A first survey of the ghost-gluon vertex in the Gribov-Zwanziger framework

[Phys.Rev. D97 (2018) 3, 034020]

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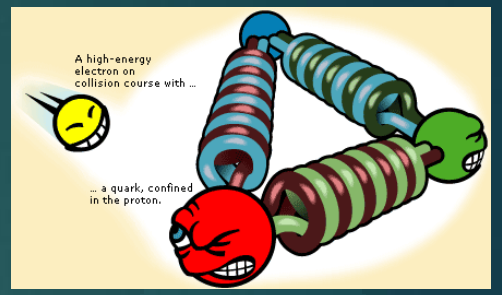
Outline

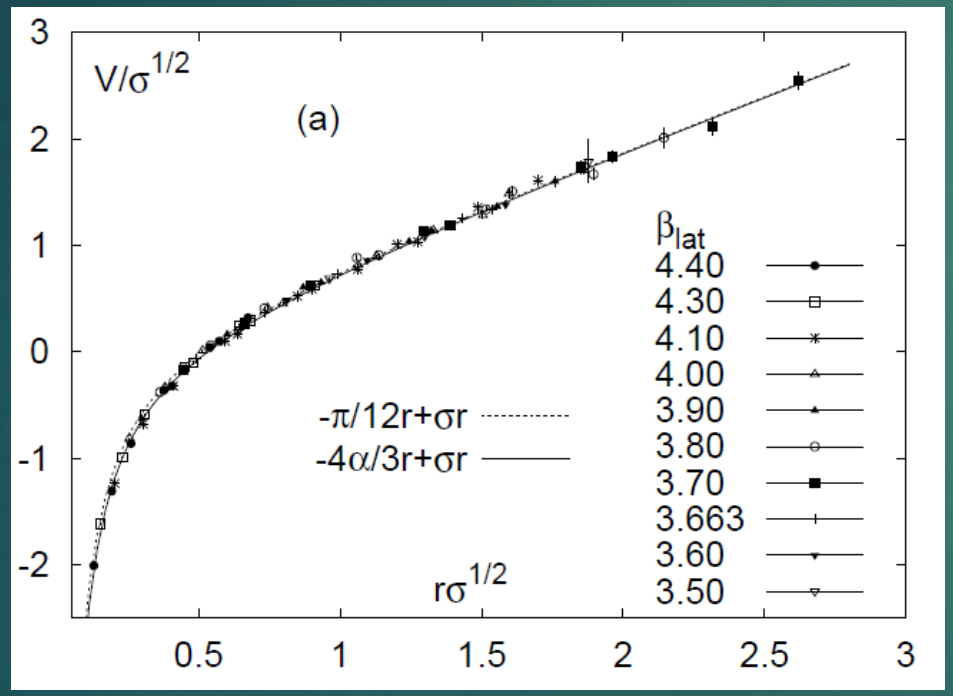
- Confinement as reflection positivity violation.
- The Gribov-Zwanziger (GZ) framework in a nutshell (somewhat technical).
- Applications: gluon propagator (old) and ghost-gluon vertex (new).

Confinement as reflection positivity violation

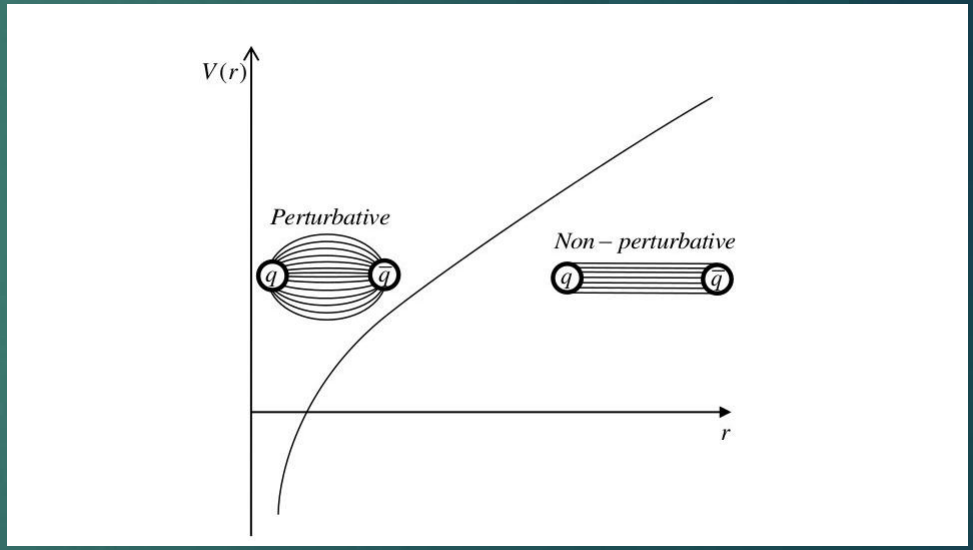
Confinement as Wilson loop area law

- Area law: $W = \exp(-\sigma RT) \Rightarrow V_{\bar{q}q} = \sigma R$ (linear potential).
- Strictly valid for infinitely heavy quarks (what about gluons?)





Coulomb + linear potential [Kaczmarek/Zantow **Phys.Rev. D71 (2005) 114510**]



Flux tube model. Figure taken from [Deldar et al., **Phys.Rev. D85 (2012) 054501**]

Confinement as positivity violation?

