

AN EMPIRICAL EQUATION OF STATE FOR NUCLEAR PHYSICS AND ASTROPHYSICS

COLLABORATORS:

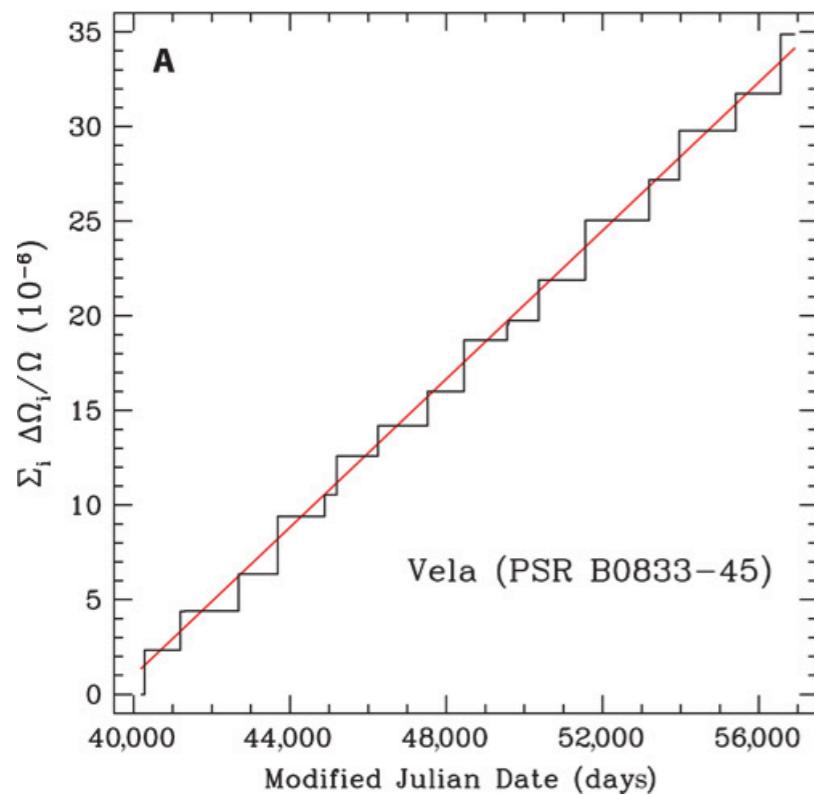
Francesca Gulminelli
Jerome Margueron
Adriana Raduta
Sofija Antic
Debora P. Menezes

Debarati Chatterjee
LPC/ENSICAEN,
CAEN, FRANCE



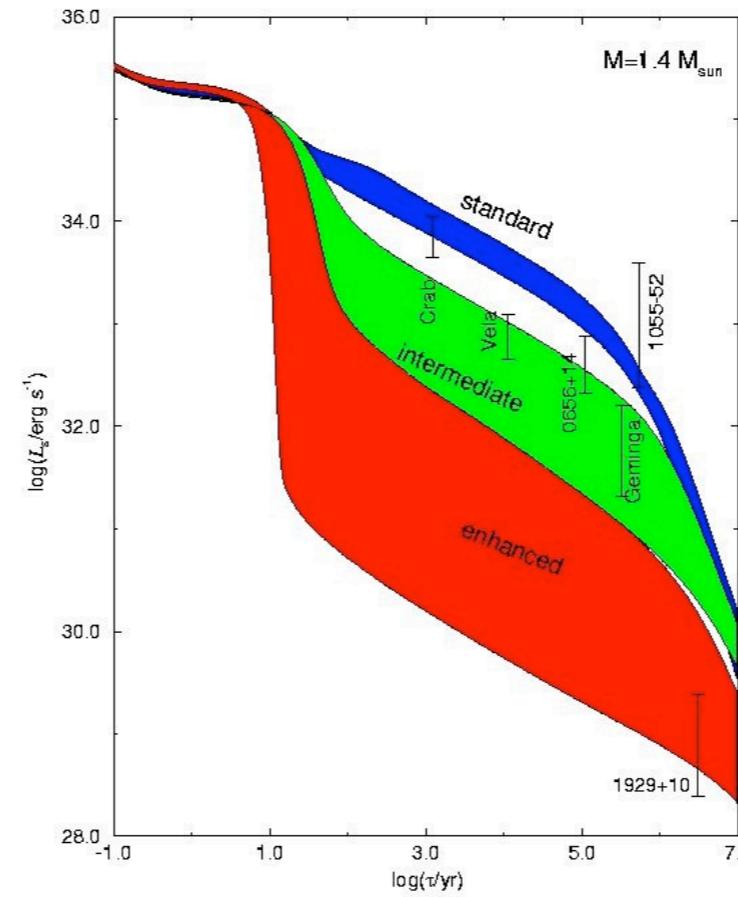
XIV HADRON PHYSICS
MARCH 18-23, 2018
FLORIANOPOLIS, BRAZIL

OBSERVABLES SENSITIVE TO CRUST-CORE PROPERTIES

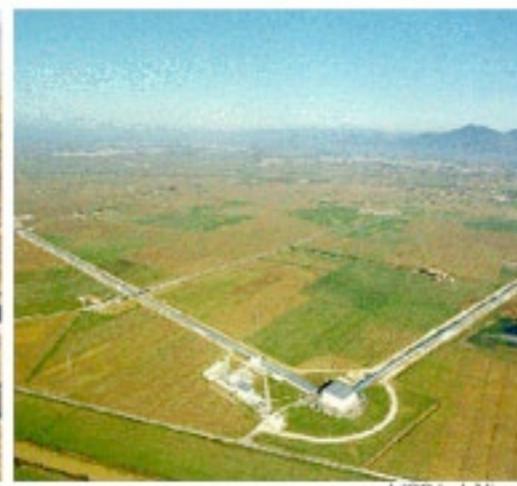


“Glitches” in spin period

Image: W. Ho

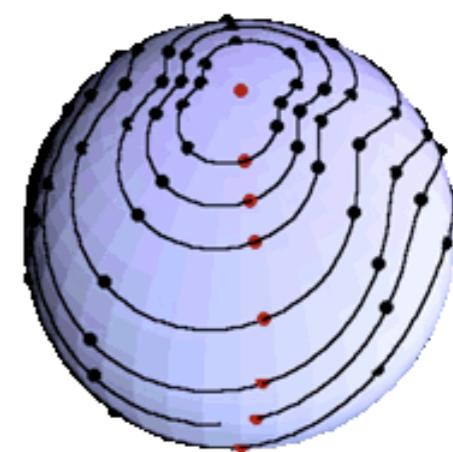


NS cooling
Image: C. Miller



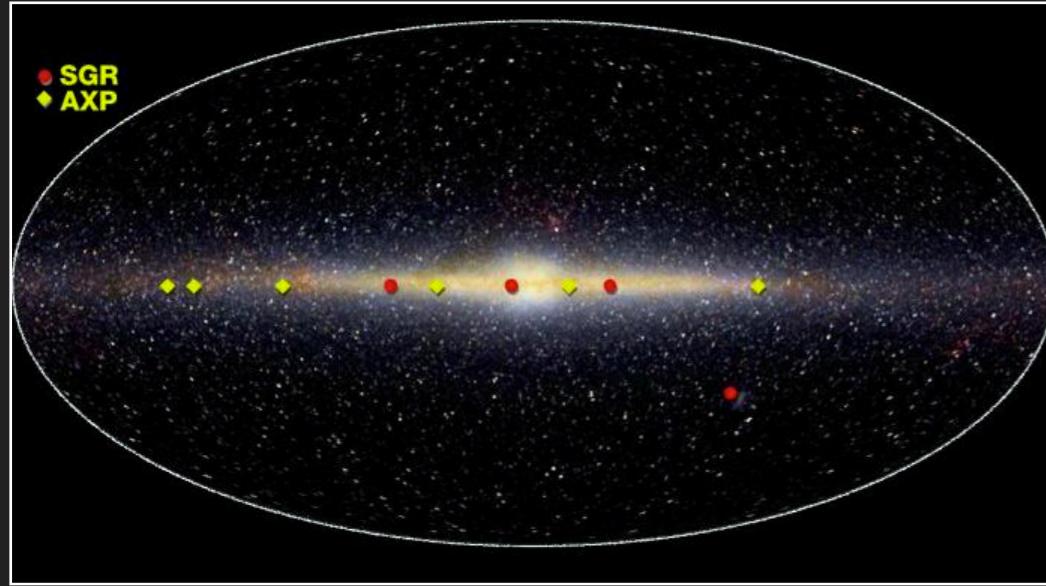
LIGO/Lab/Virgo

Gravitational wave detectors (LIGO/VIRGO)



NS oscillations
Image: L. Rezzolla

MAGNETARS



Images: NASA

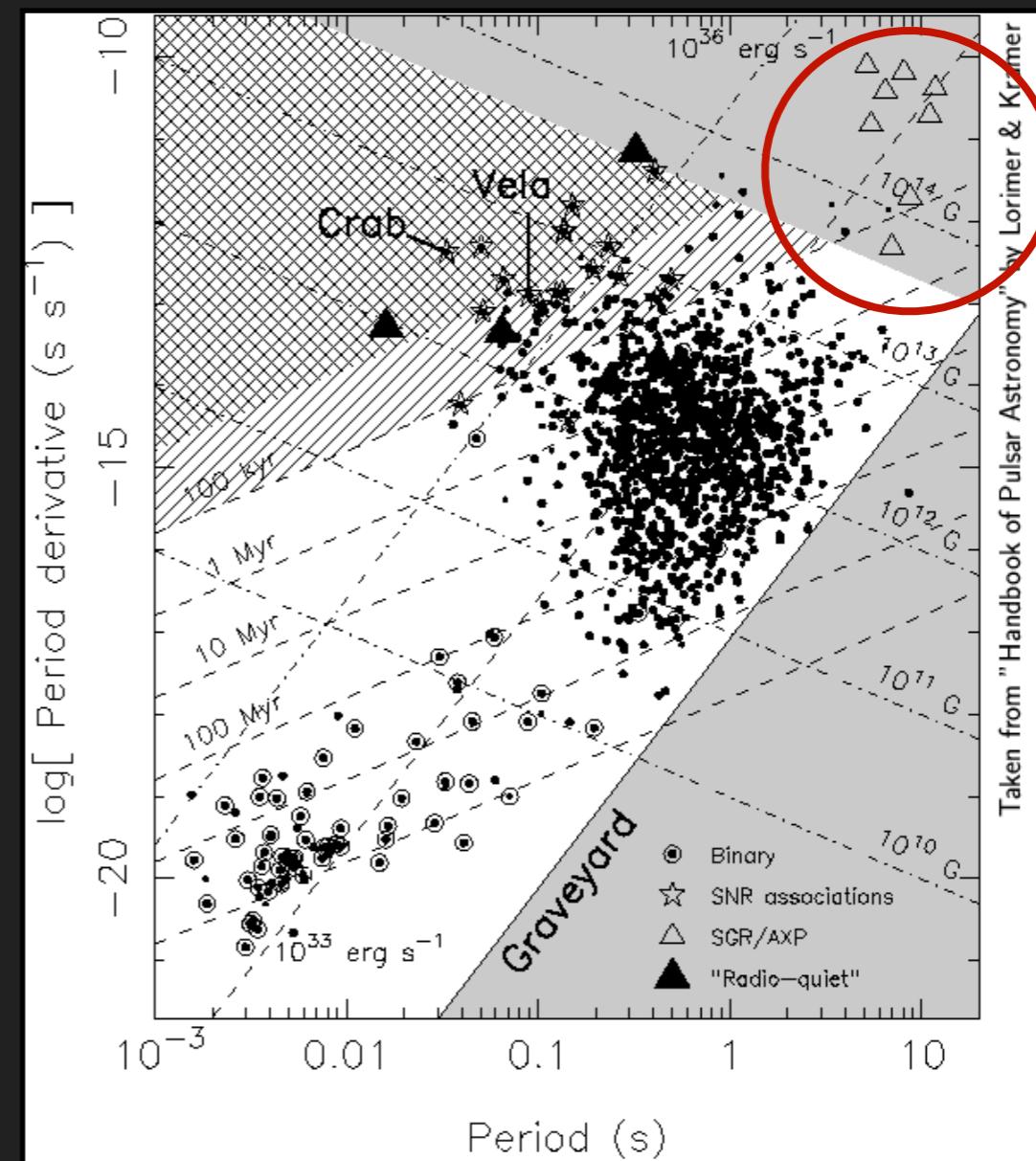
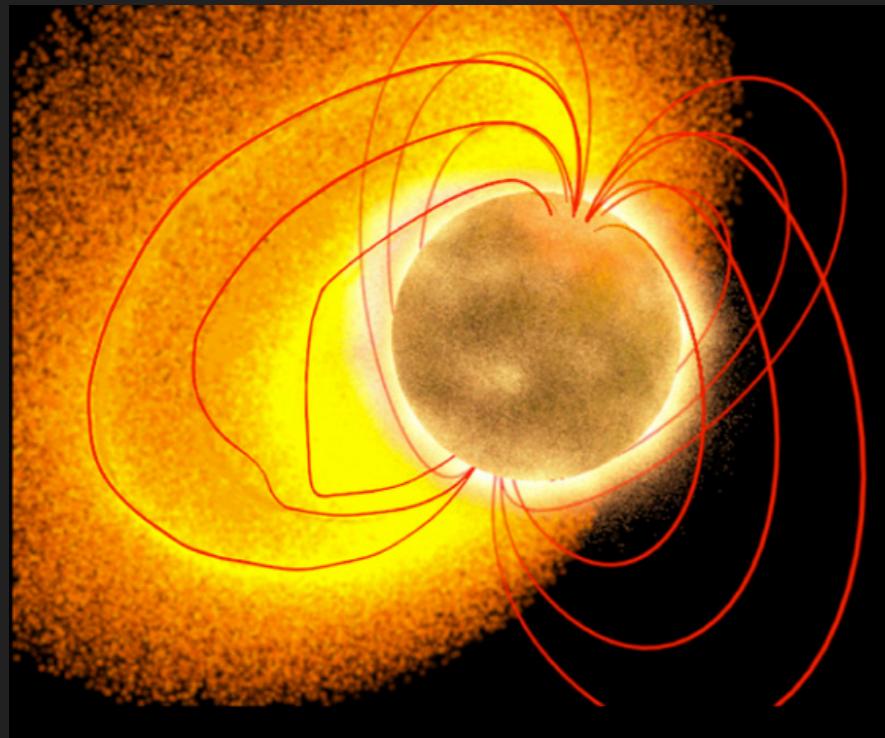


Image: Handbook of Pulsar Astronomy,
(Lorimer and Kramer)

Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

NUCLEAR EQUATION OF STATE

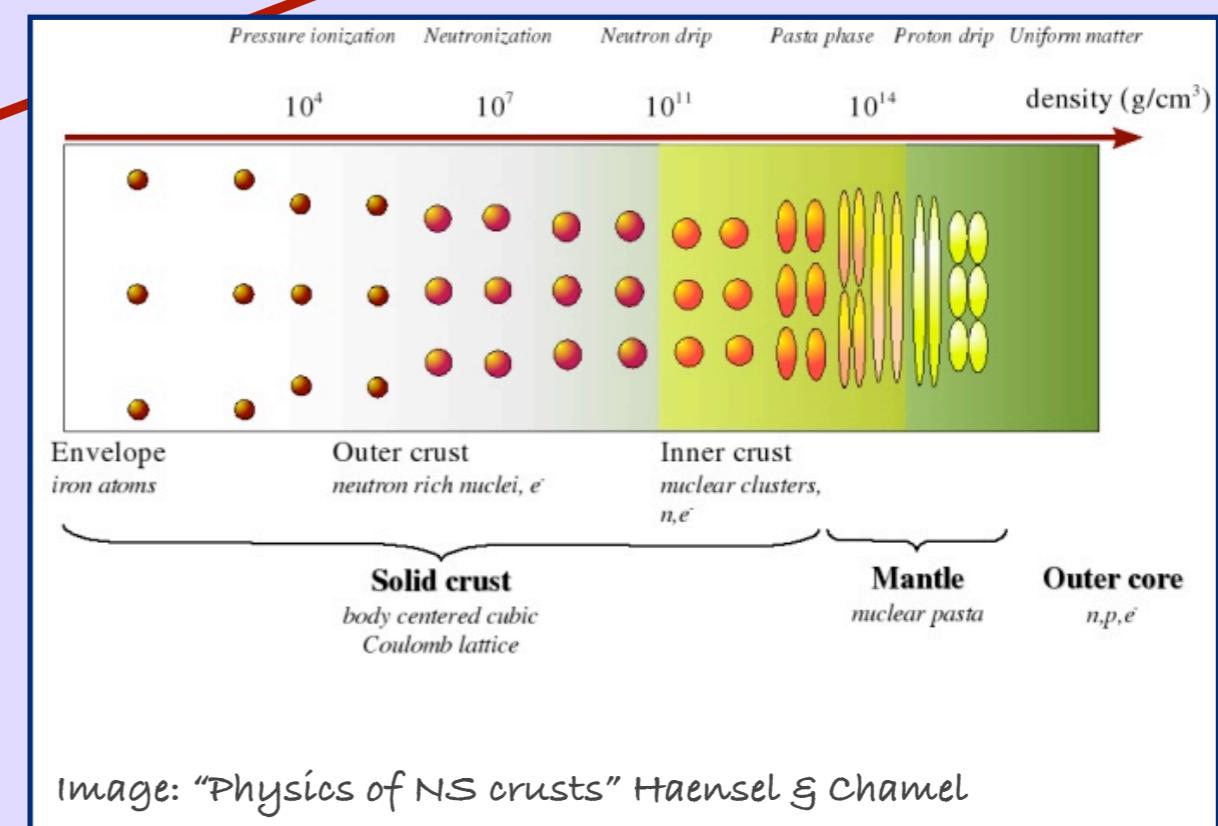
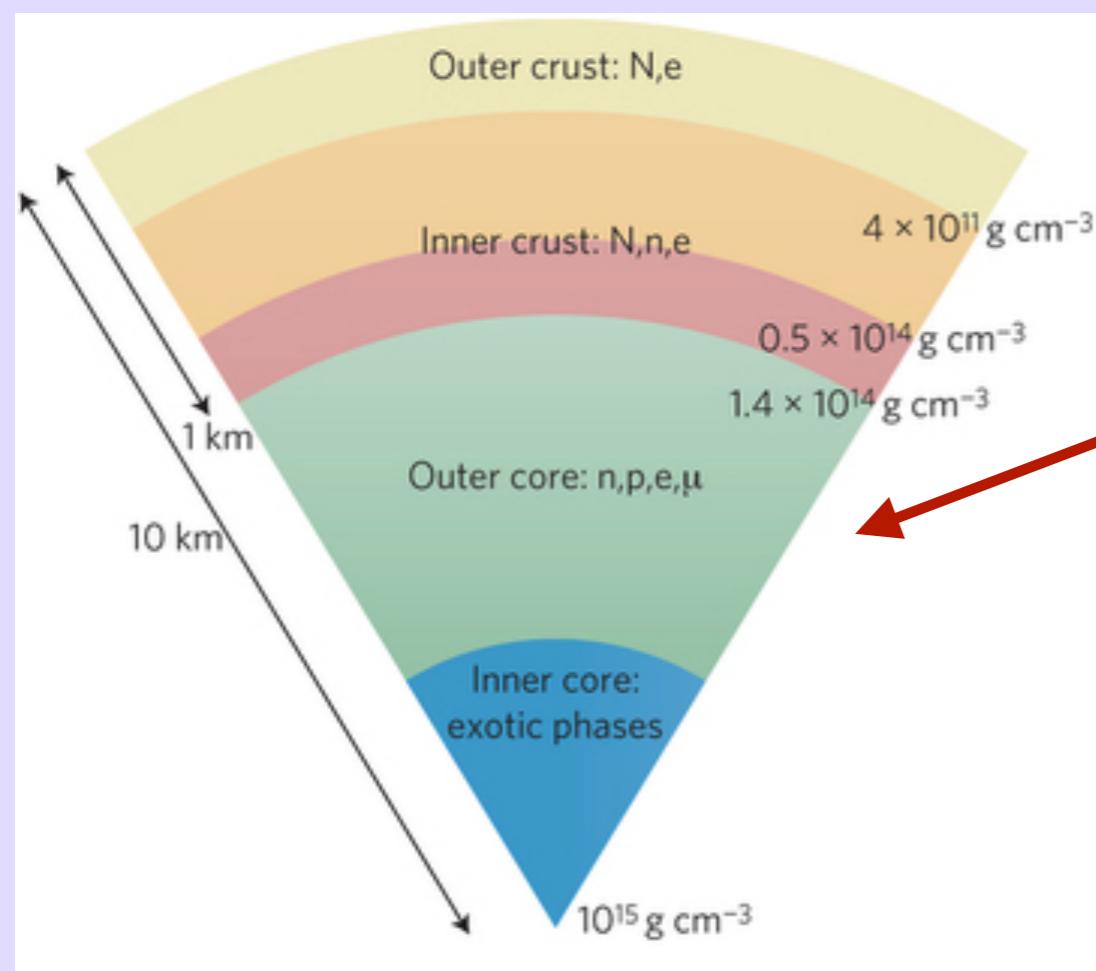
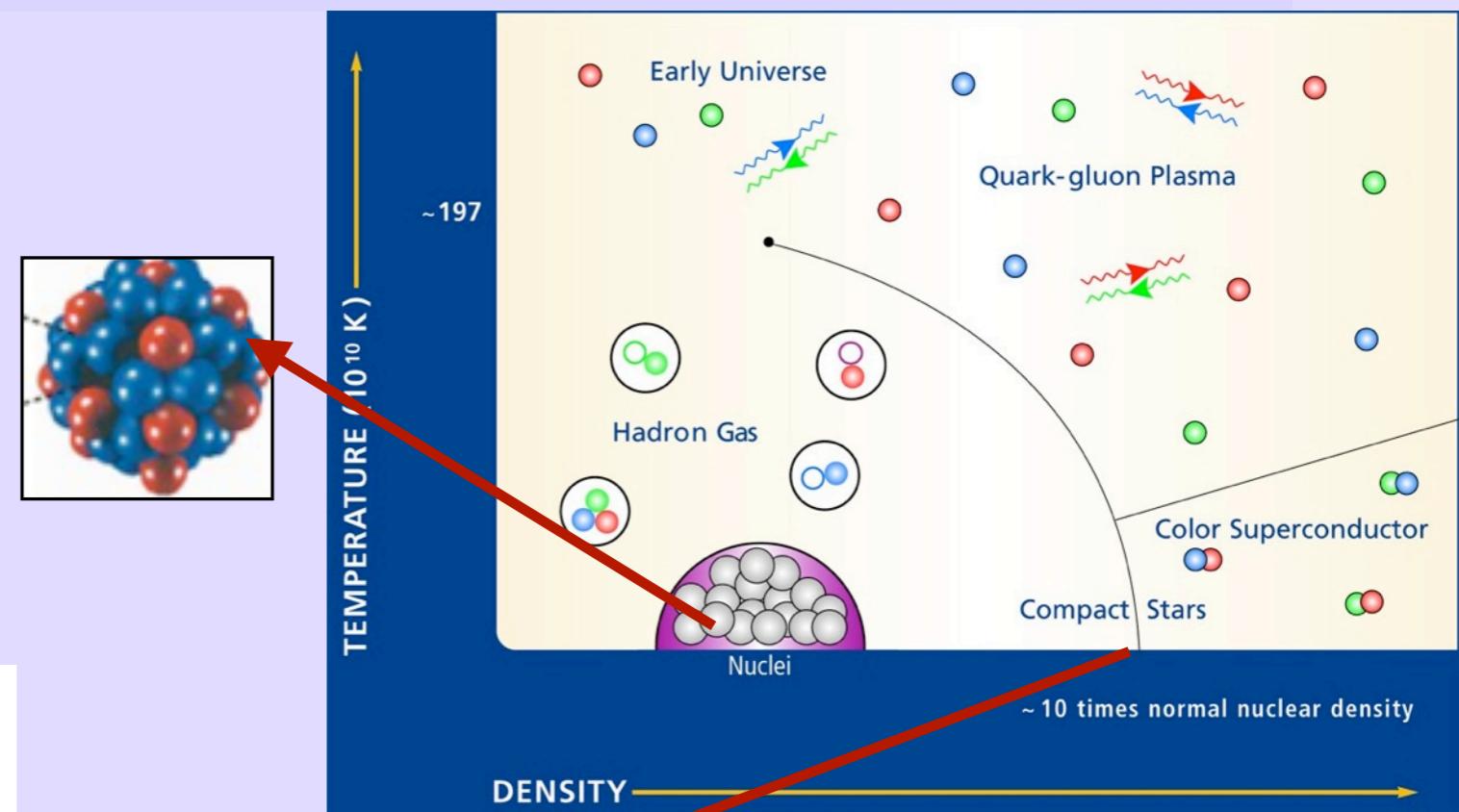
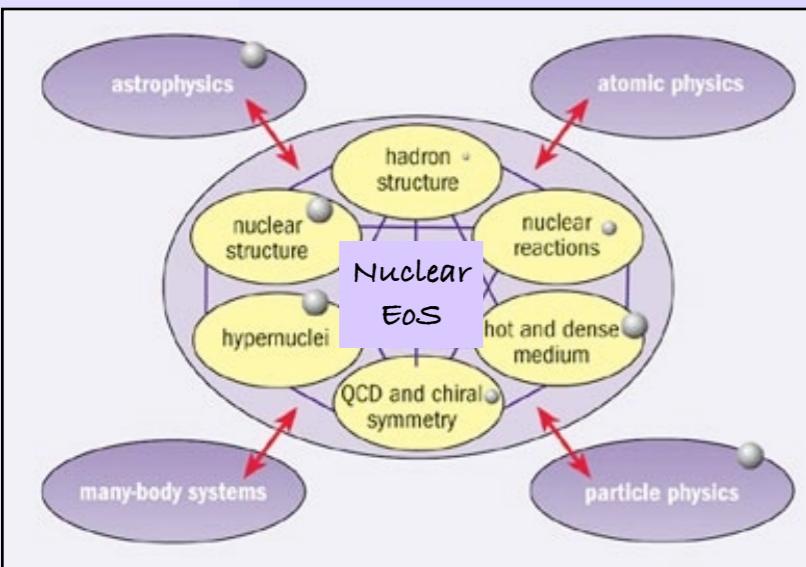
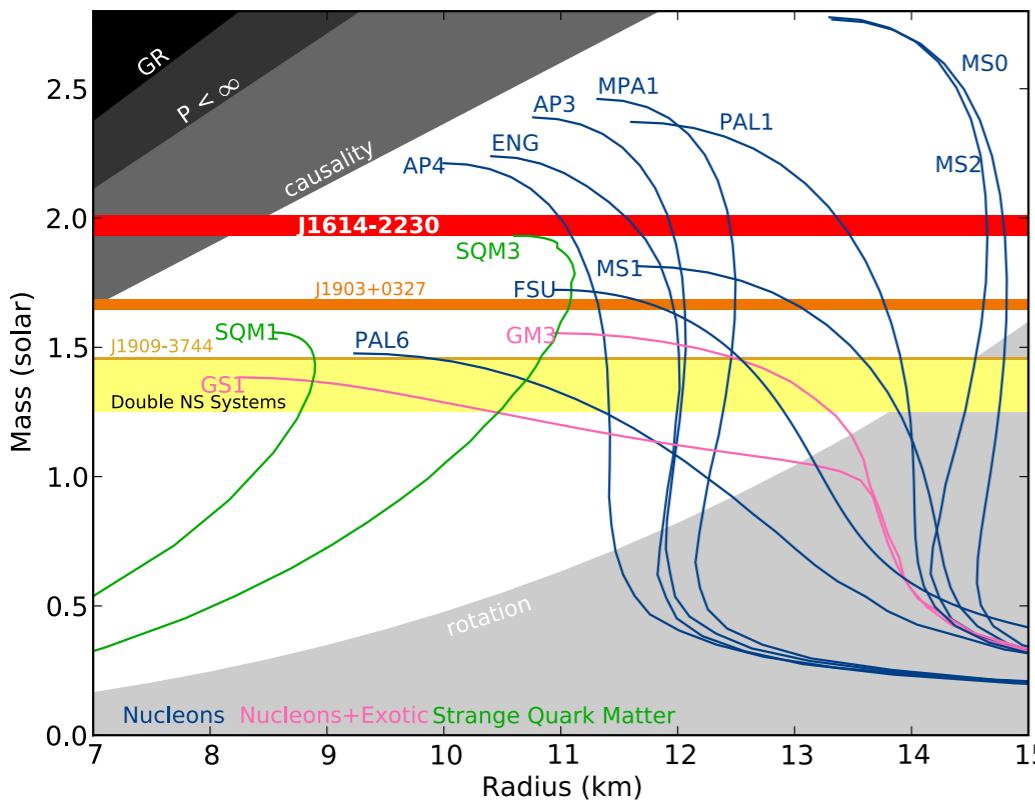


Image: "Physics of NS crusts" Haensel & Chamel

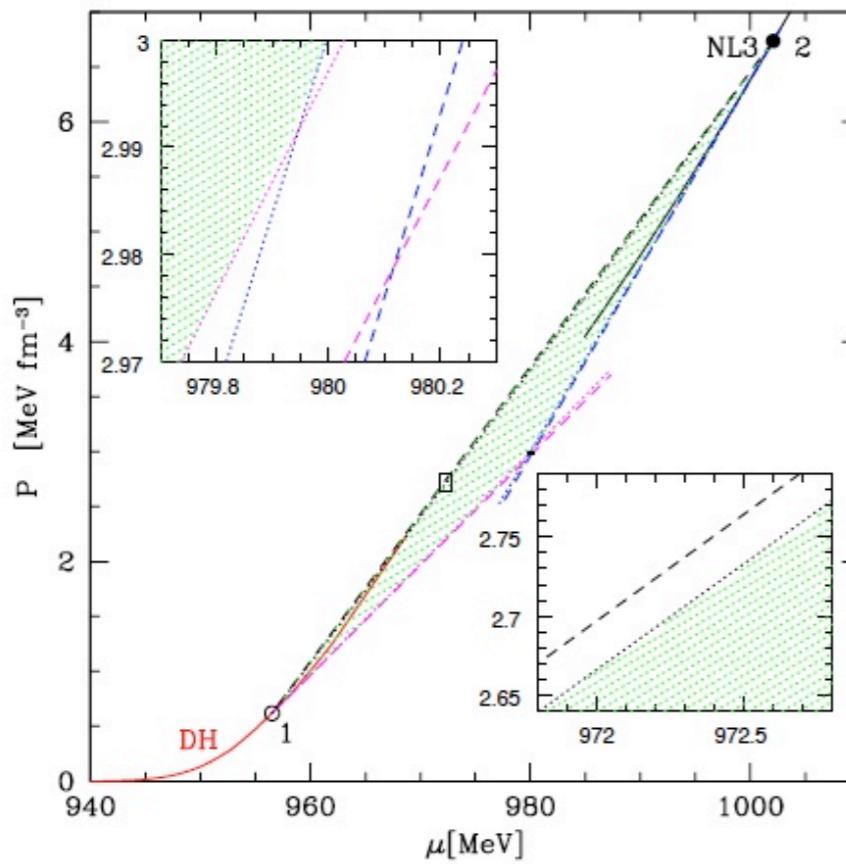
NON-UNIFIED VS UNIFIED EOS

Crust-Core Matching for non-unified EoSs

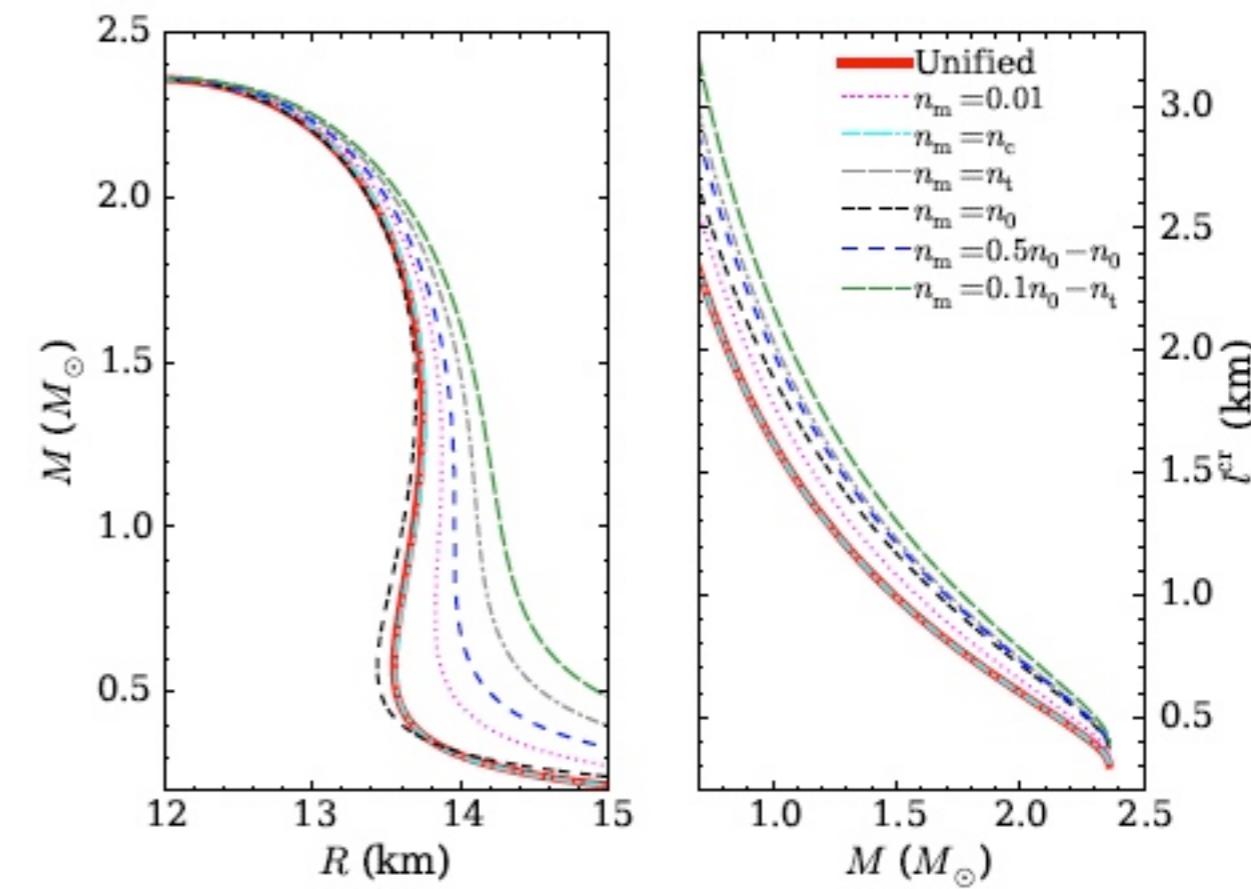
- Matching is done so that pressure is an increasing function of energy density
- different models leads to arbitrary results
- uncertainty in crust thickness upto 30% and for radius 4%



