Effects of strong magnetic fields on quark matter and neutral meson properties within nonlocal chiral quark models

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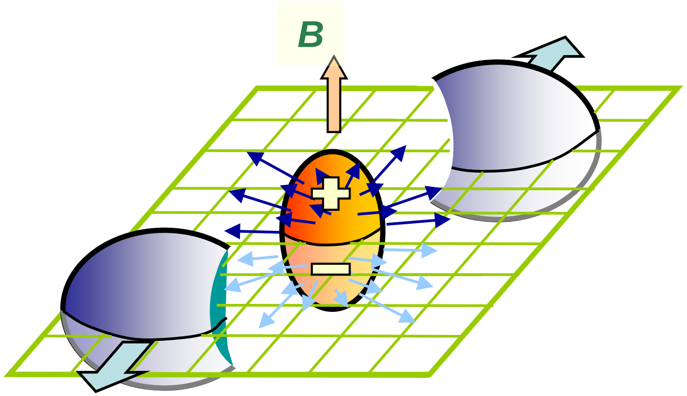
Plan of the talk

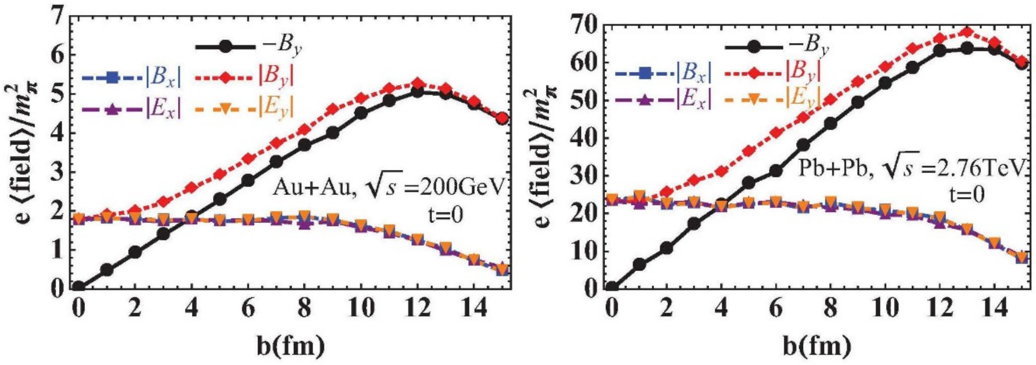
- Motivation
- External magnetic field in nonlocal NJL-like models
- Mean field results
- π^0 meson properties: m_{π^0} and f_{π^0}
- Conclusions & outlook

Motivation

Recent interest in studying how the interaction with a strong external magnetic field affects the QCD phase diagram and hadron properties

High magnetic fields in non-central relativistic heavy ion collisions





Deng, Huang (12)

Magnetars: neutron stars with high magnetic fields, $B \sim 10^{14}$ - 10^{15} G at the surface (and presumably much larger in the interior)

Duncan, Thompson (93)

$$m_\pi^2/|e| \simeq 3.4 \times 10^{18}\ \mathrm{G}$$

$$50\ m_\pi^2 \sim 1\ \mathrm{GeV}^2$$

Theoretical calculations carried out within various approaches to the dynamics of strong interactions:

- Lattice QCD techniques
- Effective models – Effective quark couplings satisfying QCD symmetry properties
  - Chiral perturbation theory
  - Linear sigma model, quark-meson model, MIT bag model
  - Holographic QCD
  - Nambu – Jona-Lasinio model (and extensions)

Recent reviews:

Kharzeev, Landsteiner, Schmitt, Yee, Lect. Not. Phys. **871**, 1 (13)

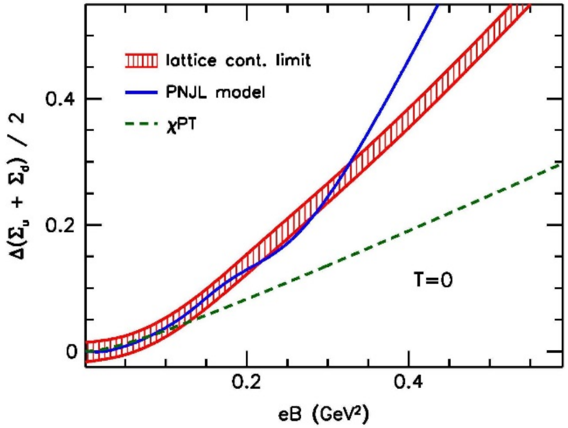
Shovkovy, Phys. Rept. **576**, 1 (15)

