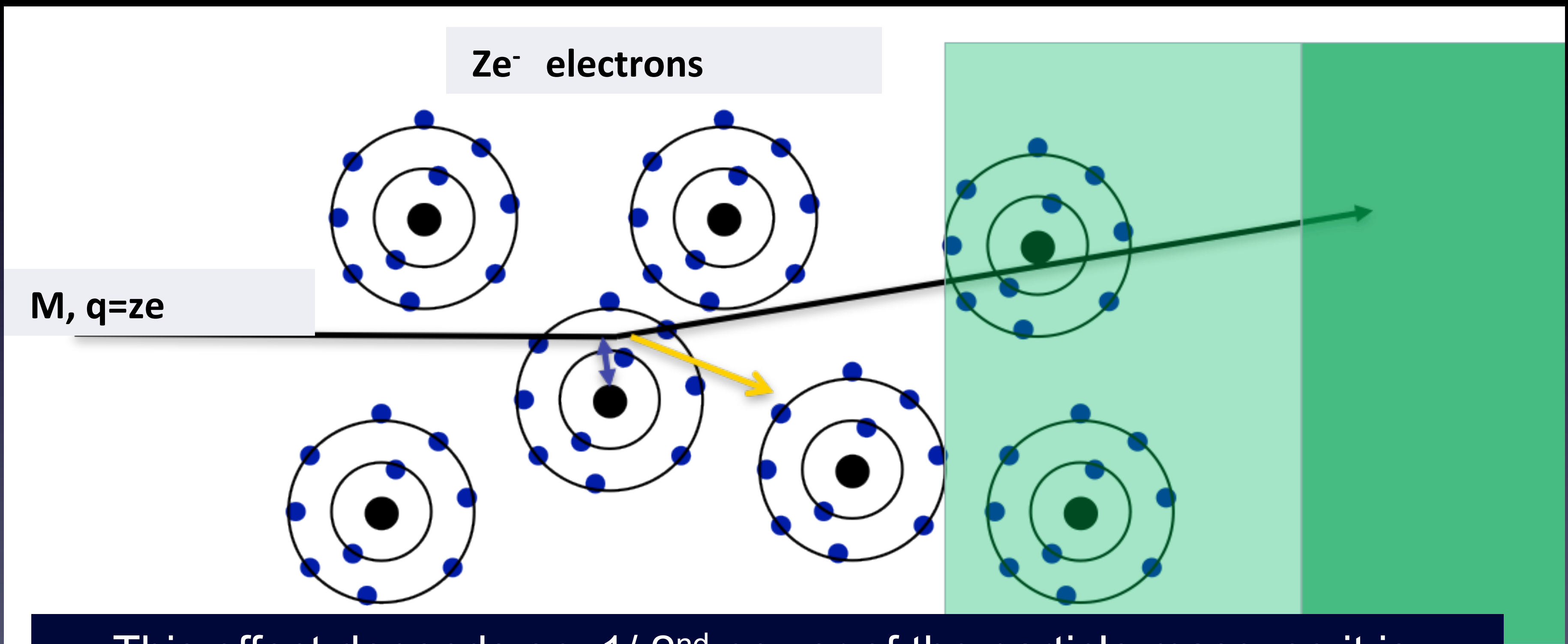




- No hidden chambers
- Now used for archeology in the Yucatan, detection of illicitly trafficked Special Nuclear Material etc.

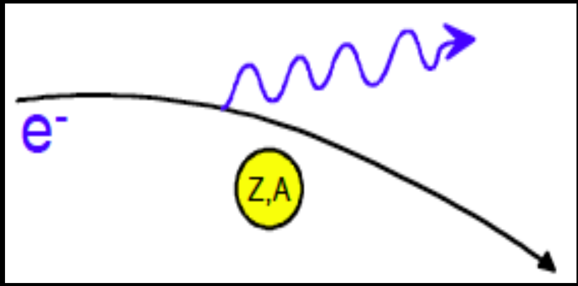
Bremsstrahlung

A charged particle of mass M and charge q=ze is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and therefore it can radiate a photon → Bremsstrahlung.



This effect depends on 1/ 2nd power of the particle mass, so it is relevant for electrons and very high energy muons

Energy loss for electrons and muons



- Bremsstrahlung=photon emission by an electron accelerated in Coulomb field of nucleus

$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A}\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}\right)^2 E \ln\frac{183}{Z^{1/3}}$$

- Dominant process for E_e > 10-30 MeV
  - energy loss proportional to $1/m^2$
  - Important mainly for electrons and h.e. muons

$$-\left\langle\frac{dE}{dx}\right\rangle_{brem} \propto \frac{E}{m^2}$$

- For electrons $$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln\frac{183}{Z^{1/3}}$$

$$\text{If } X_0 \approx \frac{A}{4\alpha N_A Z^2 r_e^2 \ln\frac{183}{Z^{1/3}}}$$

X_0 = radiation length in [g/cm^2]

$$\frac{dE}{dx} = \frac{E}{X_0}$$

$$E = E_0 e^{-x/X_0}$$

After passing a layer of material of thickness $\mathbf{X_0}$ the electron has 1/e of its initial energy.

Total energy loss and critical energy

$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$

- Critical energy

$$\left.\frac{dE}{dx}(E_c)\right|_{brems} = \left.\frac{dE}{dx}(E_c)\right|_{ion}$$

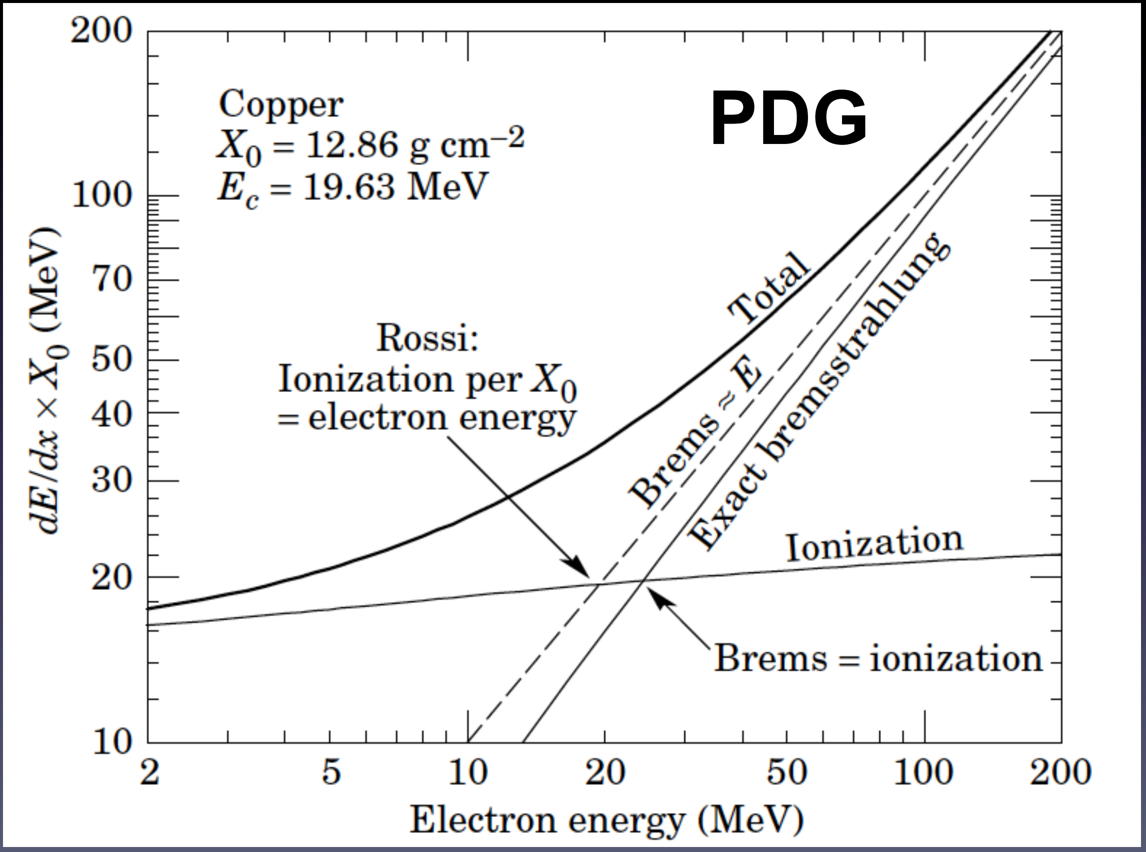
For solid and liquids

$$E_c = \frac{610\ \mathrm{MeV}}{Z+1.24}$$

For gasses

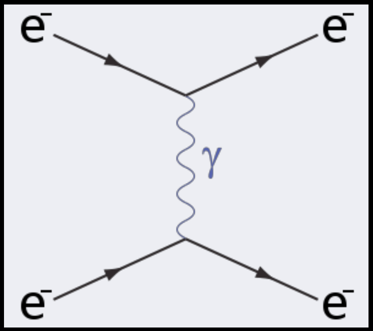
$$E_c = \frac{710\ \mathrm{MeV}}{Z+0.92}$$

