Ab initio calculation of the neutron-proton mass difference

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Budapest-Marseille-Wuppertal collaboration (BMWc)

(based mainly on Science 347 '15, PRL 111 '13, Science 322 '08)

Nucleon mass difference

Well known experimentally (PDG '14)

$$\begin{aligned}\Delta M_N &= M_n - M_p \\ &= 1.2933322(4)\,\mathrm{MeV} \\ &= 0.14\% \times M_N\end{aligned}$$

w/ $M_N = (M_n + M_p)/2$

Tiny but very important, e.g.

- required for stability of p and $^1\mathrm{H}$:

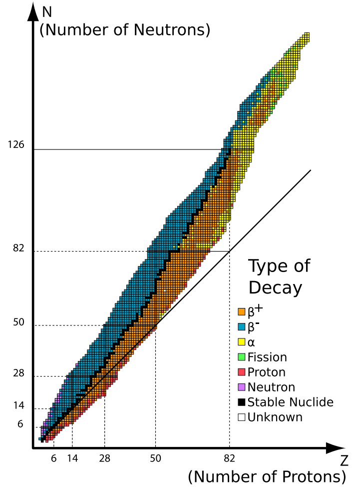
  If $M_p > M_n - m_e$ or $M_p > M_n + m_e$

  $\Rightarrow p + e^- \to n + \nu_e$

  or/and

  $\Rightarrow p \to n + e^+ + \nu_e$
- determines valley of stability through β-decay

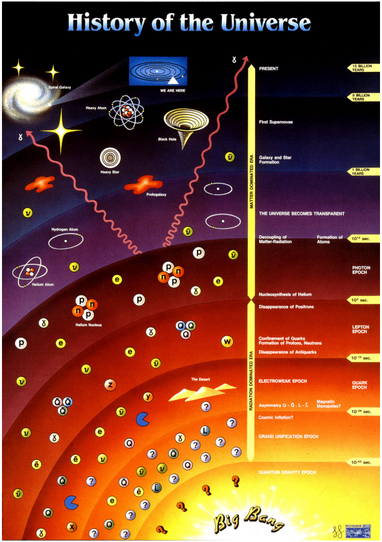
$\to$ necessary to explain baryonic matter as we know it



(Wikipedia)

Importance in the early universe

Time of interest here:
$1\ \mu s \lesssim t \lesssim 3\ \text{min}$



(CERN)

$$E_\beta = \Delta M_N - m_e - m_{\nu_e} = 0.08\% \times M_N$$

$$\downarrow$$

$$n \to p + e^- + \bar{\nu}_e \quad \text{in} \quad \tau_n \sim 15\ \text{min}$$

Critical for Big Bang nucleosynthesis (BBN)

If $\Delta M_N/M_N > 0.14\%$ and thus τ_n smaller

- $\rightarrow$ n decay before trapped and preserved in nuclei
- $\rightarrow$ easily get an universe without n !

If $0.14\% > \Delta M_N/M_N \gtrsim 0.05\%$

- $\rightarrow$ much more $^4\mathrm{He}$ and less $^1\mathrm{H}$

If $\Delta M_N/M_N < 0.05\%$, $p + e^- \to n + \nu_e$

- $\rightarrow$ universe w/ mostly n

$\rightarrow$ very finely tuned system
$\rightarrow$ goal: understand physics behind ΔM_N and similar phenomena

Isospin symmetry and its breaking

$\Delta M_N/M_N \ll 1$ because Nature has a near $SU(2)$-isospin symmetry

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \exp[i\vec{\theta}\cdot\frac{\vec{\tau}}{2}]\begin{pmatrix} u \\ d \end{pmatrix}$$

Only broken by small, often competing effects

$$3\,\frac{m_d - m_u}{M_N} \sim 1\% \qquad \text{and} \qquad (Q_u^2 - Q_d^2)\,\alpha \sim 1\%$$

Isospin breaking also crucial role in many other places, e.g.:

- Knowledge of m_u and m_d, limited by EM (e.g. FLAG 13)
- Improving indirect search for new physics

$\rightarrow$ important flavor observables that are becoming very precisely known: e.g. $\mathrm{err}(m_{ud}), \mathrm{err}(m_s) \sim 2\%$, $\mathrm{err}(m_s/m_{ud}) \lesssim 1\%$, $\mathrm{err}(F_K) \sim 1\%$, $\mathrm{err}(F_K/F_\pi) \sim 0.5\%$, $\mathrm{err}(F_+^{K\pi}(0)) \sim 0.8\%$

Can compute perturbatively in α & $(m_d - m_u)$ ... but mixing w/ nonperturbative QCD

