An effective holographic approach to QCD

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Motivation

QCD: A challenging theory

$$L_{QCD} = \bar{\psi}_f \left[i\,\gamma^\mu D_\mu - m_f \right] \psi_f - \frac{1}{2} \mathrm{Tr}\left[\mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} \right],$$

where

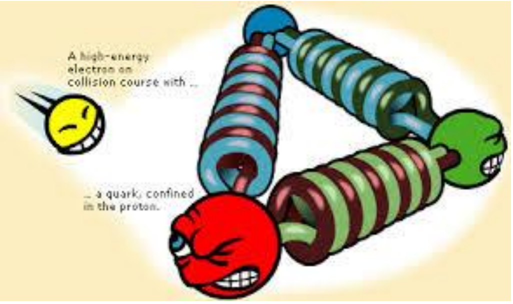
$$D_\mu = \partial_\mu - igA_\mu \;\;, \;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right].$$

- **Quarks** are Dirac spinors ψ_f with $f = (1, .., N_f)$ and $N_f = 6.$
- **Gluons** are non-Abelian gauge fields A_μ in the group $SU(N_c)$ with $N_c = 3$.

- **UV asymptotic freedom**:

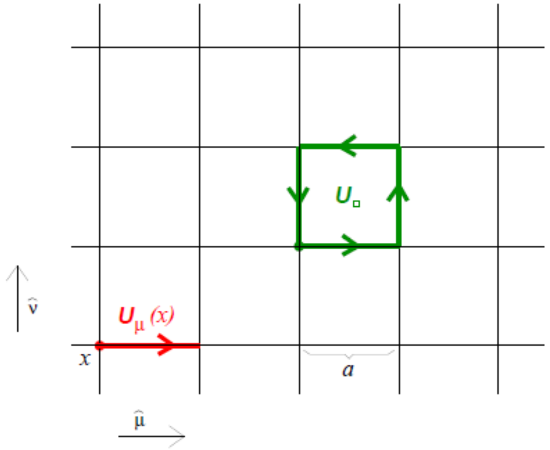
$$\beta = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3} N_c - \frac{2}{3} N_f\right] < 0$$

- **IR confinement**:



Nonperturbative approaches to QCD

- Lattice QCD: *Wilson 1974*



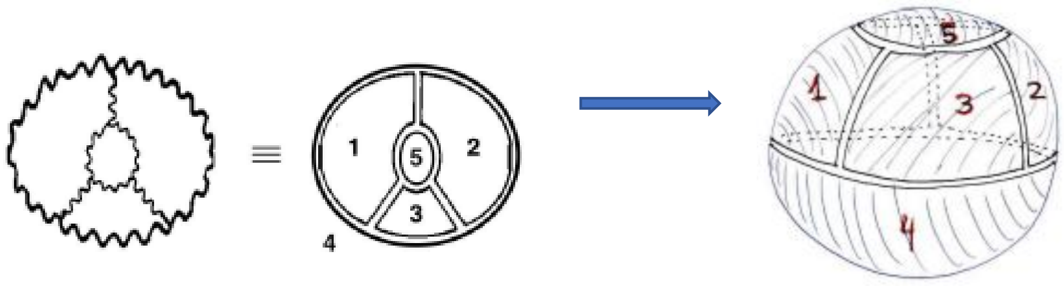
-Dyson-Schwinger equations: *Dyson 1949, Schwinger 1951*

$$\left(\frac{\delta S[\phi]}{\delta \phi}\bigg|_{\phi=\frac{\delta}{\delta J}} - \mathbf{J}\right)\mathbf{Z}[\mathbf{J}] = \mathbf{0}$$

- Other approaches: *Chiral Lagrangians, Nambu-Jona-Lasinio (NJL) model, quark-meson model, QCD sum rules, RG flows, etc.*

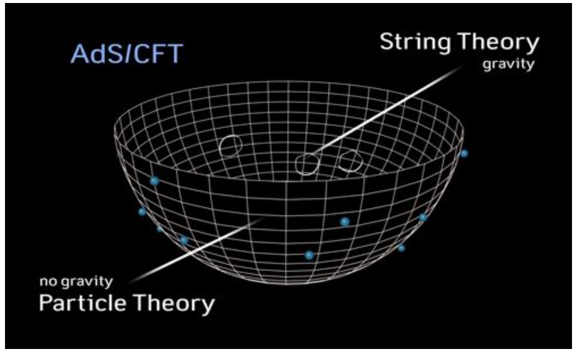
AdS/CFT and holographic QCD (HQCD)