Andersen, Naylor, Tranberg, Rev. Mod. Phys. **88**, 025001 (16)

Expected effect: "magnetic catalysis" in vacuum

The interaction with an external B field favors the dynamical breakdown of chiral symmetry

Gusynin, Miransky, Shovkovy (94/95/96)



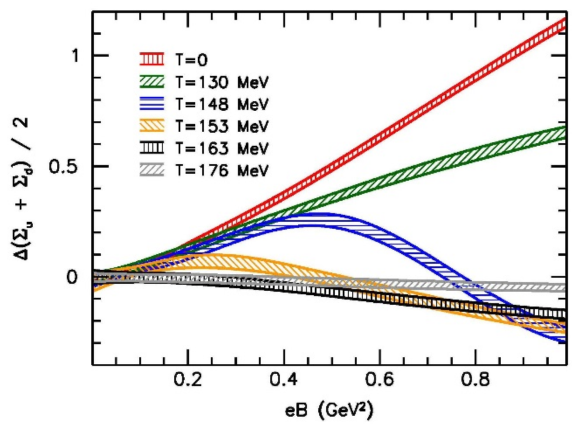
$$\Delta\Sigma_f \;=\; -\frac{2\,m_c}{S^4}\Big(\langle\bar{q}_f q_f\rangle_B - \langle\bar{q}_f q_f\rangle_0\Big)$$

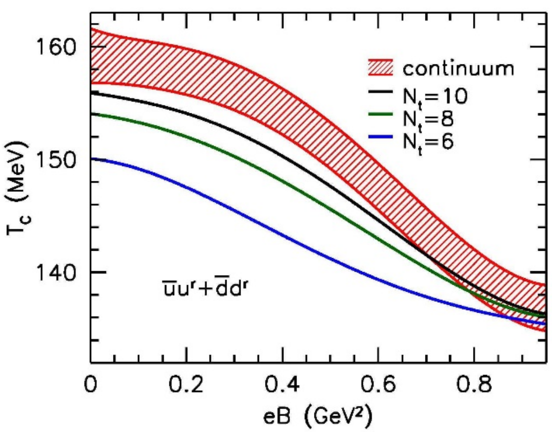
Bali, Bruckmann, Endrödi, Fodor, Katz, Schäfer (12)

Gatto, Ruggieri (11)

Cohen, McGady, Werbos (07); Andersen (12)

Unexpected: "inverse magnetic catalysis" (IMC) found in lattice QCD for finite T





Bali, Bruckmann, Endrödi, Fodor, Katz, Schäfer (12)

Bali, Bruckmann, Endrödi, Fodor, Katz, Krieg, Schäfer, Szabo (12)

Discrepancy between LQCD results and predictions from most (naïve) effective models for strong interactions: no IMC found

Various scenarios have been proposed to account for this problem:

- T and/or B dependence of effective strong coupling constants — Ayala (14); Farias (14); Ferrer (15)
- B dependence of interactions between fermions and the Polyakov Loop — Ferreira (14)
- Effects beyond mean field — Fukushima, Hidaka (13), Mao (16)
- Schwinger-Dyson approaches — Müller, Pawlowski (15); Braun, Mian, Rechenberger (16)
- Holographic QCD models — Rougemont, Critelli, Noronha (16)

Physics behind inverse magnetic catalysis at finite T not yet understood

A step towards a more realistic modeling of QCD:
Extension to NJL–like theories that include nonlocal quark interactions

Bowler, Birse (95); Blaschke et al. (95); Ripka (97); Plant, Birse (98)

Natural in the context of many approaches to low-energy quark dynamics

- ✓ No sharp momentum cut-offs $\rightarrow$ relatively low dependence on model parameters
- ✓ Momentum dependence of effective quark propagators (as obtained in LQCD)
- ✓ Successful description of vacuum meson phenomenology
- ✓ Successful description of chiral/deconfinement transitions at finite temperature