Example Copper:
$E_c \approx 610/30$ MeV ≈ 20 MeV

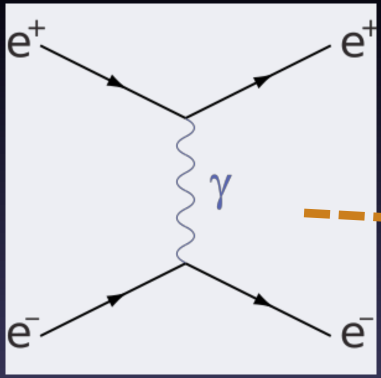


Electron energy loss

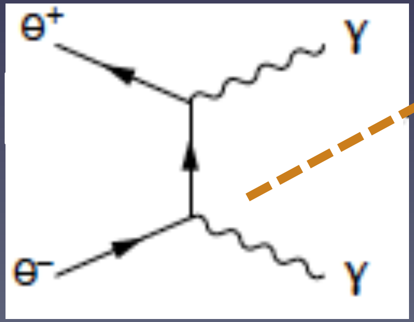
- Møller scattering

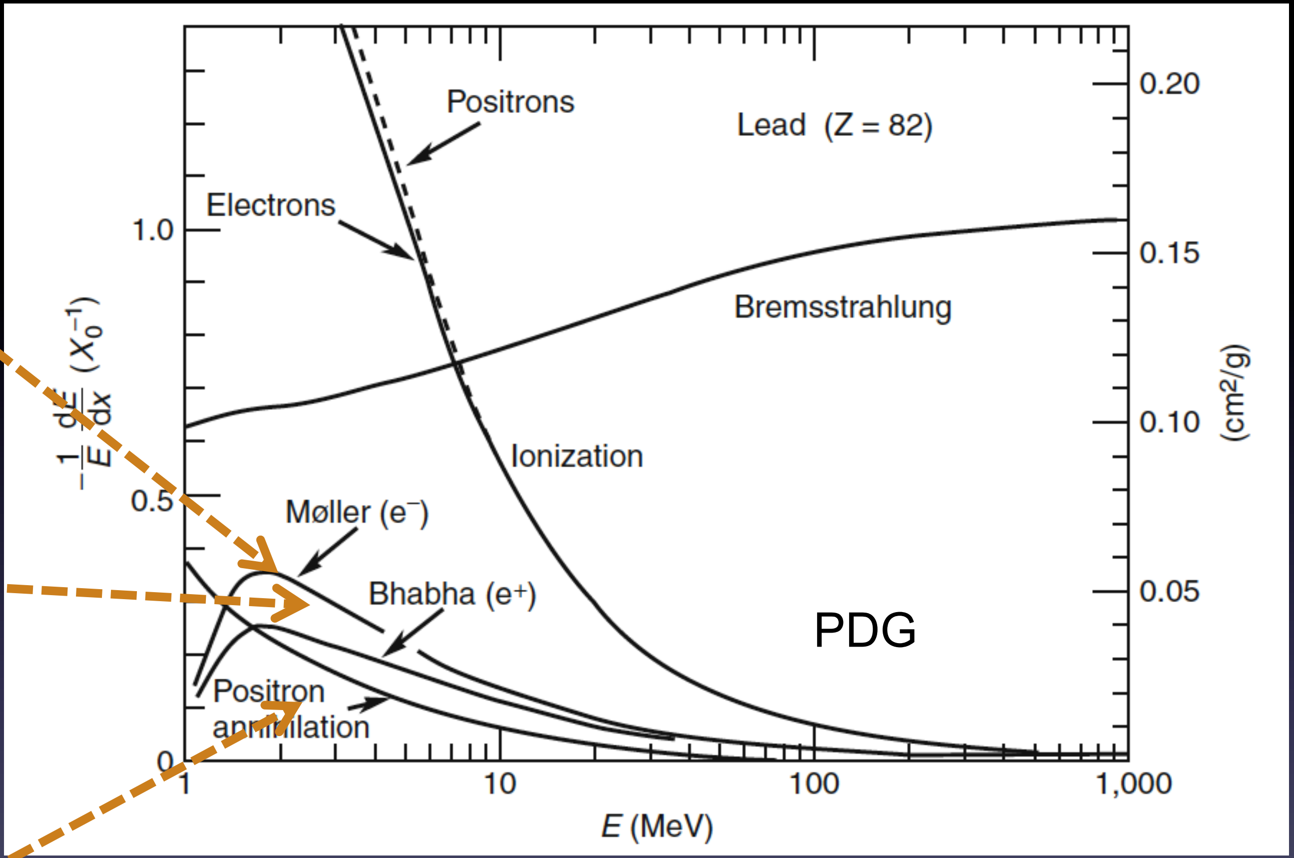


- Bhabha scattering



- Positron annihilation





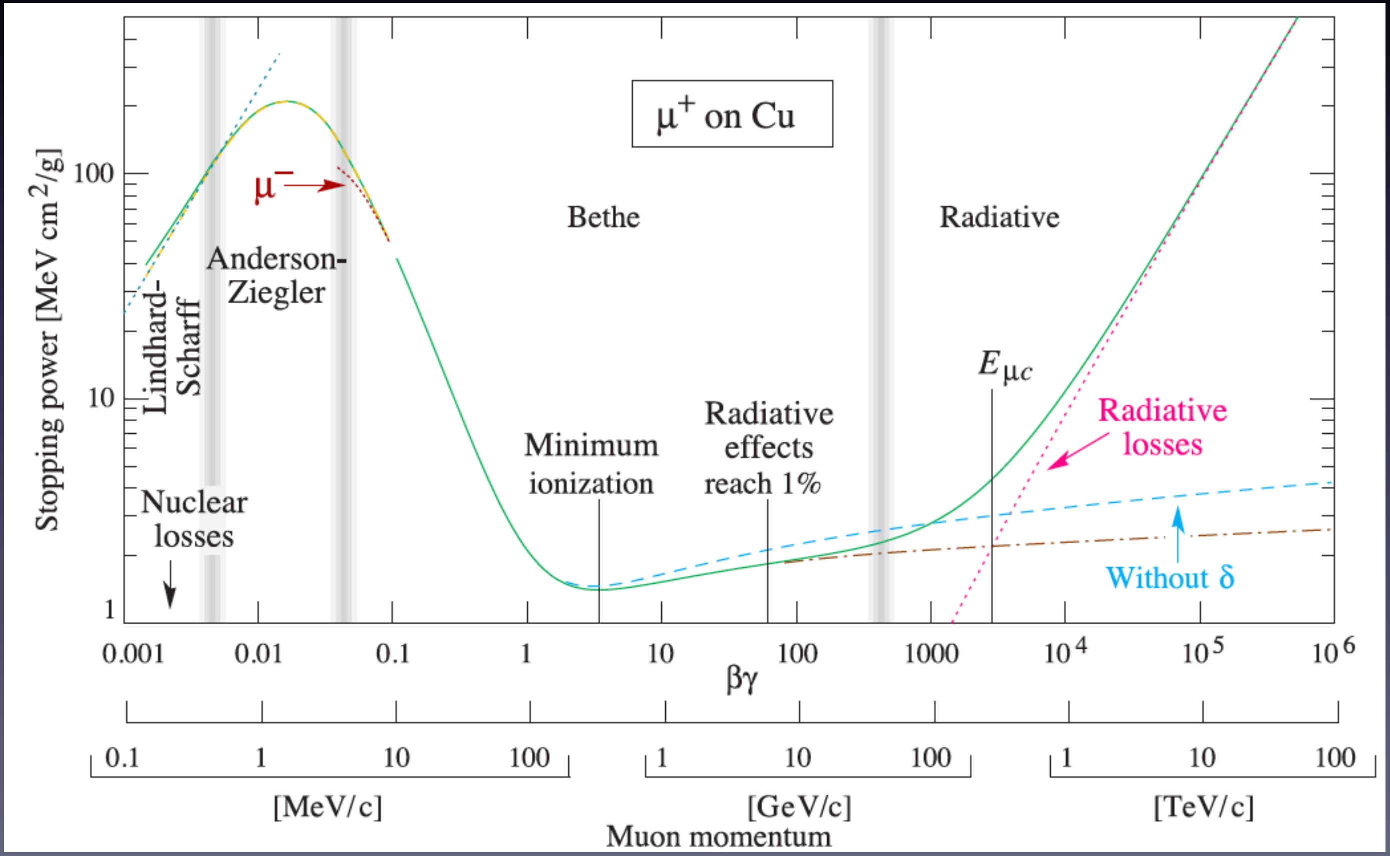
- Fractional energy loss per radiation length in lead as a function of the electron or positron energy

Energy loss summary

Since $m_\mu/m_e \approx 200$ E_c for muons ≈ 400 GeV.

$$-\left\langle \frac{dE}{dx} \right\rangle_{brem} \propto \frac{E}{m^2}$$

$$\left\langle \frac{dE}{dx} \right\rangle_{brem,\mu} \propto \frac{1}{40,000} \left\langle \frac{dE}{dx} \right\rangle_{brem,2}$$



- Muons with energies > ~10 GeV can penetrate thick layers of matter
- This is the key signature for muon identification

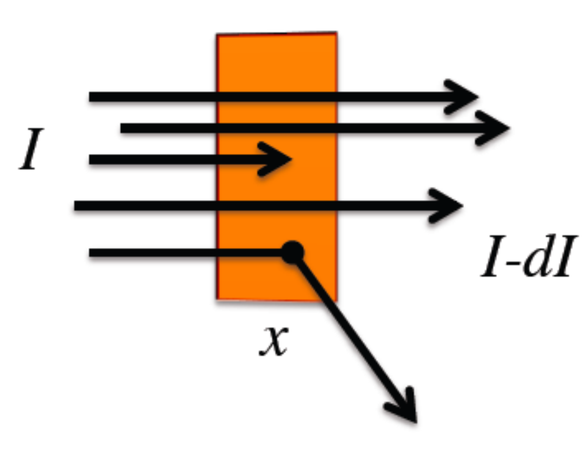
Interaction of photons with matter

- A photon can disappear or its energy can change dramatically at every interaction

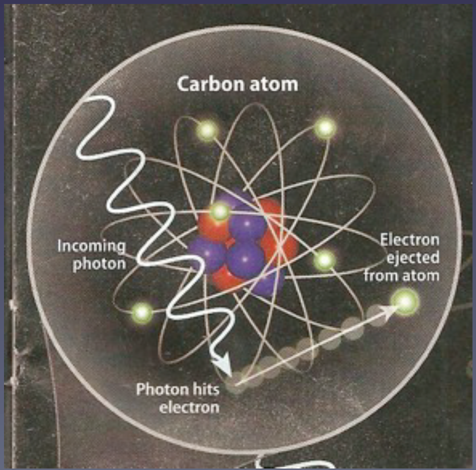
$$I(x) = I_0 e^{-\mu x} \qquad \mu = \frac{N_A}{A}\sum_{i=1}^{3}\sigma_i$$

$$\lambda = \frac{1}{\mu}$$

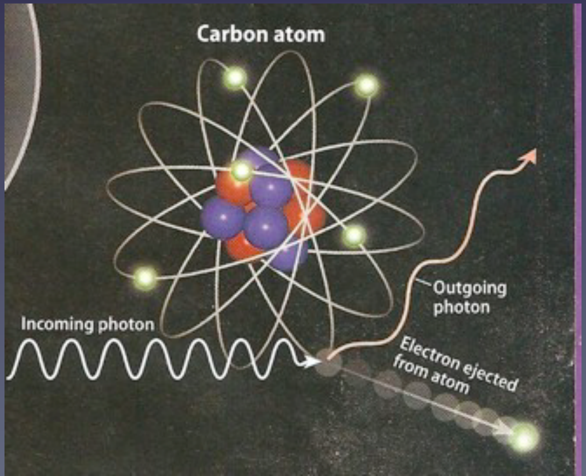
μ=total attenuation coefficient
σ_i=cross section for each process



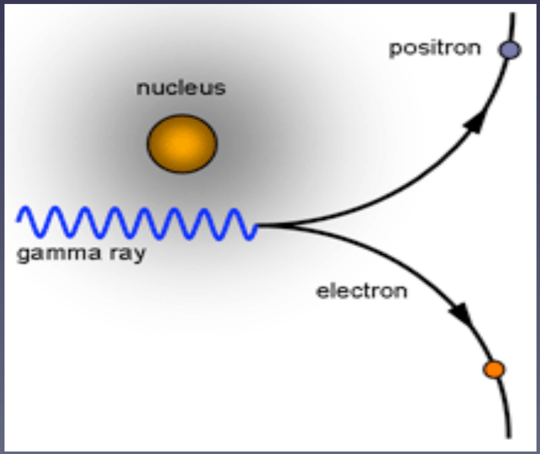
Photoelectric Effect



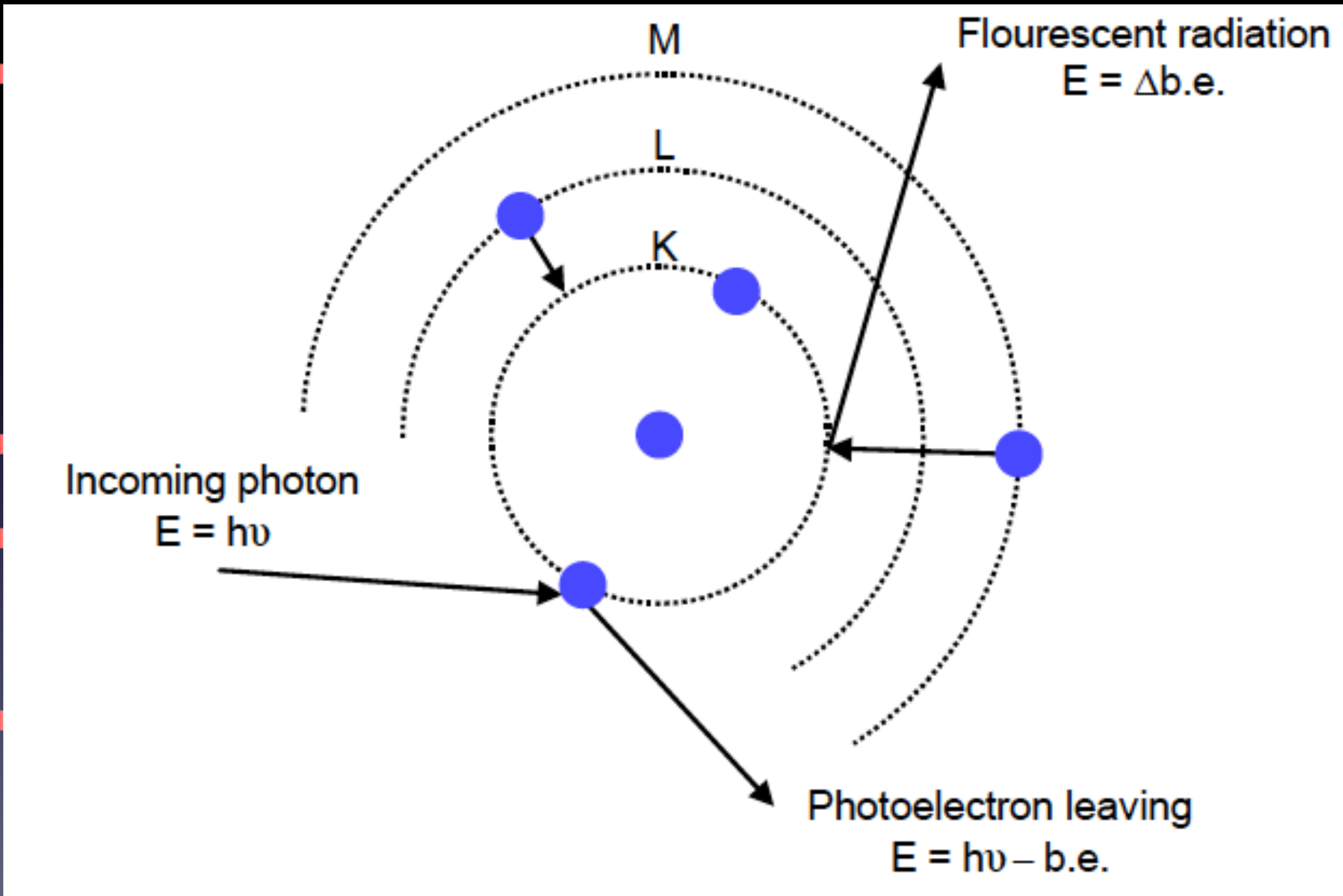
Compton Scattering



Pair production



Photoelectric effect



Compton scattering

- Best known electromagnetic process (Klein–Nishina formula)
  - for $\mathbf{E_\lambda << m_e c^2}$

