Thermodynamic consistency of quasiparticle equation of state at finite baryon density

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Abstract

- Quasiparticle model description
- Motivation
- Formalism and thermodynamic consistency
- Formal solution and Numerical results
- Concluding remarks and outlook

A brief review of quasiparticle description

- QCD predicts a phase transition between the quark-gluon dominated high energy region and the hadronic state in the low energy region
- The two states are characterized by a dramatic difference in the number of degrees of freedom
- A phenomenological approach for this phase transition which agree with the perturbative results at high energy and with the lattice QCD data at relatively low energies

A brief review of quasiparticle picture

- Appropriate degrees of freedom
- View the system as composed of non-interacting quasiparticle in background mean field
- The interactions among basic constituents is incorporated through medium dependent effective mass

- Quasiparticles of gluon plasma with temperature dependent mass (Golovizinin and Satz 1993, Peshier et al 1994)

A brief review of quasiparticle picture

- Systems of pure gauge theories for gluon plasma → inclusion of light dynamical quarks

- The temperature dependence of particle mass are usually extracted from
- Lattice QCD calculations (Karsch et al 1988, Peshier et al 1996, etc) (for gluon plasma)
- Thermal mass (Weldon 1982, Pisarski 1989, Frenkel and Taylor 1990, etc) via the pole of effective propagator
- Debye screen mass through the behavior at small momentum (Klimov 1982, Pisarski 1989, Thoma 1995, etc)

A brief review of quasiparticle picture

- Various discussions on thermodynamic/statistical consistency following Gorenstein and Yang (Gorenstein and Yang 1995) and inclusively to the case of finite chemical potential

- Peshier et al 2000, 2002
- Biro et al 2003
- Gardim and Steffens 2003, 2007
- Bannur 2007, 2008, 2012
- Ying and Su 2007, 2008, 2010
- etc

Thermodynamic consistency for quasiparticle model
(Gorenstein and Yang 1995)

$$p_{\mathrm{id}}(T,m) = -T\,\frac{d}{2\pi^2}\int_0^\infty k^2 dk\,\ln\left[1 - \exp(-\omega/T)\right]$$

$$\epsilon_{\mathrm{id}}(T,m) = \frac{d}{2\pi^2}\int_0^\infty k^2 dk\,\frac{\omega}{\exp(\omega/T) - 1}$$

$$\epsilon(T) = T\,\frac{dp(T)}{dT} - p(T)$$

- Energy density obtained from ensemble average may not be the same as that from thermodynamic relation

Thermodynamic consistency for quasiparticle model
(Gorenstein and Yang 1995)

- The solution is to visualize mass as an intermediate field which satisfies

$$\left(\frac{\partial p}{\partial c_1}\right)_{T,\mu,c_2,\ldots} = 0\ , \quad \left(\frac{\partial p}{\partial c_2}\right)_{T,\mu,c_1,\ldots} = 0\ , \ldots$$

Thermodynamic consistency for quasiparticle model
(Gorenstein and Yang 1995)

- The previous condition is achieved by introducing a specific temperature dependent bag constant

$$\left(\frac{\partial p}{\partial c_1}\right)_{T,\mu,c_2,\ldots} = 0\ , \quad \left(\frac{\partial p}{\partial c_2}\right)_{T,\mu,c_1,\ldots} = 0\ , \ldots$$

$$p(T,\mu,c_1,c_2,\ldots) = \mp T\,\frac{d}{2\pi^2}\int_0^\infty k^2dk \times \ln\left[1 \mp \exp\left(-\frac{(\omega^* - \mu)}{T}\right)\right] - B^*$$

$$\epsilon(T,\mu,c_1,c_2,\ldots) = \frac{d}{2\pi^2}\int_0^\infty \frac{k^2dk\ \omega^*}{\exp[(\omega^*-\mu)/T] \mp 1} + B^*$$

$$\frac{dB^*}{dm} = -m\frac{d}{2\pi^2}\int_0^\infty \frac{k^2dk}{\omega^*(k,T)}\,\frac{1}{\exp[\omega^*(k,T)/T]-1}$$

The study is extended to finite baryon density
(Peshier et al 2000, Bluhm 2005, 2007)

$$\frac{\partial B}{\partial \Pi_j^*} = \frac{\partial p_j(T, \mu_j; m_j^2)}{\partial m_j^2}$$

$$p(T, \mu; m_{0j}^2) = \sum_i p_i(T, \mu_i(\mu); m_i^2) - B(\Pi_j^*)$$

$$m_i^2 = m_{0i}^2 + \Pi_i^* ,$$

- The additional contribution due to the temperature/chemical potential dependence of particle mass should be canceled out by those of the bag constant
- The temperature/chemical potential dependence of the bag constant is through that of the screen mass

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$$p(T, \mu; m_{0j}^2) = \sum_i p_i(T, \mu_i(\mu); m_i^2) - B(\Pi_j^*)$$

$$\Pi_q^* = 2\,\omega_{q0}\,(m_0 + \omega_{q0})\,, \quad \omega_{q0}^2 = \frac{N_c^2 - 1}{16 N_c}\left[T^2 + \frac{\mu_q^2}{\pi^2}\right] G^2$$

$$\Pi_g^* = \frac{1}{6}\left[\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2\pi^2}\sum_q \mu_q^2\right] G^2 .$$

- The screen mass depends on the temperature/chemical potential as well as the coupling, is given by the asymptotic values of the gauge independent hard-thermal/density-loop (HTL) self-energies

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$$s_i = \left.\frac{\partial p_i(T, \mu_i; m_i^2)}{\partial T}\right|_{m_i^2}, \quad n_i = \left.\frac{\partial p_i(T, \mu_i; m_i^2)}{\partial \mu_i}\right|_{m_i^2}$$