Nonlocal NJL-like model with external B field

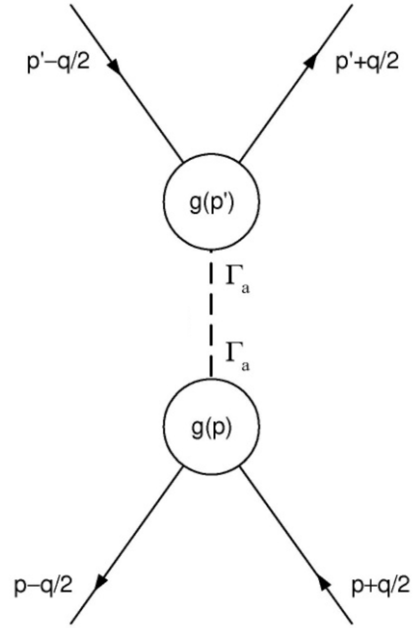
Euclidean action (two active flavors, isospin symmetry)

$$S_E = \int d^4x \left[\bar\psi(x)\,(-i\partial\!\!\!/ + m_c)\,\psi(x) - \frac{G}{2} j_a(x) j_a(x) \right]$$

Nonlocal quark-antiquark currents

$$j_a(x) = \int d^4z\; \mathcal{G}(z)\; \bar\psi\left(x + \frac{z}{2}\right)\, \Gamma_a\, \psi\left(x - \frac{z}{2}\right) \qquad \Gamma_a = (\mathbb{1}\,,\; i\gamma_5\vec\tau)$$

$\mathcal{G}(z)$: nonlocal, well behaved covariant form factor



Interaction with an external electromagnetic field:

$$\partial_\mu \;\to\; D_\mu = \partial_\mu - i\,\hat Q \mathcal{A}_\mu(x) \qquad\qquad \hat Q \;=\; e \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$$

and in addition

$$\psi(x - z/2) \;\to\; \mathcal{W}\,(x, x - z/2)\;\psi(x - z/2) \qquad\qquad \mathcal{W}(s,t) \;=\; \mathrm{P}\;\exp\left[-i\int_s^t dr_\mu\, \hat Q \mathcal{A}_\mu(r)\right]$$

Further steps:

- Hubbard-Stratonovich transformation: standard bosonization of the fermion theory. Introduction of bosonic fields σ and π_i

$$S_{\rm bos} = -\log\det \mathcal{D}_{x,x'} + \frac{1}{2G}\int d^4x \Big[\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\Big]$$

- Mean field approximation: expansion in powers of meson fluctuations

$$\begin{aligned} \sigma &= \bar{\sigma} + \delta\sigma(x) \\ \pi_i &= \delta\pi_i(x) \end{aligned}$$

- Assumption of a static, homogeneous magnetic field along the z axis

$$\mathcal{D}^{{\rm MF},f}_{x,x'} = \delta^{(4)}(x-x')\left(\Pi^f + m_c\right) + \bar{\sigma}\,\mathcal{G}(x-x')\exp\left[i\Phi_f(x,x')\right]$$

Landau gauge: $A_\mu = (0, B\,x_1, 0, 0)$, $\vec{B} = (0,0,B)$ $\qquad \Pi^f = -i\partial\!\!\!/ - q_f B\,x_1\gamma_2$

Schwinger phase $\qquad \Phi_f(x,x') = (q_f B/2)\,(x_2 - x_2')\,(x_1 + x_1')$

- Ritus transform $$\mathcal{D}^{\mathrm{MF},f}_{\bar p,\bar p\,'} = \int d^4x\; d^4x'\; \bar{\mathbb{E}}_{\bar p}(x)\; \mathcal{D}^{\mathrm{MF},f}_{x,x'}\; \mathbb{E}_{\bar p\,'}(x') \qquad \bar p = (k, p_2, p_3, p_4)$$

$$\Pi^2\; \mathbb{E}_{\bar p}(x) \;=\; \epsilon_{\bar p}\; \mathbb{E}_{\bar p}(x) \qquad\qquad \epsilon_{\bar p} = -(2k|qB| + p_3^2 + p_4^2)$$