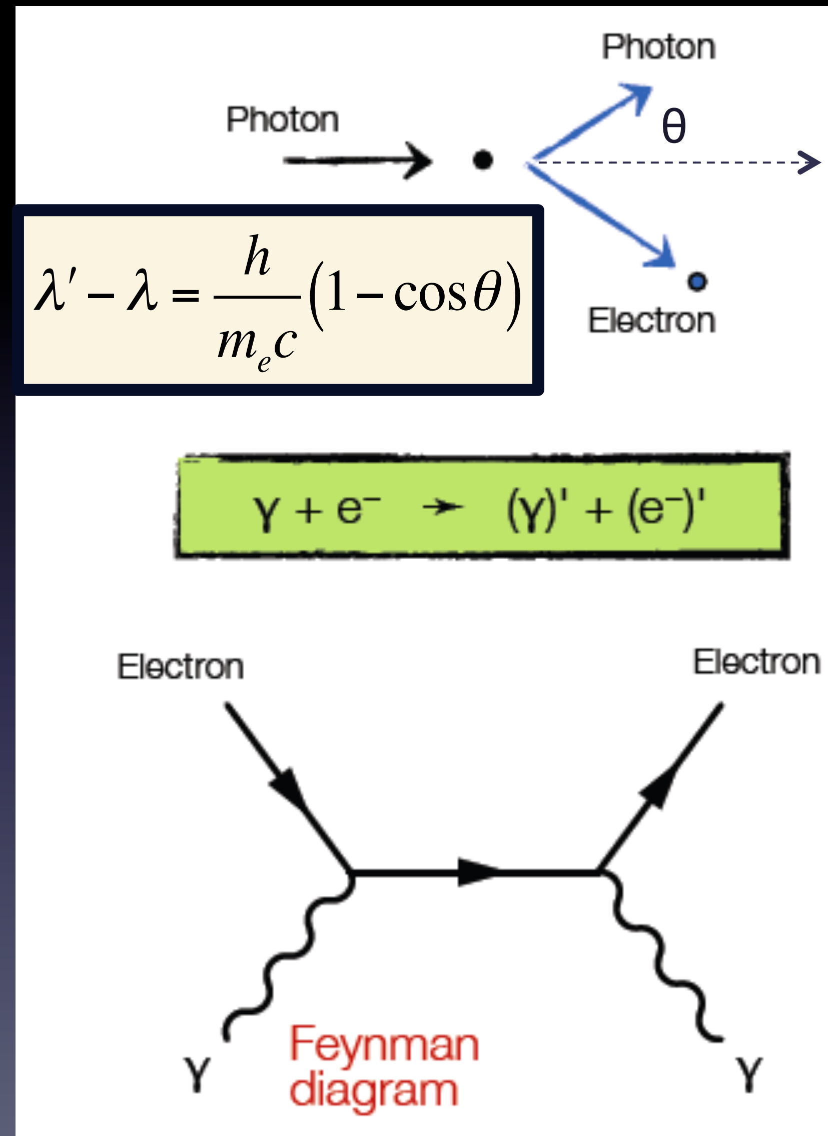
$$\sigma_c \propto \sigma_{Th}(1-\varepsilon)$$

  - for $\mathbf{E_\lambda >> m_e c^2}$

$$\sigma_c \propto \frac{\ln\varepsilon}{\varepsilon} Z$$

where

$$\sigma_{Th} = \frac{8\pi}{3r_e^2} = 0.66\ barn \qquad \varepsilon = \frac{E_\lambda}{m_e c^2}$$



Compton scattering

- From E and p conservation yields the energy of the scattered photon

$$E'_\gamma = \frac{E_\gamma}{1+\varepsilon(1-\cos\theta)}$$

$$\varepsilon = \frac{E_\lambda}{m_e c^2}$$

- Kinetic energy of the outgoing electron:

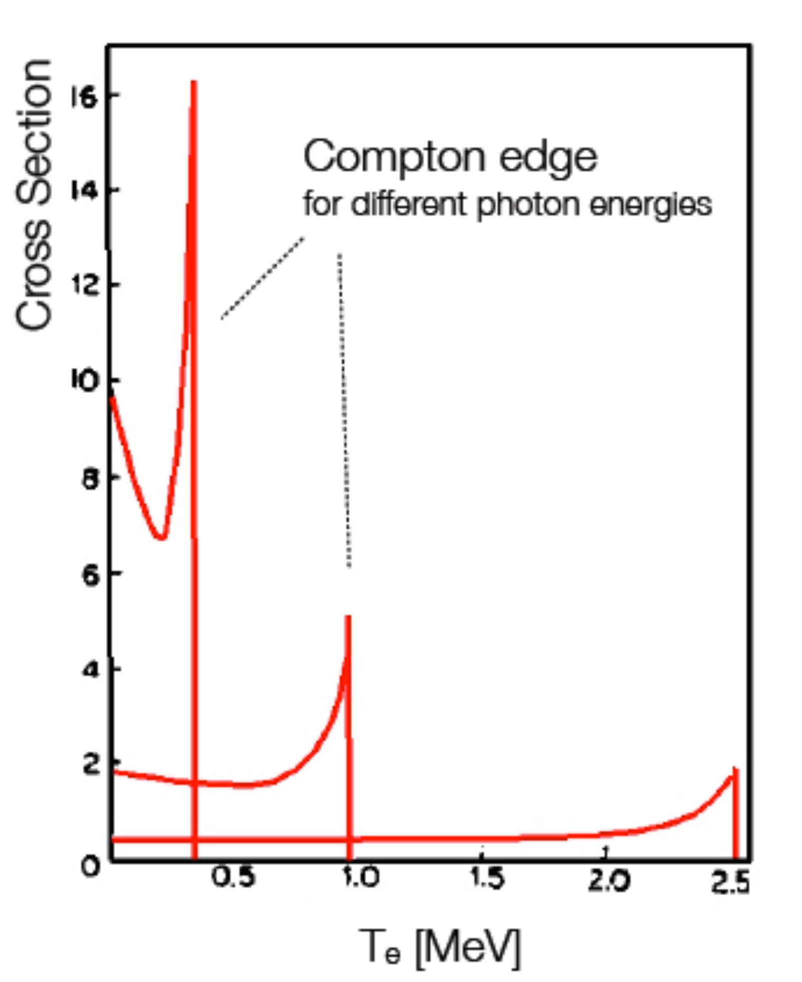
$$T_e = E_\gamma - E'_\gamma = E_\gamma \frac{\varepsilon(1-\cos\theta)}{1+2\varepsilon}$$

- The max. electron recoil is for θ=π

$$T_{max} = E_\gamma \frac{2\varepsilon}{1+2\varepsilon}$$

$$\Delta E = E_\gamma - T_{max} = E_\gamma \frac{1}{1+2\varepsilon}$$

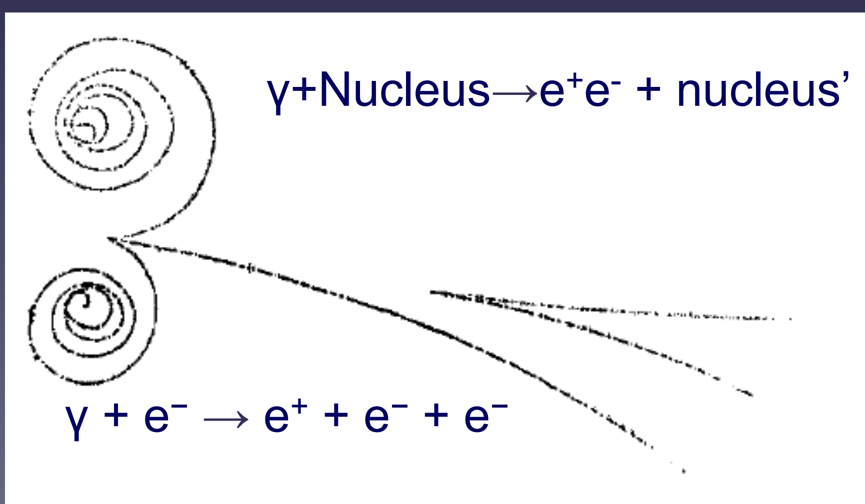
- Transfer of complete γ-energy via Compton scattering not possible

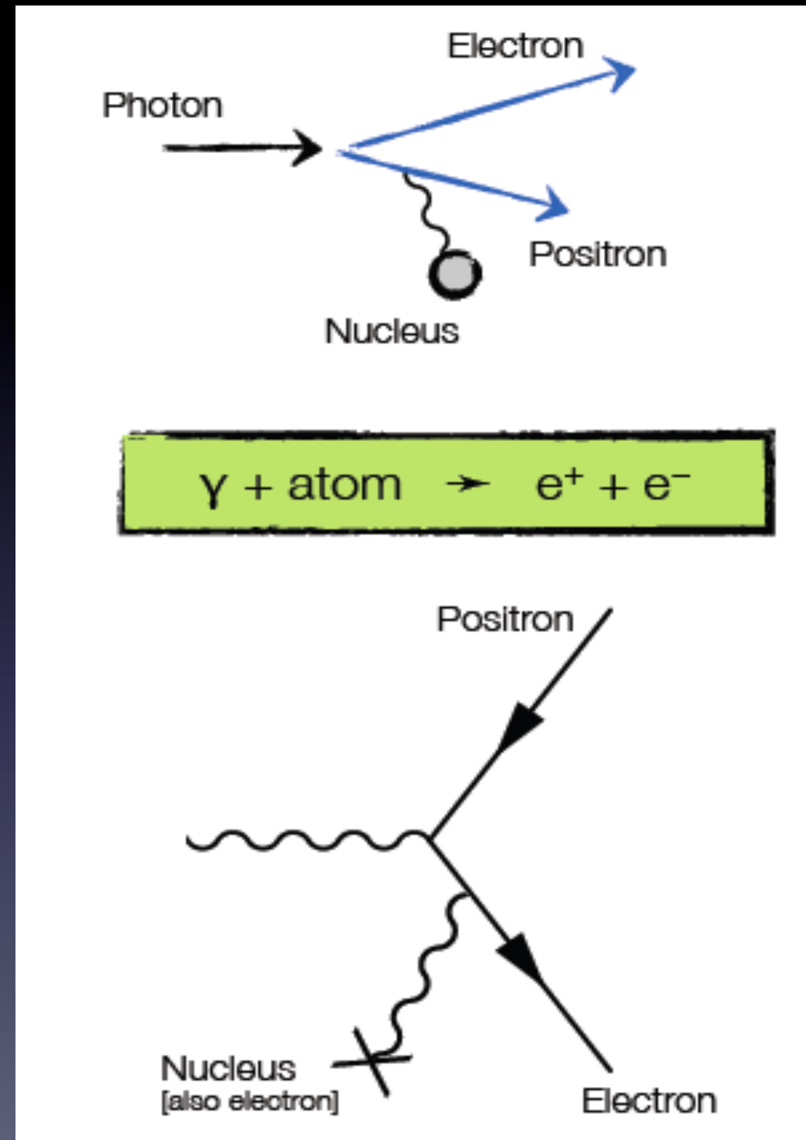


Pair production

- At E>100 MeV, electrons lose their energy almost exclusively by bremsstrahlung while the main interaction process for photons is electron–positron pair production.
- Minimum energy required for this process 2 m_e + Energy transferred to the nucleus

$$E_\gamma \geq 2m_ec^2 + \frac{2m_ec^2}{m_{Nuleus}} \geq 2m_ec^2$$





Pair production

- If $E_\lambda >> m_e c^2$

$$\sigma_{pair} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \ [\mathrm{cm}^2/\mathrm{atom}]$$

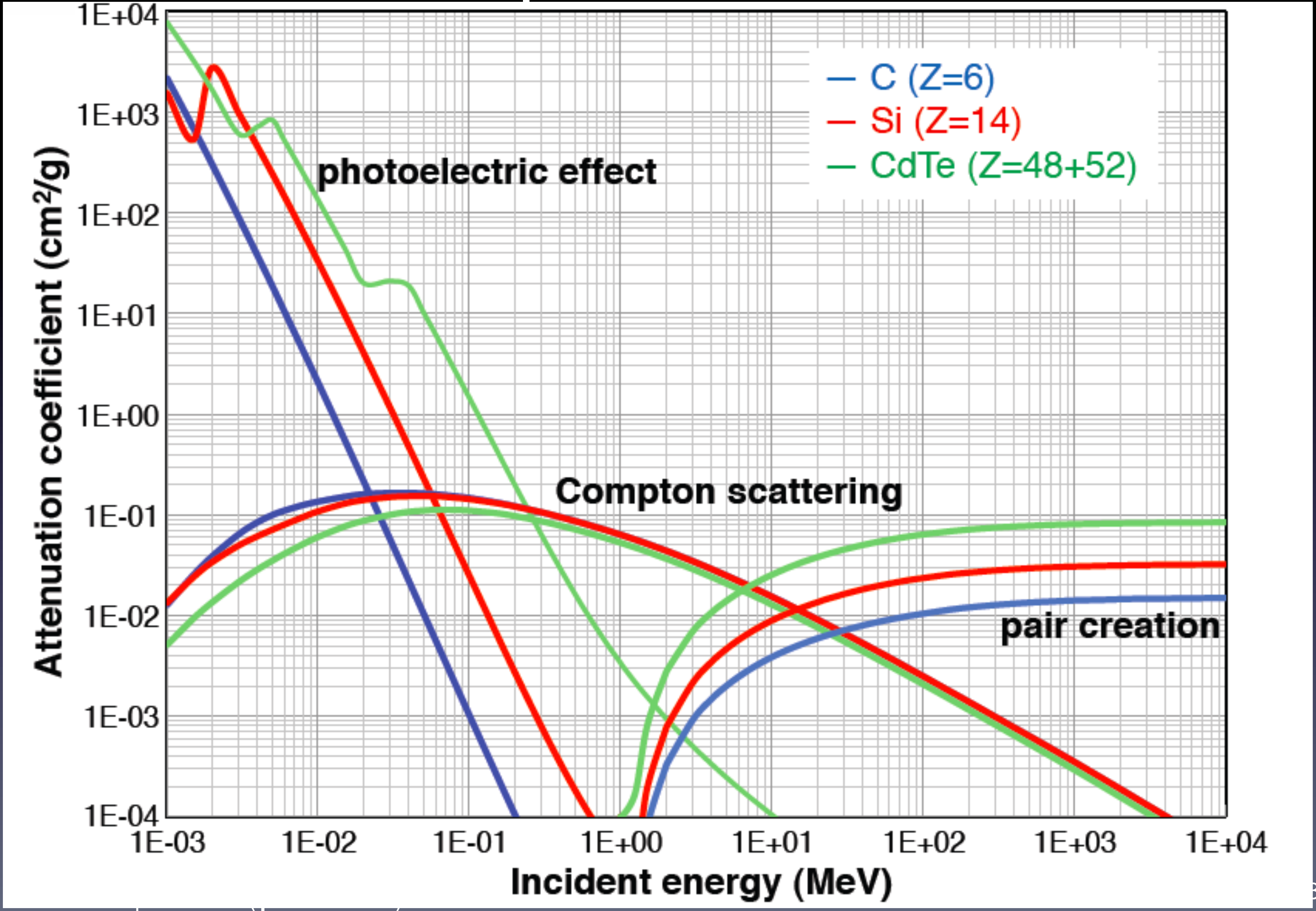
- Using as for Bremsstrahlung the radiation length and neglecting the small 1/54 term