- On the other hand, thermodynamic quantities, such as entropy and particle number can be derived from the grand canonical partition function for a grand canonical ensemble

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$$\sum_i \left[\frac{\partial n_i}{\partial m_i^2} \frac{\partial \Pi_i^*}{\partial T} - \frac{\partial s_i}{\partial m_i^2} \frac{\partial \Pi_i^*}{\partial \mu} \right] = 0$$

$$\Pi_q^* = 2\,\omega_{q0}\,(m_0 + \omega_{q0})\,, \quad \omega_{q0}^2 = \frac{N_c^2 - 1}{16 N_c} \left[T^2 + \frac{\mu_q^2}{\pi^2} \right] G^2$$

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- Therefore, Maxwell relation implies

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$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

- Therefore, one obtains a partial differential equation for the coupling constant

The study is extended to finite baryon density
(Peshier et al 2000, Bluhm 2005, 2007)

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$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

$$G^2(T, \mu = 0) = \frac{48\pi^2}{(11N_c - 2\,N_f)\ln\left(\frac{T + T_s}{T_c/\lambda}\right)^2}$$

- If one uses the running coupling in the asymptotic limit of large temperature and vanishing chemical potential as the boundary condition, one may obtain the value at finite chemical potential

The study is extended to finite baryon density
(Peshier et al 2000, Bluhm 2005, 2007)

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$$\sum_i \left[\frac{\partial n_i}{\partial m_i^2} \frac{\partial \Pi_i^*}{\partial T} - \frac{\partial s_i}{\partial m_i^2} \frac{\partial \Pi_i^*}{\partial \mu} \right] = 0$$

$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

$$G^2(T, \mu = 0) = \frac{48\pi^2}{(11 N_c - 2\, N_f) \ln \left(\frac{T + T_s}{T_c/\lambda} \right)^2}$$

- The arguments are simply based on the quasiparticle ansatz and thermodynamic consistency

The study is extended to finite baryon density
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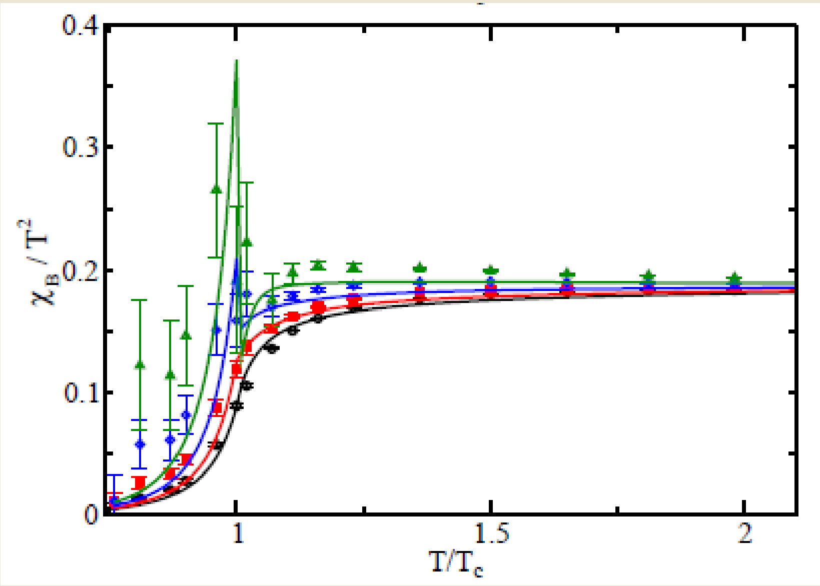

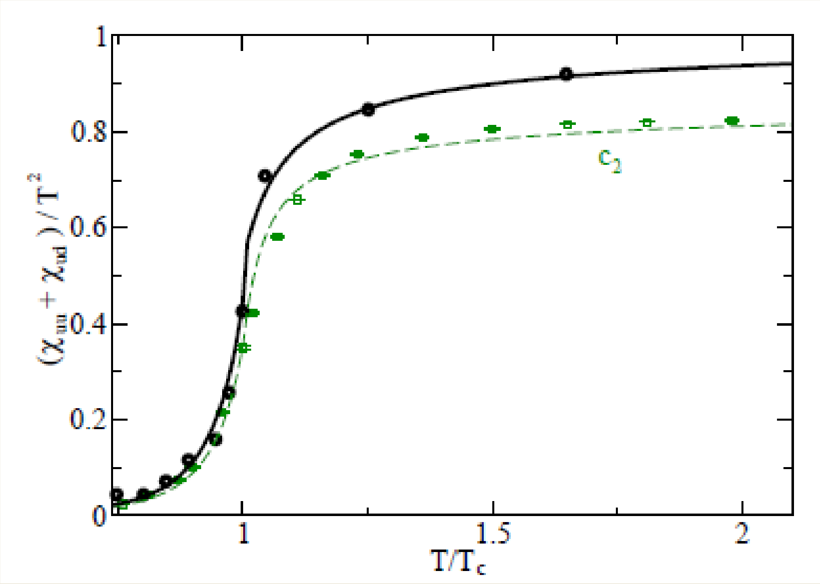

- The numerical results, inclusively those of the quark number susceptibility, are perfectly compared to those obtained by lattice QCD calculations

Thermodynamic consistency in quasiparticle model in finite chemical potential

- The additional contribution due to the temperature/chemical potential dependence of particle mass should be canceled out by those of the bag constant

$$\frac{dB}{dm} = \left.\frac{\partial p(T,\mu,m)}{\partial m}\right|_{T,\mu}$$

Thermodynamic consistency in quasiparticle model in finite chemical potential

- The condition of thermodynamic consistency is implemented assuming bag constant is a function of screen mass

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$$\frac{dB}{dm} = \left.\frac{\partial p(T,\mu,m)}{\partial m}\right|_{T,\mu}$$

$$-\frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)} \frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T]\mp 1} m$$