– diagonal in p_2, p_3, p_4 and k

Bosonized effective MF action given by

$$\frac{S^{\mathrm{MF}}_{\mathrm{bos}}}{V^{(4)}} \;=\; \frac{\bar\sigma^2}{2G} - N_c \int \frac{d^2 p_\parallel}{(2\pi)^2} \sum_{f=u,d} \frac{|q_f B|}{2\pi} \left\{ \log\left(p_\parallel^2 + M^{s_f,f\;2}_{0,p_\parallel} \right) + \sum_{k=1}^{\infty} \log\left[\left(2k|q_f B| + p_\parallel^2 + M^{+,f}_{k,p_\parallel} M^{-,f}_{k,p_\parallel} \right)^2 + p_\parallel^2 \left(M^{+,f}_{k,p_\parallel} - M^{-,f}_{k,p_\parallel} \right)^2 \right] \right\}$$

$$p_\perp = (p_1, p_2) \qquad p_\parallel = (p_3, p_4) \qquad k_\pm = k - 1/2 \pm s_f/2 \qquad s_f = \mathrm{sg}(q_f)$$

where $$M^{\lambda,f}_{k,p_\parallel} = \frac{4\pi}{|q_f B|} (-1)^{k_\lambda} \int \frac{d^2 p_\perp}{(2\pi)^2} \left[m_c + \bar\sigma\, g(p_\perp^2 + p_\parallel^2) \right] \exp(-p_\perp^2/|q_f B|)\, L_{k_\lambda}(2p_\perp^2/|q_f B|)$$

$B = 0$: $$\frac{S^{\mathrm{MF}}_{\mathrm{bos}}}{V^{(4)}} \;=\; \frac{\bar\sigma^2}{2G} - 4\, N_c \int \frac{d^4 p}{(2\pi)^4} \log\left(p^2 + M(p)^2 \right) \qquad M(p) = m_c + \bar\sigma\, g(p^2)$$

NJL : $M = m_c + \bar\sigma$

- Minimization of $S_{\rm bos}$ $\Rightarrow$ gap equation for $\bar\sigma$

- Calculation of quark – antiquark condensates $\langle \bar q_f \, q_f \rangle_B \;=\; -\,\mathrm{Tr}\left(\mathcal{D}^{\rm MF}_{x,x'}{}^{-1}\right)$

- Model parametrization

  Simplest choice: Gaussian form factor $g(p^2) \;=\; e^{-p^2/\Lambda^2}$

  Model inputs: free parameters G, m_c and Λ that lead to phenomenological values of m_π, f_π and $\Psi_0 \;=\; (-\langle \bar q_f q_f \rangle_0)^{1/3}$
  (e.g. $\Psi_0 = 230$ MeV $\Rightarrow$ $m_c = 6.5$ MeV, $\Lambda = 680$ MeV, $G\Lambda^2 = 23.7$)

