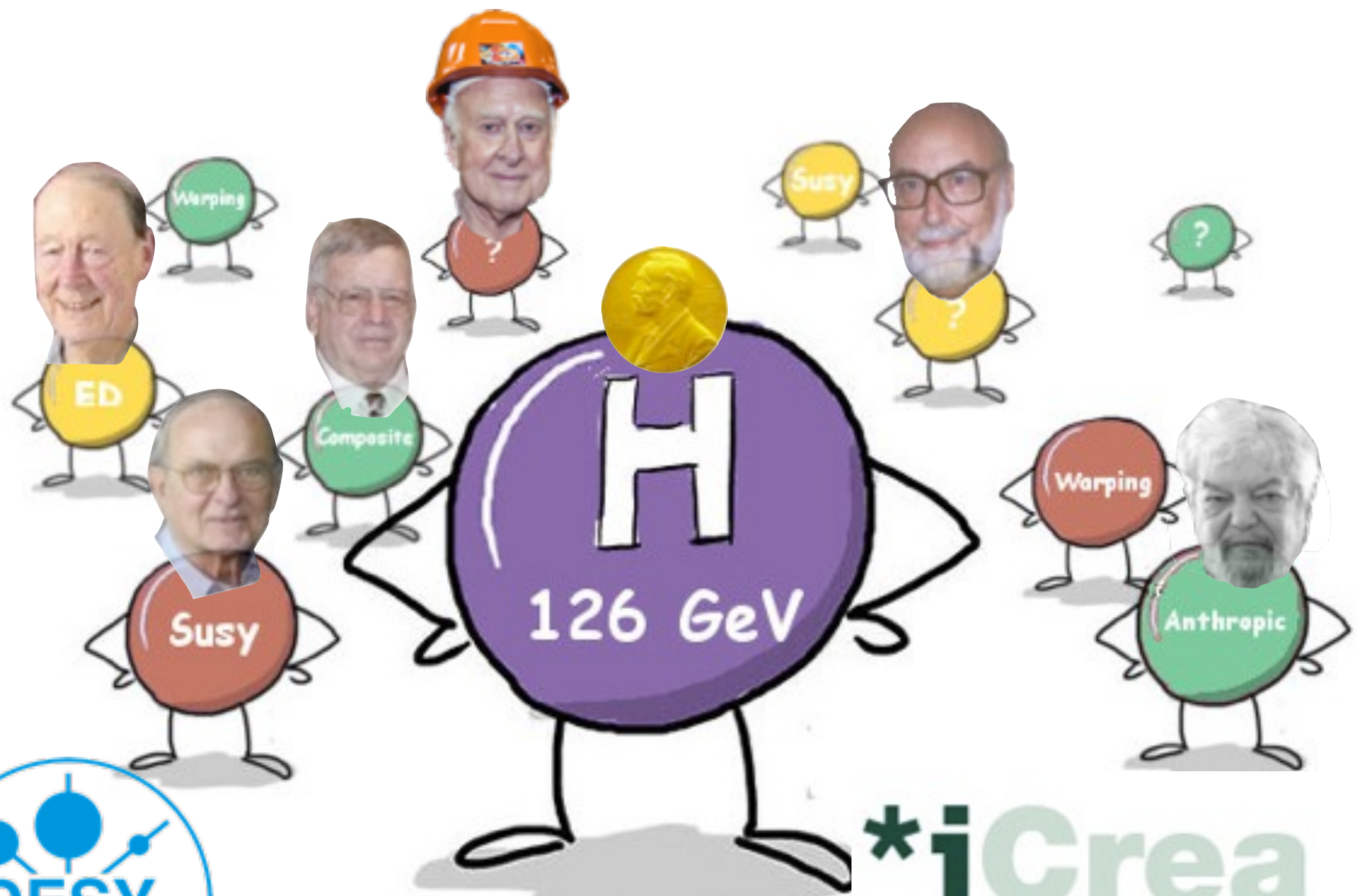
Beyond the Standard Model

CERN summer student lectures 2015



Lecture 4/5

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DESY

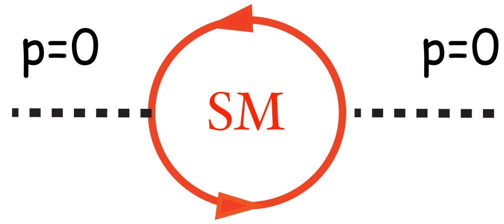
*iCrea INSTITUCIÓ CATALANA DE RECERCA I ESTUDIS AVANÇATS

Outline

- Monday
  - general introduction, units
- Tuesday
  - Higgs physics as a door to BSM
- Wednesday
  - Higgs and Naturalness: small and large numbers in a quantum world
- Thursday
  - grand unification, proton decay
  - supersymmetry
  - extra dimensions
- Friday
  - cosmological interplay

Higgs couplings as a test of naturalness

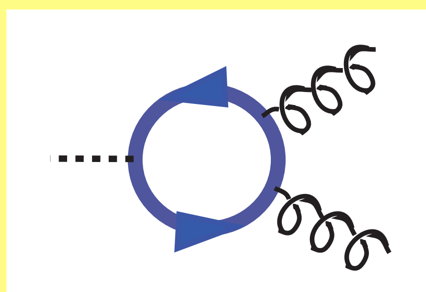
$$-(125\ \text{GeV})^2 \left(\frac{\Lambda}{600\ \text{GeV}}\right)^2 \qquad\qquad \frac{g_*^2}{16\pi^2}\Lambda^2$$

$\delta m_H^2 =$  $+$  $\sim m_H^2$

generically

charged particles

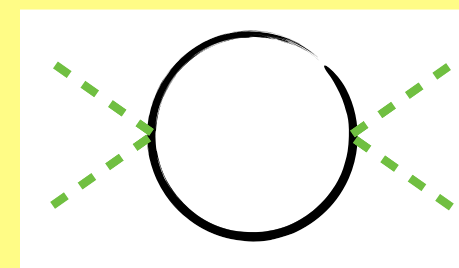
neutral particles



$$\frac{g_s^2 g_*^2}{16\pi^2}\frac{1}{m_*^2}|H|^2 G_{\mu\nu}^2 \qquad \frac{e^2 g_*^2}{16\pi^2}\frac{1}{m_*^2}|H|^2 F_{\mu\nu}^2$$

$$\frac{\Delta BR(h\to\gamma\gamma, Z\gamma, gg)}{\text{SM}} \sim \frac{g_*^2 v^2}{m_*^2}$$

Colorful naturalness probed @ LHC



$$\frac{g_*^2}{16\pi^2}\frac{1}{m_*^2}\left(\partial_\mu |H|^2\right)^2$$

$$BR(h\to ii) = BR_{\text{SM}} \qquad \Gamma = \left(1 - \frac{g_*^2 v^2}{16\pi^2 m_*^2}\right)\Gamma_{\text{SM}}$$

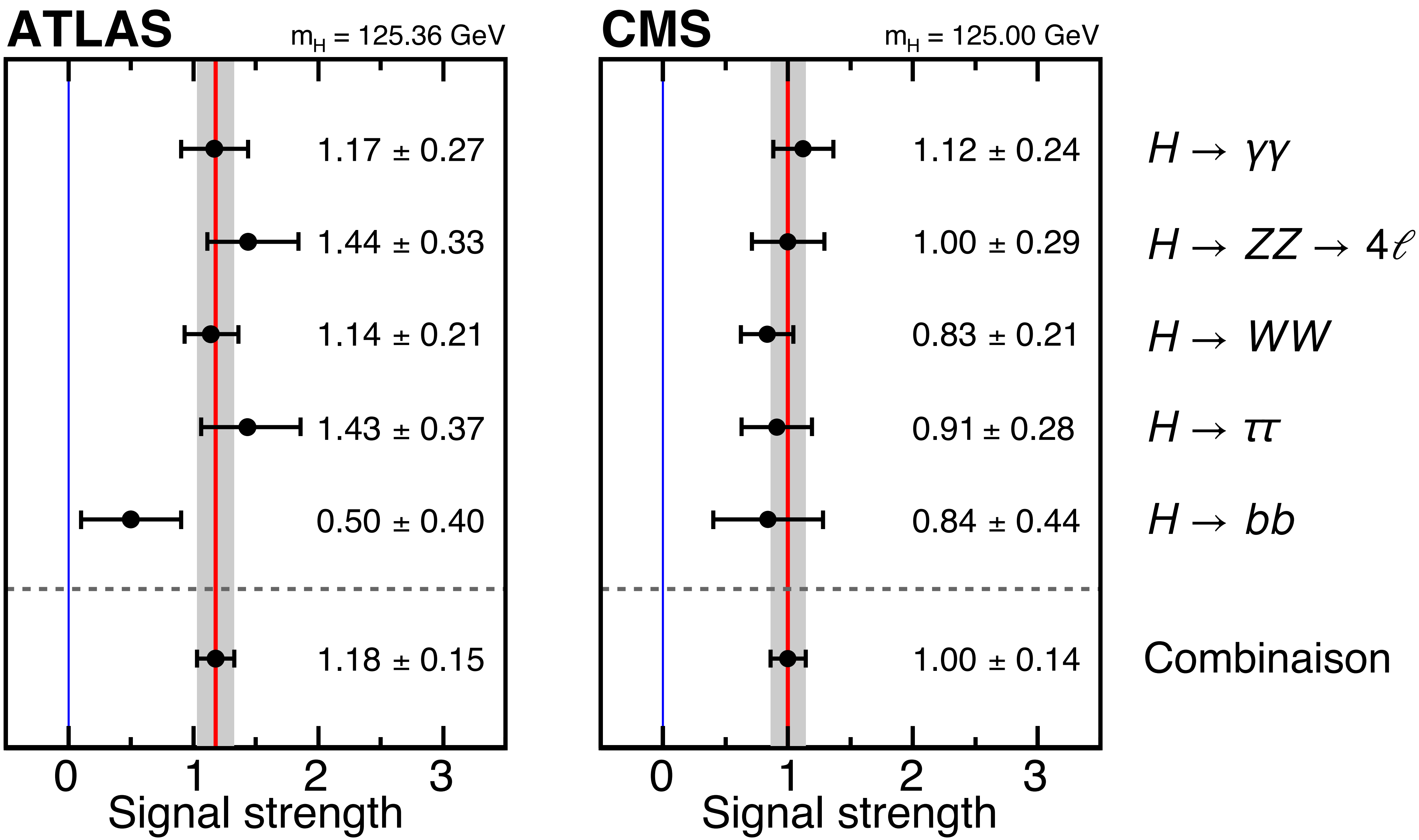
$$\delta\sigma_{Zh} = -\frac{g_\star^2}{8\pi^2}\frac{v^2}{m_\star^2}$$