- If one writes down the r.h.s. of the equation, one finds the explicit form of temperature and chemical potential dependence of the expression

Thermodynamic consistency in quasiparticle model in finite chemical potential

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$$\frac{dB}{dm} = \left.\frac{\partial p(T,\mu,m)}{\partial m}\right|_{T,\mu}$$

$$-\frac{g}{2\pi^2}\int_0^\infty \frac{k^2dk}{\omega^*(k,T,\mu)}\frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T]\mp 1}m$$

- The explicit form of temperature and chemical potential dependence of the r.h.s. implies that either bag constant is also an explicit function of temperature/chemical potential, or one cannot freely choose the form of particle mass as a function of those variables

Thermodynamic consistency in quasiparticle model in finite chemical potential

- The condition of thermodynamic consistency is implemented assuming bag constant is a function of screen mass

$$\frac{dB}{dm} = \left.\frac{\partial p(T,\mu,m)}{\partial m}\right|_{T,\mu}$$

$$-\frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)} \frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T] \mp 1} m$$

- The former is not possible, because it implies extra terms for thermodynamical quantities which breaks the thermodynamic consistency, also it has been proven that it is not the case (Biro et al 2003)

Thermodynamic consistency in quasiparticle model in finite chemical potential

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$$\frac{dB}{dm} = \left.\frac{\partial p(T,\mu,m)}{\partial m}\right|_{T,\mu}$$

$$-\frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)} \frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T] \mp 1} m$$

- Therefore, we conclude that there exist an extra constrain for the form of screen mass, again, owing to the thermodynamic consistency

Thermodynamic consistency in quasiparticle model in finite chemical potential

$$\frac{\partial B}{\partial T} = \left.\frac{\partial B}{\partial m}\right|_{T,\mu} \frac{\partial m}{\partial T} + \left.\frac{\partial B}{\partial T}\right|_{m,\mu} = -\frac{g}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)} \frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T] \mp 1} m \frac{\partial m}{\partial T}.$$

$$\frac{\partial B}{\partial \mu} = \left.\frac{\partial B}{\partial m}\right|_{T,\mu} \frac{\partial m}{\partial \mu} + \left.\frac{\partial B}{\partial \mu}\right|_{m,T} = -\frac{g}{2\pi^2} \int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)} \frac{1}{\exp[(\omega^*(k,T,\mu)-\mu)/T] \mp 1} m \frac{\partial m}{\partial \mu}.$$

$$\frac{\partial m}{\partial \mu} = \frac{T}{(\omega^*(k,T,\mu)-\mu)} \frac{\partial m}{\partial T}.$$

- Gorenstein and Yang's criterion + Maxwell relation for the bag constant leads to a partial differential equation which can be solved by the method of characteristics

Thermodynamic consistency in quasiparticle model in finite chemical potential

$$\frac{\partial m}{\partial \mu} = \frac{T}{(\omega^*(k,T,\mu)-\mu)} \frac{\partial m}{\partial T}.$$

$$w = \omega^* - \mu$$

$$w\frac{\partial w}{\partial \mu} - T\frac{\partial w}{\partial T} + w = 0,$$

$$\frac{d\mu}{w} = \frac{dT}{-T} = \frac{dw}{-w}.$$

$$d(\mu + w) = 0,$$

$$d\left[\ln\left(\frac{w}{T}\right)\right] = d\left(\frac{w}{T}\right) = 0.$$

$$F\left(\frac{w}{T}, (w+\mu)\right) = 0.$$

- Method of characteristics

Thermodynamic consistency in quasiparticle model in finite chemical potential

$$\frac{\partial m}{\partial \mu} = \frac{T}{(\omega^*(k,T,\mu)-\mu)}\frac{\partial m}{\partial T}.$$

$$w = \omega^* - \mu$$

$$w\frac{\partial w}{\partial \mu} - T\frac{\partial w}{\partial T} + w = 0,$$

$$\frac{d\mu}{w} = \frac{dT}{-T} = \frac{dw}{-w}.$$

$$d(\mu + w) = 0,$$

$$d\left[\ln\left(\frac{w}{T}\right)\right] = d\left(\frac{w}{T}\right) = 0.$$

$$F\left(\frac{w}{T}, (w+\mu)\right) = 0.$$

- (Unfortunately) the above analytic solution only serves for the case where one ignores antiparticles

Thermodynamic consistency in quasiparticle model in finite chemical potential

$$\left\{\frac{\exp[(\omega^*-\mu)/T]T}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}-\text{c.t.}\right\}\frac{\partial m}{\partial T}=\left\{\frac{\exp[(\omega^*-\mu)/T](\omega^*-\mu)}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}+\text{c.t.}\right\}\frac{\partial m}{\partial \mu}.$$

$$\left.\frac{\partial \mu}{\partial T}\right|_{\omega^*}=\frac{a(\mu,T,\omega^*)}{b(\mu,T,\omega^*)},$$

$$a(\mu,T,\omega^*)=\left\{\frac{\exp[(\omega^*-\mu)/T](\omega^*-\mu)}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}+\text{c.t.}\right\},$$

$$b(\mu,T,\omega^*)=-\left\{\frac{\exp[(\omega^*-\mu)/T]T}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}-\text{c.t.}\right\}.$$

- When considering antiparticles, the solution is a surface in temperature-chemical potential coordinates which consists of characteristic curves for given omega star.

