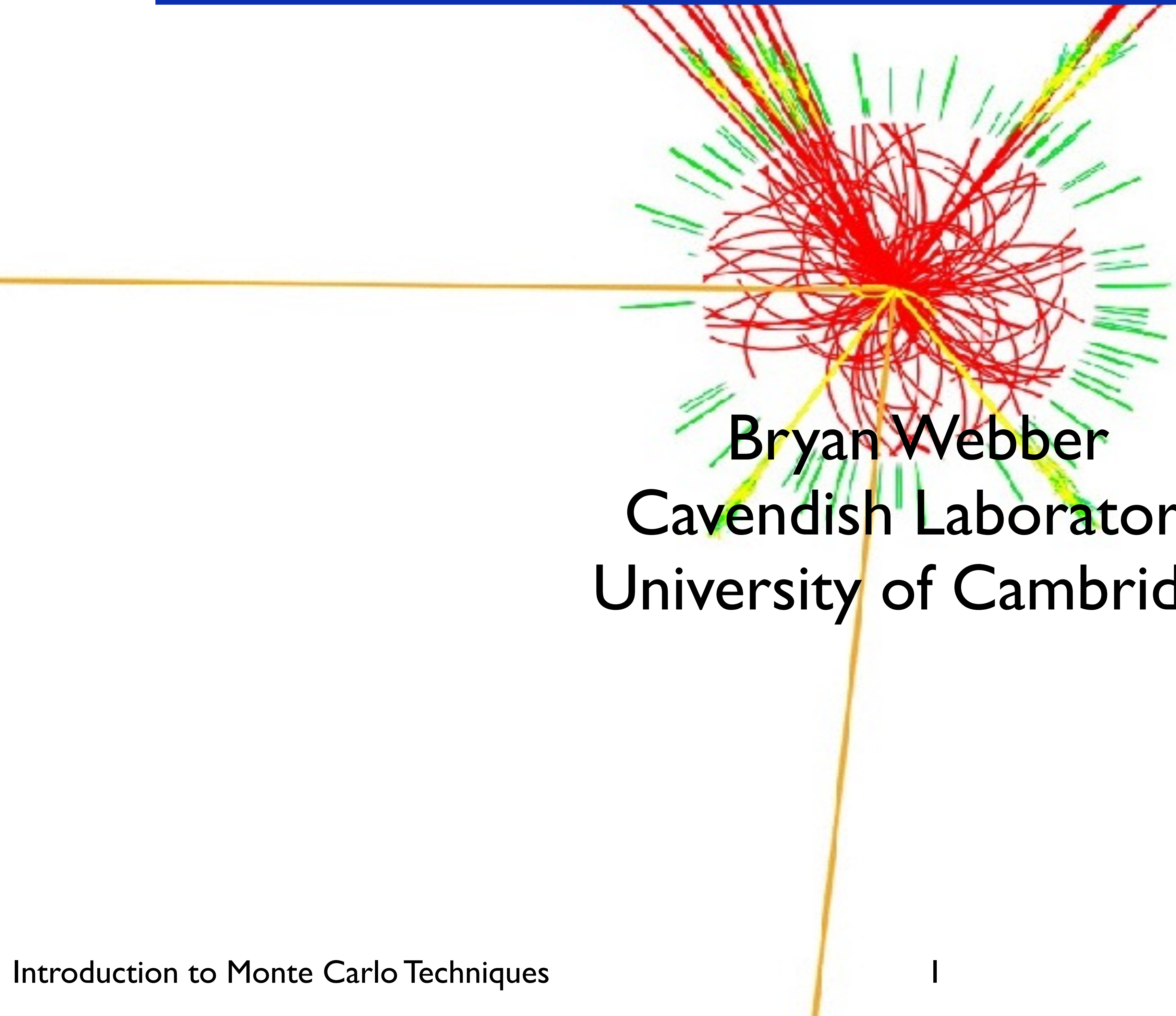
Introduction to Monte Carlo Techniques

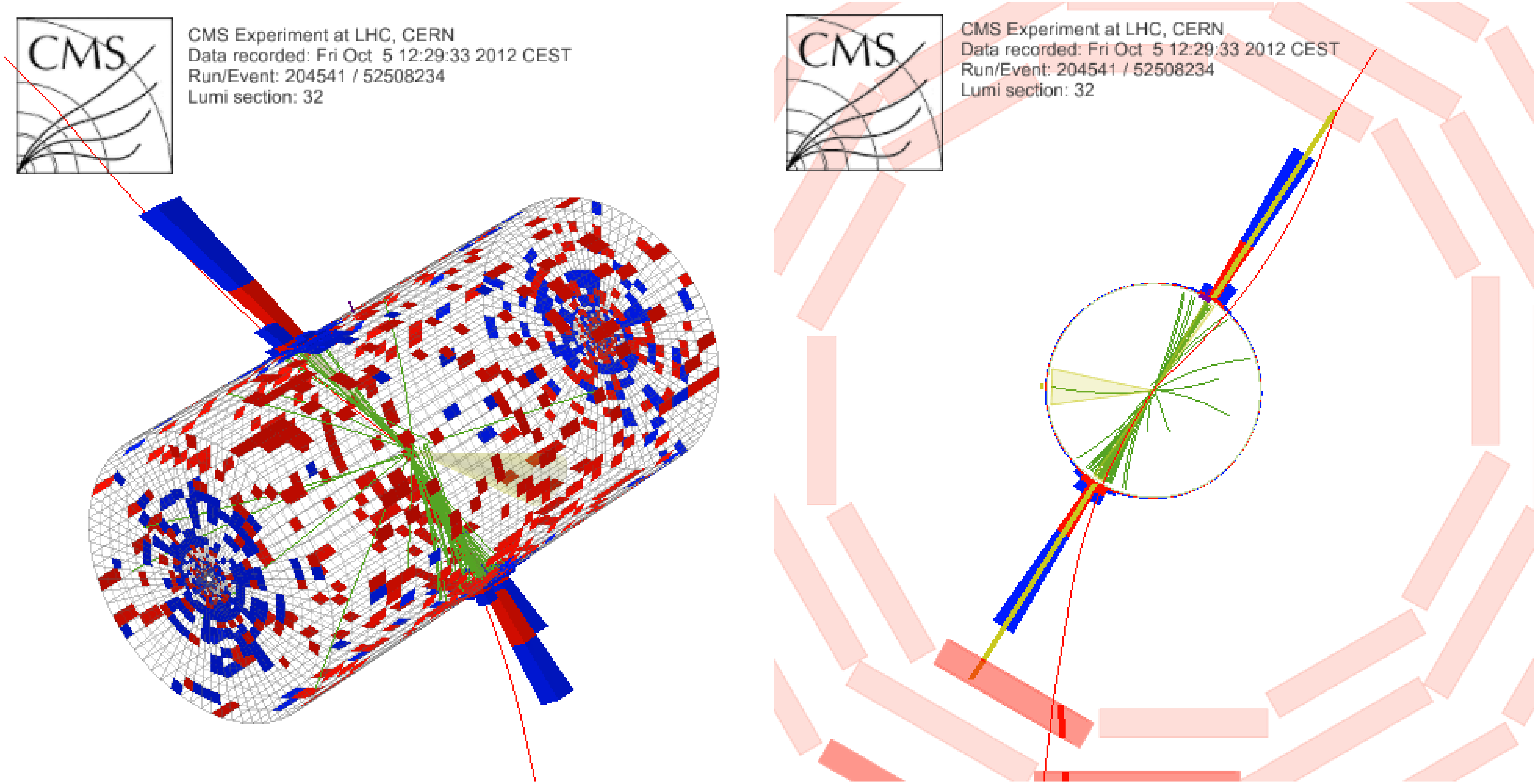


Bryan Webber
Cavendish Laboratory
University of Cambridge

Introduction to Monte Carlo

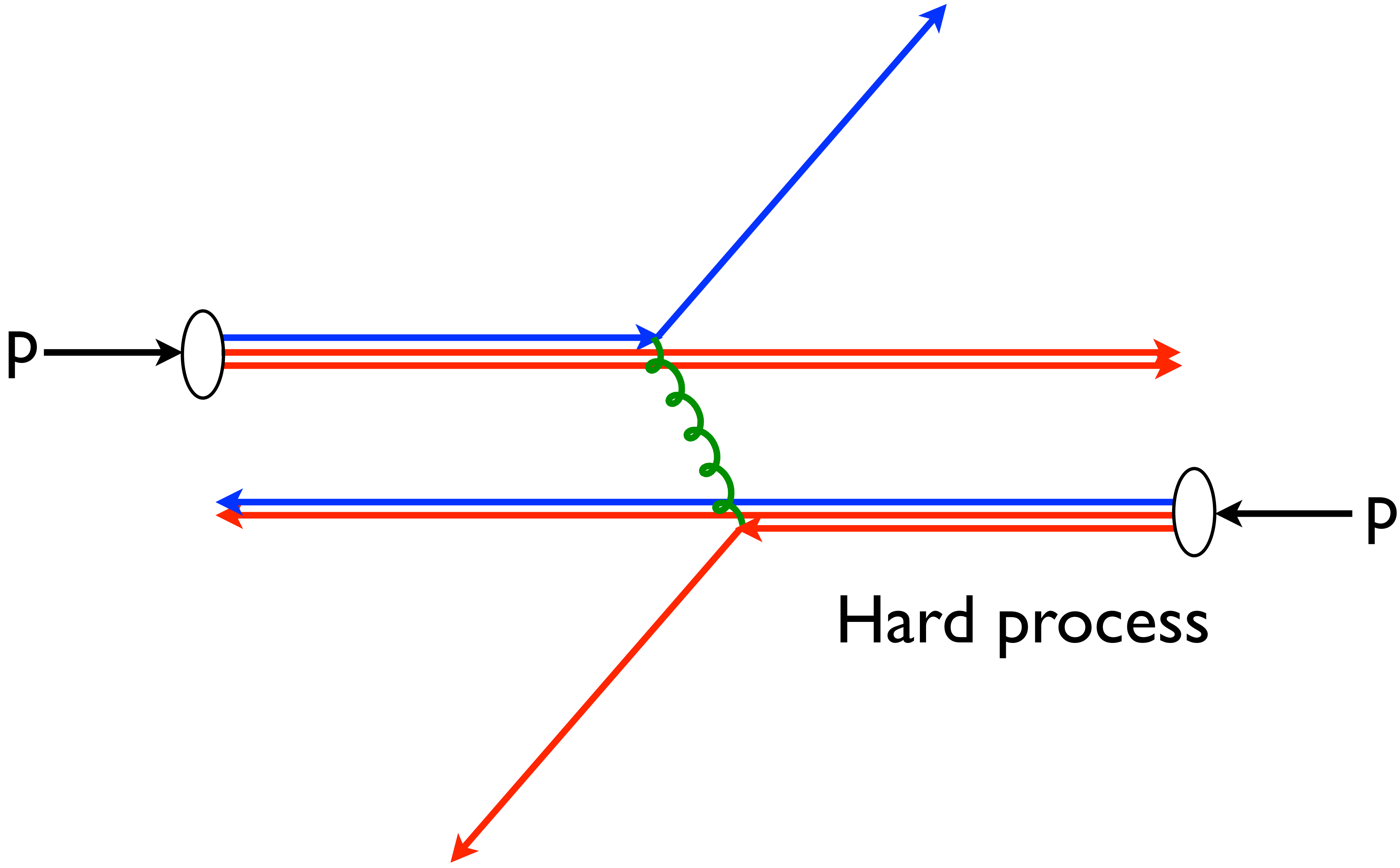
- Lecture 1: The Monte Carlo method
  - theoretical foundations and limitations
  - parton-level event generation
- Lecture 2: Hadron-level event generation
  - parton showering
  - hadronization and underlying event
  - sample of results