nice to be able to measure Zh & Γ

Neutral naturalness (invisible?) @ LHC

Higgs couplings measurements

the precise characterization of the Higgs is on its way



Higgs couplings measurements

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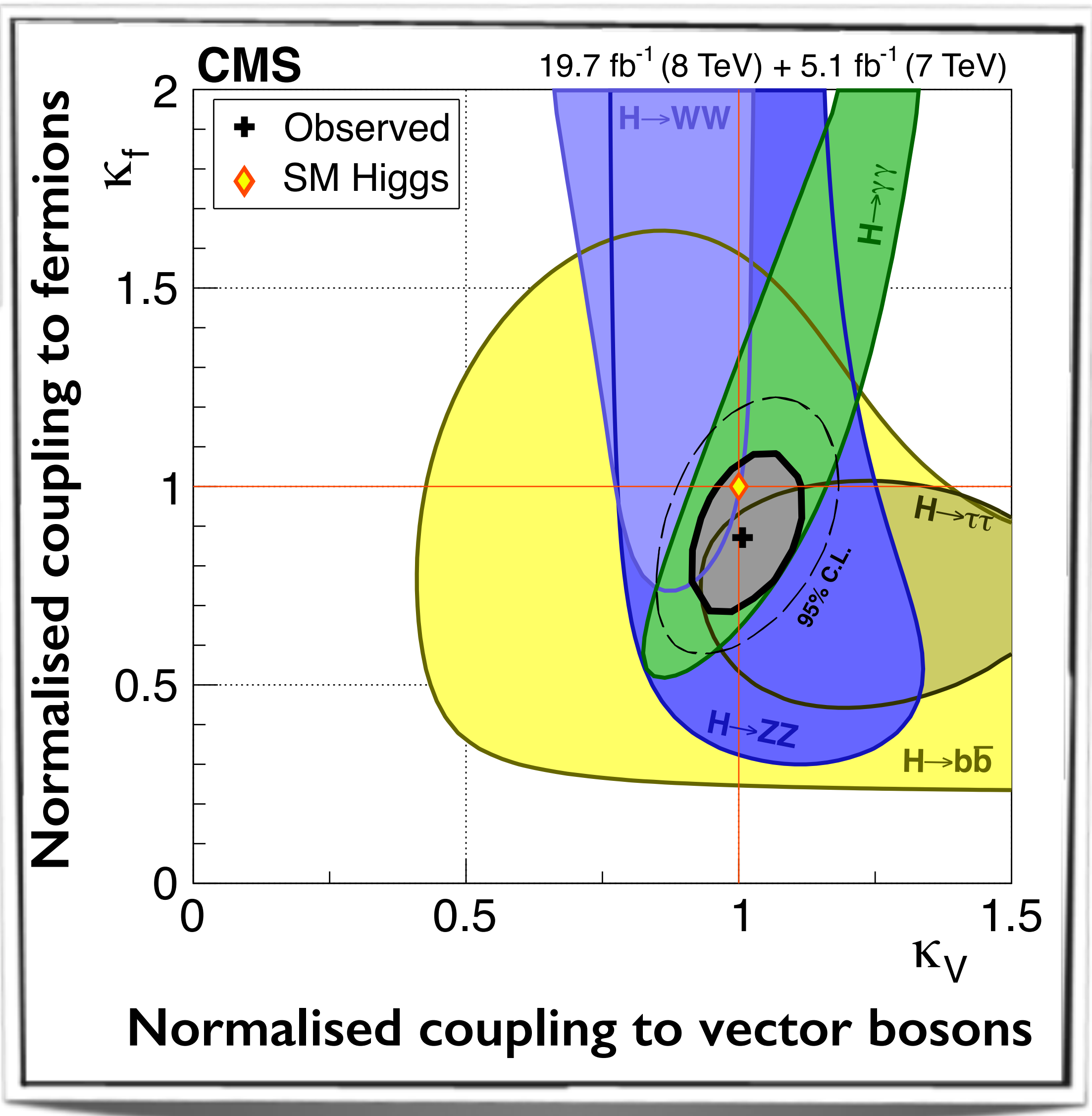
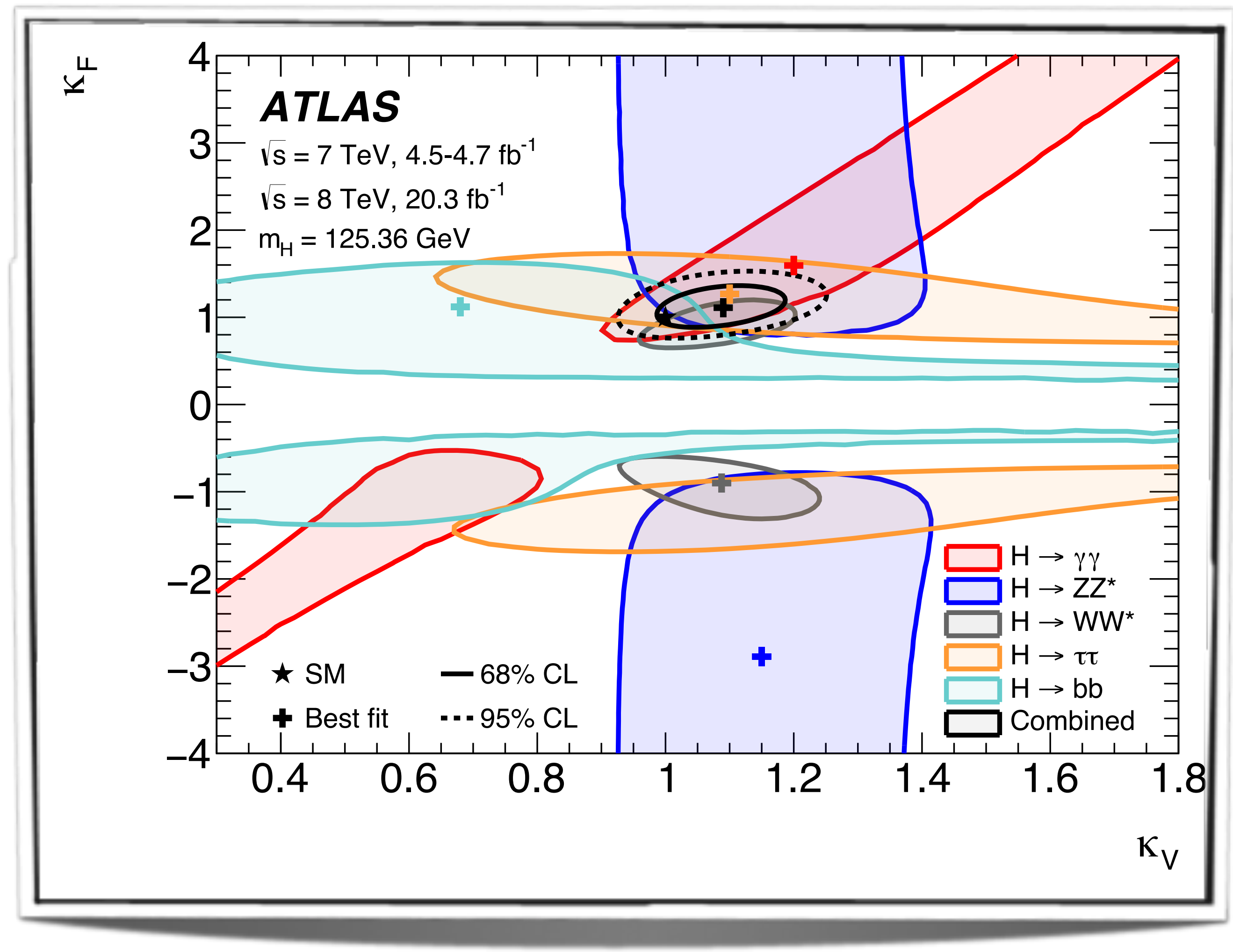
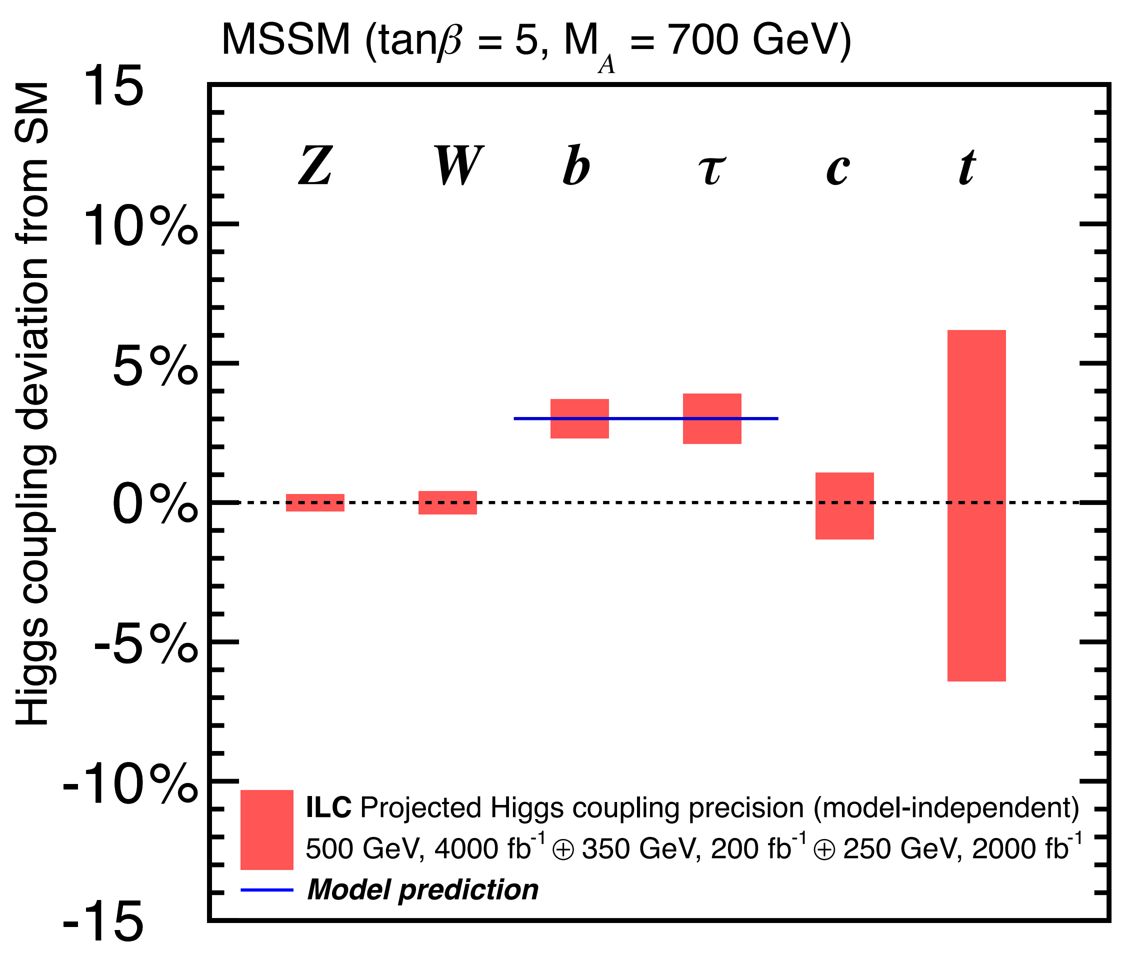




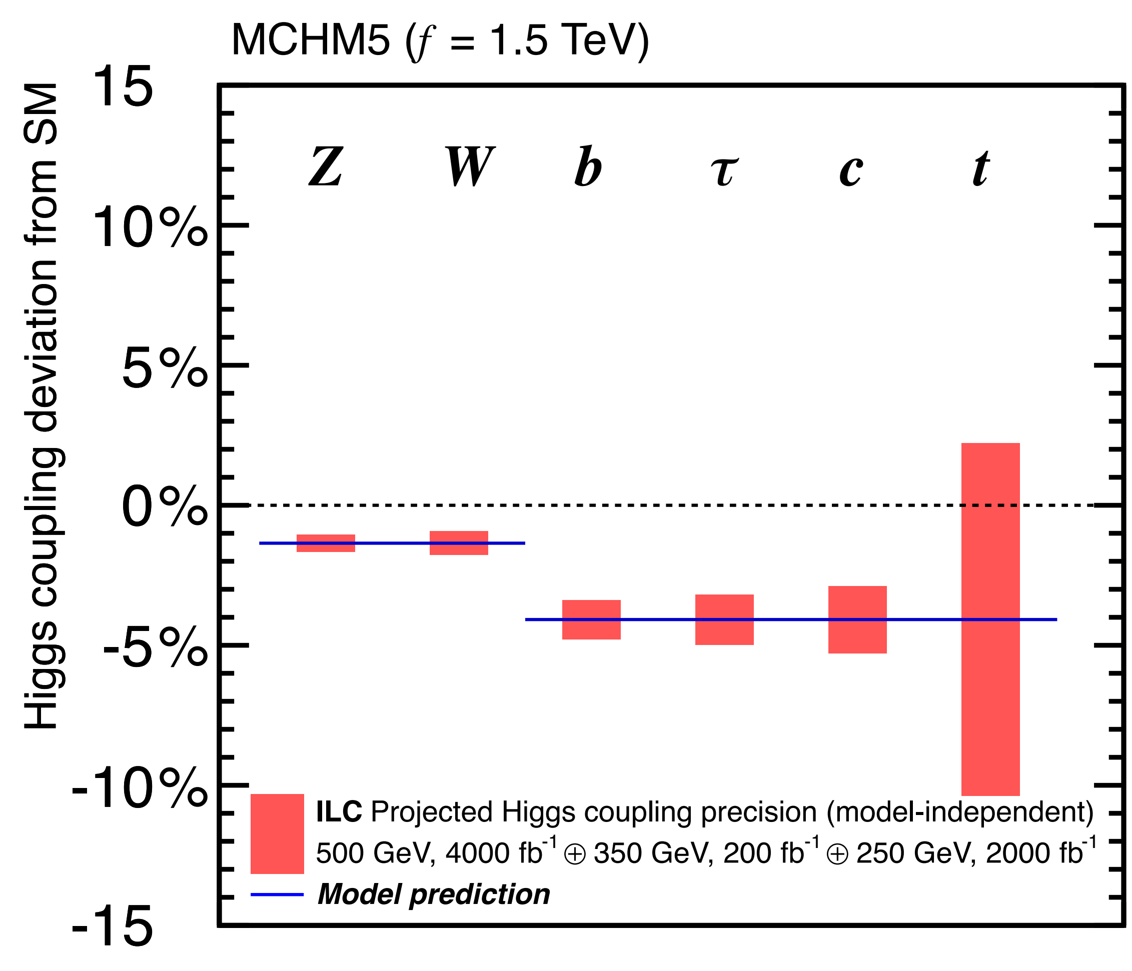
Higgs couplings and model discriminations