Thermodynamic consistency in quasiparticle model in finite chemical potential

$$\left\{\frac{\exp[(\omega^*-\mu)/T]T}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}-\text{c.t.}\right\}\frac{\partial m}{\partial T}=\left\{\frac{\exp[(\omega^*-\mu)/T](\omega^*-\mu)}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}+\text{c.t.}\right\}\frac{\partial m}{\partial \mu}.$$

$$\left.\frac{\partial \mu}{\partial T}\right|_{\omega^*}=\frac{a(\mu,T,\omega^*)}{b(\mu,T,\omega^*)},$$

$$a(\mu,T,\omega^*)=\left\{\frac{\exp[(\omega^*-\mu)/T](\omega^*-\mu)}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}+\text{c.t.}\right\},$$

$$b(\mu,T,\omega^*)=-\left\{\frac{\exp[(\omega^*-\mu)/T]T}{(\exp[(\omega^*-\mu)/T]\mp 1)^2}-\text{c.t.}\right\}.$$

- When considering antiparticles, the solution is a surface in temperature-chemical potential coordinates which consists of characteristic curves for given omega star.
- While the boundary condition is given by lattice data at vanishing chemical potential

The quasiparticle model employed in the present study

- Present work: Energy density and particle number are expressed in terms of ensemble average which is consistent with those obtained by grand canonical partition function (Gorenstein et al 1995, 2010)

$$Q_G = \langle \exp[-\alpha \hat{N} - \beta \hat{H}_{\text{eff}}] \rangle,$$

$$\hat{H}_{\text{eff}} = \hat{H}_{\text{id}} + E_0 + E_1.$$

$$\hat{H}_{\text{id}} = \sum_j \sum_{\mathbf{k}} \omega(\mathbf{k}) a^{\dagger}_{\mathbf{k},j} a_{\mathbf{k},j},$$

- There is additional term E1 which is linear in temperature

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$$\varepsilon = \frac{\langle E \rangle}{V} = -\frac{1}{V}\frac{\partial \ln Q_G}{\partial \beta} = \epsilon_{\text{id}} + \frac{E_0}{V} + \frac{E_1}{V} + \frac{1}{V}\langle \beta \frac{\partial E_1}{\partial \beta} \rangle = \epsilon_{id} + B.$$

$$\epsilon_{\text{id}} = \frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk \omega^*(k,T,\mu)}{\exp[(\omega^*(k,T,\mu)-\mu)/T] \mp 1} + \text{c.t.},$$

$$\lim_{V\to\infty} \frac{E_0}{V} = B$$

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$$p = \frac{1}{\beta}\frac{\partial \ln Q_G}{\partial V} = \frac{1}{\beta}\frac{\ln Q_G}{V} = p_{\rm id} - B - \frac{E_1}{V}.$$

- There is one more free parameter in the expression of pressure

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$$\varepsilon = \frac{\langle E \rangle}{V} = -\frac{1}{V}\frac{\partial \ln Q_G}{\partial \beta} = \epsilon_{\mathrm{id}} + \frac{E_0}{V} + \frac{E_1}{V} + \frac{1}{V}\langle \beta \frac{\partial E_1}{\partial \beta} \rangle = \epsilon_{id} + B.$$

$$p = \frac{1}{\beta}\frac{\partial \ln Q_G}{\partial V} = \frac{1}{\beta}\frac{\ln Q_G}{V} = p_{\mathrm{id}} - B - \frac{E_1}{V}.$$

$$n = \frac{\langle N \rangle}{V} = -\frac{1}{V}\frac{\partial \ln Q_G}{\partial \alpha} = n_{\mathrm{id}},$$

The quasiparticle model employed in the present study

- We note that the derived quantities must satisfy the "first law of thermodynamics", therefore any other quantity can be unambiguously derived from thermodynamic potential

The quasiparticle model employed in the present study

- We note that the derived quantities must satisfy the "first law of thermodynamics", therefore any other quantity can be unambiguously derived from thermodynamic potential
- In fact, one can readily verify that

$$dq = -\langle N\rangle d\alpha - \langle E\rangle d\beta - \beta p dV.$$

- By matching

$$\beta = \frac{1}{k_B T},$$
$$\alpha = -\frac{\mu}{k_B T},$$
$$q + \alpha N + \beta E = \frac{S}{k_B}.$$

- One has

$$d\langle E\rangle = TdS - pdV + \mu d\langle N\rangle.$$

The quasiparticle model employed in the present study

- Indeed, one can readily verify that the quantities obtained by ensemble average indeed furnish the “first law of thermodynamics”, and subsequently one can derive the thermodynamic potential and thus any other quantity without any ambiguity.

Numerical Results

- Thermodynamical consistency check: we compare our results with an approach which assumes the form of screen mass inspired by HTL results

$$m_g^2 = \frac{3}{2}\omega_p^2$$

$$m_q^2 = (m_{q0} + m_f)^2 + m_f^2$$

$$\omega_p^2 = a_g^2 g^2 \frac{n_g}{T} + \sum_q a_q^2 g^2 \frac{n_q}{T},$$

$$m_f^2 = b_q^2 g^2 \frac{n_q}{T},$$

$$\alpha_s(T) \equiv \frac{g^2}{4\pi} = \frac{6\pi}{(33-2n_f)\ln(T/\Lambda_T)}\left(1 - \frac{3(153-19n_f)}{(33-2n_f)^2}\frac{\ln(2\ln(T/\Lambda_T))}{\ln(T/\Lambda_T)}\right).$$