$\Rightarrow$ **nonperturbative QCD tool**
$\Rightarrow$ **include QED and** $m_u \neq m_d$

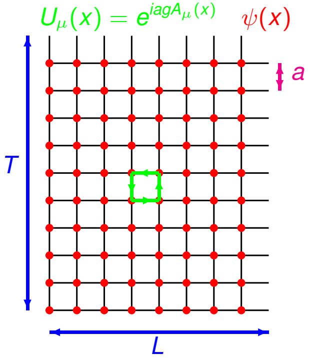
What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires $\geq$ 104 numbers at every point of spacetime
$\rightarrow \infty$ number of numbers in our continuous spacetime
$\rightarrow$ must temporarily "simplify" the theory to be able to calculate *(regularization)*
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O\rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G-\int\bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G}\,\det(D[M])\, O[U]_{\mathrm{Wick}}$$

- $\mathcal{D}Ue^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs $\rightarrow$ evaluate numerically using stochastic methods



LQCD is QCD when $m_q \rightarrow m_q^{\mathrm{phys}}$, $a \rightarrow 0$ (after renormalization), $L \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical challenge

Challenges of a full lattice calculation

To make contact with experiment need:

- **A valid approximation to the SM**
  - $\rightarrow$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1)
  - $\rightarrow$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ (N_f=4$\times$1) & EM (N_f=4$\times$1 + QED)
- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\rightarrow$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \to 135\,\mathrm{MeV}$
- **a $\rightarrow$ 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n,\ (am_q)^n,\ (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\rightarrow$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \to 0$
- **L $\rightarrow \infty$** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadron pties $\sim 1/L^n$ for resonances, QED, ...
  - $\rightarrow$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \to \infty$
- These requirements $\Rightarrow O(10^9)$ **dofs** that have to be integrated over
- **Renormalization** : best done nonperturbatively
- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{conf}}}$, reduce w/ $N_{\mathrm{conf}} \to \infty$

Challenges of a full lattice calculation (cont'd)

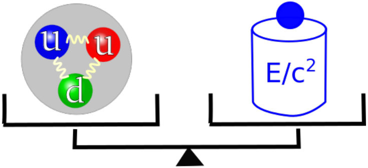
- If one or more of these ingredients are missing, calculation can only give qualitative results
- Difficulty: our algorithms typically loose effectiveness as the physical limit is approached
- Response: algorithmic and methodological improvements (Sexton et al '92, Hasenbusch '01, Urbach et al '06, Lüscher '04, Del Debbio et al '06, Lüscher '07, BMWc '08, Blum et al '12, Frommer et al '13, ...) and Pflop/s supercomputers

  $\Rightarrow$ possible to compute simple quantities w/ %-level accuracy
- Still need large # of large simulations over large range of parameters

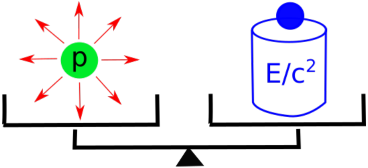
  $\Rightarrow$ only a few full QCD calculations exist

Where does mass of ordinary matter come from?

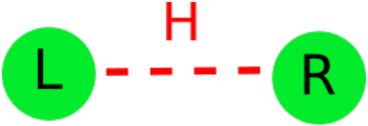
STRONG $> 95\%$



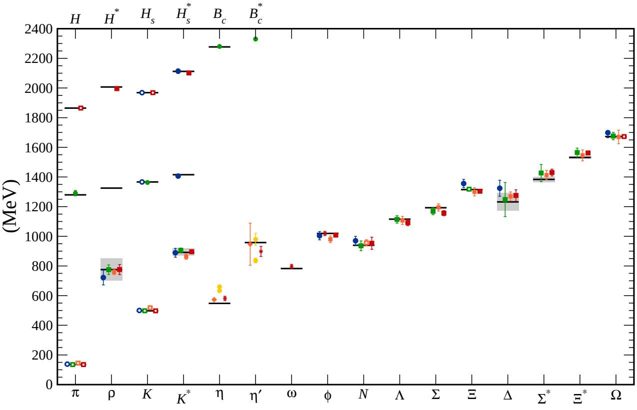
ELECTROMAGNETIC $< 1\%$



HIGGS $< 4\%$



Show how all three combine to give experimental $M_n - M_p$



- From Kronfeld '12
- Light hadrons: BMWc Science '08, MILC '04-'10, PACS-CS '09, QCDSF '11
- η, η': RBC/UKQCD '10, HadSpec '11, HPQCD '12
- ω: HadSpec '11
- Heavy-light (b-light shifted by $-4\,\mathrm{GeV}$): MILC '11, HPQCD '11, Mohler et al '11
- Also ETM '14, ...