- The large N_c limit:



't Hooft 1974

- The AdS/CFT correspondence :



Maldacena 1997

$$Z_{CFT}[\phi_0] = Z_{AdS}[\phi]$$

Gubser, Klebanov and Polyakov 1998,

Witten 1998

From AdS/CFT to HQCD

- The top-down approach:

4-d $\boldsymbol{N = 4}$ super Yang-Mills ⟷ IIB string theory in $\boldsymbol{AdS_5 \times S^5}$

↕ ↕

4-d QCD-like theories ⟷ deformed string theories

Klebanov-Witten (1998), Klebanov-Strassler (2000), Maldacena-Nunez (2000), Sakai-Sugimoto (2004),...

- The bottom-up approach:

4-d **CFT** ⟷ $\boldsymbol{AdS_5}$

↕ ↕

4-d QCD as a deformed **CFT** ⟷ $\boldsymbol{AdS_5}$ deformed by fields

Polchinski-Strassler (2000), Erlich et al (2005), Karch et al (2006), Csaki-Reece (2006), Gursoy et al (2007),...

Dilaton-Gravity and Improved Holographic QCD (IHQCD)

Gursoy, Kiritsis and Nitti 2007

In QCD **conformal symmetry breaking** is described by the trace anomaly

$$\langle T^\mu_\mu \rangle = \frac{\beta}{2\lambda^2} \langle \mathrm{Tr} F^2 \rangle.$$

The **YM** operator $\mathbf{TrF^2}$ maps to a 5-d scalar field $\boldsymbol{\Phi}$ (the **dilaton**).

This field is responsible for **deforming AdS**.

The IHQCD model arises from 5-d **Dilaton-Gravity**

$$S = M_p^3 N_c^2 \int d^5x \sqrt{-g} \left[R - \frac{4}{3} g^{mn} \partial_m \Phi \partial_n \Phi + \mathrm{V}(\Phi) \right],$$

and we take the ansatz

$$ds^2 = e^{2A(z)}[-dt^2 + dx_i^2 + dz^2] \quad , \quad \Phi = \Phi(z).$$

for the metric and dilaton. The equations then reduce to

$$A'^2 - A'' = \frac{4}{9} \Phi'^2 \quad , \quad 3A'^2 + A'' = \frac{V}{3} e^{2A}.$$

- It is convenient to define the quantity $\zeta(z) := exp[-A(z)]$

Then the first eq. becomes $$\zeta'' - \frac{4}{9}\Phi'^2\zeta = 0$$

- Introducing a superpotential $W[\Phi]$ the **2nd order** differential equations become **1st order**:

$$\zeta' = \frac{4}{9}W \ , \quad \zeta\Phi' = \frac{dW}{d\Phi} \ , \quad V = \frac{64}{27}W^2 - \frac{4}{3}\frac{dW}{d\Phi}.$$

The **RG flow** in the bulk is dictated by the **`holographic beta function'** :

$$\beta_\Phi = \frac{d\Phi}{dA} = -\frac{9}{4}\frac{\mathrm{dLog[W]}}{\mathrm{d}\Phi}.$$

- In **IHQCD** the dilaton Φ is mapped to the **YM** coupling λ by

$$\lambda = e^{\Phi} \longrightarrow \beta_\lambda = \lambda\,\beta_\Phi$$

<u>**UV constraint:**</u> $W[\Phi]$ is built to reproduce the **2-loop perturbative QCD** beta function

$$\beta_\lambda = -b_0\lambda^2 - b_1\lambda^3.$$

The IR constraint

A **Nambu-Goto** string is defined in the **string frame**, where the metric takes the form

$$ds_{SF}^2 = e^{2A_s(z)}\left[-dt^2 + dx_i^2 + dz^2\right]$$

with

$$A_s = A + \frac{2}{3}\Phi.$$

Confinement criterion: $f(z^*) > 0$ where $f = \exp[2A_s]$ and z^* is the minimum of $f(z)$.