$$X_0 = \frac{A}{4\pi N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

$$\sigma_{pair} = \frac{7}{9} \frac{N_A}{A} \frac{1}{X_0}$$

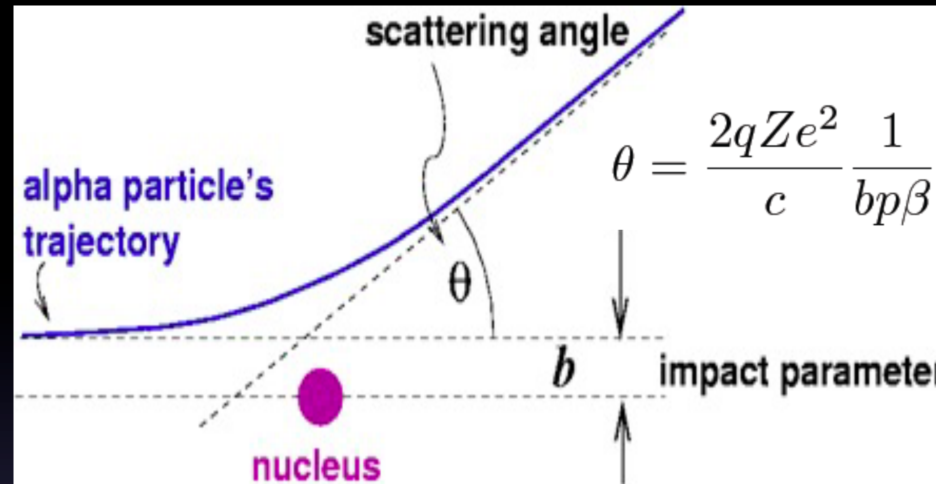
	ρ [g/cm³]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Luft	$1.2 \cdot 10^{-3}$	$30 \cdot 10^{3}$

Interaction of photons with matter



Multiple scattering

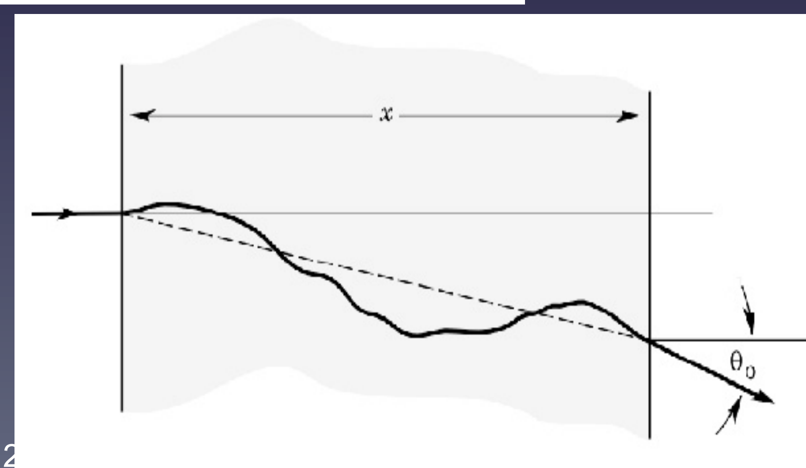
- A particle passing through material undergoes also multiple deflections due to Coulomb scattering with the nuclei



  - The scattering angle as a function of the thickness x is

$$\theta_{\rm rms}^{\rm proj} = \sqrt{\langle\theta^2\rangle} = \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}}\left[1 + 0.038 \ln(x/X_0)\right]$$

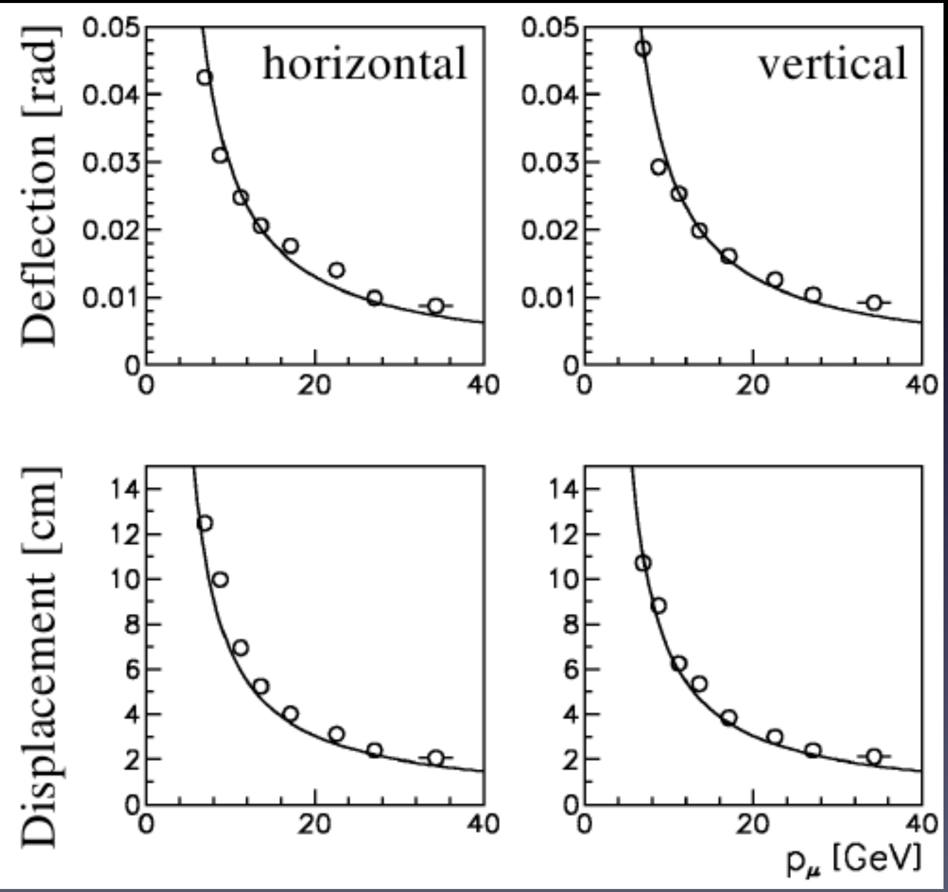
  - Where:
    - p (in MeV/c) is the momentum,
    - βc the velocity,
    - z the charge of the scattered particle
    - x/X_0 is the thickness of the medium in units of radiation length (X_0).



Multiple scattering

- Particularly relevant for μ in high-energy physics, but also common for low-energy e
- Hadrons generally undergo nuclear interactions before multiple scattering and energy loss become significant.
- Example: muon with E=14 GeV
  $\theta_0 \sim 13.6 / 14\text{x}10^3 \sqrt{(x / x_0)}$
  ~ 1 mRad $\sqrt{(x / x_0)}$
  Iron X_0 = 1.8 cm ; μ at E=10 GeV after 100 cm Fe :
  $\theta_0 \sim 13.6 / 10^4 \sqrt{(100/1.8)} \sim 10$ mRad

Example of Multiple scattering: Muons before and after 320 radiation lengths



Building your detector

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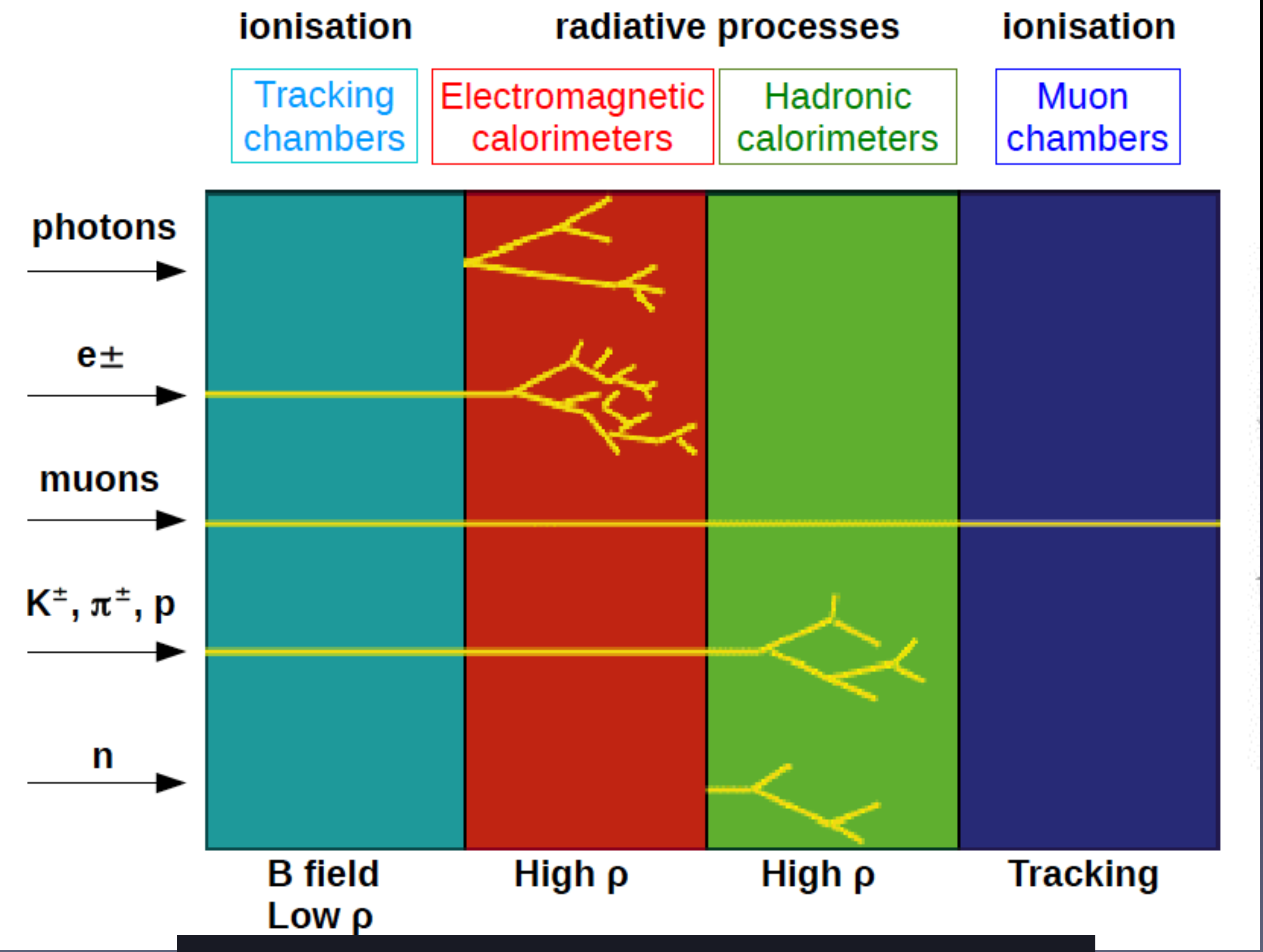
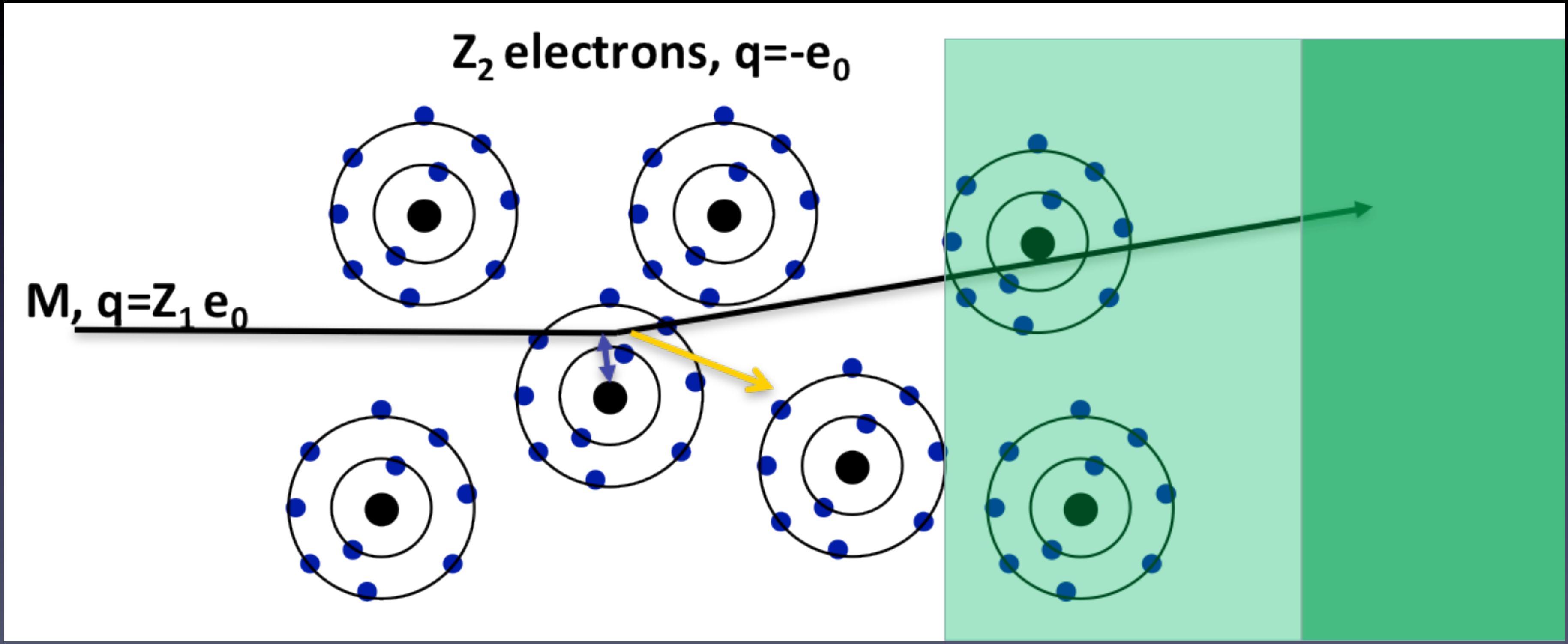