A high-mass dijet event

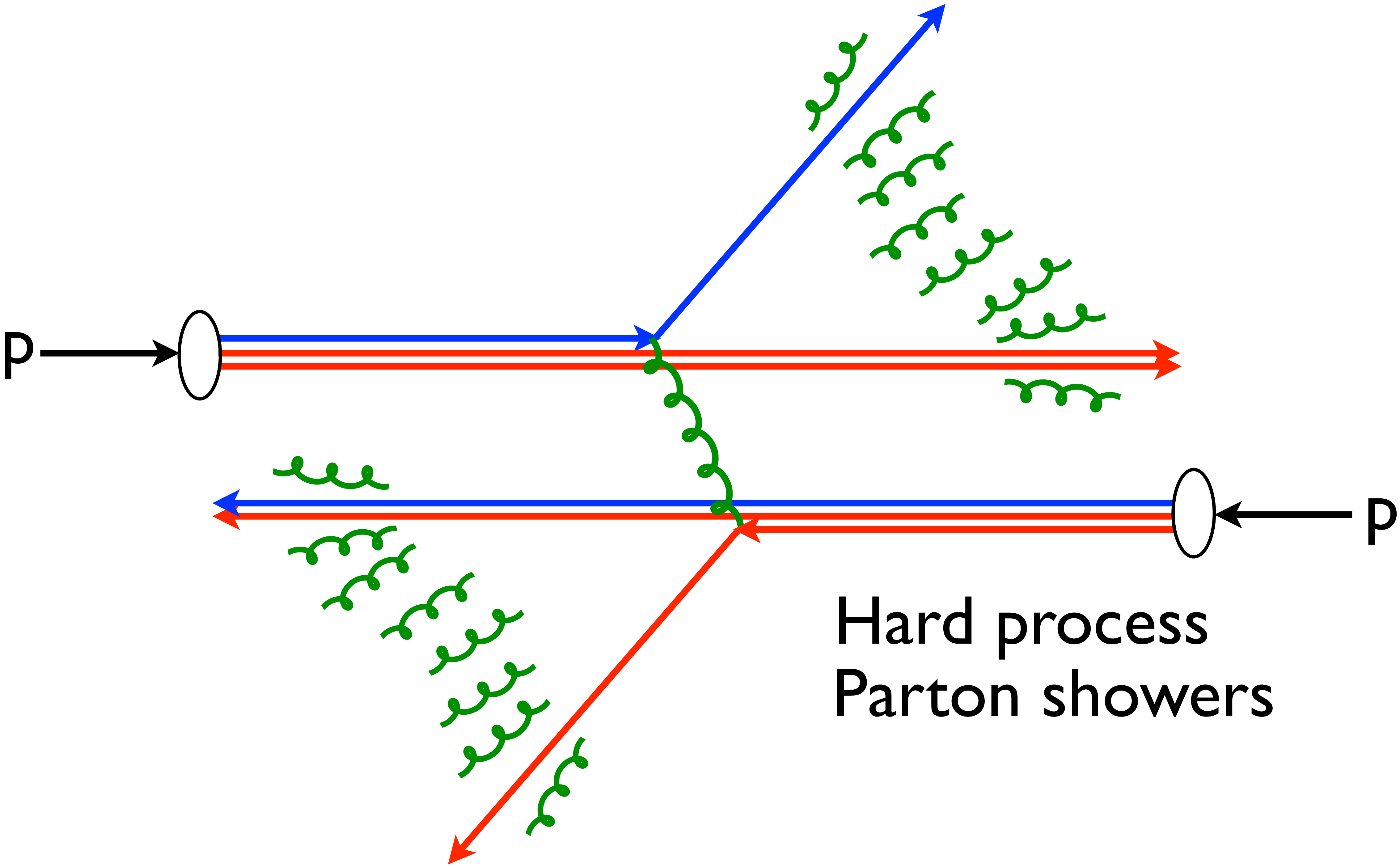


CMS PAS
EXO-12-059

LHC dijet

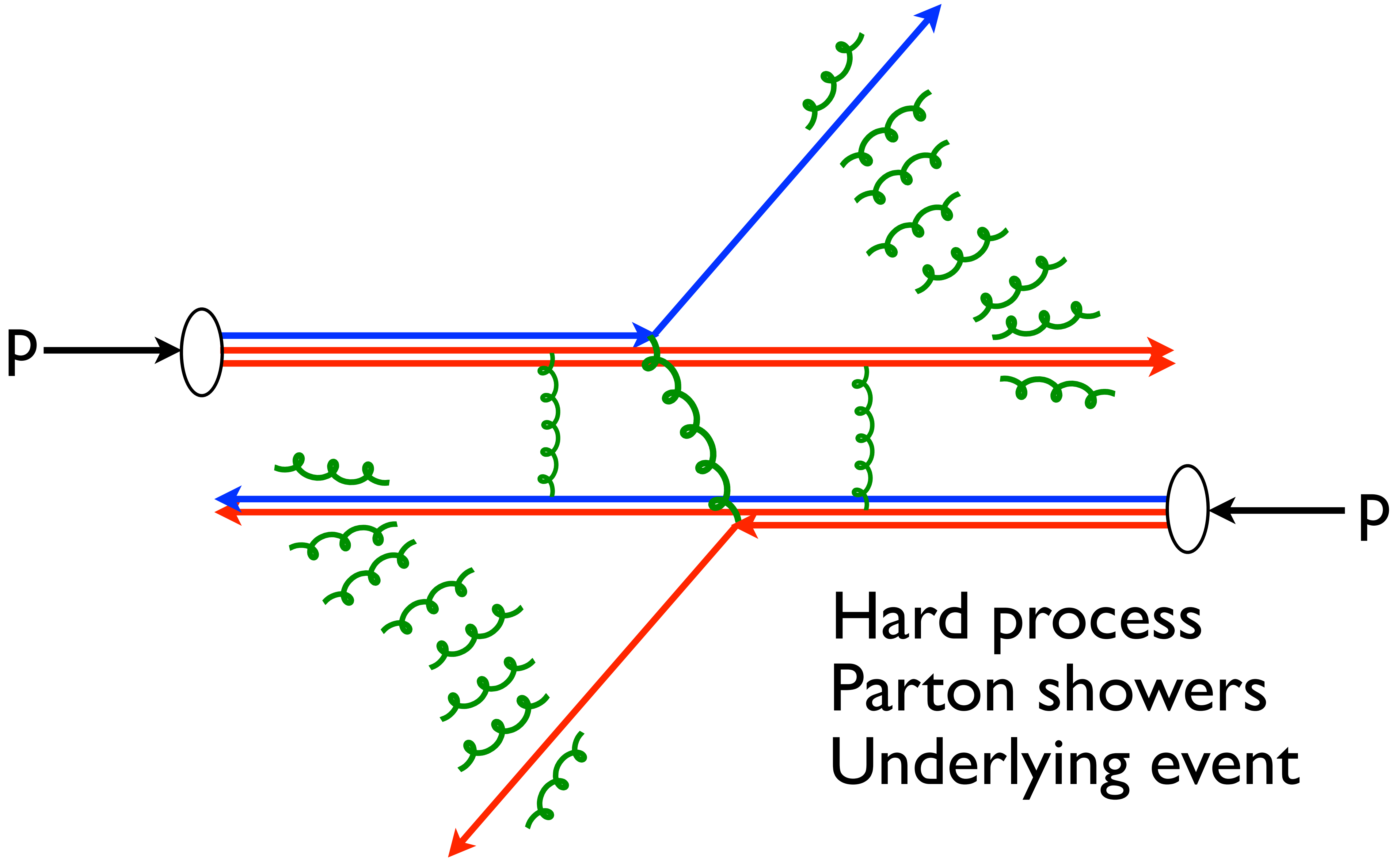


LHC dijet



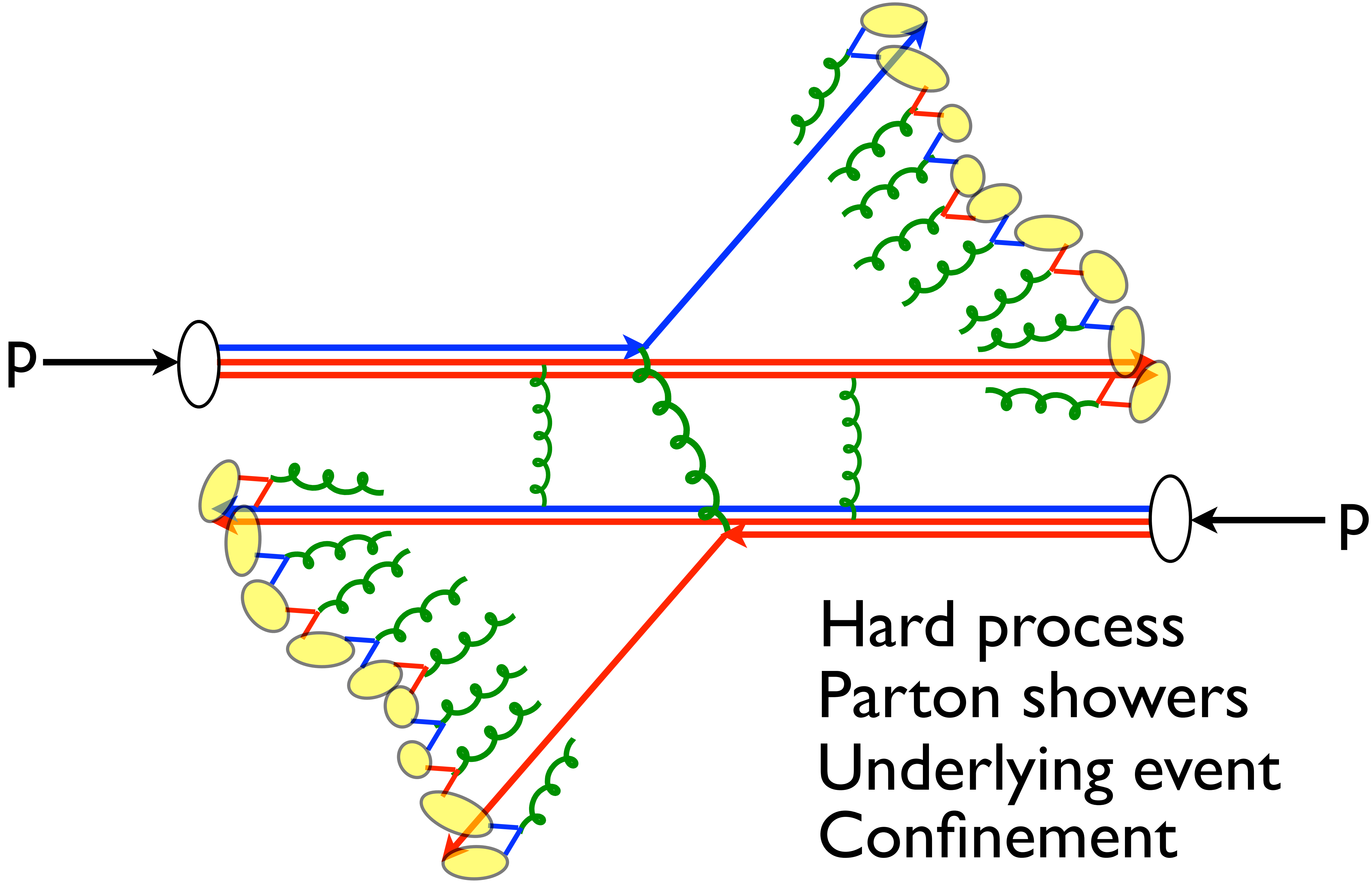
Hard process
Parton showers

LHC dijet



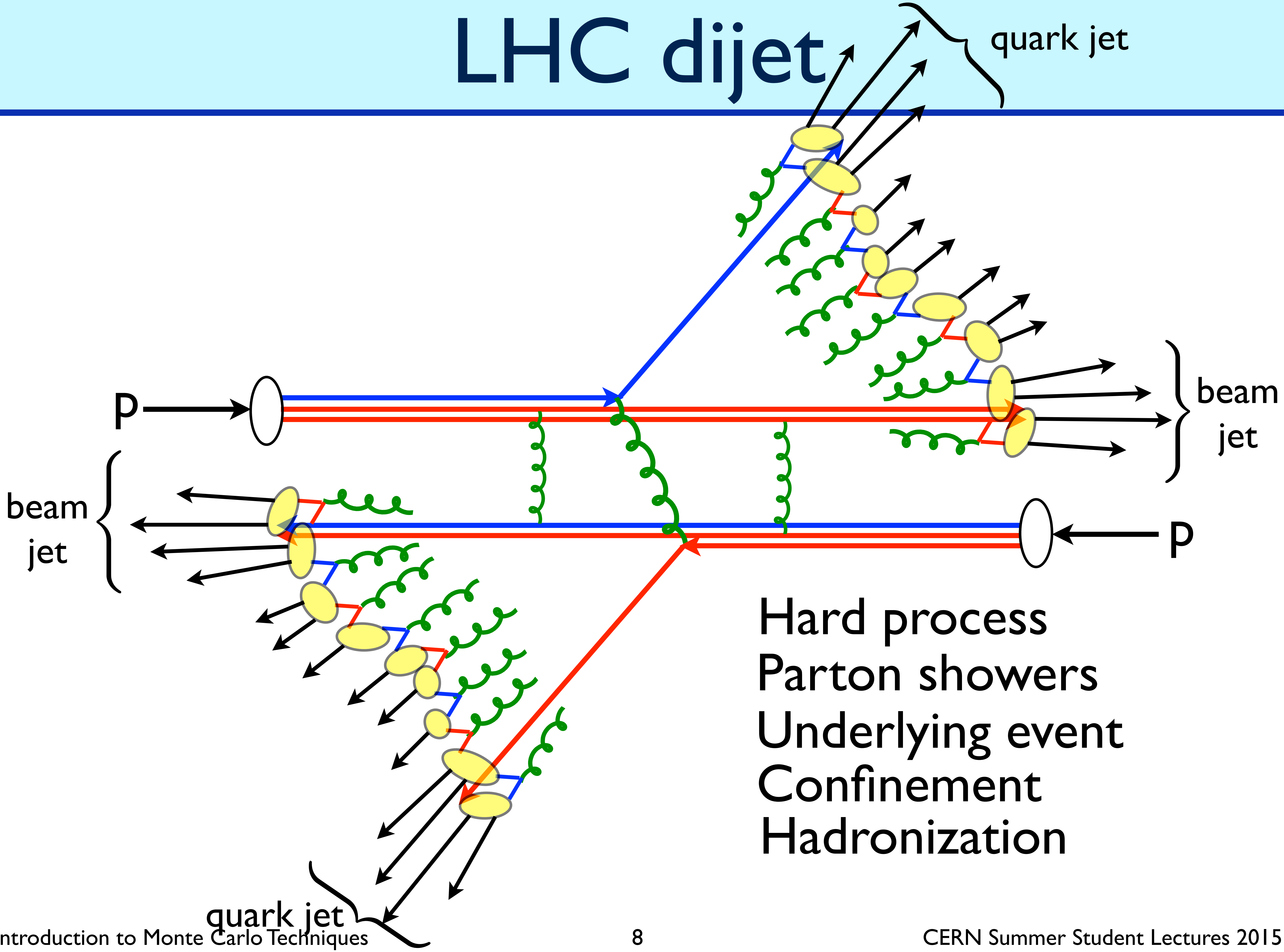
Hard process
Parton showers
Underlying event

LHC dijet



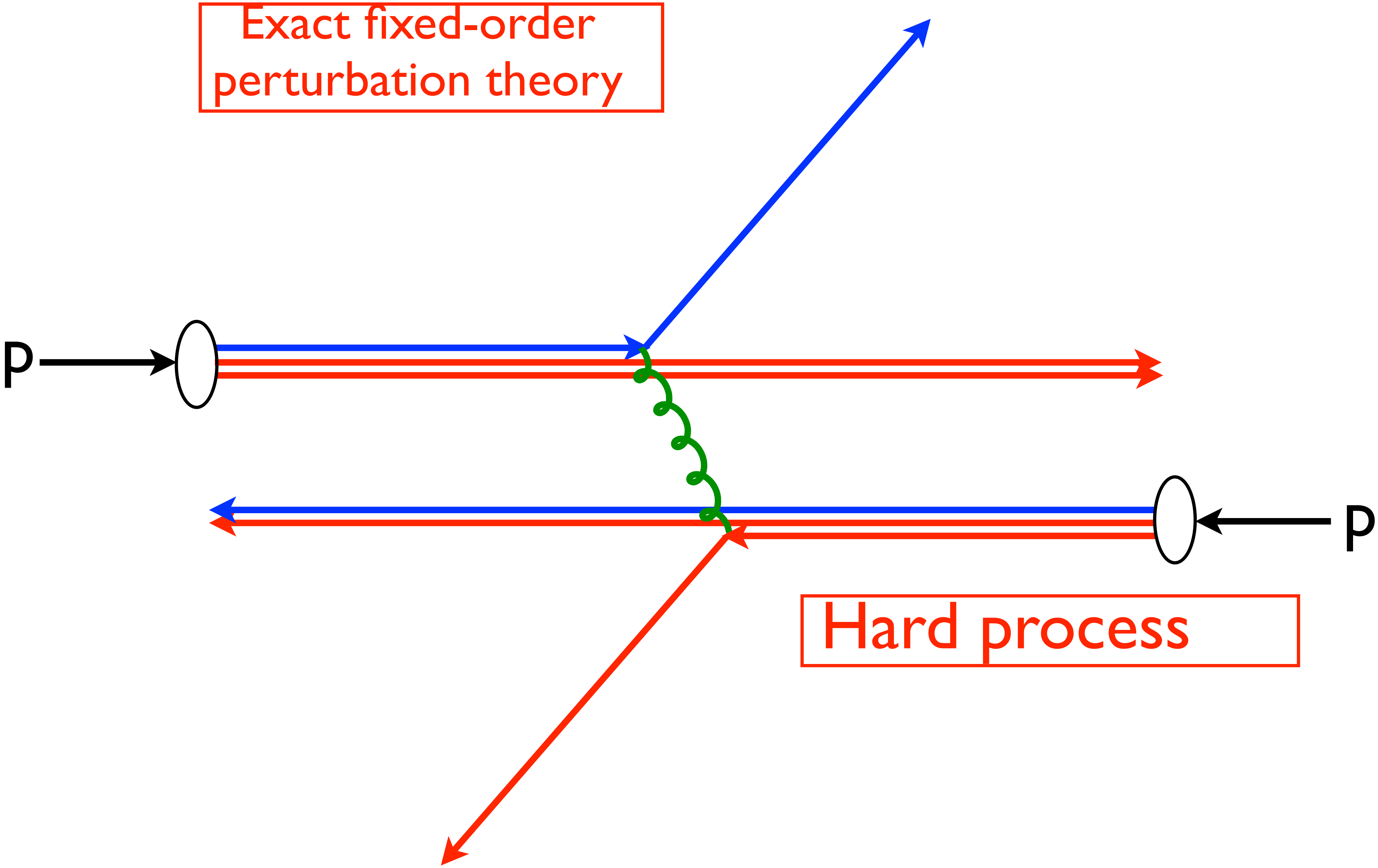
Hard process
Parton showers
Underlying event
Confinement

LHC dijet



Hard process
Parton showers
Underlying event
Confinement
Hadronization

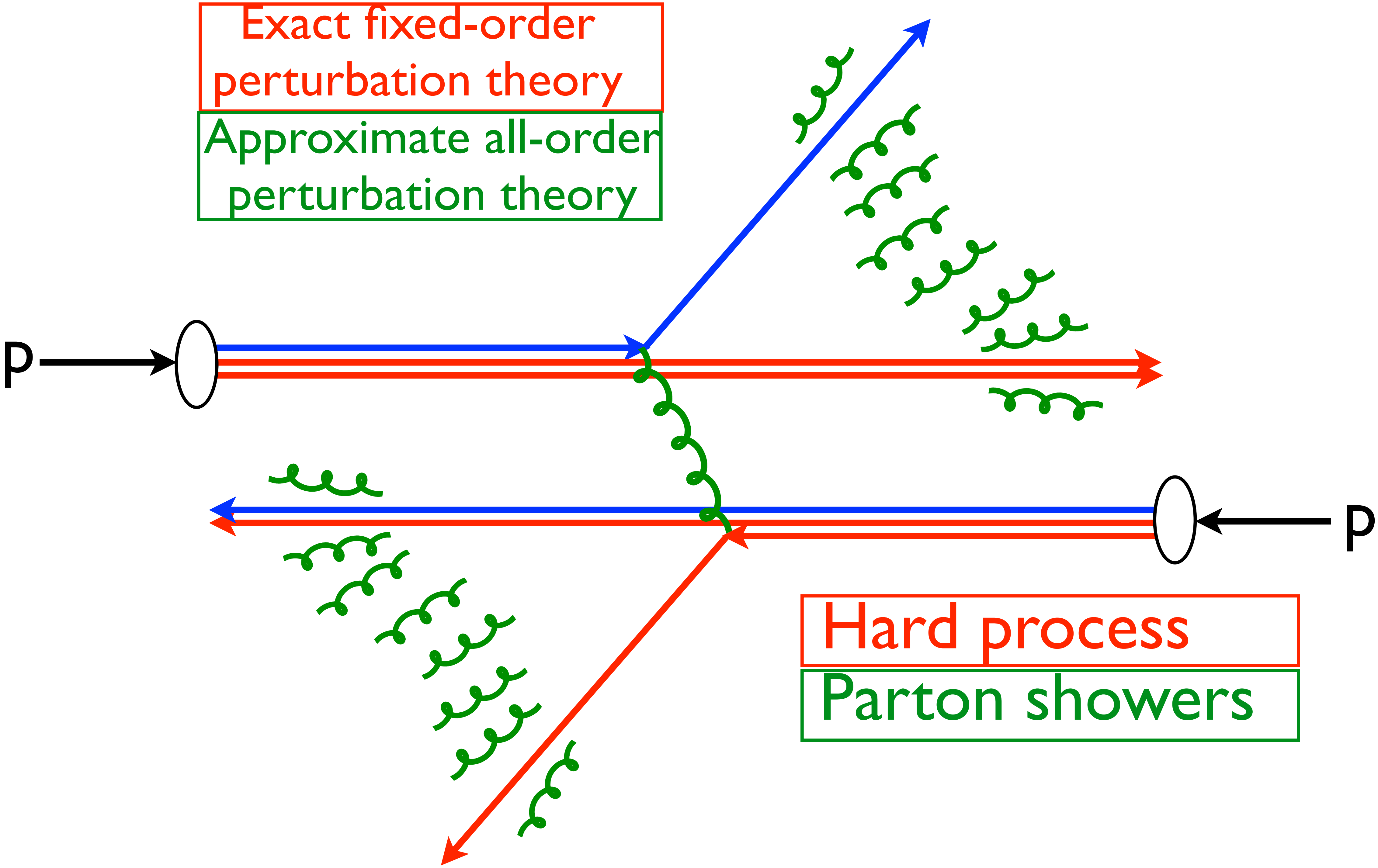
Theoretical Status



Exact fixed-order perturbation theory

Hard process

Theoretical Status



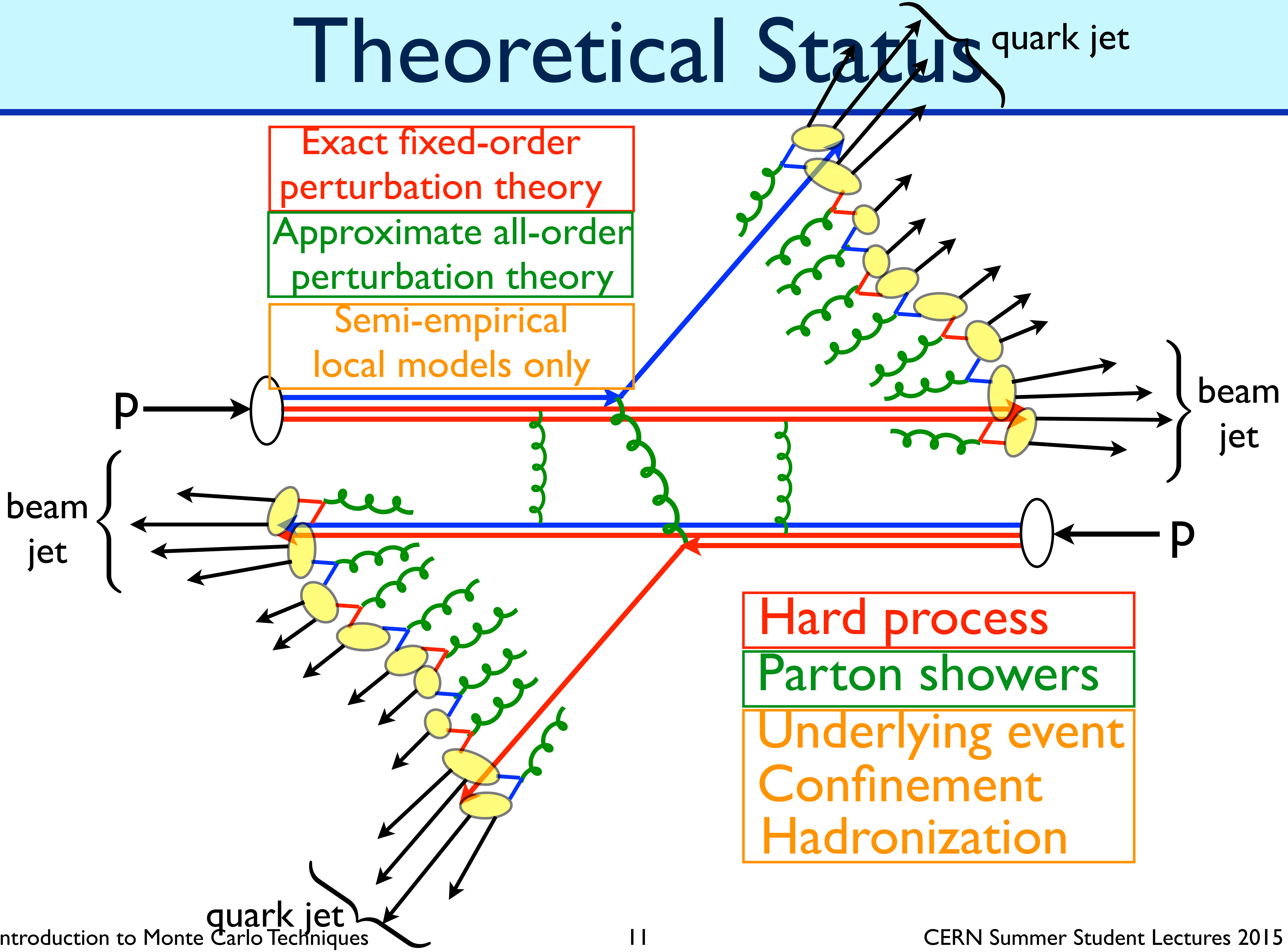
Exact fixed-order perturbation theory

Approximate all-order perturbation theory

Hard process

Parton showers

Theoretical Status



Exact fixed-order perturbation theory

Approximate all-order perturbation theory

Semi-empirical local models only

Hard process

Parton showers

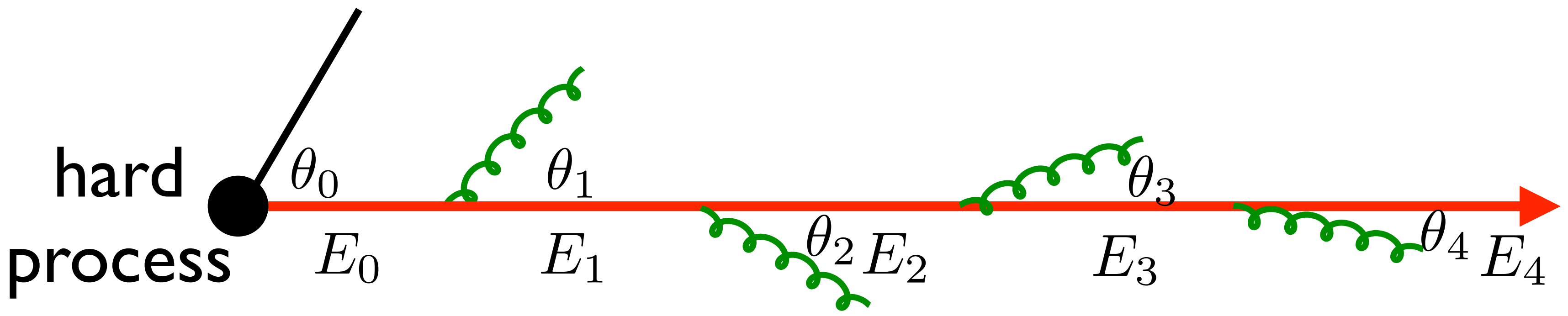
Underlying event
Confinement
Hadronization