$\rightarrow$ QCD mass generation mechanism checked at few % level

$\rightarrow$ impressive validation of nonperturbative QCD

Including isospin breaking on the lattice

$$S_{\rm QCD+QED} = S^{\rm iso}_{\rm QCD+QED} + \frac{1}{2}(m_u - m_d)\int(\bar{u}u - \bar{d}d) + ie\int A_\mu j_\mu$$

with $j_\mu = \bar{q}Q\gamma_\mu q$

(1) operator insertion method

$$\langle\mathcal{O}\rangle_{\rm QCD+QED} = \langle\mathcal{O}\rangle^{\rm iso}_{\rm QCD} - \underbrace{\frac{1}{2}(m_u - m_d)\langle\mathcal{O}\int(\bar{u}u - \bar{d}d)\rangle^{\rm iso}_{\rm QCD}}_{(a)}$$

$$+ \underbrace{\frac{1}{2}e^2\langle\mathcal{O}\int_{xy} j_\mu(x)D_{\mu\nu}(x-y)j_\nu(y)\rangle^{\rm iso}_{\rm QCD}}_{(b)} + \text{hot}$$

(2) direct method

Include $m_u \neq m_d$ and QED directly in simulation

Including isospin breaking on the lattice (cont'd)

What has been done:

- $m_u \neq m_d$ in valence only (MILC '09, Blum et al '10, Laiho et al '11, QCDSF/UKQCD '12, BMWc '10-, ...)
  - ✓ no new simulations
  - × error of $O(\alpha) \Rightarrow$ use phenomenology
- (a) (RM123 '12) and (b) (RM123 '13) of operator insertion method tried w/out quark-disconnected contributions
  - ✓ no new simulations
  - × error of $O(\alpha(m_s - m_{ud})/(N_c M_{\text{QCD}}))$
- QED & $m_u \neq m_d$ in valence only (Eichten et al '97, Blum et al '07, '10, BMWc '10-, MILC '10-)
  - ✓ no new simulations
  - × error of $O(\alpha(m_s - m_{ud})/(N_c M_{\text{QCD}}))$
- QED (Blum et al '12) & $m_u \neq m_d$ (PACS-CS '12) in sea w/ reweighting
  - ✓ as good as full simulation
  - × exponentially expensive in the volume
  - × only tried w/ low statistics in a single simulation $\rightarrow$ not very conclusive

QCD + QED à la BMWc

Borsanyi et al (BMWc), Science 347 (2015)

First full QCD + QED calculation w/ non-degenerate u, d, s, c quarks

- 41 large statistics simulations with $m_u \neq m_d$
  $\rightarrow$ 41 m_u, m_d, m_s, m_c combinations w/ pion masses $M_\pi = 195 \nearrow 420\,\mathrm{MeV}$ (sufficient for light hadron masses cf. Science '08)
- 5 values of $e = 0 \nearrow 1.4$ (physical ~ 0.3)
- 4 lattice spacings $a = 0.06 \nearrow 0.10\,\mathrm{fm}$
- 11 volumes w/ $L = 2.1 \nearrow 8.2\,\mathrm{fm}$

$\rightarrow$ fully controlled calculation of per mil, $M_n - M_p$ effect w/ total error $< 20\%$

QCD+QED challenges

In addition to usual challenges:

- formulate QED in a finite box (long-range interactions)
  $\to$ photon zero mode subtraction (Hayakawa et al ’08, BMWc ’14)

- subtract large finite-volume effects (“soft” photons)
  $\to$ new low-E theorem (BMWc ’14, Davoudi et al ’14): leading $1/L$ and $1/L^2$ depend only on particle charge and mass w/ known coefficients
  $\to$ pure QED simulations to check

- consistently renormalize QCD+QED theory on the lattice
  $\to$ renormalize α using Wilson flow (BMWc ’14, Lüscher ’10)

- avoid unwanted phase transitions of lattice QED
  $\to$ use non-compact formulation (Duncan et al ’96)

- fight large autocorrelations of QED field
  $\to$ Fourier accelerated algorithm (BMWc ’14)

- fight large noise/signal ratio
  $\to$ larger than physical e (Duncan et al ’96)