http://pdg.lbl.gov/2014/AtomicNuclearProperties/

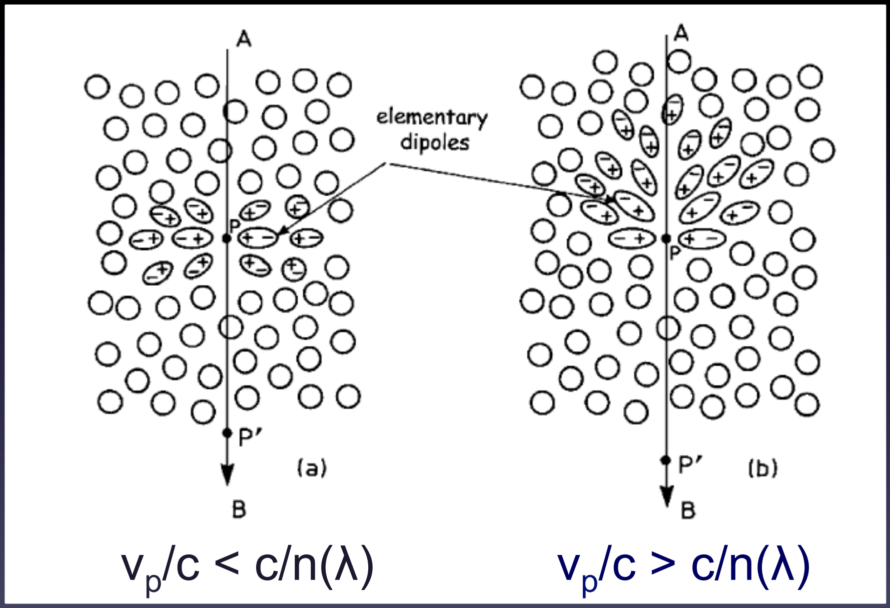
Energy loss by photon emission

- Emission of Cherenkov light
- Emission of transition radiation



Cherenkov emission

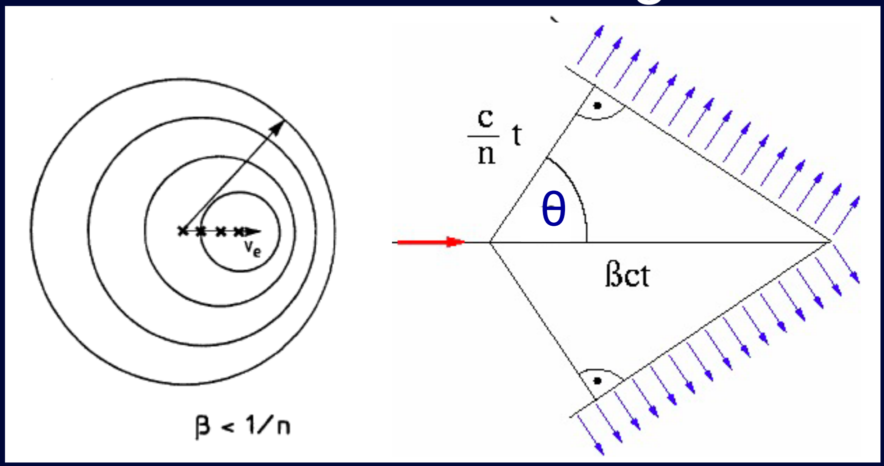
- If the velocity of a particle is such that **$\beta = v_p/c > c/n(\lambda)$** where $n(\lambda)$ is the index of refraction of the material, a pulse of light is emitted around the particle direction with an opening angle ($\boldsymbol{\theta_c}$)



Symmetric dipoles

coherent wavefront

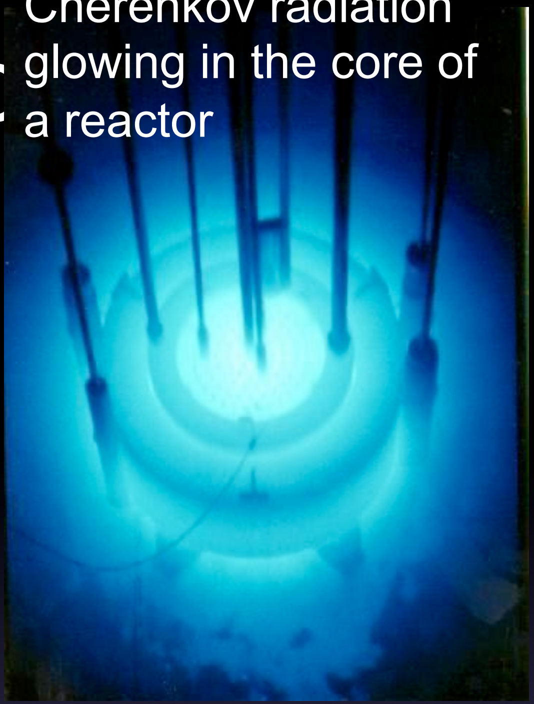
Cherenkov angle



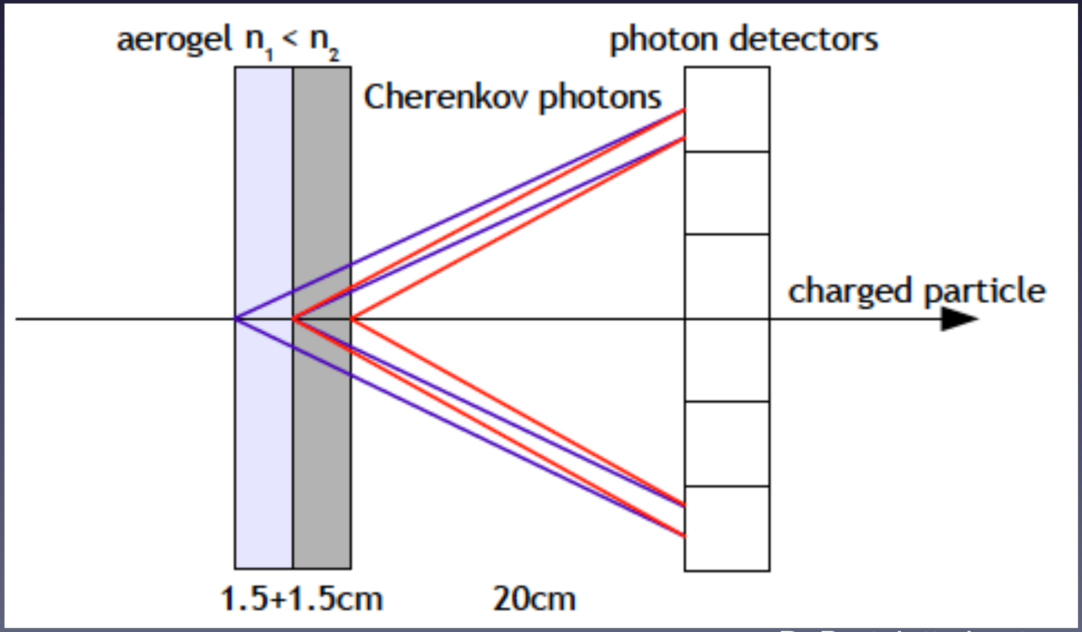
$$\cos\theta_c = \frac{1}{n(\lambda)\beta}$$

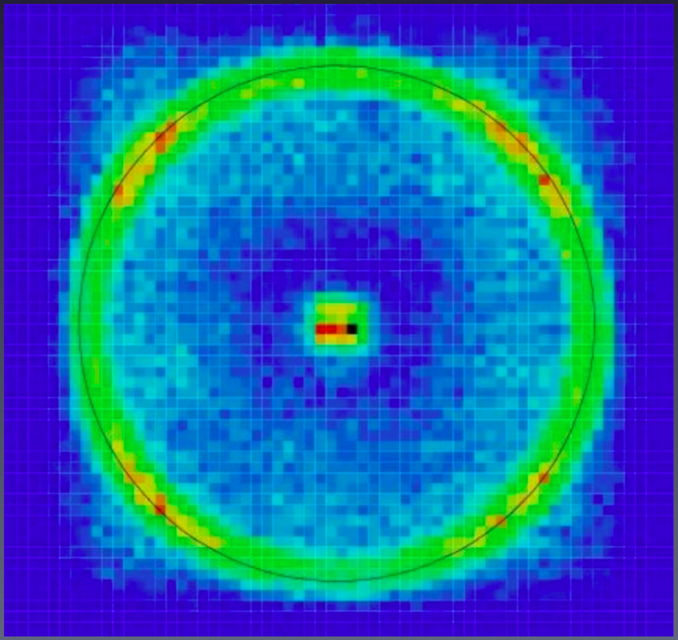
- The threshold velocity is $\beta_c = 1/n$
- At velocity below β_c no light is emitted

Cherenkov photon emission



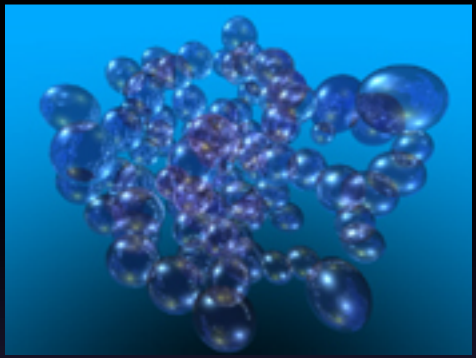
- Cherenkov emission is a weak effect and causes no significant energy loss (<1%)
- It takes place only if the track L of the particle in the radiating medium is longer than the wavelength λ of the radiated photons.
- Typically O(1-2 keV / cm) or O(100-200) visible photons /cm





Cherenkov radiators

Material	n-1	β_c	θ_c	photons/cm
solid natrium	3.22	0.24	76.3	462
Lead sulfite	2.91	0.26	75.2	457
Diamond	1.42	0.41	65.6	406
Zinc sulfite	1.37	0.42	65	402
silver chloride	1.07	0.48	61.1	376
Flint glass	0.92	0.52	58.6	357
Lead crystal	0.67	0.6	53.2	314
Plexiglass	0.48	0.66	47.5	261
Water	0.33	0.75	41.2	213
Aerogel	0.075	0.93	21.5	66
Pentan	1.70E-03	0.9983	6.7	7
Air	2.90E-03	0.9997	1.38	0.3
He	3.30E-05	0.999971	0.46	0.03

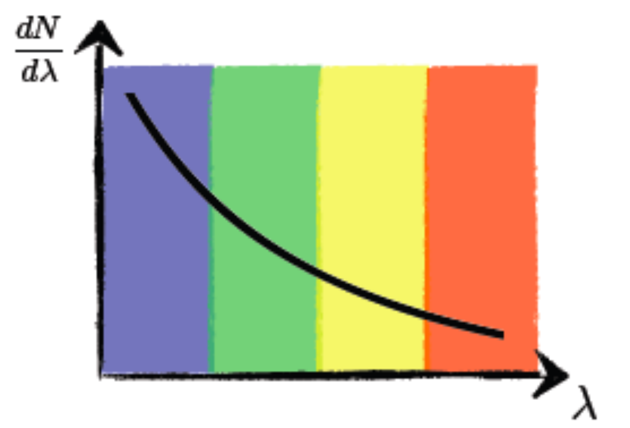


Silica Aerogel

Cherenkov photon emission

- The number of Cherenkov photons produced by unit path length by a charged particle of charge **z is**

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_c$$



- Note the wavelength dependence ~ 1/**λ**^{2}
- The index of refraction n is a function of photon energy E=hv, as is the sensitivity of the transducer used to detect the light.
- Therefore to get the number of photon we must integrate over the sensitivity range:

$$\frac{d^2N}{dx} = \int_{350\,nm}^{550\,nm} d\lambda \frac{dN}{d\lambda dx} = 475 z^2 \sin\theta_c \quad \text{photons/cm}$$