The pattern of Higgs coupling deviations is a signature of the underlying dynamics beyond the Standard Model

Supersymmetry
(MSSM)



Composite Higgs
(MCHM5)



ILC Physics WG, '15

Higgs couplings and model discriminations

The pattern of Higgs coupling deviations is a signature of the underlying dynamics beyond the Standard Model

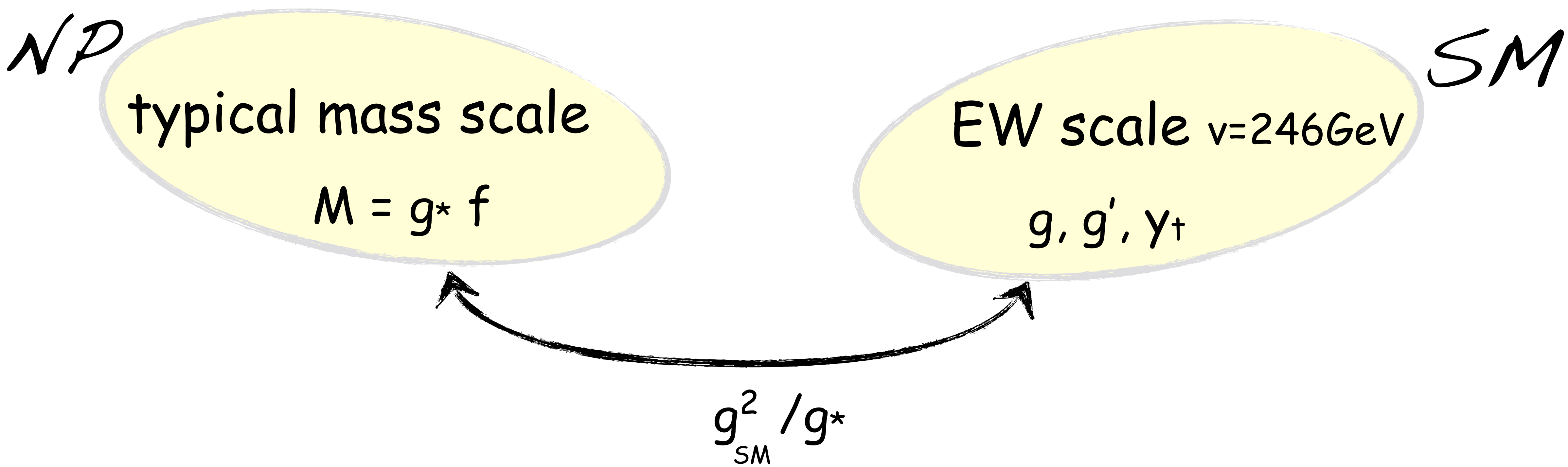
~~ expected largest relative deviations ~~

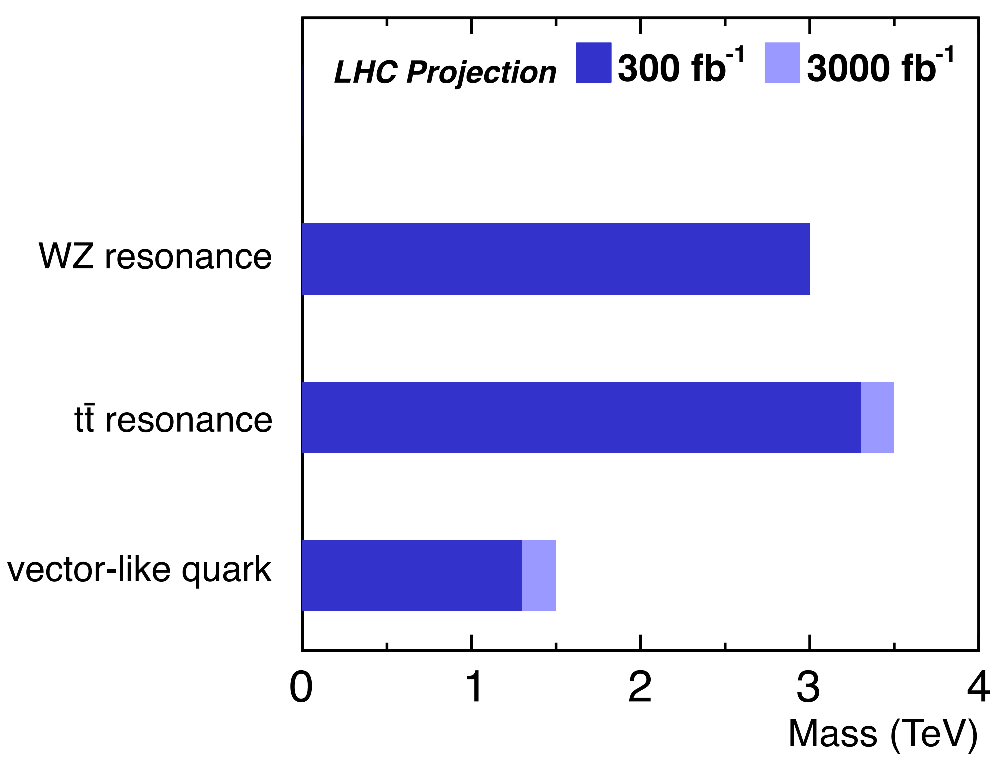
	hff	hVV	hγγ	hγZ	hGG	h^3
MSSM	√		√	√	√	
NMSSM	√	√	√	√	√	
PGB Composite	√	√		√		√
SUSY Composite	√	√	√	√	√	√
SUSY partly-composite			√	√	√	√
"Bosonic TC"						√
Higgs as a dilaton			√	√	√	√

A. Pomarol, Naturalness '15

Higgs & New Physics

Precision /indirect searches (high lumi.) vs. direct searches (high energy)





LHC direct search

- Precision Higgs study: $\xi \equiv \frac{\delta g}{g} = \frac{v^2}{f^2}$
- Direct searches for resonances: $m_\rho \approx g_* f$

Which one is doing best?
it depends on value of g_*

Higgs & New Physics

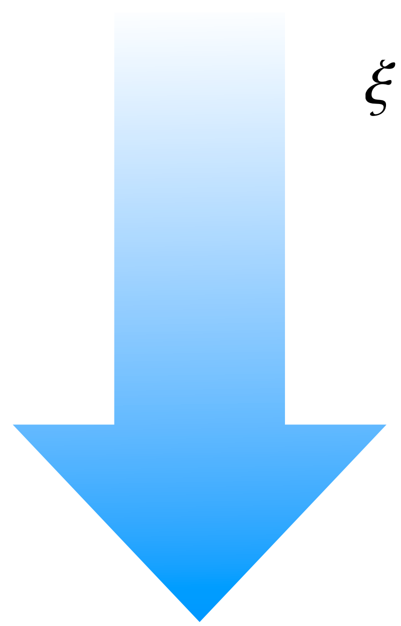
Precision /indirect searches (high lumi.) vs. direct searches (high energy)

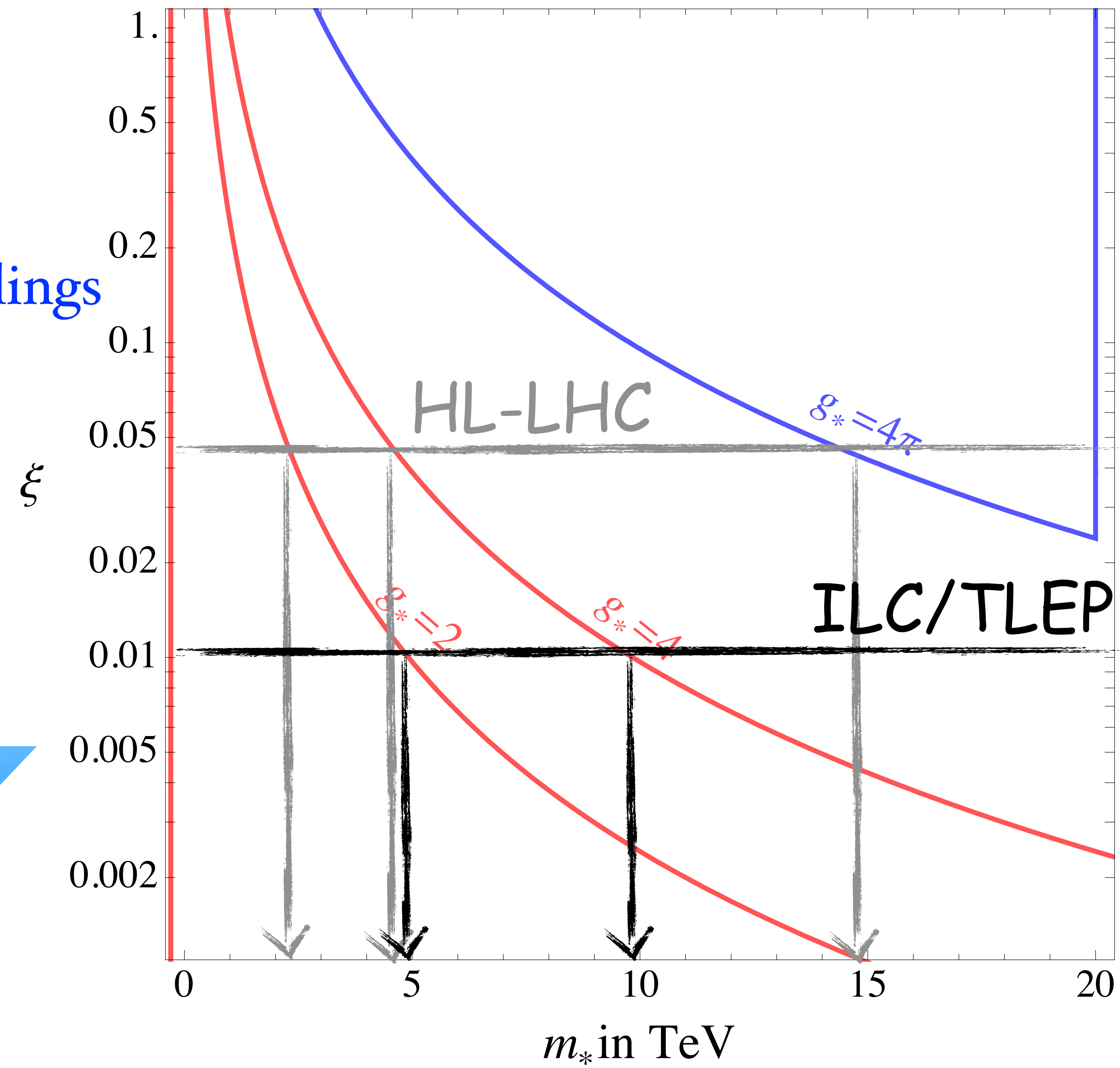
Mass reach:

ee machines are best to test strongly coupled NP

pp machines are best to test weakly coupled NP

Higgs couplings





Rattazzi, BSM@100TeV, CERN '14

direct searches

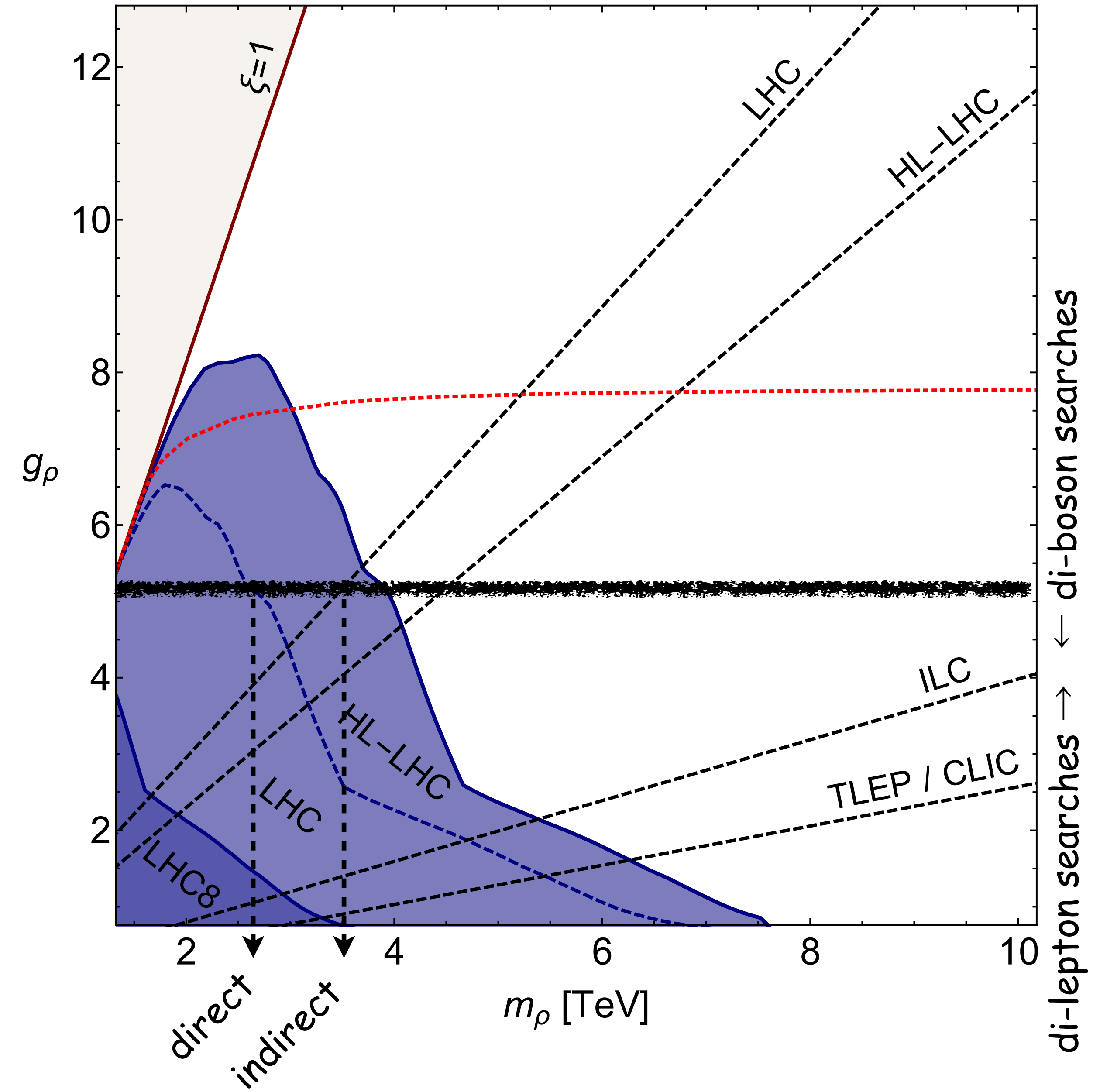
Higgs & New Physics

Precision /indirect searches (high lumi.) vs. direct searches (high energy)

DY production xs of resonances decreases as $1/g_\rho^2$

Torre, Thamm, Wulzer '15

Collider	Energy	Luminosity	ξ [1σ]
LHC	14 TeV	300 fb^{-1}	$6.6 - 11.4 \times 10^{-2}$
LHC	14 TeV	3 ab^{-1}	$4 - 10 \times 10^{-2}$
ILC	250 GeV + 500 GeV	250 fb^{-1} 500 fb^{-1}	4.8-7.8 $\times 10^{-3}$
CLIC	350 GeV + 1.4 TeV + 3.0 TeV	500 fb^{-1} 1.5 ab^{-1} 2 ab^{-1}	2.2 $\times 10^{-3}$
TLEP	240 GeV + 350 GeV	10 ab^{-1} 2.6 ab^{-1}	2 $\times 10^{-3}$



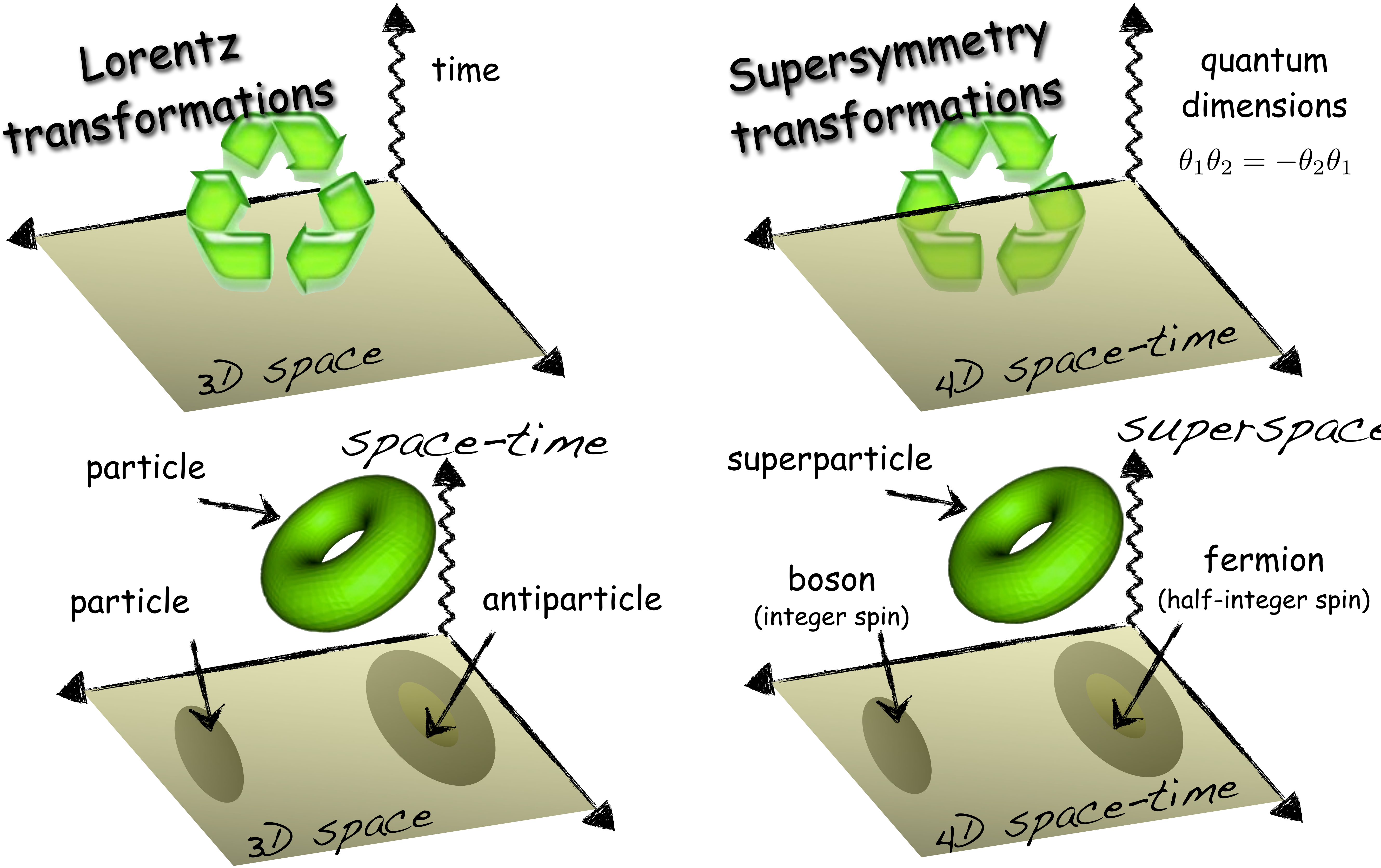
- **complementarity:**
  - direct searches win at small couplings
  - indirect searches probe new territory at large coupling

e.g.
indirect searches at LHC over-perform direct searches for g > 4.5
indirect searches at ILC over-perform direct searches at HL-LHC for g > 2

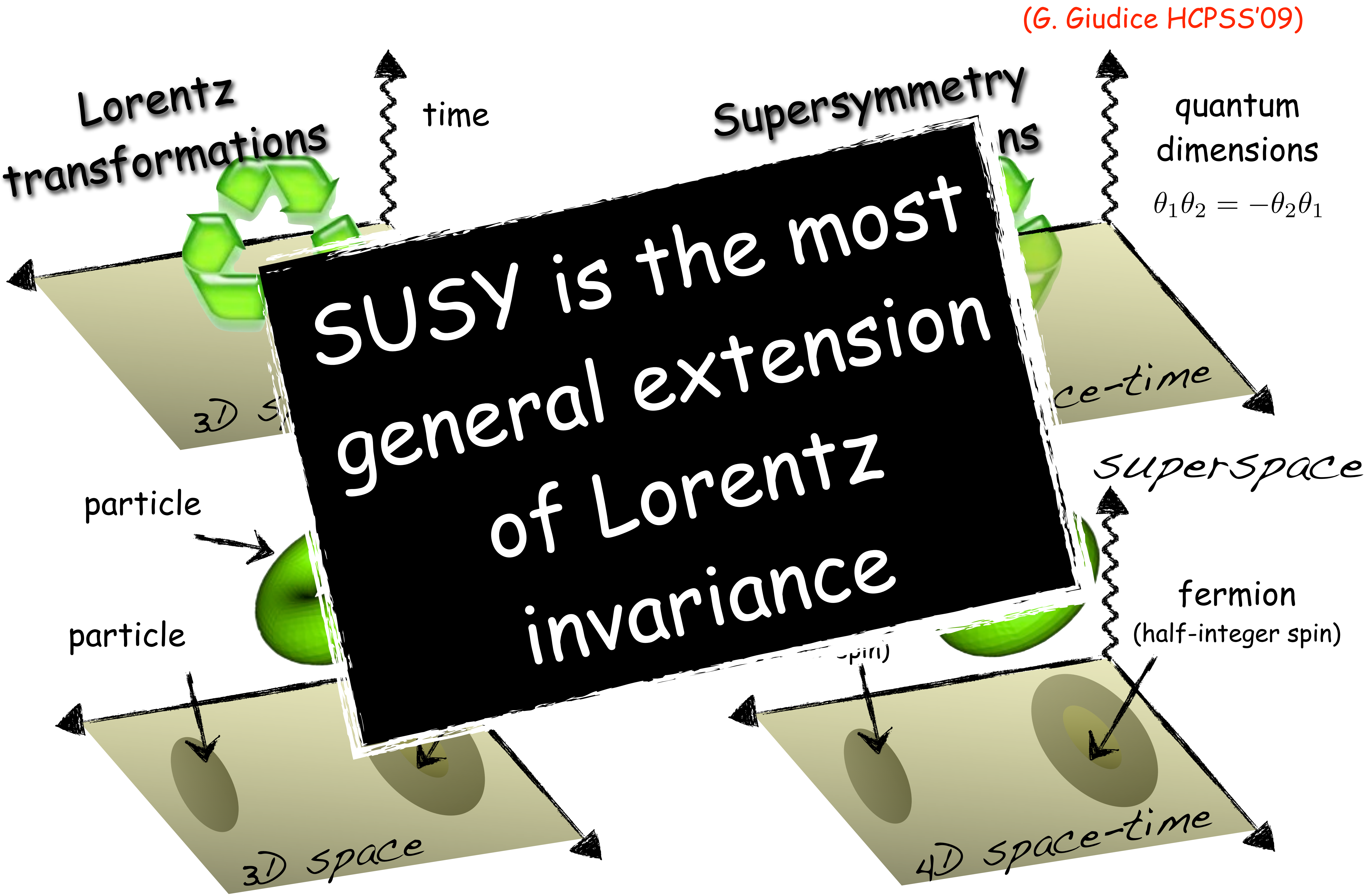
Supersymmetry

SUSY: a quantum space-time

(G. Giudice HCPSS'09)