Large z (**IR**) dilaton asymptotics:

$$\Phi = \mathrm{C}\, \mathrm{z}^{\alpha} \longrightarrow A_s = \frac{\alpha-1}{2}\log[\mathrm{z}].$$

Confinement when $\alpha \geq 1$.

- Dilaton-Gravity perturbations ⟷ **QCD glueballs** (spin 0 and 2)

Asymptotic linear spectrum when $\alpha = 2$.

The corresponding **IR** potential is

$$V(\Phi) \sim \Phi^{\frac{1}{2}} \exp\left[\frac{4}{3}\Phi\right].$$

The effective holographic approach to QCD (EHQCD)

- Consider the trace Ward identity of a **deformed CFT**

$$\langle T^{\mu}_{\mu} \rangle = (\Delta - 4)\phi_0 \langle O \rangle,$$

where Δ is the conformal dimension of the **operator** O that deforms the CFT.

The **AdS/CFT dictionary** maps O to a 5-d scalar field Φ with mass

$$M^2 = \Delta(\Delta - 4).$$

- Consider $\Delta = 4 - \epsilon$. The relevant operator O plays the role of a $\mathrm{Tr}F^2$ after renormalization ⟶ **Effective FT** approach to **QCD**

Gubser, Nellore, Pufu and Rocha 2008

In the **UV** the field Φ behaves as

$$\Phi = \phi_0 z^{\Delta_-} + G z^{\Delta_+},$$

where $\Delta_- = \epsilon$ and $\Delta_+ = 4 - \epsilon$.

This is responsible for the AdS deformation (backreaction)

$$A(z) = -\log[z] - \frac{2\,\Delta_-}{9(1 + 2\,\Delta_-)}\phi_0^2\, z_0^{2\Delta_-} - \frac{2\Delta_-\Delta_+}{45}\phi_0 G\, z^4 - \frac{2\Delta_+}{9(1 + 2\Delta_+)} G^2\, z^{2\Delta_+} - \cdots$$

The corresponding holographic beta function behaves as

$$\beta_\Phi = -\Delta_- \phi_0 z^{\Delta_-} - \Delta_+ G\, z^{\Delta_+}.$$

A simple EHQCD model *B-B, Boschi-Filho, Miranda, Mamani and Zanchin 2017*

- Consider the UV/IR interpolation

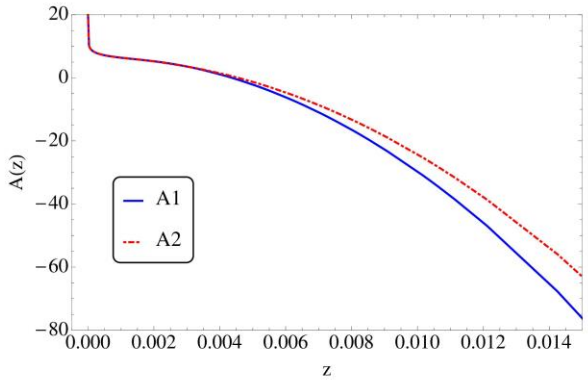
$$\Phi(z) = \hat{\phi}_0 (\Lambda\, z)^\epsilon + \frac{(\Lambda z)^{4-\epsilon}}{1 + (\Lambda z)^{2-\epsilon}} \ ,$$

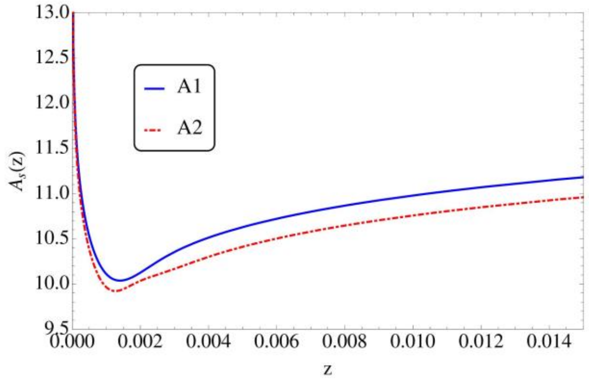
This is one of many possible interpolations. We call this model **A1**.

- The original **source** and **VEV** coefficients are given by

$$\phi_0 = \hat{\phi}_0 \Lambda^\epsilon \quad , \qquad G = \Lambda^{4-\epsilon} \, .$$

The parameter Λ behaves as an auxiliary field of conformal dimension **1**. The metric warp factor **A(z)** is solved numerically.