Lattimer and Prakash, (2010)



Fortin et al., (2016)



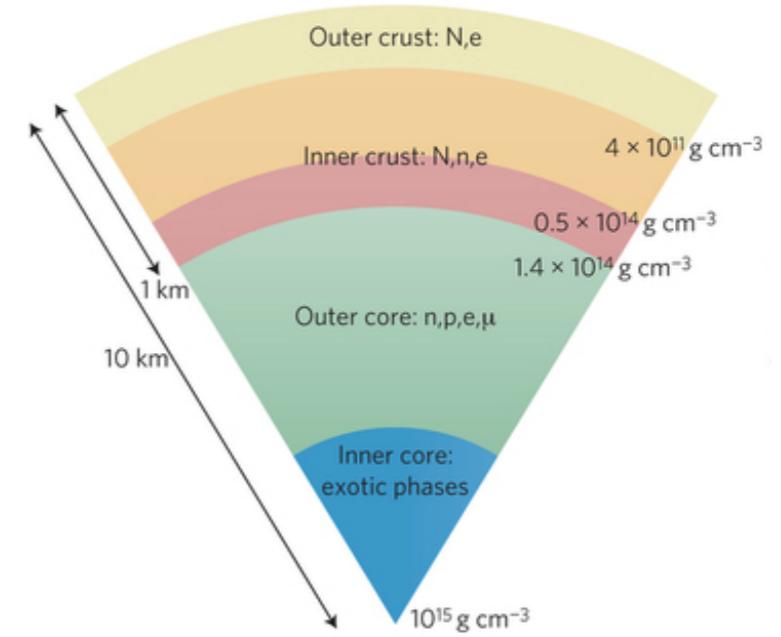
A Unified Model

- ▶ In density functional theory, the energy density functional

$$\mathcal{H} = \mathcal{H} \left[\rho_q(r), \nabla_{q'}^k \rho_q(r) \right]$$

- ▶ In general, there are an infinite number of gradients
- ▶ In HNM $k = 0$ (density terms only) \Rightarrow "Thomas-Fermi approximation"
- ▶ In finite nuclei, non-zero k (say 2) \Rightarrow "Extended-Thomas-Fermi approximation" of order k

e.g. Non-relativistic (Skyrme, BHF),
relativistic (RMF, RHF)



A Unified Model

- ▶ In density functional theory, the energy density functional

$$\mathcal{H} = \mathcal{H} \left[\rho_q(r), \nabla_{q'}^k \rho_q(r) \right]$$

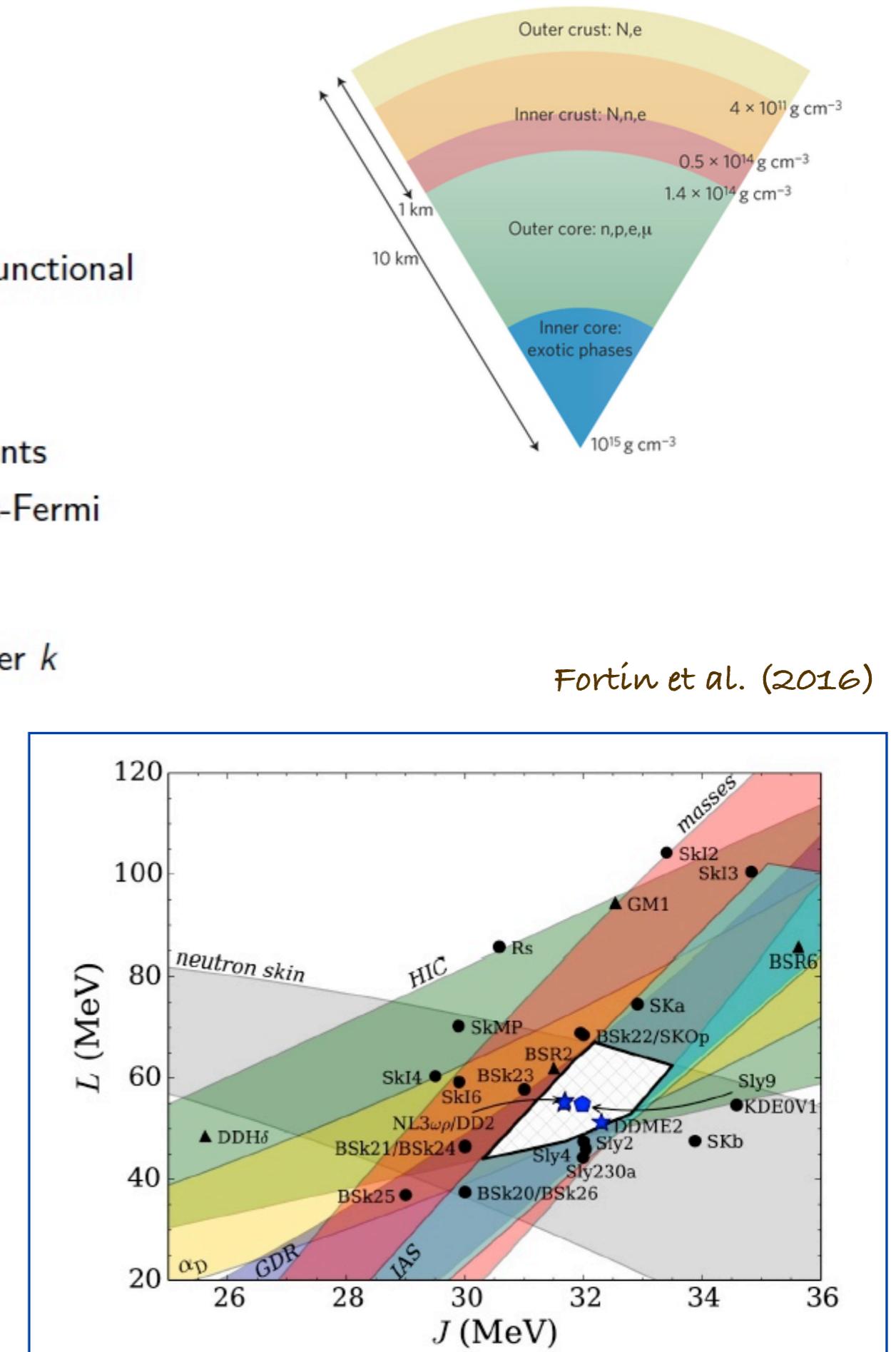
- ▶ In general, there are an infinite number of gradients
 - ▶ In HNM $k = 0$ (density terms only) \Rightarrow "Thomas-Fermi approximation"
 - ▶ In finite nuclei, non-zero k (say 2) \Rightarrow "Extended-Thomas-Fermi approximation" of order k

Fortin et al. (2016)

e.g. Non-relativistic (Skyrme, BHF),
relativistic (RMF, RHF)

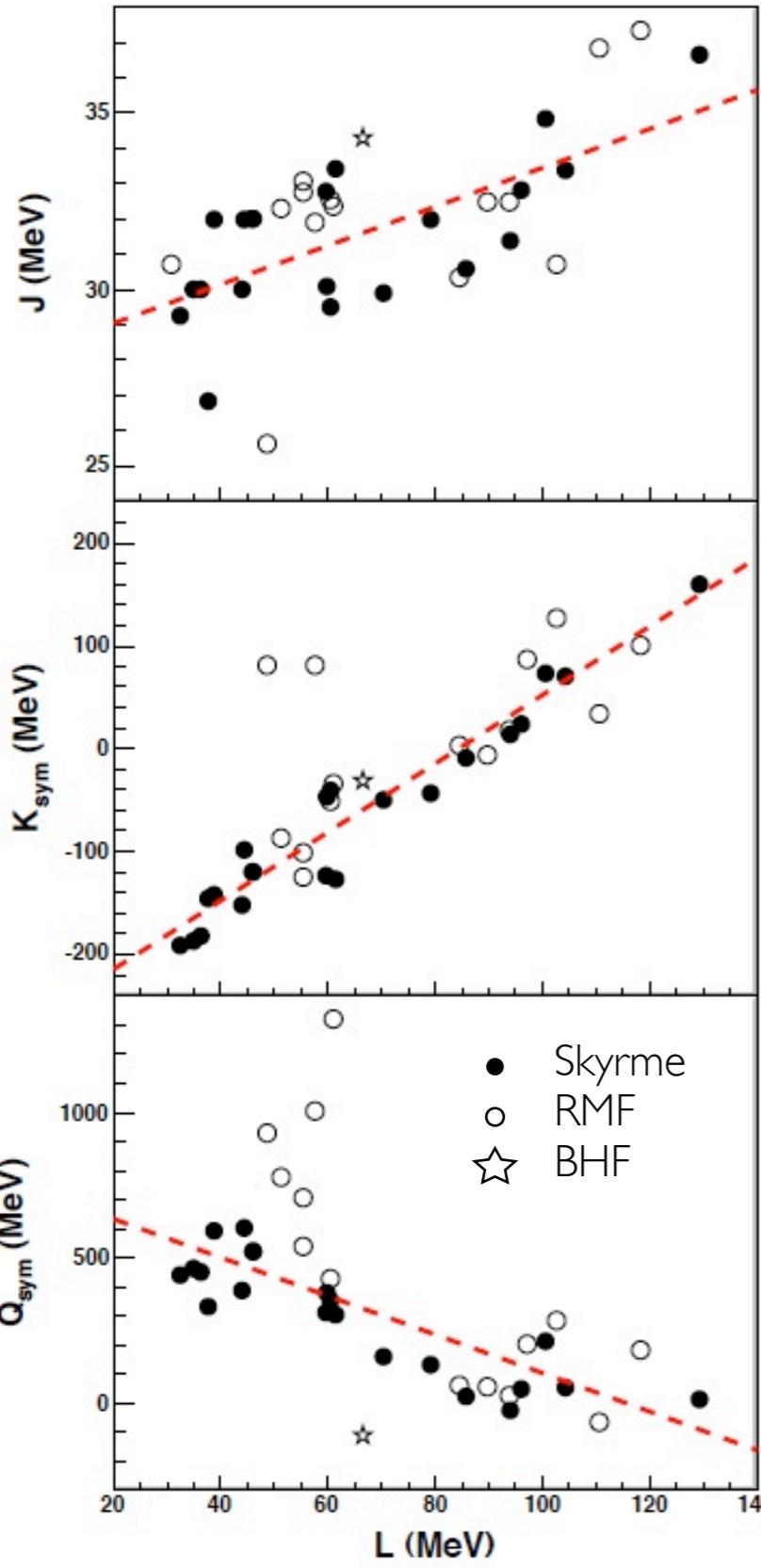
Constraints in J-L plane

- From n skin thickness of ^{208}Pb
 - From HIC
 - From electric dipole polarizability α_D
 - From giant dipole resonance (GDR) of ^{208}Pb
 - From measured nuclear masses
 - From isobaric analog states (IAS)

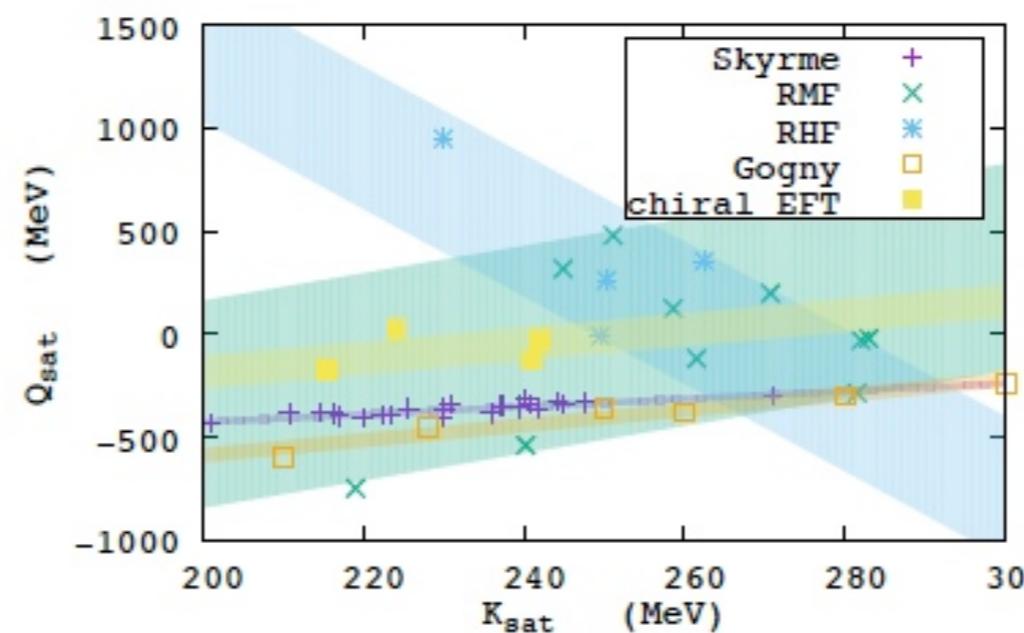


WHY WE NEED A MODEL INDEPENDENT EOS

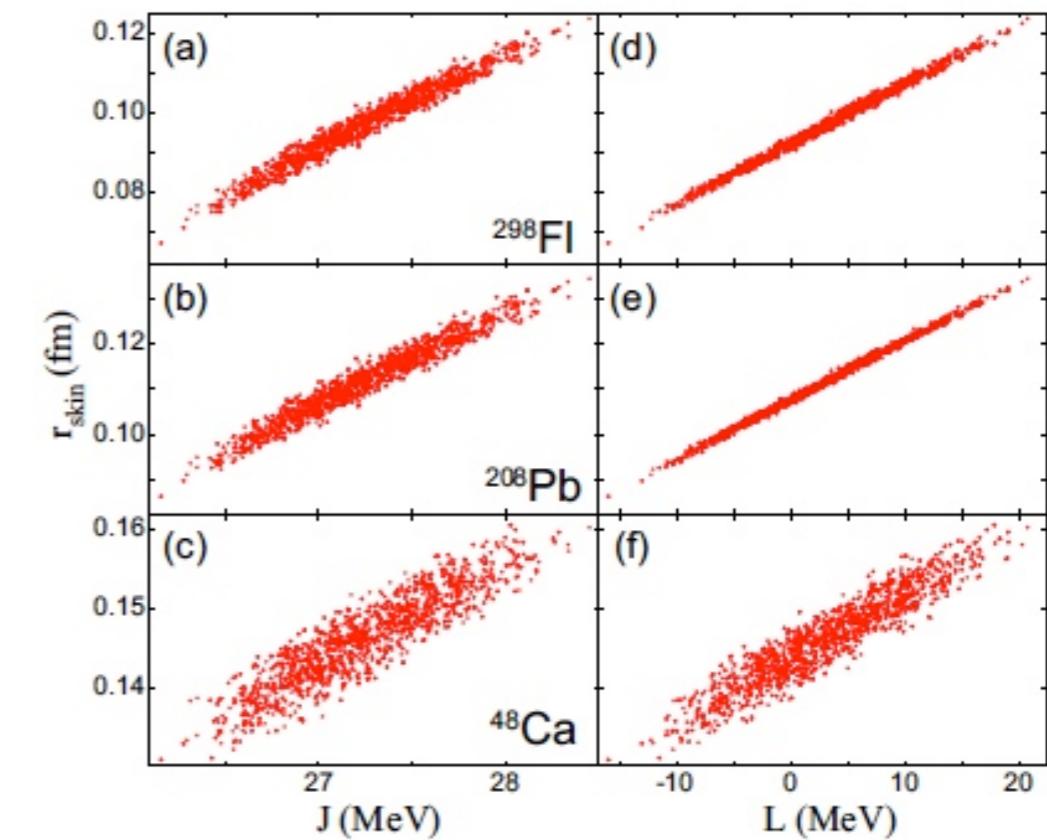
Spurious correlations!