SUSY: a quantum space-time



SUSY 1.0.1

Wess, Zumino '74

fermion ⇔ boson

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$

- susy transformations:

$$\delta\phi = \bar{\epsilon}\psi$$
$$\delta\psi = -i\left(\gamma^\mu \partial_\mu \phi\right)\epsilon$$

$$\delta\mathcal{L} = \text{total derivative}$$

- susy algebra:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = -i\left(\bar{\epsilon_2}\gamma^\mu \epsilon_1\right)\partial_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

susy^2 = 4D translation

How to introduce interactions?

Superspace

$$\left(x^\mu, \theta, \bar{\theta}\right)$$

usual 4D space-time coordinates → x^μ

new fermionic/Grassmanian coordinates → $\theta, \bar{\theta}$

A general superfield can be Taylor-expanded in the superspace

$$F(x,\theta,\bar\theta) = f(x) + \theta\chi(x) + \bar\theta\bar\chi(x) + \theta\theta m(x) + \bar\theta\bar\theta\bar m(x) + \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta d(x)$$

complex spin-0 fields:	$f(x), m(x), \bar m(x), d(x)$	4x2=8 real off-shell degrees of freedom
complex spin-1 fields:	$v_\mu(x)$	1x8=8 real off-shell degrees of freedom
Weyl spin-1/2 fields:	$\chi(x), \bar\chi, \lambda(x), \bar\lambda(x)$	4x4=16 real off-shell degrees of freedom

- **Chiral superfield** $\bar D_{\dot\alpha} F = 0$

  covariant derivative

  ie commute with supersymmetry

  ⇒ $F = \phi(x) + \theta\psi(x) + \theta\theta f(x)$

	$\phi(x)$	$\psi(x)$	$f(x)$
off-shell dof	2	4	2
on-shell dof	2	2	0

- **Vector superfield**

  $F = F^\dagger$ ⇒ $F = \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta d(x)$

	$v_\mu(x)$	$\lambda(x)$	$d(x)$
off-shell dof	3	4	1
on-shell dof	2	2	0

MSSM - Matter Content

		particles	Sparticles	
Chiral superfields	quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R d_R	squarks	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$ $\tilde{u}_R$ $\tilde{d}_R$
	leptons	$\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$ e_R	sleptons	$\begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}$ $\tilde{e}_R$
	Higgs doublets	H_1 (hypercharge $= -1$) H_2 (hypercharge $= +1$)	Higgsinos	$\tilde{H}_1$ $\tilde{H}_2$
vector superfields		$W^\pm_\mu, W^3_\mu$	winos	$\tilde{\omega}^\pm, \tilde{\omega}^3$
		B_μ	bino	$\tilde{b}$
		$G^A_\mu \quad A = 1, \ldots, 8$	gluinos	$\tilde{g}^A$

(G. Giudice HCPSS'09)

SUSY Interactions - Superpotential

superpotential W = holomorphic fct of chiral superfields

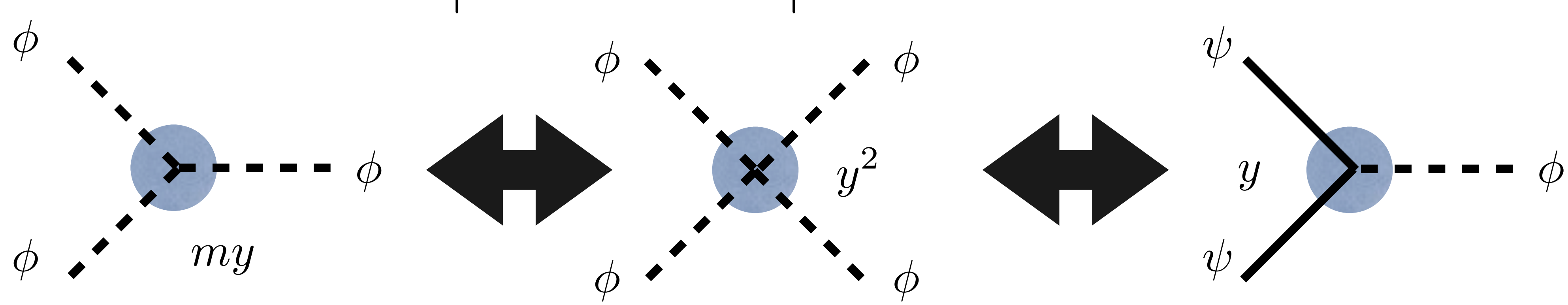
$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \left|\frac{\partial W}{\partial \phi}\right|^2_{|\theta=0} - \frac{1}{2}\frac{\partial^2 W}{\partial \phi^2}_{|\theta=0} \psi\psi + h.c.$$

is invariant under susy

example: susy Yukawa interaction

$$W = \frac{1}{2}m\phi^2 + \frac{1}{3!}y\phi^3 \qquad \partial_\phi W = m\phi + \frac{1}{2}y\phi^2 \qquad \partial^2_\phi W = m + y\phi$$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \left|m\phi + \frac{1}{2}y\phi^2\right|^2 - \frac{1}{2}\left(m + y\phi\right)\psi\psi + h.c.$$



will be modified by soft susy breaking

will survive soft susy breaking

MSSM Superpotential

the most general ("renormalizable") superpotential of the MSSM

$$W = H_u Q D + H_u Q U + H_d L E + \mu H_u H_d + LQD + UDD + LLE + \mu_L L H_u$$

$\not{B}, \not{L}$

lead to fast p decay

R parity forbids all the dangerous terms

superfields

$Q, D, U, L : -1$

$H_u, H_d : +1$

R-parity

doesn't commute with susy

$\theta : -1$

fields

$\phi_{\rm SM} : +1$

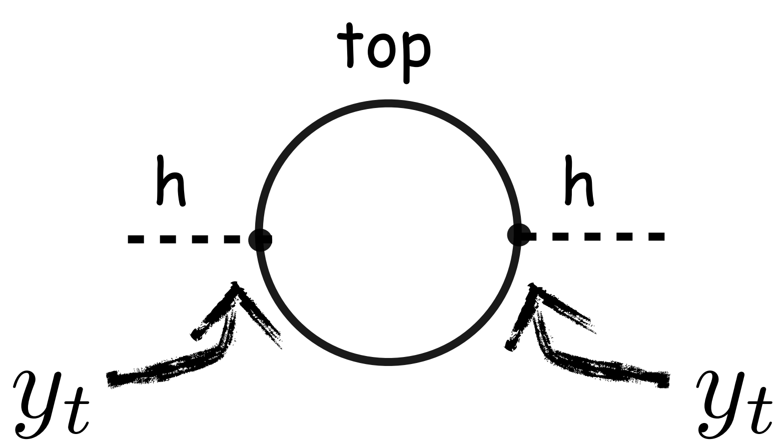
$\phi_{\rm superpartner} : -1$

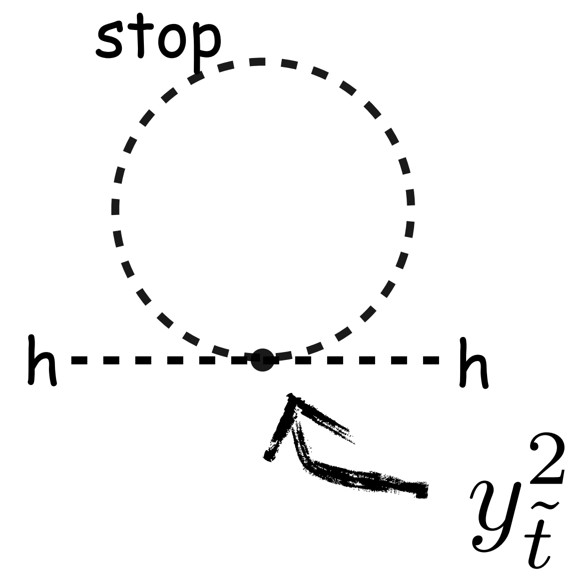
nice consequences:

- superpartners are pair-produced
- Lightest Supersymmetric Particle is stable → DM?

SUSY and the (big) hierarchy problem

(DE Kaplan HCPSS'07)





$$\delta m_H^2 \propto \left(y_t^2 - y_{\tilde{t}}^2\right)\Lambda^2 + \left(m_t^2 - m_{\tilde{t}}^2\right)\log\Lambda$$

$y_t \neq y_{\tilde{t}}$	$m_t \neq m_{\tilde{t}}$
⇩	⇩
Λ^2 dv	$\log\Lambda$ dv
hard susy breaking	soft susy breaking

SUSY biggest pb: how to dynamically generate soft breaking terms compatible with exp constraints?

SUSY little hierarchy problem

SUSY needs new (super)particles that haven't been seen (yet?)

SUSY (at least MSSM) predicts a (very) light Higgs

$$V = (|\mu|^2 + m_{H_u}^2)\left|H_u^0\right|^2 + (|\mu|^2 + m_{H_d}^2)\left|H_d^0\right|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8}\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2$$

$$m_h^2 = m_Z^2 \cos^2 2\beta$$

tree-level

excluded

SUSY little hierarchy problem

SUSY needs new (super)particles that haven't been seen (yet?)
SUSY (at least MSSM) predicts a (very) light Higgs

$$V = (|\mu|^2 + m_{H_u}^2)\left|H_u^0\right|^2 + (|\mu|^2 + m_{H_d}^2)\left|H_d^0\right|^2 - B(H_u^0 H_d^0 + c.c.) + \frac{g^2 + g'^2}{8}\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2$$