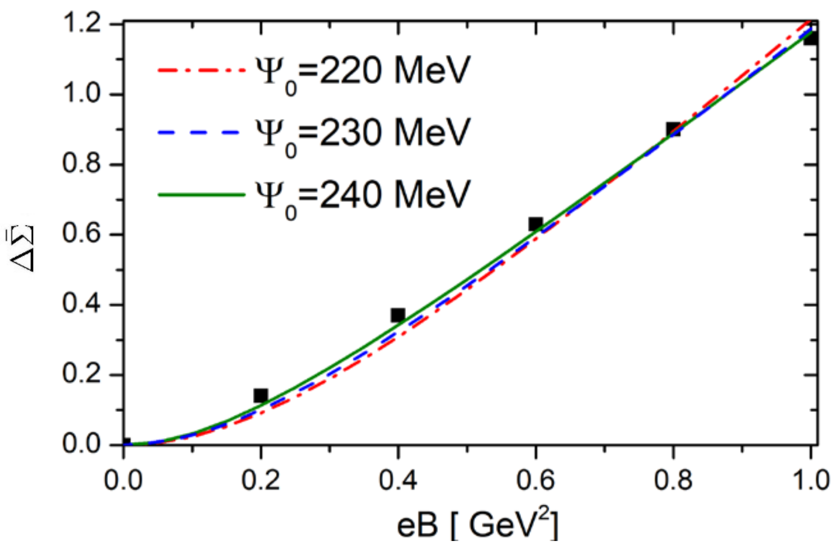
- Extension to finite temperature

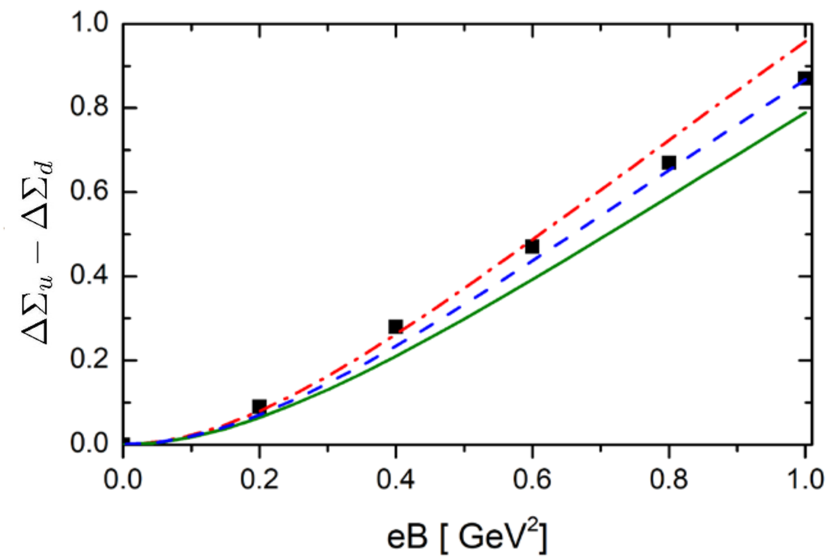
  Grand canonical thermodynamic potential, gap equation, condensates obtained trhrough the standard Matsubara formalism

$$\int \frac{d^2 p_\parallel}{(2\pi)^2}\, F(p_\parallel^2) \;\to\; T \sum_{n=-\infty}^{\infty} \int \frac{dp_3}{2\pi}\, F(p_3^2 + \omega_n^2) \qquad\qquad \omega_n = (2n+1)\pi T$$

Results

$T = 0$: behavior of quark-antiquark condensates as functions of eB





- Magnetic catalysis at $T = 0$
- Good agreement with LQCD
- Little dependence on the parametrization

$$\Delta\Sigma_f = -\frac{2\,m_c}{S^4}\Big(\langle\bar{q}_f q_f\rangle_B - \langle\bar{q}_f q_f\rangle_0\Big)$$

$$\Delta\bar{\Sigma} = (\Delta\Sigma_u + \Delta\Sigma_d)/2$$

Finite T : chiral restoration transition

Behavior of condensates vs. T for given values of eB

New ingredient: quarks coupled to a background color field A_4 + pure gauge potential $\mathcal{U}(\Phi, T)$

Traced Polyakov loop Φ taken as order parameter of deconfinement transition
- related with breakdown of Z(3) center symmetry of SU(3)$_C$ -