- Another confinement criterion: positivity violation.

- Källén-Lehmann (spectral) representation of the propagator (momentum space):

$$D(p) = \langle \varphi(p)\varphi(-p) \rangle = \int_0^\infty \frac{d\tau}{2\pi} \frac{\rho(\tau)}{p^2 + \tau}$$

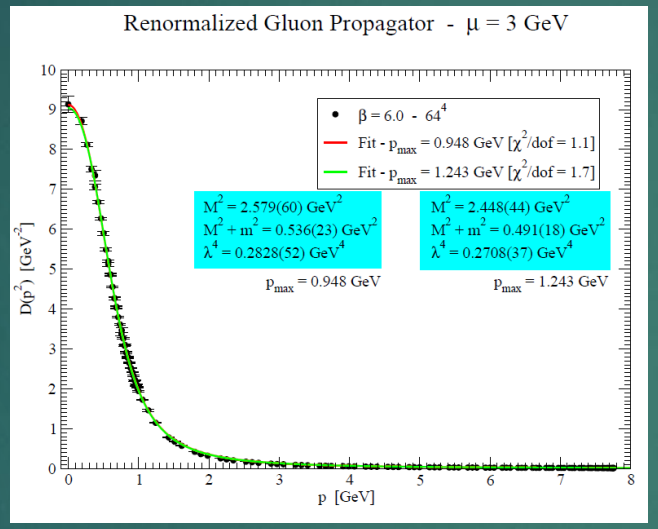
- Asymptotic states → $\rho(\tau) > 0$ for $\tau > 0$.
- Example: free particle of mass m → $\boxed{\rho_{Free}(\tau) = 2\pi\delta(\tau - m^2)}$

- Positivity violation: if $\rho(\tau) < 0$ for some $\tau > 0 \Rightarrow$ no asymptotic states.

Confinement as positivity violation?

- Nonperturbative gluon propagator: Lattice, SDE, FRG, and RGZ are all compatible with positivity violation. → a sign of confinement(?)

D. Dudal *et al.*,
[Phys.Rev. D81 (2010) 074505]



$$D_{fit}(p) = Z\frac{p^2 + a}{p^4 + bp^2 + c} = D + D^*$$

(sum of complex propagators,
with possibly complex masses)

- "Handwaving argument": complex mass → no propagation.

$$\langle \varphi(\vec{x}, t)\varphi(\vec{0}, 0)\rangle \sim \exp(-i\omega t)\exp(-i\vec{p}\cdot\vec{x}) \rightarrow \text{decaying exp, with } \omega \in C.$$

- Expectation: physical (colorless) bound states should have propagators with $\rho(\tau) > 0 \;\; \forall\, \tau$. [Gluballs: D. Dudal *et al.*, **Phys.Rev.Lett. 106 (2011) 062003**, rho meson: D. Dudal *et al.* **Annals Phys. 365 (2016) 155-179**].

The quantization of gauge theories and Gribov's problem

Gribov problem and Gribov horizon

- Generating functional = Functional integral over <u>all</u> field configs.

$$Z[J] = \int [D\phi] \exp\left[-S[\phi] + \int_X J(x)\phi(x)\right]$$

- Gauge fields → gauge copies, which have to be removed.
- Usual removal of gauge copies: Faddeev-Popov procedure, which is okay for high energies (low g).
- For low energies: FP is not sufficient (more gauge copies) → Gribov Problem.
- How can we try to deal with these gauge copies at low energies?

[L. D. Faddeev and V. N. Popov **Phys.Lett. 25B (1967) 29-30**]

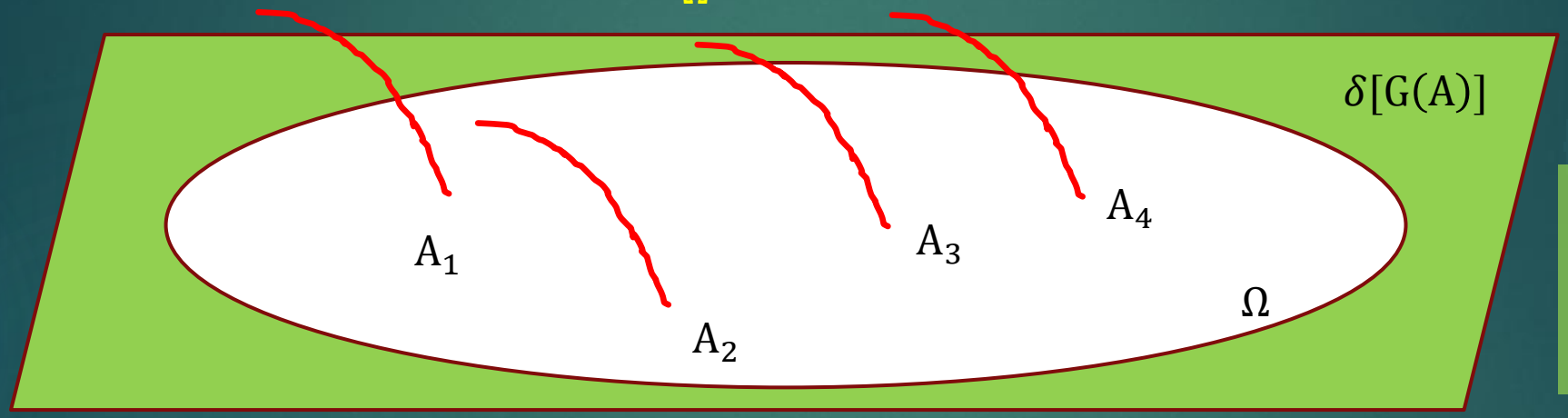
[V. N. Gribov **Nucl.Phys. B139 (1978) 1**]

