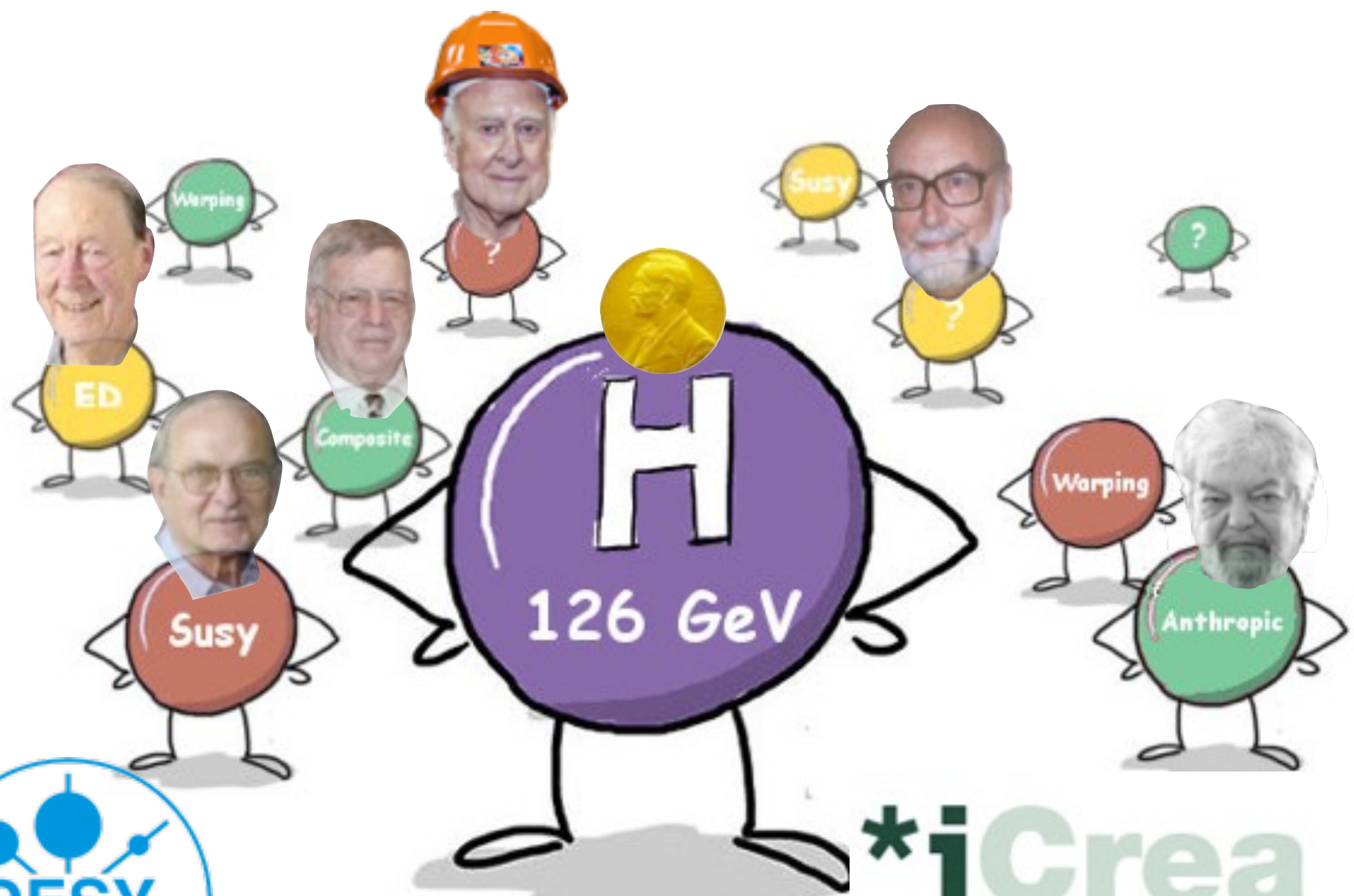
Beyond the Standard Model

CERN summer student lectures 2015



Exercises 4/5

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SUSY 1.0.1

Show that the following free Lagrangian

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$

is invariant (up to a total derivative) under the SUSY transformations

$$\delta\phi = \bar{\epsilon}\psi \qquad \text{and} \qquad \delta\psi = -i\left(\gamma^\mu \partial_\mu \phi\right)\epsilon$$

Compute the commutator of two successive SUSY transformations to derive the susy algebra

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = -i\left(\bar{\epsilon_2}\gamma^\mu \epsilon_1\right)\partial_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