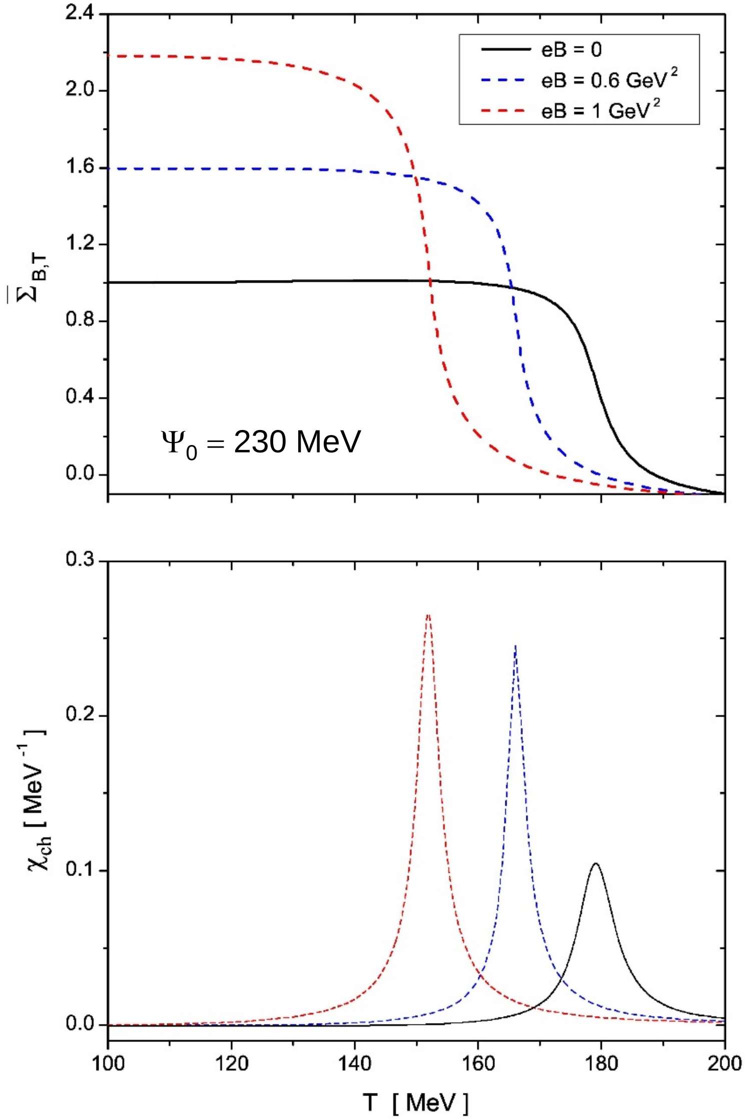
Fukushima (04); Megias, Ruiz Arriola, Salcedo (06); Roessner, Ratti, Weise (07)

Numerical results for critical temperatures at $B = 0$:

Ψ_0 [MeV]	220	230	240
T_c(0) [MeV]	182	179	177

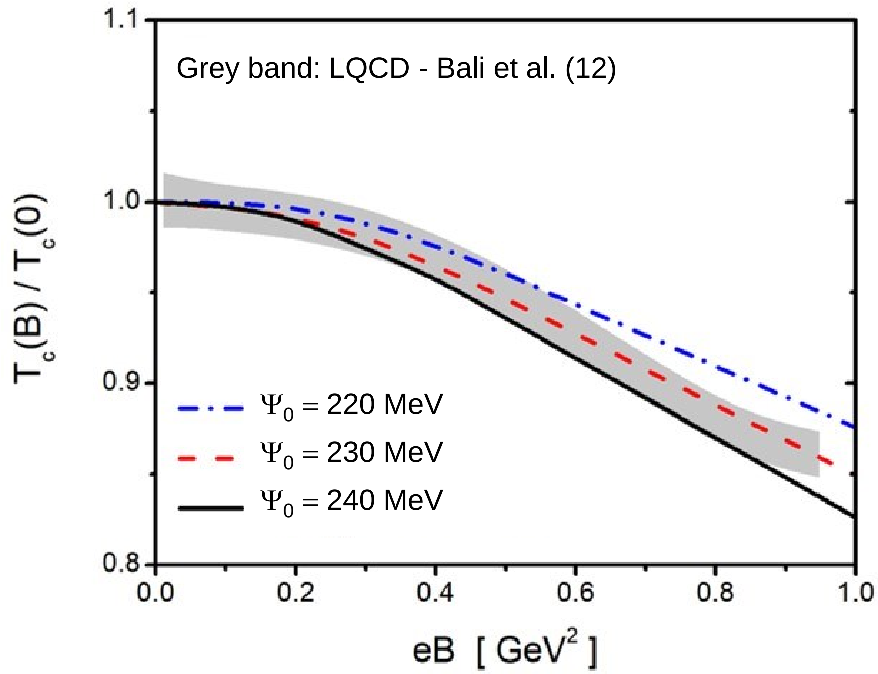
LQCD : T_c (0) $\sim$160 - 170 MeV

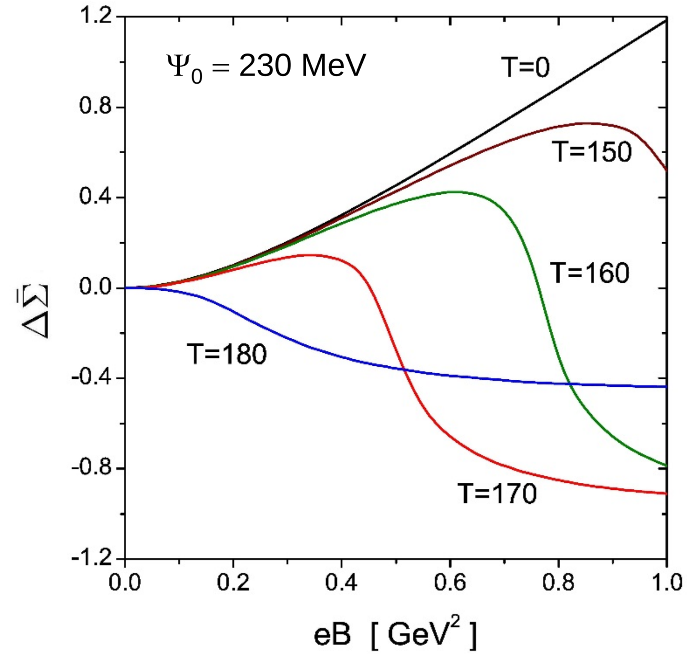
- Crossover chiral restoration transitions
- T_c is a decreasing function of eB



$\chi_{\mathrm{ch}} \equiv -d\,\Delta\bar{\Sigma}(T,B)/dT$

Behavior of critical temperature and quark condensates as functions of eB





- Inverse magnetic catalysis
- Agreement with LQCD
- No need of ad-hoc parameters / assumptions

Gaussian form factor: $$M^{\pm,f}_{k,p_\parallel} = m_c + \bar{\sigma} \frac{\left(1-|q_f B|/\Lambda^2\right)^{k_\pm}}{\left(1+|q_f B|/\Lambda^2\right)^{k_\pm+1}} e^{-p_\parallel^2/\Lambda^2}$$

Beyond mean field: π^0 meson properties

Quadratic terms in the bosonic action for the π^0 meson

$$S_{\rm bos}^{\rm (quad)}|_{\pi^0} \;=\; \frac{1}{2}\int dx\,dx'\;\delta\pi^0(x)\,\delta\pi^0(x')\left[\frac{1}{G}\,\delta^{(4)}(x-x')\;+\;J_{\pi^0}(x,x')\right]$$

where $$J_{\pi^0}(x,x') \;=\; \sum_{f=u,d}\int dy\,dy'\,\mathcal{G}(y)\,\mathcal{G}(y')\;{\rm Tr}\left[\left(\mathcal{D}^{{\rm MF},f}_{x'_-,x_+}\right)^{-1}\gamma_5\,e^{i\Phi_f(x_+,x_-)}\left(\mathcal{D}^{{\rm MF},f}_{x_-,x'_+}\right)^{-1}\gamma_5\,e^{i\Phi_f(x'_+,x'_-)}\right]$$

$$x_\pm^{(\prime)} = x^{(\prime)} \pm y^{(\prime)}/2 \qquad\qquad \left(\mathcal{D}^{{\rm MF},f}_{x,x'}\right)^{-1} \;=\; e^{i\Phi_f(x,x')}\int\frac{d^4p}{(2\pi^4)}\;e^{ip(x-x')}\,S_f(p_\parallel,p_\perp)$$

(local NJL: Schwinger phases cancel)

Fourier transformation of meson fields, Ritus representation of MF quark propagators

In momentum space, we get $\quad J_{\pi^0}(p,p') \;=\; (2\pi)^4\,\delta^{(4)}(p'-p)\;\tilde{J}_{\pi^0}(p_\perp^2,p_\parallel^2)$

$$S_{\rm bos}^{\rm (quad)}|_{\pi^0} \;=\; \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\;\delta\pi_3(p)\,\delta\pi_3(-p)\left[\frac{1}{G}\;+\;\tilde{J}_{\pi^0}(p_\perp^2,p_\parallel^2)\right]$$

π^0 mass given by

$$\frac{1}{G} + \tilde{J}_{\pi^0}(0,-m_{\pi^0}^2) \;=\; 0$$

Pion decay constant: need to calculate the matrix element of the axial current between the vacuum and the one-π^0 state

$$\langle 0|\mathcal{J}^{\mu}_{A3}|\pi^0(p)\rangle \;=\; ie^{-ipx}\, f(p_\perp^2, p_\parallel^2)\, p^\mu \;+\; \ldots$$ more terms!