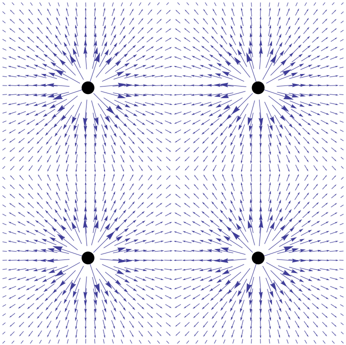
QCD+QED challenges

- finding asymptotic time-range for hadron mass extractions
  $\rightarrow$ method based on Kolmogorov-Smirnov test (BMWc '14)
- robust estimation of systematic errors
  $\rightarrow$ improve Science '08 method using Akaike information criterion (BMWc '14)
  $\rightarrow$ 4 fully independent analyses including a blind one
- unprecedented precision required ($\sim \times 1000$ more statistics for ΔM_N than for M_N while Moore's law only gives $\sim \times 5$)
  $\rightarrow$ $O(10k)$ trajectories/ensemble, $O(500)$ sources/configuration, using 2-level multigrid inverter (Frommer et al '13) and variance reduction technique (Blum et al '13)

49pp appendix summarizing and validating theoretically and numerically all of these new methods (arXiv:1406.4088)

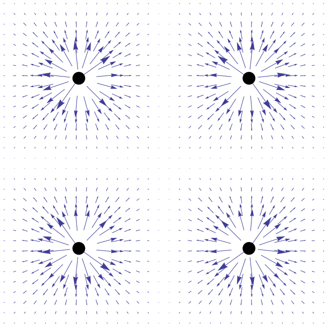
QED in finite volume

EM field of a point charge cannot be made periodic & continuous



$\longrightarrow$

Introduce small modification of QED e.o.m. $\sim 1/L^3$ which makes this possible



- Induces finite-volume effects $\sim \alpha/L$ that must be subtracted
  $\rightarrow$ small on QCD quantities but significant for isospin splittings

Finite-volume QED and zero-mode problem

A $T \times L^3$ spacetime with periodic BCs has the topology of a four-torus

On four-torus **zero mode**, $\tilde{A}_\mu(k=0)$, of photon field is troublesome:

- usual perturbative calculations are not well defined

$$\alpha \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \cdots \qquad \longrightarrow \qquad \frac{\alpha}{TL^3} \sum_k \frac{1}{k^2} \cdots$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

possible IR divergences
but not in physical qties

contains a straight 1/0!

- HMC algorithm is ineffective in updating the zero mode

Finite-volume QED and zero-mode problem

Problem can be solved by **removing zero mode(s)**

$\rightarrow$ modification of $\tilde{A}_\mu(k)$ on set of measure zero

$\rightarrow$ does not change infinite-volume physics

$\rightarrow$ physically equivalent to adding a canceling uniform charge distribution

- different schemes $\rightarrow$ different finite-volume behaviors
- some schemes more interesting than others

QED_{TL} zero-mode subtraction

- Set $\tilde{A}_\mu(k=0)=0$ on $T\times L^3$ four-torus (Duncan et al '96)
- Used in most previous studies
- Violates reflection positivity!

  $\rightarrow$ no hermitian Hamiltonian, states w/ non-positive norm

  $\rightarrow$ divergences when L fixed, $T\to\infty$

$$\frac{\alpha}{TL^3}\sum_{k\neq 0}\frac{1}{k^2}\cdots \quad \xrightarrow[T\to+\infty,L\text{ fixed}]{} \quad \alpha\int\frac{dk_0}{2\pi}\frac{1}{L^3}\sum_{\vec{k}}\frac{1}{k^2}\cdots$$

Checked analytically in 1-loop spinor (also scalar) QED calculation

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1-q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\left[1-\frac{\pi}{2\kappa}\frac{T}{L}\right]\right)\right.\right.$$
$$\left.\left.-\frac{3\pi}{(mL)^3}\left[1-\frac{\coth(mT)}{2}\right]-\frac{3\pi}{2(mL)^4}\frac{L}{T}\right]\right\}$$

with $\kappa=2.837\cdots$, up to exponentially-suppressed corrections

QED$_L$ zero-mode subtraction

- Set $\tilde{A}_\mu(k_0, \vec{k}=0) = 0$ on $T \times L^3$ four-torus for all $k_0 = 2\pi n_0/T$, $n_0 \in \mathbb{Z}$
- Used here (orginally suggested in Hayakawa & Uno '08)
- Satisfies reflection positivity

  $\rightarrow$ fixing to Coulomb gauge, $\vec{\nabla} \cdot \vec{A} = 0$, ensures existence of Hamiltonian