MSSM superpotential

the MSSM matter content fit into chiral multiplet with the following quantum numbers under SU(3)xSU(2)xU(1)

Particules	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$L_L^i = \begin{cases} N^i = (\nu^i, \tilde{\nu}^i) \\ E_L^i = (l_L^i, \tilde{l}_L^i) \end{cases}$	1	2	1/2
$E^i = (CP(l_R^i), CP(\tilde{l}_R^i))$	1	1	-1
$Q_L^i = \begin{cases} U_L^i = (u_L^i, \tilde{u}_L^i) \\ D_L^i = (d_L^i, \tilde{d}_L^i) \end{cases}$	3	2	$-1/6$
$U_R^i = (CP(u_R^i), CP(\tilde{u}_R^i))$	$\bar{3}$	1	2/3
$D_R^i = (CP(d_R^i), CP(\tilde{d}_R^i))$	$\bar{3}$	1	$-1/3$
$H_d = \begin{cases} (h_d^0, \tilde{h}_d^0) \\ (h_d^-, \tilde{h}_d^-) \end{cases}$	1	2	1/2
$H_u = \begin{cases} (h_u^+, \tilde{h}_u^+) \\ (h_u^0, \tilde{h}_u^0) \end{cases}$	1	2	-1/2

Show the most general renormalizable superpotential is

$$W = H_u Q D + H_u Q U + H_d L E + \mu H_u H_d + L Q D + U D D + L L E + \mu_L L H_u$$

Search for a symmetry that would forbid the last 4 terms that violate baryon or lepton numbers

SU(5) GUT

- Anomaly cancelation: show that the SU(5) model is anomaly free
  - Recall: the anomaly is proportional to sum over the chiral representations of the coefficients A(R) defined by

$$\mathrm{Tr}_R\, T^A\{T^B, T^C\} = A(R) d^{ABC}$$

where the totally symmetric coefficients dABC are defined in the fundamental representation by

$$T^A\{T^B, T^C\} = 2d^{ABC}$$

  - Hint: A($\bar{5}$) and A(10) will be computed using a simple U(1) factor of SU(5), eg. the electric charge or the "hypercharge"

- When the fermion masses are generated with a Higgs in a 5 of SU(5), show that the theory is invariant under the global B-L symmetry

- The breaking pattern SU(5) $\xrightarrow{\langle\phi\rangle}$ SU(3)xSU(2)xU(1) $\xrightarrow{\langle H\rangle}$ U(1)$_{em}$ can be achieved using an adjoint Higgs, $\langle\phi\rangle = v_1 \mathrm{diag}(-3,-3,2,2,2)$, and a Higgs transforming as a 5 of SU(5), $\langle H\rangle = (0, v_2/\sqrt{2}, 0, 0, 0)$. From the potential

$$V = -m_1^2 \mathrm{Tr}\phi^2 + \lambda_1 \left(\mathrm{Tr}\phi^2\right)^2 + \lambda_2 \mathrm{Tr}\phi^4 - m_2^2 H^\dagger H + \lambda_3 \left(H^\dagger H\right)^2 + \lambda_4 H^\dagger H \mathrm{Tr}\phi^2 + \lambda_5 H^\dagger \phi^2 H$$

  - find the value of v_1
  - compute the mass spectrum of the scalar fields. Interpret the result (Goldstone theorem).
  - Notice that to keep only a light doublet at the weak scale, a fine-tuning is needed. This is known as the doublet-triplet splitting problem

β function, gauge coupling running

The one-loop β function giving the running of the coupling constant of an SU(N) gauge symmetry is given by

$$\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2} b_0 g^3 \qquad \text{ie} \qquad \frac{d\alpha}{d\log\mu} = -\frac{1}{2\pi} b_0 \alpha^2$$

where the coefficient b_0 is computed to

$$b_0 = \frac{11}{3} T_2(\texttt{spin-1}) - \frac{2}{3} T_2(\texttt{chiral spin-1/2}) - \frac{1}{3} T_2(\texttt{complex spin-0})$$

$T_2(R)$ is defined from the traces of the product of two generators of SU(N) in the representation R

$$\mathrm{Tr}\left(T^a(R) T^b(R)\right) = T_2(R)\delta^{ab}$$

1/ Compute the b_0 coefficients for SU(3)xSU(2)xU(1) with the particle content of the SM

2/ Compute the b_0 coefficients for SU(3)xSU(2)xU(1) with the particle content of the MSSM

3/ Compute M_{GUT} and α_{GUT} in the SM and in the MSSM

4/ Compute $\sin^2\theta_W$ in the SM and in the MSSM