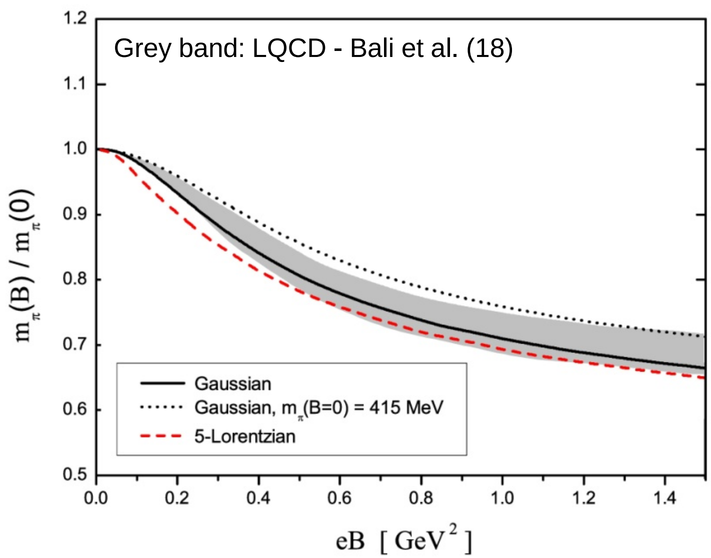
Auxiliary axial gauge field $\mathcal{W}_3{}^{\mu}$ – Expansion of the effective action to LO in π^0 and $\mathcal{W}_3{}^{\mu}$

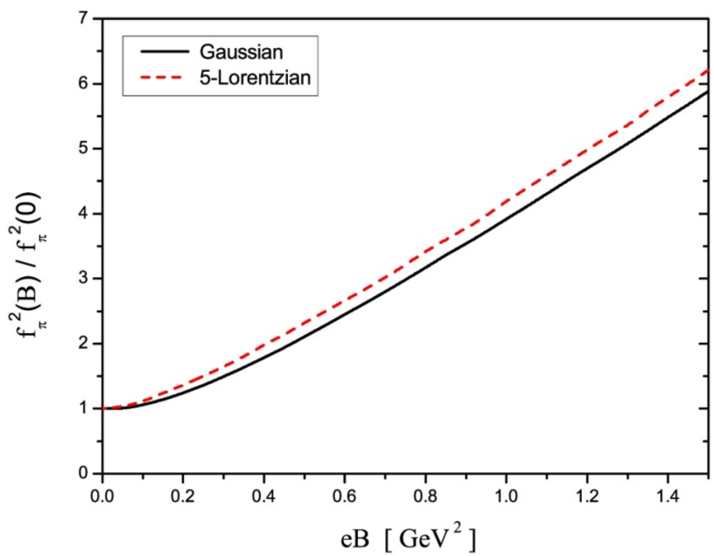
π^0 decay constant obtained from $f_{\pi^0} = f(0, -m^2_{\pi^0})$

✓ Consistency with ChPT results: GT relation, GOR relation, $\pi^0\gamma\gamma$ coupling

Numerical results:

Lorentzian: $g(p^2) \;=\; \dfrac{1}{1+(p^2/\Lambda^2)^n}$





- Agreement with LQCD – no ad-hoc parameters / assumptions

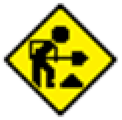
Summary & outlook

We have studied the behavior of quark matter under strong external magnetic fields within quark models that include effective nonlocal interactions. These models can be viewed as an improvement of the NJL model towards a more realistic description of QCD.

Main results:

- At $T = 0$, quark-antiquark condensates get increased with B ("magnetic catalysis")
- At finite T, crossover chiral restoration transitions
- For T close to chiral restoration, non monotonic behavior of condensates with B $\Rightarrow$ decrease of the transition temperature when B is increased ("inverse magnetic catalysis") No ad-hoc assumptions. Little dependence on form factors and model parameters.
- Behavior of π^0 mass in agreement with LQCD predictions

In progress / to be done:



- Behavior of π^0 mass with T
- Mass and decay constant of charged pions (technical difficulties: Schwinger phases, breakdown of traslational invariance – charged pions cannot be at rest!)
- Nonzero chemical potential (critical end point, $T = 0$ region of the phase diagram, ...)