The glueball spectrum

- The Dilaton-Gravity equations are **linearised** by taking

$$\boldsymbol{\Phi \to \Phi + \chi} \quad , \qquad \boldsymbol{g_{mn} \to e^{2A(z)}[\eta_{mn} + h_{mn}]} \,,$$

and expanding at **1st** order in the perturbations $\boldsymbol{\chi}$ and $\boldsymbol{h_{mn}}$. The equations for $\boldsymbol{\chi}$ and $\boldsymbol{h_{mn}}$ take the form

$$\boldsymbol{R^{(1)}_{mn} = \frac{8}{3}\partial_{(m}\Phi\partial_{n)}\chi - \frac{1}{3}e^{2A}[V\, h_{mn} + (\partial_\Phi V)\chi\, \eta_{mn}]} \,,$$

$$\boldsymbol{\left(\nabla^2\Phi\right)^{(1)} = -\frac{1}{2}\left(\partial^2_\Phi V\right)\chi} \,,$$

where $\boldsymbol{R^{(1)}_{mn}}$ and $\boldsymbol{\left(\nabla^2\Phi\right)^{(1)}}$ are the **1st** order expansion of the **Ricci tensor** and **scalar Laplacian** respectively.

$\boldsymbol{h_{mn}}$ can be decomposed as $\boldsymbol{(h_{zz}, h_{z\mu}, h_{\mu\nu})}$. The fields $\boldsymbol{h_{z\mu}}$ and $\boldsymbol{h_{\mu\nu}}$ can further be decomposed in Lorentz irreducible rep. In the end one arrives at 2 independent eqs

$$\boldsymbol{\left[\partial_z^2 + 3\,A'\partial_z + \Box\right]h^{TT}_{\mu\nu} = 0} \,, \qquad \textbf{(spin 2)}$$

$$\boldsymbol{\left[\partial_z^2 + (3\,A' + 2\,\frac{\beta_\Phi{}'}{\beta_\Phi})\partial_z + \Box\right]\xi = 0}. \qquad \textbf{(spin 0)}$$

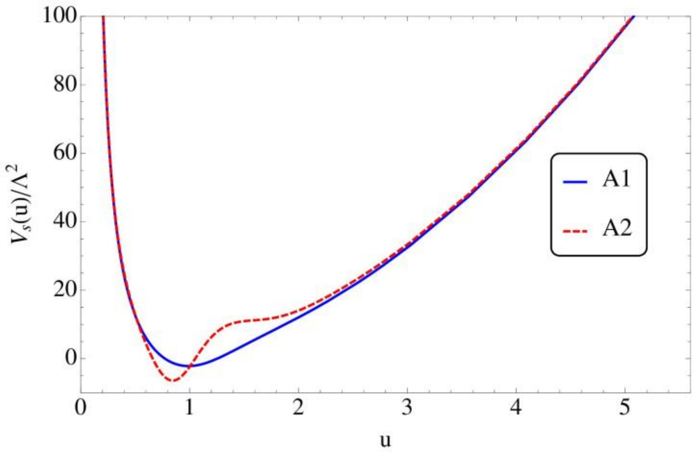
These equations can be written in a **Schrödinger** form:

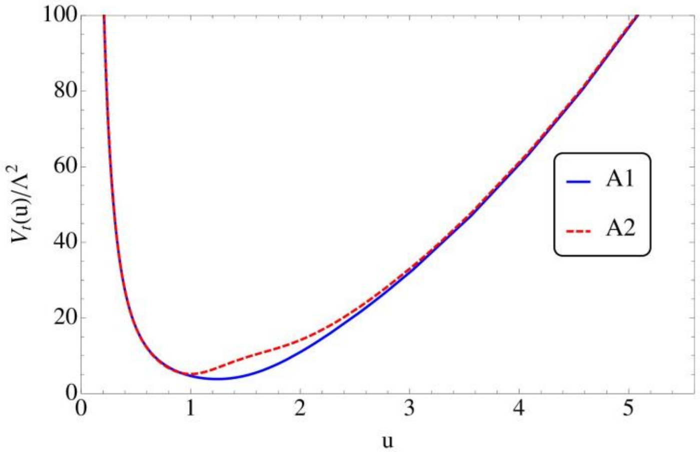
$$\left[\partial_z^2 + m_s^2 - V_s\right]\psi_s = 0\,,$$
$$\left[\partial_z^2 + m_t^2 - V_t\right]\psi_t = 0\,,$$

with effective potentials

$$V_{s,t} = \partial_z^2 B_{s,t} + \left(\partial_z B_{s,t}\right)^2\,,$$
$$B_s = \frac{3}{2}A + Log[\beta_\Phi]\,,\quad B_t = \frac{3}{2}A.$$

→ **Glueballs** as bound states in a **potential well**.