Ducoin et al. PRC 2011



Margueron, Casali,
Gulminelli,
PRC 97, 025805
(2018)



Reinhard and Nazarewicz PRC 2016

EMPIRICAL “META-MODEL”

- ▶ $\mathcal{H}/\rho = e = e_{IS} + e_{IV}\delta^2 + \mathcal{O}(\delta^4)$
- ▶ In terms of $x = \frac{\rho - \rho_{sat}}{3\rho_{sat}}$,
 $e_{IS} = E_{sat} + \frac{K_{sat}}{2!}x^2 + \frac{Q_{sat}}{3!}x^3 + \frac{Z_{sat}}{4!}x^4 + \mathcal{O}(x^5)$, where
 $K_{sat} = \left. \frac{\partial^2 e}{\partial x^2} \right|_{x=0, \delta=0}$
- ▶ $e_{IV} = J_{sym} + L_{sym}x + \frac{K_{sym}}{2!}x^2 + \frac{Q_{sym}}{3!}x^3 + \frac{Z_{sym}}{4!}x^4 + \mathcal{O}(x^5)$
where
 - symmetry energy : $J_{sym} = e_{IV}(x=0) = \frac{1}{2} \left. \frac{\partial^2 e}{\partial \delta^2} \right|_{\delta=0}$
 - slope of the symmetry energy : $L_{sym} = \left. \frac{\partial J_{sym}}{\partial x} \right|_{x=0}$
 - curvature of symmetry energy : $K_{sym} = \left. \frac{\partial^2 J_{sym}}{\partial x^2} \right|_{x=0}$

EMPIRICAL COEFFICIENTS: REFERENCE PARAMETERS

Margueron, Casali, Gulminelli, PRC 97, 025805 (2018)

Model	fixed		Explore inside small interval		Consider large interval		ρ_0 fm ⁻³	E_0 MeV	K_0 MeV	Q_0 MeV	Z_0 MeV	E_{sym} MeV	L_{sym} MeV	K_{sym} MeV	Q_{sym} MeV	Z_{sym} MeV
	σ	Average	σ	Average	σ	Average										
Skyrme	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99						
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23					
RMF	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83						
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34					
RHF	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49						
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74					
Average	0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77						
	σ	0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35					

$$\begin{aligned}
 e &= E_{sat} + E_{sym}\delta^2 + L_{sym}\delta^2x + \frac{1}{2!}(K_{sat} + K_{sym}\delta^2)x^2 \\
 &+ \frac{1}{3!}(Q_{sat} + Q_{sym}\delta^3)x^3 + \frac{1}{4!}(Z_{sat} + Z_{sym}\delta^4)x^4
 \end{aligned}$$

N = 2

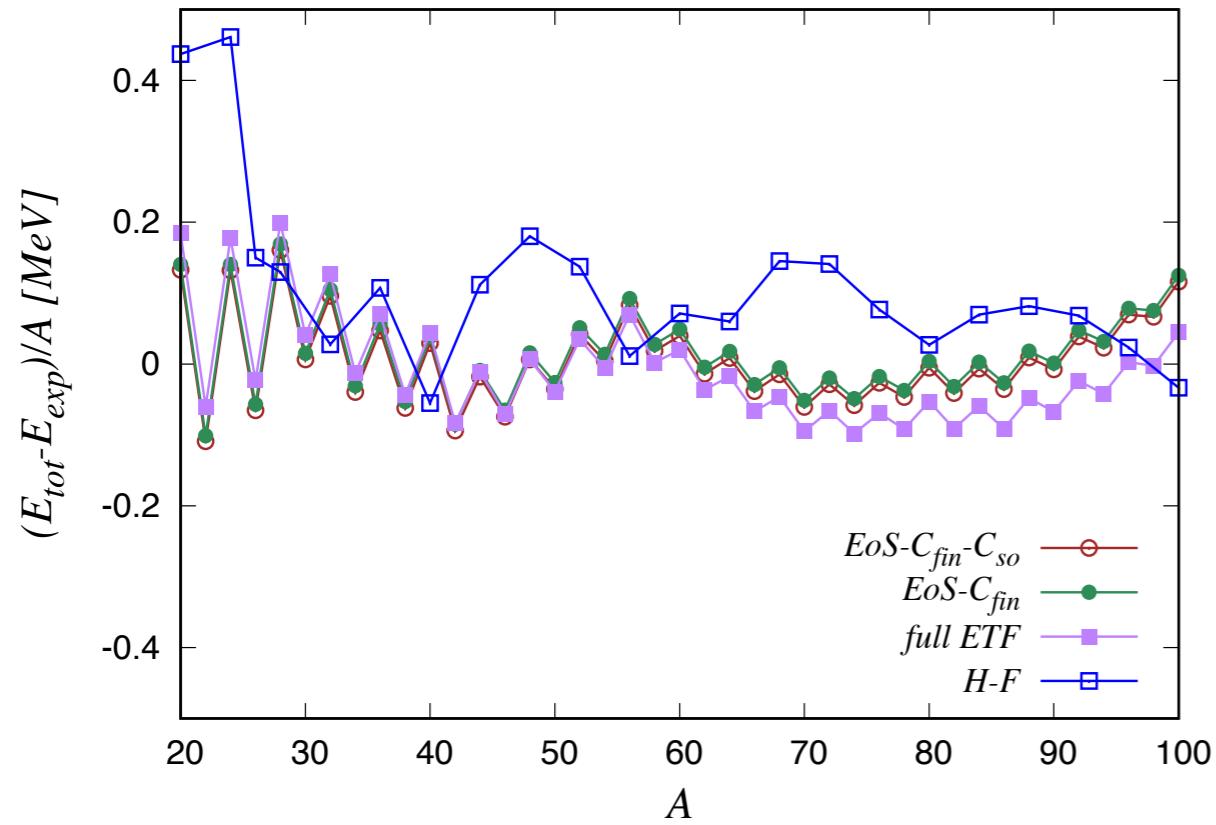
N = 3

N = 4

“Prior” distribution of
Bayesian analysis

APPLICATION TO NUCLEAR PHYSICS : NUCLEI

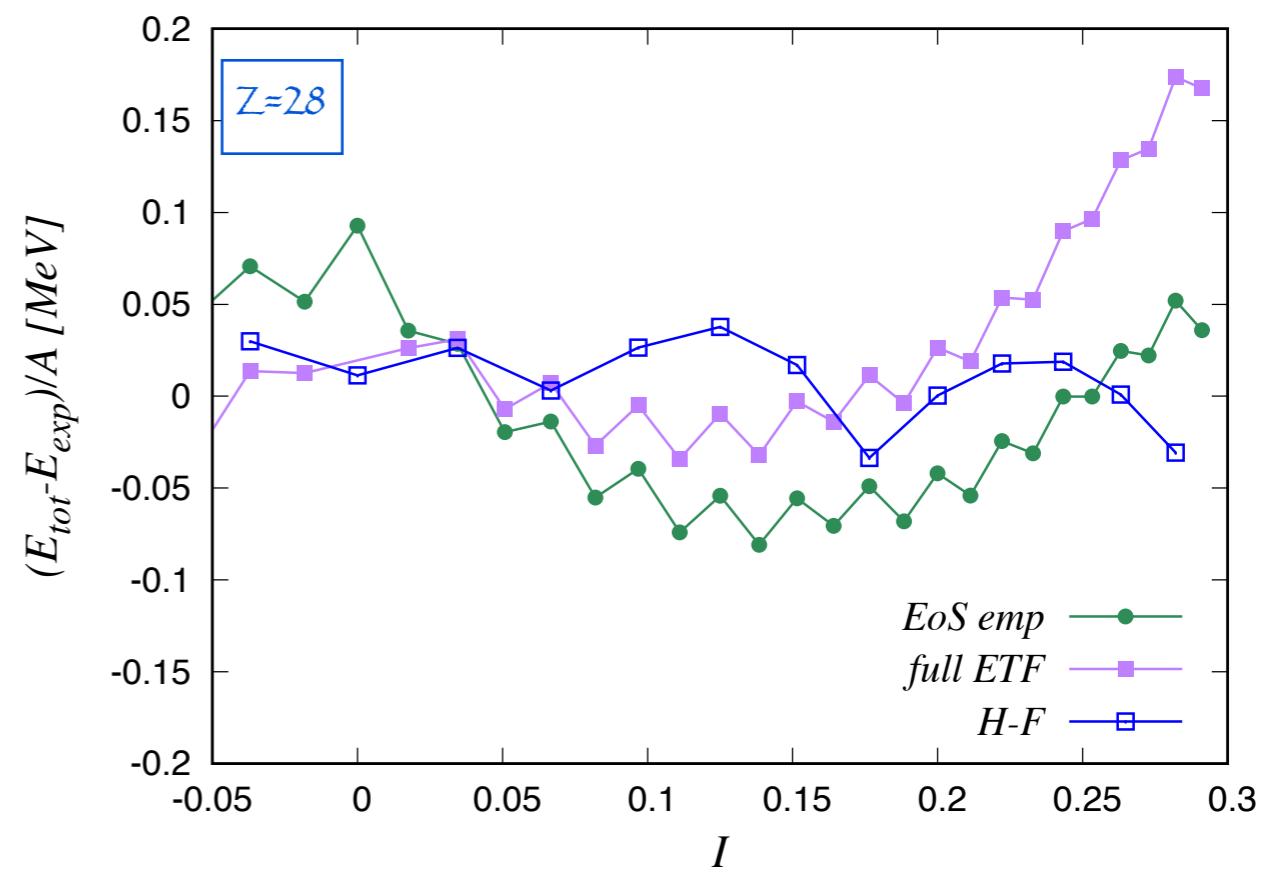
$$\mathcal{H}_{ETF}(\rho_q(r)) = \mathcal{H}_{bulk}(\rho_q(r)) + C_{fin}^{eff}(\nabla \rho)^2$$



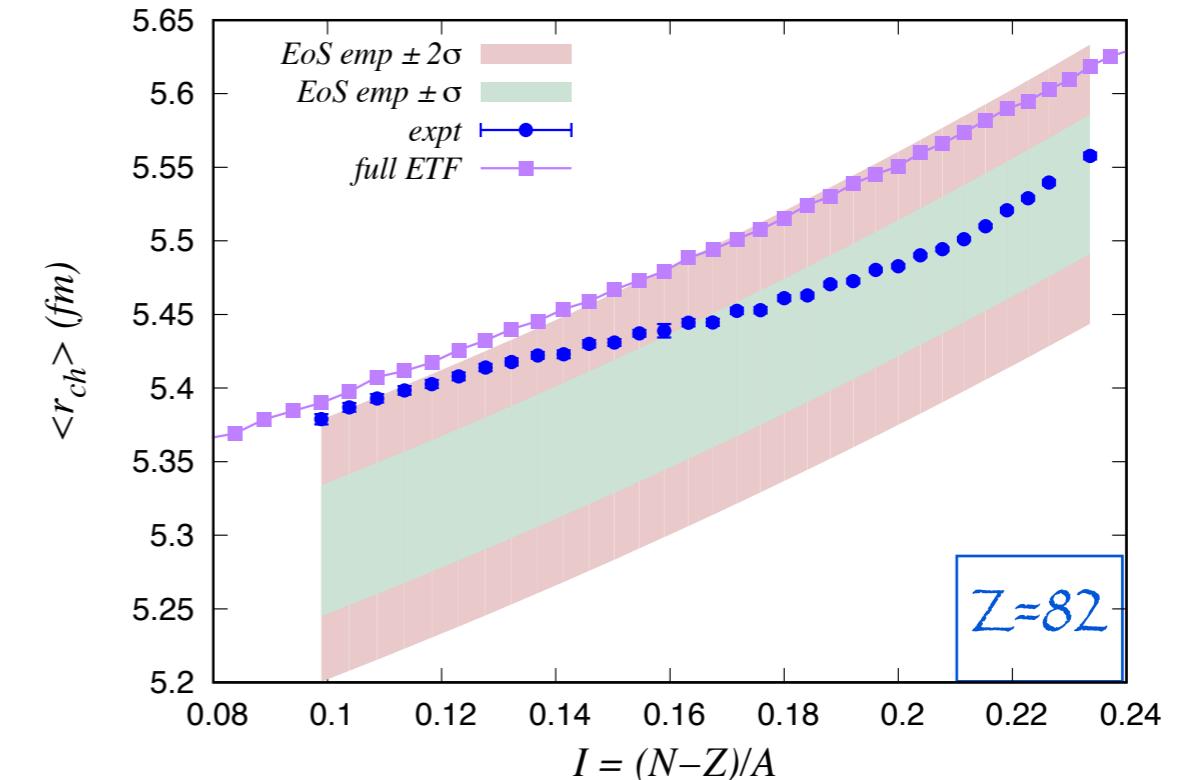
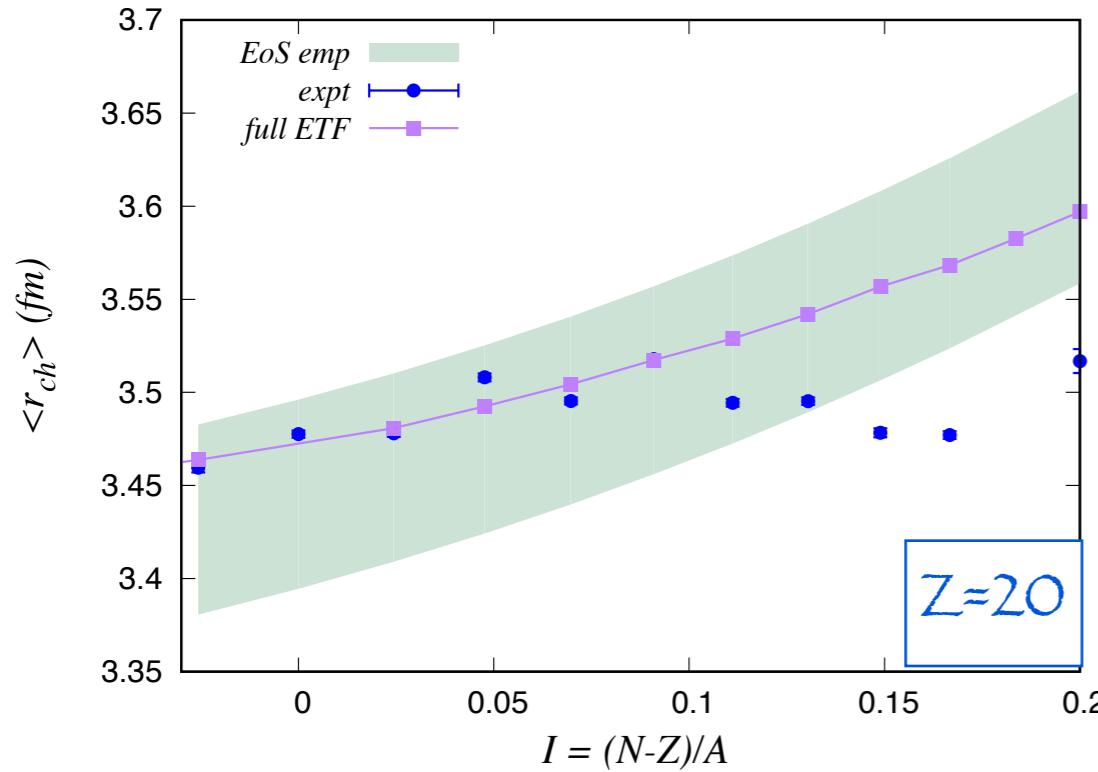
AME (2012)
Mass tables