Gribov problem and Gribov horizon

- Restrict path integral to some region of the functional space of A: Gribov's region Ω.

$$Z[J] = \int_{\Omega} [All\ fields] exp\left(-S[\Phi] + \int J \cdot \Phi\right)$$

[V. N. Gribov
Nucl.Phys. B139 (1978) 1]



Gauge fixing: $\delta[G(A)]$
Gribov region: Ω
Gribov horizon: $\partial\Omega$
Gribov parameter: γ

- Restriction to the Gribov region Ω → Constraint → Add a Lagrange multiplier to the action ($\gamma \neq 0$: Gribov parameter, to be fixed by physical info).
- Lagrange multiplier method: add constraint ("horizon function") to the action.

$$S_H = \gamma^4 \int d^4x \;\; g^2 f^{abc} A_\mu^a [M^{-1}]^{be} f^{dec} A_\mu^d$$

Faddeev-Popov operator:
$M^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c\right)$

The Refined Gribov-Zwanziger (RGZ) action: an effective theory for QCD

The Refined Gribov-Zwanziger action

Dudal *et al.*
[Phys.Rev. D78 (2008) 065047]

$$S_{RGZ} = S_{Yang-Mills} + S_{Gauge-fixing} + S_{Gribov\ Horizon} + S_{Condensates}$$

$$S_{Yang-Mills} + S_{Gauge-fixing} = \int d^d x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{c}^a M^{ab} c^b + b^a (\partial_\mu A^a_\mu)\right]$$

$$S_{Gribov\ Horizon} = \int d^d x \left[\bar{\varphi}^{ab}_\mu (M^{bc}) \varphi^{ca}_{\mu\nu} + g\gamma^2 f^{abc} A^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) - \bar{\omega}^{ab}_\mu (M^{bc}) \omega^{ca}_{\mu\nu}\right]$$

$$S_{Condensates} = \int d^d x \left[\mu^2 \bar{\varphi}^{ab}_\mu \varphi^{ca}_{\mu\nu} - \mu^2 \bar{\omega}^{ab}_\mu \omega^{ca}_{\mu\nu} + \frac{1}{2} m^2 A^a_\mu A^a_\mu\right]$$

The parameters γ, μ^2, and m^2 can be related to Λ_{QCD}.

ω, φ: auxiliary fields (added to write Gribov's action as a local QFT)

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$M^{ab} = -\partial_\mu (\delta^{ab} \partial_\mu - g f^{abc} A^c_\mu)$$

γ can be self-consistently calculated via a gap equation as $\gamma(g_s, \mu_{renorm})$.

Tree-level RGZ propagators

- Using standard QFT techniques, one finds tree-level propagators

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle = \frac{p^2+\mu^2}{p^4+(m^2+\mu^2)p^2+m^2\mu^2+2Ng^2\gamma^4}\delta^{ab}\mathcal{P}_{\mu\nu}$$

$$\langle A^a_\mu(p)\varphi^{bc}_\nu(-p)\rangle = \langle A^a_\mu(p)\bar{\varphi}^{bc}_\nu(-p)\rangle = \frac{g\gamma^2 f^{abc}}{p^4+p^2(m^2+\mu^2)+m^2\mu^2+2Ng^2\gamma^4}\mathcal{P}_{\mu\nu}$$

$$\langle \bar{c}^a(p)c^b(-p)\rangle = \frac{1}{p^2}\delta^{ab}$$

N.B.: Many propagators omitted. Only these will be used for the vertex calculation at 1-loop.

Tree-level RGZ gluon propagator

- RGZ tree-level propagator (Landau gauge):

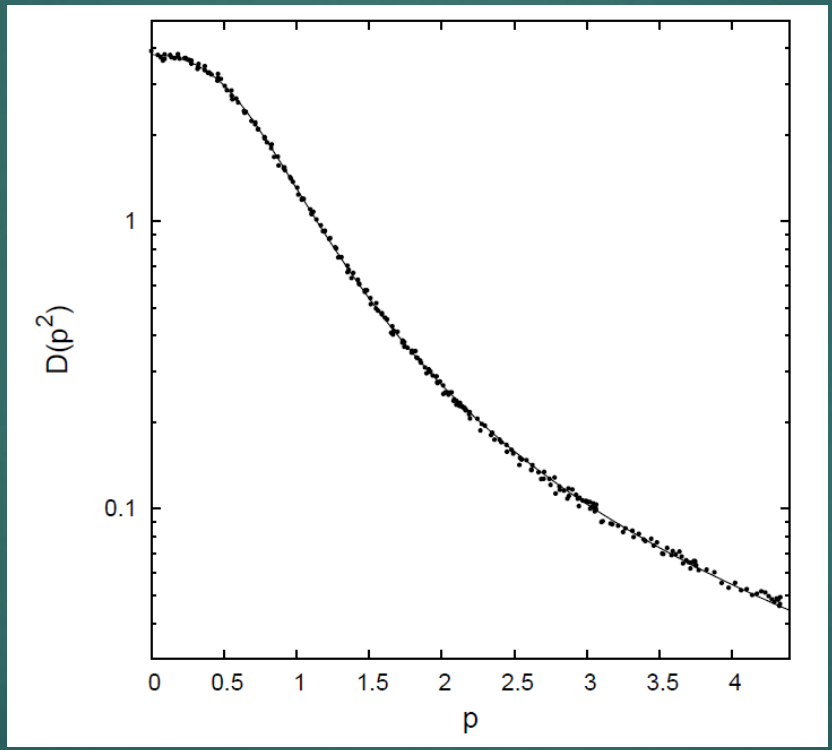
$$\langle A_\mu^a(p) A_\nu^b(-p) \rangle = \frac{(p^2+\mu^2)\delta^{ab} P_{\mu\nu}(p)}{(p^2+\mu^2)(p^2+m^2)+2g^2N\gamma^4} = \left(\frac{R_+}{p^2+m_+^2} + \frac{R_-}{p^2+m_-^2}\right)\delta^{ab} P_{\mu\nu}(p)$$