Example: The case $\epsilon = 0.01$

- We use lattice data for the first two scalar glueball masses $(m_{0^{++}}, m_{0^{++*}})$ to fix $\hat{\phi}_0$ and Λ.

Model	$\tilde{\phi}_0$	Λ (MeV)	ϕ_0 (MeV$^\epsilon$)	G (MeV$^{4-\epsilon}$)	C (MeV^2)
A1	53.79	736	57.46	2.75×10^{11}	5.42×10^5
A2	49.41	682	52.75	2.03×10^{11}	4.65×10^5
B1	48.40	668	51.65	1.86×10^{11}	4.46×10^5
B2	46.57	709	49.73	2.36×10^{11}	5.02×10^5

Then the model predicts the other glueball masses

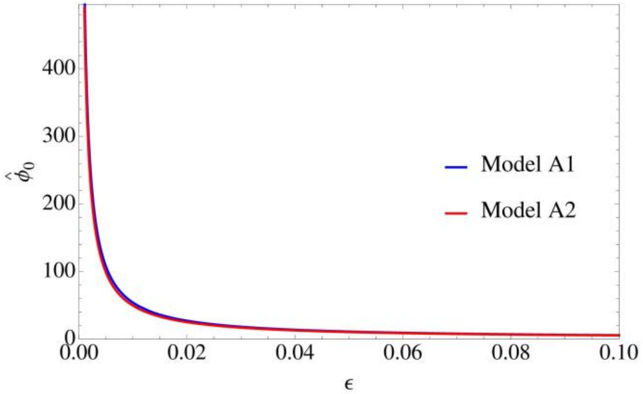
n	A1	A2	B1	B2	IHQCD [4]	Lattice [52]
0^{++}	1475	1475	1475	1475	1475	1475(30)(65)
0^{++*}	2755	2755	2755	2755	2753	2755(70)(120)
0^{++**}	3507	3376	3361	3449	3561	3370(100)(150)
0^{++***}	4106	3891	3861	4019	4253	3990(210)(180)
0^{++****}	4621	4349	4313	4514	4860	
$0^{++*****}$	5079	4762	4721	4956	5416	
2^{++}	2075	2180	2182	2130	2055	2150(30)(100)
2^{++*}	2945	2899	2887	2943	2991	2880(100)(130)
2^{++**}	3619	3468	3444	3568	3739	
2^{++***}	4185	3962	3928	4102	4396	
2^{++****}	4680	4404	4365	4576	5530	
$2^{++*****}$	5127	4807	4763	5006		

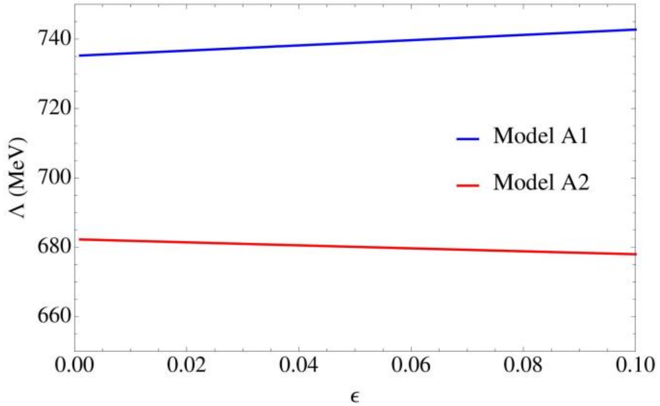
Evolution of the parameters as $\epsilon \to 0$

- The **CFT deformation** is of the form $L_{CFT} + \phi_0\, O\,,$

where the coupling ϕ_0 has conformal dimension ϵ.