QCD Factorization

$$\sigma_{pp\to X}(E_{pp}^2) = \int_0^1 \underbrace{dx_1\,dx_2}_{\substack{\text{momentum}\\ \text{fractions}}}\; \underbrace{f_i(x_1,\mu^2)\,f_j(x_2,\mu^2)}_{\substack{\text{parton}\\ \text{distributions}\\ \text{at scale } \mu^2}}\; \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2 E_{pp}^2,\mu^2)}_{\substack{\text{hard process}\\ \text{cross section}}}$$

- Jet formation and underlying event take place over a much longer time scale, with unit probability
- Hence they cannot affect the cross section
- Scale dependences of parton distributions and hard process cross section are perturbatively calculable, and cancel order by order

Parton Shower

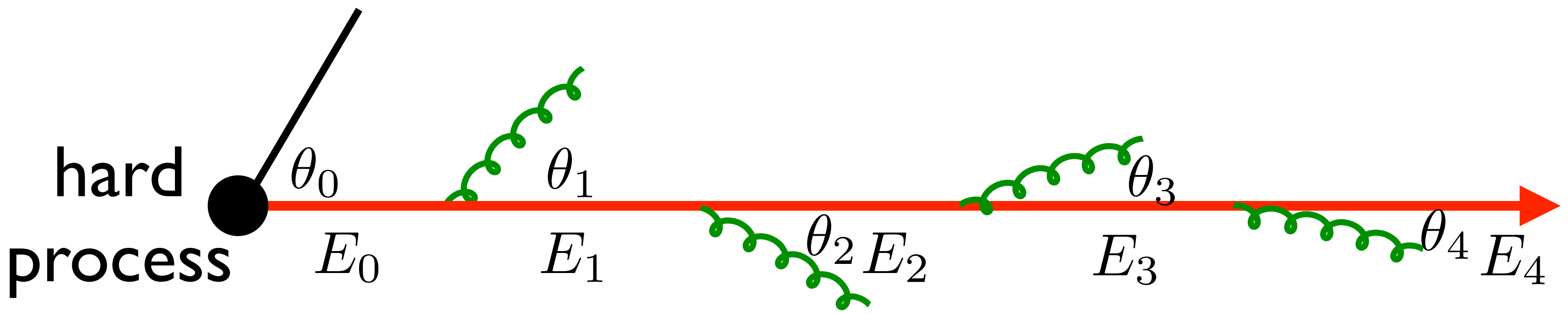


- Shower = sequence of emissions with decreasing angles and energies
- Approximation: keep only contributions $\propto 1/\theta$

$$d^2\mathcal{P} = \frac{\alpha_\mathrm{s}}{\pi}\frac{d\theta}{\theta}P(z)\,dz \qquad\qquad z = \frac{E_{i+1}}{E_i}$$

- For very small energy and/or angle, emission is “unresolvable”

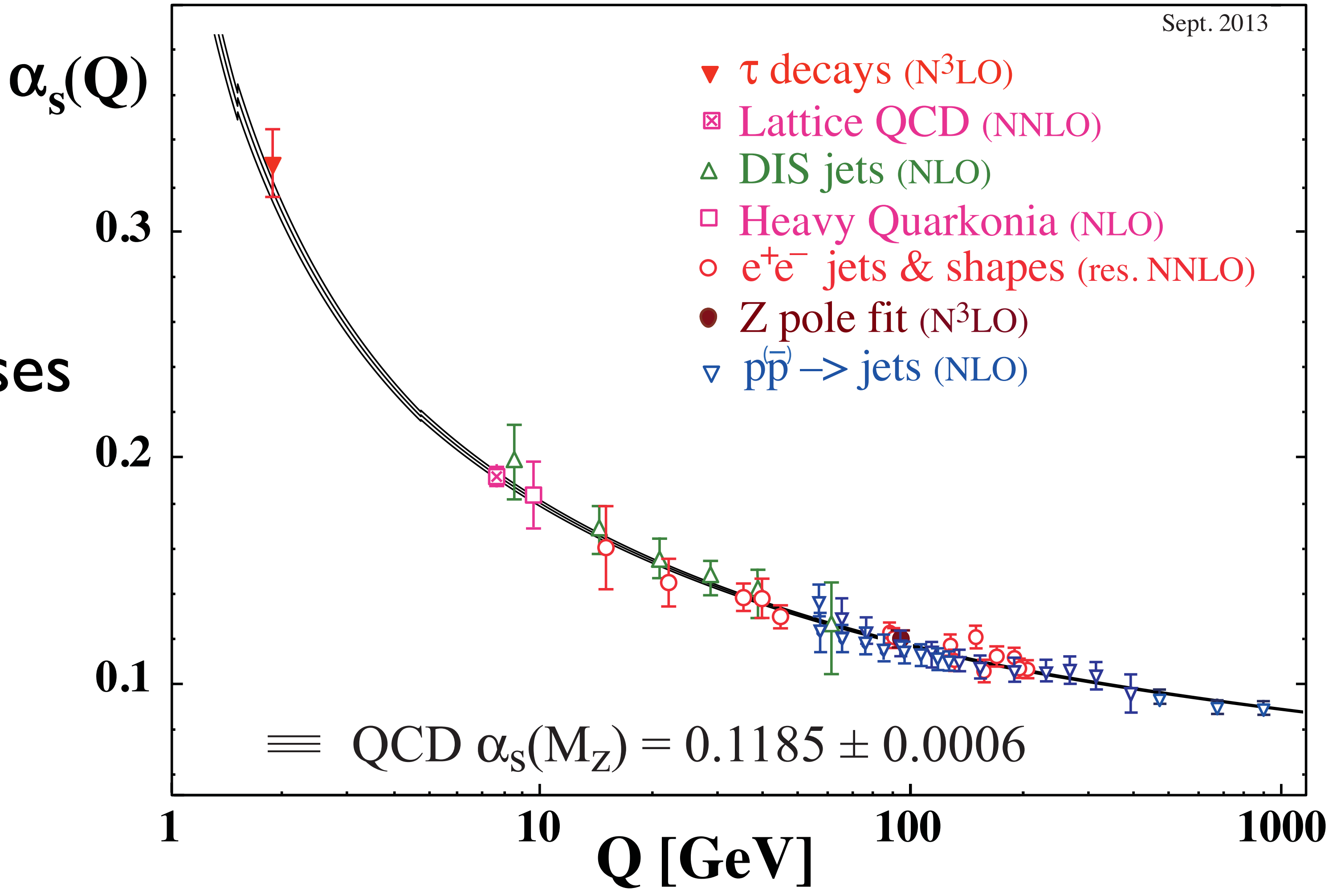
Parton Shower



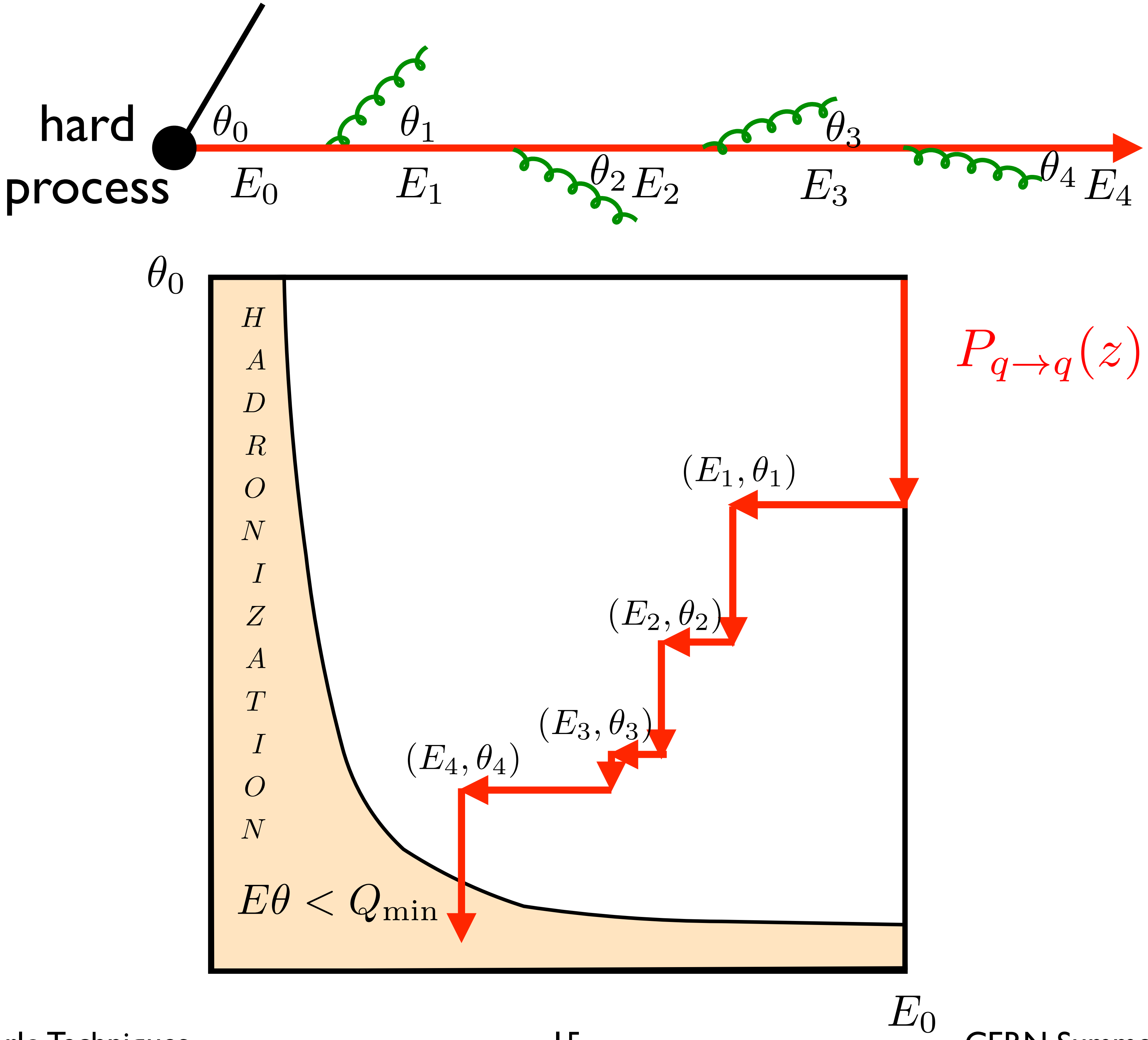
$$d^2\mathcal{P} = \frac{\alpha_s}{\pi}\frac{d\theta}{\theta}P(z)\,dz$$

- α_s increases as $Q = E\theta$ decreases
- When $Q < Q_{\min} \sim 1\ \mathrm{GeV}$

$\alpha_s \sim 1$ ➞ hadronization

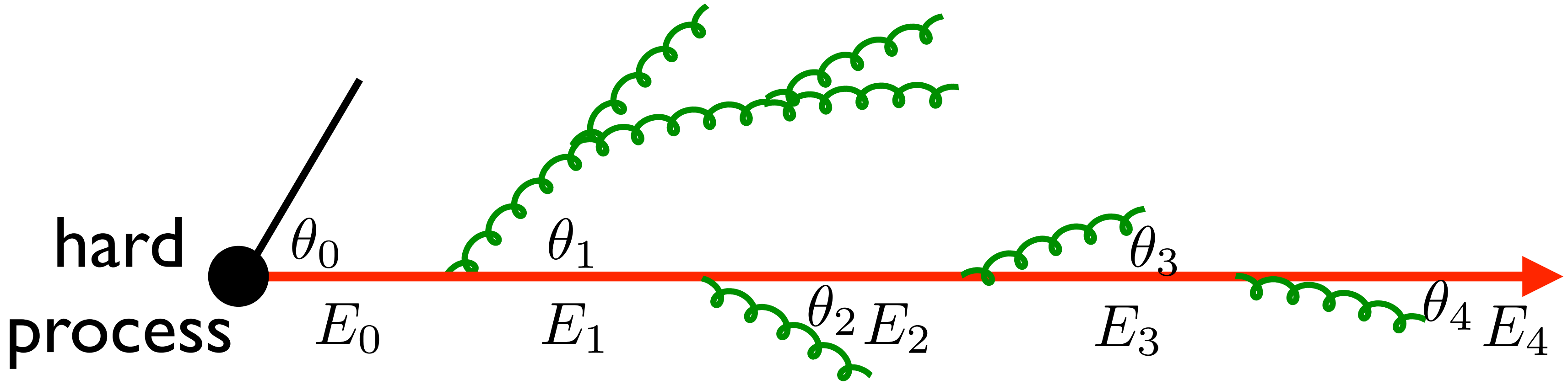


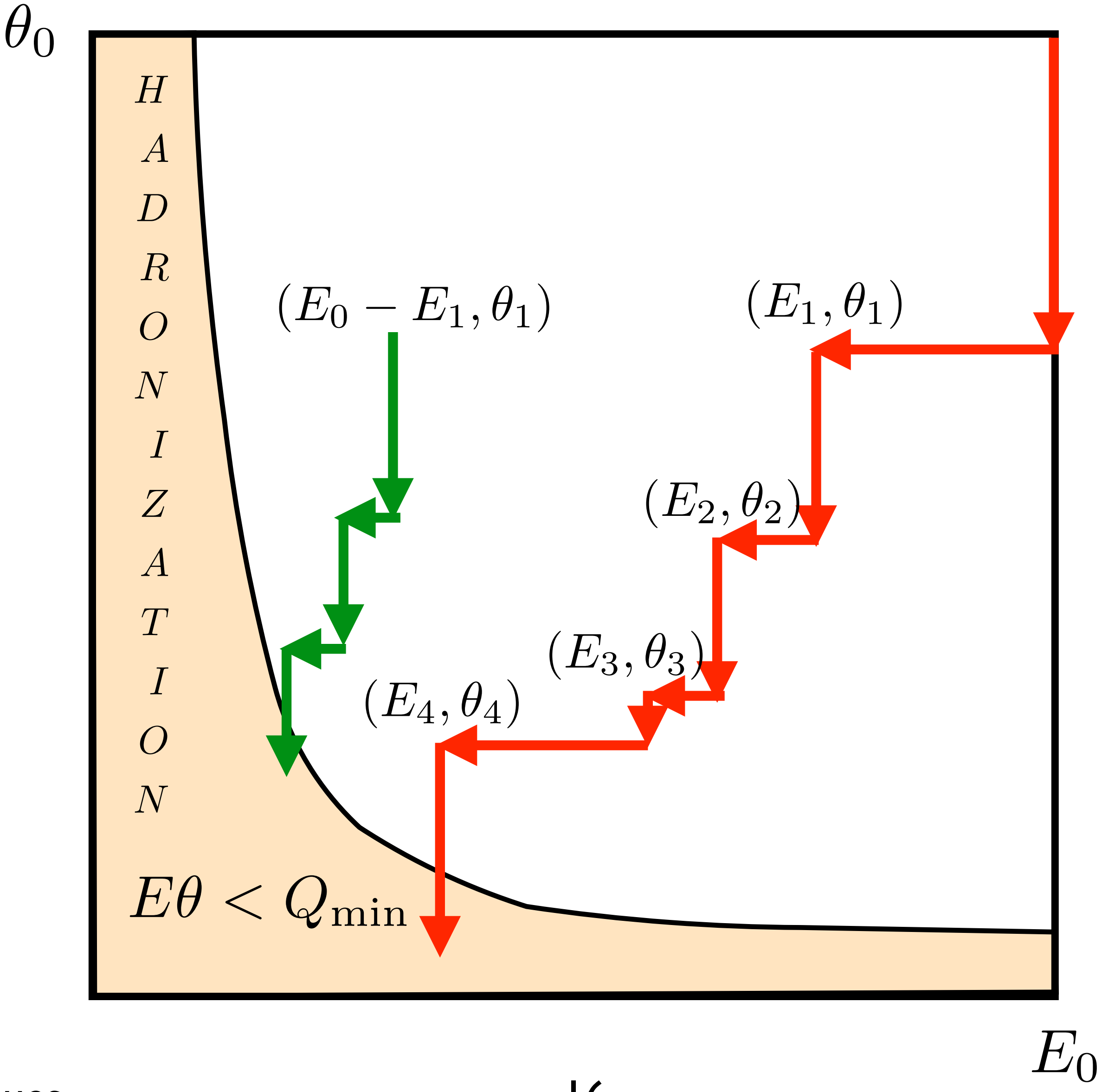
Parton Shower Evolution



$$P_{q\to q}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

Parton Shower Evolution



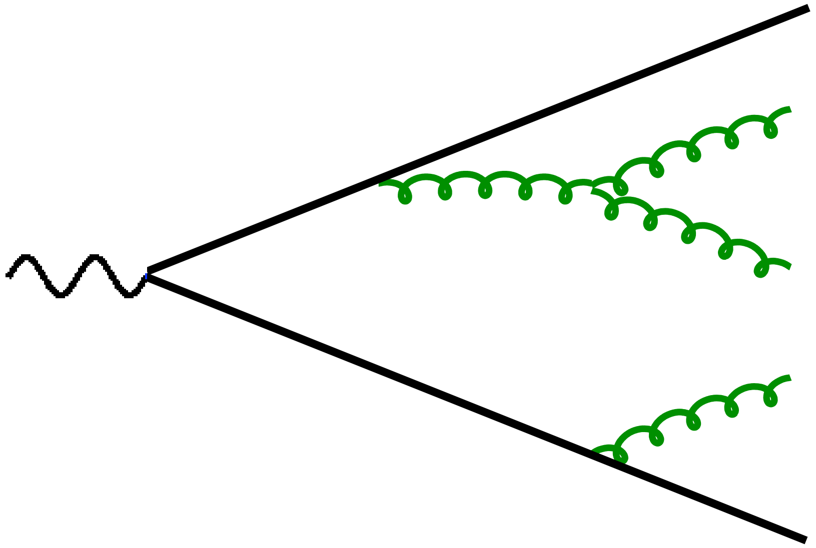


$$P_{q\to q}(z) = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{g\to g}(z) = 3\frac{1+z^4+(1-z)^4}{z(1-z)}$$

Hadronization Models

- In parton shower, relative transverse momenta evolve from a high scale Q towards lower values
- At a scale near Λ_{QCD}~200 MeV, perturbation theory breaks down and hadrons are formed
- Before that, at scales ~ few x Λ_{QCD}, there is universal preconfinement of colour
- Colour, flavour and momentum flows are only locally redistributed by hadronization