- Spectral function representation:

$$\rho(\tau) = 2\pi R_+ \delta(\tau - m_+^2) + 2\pi R_- \delta(\tau - m_-^2)$$

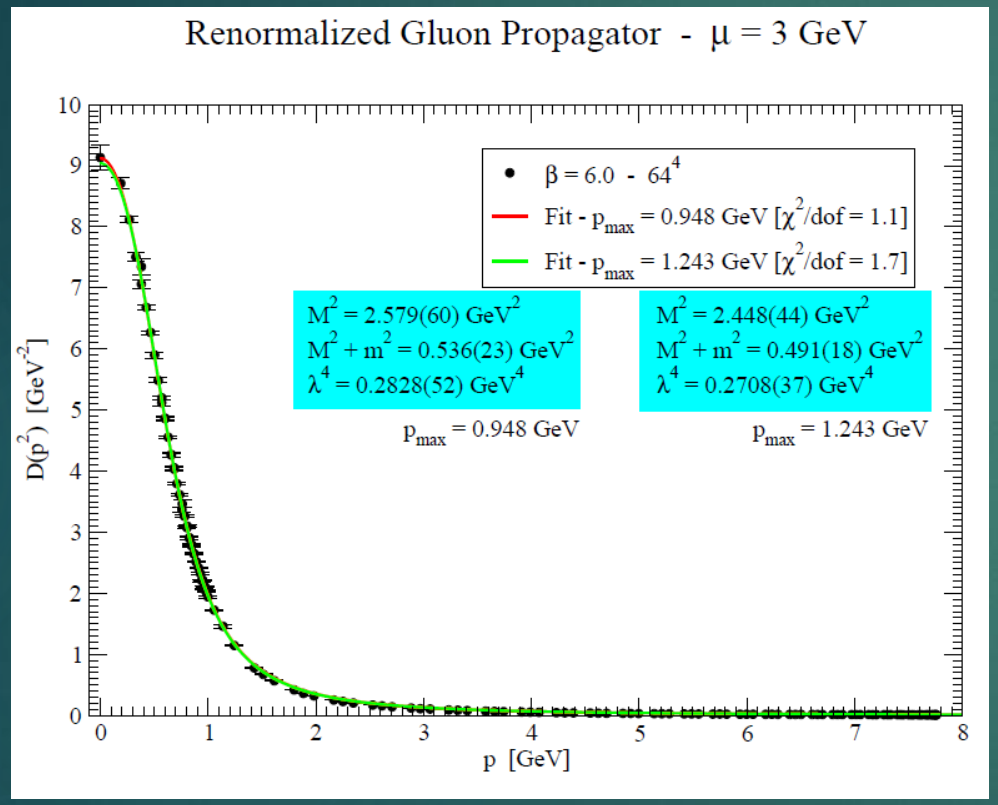
where $R_\pm$ is either negative or complex → positivity violation → gluons not present in the physical spectrum (confined).