one-loop level

$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

$$m_Z^2/2 = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

$m_H > 115$ GeV $\Rightarrow \tilde{m}_t > 1$ TeV

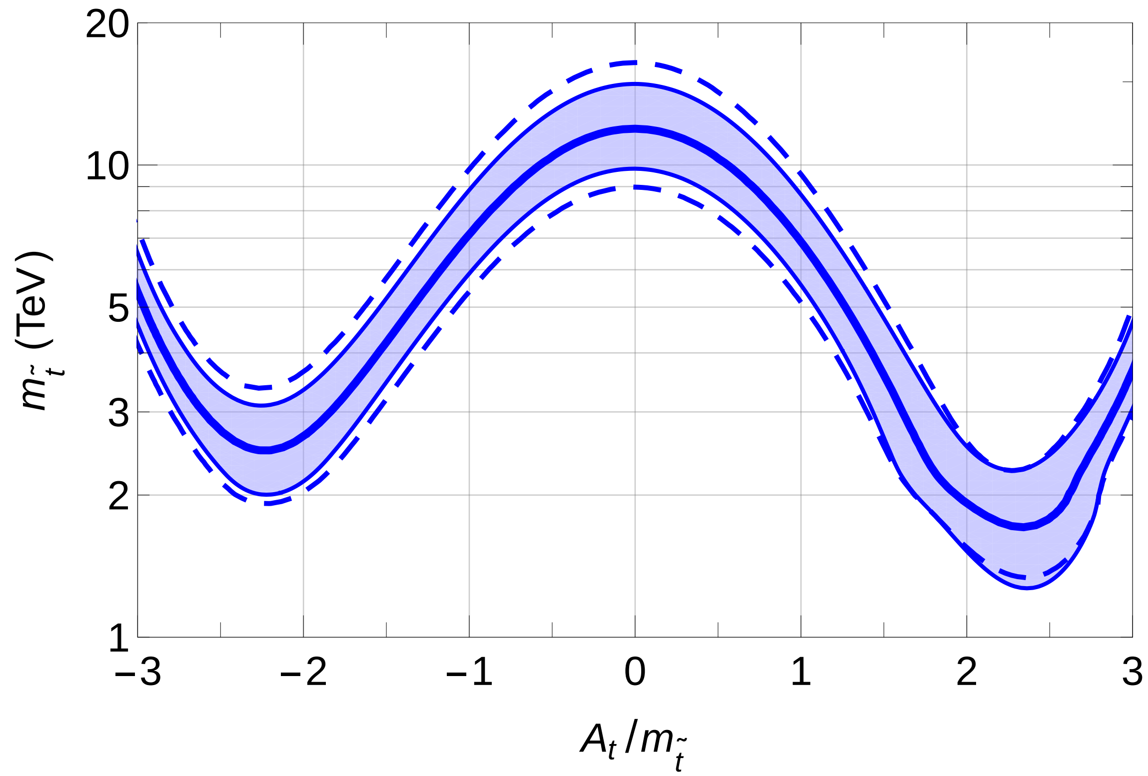
fine-tuned

$$\delta m_{H_u}^2 = -\frac{3\sqrt{2} G_F m_t^2 m_{\tilde{t}}^2}{4\pi^2} \log \frac{\Lambda}{m_{\tilde{t}}}$$

requires some fine-tuning O(1%) in m_Z

susy little hierarchy problem

The MSSM Higgs mass and stop searches

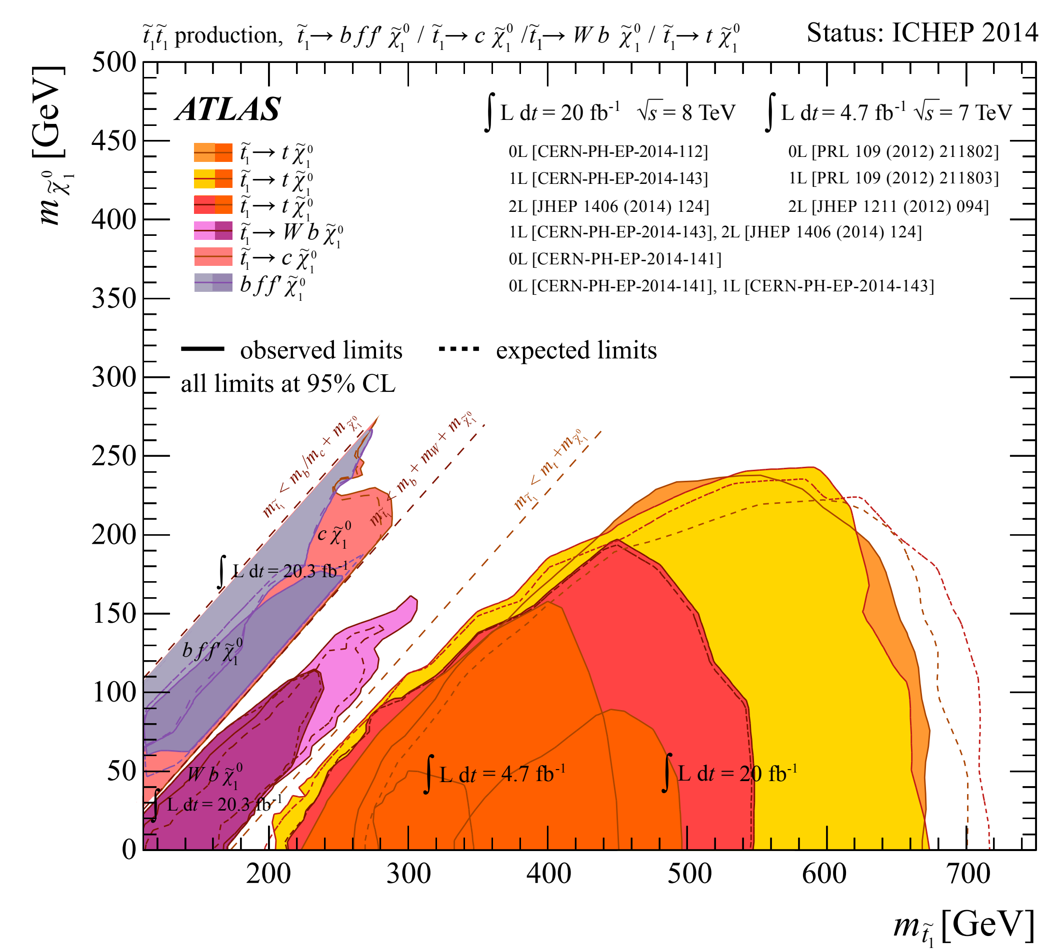


Pardo Vega, Villadoro '15 + many others

One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Figure 5: *Allowed values of the OS stop mass reproducing $m_h = 125$ GeV as a function of the stop mixing, with $\tan\beta = 20$, $\mu = 300$ GeV and all the other sparticles at 2 TeV. The band reproduce the theoretical uncertainties while the dashed line the 2σ experimental uncertainty from the top mass. The wiggle around the positive maximal mixing point is due to the physical threshold when $m_{\tilde t}$ crosses $M_3 + m_t$.*

Current and future bounds on stop mass



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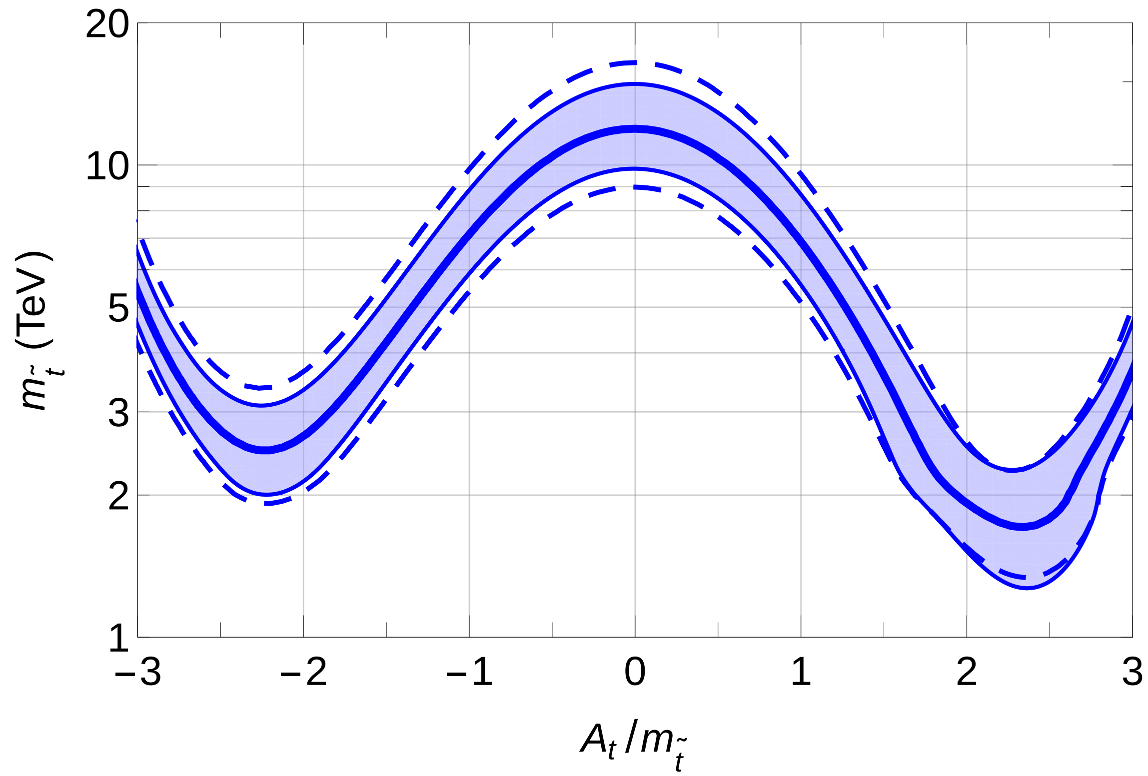

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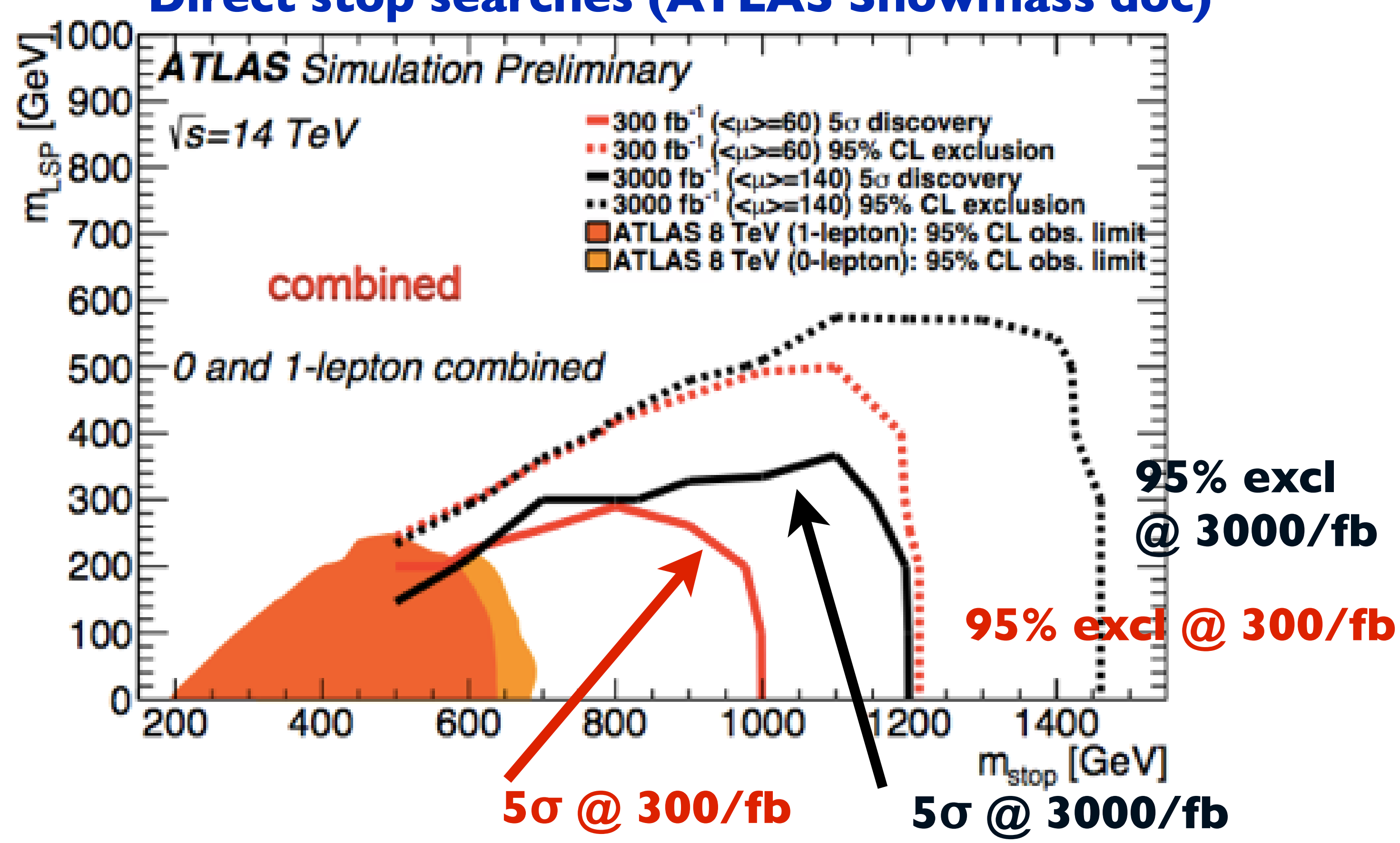
One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