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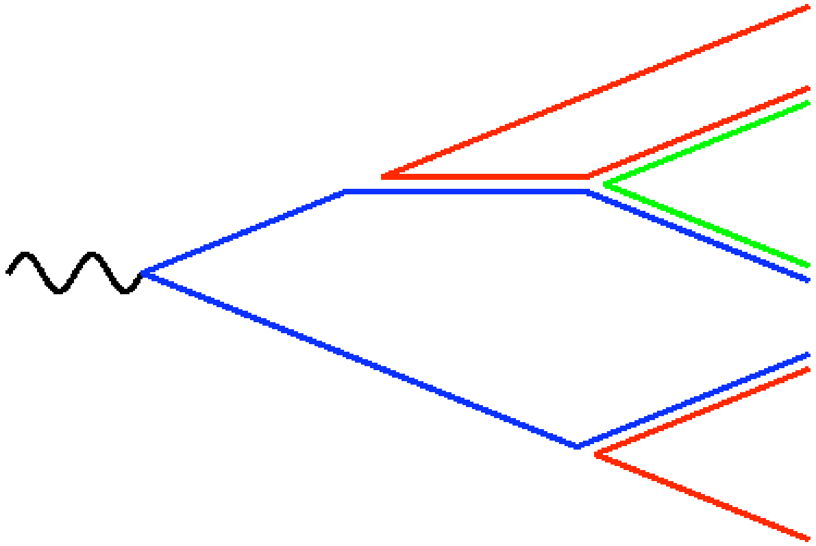

String Hadronization Model

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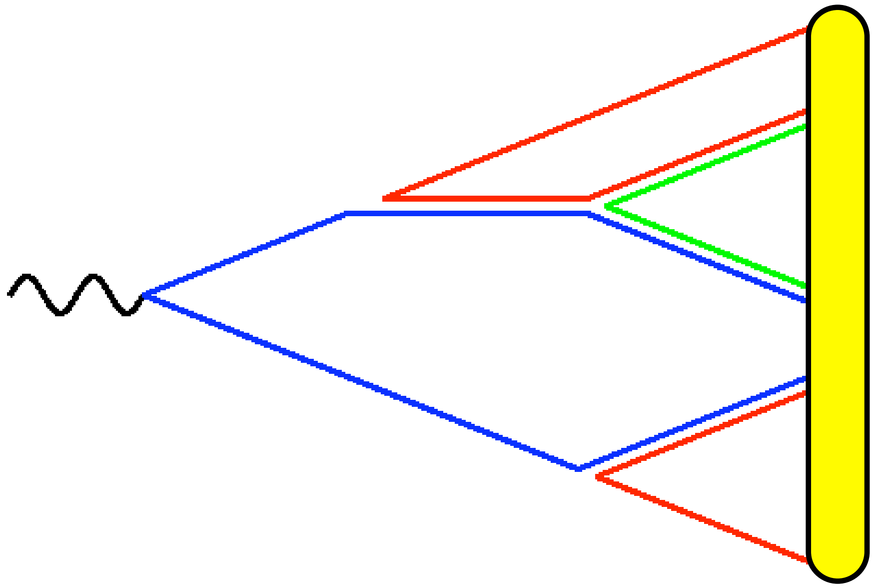

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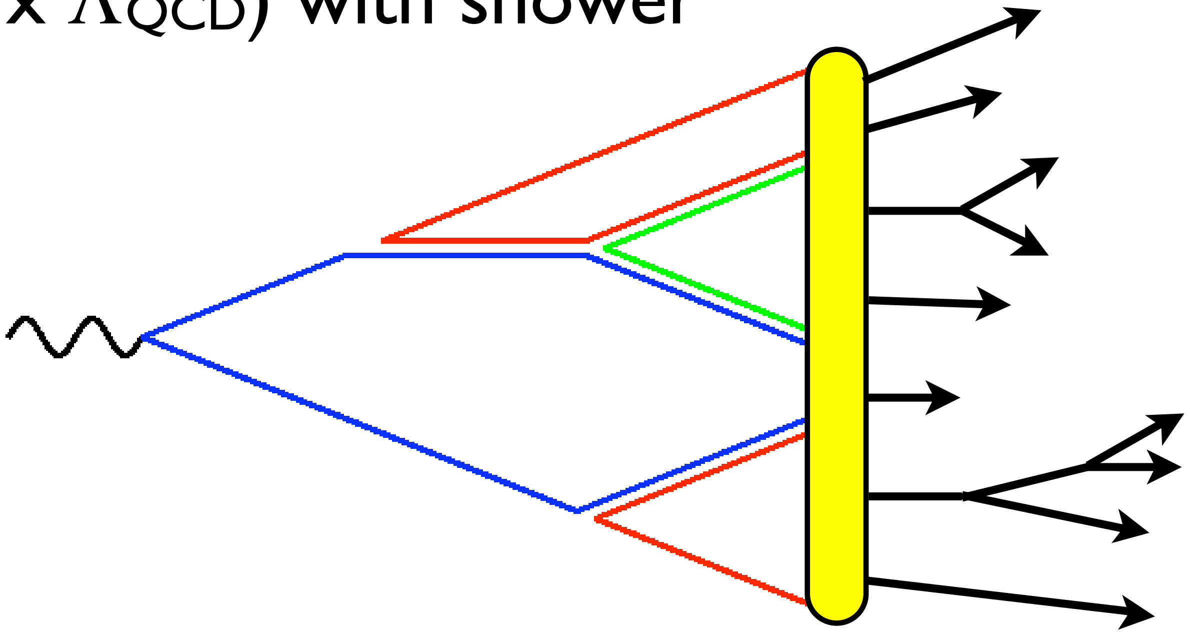
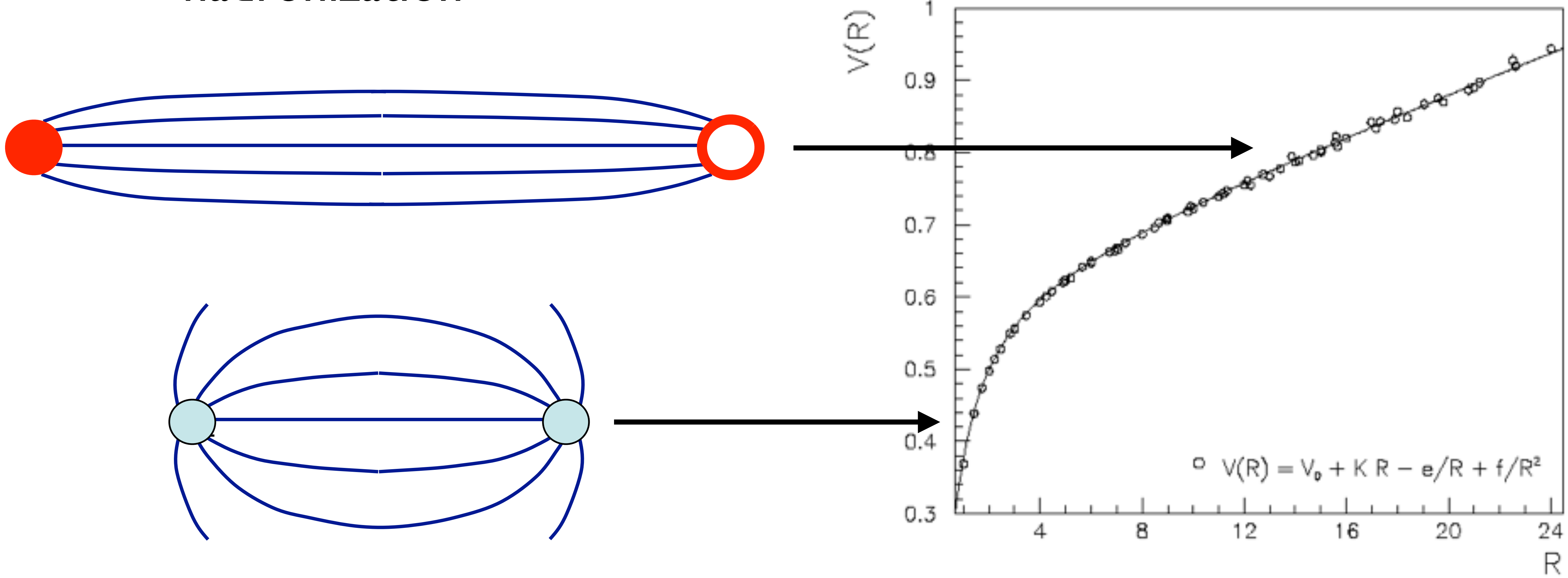


String Hadronization Model

- At short distances (large Q), QCD is like QED: colour field lines spread out (1/r potential)
- At long distances, gluon self-attraction gives rise to colour string (linear potential, quark confinement)
- Intense colour field induces quark-antiquark pair creation: hadronization



Cluster Hadronization Model

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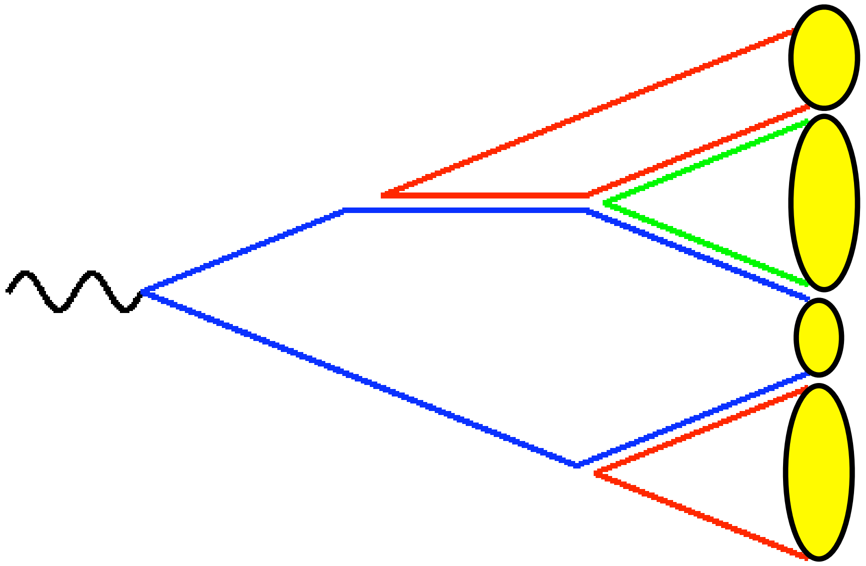
- Decay of preconfined clusters provides a direct basis for hadronization



Cluster Hadronization Model

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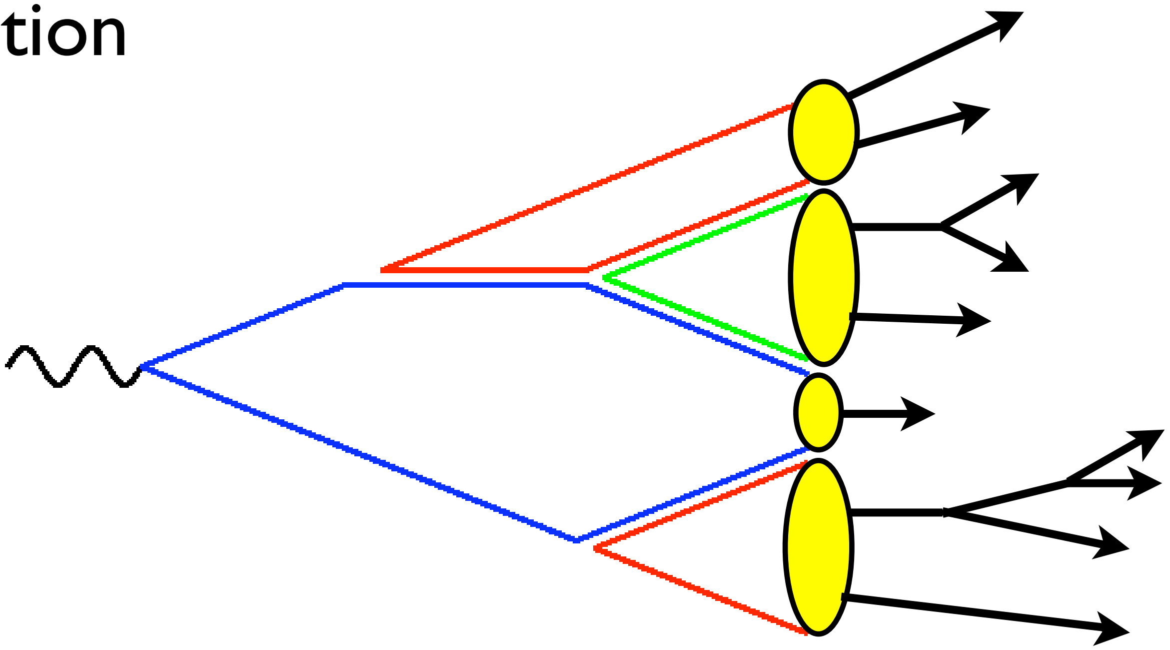
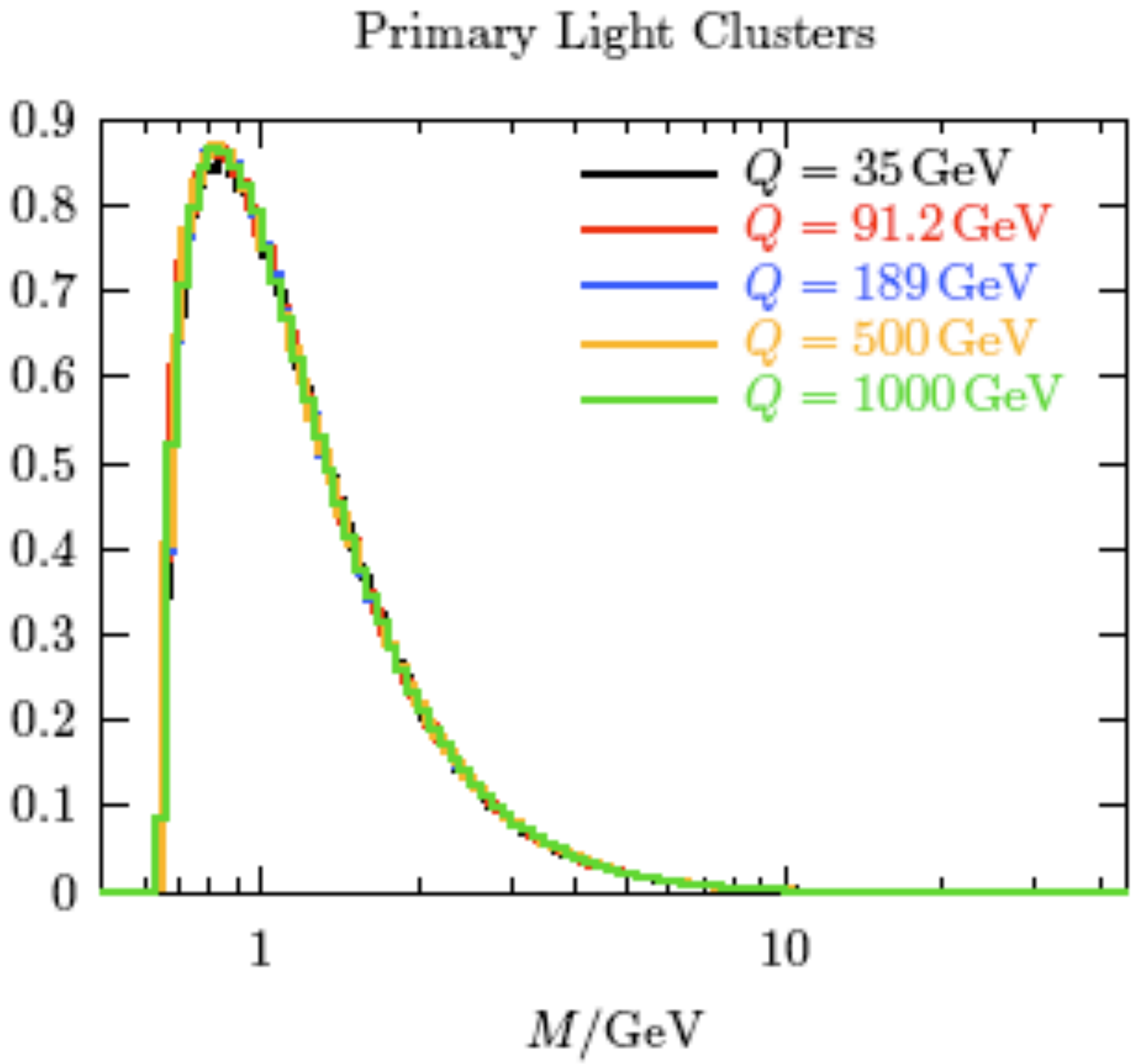


Cluster Hadronization Model

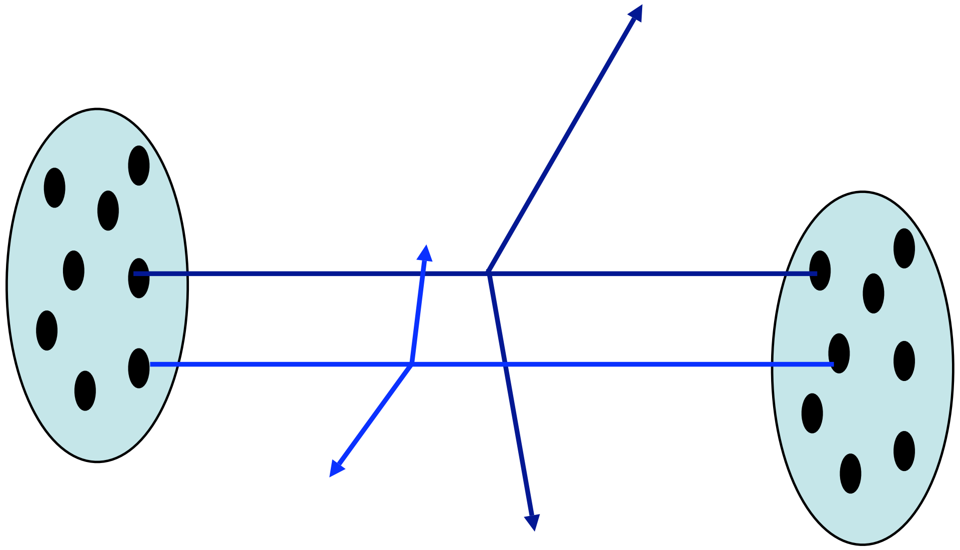


- Mass distribution of preconfined clusters is universal
- Phase-space decay model for most clusters
- High-mass tail decays anisotropically (string-like)

Hadronization Status

- No fundamental progress since 1980s
  - Available non-perturbative methods (lattice, AdS/QCD, ...) are not applicable
- Less important in some respects in LHC era
  - Jets, leptons and photons are observed objects, not hadrons
- But still important for detector effects
  - Jet response, heavy-flavour tagging, lepton and photon isolation, ...