Aymard, Gulminelli, Margueron (2016)

Energy residuals vs A
for symmetric and asymmetric nuclei



PREDICTION OF NUCLEAR OBSERVABLES



charge Radii expt: ADNDT, Marinova and Angelis (2013)

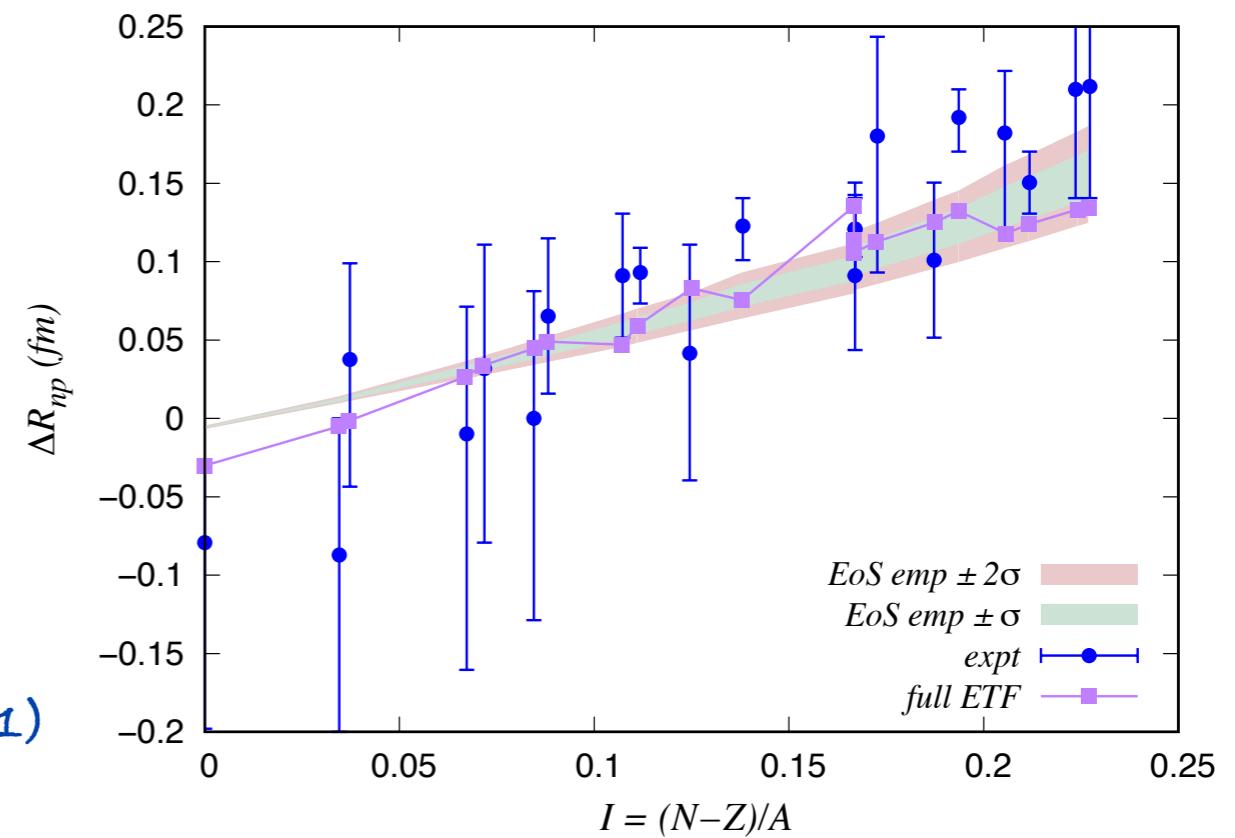
charge radii

$$\langle r^2 \rangle_{ch}^{1/2} = [\langle r^2 \rangle_p + S_p^2]^{1/2}$$

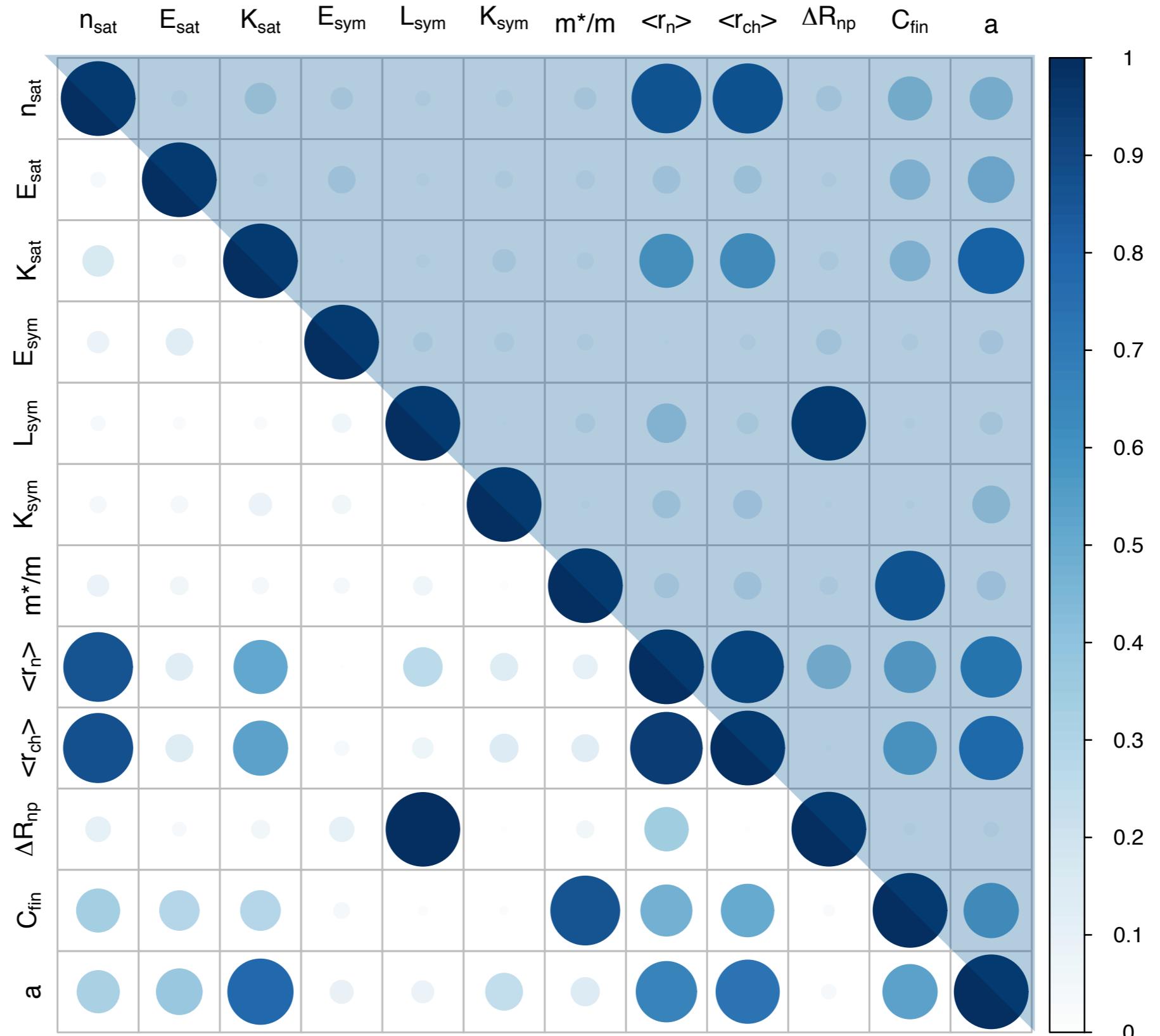
neutron skin

$$\Delta R_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$$

Trzcińska et al. (2001)



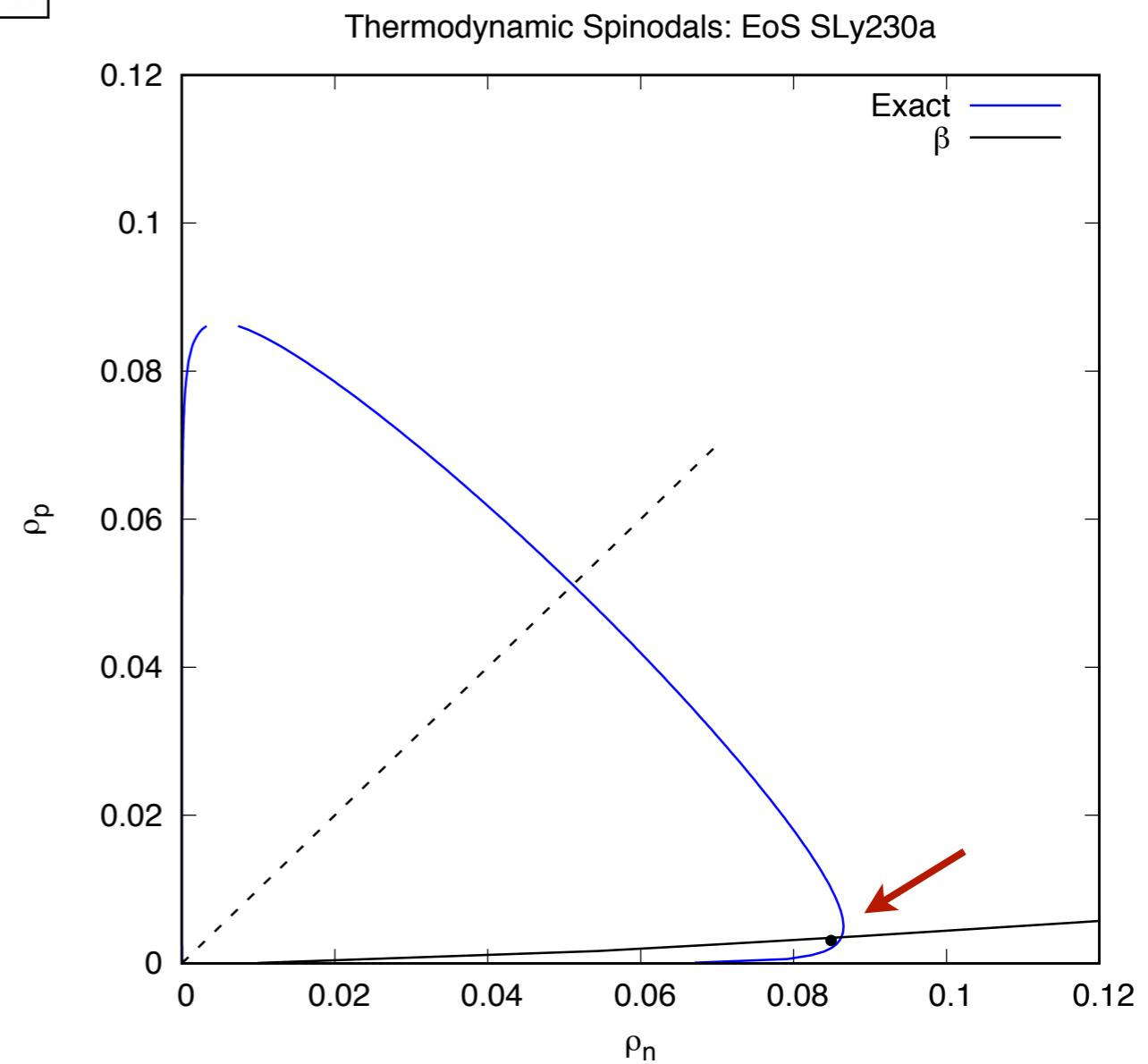
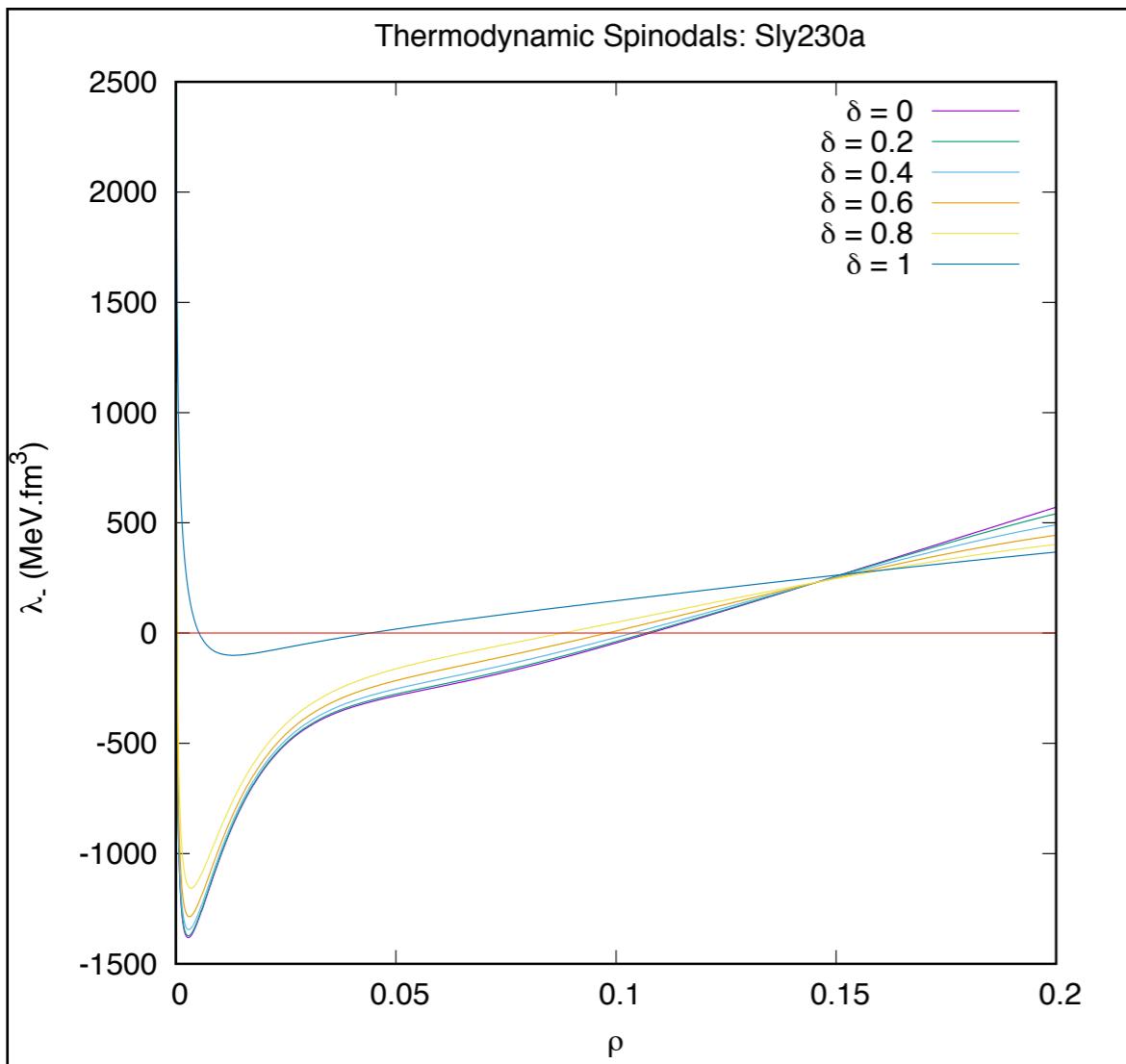
CORRELATIONS: EMPIRICAL PARAMETERS & NUCLEAR OBSERVABLES



CRUST-CORE PHASE TRANSITION

NM

$$C_{NM}^h = \begin{pmatrix} \partial^2 f / \partial \rho_n^2 & \partial^2 f / \partial \rho_n \partial \rho_p \\ \partial^2 f / \partial \rho_p \partial \rho_n & \partial^2 f / \partial \rho_p^2 \end{pmatrix} = \begin{pmatrix} \partial \mu_n / \partial \rho_n & \partial \mu_n / \partial \rho_p \\ \partial \mu_p / \partial \rho_n & \partial \mu_p / \partial \rho_p \end{pmatrix}$$