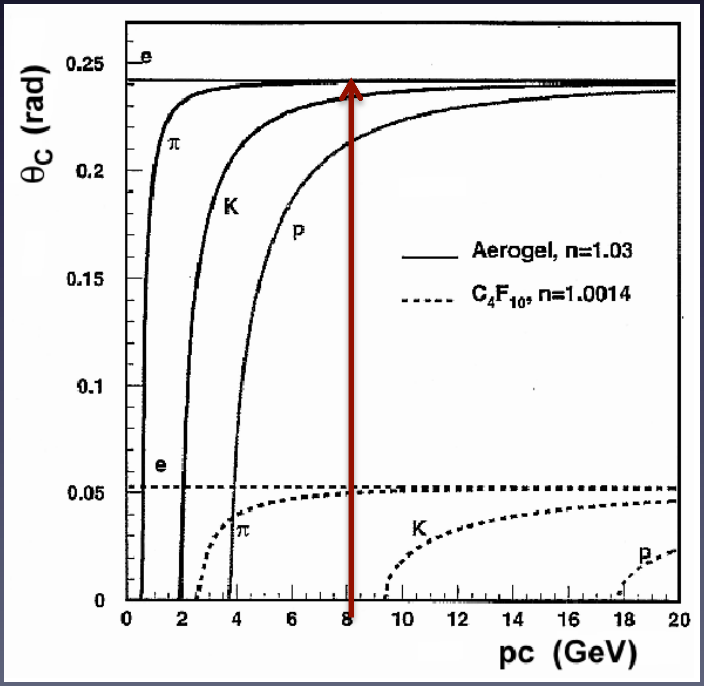
Threshold Cherenkov Counter

- Combination of threshold Cherenkov counters can identify particles

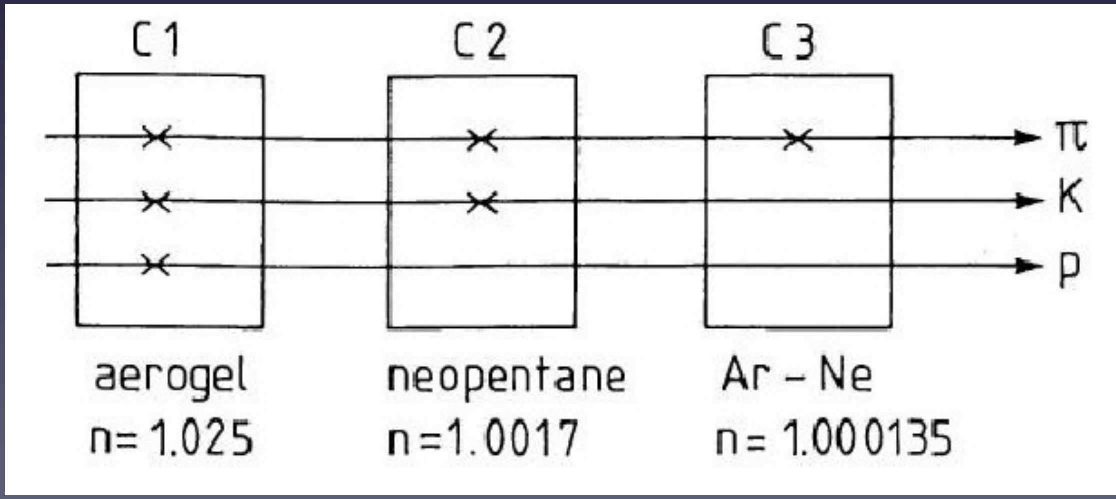
$$p \propto m\gamma\beta = \frac{m\beta}{\sqrt{1-\beta^2}} \qquad m_1 > m_2 \Rightarrow v_1 < v_2$$

$$m_{th} = \frac{p\sqrt{1-\beta_{th}^2}}{\beta_{th}} \quad \Rightarrow \quad n = \sqrt{\frac{m_{tr}^2}{p^2}+1}$$

If $m>m_{th}$ no light

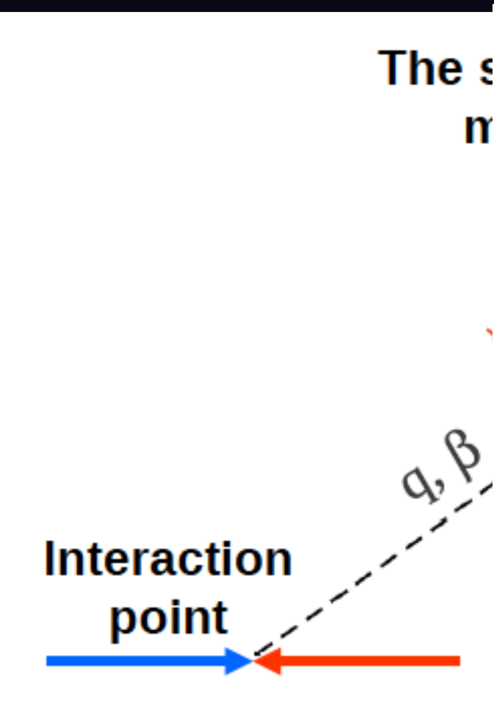


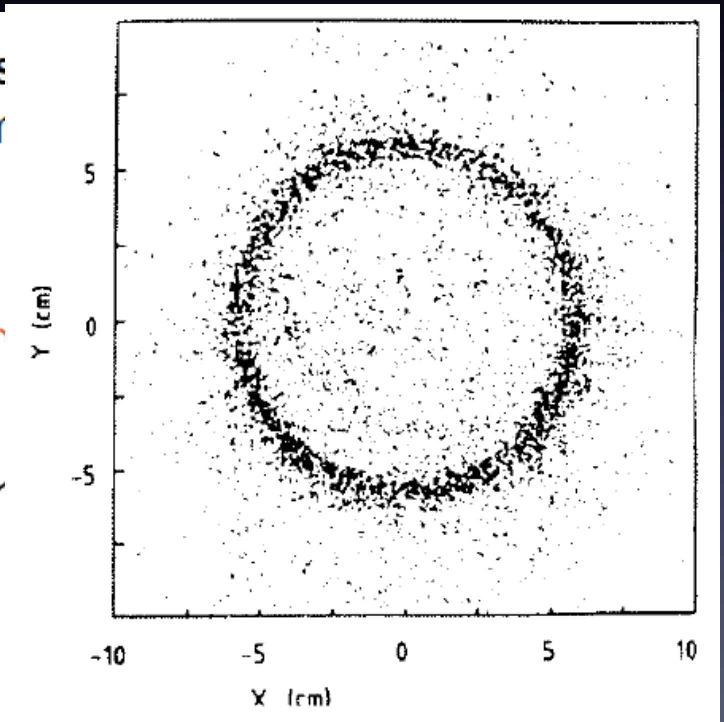
Example

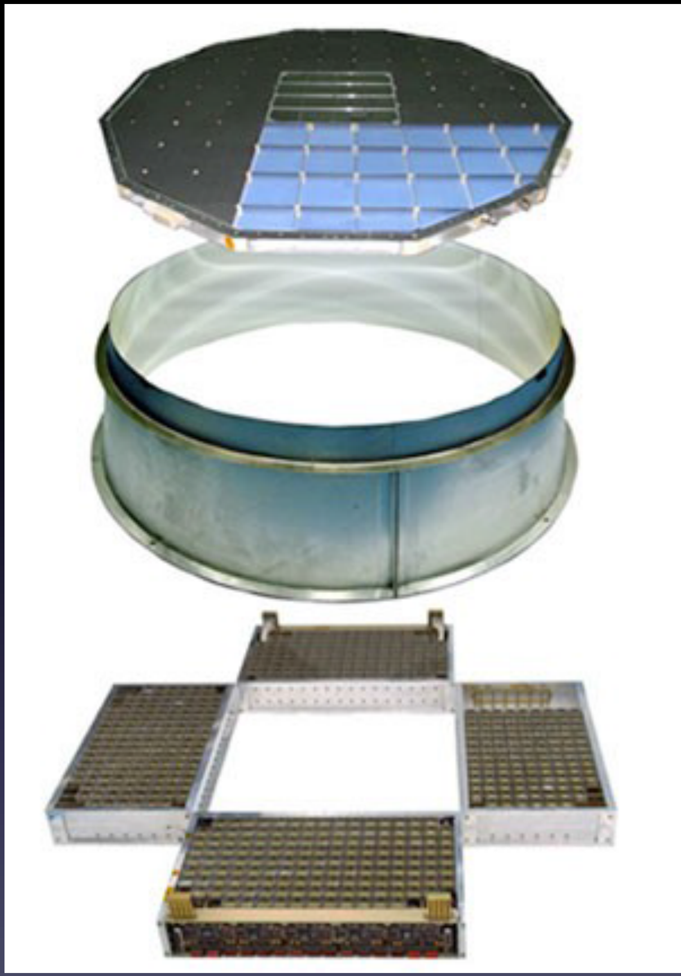


Ring Imaging Cherenkov counters (RICH)

- Particles pass through a radiator, the radiated photons may be directly collected by (or are focused by a mirror onto) a position-sensitive photon detector.
- The velocity is determined by a measurement of the radius r of the ring, on which the photons are detected



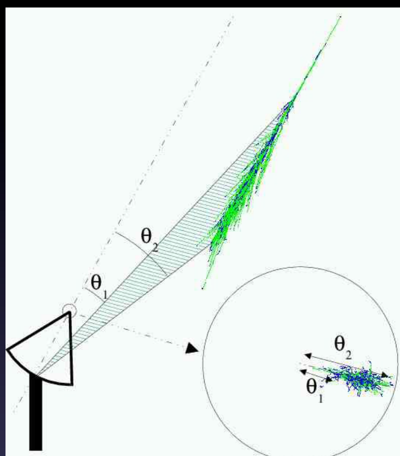


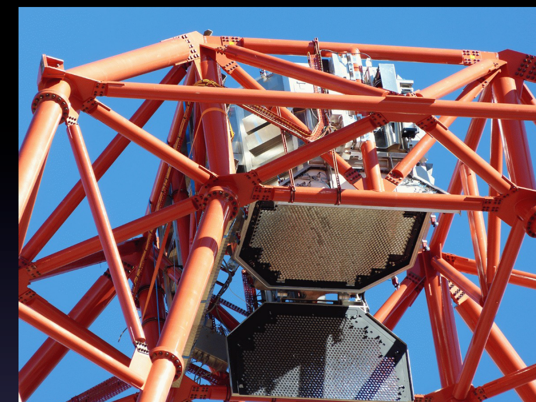


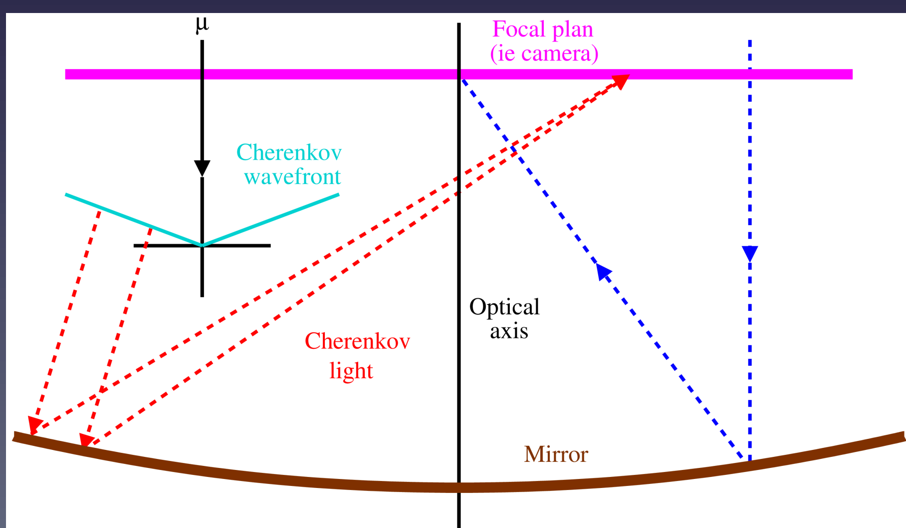
AMS RICH exploded view: the radiator, the conical mirror and the PMTs matrix

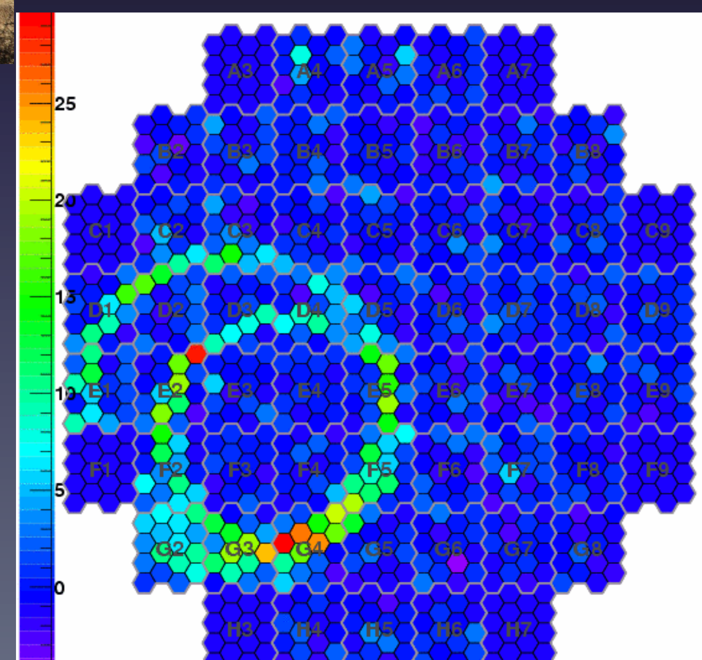
Cherenkov Radiations and ground based gamma-ray telescopes

- Principle of Air Cherenkov Telescope (ACT)