Numerical Results

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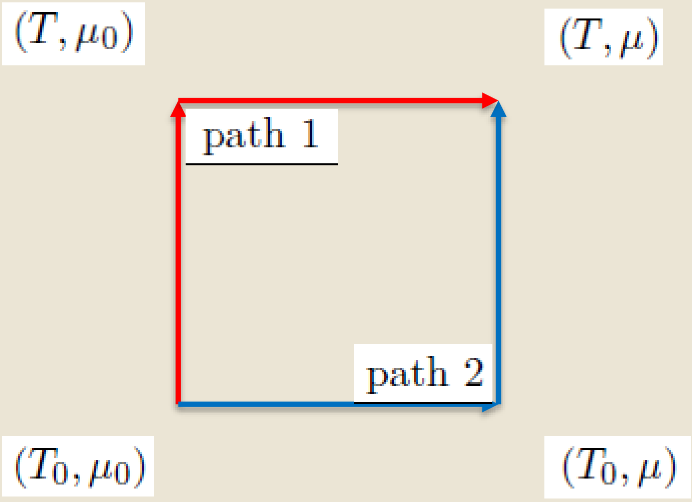
$$\frac{\partial B}{\partial T} = \frac{\partial B}{\partial m}\bigg|_{T,\mu}\frac{\partial m}{\partial T} + \frac{\partial B}{\partial T}\bigg|_{m,\mu} = -\frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)}\frac{1}{\exp[(\omega^*(k,T,\mu) - \mu)/T] \mp 1} m\frac{\partial m}{\partial T}.$$

$$\frac{\partial B}{\partial \mu} = \frac{\partial B}{\partial m}\bigg|_{T,\mu}\frac{\partial m}{\partial \mu} + \frac{\partial B}{\partial \mu}\bigg|_{m,T} = -\frac{g}{2\pi^2}\int_0^\infty \frac{k^2 dk}{\omega^*(k,T,\mu)}\frac{1}{\exp[(\omega^*(k,T,\mu) - \mu)/T] \mp 1} m\frac{\partial m}{\partial \mu}.$$

Numerical Results

- Thermodynamical consistency check: we compare our results with an approach which assumes the form of screen mass inspired by HTL results
- We compare the calculated bag constant by using different paths in the temperature-chemical potential plane



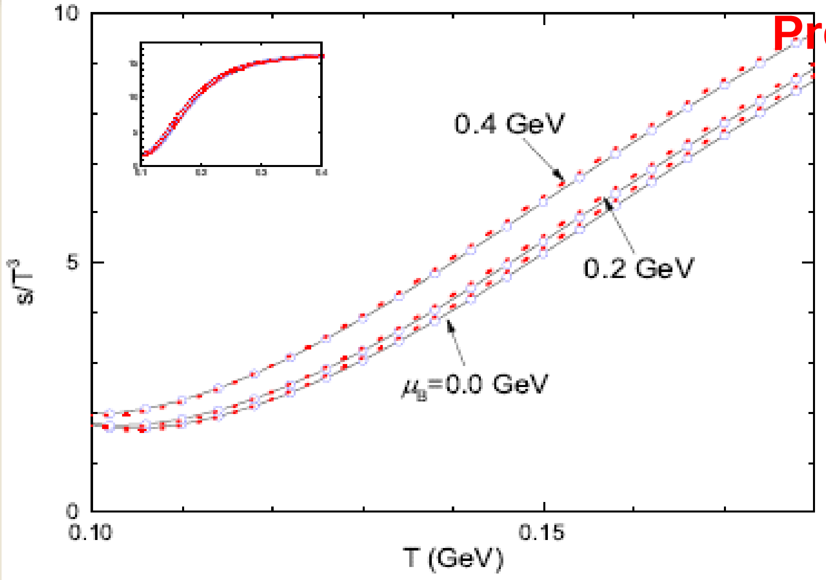
Numerical Results

- We compare the calculated bag constant with a more straightforward calculation which assumes the form of screen mass inspired by HTL results

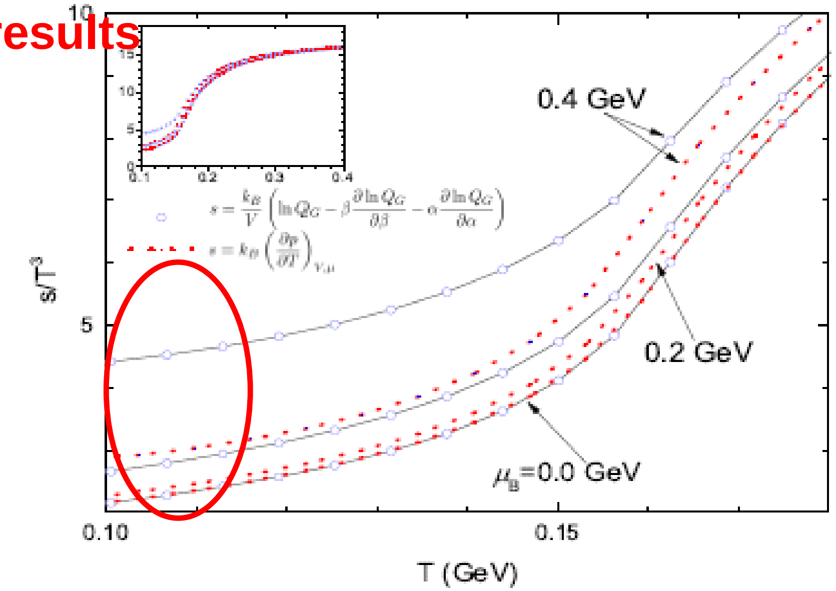
T	μ	the present model		discrepancy	preassumed form for $m(T,\mu)$		discrepancy
		path 1	path 2		path 1	path 2	
0.1GeV	0.1GeV	-6.63×10^{-3}	-6.63×10^{-3}	0.043%	1.52×10^{-2}	1.32×10^{-2}	7.11%
0.15GeV	0.1GeV	-3.34×10^{-3}	-3.34×10^{-3}	0.047%	9.27×10^{-3}	8.78×10^{-3}	2.72%
0.20GeV	0.1GeV	2.49×10^{-2}	2.49×10^{-2}	0.010%	1.04×10^{-1}	1.05×10^{-1}	0.43%
0.1GeV	0.2GeV	-6.84×10^{-3}	-6.83×10^{-3}	0.072%	2.15×10^{-2}	1.37×10^{-2}	22.2%
0.15GeV	0.2GeV	-2.65×10^{-3}	-2.65×10^{-3}	0.062%	1.52×10^{-2}	1.34×10^{-2}	6.35%
0.20GeV	0.2GeV	2.65×10^{-2}	2.65×10^{-2}	0.002%	1.05×10^{-1}	1.08×10^{-1}	1.56%
0.1GeV	0.3GeV	-7.09×10^{-3}	-7.12×10^{-3}	0.240%	3.11×10^{-2}	1.47×10^{-2}	35.9%
0.15GeV	0.3GeV	-1.43×10^{-3}	-1.44×10^{-3}	0.196%	2.43×10^{-2}	2.07×10^{-2}	8.00%
0.20GeV	0.3GeV	2.92×10^{-2}	2.92×10^{-2}	0.066%	1.07×10^{-1}	1.13×10^{-1}	2.97%