Thermodynamical spinodals

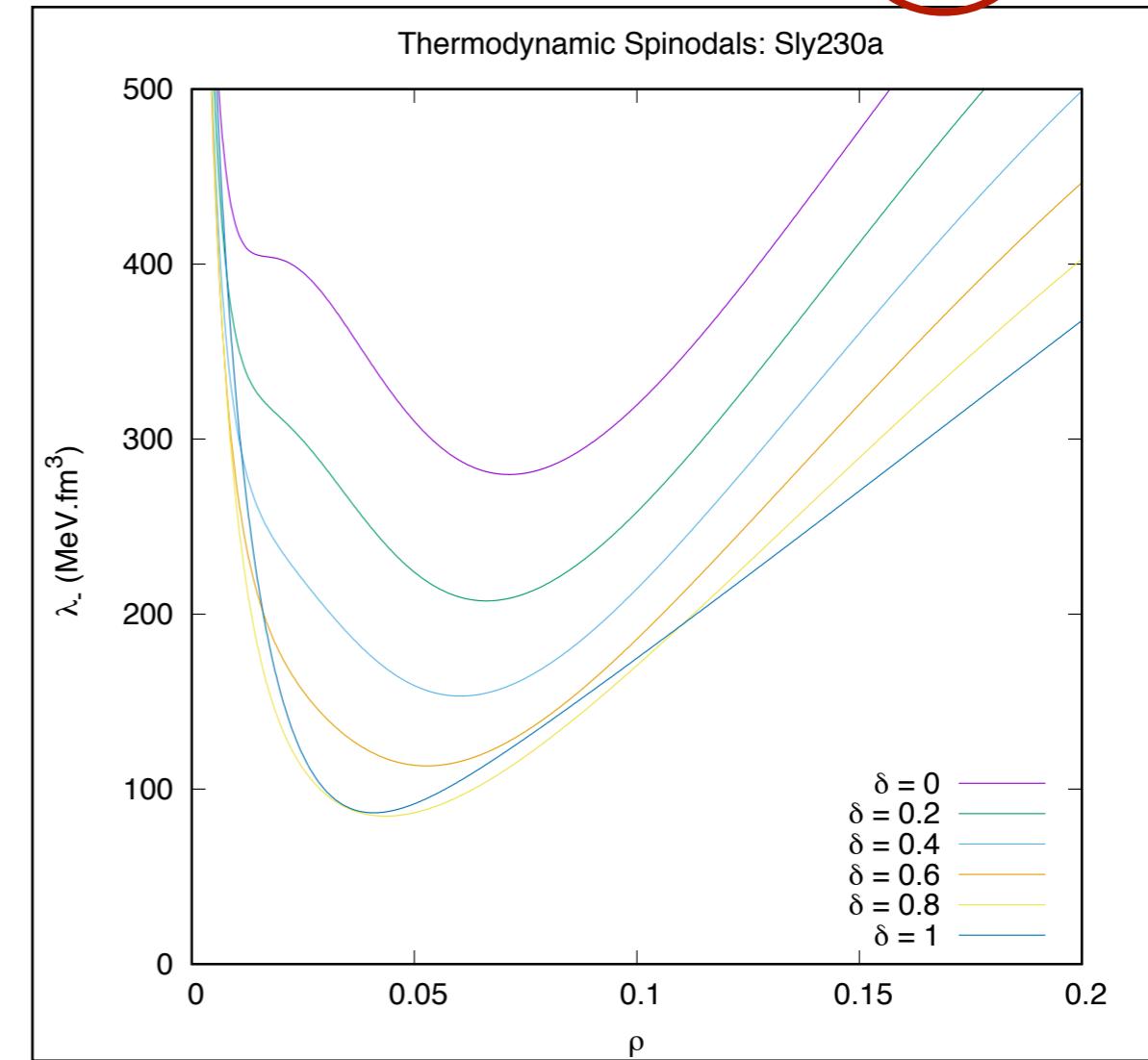
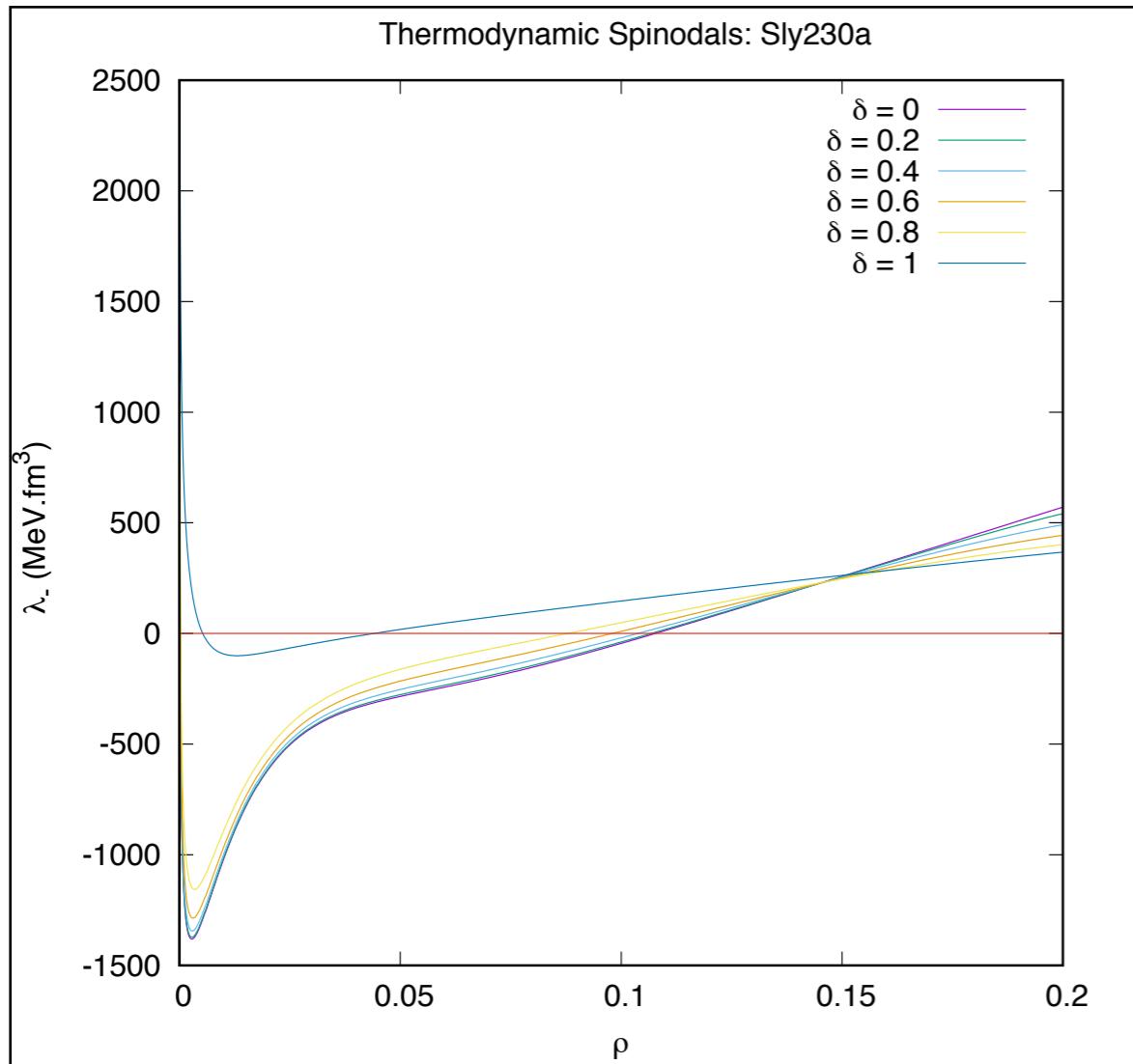
CRUST-CORE PHASE TRANSITION

NM

$$C_{NM}^h = \begin{pmatrix} \partial^2 f / \partial \rho_n^2 & \partial^2 f / \partial \rho_n \partial \rho_p \\ \partial^2 f / \partial \rho_p \partial \rho_n & \partial^2 f / \partial \rho_p^2 \end{pmatrix} = \begin{pmatrix} \partial \mu_n / \partial \rho_n & \partial \mu_n / \partial \rho_p \\ \partial \mu_p / \partial \rho_n & \partial \mu_p / \partial \rho_p \end{pmatrix}$$

NMe

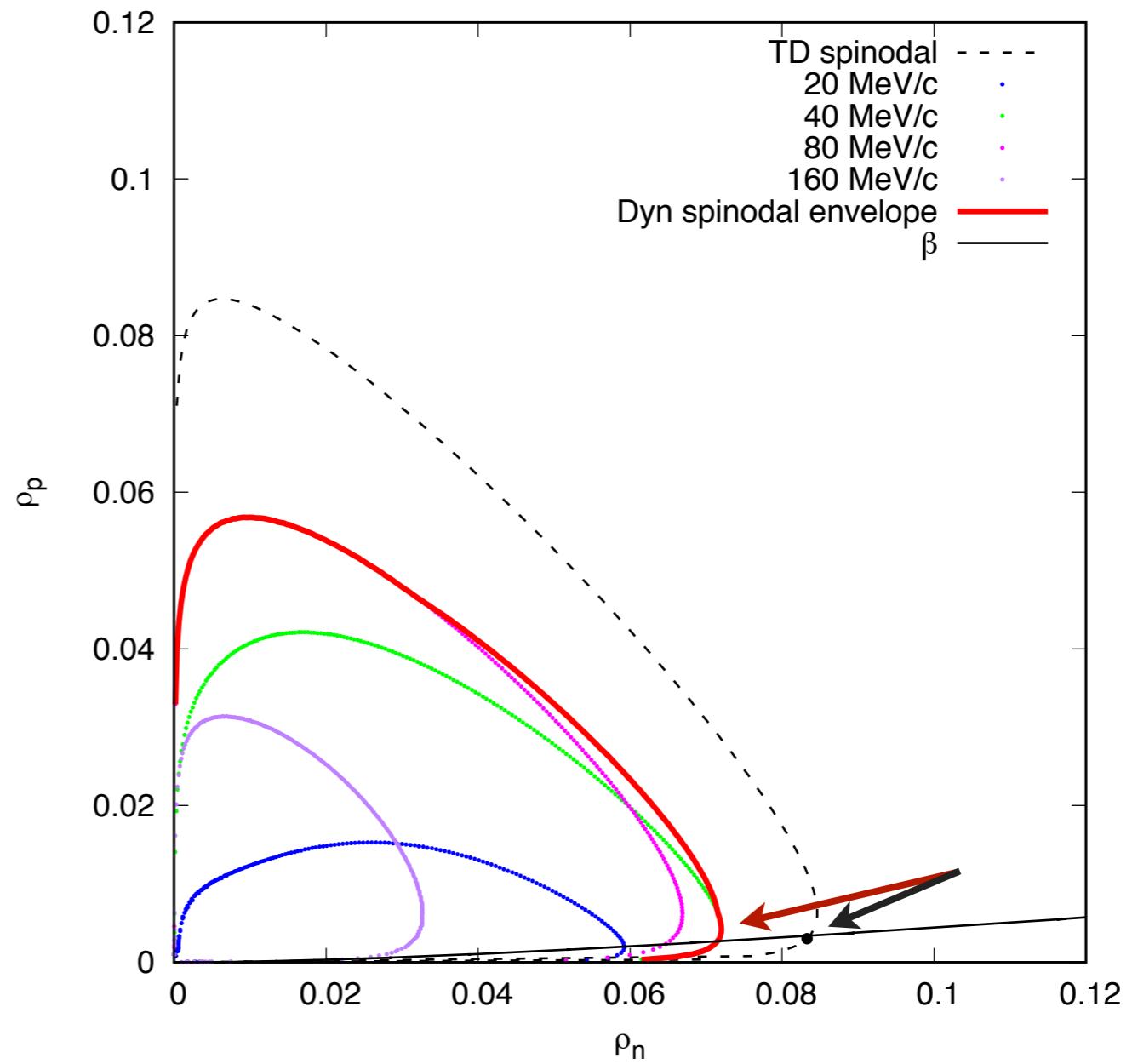
$$C_{NMe}^h = \begin{pmatrix} \partial \mu_n / \partial \rho_n & \partial \mu_n / \partial \rho_p \\ \partial \mu_p / \partial \rho_n & \partial \mu_p / \partial \rho_p + \partial \mu_e / \partial \rho_e \end{pmatrix}$$



Thermodynamical spinodals

CRUST-CORE PHASE TRANSITION

Finite size fluctuations:
Dynamical spinodals



NMe

$$\mathcal{C}^f = \begin{pmatrix} \partial\mu_n/\partial\rho_n & \partial\mu_n/\partial\rho_p & 0 \\ \partial\mu_p/\partial\rho_n & \partial\mu_p/\partial\rho_p & 0 \\ 0 & 0 & \partial\mu_e/\partial\rho_e \end{pmatrix} + k^2 \begin{pmatrix} 2C_{nn}^f & 2C_{np}^f & 0 \\ 2C_{pn}^f & 2C_{pp}^f & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{4\pi^2 e^2}{k^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Ref: Thesis, Camille Ducoin

bulk

density-gradient

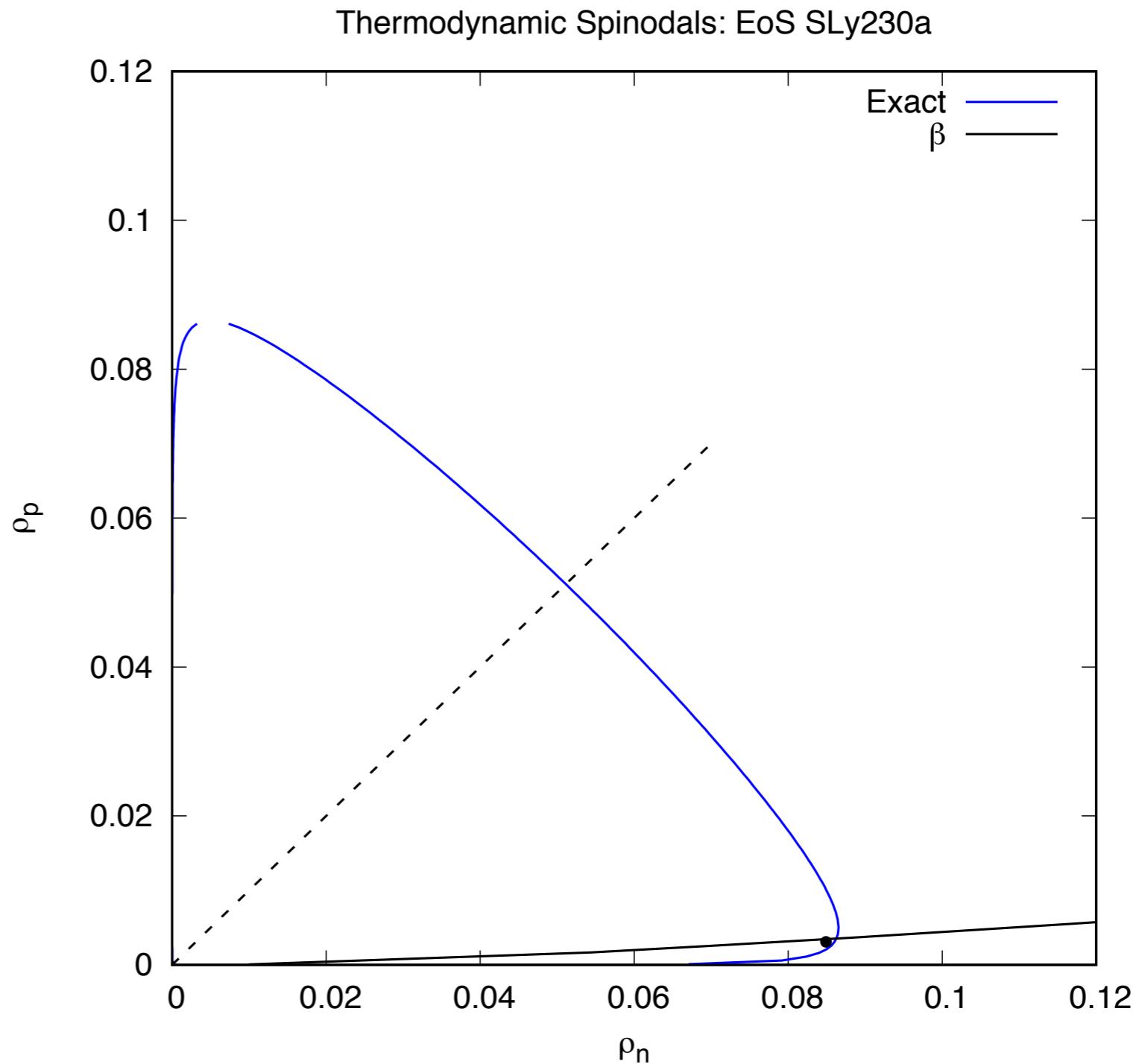
Coulomb

APPLICATION OF META-MODEL IN ASTROPHYSICS: CCPT

$$C = \begin{bmatrix} \frac{\partial \mu_n}{\partial \rho_n} & \frac{\partial \mu_n}{\partial \rho_p} \\ \frac{\partial \mu_p}{\partial \rho_n} & \frac{\partial \mu_p}{\partial \rho_p} \end{bmatrix}$$