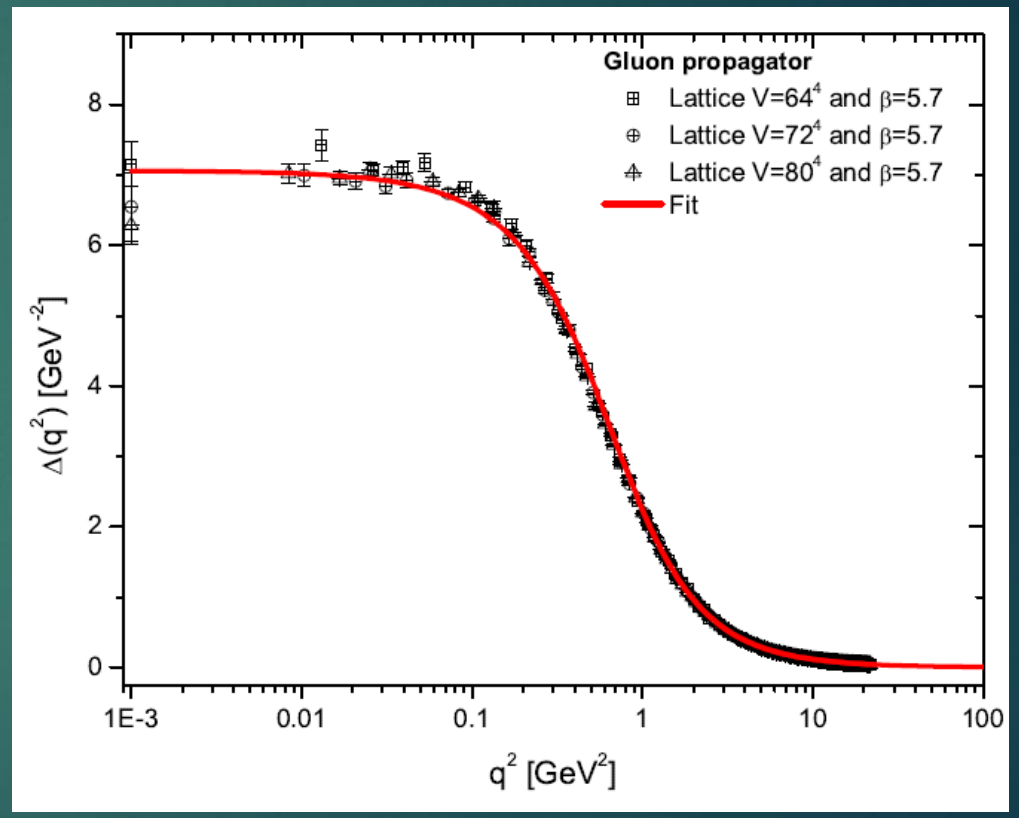
The RGZ action and nonperturbative gluon propagator: SU(2)



[Lattice vs. RGZ (fitted – almost invisible – solid line)
Cucchieri *et al.*, **Phys.Rev. D85 (2012) 094513**]

The RGZ action and nonperturbative gluon propagators: SU(3)





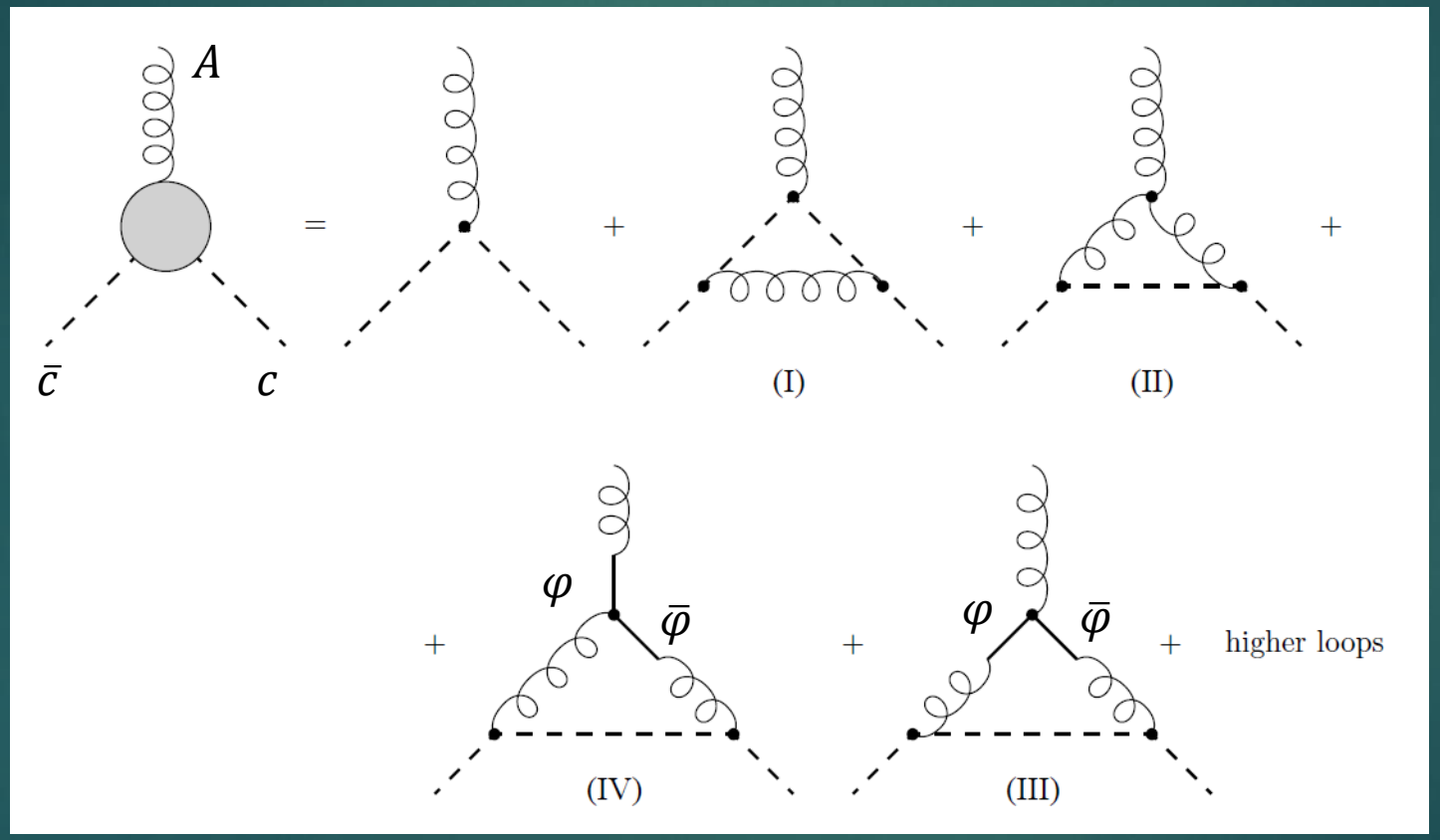
RGZ vs. Lattice
[Dudal et al.
Phys.Rev. D81 (2010) 074505]