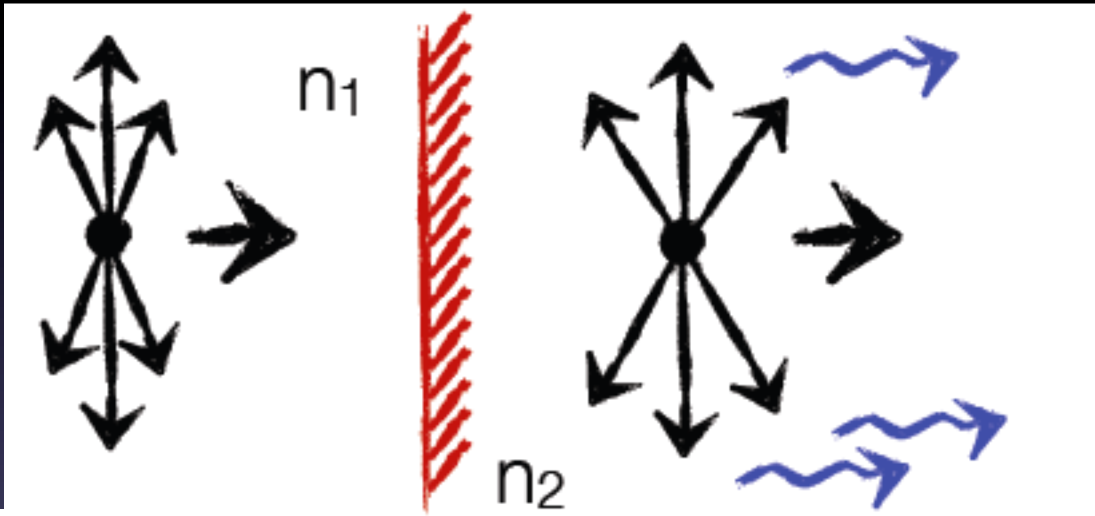


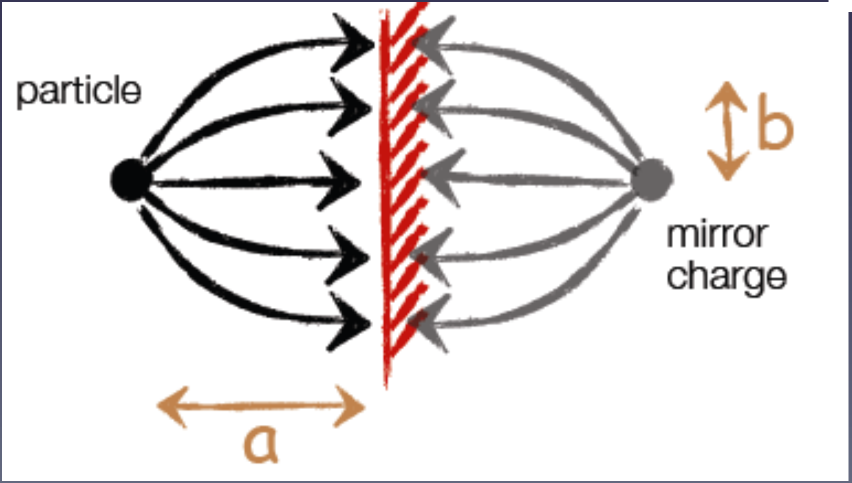


Transition radiation

- Transition radiation occurs if a relativist particle (large **γ**) passes the boundary between two media with different refraction indices ($n_1 \neq n_2$) [predicted by Ginzburg and Frank 1946; experimental confirmation 70ies]

- Effect can be explained by re-arrangement of electric field
- A charged particle approaching a boundary creates a dipole with its mirror charge

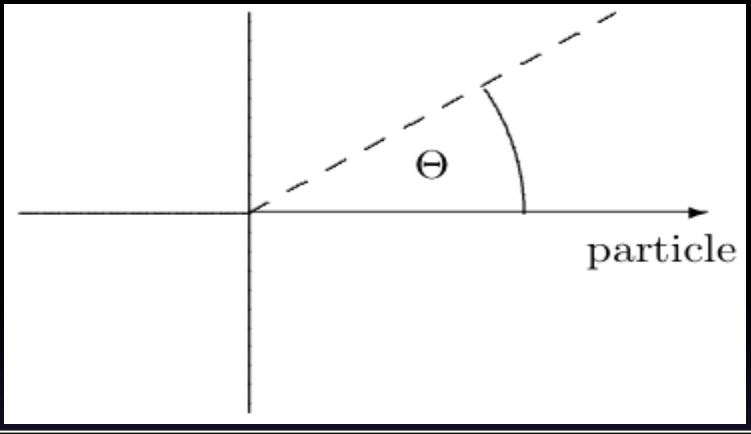




The time-dependent dipole field causes the emission of electromagnetic radiation

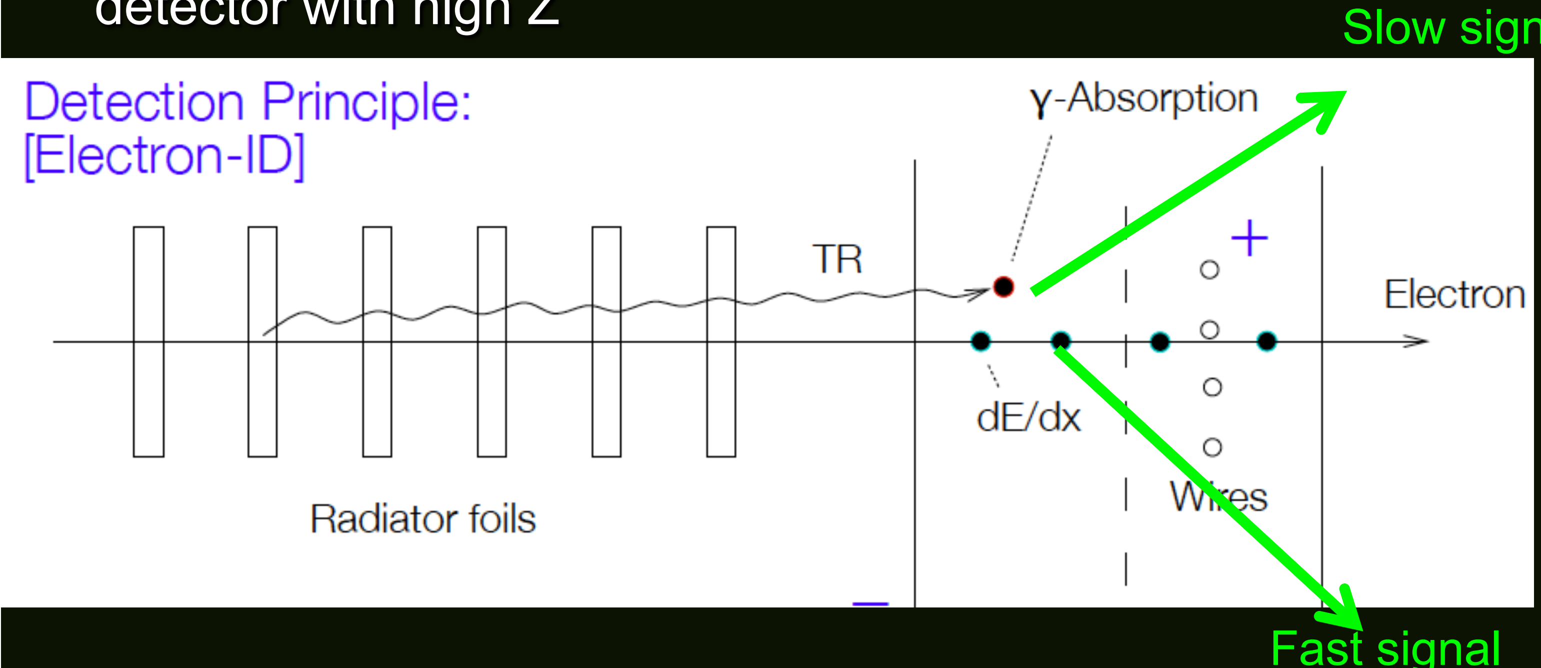
$$S = \frac{1}{3}\alpha z^2 \gamma \hbar \omega_P \quad (\hbar \omega_P \approx 28.8\sqrt{\frac{Z\rho}{A}} eV)$$

Transition Radiation



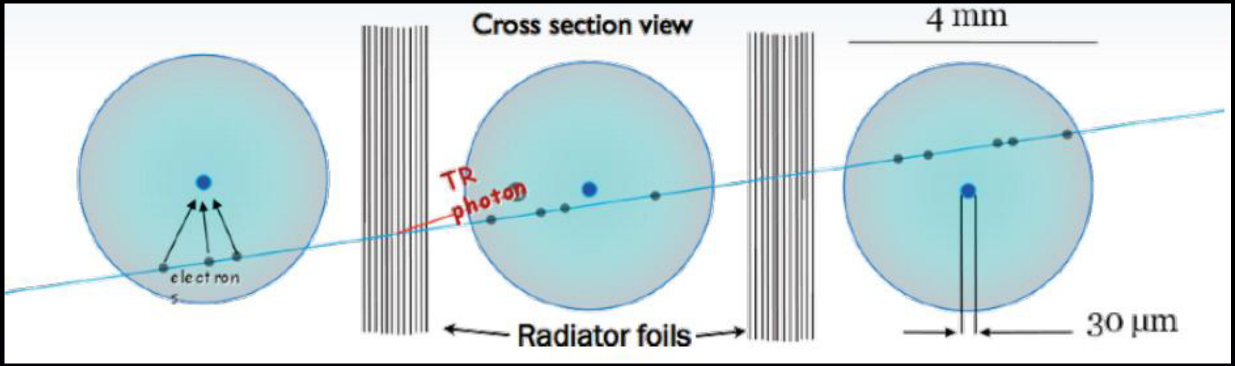
- Typical emission angle: $\theta = 1/\gamma$
- Energy of radiated photons: $\sim \gamma$
- Number of radiated photons: αz^2
- Effective threshold: $\gamma > 1000$

- Use stacked assemblies of low Z material with many transitions and a detector with high Z

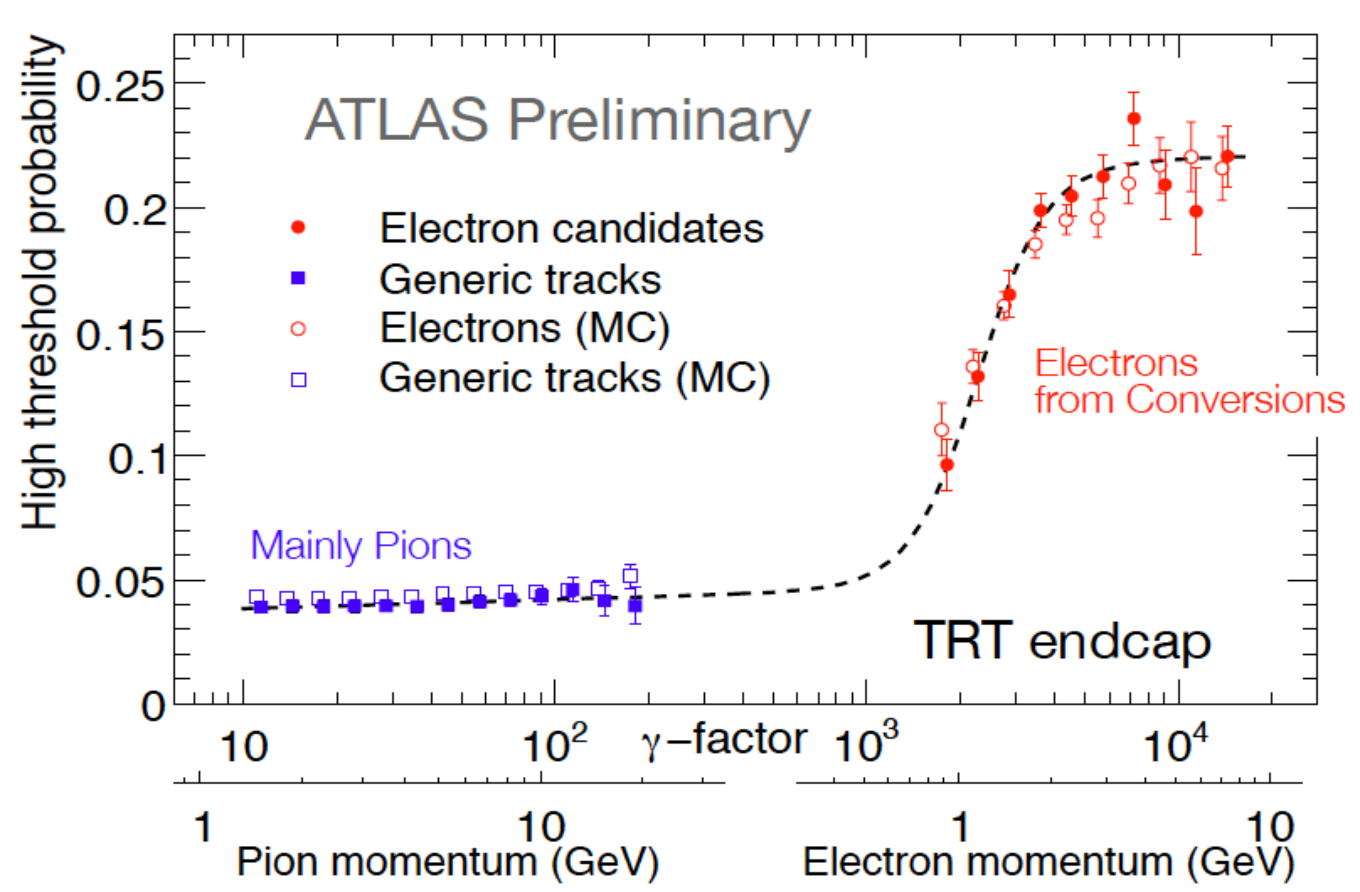


Note: Only X-ray (E>20keV) photons can traverse the many radiators without being absorbed