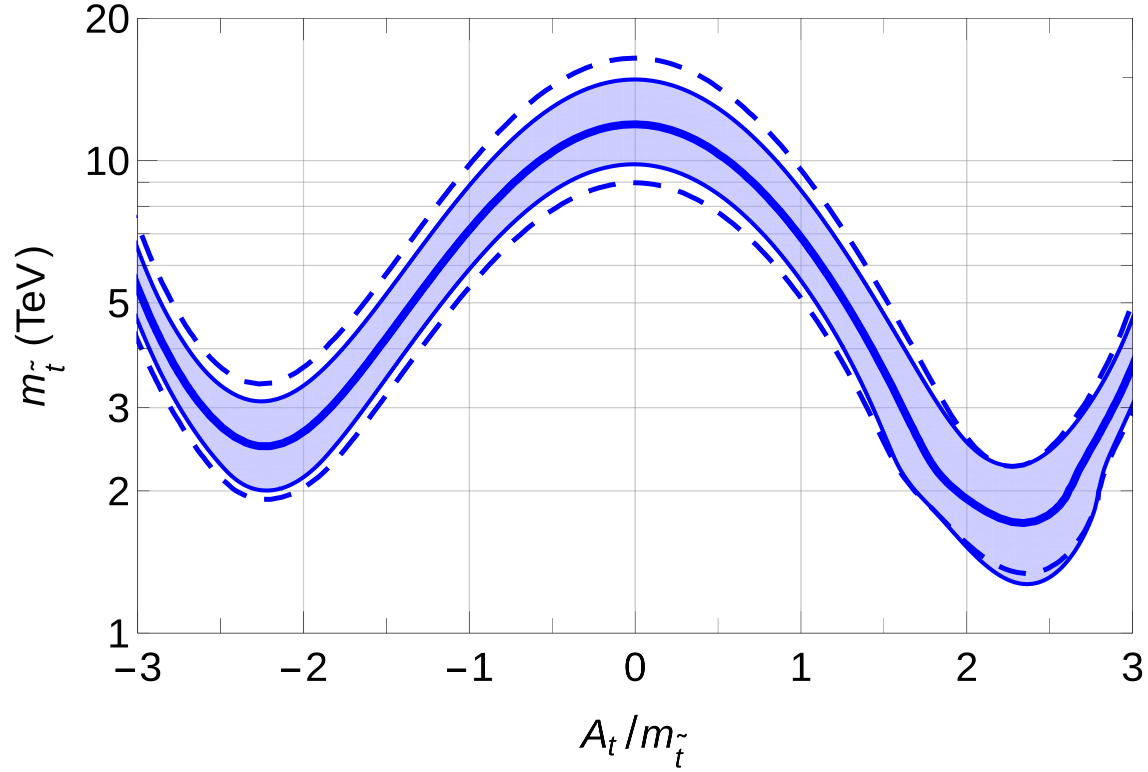
Direct stop searches (ATLAS Snowmass doc)

Current and future bounds on stop mass



HL-LHC (2030)

The MSSM Higgs mass and stop searches



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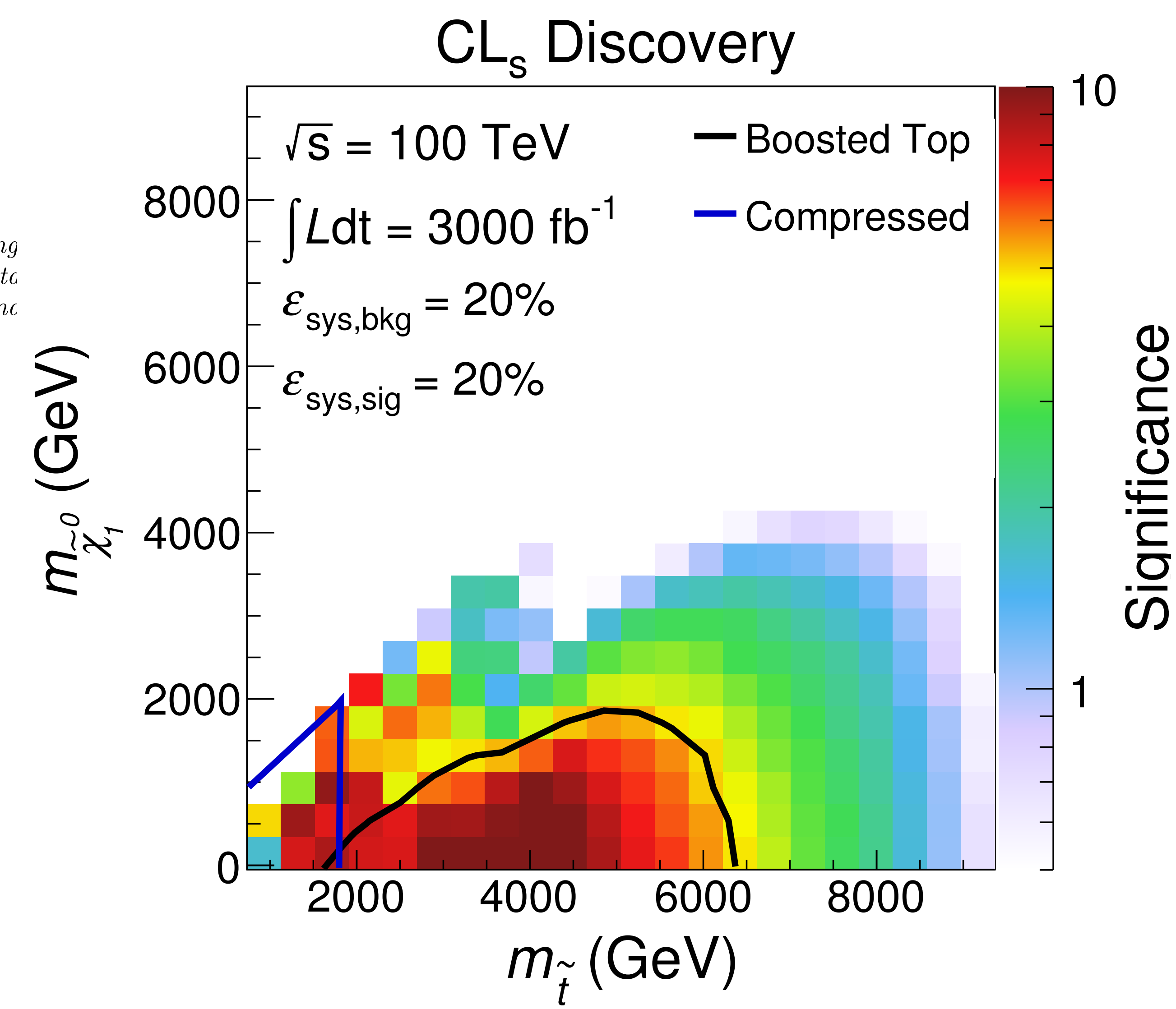
Pardo Vega, Villadoro '15 + many others

One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Current and future bounds on stop mass

FCC-hh @ 100TeV (2050)



Solving the susy little hierarchy pb

Various proposals on the market:

- singlet extensions of the Higgs sector: NMSSM and friends
  Fayet '75 + O(500) papers
- gauge extensions with new non-decoupled D-terms:
  Batra, Delgado, Kaplan, Tait '03 + O(10) papers
- low scale susy breaking mediation (Λ~100 TeV)
- double protection: (super-little) Higgs as a Goldstone boson
  Birkedal, Chacko, Gaillard '04 + O(20) papers
- add higher dimensional terms: BMSSM Dine, Seiberg, Thomas '07

$$W_{\rm BMSSM} = \frac{\lambda_1}{M}(H_u H_d)^2 + \frac{\lambda_2}{M}\mathcal{Z}_{\rm soft}(H_u H_d)^2$$

  - allow for much lighter susy particles
  - window for MSSM baryogenesis extended and more natural
  - LSP can account for DM relic density in larger region of parameter space

- ... your own model?

Saving SUSY

SUSY is Natural
but not plain vanilla

- ~~CMSSM~~
- pMSSM
- NMSSM
- Hide SUSY, e.g. smaller phase space
  - reduce production (eg. split families) Mahbubani et al
  - reduce MET (e.g. ~~R-parity~~, compressed spectrum) Csaki et al
  - dilute MET (decay to invisible particles with more invisible particles)
  - soften MET (stealth susy, stop -top degeneracy) Fan et al

$LHC_{100fb^{-1}}$ will tell!

Good coverage of

hidden natural susy

- mono-top searches (DM, flavored naturalness - mixing among different squark flavors-, stop-higgsino mixings)
- mono-jet searches with ISR recoil (compressed spectra)
- precise tt inclusive measurement+ spin correlations (stop → top + very soft neutralino)
- multi-hard-jets (RPV, hidden valleys, long decay chains)

Grand Unified Theory: SM vs MSSM

Evolution of coupling constants

Classical physics: the forces depend on distances

Quantum physics : the charges depend on distances

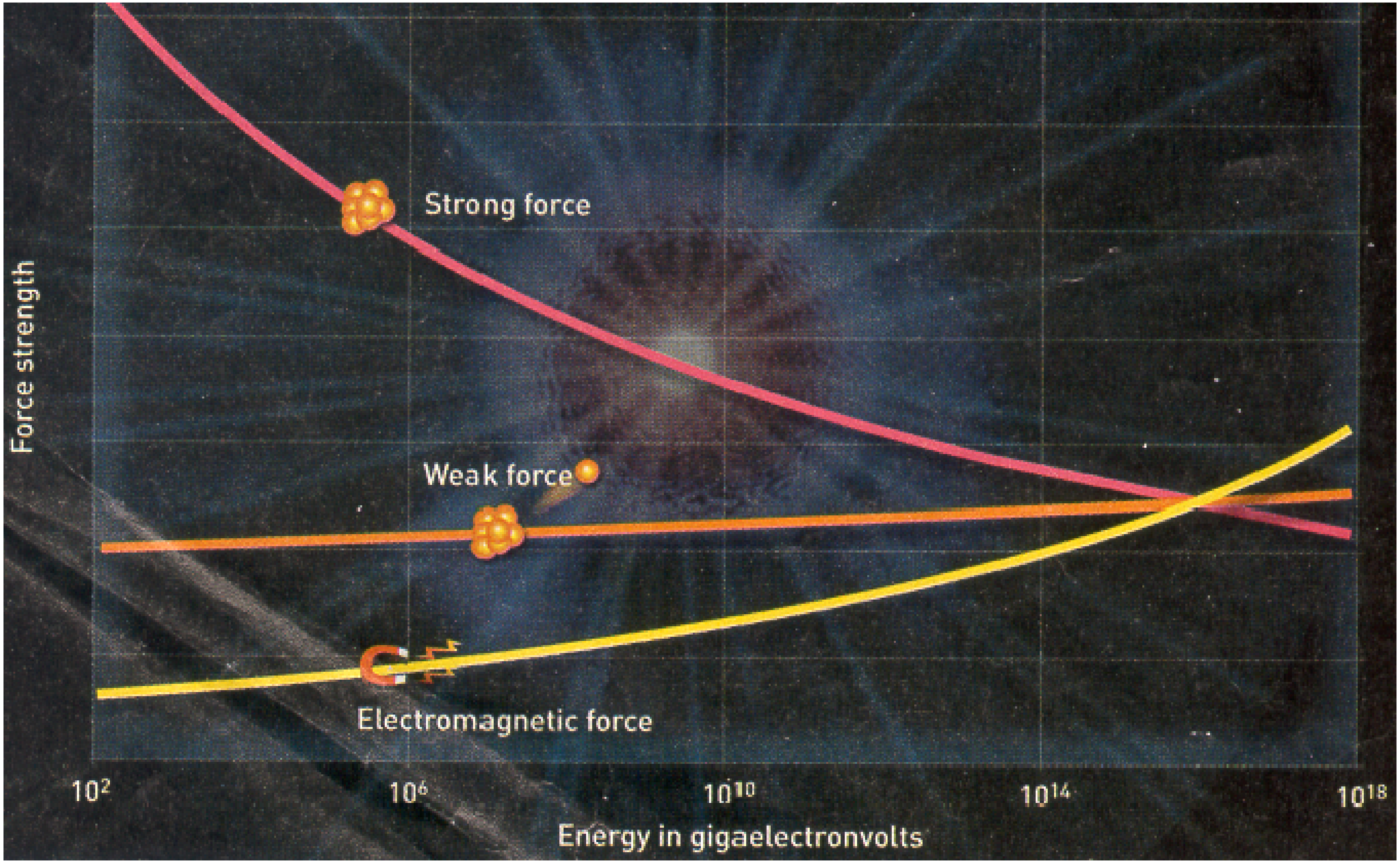
QED: virtual particles screen the electric charge: $\alpha \searrow$ when d $\nearrow$