Underlying Event (MPI)



- Multiple parton interactions in same collision
  - Depends on density profile of proton
- Assume QCD 2-to-2 secondary collisions
  - Need cutoff at low p_T
- Need to model colour flow
  - Colour reconnections are necessary

Sample of Event Generator Results

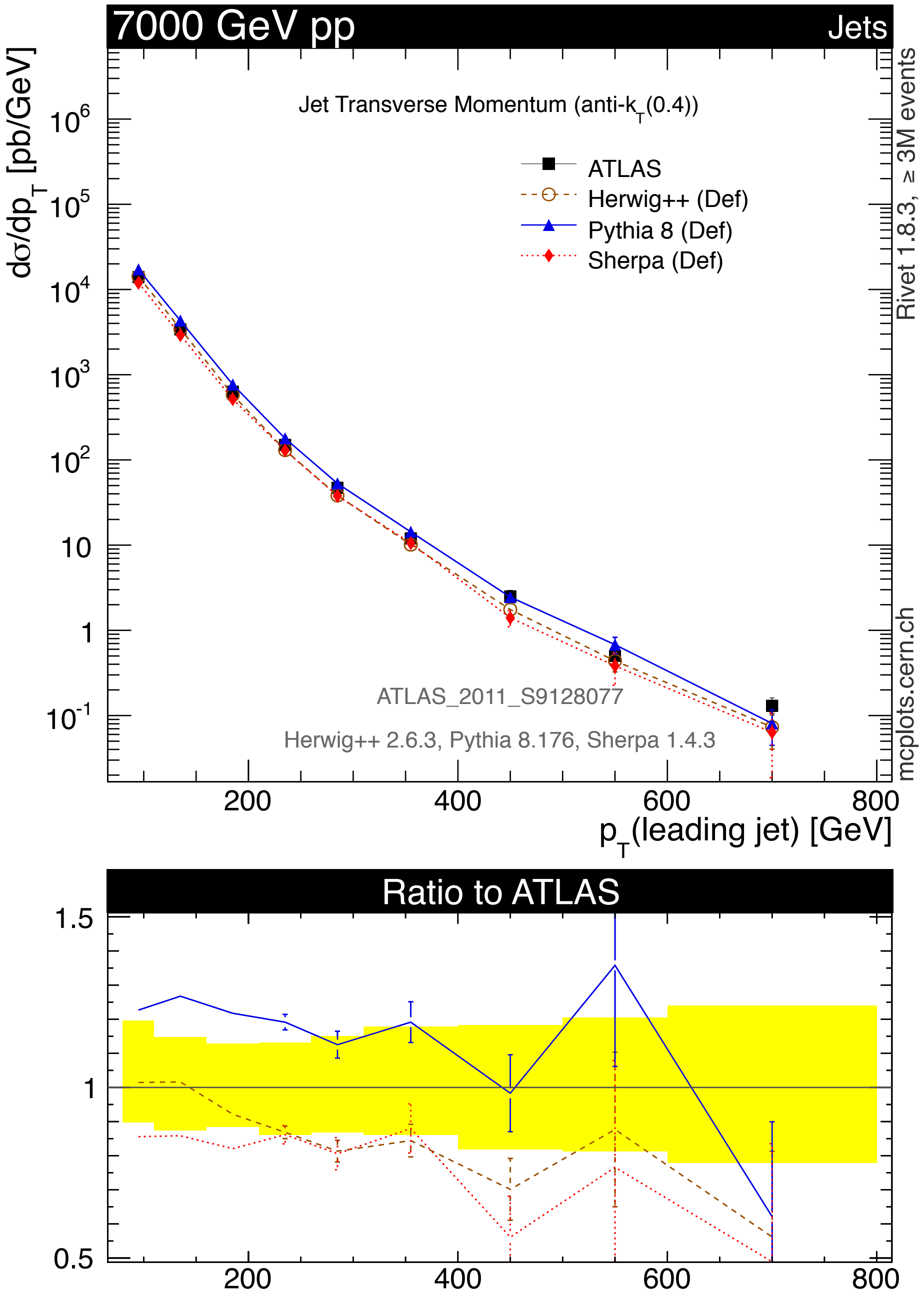
MC Event Generators

- **HERWIG** http://projects.hepforge.org/herwig/
  - Angular-ordered parton shower, cluster hadronization
  - v6 Fortran; Herwig++
- **PYTHIA** http://www.thep.lu.se/~torbjorn/Pythia.html
  - k_t-ordered parton shower, string hadronization
  - v6 Fortran; v8 C++
- **SHERPA** http://projects.hepforge.org/sherpa/
  - Dipole-type parton shower, cluster hadronization
  - C++

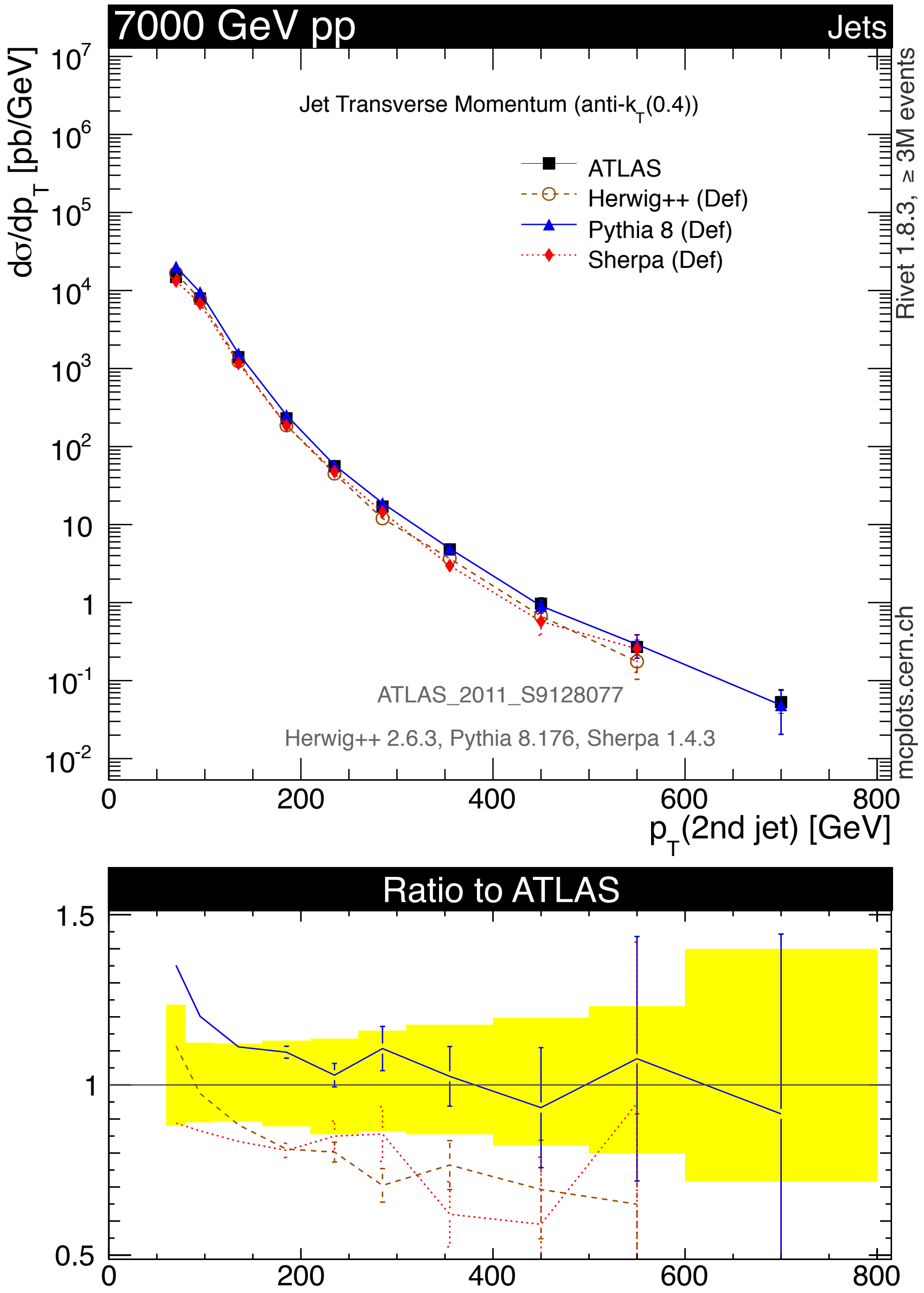
"General-purpose event generators for LHC physics",
A Buckley et al., arXiv:1101.2599, Phys. Rept. 504(2011)145