Transition radiation detector (ATLAS)

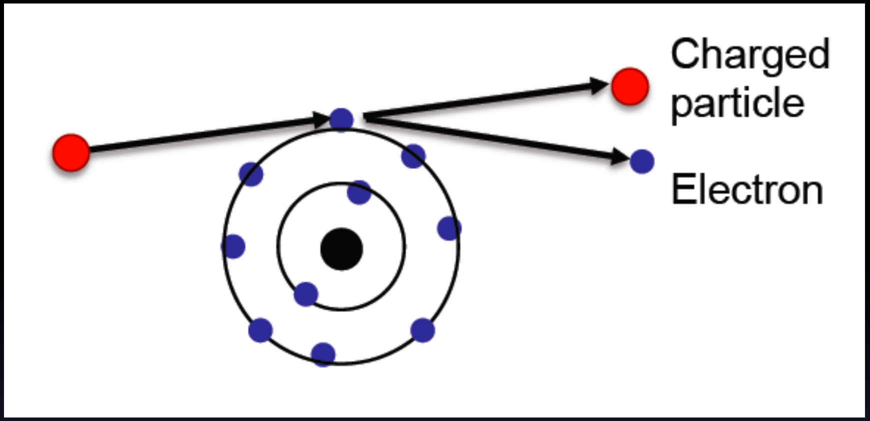


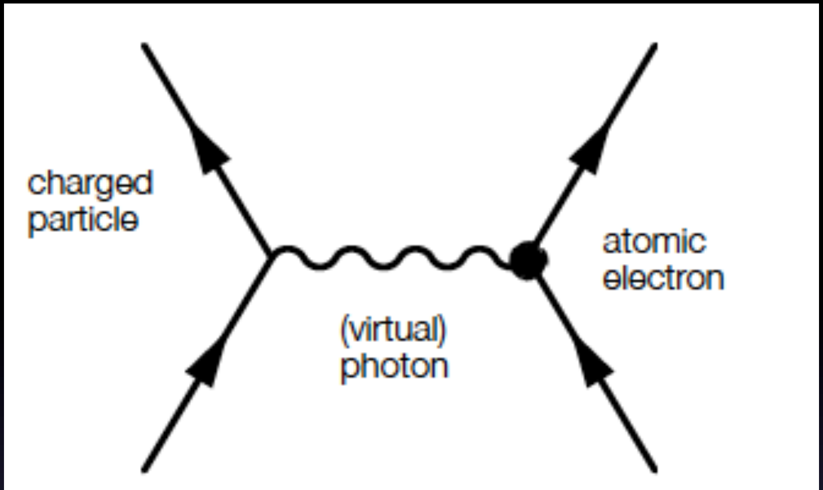
370,000 drift tubes. Each layer of straws interleaved with polypropylene as a radiator



- BACKUP information

Energy loss by ionization





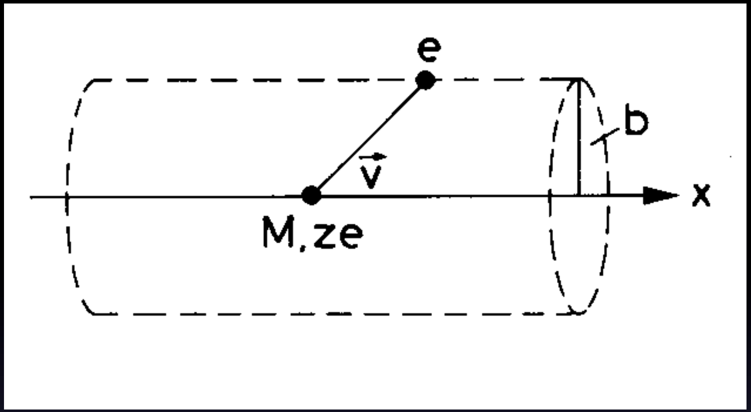
- First calculate for $Mc^2 \gg m_e c^2$:
- Energy loss for heavy charged particle [dE/dx for electrons more complex]
- The trajectory of the charged particle is unchanged after scattering

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

a= material dependent

Bohr's Classical Derivation [1913]

- Particle with charge Ze and velocity v moves through a medium with electron density n.
- Electrons considered free and initially at rest
- The momentum transferred to the electron is:



$$\Delta p_{\perp} = \int F_{\perp} dt = \int F_{\perp} \frac{dt}{dx} dx = \int F_{\perp} \frac{dx}{v}$$

$\Delta p_{\parallel}$: averages to zero because of symmetry

$$\text{Gauss'Law: } \int E_{\perp}(2\pi b)dx = 4\pi(ze)$$

$$\int E_{\perp} dx = \frac{4ze}{b}$$

$$F_{\perp} = eE_{\perp}$$

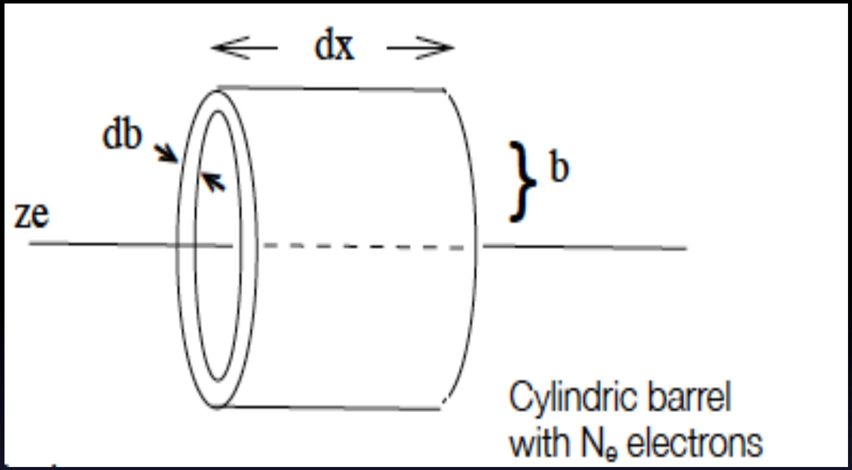
$$\Delta p_{\perp} = e\int E_{\perp} \frac{dx}{v}$$

$$\Delta p_{\perp} = \frac{2ze^2}{bv}$$

Bohr's Classical Derivation

- Energy transfer to a single electron with an impact parameter b

$$\Delta E(b) = \frac{\Delta p^2}{2m_e} \qquad \Delta p_\perp = \frac{2ze^2}{bv}$$



- Consider Cylindric barrel: N_e=**n**(2**π**b)·db dx
- Energy loss per path length dx for distance between b and b+db in medium with electron density n:

Energy loss

$$-dE(b) = \frac{\Delta p^2}{2m_e} 2\pi n b\, db\, dx = \frac{\left(2ze^2\right)^2}{2m_e\left(bv\right)^2} 2\pi n b\, db\, dx = \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b}dx$$

- Diverges for b➔0. Integrate in [b_{min}, b_{max}]

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2}\int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2} \ln\frac{b_{\max}}{b_{\min}}$$

Bohr's Classical Derivation

- Determination of relevant range [b_{min}, b_{max}]:
- [Arguments: $b_{min} > \lambda_e$, i.e. de Broglie wavelength; $b_{max} < \infty$ due to screening ...]

$$b_{\min} = \lambda_e = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_e v}$$

$$b_{\min} = \frac{\gamma v}{\langle v_e \rangle} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e c^2 \beta^2} n \ln \frac{m_e c^2 \beta^2 \gamma}{2\pi\hbar \langle v_e \rangle}$$

Deviates by factor 2 from QM derivation

Electron density n=**NA**·**ρ**·**Z/A**

Effective Ionization potential I=h <$\mathbf{v}_e$>

Bohr Calculation of dE/dx

- Stopping power

$$-\frac{dE}{dx}=\frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2}\ln\frac{b_{\max}}{b_{\min}}$$

- Determination of the relevant range [b_{min}, b_{max}]:
  - b_{min} : Maximum kinetic energy transferred Bohr formula

$$W_{\max}=\frac{1}{2}\gamma^2 m_e(2v)^2=2m_e c^2\beta^2\gamma^2 \qquad b_{\min}=\frac{ze^2}{\gamma m_e v^2}$$

  - b_{max} :particle moves faster than e in the atomic orbit. Electrons are bound to atoms with average orbital frequency <v_e>. Interaction time has to be ≤ <1/v_e>

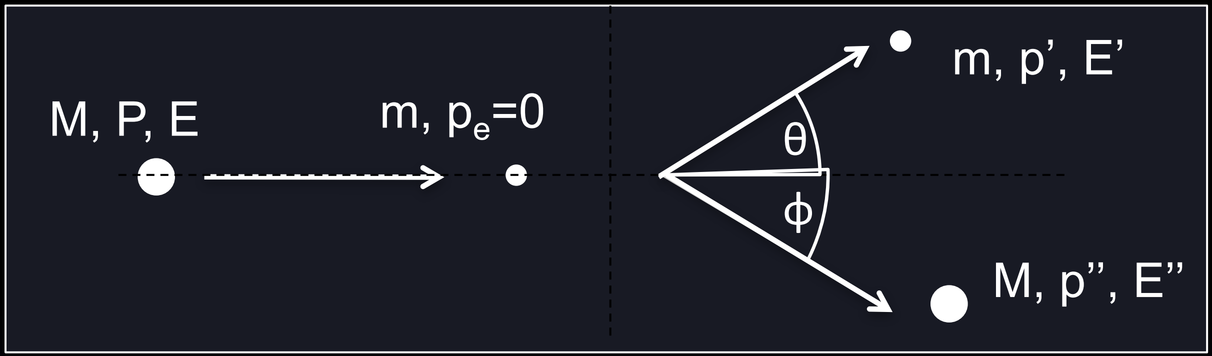
$$b_{\max}=\frac{\gamma v}{\langle v_e\rangle}$$

or distance at which the kinetic energy transferred is minimum W_{min}= I (mean ionization potential)

- We can integrate in this interval an derive the classical Bohr formula

$$-\frac{dE}{dx}=\frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2}\ln\frac{\gamma^2 m v^3}{ze^2\langle v_e\rangle}=\frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2}\ln\left(\frac{2m_e c\beta^2\gamma^2}{I}\right)$$

Relativistic Kinematic



$$\text{Energy conservation: } \sqrt{p^2c^2+M^2c^4}+mc^2=\sqrt{p''^2c^2+M^2c^4}+\sqrt{p'^2c^2+m^2c^4}$$

$$\text{momentum conservation: } p=p'\cos\theta+p''\cos\phi$$

$$0=p'\sin\theta+p''\sin\phi$$

$$p''^2=p'^2+p^2-2pp'\cos\theta$$

- Using energy and momentum conservation we can find the kinetic energy

$$\varepsilon'=\sqrt{p'^2c^2+m^2c^4}-mc^2=\frac{2mc^2p^2c^2\cos^2\theta}{mc^2+\sqrt{p^2c^2+M^2c^4}-p^2c^2\cos^2\theta}$$

- The maximum energy transfer is

$$\varepsilon'_{\max}=\frac{2mp^2}{m^2+M^2+2mE/c^2}$$

A few examples

What is the deposited energy for a 10 GeV μ passing through a 1 cm thick scintillator ?

p= γmv = βγmc ⇒βγ=p/mc ~ 10 GeV / 106 MeV ~ 100 ® 10 GeV μ ≡ MIP

dE/dx' ~ 2 MeV g-1 cm2 ⇒ ΔE = ρ dE / dx' Δx with ρ ~ 1 g / cm3 for plastic scintillator

⇒ ΔE ~ 1 g / cm3 °— 2 MeV / g cm2 °— 1 cm = 2 MeV

What is the deposited energy for a 10 GeV μ passing through a 1 cm thick cloud chamber ?

ρ ~ 0.001 g / cm3 ⇒ ΔE = 0.001 °— 2 °— 1 = 2 keV

What should be the thickness of a concrete wall (ρ ~ 2.5 g / cm3) to stop a 450 GeV proton beam ?

βγ=p/mc ~ 450 ® 450 GeV p ≡ MIP ⇒ dE/dx' ~ 2 MeV g-1 cm2 ⇒ ΔE = 5 MeV/cm

⇒ e = 450 000/5 = 90 000 cm = 900 m

Nb : nuclear interactions have been neglected here …

What thickness of air (ρ ~ 1 g / cm3 = 10-3 g / l) stops a 30 MeV/c α particle ?

βγ=p/mc ~ 30 MeV/ 3700 MeV = 10-2 ≪ 3 !

⇒ dE/dx' ~ 8 MeV g-1 cm2 °— (0.01)-5/3 ~ 17 235 MeV g-1 cm2

⇒ ΔE = 17 MeV/cm in air ⇒ e = 30 MeV/ 17 MeV ~ 1.74 cm

Cherenkov Radiation – Momentum Dependence

- Cherenkov angle θ and number of photons N grows with β
- Asymptotic value for β=1: $\cos \theta_{max} = 1/n$; $N_\infty = x \cdot 370 / \text{cm} \, (1-1/n^2)$