  $\rightarrow$ well defined asymptotic states

  $\rightarrow$ well defined $T, L \rightarrow \infty$ limit

Checked analytically in 1-loop spinor (and scalar) QED calculation

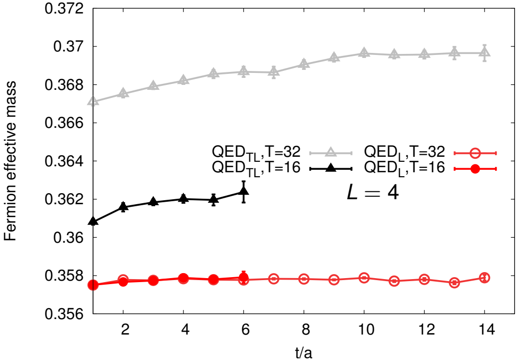
$$m(T,L) \underset{T,L\rightarrow+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$

with $\kappa = 2.837\cdots$, up to exponentially-suppressed corrections

$\Rightarrow$ only inverse powers of L and no powers in T

QED_{TL} vs QED_L: numerical tests

Numerical studies in pure spinor QED (w/out QCD, $e = \sqrt{4\pi/137}$, $am = 0.4$, $L/a = 4$)



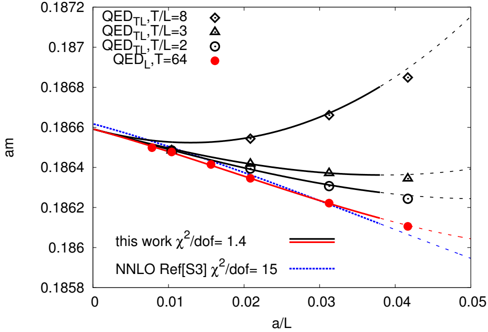
QED_{TL}, as expected, has:

- no clear mass plateaux
- mass increases w/ T

As predicted, QED_L has none of these problems:

- ground state dominates at large t/a
- T-independent mass

Test pure QED simulations against our 1-loop finite-volume predictions (w/out QCD, $e = \sqrt{4\pi/137}$, $am = 0.2$, $L/a = 24, \cdots, 128$)



- Excellent agreement
- Both schemes give the same result in infinite volume
- QED_L cleaner and has more controlled infinite-volume limit
- Resolve discrepancy w/ Davoudi et al '14 on $1/L^3$ term in our favor, numerically here and analytically in Fodor et al, arXiv:1502.06921

QED_L finite-volume effects for composite particles

In our point spinor and scalar QED_L calculations find

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\frac{\kappa}{2mL}\left[1+\frac{2}{mL}\right] + \mathcal{O}\left(\frac{\alpha}{L^3}\right)\right\}$$

independent of particle spin (w/ $\kappa = 2.837\cdots$)

Same result found for:

- Mesons in $SU(3)$ PQ χPT (Hayakawa et al '08)
- Mesons/baryons in non-relativistic EFT (Davoudi et al '14)

$\rightarrow$ leading $1/L$ and $1/L^2$ terms independent of particle spin and structure?

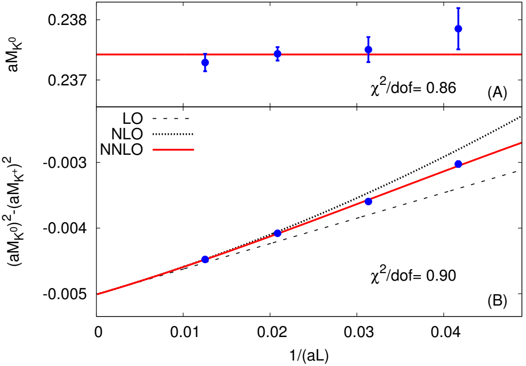
For a general field theory, this universality **follows from Ward identities** (BMWc '14), using Lüscher '86 and assuming:

- the photon is the only massless asymptotic state
- the charged particle considered is stable and non-degenerate in mass

$\rightarrow$ leading FV effects can be removed analytically

FV effects in kaon masses

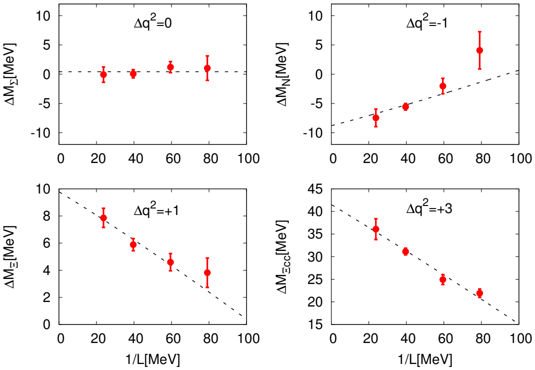
Dedicated FV study w/ $L = 2.4 \nearrow 8.2\,\mathrm{fm}$ and other parameters fixed (bare $\alpha \sim 1/10$, $M_\pi = 290\,\mathrm{MeV}$, $M_{K^0} = 450\,\mathrm{MeV}$, $a = 0.102\,\mathrm{fm}$)