Jets

Jet pT



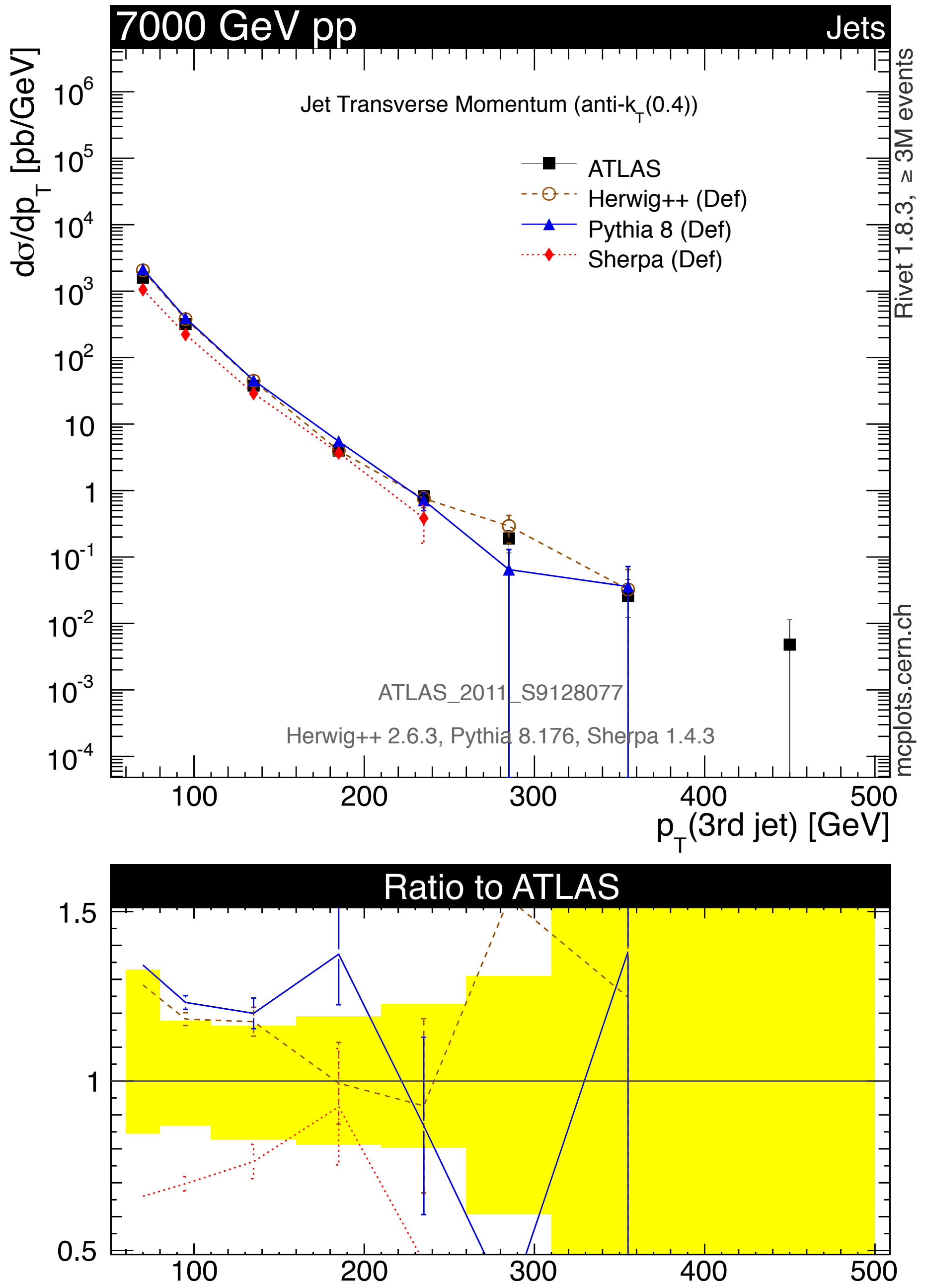
Leading jet

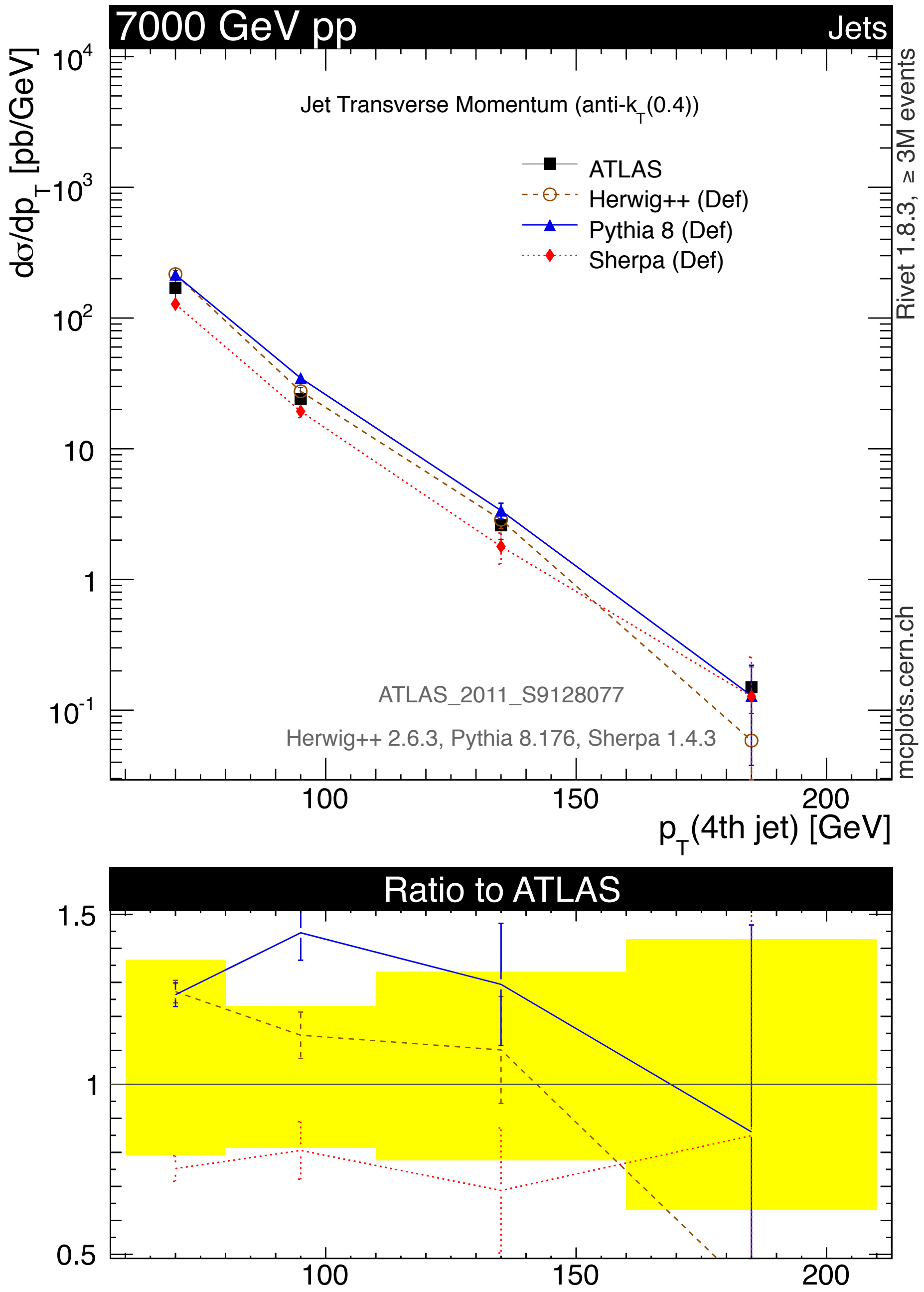


Second jet

http://mcplots.cern.ch

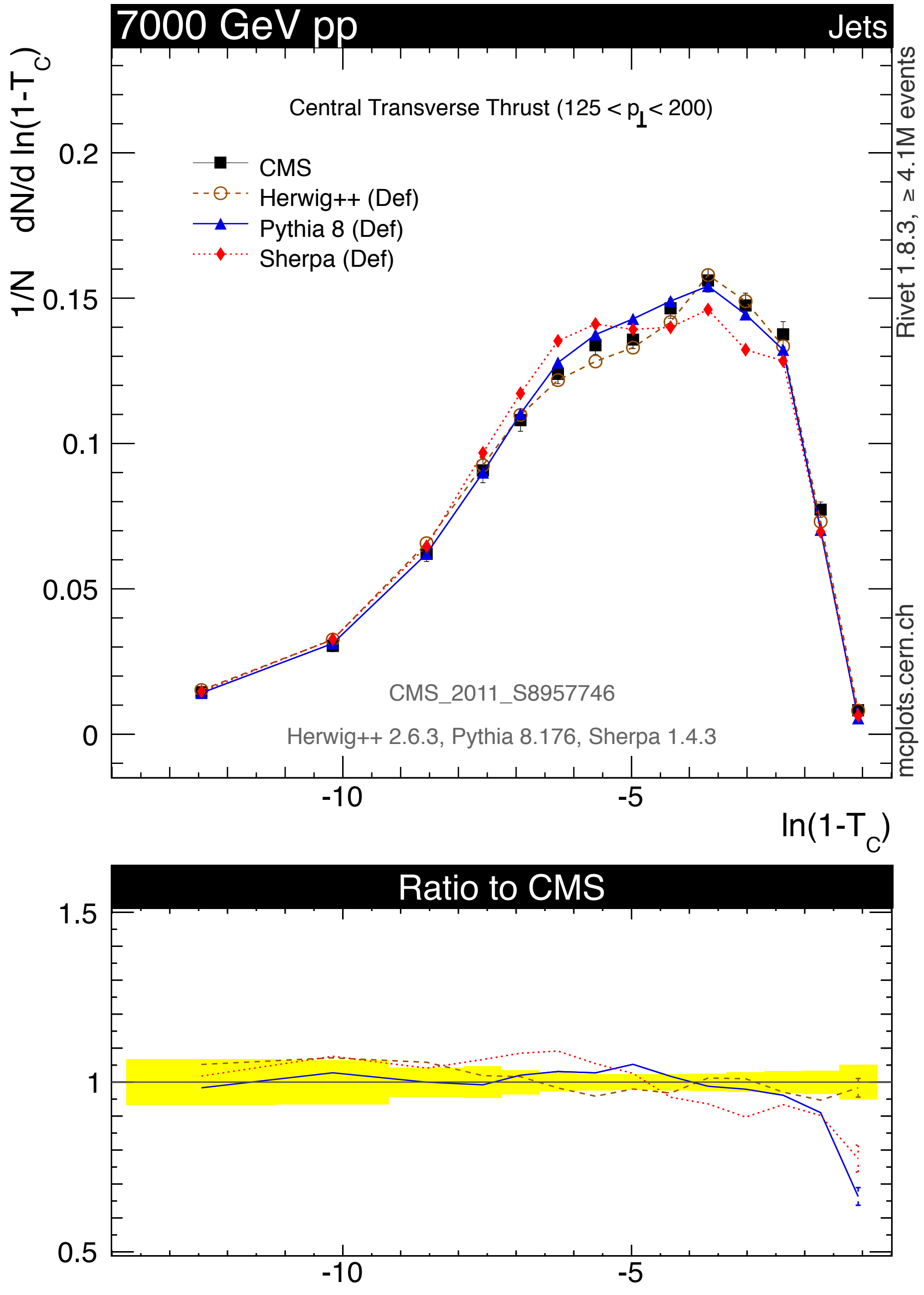
Jet p_T

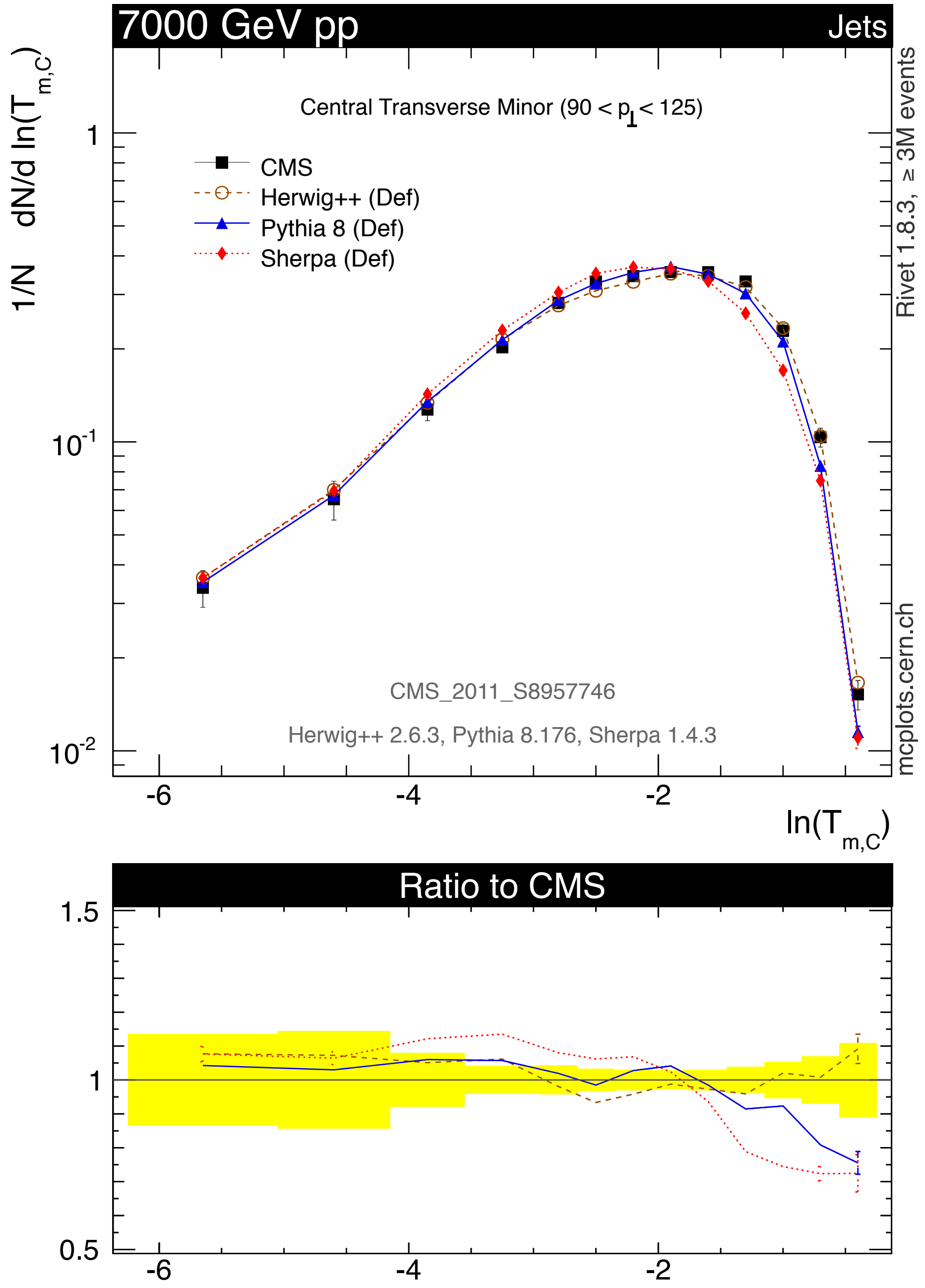




Extra jets from parton showers

Jet event shapes

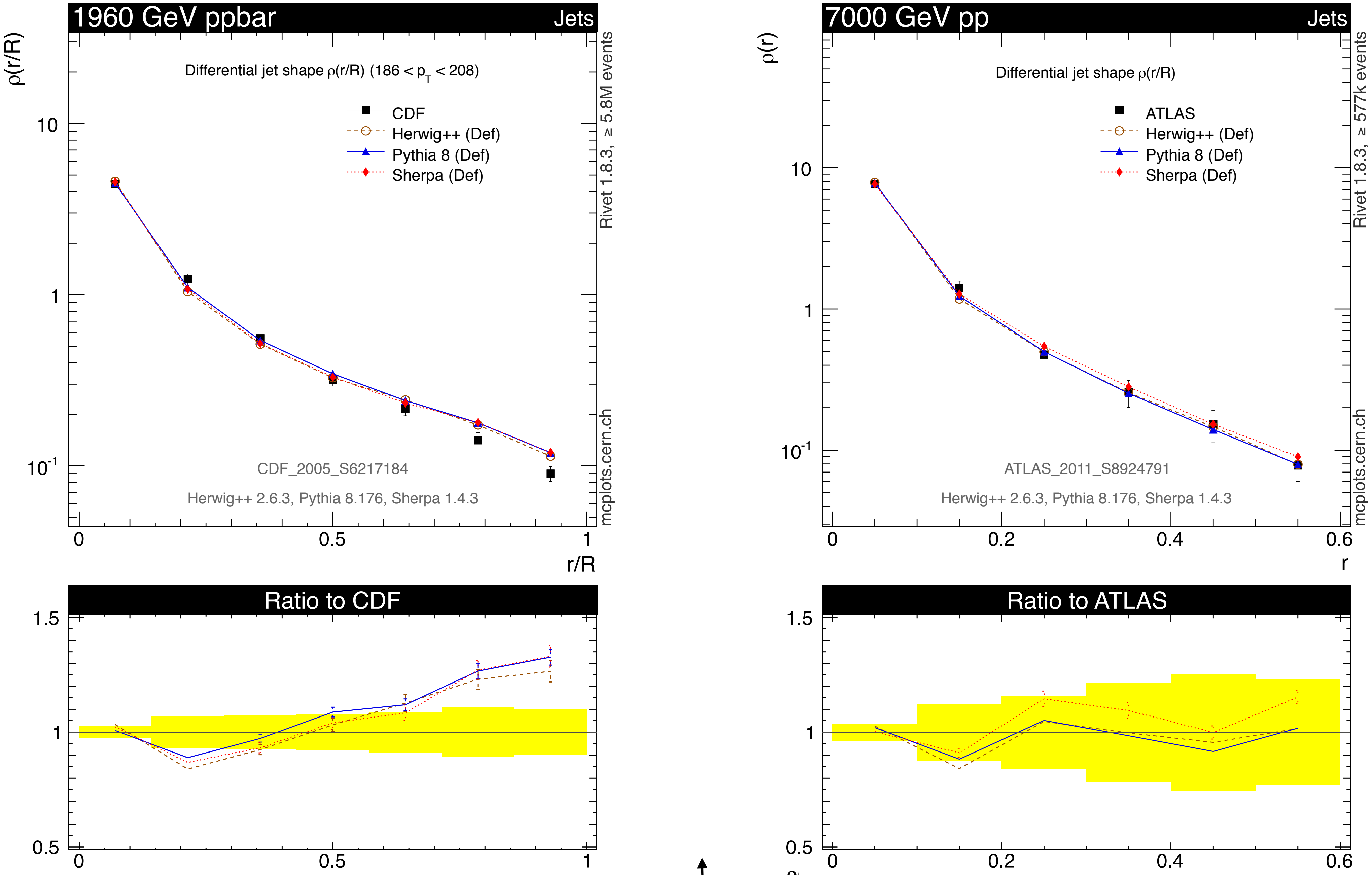


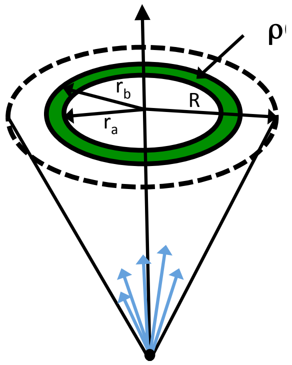


$$T_c \equiv \max_{\hat{n}_\mathrm{T}} \frac{\sum_i |\vec{p}_{\perp,i} \cdot \hat{n}_\mathrm{T}|}{\sum_i p_{\perp,i}}$$

$$T_{m,\mathcal{C}} \equiv \frac{\sum_i |\vec{p}_{\perp,i} \times \hat{n}_{\mathrm{T},\mathcal{C}}|}{\sum_i p_{\perp,i}}$$

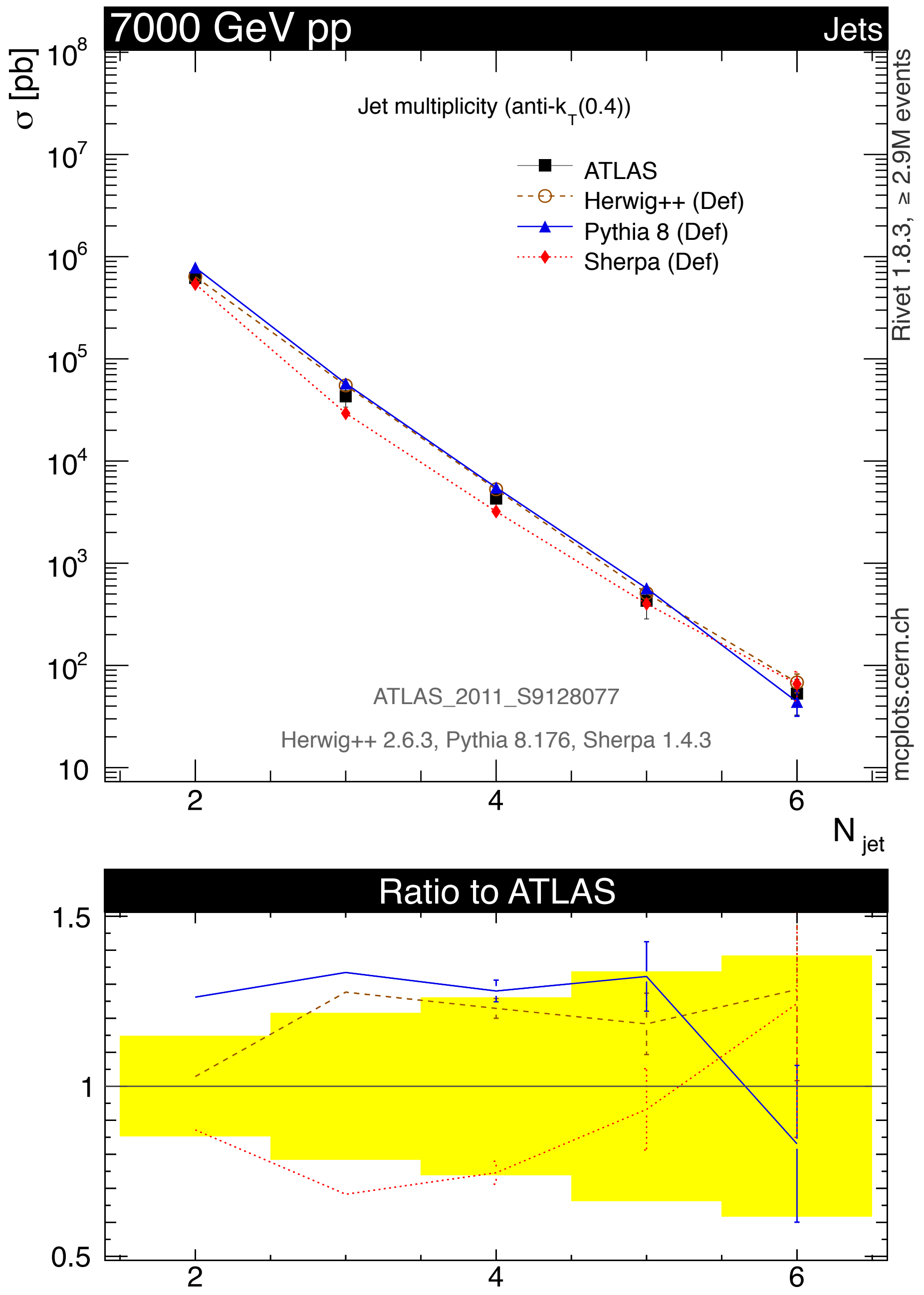
Jet profile



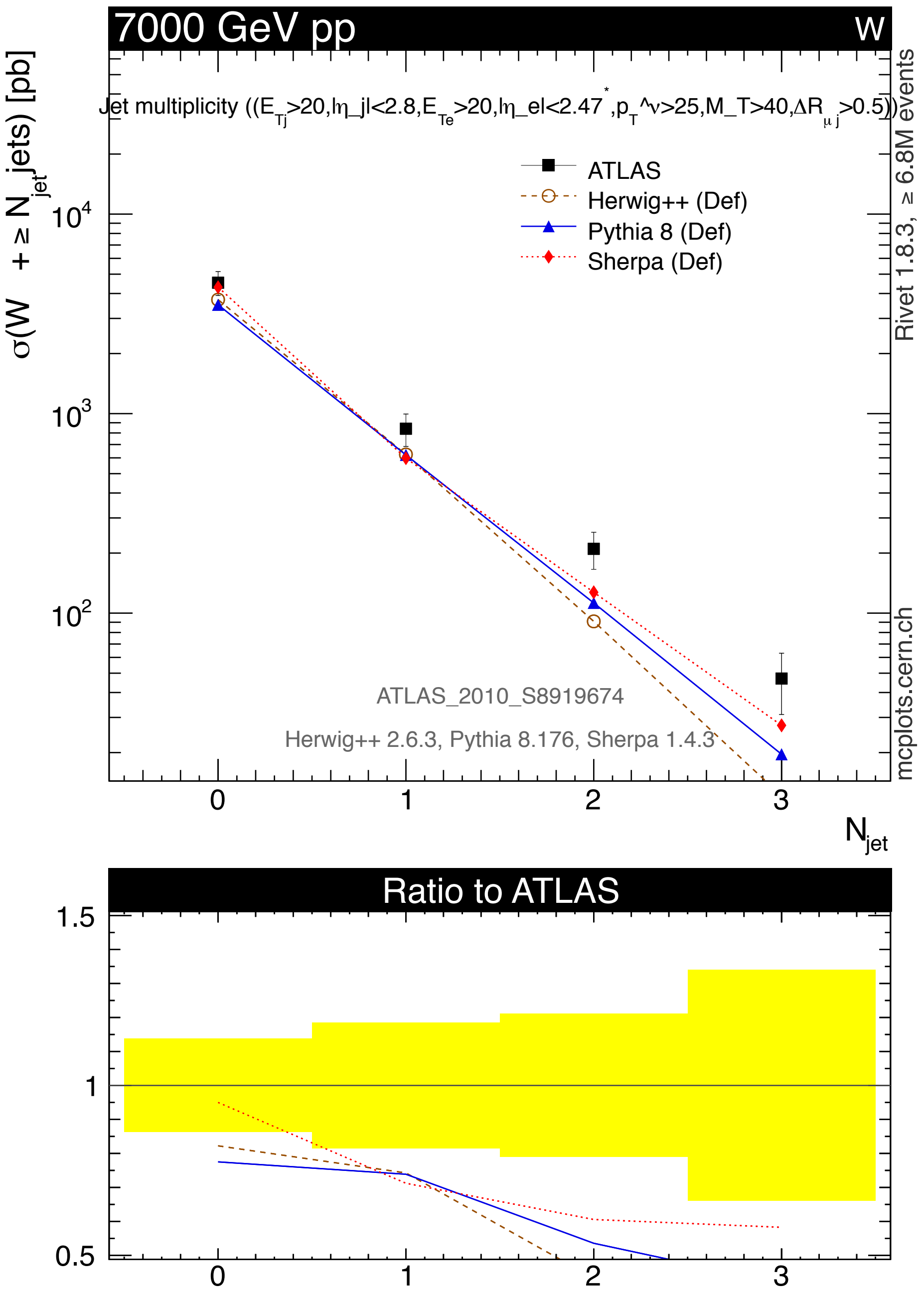


$$\rho(r) = \frac{1}{\delta r}\frac{\sum_{r_a<r_i<r_b} p_{T,i}}{\sum_{r_i<R} p_{T,i}}$$