Keeping fixed glueball masses $(m_{0^{++}}, m_{0^{++*}})$ we find the ϵ dependence of the parameters $\hat{\phi}_0$ and Λ.





In the limit $\epsilon \to 0$ the operator O should approximate the YM operator TrF^2 . The dimensionless coupling $\hat{\phi}_0$ indeed behaves qualitatively as $1/\lambda$.

The parameter Λ is almost constant. The glueball spectrum also remain approximately constant.

Large-Nc QCD as a deformed CFT ?

- Consider the large N_c **YM** Lagrangian $$\boldsymbol{L_{YM} = \frac{1}{\lambda}(\frac{1}{2}\mathrm{Tr}\,F^2)}.$$

This Lagrangian becomes a CFT deformation $\boldsymbol{\phi_0 O}$ if $$\boldsymbol{\phi_0 \equiv \frac{\Lambda^{\epsilon}}{\lambda} \quad , \quad O \equiv \frac{1}{2}\frac{\mathrm{Tr}F^2}{\Lambda^{\epsilon}}}.$$

From this dictionary one finds the **QCD gluon condensate**

$$\boldsymbol{\langle \mathrm{Tr}F^2 \rangle^{ren} = \frac{32}{15} M^3 N_c^2 (4-\epsilon)\, \Lambda^4}.$$

The vacuum energy density

- From the on-shell dilaton-gravity action we find the **QCD vacuum energy density**

$$\boldsymbol{< T^{00} >^{ren} = -\frac{4}{15} M^3 N_c^2\, \epsilon(4-\epsilon)\phi_0 G}\,.$$

The trace anomaly and an emergent beta function

- Our model is a special case of an **RG flow** of **deformed CFTs**.

As a consequence we obtain the trace Ward identity $\langle T^{\mu}{}_{\mu}\rangle^{ren} = -\epsilon\phi_0\langle O\rangle^{ren}$

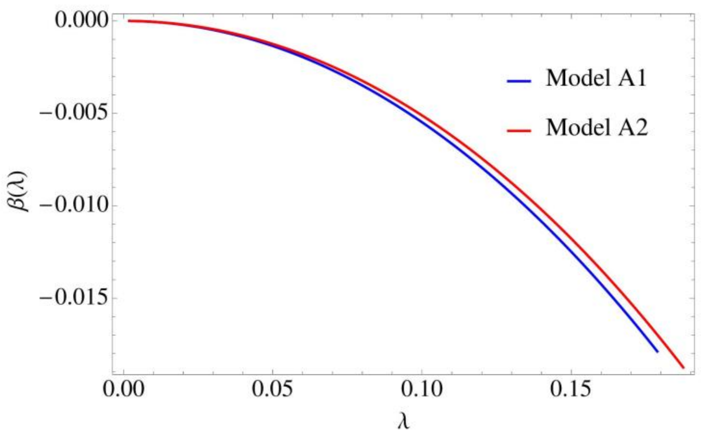
Using the map described above we rewrite this anomaly as $\langle T^{\mu}{}_{\mu}\rangle^{ren} = -\frac{\epsilon}{2\lambda}\langle \mathrm{TrF}^2\rangle^{ren}.$

This Ward identity becomes the QCD scale anomaly if we identify ϵ with λ by the relation

$$\epsilon \equiv -\frac{\beta_\lambda}{\lambda}.$$

The evolution of $\hat{\phi}_0$ with ϵ does lead to a beta function similar to the pQCD beta function, i.e

$$\beta_\lambda = -b_0\lambda^2 - \mathrm{b}_1\lambda^3 .$$



Conclusions

- EHQCD treats QCD in terms of a CFT deformed by relevant operator in the UV.
- The IR constraint is the same used in the IHQCD approach, namely a quadratic dilaton. The glueball spectrum in the model is consistent with lattice QCD data.
- We proposed a map to go from the deformed CFT Lagrangian to the QCD Lagrangian and extract the QCD vacuum energy and the gluon condensate. Using this map for the trace anomaly leads to an emergent beta function similar to the pQCD result.

- **Open problems:** Build a bridge between this approach and the IHQCD approach. Find the Callan-Symanzik equations to describe the dual RG flow.
- **Extensions:** Mesons and chiral symmetry breaking, finite T plasma and the trace anomaly, higher spin glueballs and the pomeron, ...