QCD: virtual particles (quarks and *gluons*) screen the strong charge:
$\alpha_s \nearrow$ when d $\nearrow$

'asymptotic freedom'

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11 N_c}{6} + \frac{N_f}{3}\right)$$

Grand Unified Theories



A single form of matter
A single fundamental interaction

SU(5) GUT: Gauge Group Structure

$SU(3)_c \times SU(2)_L \times U(1)_Y$: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

How can you ever remember all these numbers?

$SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5)$

SU(5)
Adjoint rep.

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

additional U(1) factor that commutes with SU(3)xSU(2)

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right)$$

$$\bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$$

$$\bar{5} = L + d_R^c$$

$$10 = (5 \times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$$

$$10 = u_R^c + Q_L + e_R^c$$

$$T^{12} = \sqrt{\frac{3}{5}} Y \qquad g_5 \sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$$\sin^2\theta_W = \frac{3}{8}$$ @ M_{GUT}

SU(5) GUT: Gauge Group Structure

SU(3)$_c$xSU(2)$_L$xU(1)$_Y$: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \ldots \quad = (1,1)_1$$

How can you even ...

the SM matter fits nicely into representations of SU(5), even more nicely into SO(10) unification baryon-lepton

Ac...

Tr(...

$$\begin{pmatrix} & \\ \hline & \begin{matrix} -1/3 & & \\ & -1/3 & \\ & & -1/3 \end{matrix} \end{pmatrix}$$

$\ldots = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$
$\bar{5} = L + d_R^c$

$10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$
$10 = u_R^c + Q_L + e_R^c$

$$T^{12} = \sqrt{\frac{3}{5}}Y \qquad g_5\sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

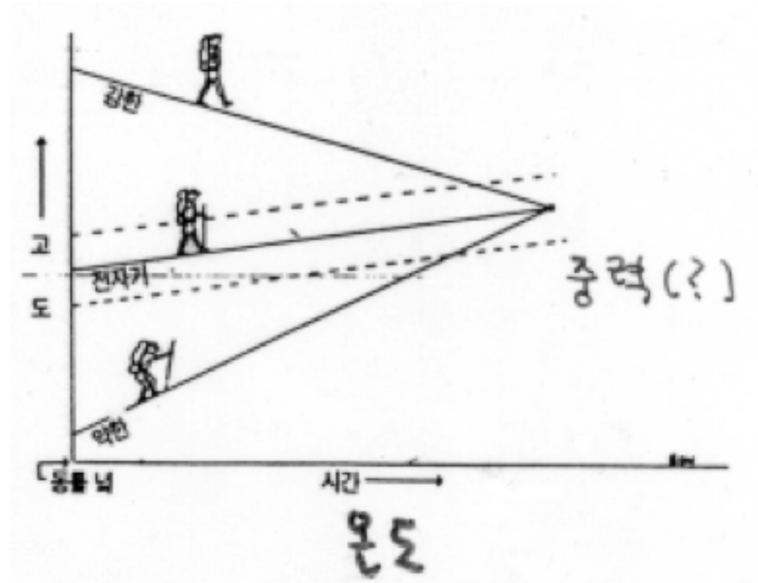
$$g_5 T^{12} = g' Y$$

$$\sin^2\theta_W = \frac{3}{8} \text{ @ M}_{GUT}$$

SU(5) GUT: SM β fcts

g, g′ and g_s are different but it is a low energy artifact!

$$\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2} b g^3 + \ldots$$



$$\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2}\ln\frac{Q^2}{Q_0^2}$$

$$b = \frac{11}{3} T_2(\texttt{spin-1}) - \frac{2}{3} T_2(\texttt{chiral spin-1/2}) - \frac{1}{3} T_2(\texttt{complex spin-0})$$

$$\mathrm{Tr}\left(T^a(R) T^b(R)\right) = T_2(R)\delta^{ab} \qquad T_2(\texttt{fund}) = \frac{1}{2} \qquad T_2(\texttt{adj}) = N$$

$$b_{SU(3)} = \underbrace{\frac{11}{3}\times 3}_{g} - \frac{2}{3}\left(\underbrace{\frac{1}{2}\times 2\times 3}_{Q_L} + \underbrace{\frac{1}{2}\times 1\times 3}_{u_R^c} + \underbrace{\frac{1}{2}\times 1\times 3}_{d_R^c}\right) = 7$$

$$b_{SU(2)} = \underbrace{\frac{11}{3}\times 2}_{W^\pm, Z} - \frac{2}{3}\left(\underbrace{\frac{1}{2}\times 3\times 3}_{Q_L} + \underbrace{\frac{1}{2}\times 1\times 3}_{L}\right) - \underbrace{\frac{1}{3}\times\frac{1}{2}}_{H} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3}\left(\underbrace{\left(\frac{1}{6}\right)^2 3\times 2\times 3}_{Q_L} + \underbrace{\left(-\frac{2}{3}\right)^2 3\times 3}_{u_R^c} + \underbrace{\left(\frac{1}{3}\right)^2 3\times 3}_{d_R^c} + \underbrace{\left(-\frac{1}{2}\right)^2 2\times 3}_{L} + \underbrace{(1)^2\times 3}_{e_R^c}\right) - \frac{1}{3}\left(\frac{1}{2}\right)^2\times 2 = -\frac{41}{6}$$

$$\Rightarrow \quad b_{T^{12}} = -\frac{41}{10}$$

SU(5) GUT: low energy consistency condition

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ ⟵ experimental inputs

b_3, b_2, b_1 ⟵ predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation for unification

$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 7 \times 10^{14} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

self-consistent computation:
- $M_{GUT} < M_{Pl}$ safe to neglect quantum gravity effects
- $\alpha_{GUT} \ll 1$ perturbative computation

SU(5) GUT: SM vs MSSM β fcts

chiral superfield

complex spin-0

Weyl spin-1/2

in same representation R of gauge group

vector superfield

Weyl spin-1/2

real spin-1

in same representation V of gauge group

$$b = \frac{11}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{vector}) - \frac{2}{3} T_2(\texttt{chiral}) - \frac{1}{3} T_2(\texttt{chiral}) = 3\, T_2(\texttt{vector}) - T_2(\texttt{chiral})$$

MSSM Chiral Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad U = (\bar{3},1)_{-2/3}, \quad D = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad E = (1,1)_1, \quad H_u = (1,2)_{1/2}, \quad H_d = (1,2)_{-1/2}$$

$$b_{SU(3)} = \overset{g}{3 \times 3} - \left(\overset{Q_L}{\frac{1}{2} \times 2 \times 3} + \overset{U}{\frac{1}{2} \times 1 \times 3} + \overset{D}{\frac{1}{2} \times 1 \times 3} \right) = 3$$

$$b_{SU(2)} = \overset{W^\pm, Z}{3 \times 2} - \left(\overset{Q_L}{\frac{1}{2} \times 3 \times 3} + \overset{L}{\frac{1}{2} \times 1 \times 3} \right) - \overset{H_u}{\frac{1}{2}} - \overset{H_d}{\frac{1}{2}} = -1$$

$$b_Y = -\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3 \times 2 \times 3} + \overset{U}{\left(-\frac{2}{3}\right)^2 3 \times 3} + \overset{D}{\left(\frac{1}{3}\right)^2 3 \times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2 \times 3} + \overset{E}{(1)^2 \times 3} \right) - \overset{H_u}{\left(\frac{1}{2}\right)^2 \times 2} - \overset{H_d}{\left(\frac{1}{2}\right)^2 \times 2} = -11 \quad \Rightarrow \quad b_{T^{12}} = -\frac{33}{5}$$

SU(5) GUT: MSSM GUT

$$b_3 = 3, \quad b_2 = -1, \quad b_1 = -33/5$$

low-energy consistency relation for unification

$$\sin^2\theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1}\frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)} \approx 0.23$$

squarks and sleptons form complete SU(5) reps ➙ they don't improve unification!
gauginos and higgsinos are improving the unification of gauge couplings

GUT scale predictions

$$M_{GUT} = M_Z \exp\left(2\pi\frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 2\times 10^{16}\ \mathrm{GeV}$$

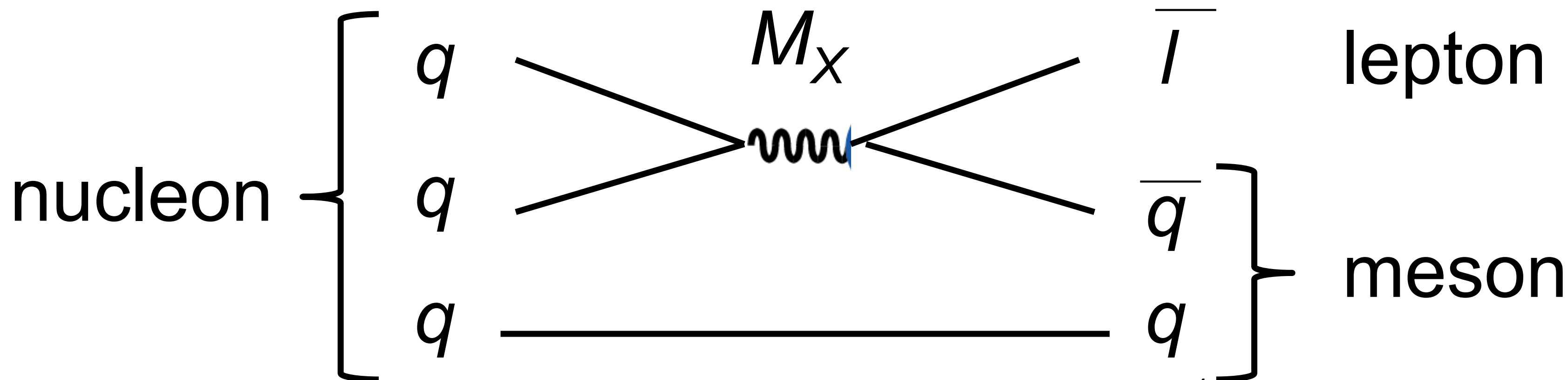
$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 24.3$$

Proton Decay

(G. Giudice SSLP'15)

in GUT, matter is unstable

decay of proton mediated by new SU(5)/SO(10) gauge bosons



$$\text{GUT:}\quad \tau_p(p\to e^+\pi^0)=\left(\frac{M_X}{10^{15}\ \text{GeV}}\right)^4 10^{31-32}\ \text{yr}$$

$$\text{Exp:}\quad \tau_p(p\to e^+\pi^0)>8.2\times10^{33}\ \text{yr}$$