NM

	Sly230a
ρ_0 (fm^{-3})	0.16
E/A_0 (MeV)	-15.99
K_∞ (MeV)	230.9
m^* (MeV)	0.7
a_s (MeV)	32.04

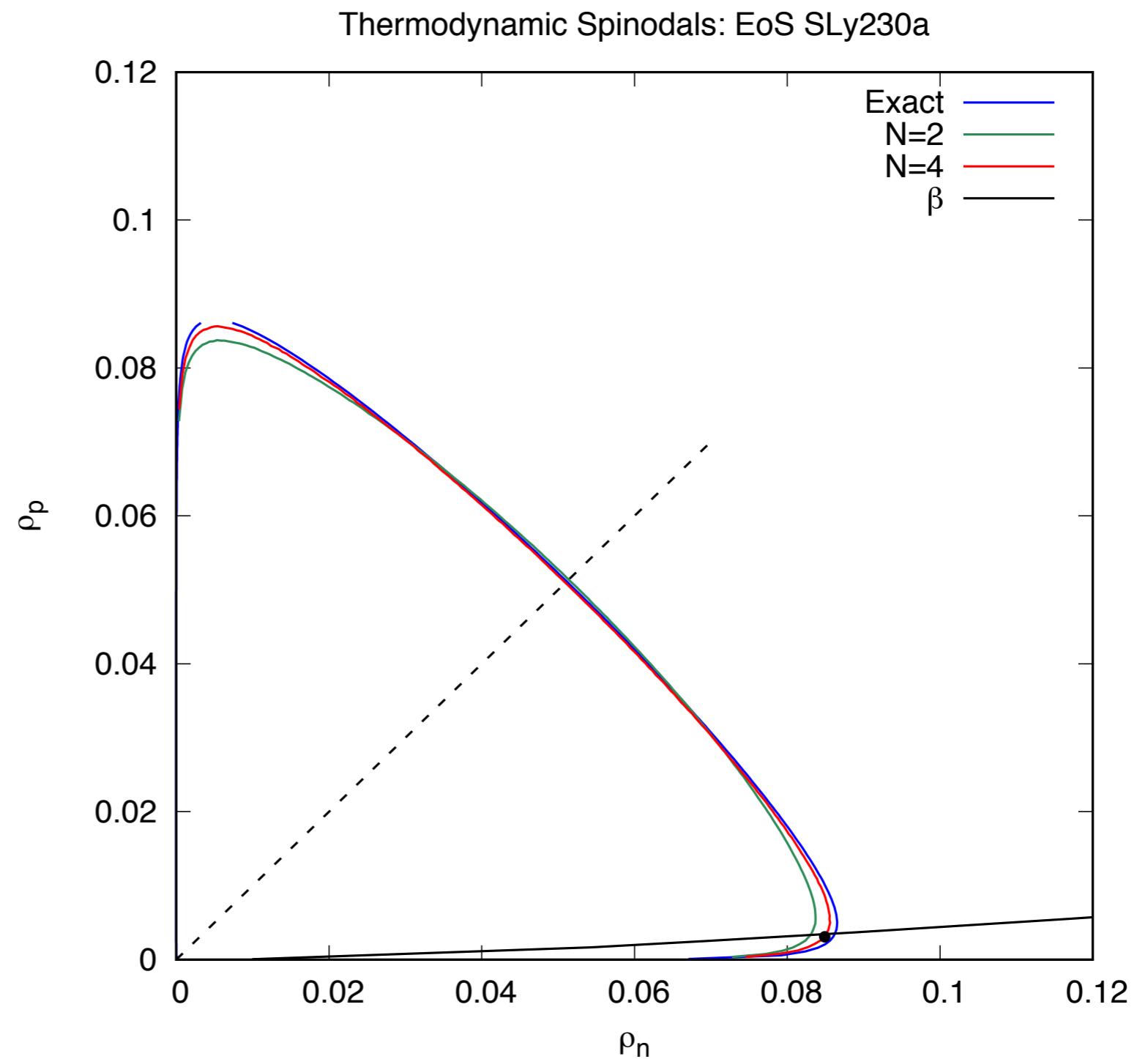


Thermodynamical spinodals

APPLICATION OF META-MODEL IN ASTROPHYSICS: CCPT

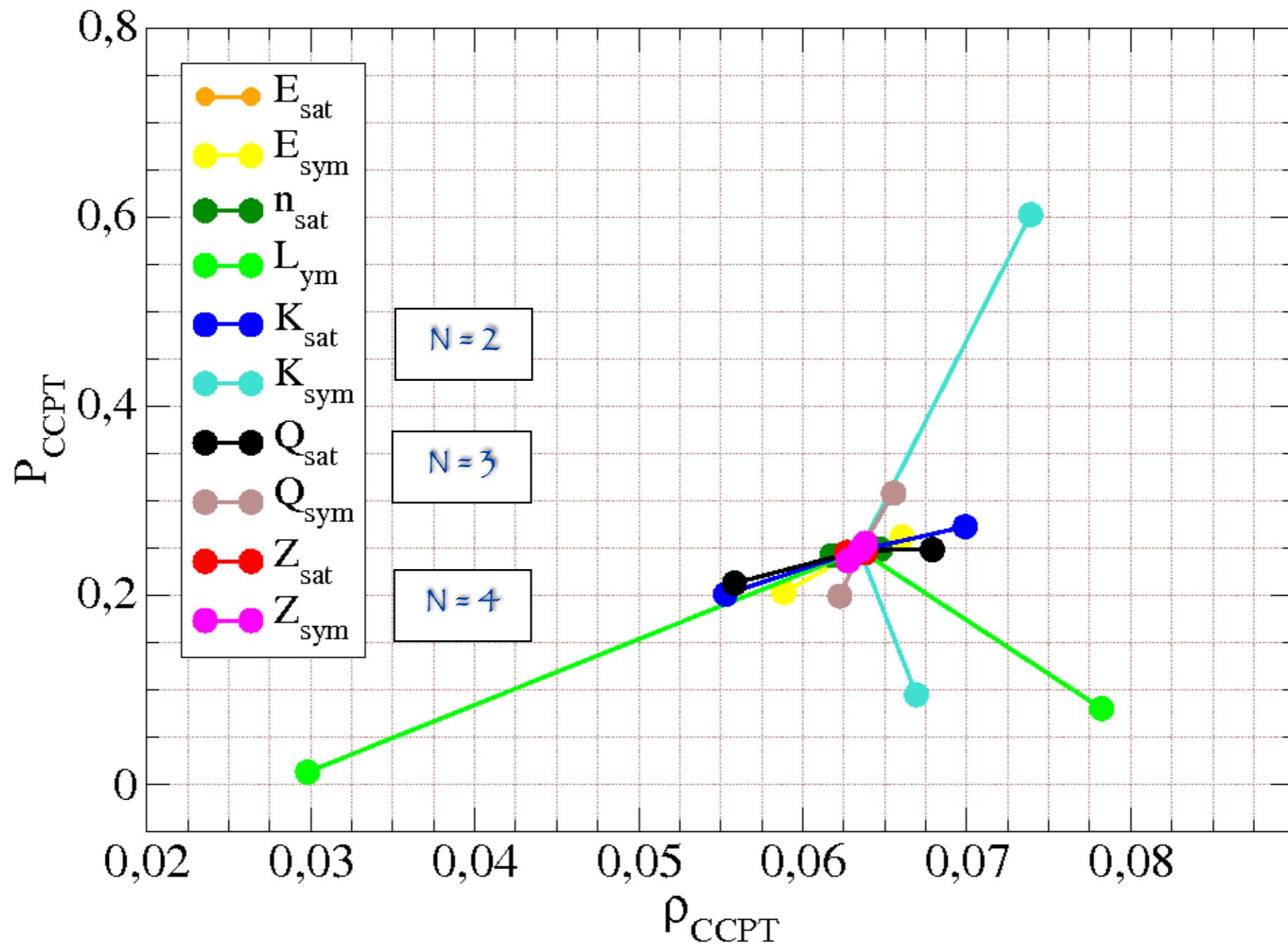
$$C = \begin{bmatrix} \frac{\partial \mu_n}{\partial \rho_n} & \frac{\partial \mu_n}{\partial \rho_p} \\ \frac{\partial \mu_p}{\partial \rho_n} & \frac{\partial \mu_p}{\partial \rho_p} \end{bmatrix}$$

NM

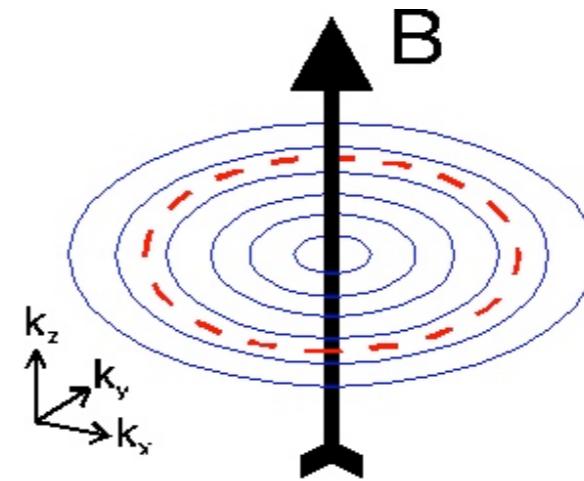
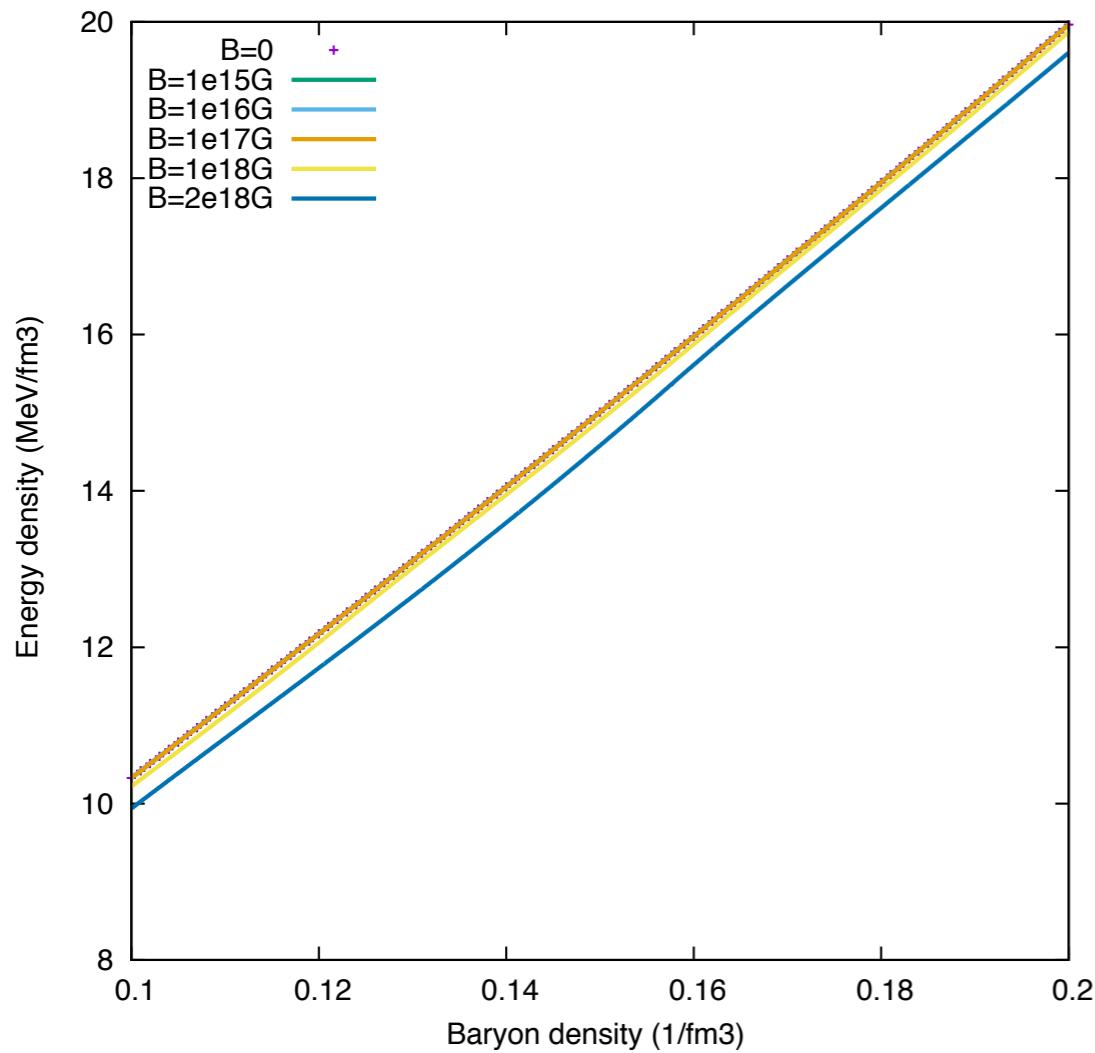