See also
[arXiv:1803.02281]

Schwinger-Dyson Equation vs. Lattice
[Aguilar et al.
Phys.Rev. D87 (2013) no.11, 114020]

The one-loop ghost-gluon vertex in the RGZ model

The one-loop ghost-gluon vertex in the RGZ model



Vertices and propagators with the φ field correspond to contributions from the Gribov Horizon.

- These diagrams can be calculated analytically (ugly expressions).

The one-loop ghost-gluon vertex in the RGZ model

- Full tensor structure: complicated. E.g.: Ball-Chiu decomposition.

$$\begin{aligned}\Gamma_{\nu\mu}(p,k,r) = {} & \delta_{\mu\nu} a(r^2,k^2,p^2) \\ & - r_\nu p_\mu b(r^2,k^2,p^2) + k_\nu r_\mu c(r^2,k^2,p^2) \\ & + r_\nu k_\mu d(r^2,k^2,p^2) + k_\nu k_\mu e(r^2,k^2,p^2)\end{aligned}$$

Taken from
[M. Pelaez et al.
Phys.Rev. D88 (2013) 125003]

- Soft gluon limit: $r \to 0$.

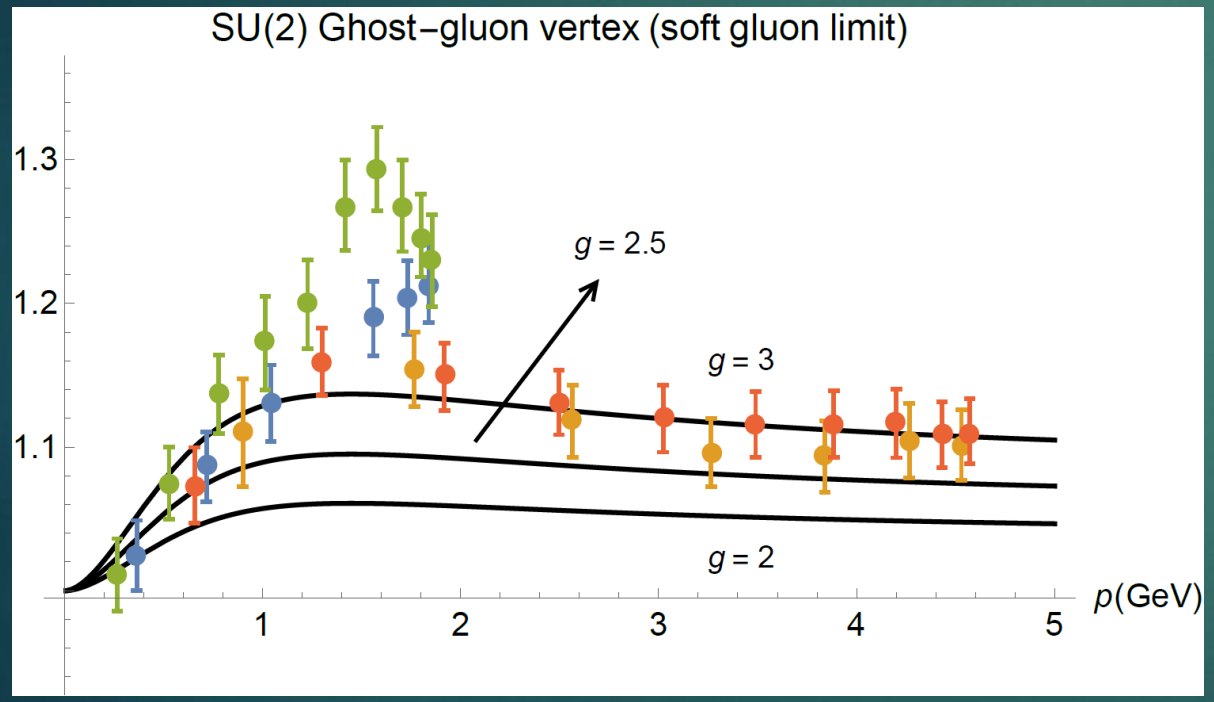
$$\begin{aligned}\Gamma_{\nu\mu}(p,k,r) = {} & \delta_{\mu\nu} a(r^2,k^2,p^2) \\ & \cancel{- r_\nu p_\mu b(r^2,k^2,p^2)} + \cancel{k_\nu r_\mu c(r^2,k^2,p^2)} \\ & \cancel{+ r_\nu k_\mu d(r^2,k^2,p^2)} + k_\nu k_\mu e(r^2,k^2,p^2)\end{aligned}$$