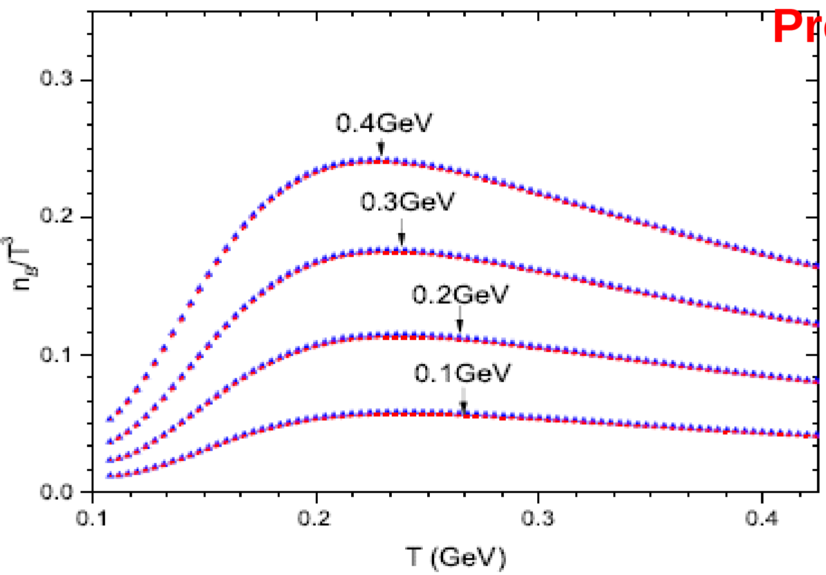
Preliminary results

Numerical Results



Preliminary results





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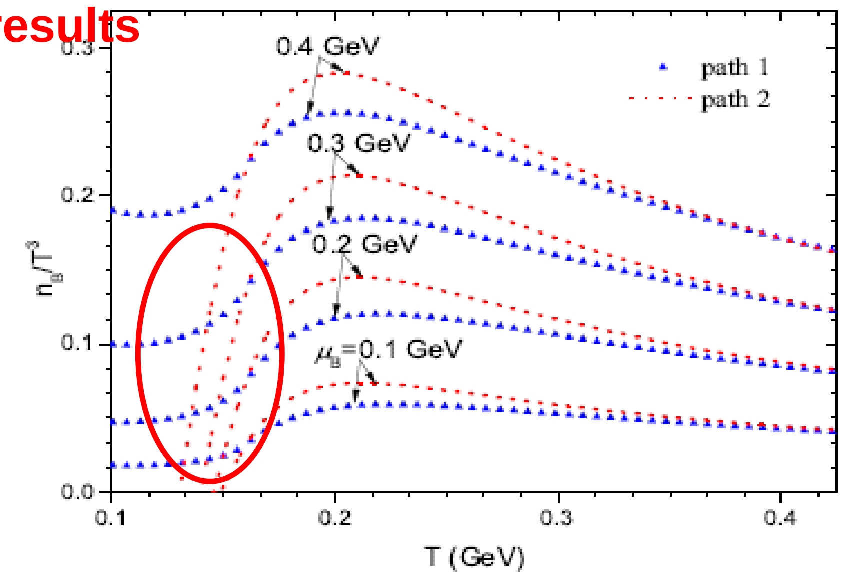