Jet multiplicity

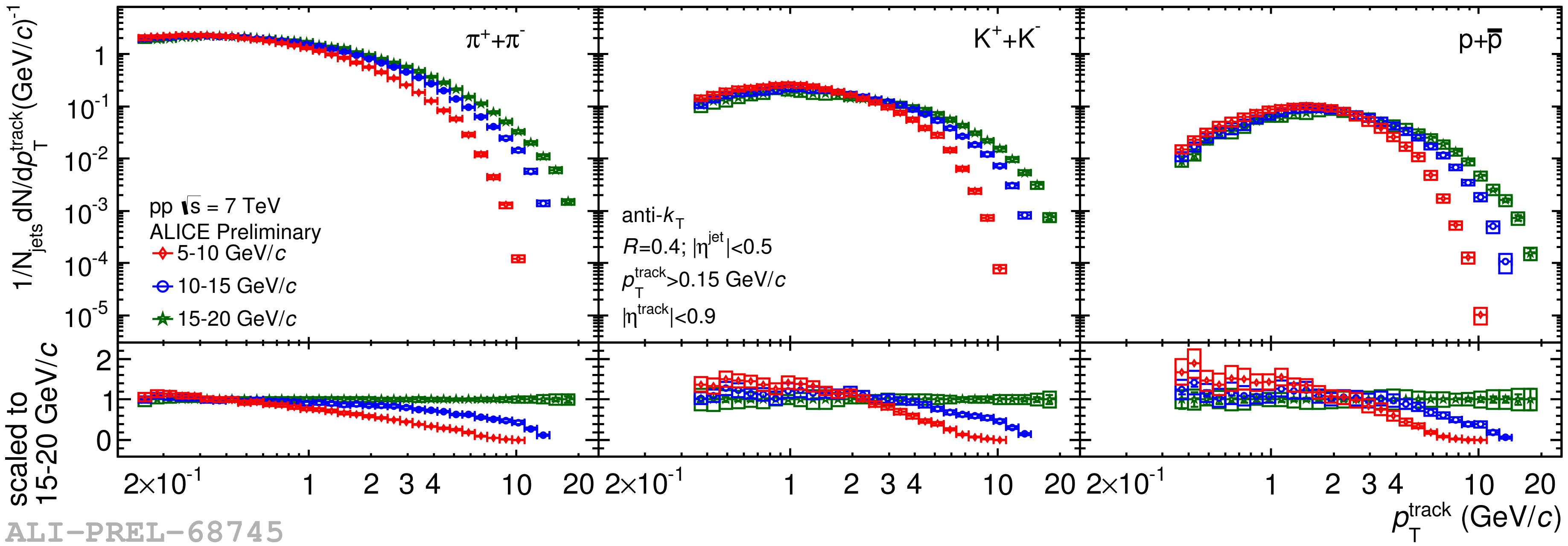


Multijets



W+jets

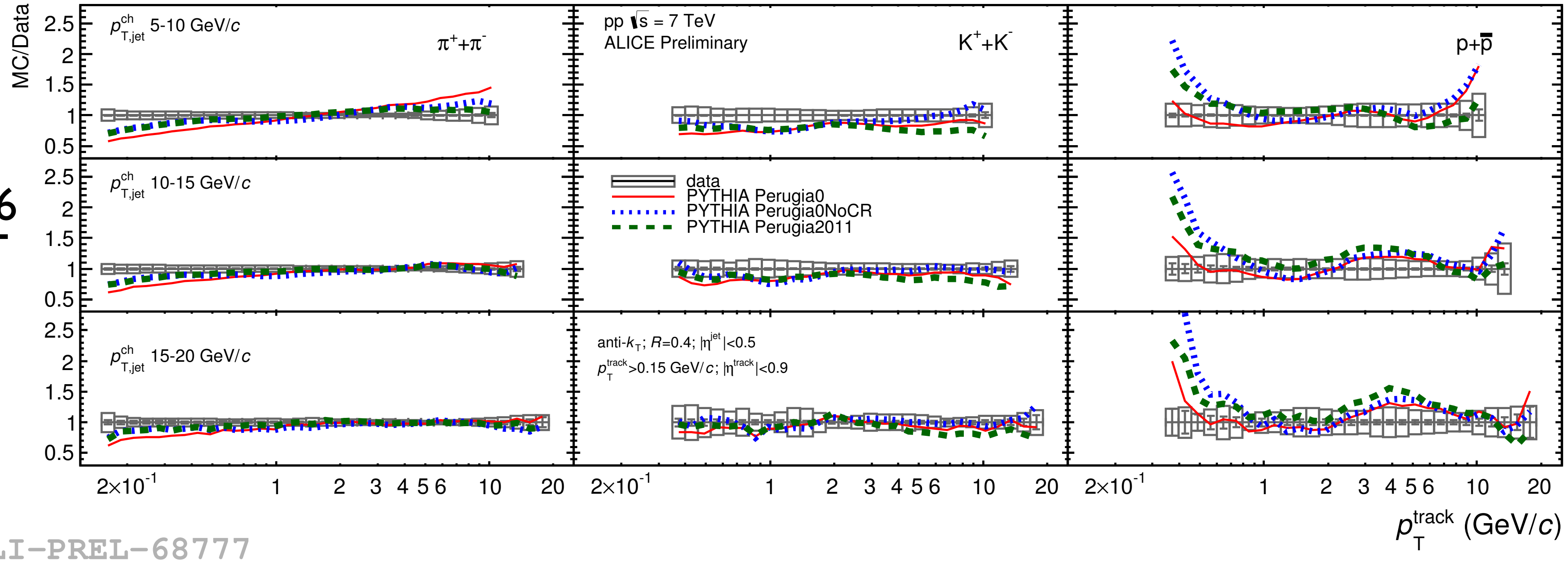
Charged particles in jets



ALI-PREL-68745

PYTHIA6

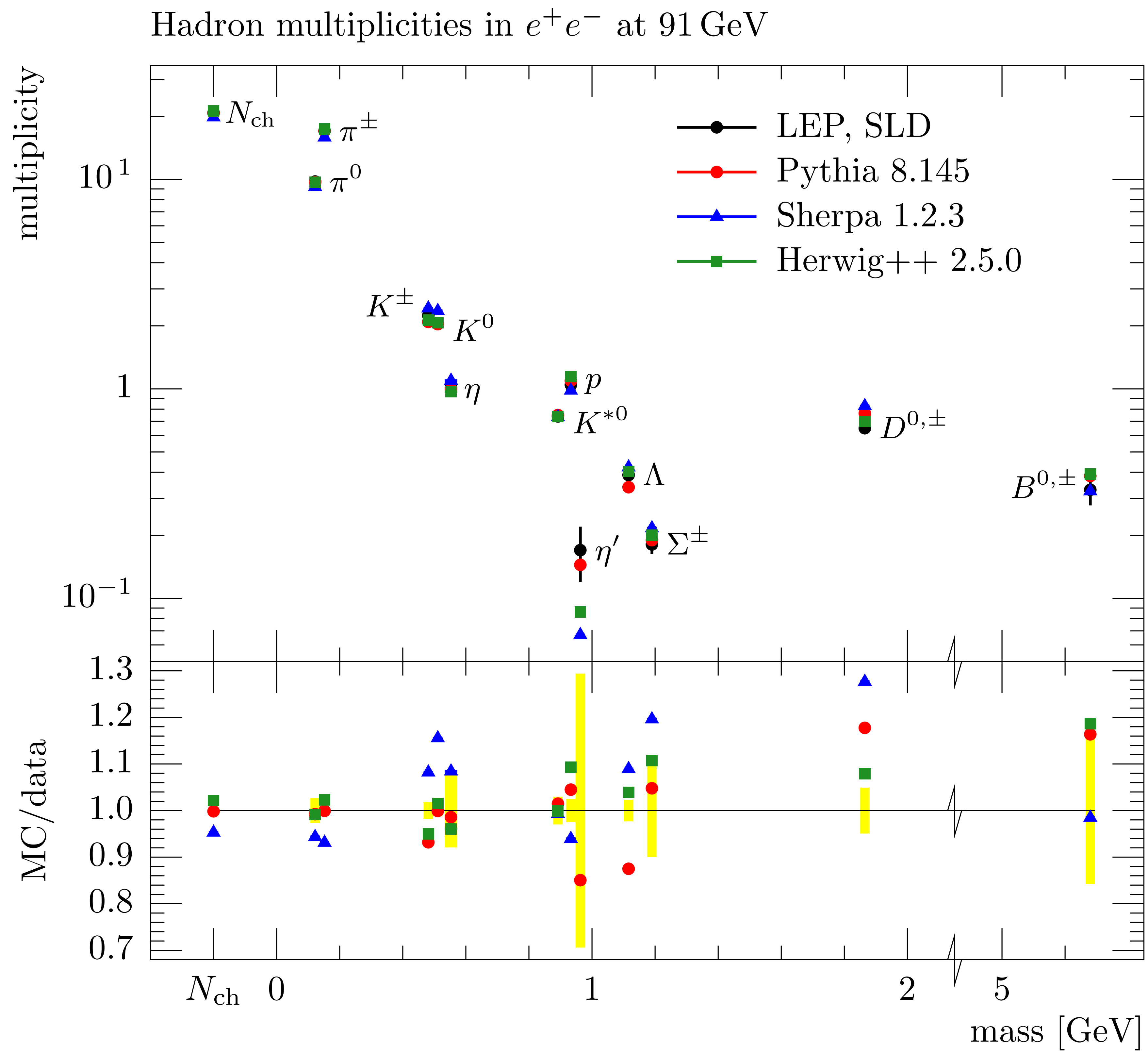
Data



ALI-PREL-68777

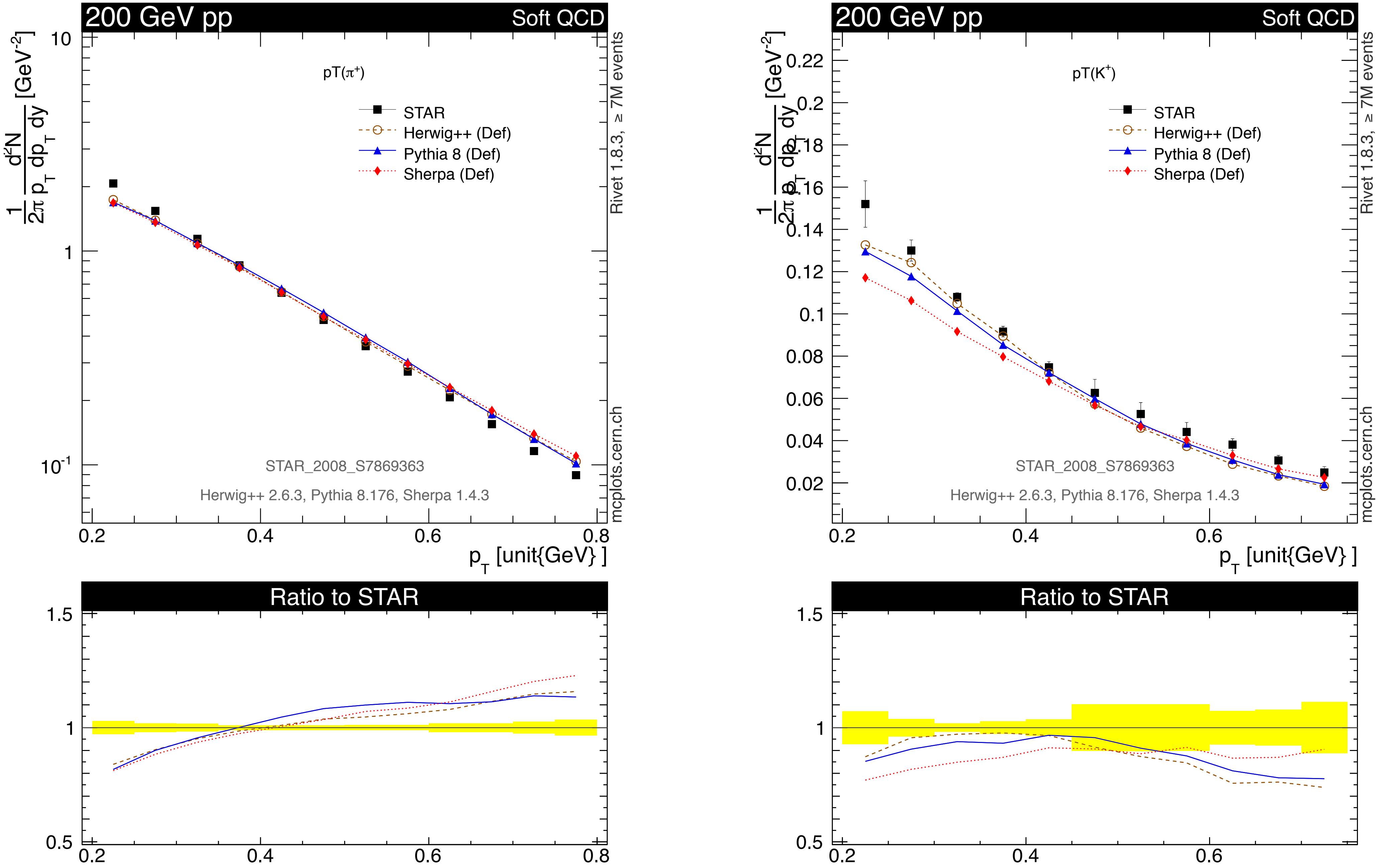
ALICE, arXiv:1408.5723

Multiplicities in Z^0 decay



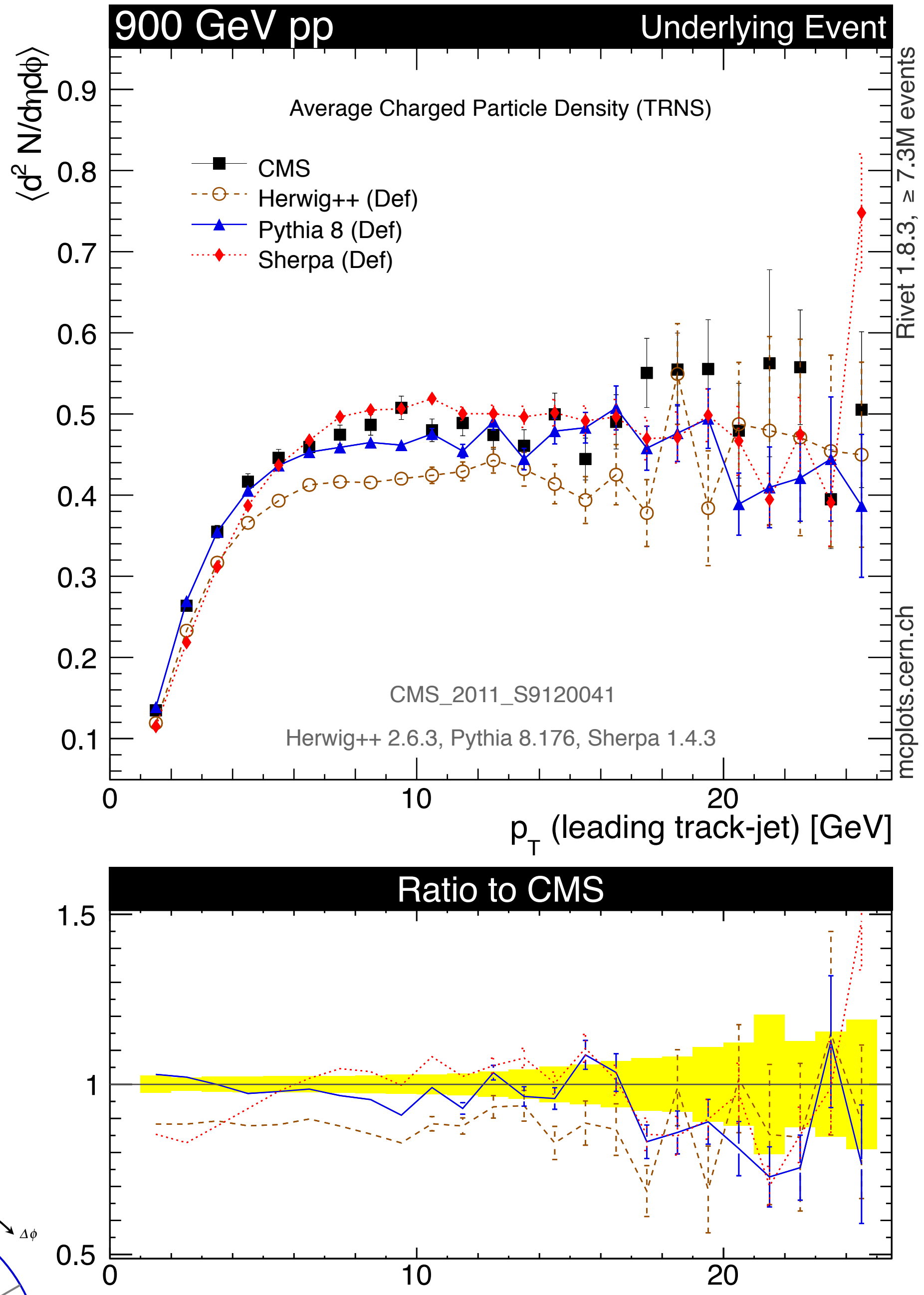
Min Bias and Underlying Event

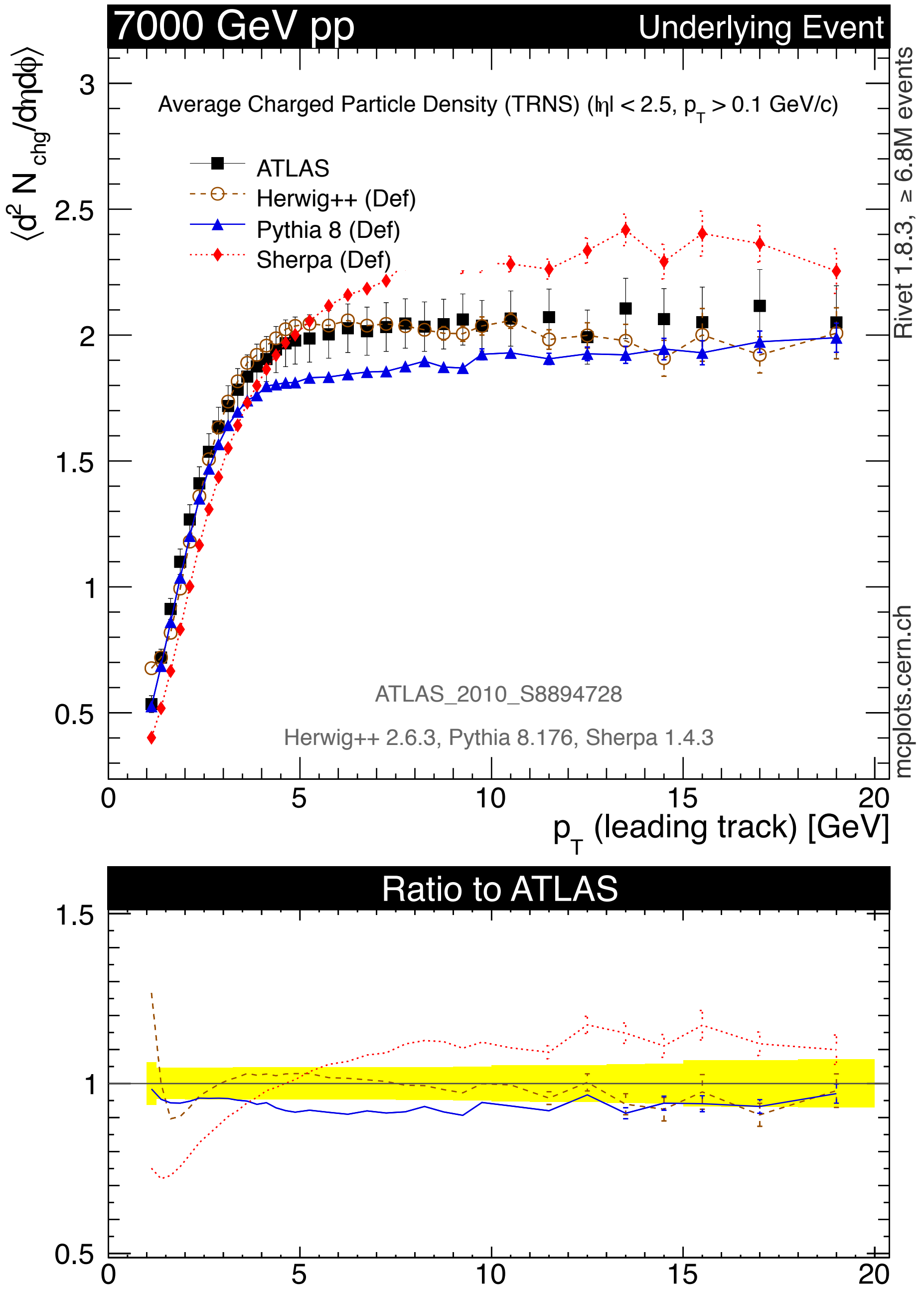
Min bias $p_T(\pi^+, K^+)$

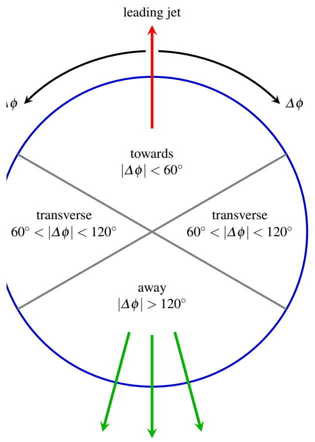


- Min bias = all scattering events

Underlying Event

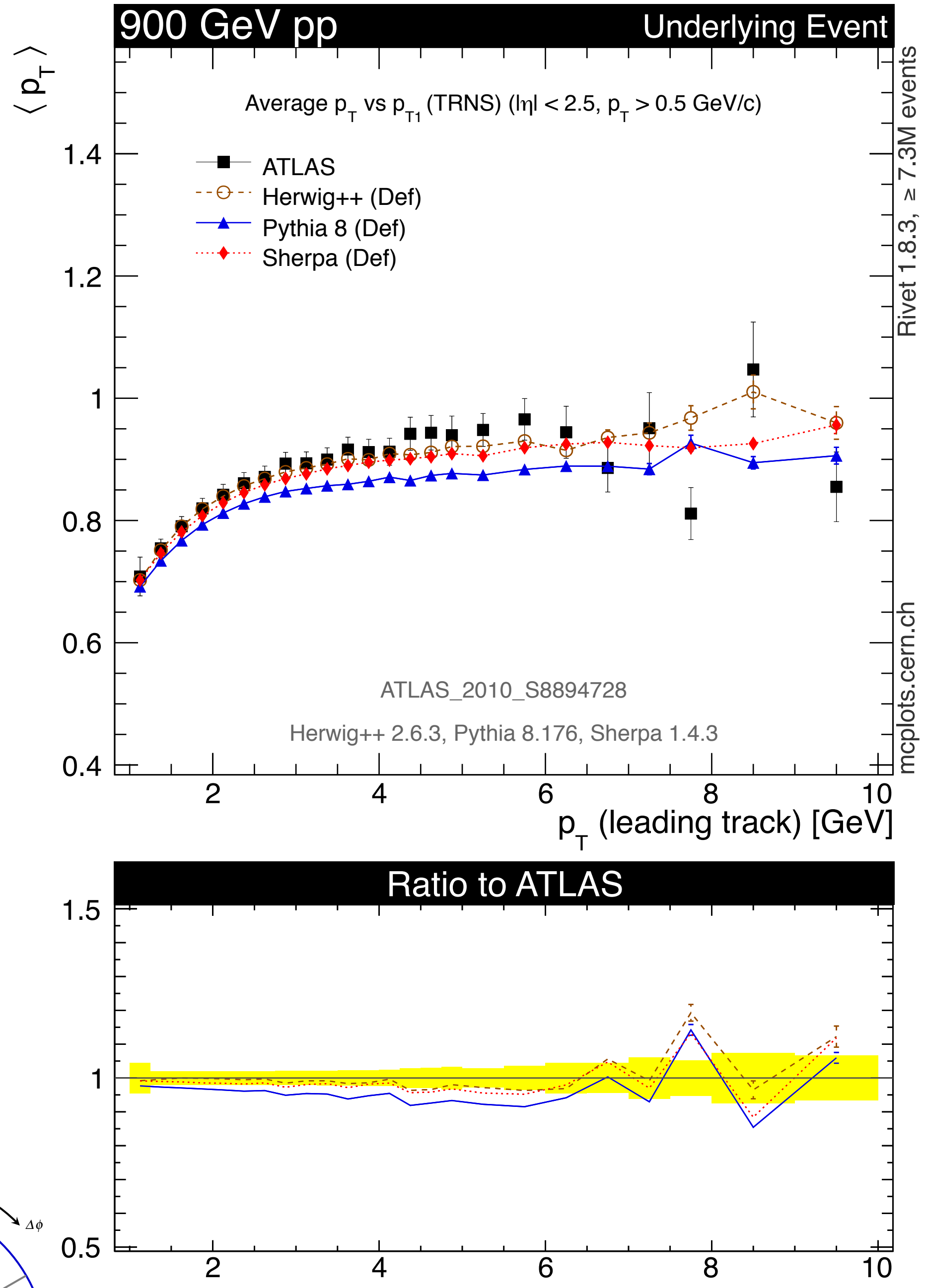


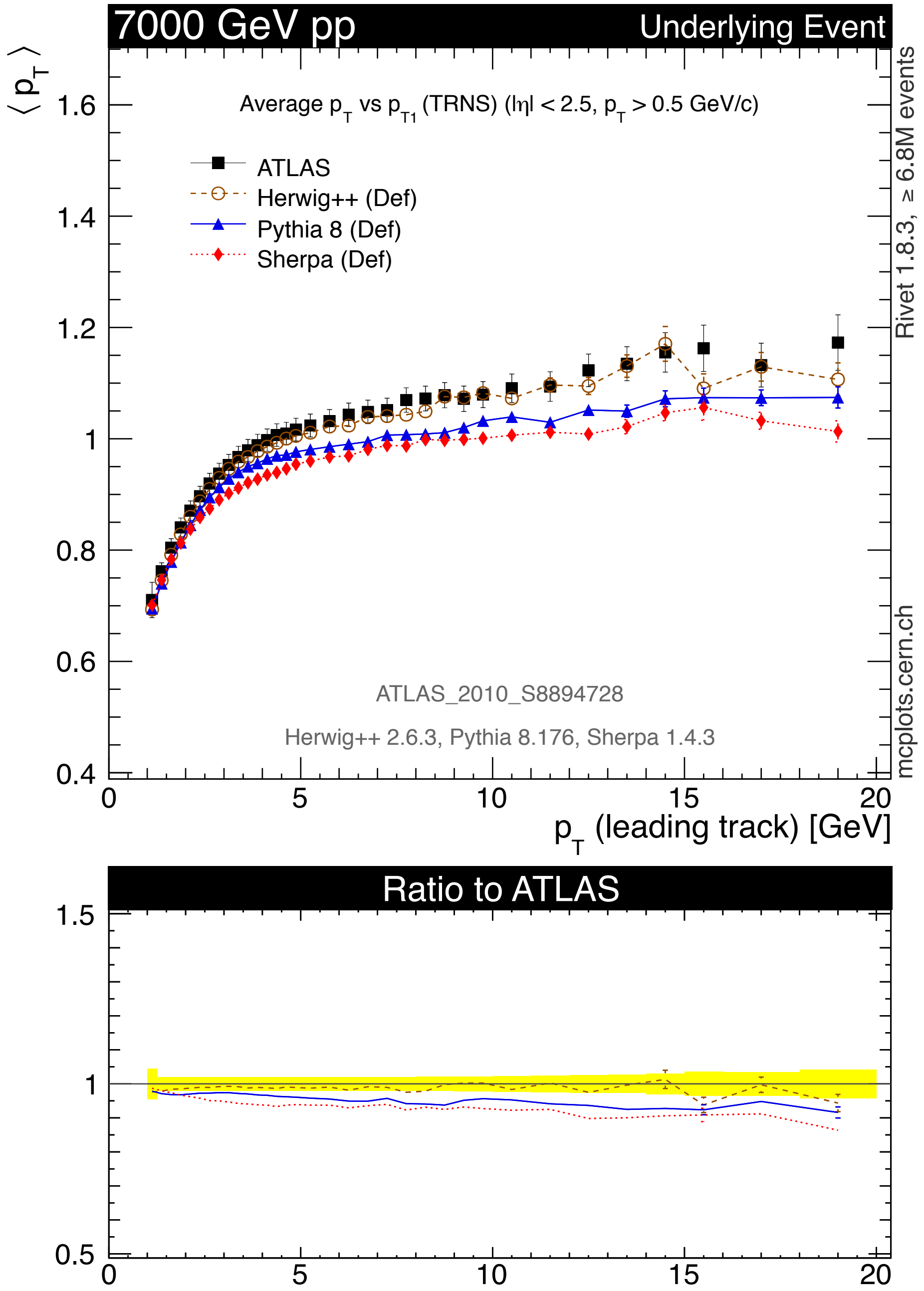


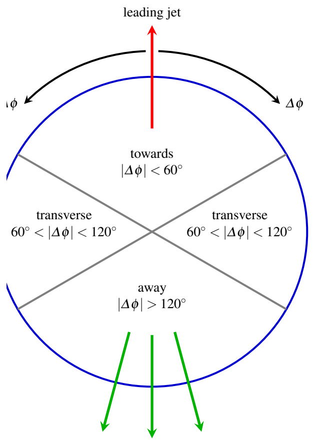


Transverse charged particle density

Underlying Event





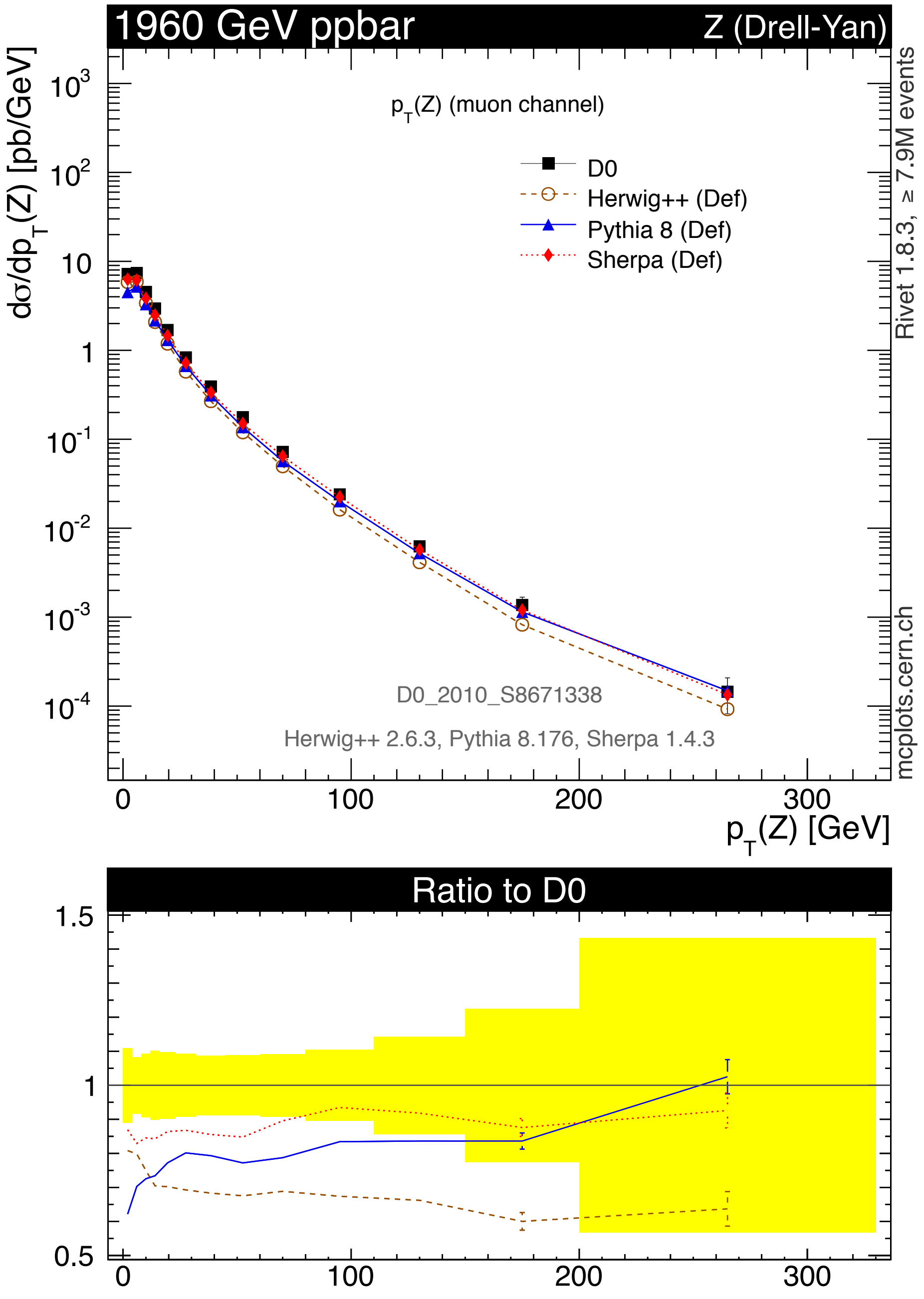


Transverse mean p_T