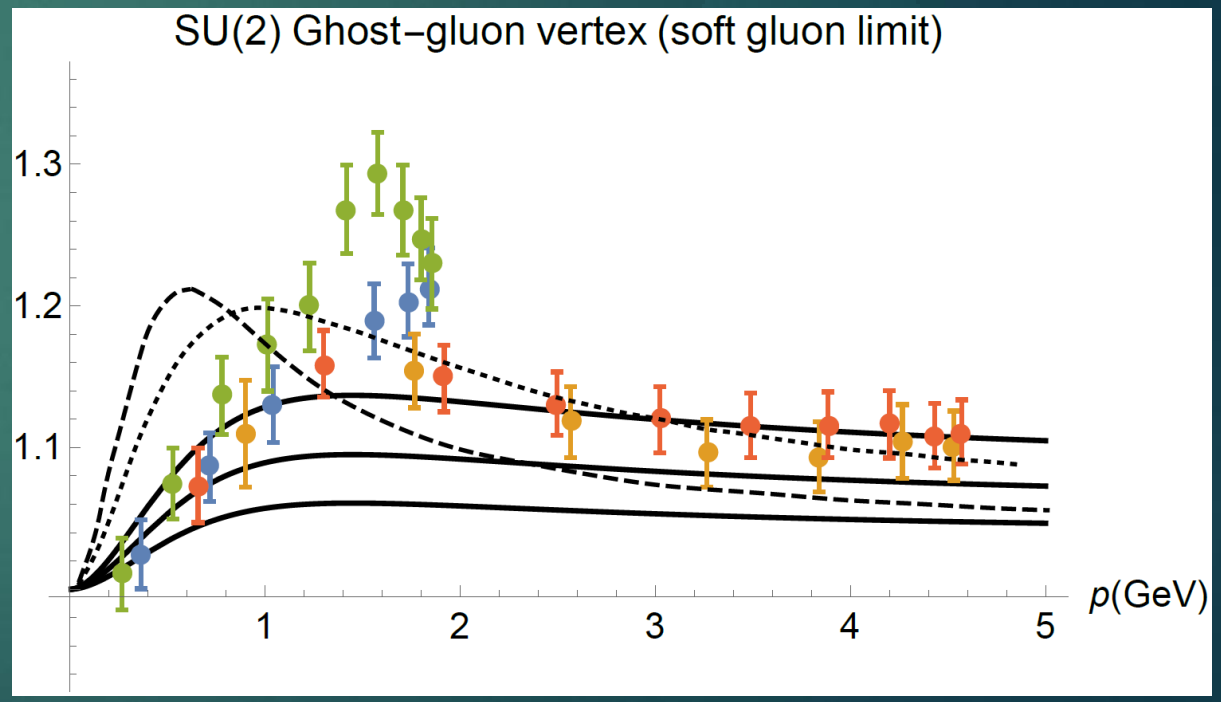
Soft gluon:
$r = 0 \;\Rightarrow\; p = -k$

The one-loop ghost-gluon vertex in the RGZ model: SU(2)

[Phys.Rev. D97 (2018) 3, 034020]

- (Tree-level) RGZ parameters fitted from lattice gluon propagator. [A. Cucchieri *et al.*, **Phys. Rev. D 85, 094513 (2012)**]