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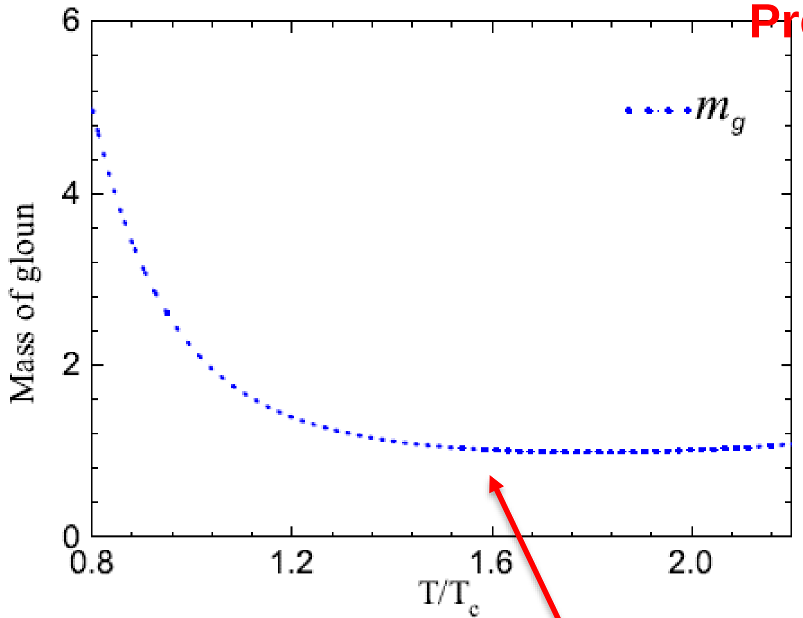
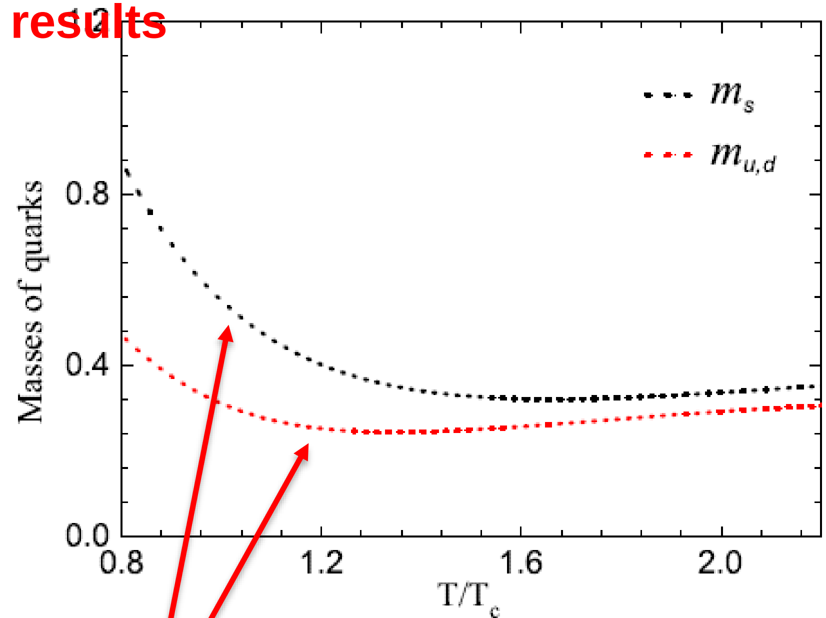
$$\chi_2^{ab} = \frac{T}{V}\frac{1}{T^2}\left.\frac{\partial^2 \ln Q_G(T,\mu_u,\mu_d)}{\partial\mu_a\partial\mu_b}\right|_{\mu_a=\mu_b=0}$$

- Then use the temperature dependent gluon screen mass and E1 to reproduce pressure and energy density at zero baryon density

Preliminary results





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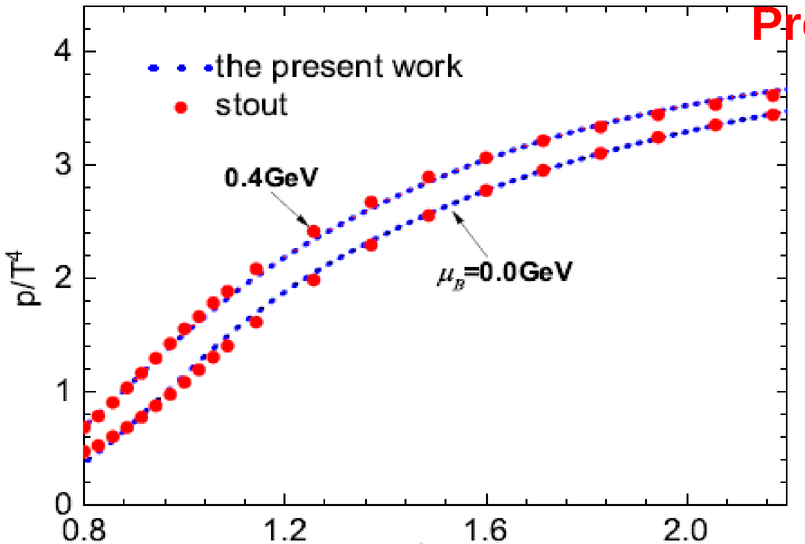