- M_{K^0} has no significant volume dependence
- $M^2_{K^0} - M^2_{K^+}$ well described by universal $1/L$, $1/L^2$ and fitted $1/L^3$ terms

FV effects in baryon masses

Dedicated FV study w/ $L = 2.4 \nearrow 8.2\,\mathrm{fm}$ and other parameters fixed (bare $\alpha \sim 1/10$, $M_\pi = 290\,\mathrm{MeV}$, $M_{K^0} = 450\,\mathrm{MeV}$, $a = 0.102\,\mathrm{fm}$)



- $\Delta M_\Sigma = M_{\Sigma^+} - M_{\Sigma^-}$ shows no volume dependence ($\Delta q^2 = 0$)
- Strategy: fix universal $1/L$, $1/L^2$ terms and add $1/L^3$ if required

How to fix α to its physical value?

- Simulation done in terms of $\alpha_{\rm bare}$: what is $\alpha_{\rm ren}$?

- Use “Wilson” flow (Lüscher ’10) (discretized version of):

$$\frac{\partial B_\mu(\tau;x)}{\partial\tau} = -\frac{\delta S[B]}{\delta B_\mu(\tau;x)}, \qquad E(\tau) = e_0^2\tau^2\langle F_{\mu\nu}^{(B)}(\tau;x)F_{\mu\nu}^{(B)}(\tau;x)\rangle$$

  w/ $B_\mu(\tau=0;x) = A_\mu(x)$

- Have: $E(\tau) = \frac{3\alpha_{\rm ren}}{8\pi}\left\{1+\alpha_{\rm ren}f(\tau)+O(\alpha_{\rm ren}^2)\right\} \xrightarrow{\tau\to\infty} \frac{3\alpha_{\rm ren}}{8\pi}$

  finite for $\tau > 0$ w/ $\alpha_{\rm ren}$ the fine structure cst in the Thomson limit

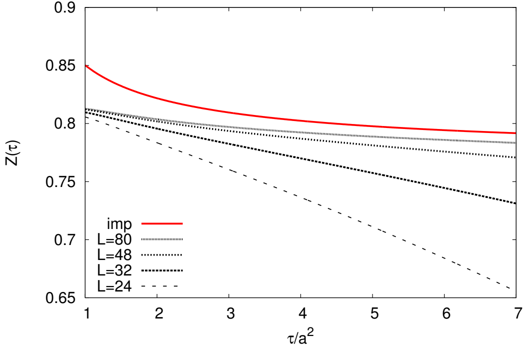
- Define: $\alpha_{\rm ren}(\tau) = Z(\tau)\alpha_{\rm bare}$ w/ $Z(\tau) = E(\tau)/E_{\rm LO}(\tau)$

- Can correct sizeable FV effects by considering $E_{\rm LO}(\tau)$ on FV lattice

- Choose renormalization scale $(8\tau)^{-1/2} \simeq 280 \nearrow 525\,\mathrm{MeV}$ and match $\alpha_{\rm ren}(\tau)$ to Thomson limit at physical value of $\alpha_{\rm ren}$

Running of $\alpha_{\text{ren}}(\tau)$

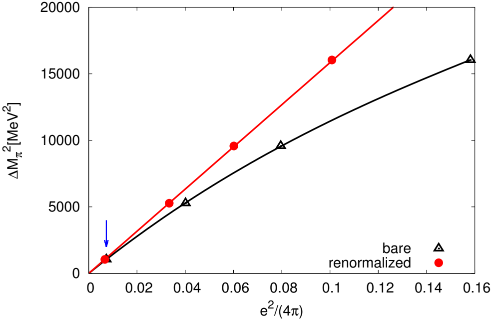
Dedicated FV study w/ $L = 2.4 \nearrow 8.2\,\mathrm{fm}$ and other parameters fixed (bare $\alpha \sim 1/10$, $M_\pi = 290\,\mathrm{MeV}$, $M_{K^0} = 450\,\mathrm{MeV}$, $a = 0.102\,\mathrm{fm}$)



- Volume dependence corrected by FV $E_{\text{LO}}(\tau)$
- Smooth approach to Thomson limit

Interpolation to physical α_{ren}

Dedicated study with $\alpha_{\text{bare}} \in [1/137, 0.16]$ and fixed $M_\pi = 290\,\text{MeV}$, $a = 0.102\,\text{fm}$ on $32^3 \times 64$ lattices ($(8\tau)^{-1/2} = 400\,\text{MeV}$)