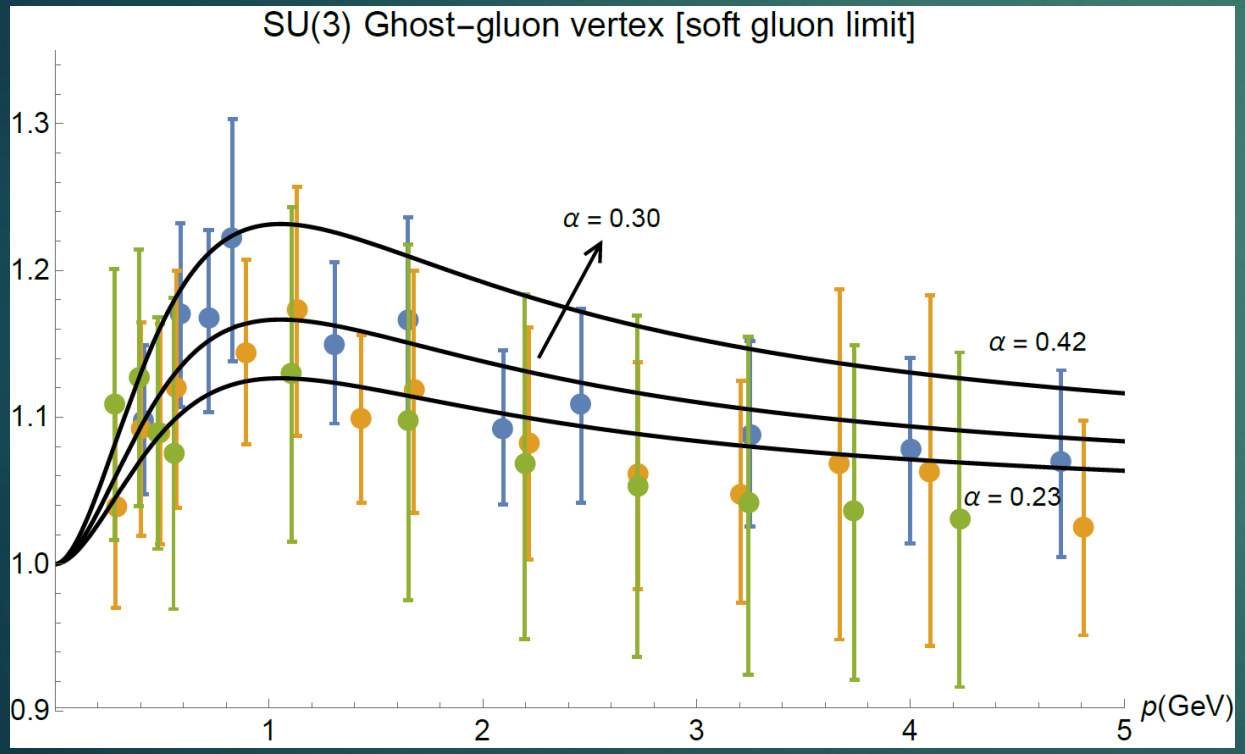


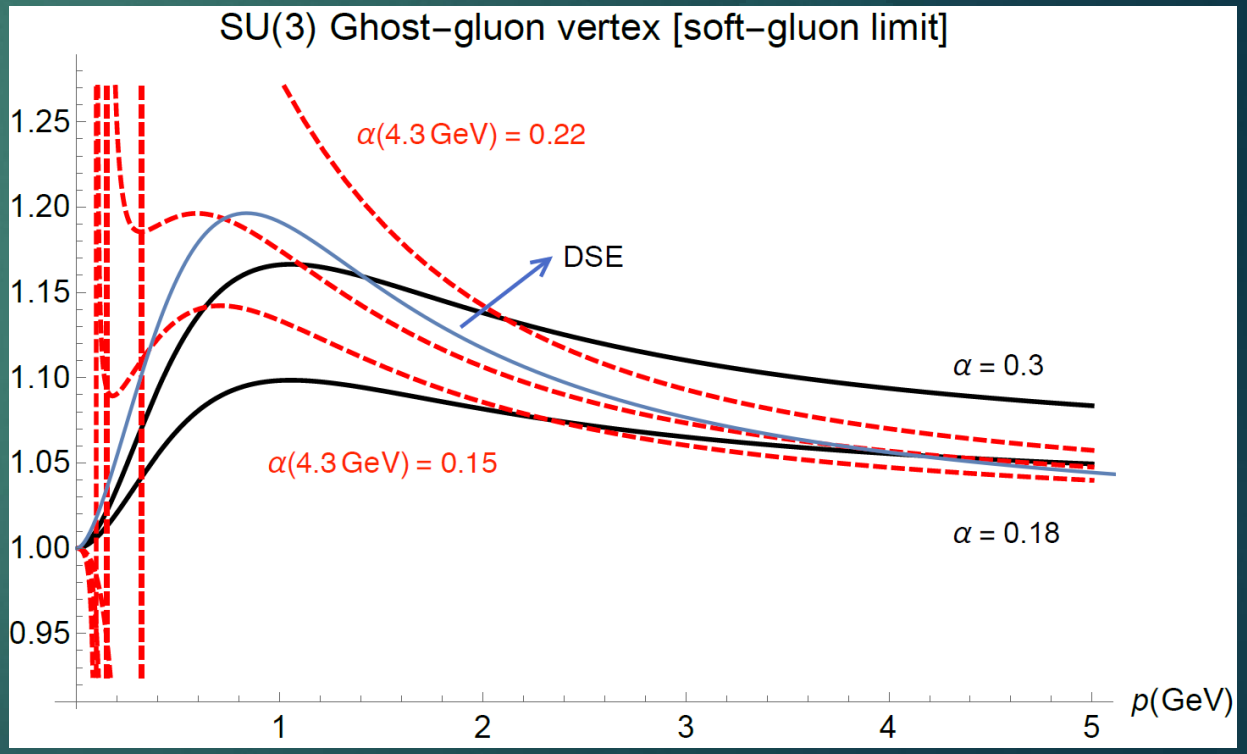
Lattice data points from [A. Cucchieri *et al.*, **Phys. Rev. D 77, 094510 (2008)**]

The one-loop ghost-gluon vertex in the RGZ model: SU(3)

[Phys.Rev. D97 (2018) 3, 034020]

- (Tree-level) RGZ parameters fitted from lattice gluon propagator. [O. Oliveira and P. J. Silva, **Phys. Rev. D 86, 114513 (2012)**]





Lattice data points from [E. M. Ilgenfritz *et al.*, **Braz. J. Phys. 37, 193 (2007)**]
DSE points from [A. C. Aguilar *et al.*, **Phys. Rev. D 87, 114020 (2013)**]

Summary and plans for the future

- Covariantly quantized gauge theories have to deal with the Gribov problem as the infrared is approached.
- The RGZ effective theory displays a nice agreement with lattice gluon propagator. (Perturbative expansion around a nontrivial vacuum?)
- Using lattice-fitted parameters for the tree-level RGZ propagator (and only that!), we calculated the ghost-gluon vertex at the soft gluon limit $(k_{gluon} \to 0)$: qualitative agreement with other methods.

To come next...

- Ghost-gluon vertex for any momentum configuration (complicated tensor structure: hard work, but feasible).
- Other YM vertices: AAA and $AAAA$.
- Quark-gluon vertex.
- Explore other covariant gauges (LCG) with the BRST invariant version of RGZ.

A UERJ (R)ESISTE !

UERJ group at Hadron Physics '18:
Carlos Mena, Apóllo Silva, Bruno Mintz, Rodrigo Terin and Leonardo Moreira.
(+ Duive van Egmond and Ozório Neto not in the photo)