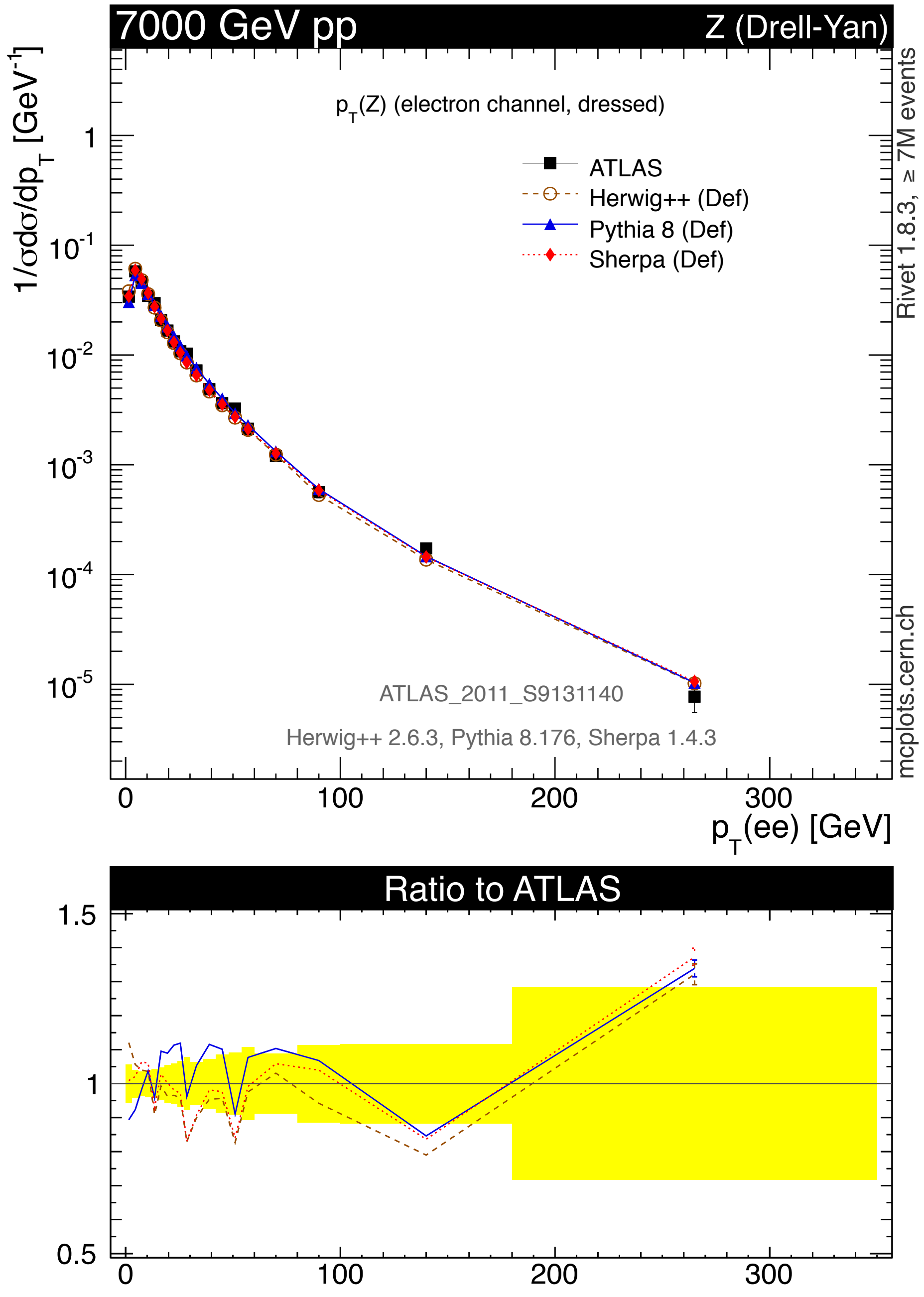
Vector Bosons

Z^0 p_T

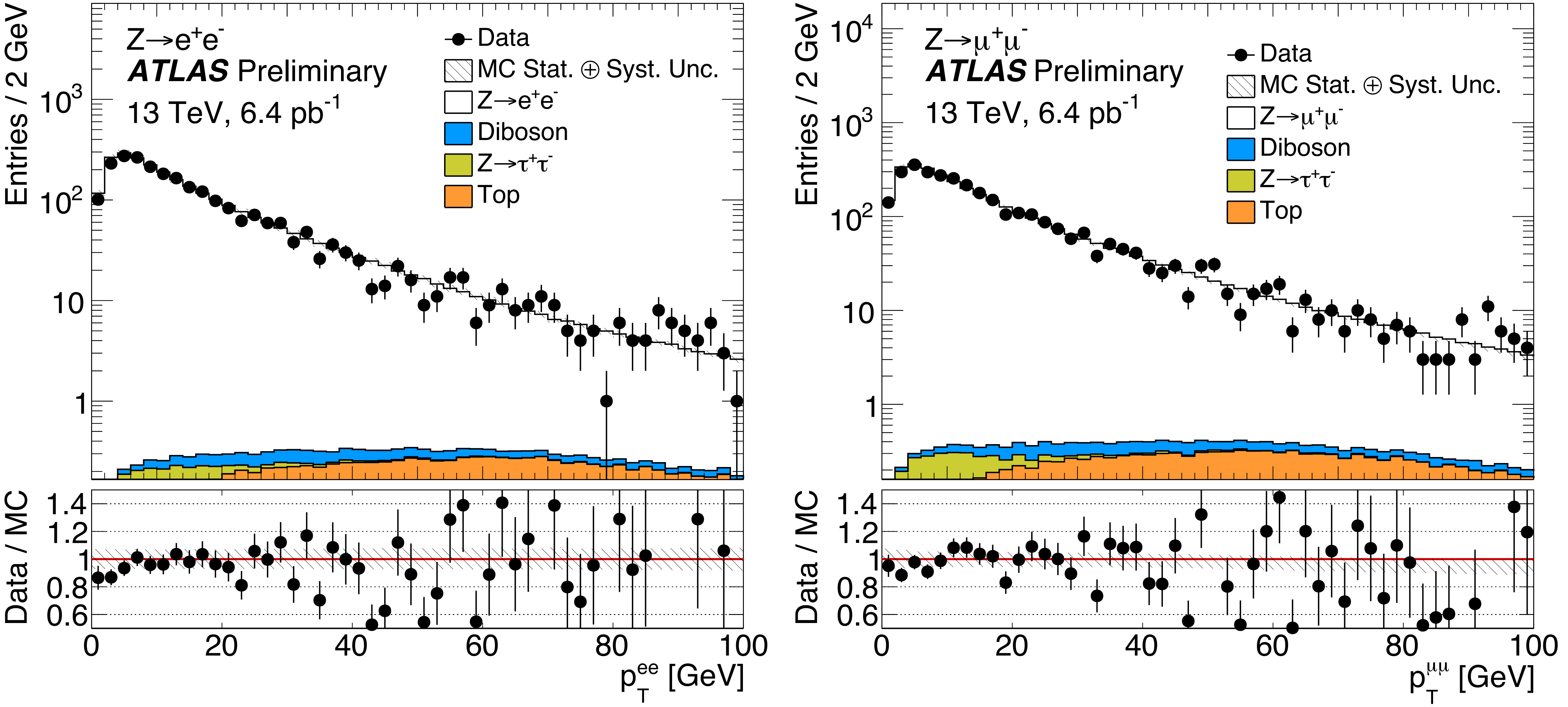
Absolute normalization



Normalized to data



Z^0 pT (13 TeV)



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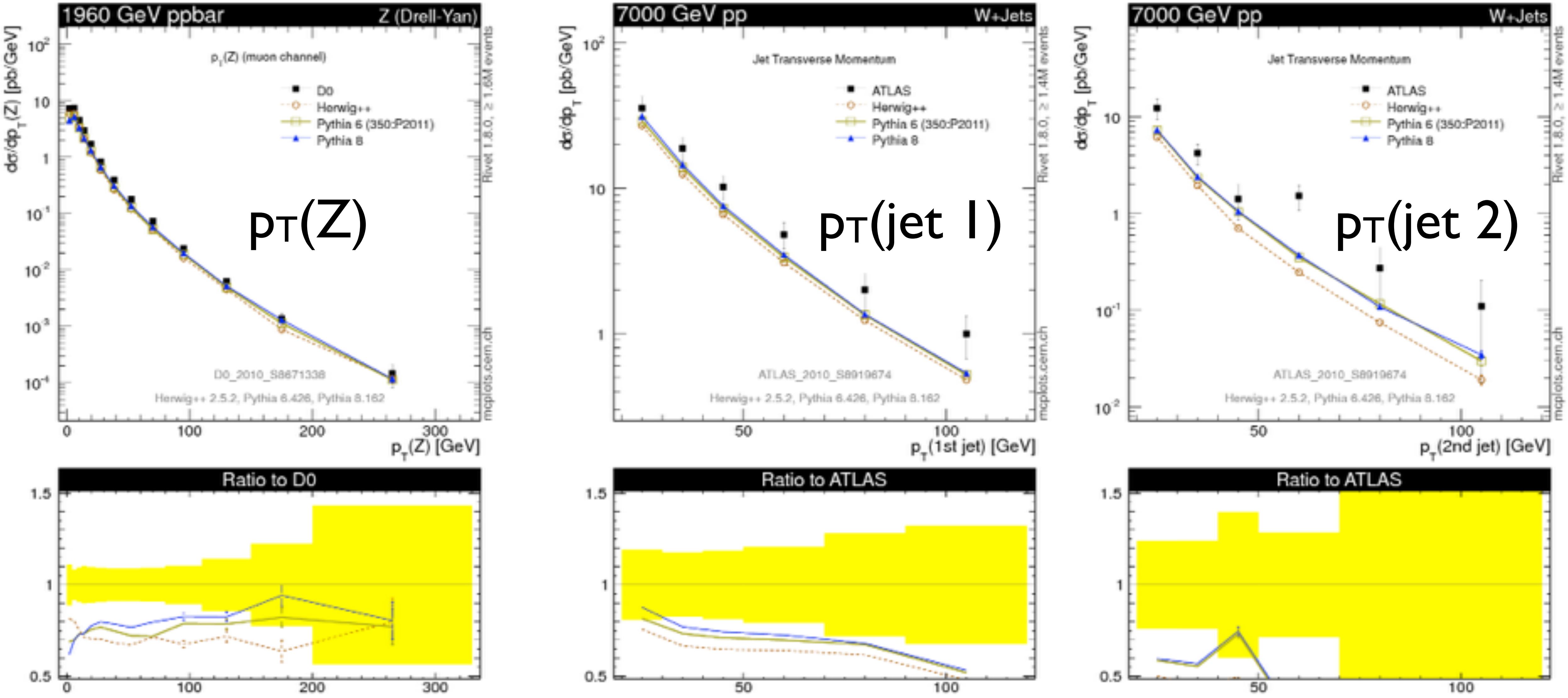
Limitations of LO+parton shower

- Hard process: $q\bar{q} \rightarrow Z^0/W^\pm$



- Leading-order (LO) normalization ➡ need next-to-LO (NLO)
- Worse for high pT and/or extra jets ➡ need multijet merging

Summary of Lecture 2

- Parton shower keeps largest small-angle contribution
- Shower gives preconfinement of colour
- This allows local model of hadronization
- String and cluster models both still viable
- Underlying event due to multiple interactions
- Sample of event generator results
- Further improvements (matching & merging) now used

Thanks for listening!