- $\Delta M_\pi^2 = M_{\pi^+}^2 - M_{“\pi^0”}^2$ is not linear in α_{bare}
- Becomes so in terms of $\alpha_{\text{ren}}(\tau)$ renormalized around scale of processes involved

$\Rightarrow$ simulate for 5 values $\alpha_{\text{bare}} \in [0, 0.16]$

$\Rightarrow$ interpolate linearly in $\alpha_{\text{ren}}(\tau)$ to physical value

Sketch of analysis

- Mass splittings on 41 ensembles modeled by

$$\Delta M_X = F_X(M_{\pi^+}, M_{K^0}, M_{D^0}, L, a) \cdot \alpha_{\rm ren} + G_X(M_{\pi^+}, M_{K^0}, M_{D^0}, a) \cdot \Delta M_K^2$$

  - F_X, G_X parametrize m_{ud}, m_s, m_c, , L and a dependences
  - Results at physical point obtained by setting M_{π^+}, M_{K^0},M_{D^0} to their physical values, $L \to \infty$ and $a \to 0$, w/ a determined by M_{Ω^-}

- Central value and systematic error estimation
  - Carry out $O(500)$ equally plausible analyses, differing in time-fit ranges for M_X determinations, functional forms for $F_X, G_X, \ldots$
  - Use Akaike information criterion

$$\mathrm{AIC} = \chi^2_{\rm min} + 2k$$

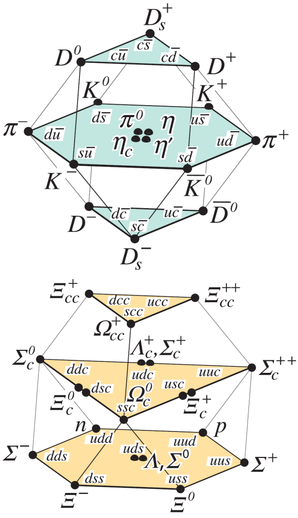
  - Weight different analyses w/

$$\exp\left[-(\mathrm{AIC} - \mathrm{AIC}_{\rm min})/2\right]$$

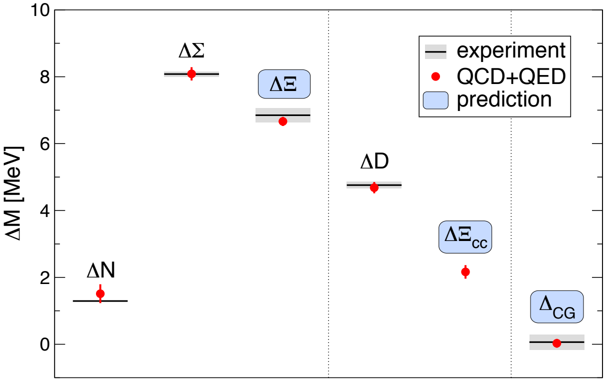
  - central value = weighted mean, syst. error = (weighted variance)$^{1/2}$
  - Final results with other weights or median and distribution width consistent
  - Blind analysis gave consistent results too

- Statistical error from variance of central values from 2000 bootstrap samples

Results for isospin mass splittings



(PDG '14)



- 5σ signal for $M_n - M_p$
- 3 predictions
- $\Delta_{CG} = \Delta N - \Delta\Sigma + \Delta\Xi = O(\alpha(m_s - m_{ud}), \delta m(m_s - m_{ud})^2)$ (Coleman-Glashow relation)
- Full calculation: all systematics are estimated

Strong + Higgs + Electromagnetism = Experiment

Separation of QED and $(m_d - m_u)$ contributions

- At LO in α and $\delta m \equiv (m_d - m_u)$ can separate

$$\Delta M_X = \Delta_{\mathrm{QED}} M_X + \Delta_{\mathrm{QCD}} M_X$$

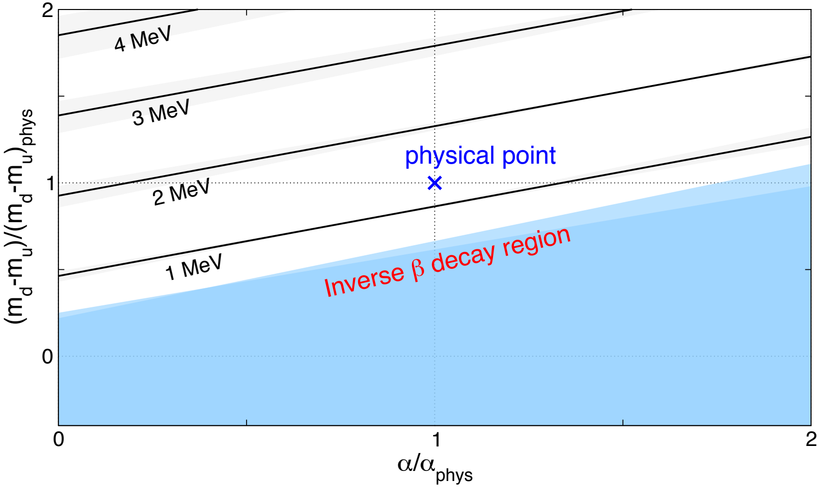
w/ first term $\propto \alpha$ and second $\propto \delta m$