Thermodynamical spinodals: effect of HO terms of MM

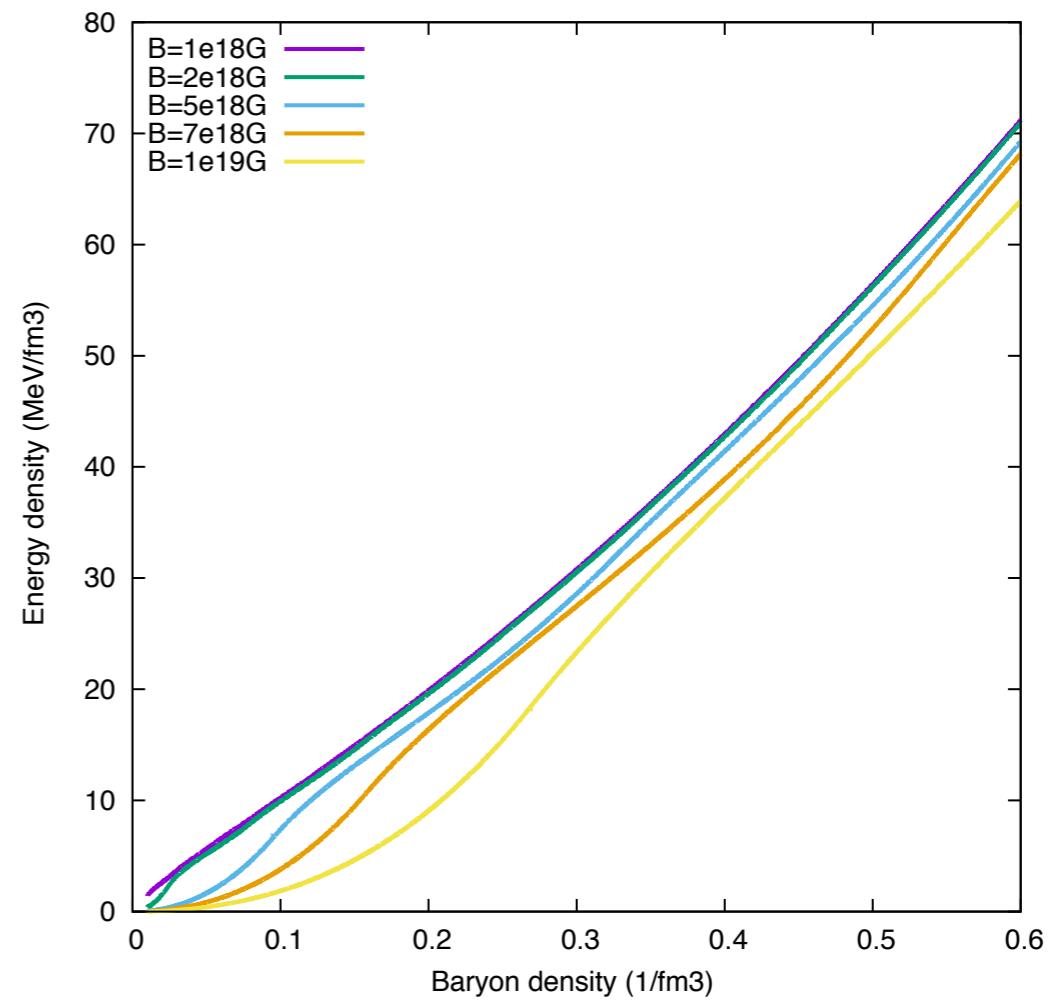
SENSITIVITY TO EMPIRICAL PARAMETERS



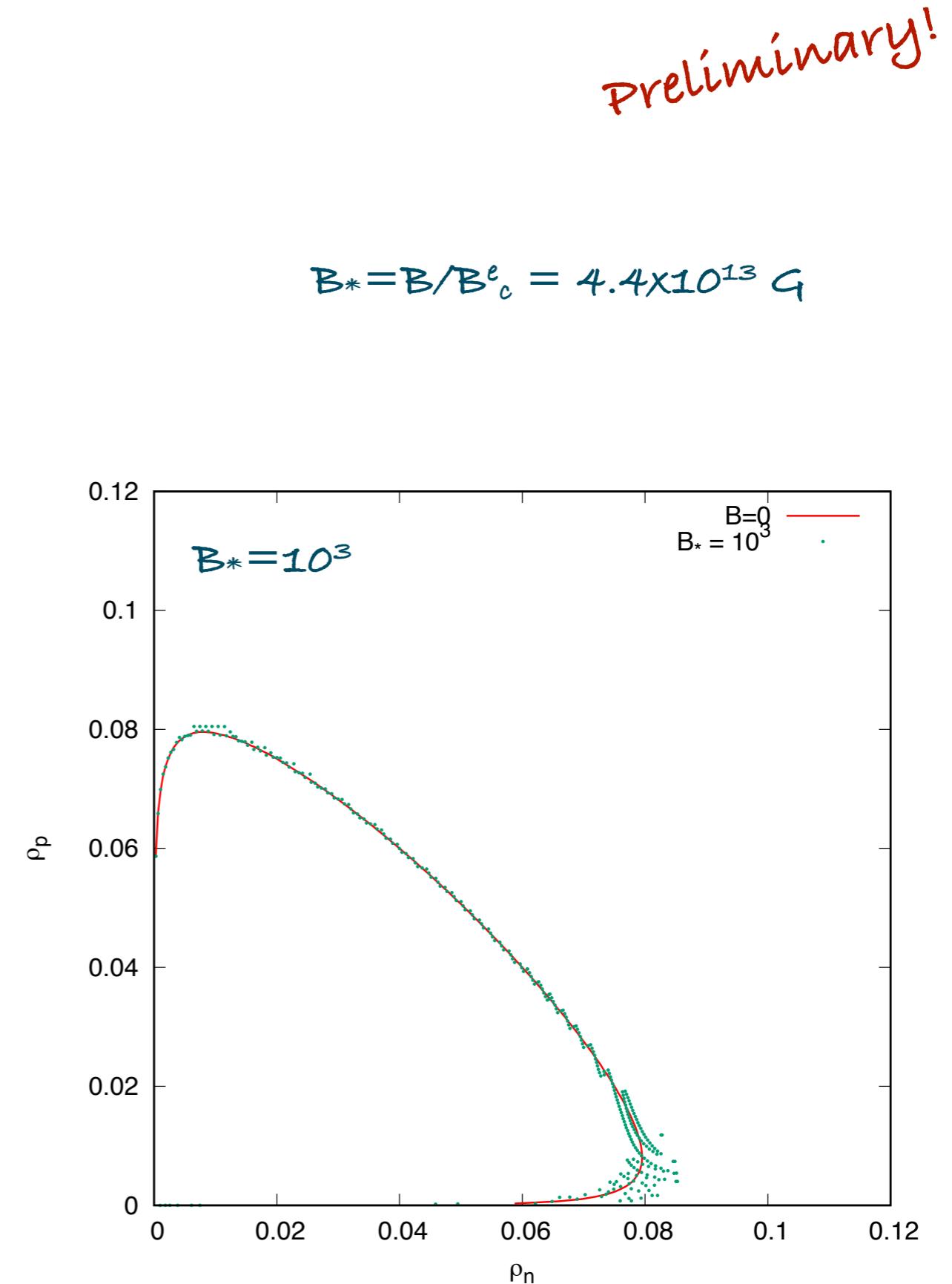
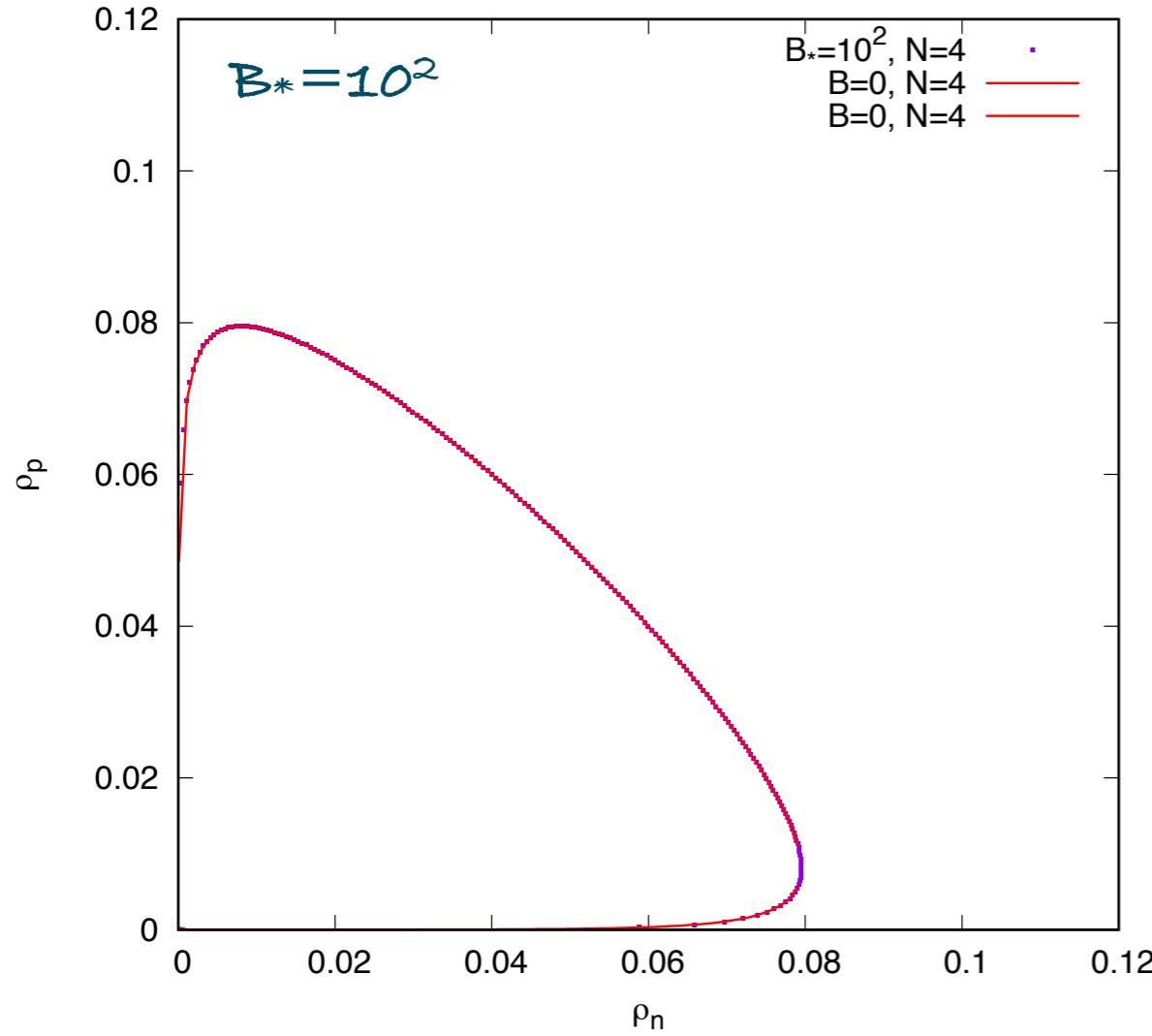
THE NEXT STEP : EFFECT OF MAGNETIC FIELD



$$E_{n,p_z} = (n + 1/2)\hbar\omega + \frac{p_z^2}{2m}$$

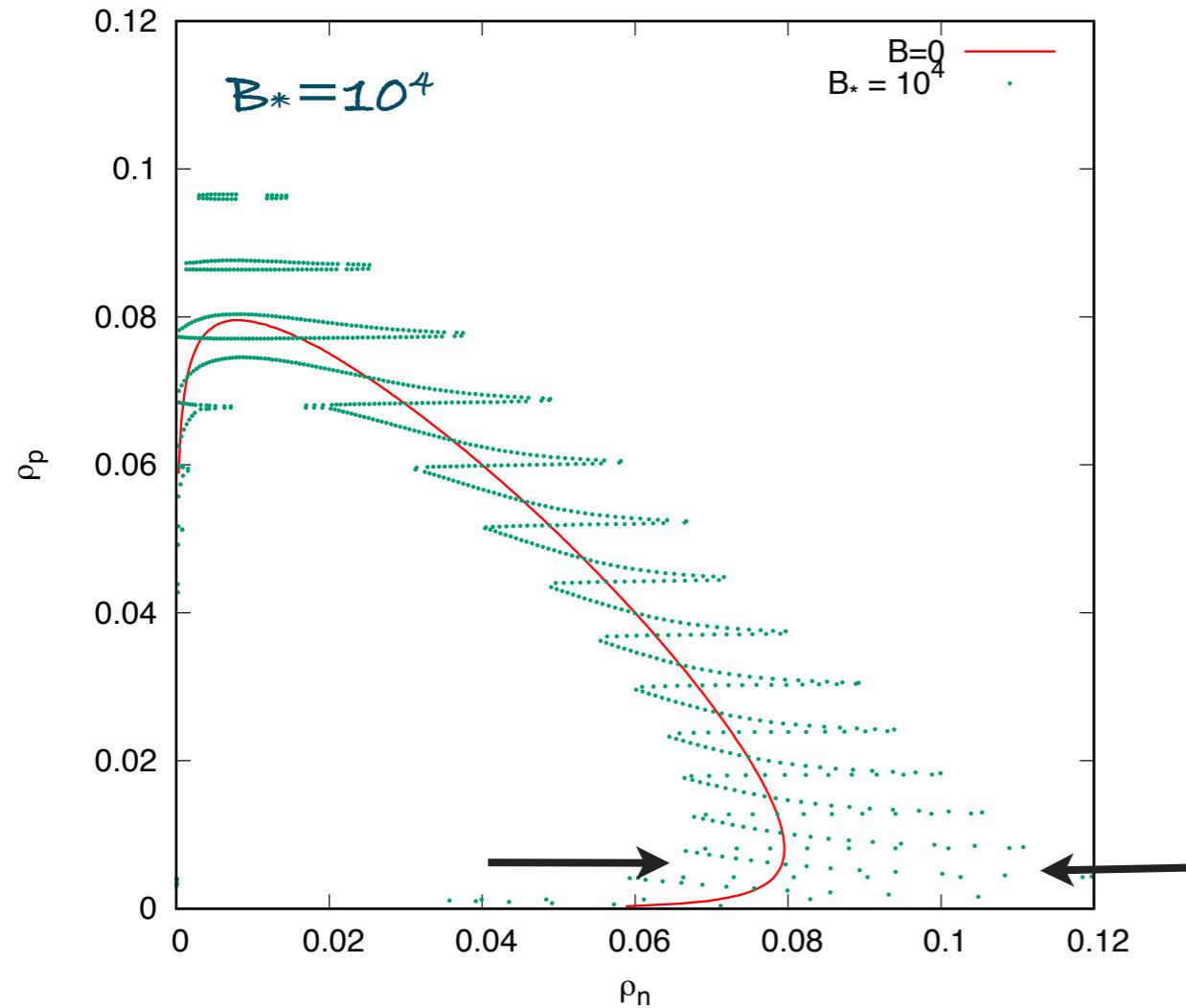


EFFECT OF MAGNETIC FIELD ON THERMODYNAMIC SPINODALS



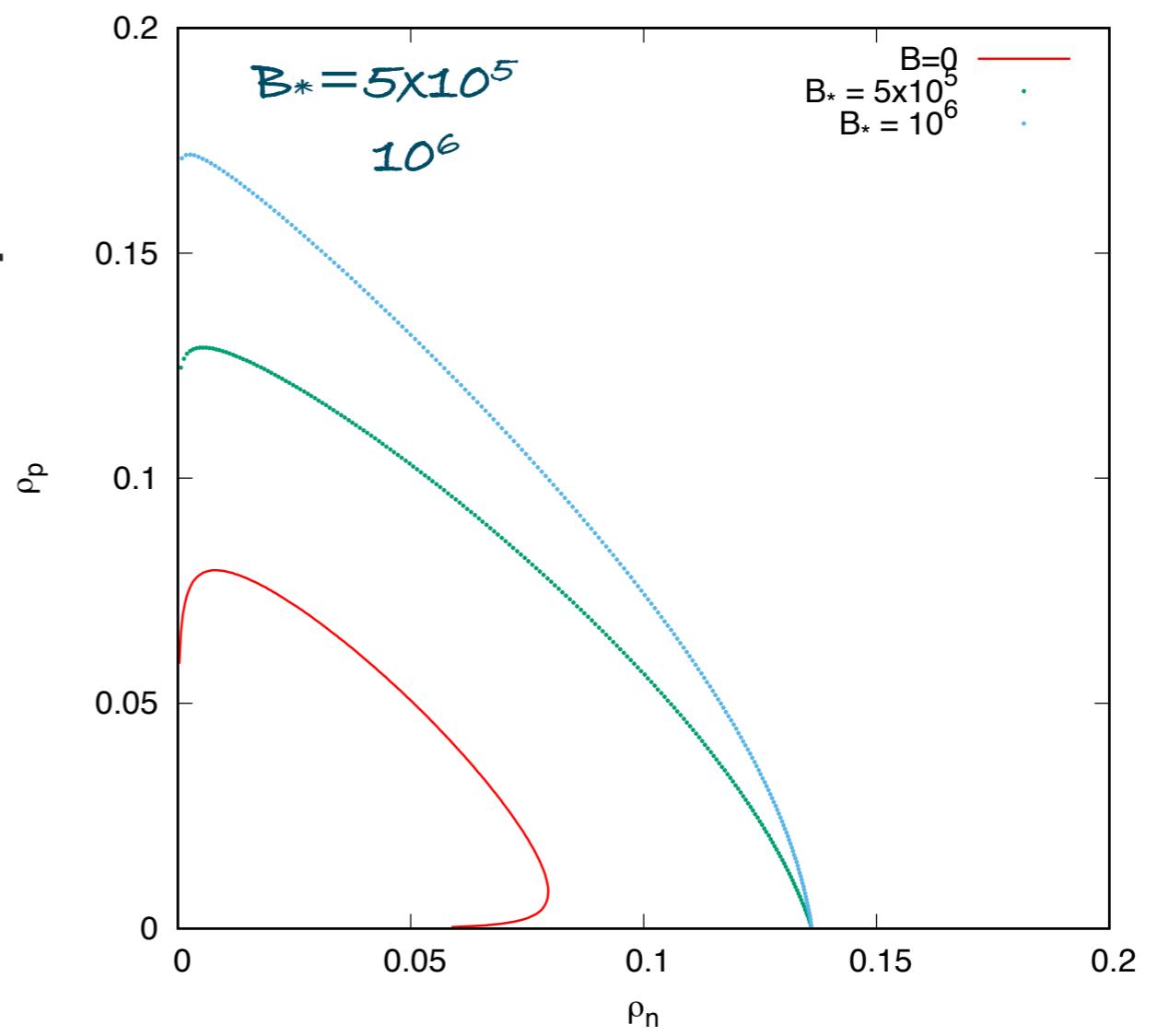
Ref: Thesis, Jianjun Fang

EFFECT OF MAGNETIC FIELD ON THERMODYNAMIC SPINODALS



Preliminary!

$$B_* = B/B_{c}^e = 4.4 \times 10^{13} \text{ G}$$



Ref: Thesis, Jianjun Fang

SUMMARY

- We propose an empirical “MetaModel” to describe homogeneous nuclear matter as well as asymmetric nuclei within the same formalism
- DFT in the ETF approximation to construct an energy functional for HNM and clusterized matter
- In HNM, the coefficients of the energy functional directly related to experimentally determined empirical parameters $\{\rho_{\text{sat}}, \lambda_{\text{sat}}, K_{\text{sat}}, J_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}\}$ and m^*
- In clusterized matter, a single extra parameter required (C_{fin}) to reproduce the experimental measurements of nuclear masses
- We apply this MetaModel to calculate the thermodynamical instability (spinodal) region that determines the crust-core phase transition in neutron stars
- We study the influence of the uncertainty in empirical parameters on the crust-core phase transition
- We investigate the influence of strong magnetic fields on the crust-core phase transition
- This may have important consequences on the crust thickness, radii, moment of inertia and other astrophysical observables