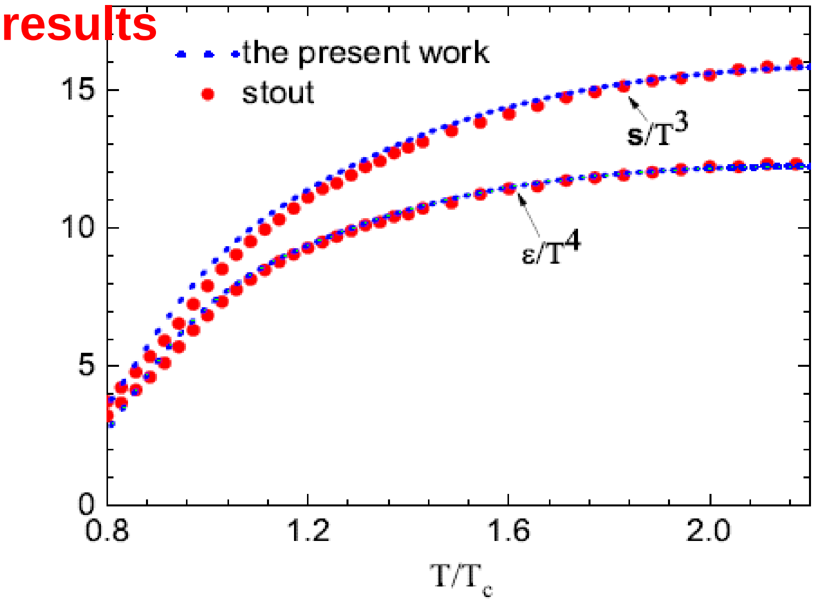

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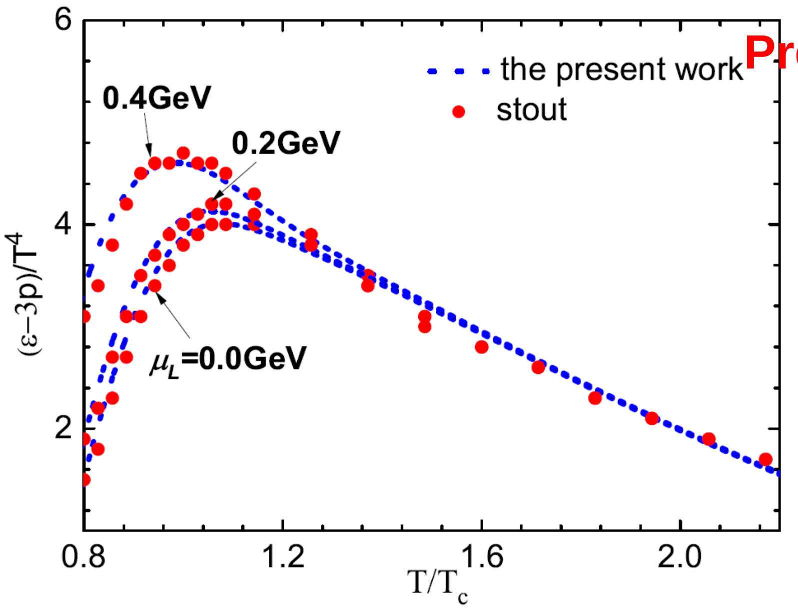
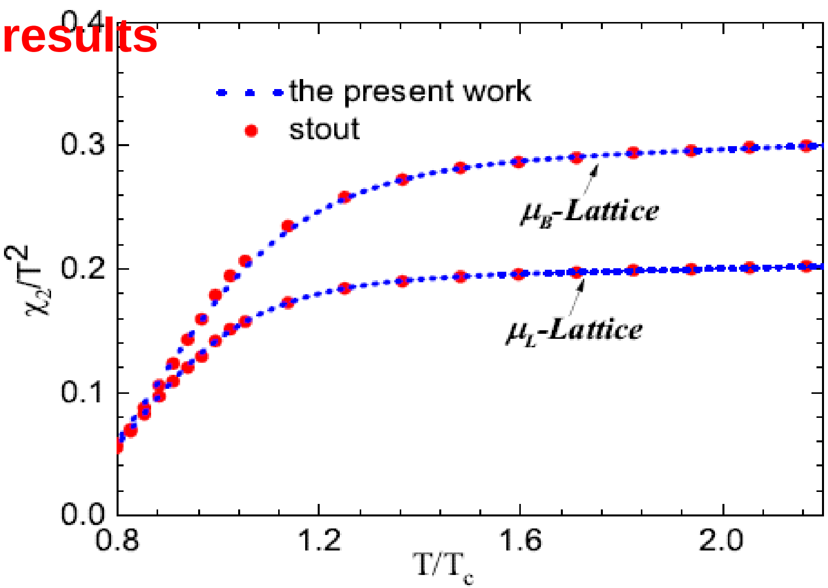
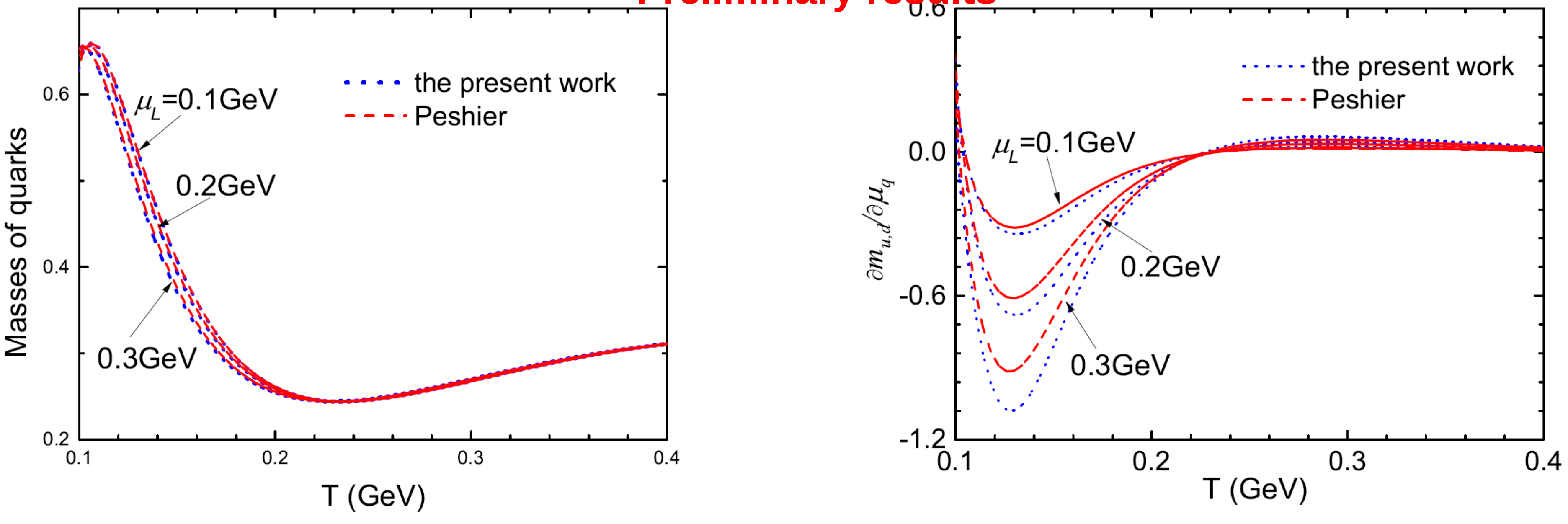




Numerical Results

- Results of Peshier et. al can be obtained by considering another possible solution where particle mass is only a function of temperature and chemical potential

Preliminary results



Concluding remarks

- We review the thermodynamic consistency in quasiparticle model with finite chemical potential and suggest that there is an possible solution where the screen mass is also a function of momentum
- The treatment is essentially based on the quasiparticle ansatz and arguments concerning thermodynamical consistency
- Our results shows that the quasiparticle ansatz is consistent with the lattice QCD data
- Future plan: inclusion of critical end point, particle number fluctuations via hydro…

谢谢!

thanks!