- Intrinsic scheme ambiguity of $O(\alpha\delta m, \alpha^2, \delta m^2, \alpha m_{ud})$
- ΔM_Σ largely dominated by δm contribution

$\rightarrow$ use $\Delta M_\Sigma \equiv 0$ to define $m_d = m_u$ point
$\rightarrow$ sufficient for current level of precision

	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{\mathrm{CG}} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Beginning of first principle answer to: what would the universe be made of if fundamental constants were different?



Conclusions

- Have now a good theoretical understanding of QCD+QED on a finite lattice
- Powerful theorem determines coefficients of leading $1/L$ and $1/L^2$ finite-volume (FV) corrections

  $\Rightarrow$ large QED FV effects can be extrapolated away reliably and precisely
- Have all of the algorithms required to reliably simulate QCD+QED
- Our QCD+QED simulations w/ u, d, s, c sea quarks and $m_u \neq m_d$

  $\rightarrow$ full description low-energy standard model w/ theoretical precision of $O(\alpha^2, 1/N_c m_b^2) \sim 10^{-3}$

  $\rightarrow$ increase in accuracy $\sim \times 10$ compared to state-of-the-art $N_f = 2+1$ simulations with intrinsic errors of $O(\alpha, \delta m, 1/N_c m_c^2) \sim 10^{-2}$
- Isosplittings in hadron spectrum determined accurately w/ full control over uncertainties
- Confirm: Strong $+$ Higgs $+$ Electromagnetism $=$ Experiment
- Make first principle anthropics possible

Outlook

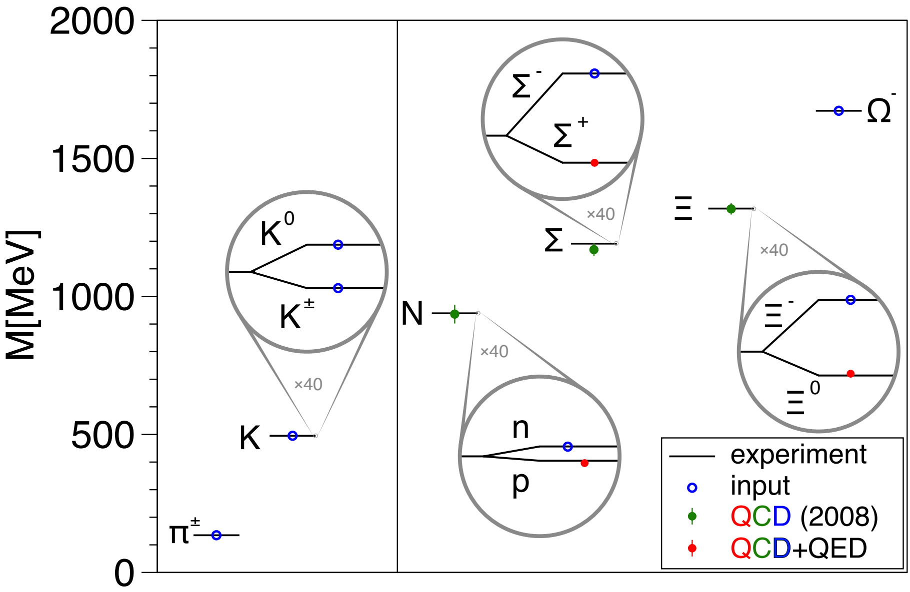
- Fully controlled computation of the u & d quark masses
- Isospin corrections to hadronic matrix elements (e.g. K_{ℓ_2}, K_{ℓ_3}, $K \to \pi\pi, \dots$)

  $\to$ bring indirect search for new physics to new level
- QCD+QED to compute hadronic corrections to anomalous magnetic moment of the μ, $a_\mu = (g_\mu - 2)/2$

  $\to$ currently $> 3\sigma$ deviation between SM and experiment w/ $\sim$matched errors

  $\to$ need to bring SM calculation to new level in view of new experiments $\gtrsim$ 2017 that will reduce error by 4
- ...

Progess since 2008



(Sivan)

Practical application (cont'd)

(PCE)