Effects of strong magnetic fields on quark matter and π^0 properties within nonlocal chiral quark models

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We study the behavior of strongly interacting matter under a strong external magnetic field in the context of chiral quark models that include nonlocal interactions. In particular, we analyze the influence of a constant magnetic field on the chiral quark condensates at zero and finite temperature, studying the deconfinement and chiral restoration critical temperatures and discussing the observed "magnetic catalysis" and "inverse magnetic catalysis" effects. In addition, we analyze in this framework the behavior of the π^0 mass and decay constant. The predictions of nonlocal chiral quark models are compared with results obtained in lattice QCD.

I. INTRODUCTION

The study of the behavior of strongly interacting matter under intense external magnetic fields has gained significant interest in the last years. The corresponding theoretical analyses require in general to deal with quantum chromodynamics (QCD) in nonperturbative regimes, therefore most studies are based either in the predictions of effective models or in the results obtained from lattice QCD (LQCD) calculations. In fact, in view of the theoretical difficulty, most works concentrate on the situations in which one has a uniform and static external magnetic field.

In this work we study the features of QCD phase transitions and the properties of the π^0 meson under an intense external magnetic field $\vec{B}$ (for recent reviews on this subject see e.g. Refs. [1, 2]). At zero temperature, both the results of low-energy effective models of QCD and LQCD calculations indicate that light quark-antiquark condensates should behave as increasing functions of B, which is usually known as "magnetic catalysis". On the contrary, close to the chiral restoration temperature, LQCD calculations carried out with realistic quark masses [3, 4] show that the condensates behave as nonmonotonic functions of B, and this leads to a decrease of the transition temperature when the magnetic field is increased. This effect is known as "inverse magnetic catalysis" (IMC). In addition, LQCD calculations predict an entanglement between the chiral restoration and deconfinement critical temperatures [3]. These findings have become a challenge to model calculations. Indeed, most naive effective approaches to low energy QCD predict that the chiral transition temperature should grow with B, i.e., they do not find IMC. In this contribution we discuss this issue in the framework of nonlocal chiral quark models [5, 6]. It is seen that nonlocal models are able to describe, at the mean field level, not only the IMC effect but also the entanglement between chiral restoration and deconfinement transition temperatures. Moreover, within these models we study the mass and decay constant of the π^0 meson [7], showing that the behavior of these quantities with the external magnetic field is also in agreement with LQCD results. The models considered here are a sort of nonlocal extensions of the Nambu−Jona-Lasinio (NJL) model that intend to provide a more realistic effective approach to QCD. Actually, nonlocality arises naturally in the context of successful descriptions of low-energy quark dynamics, and it has been shown [8] that nonlocal models can lead to a momentum dependence in quark propagators that is consistent with LQCD results. Moreover, in this framework it is possible to obtain an adequate description of the properties of light mesons at both zero and finite temperature (see e.g. [9, 10] and references therein).

The article is organized as follows. In Sect. II we introduce the formalism to deal with a nonlocal NJL-like model in the presence of the magnetic field at zero temperature. Then we extend this formalism to a finite temperature system, including the coupling to a background gauge field, and sketch the analytical calculations required to study π^0 meson properties. In Sect. III we quote our numerical results, discussing the behavior of the different relevant quantities as functions of the magnetic field and/or temperature. Finally, in Sect. IV we summarize our results and present our conclusions.

II. THEORETICAL FORMALISM

Let us start by stating the Euclidean action for our nonlocal NJL-like two-flavor quark model,

$$S_E = \int d^4x \, \left\{ \bar{\psi}(x) \left(-i\partial\!\!\!/ + m_c\right) \psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\} . \tag{1}$$

where m_c is the current quark mass (same for u and d quarks). The currents $j_a(x)$ are given by

$$j_a(x) = \int d^4z\ \mathcal{G}(z)\ \bar{\psi}(x+\frac{z}{2})\ \Gamma_a\ \psi(x-\frac{z}{2})\ , \quad (2)$$

where $\Gamma_a = (\mathbb{1}, i\gamma_5\vec{\tau})$, and the function $\mathcal{G}(z)$ is a nonlocal form factor that characterizes the effective interaction. We introduce now in the effective action a coupling to an external electromagnetic gauge field $\mathcal{A}_\mu$. For a local theory this can be done by performing the replacement

$$\partial_\mu \ \rightarrow\ D_\mu \equiv \partial_\mu - i\,\hat{Q}\mathcal{A}_\mu(x)\ , \quad (3)$$

where $\hat{Q} = \mathrm{diag}(q_u, q_d)$, with $q_u = 2e/3$, $q_d = -e/3$, is the electromagnetic quark charge operator. In the case of the nonlocal model under consideration, the inclusion of gauge interactions requires also a change in the nonlocal currents in Eq. (2), namely

$$\psi(x-z/2)\ \rightarrow\ \mathcal{W}\,(x, x-z/2)\ \psi(x-z/2)\ , \quad (4)$$

and the corresponding change for $\bar{\psi}(x+z/2)$ [8]. Here the function $\mathcal{W}(s,t)$ is defined by

$$\mathcal{W}(s,t)\ =\ \mathrm{P}\ \exp\left[-i\int_s^t dr_\mu\,\hat{Q}\,\mathcal{A}_\mu(r)\right]\ , \quad (5)$$

where r runs over a path connecting s with t. As it is usually done, we take it to be a straight line.

To proceed we bosonize the fermionic theory, introducing scalar and pseudoscalar fields $\sigma(x)$ and $\vec{\pi}(x)$ and integrating out the fermion fields. The bosonized action can be written as

$$S_{\mathrm{bos}} = -\ln\det\mathcal{D}_{x,x'} + \frac{1}{2G}\int d^4x\,\left[\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\right]\ , \quad (6)$$

where

$$\mathcal{D}_{x,x'} = \delta^{(4)}(x-x')\left(-i\not{D} + m_c\right)\mathcal{G}(x-x')\,\gamma_0\ \times \\ \mathcal{W}(x,\bar{x})\,\gamma_0\left[\sigma(\bar{x}) + i\,\gamma_5\,\vec{\tau}\cdot\vec{\pi}(\bar{x})\right]\mathcal{W}(\bar{x},x')\ , \quad (7)$$

with $\bar{x} = (x+x')/2$. We consider the case of a constant and homogenous magnetic field orientated along the 3-axis, choosing the Landau gauge, in which one has $\mathcal{A}_\mu = B\,x_1\,\delta_{\mu 2}$. In addition, we assume that the field σ has a nontrivial translational invariant mean field value $\bar{\sigma}$, while the mean field values of pseudoscalar fields π_i are zero. In this way, within the mean field approximation (MFA) we get

$$\mathcal{D}^{\mathrm{MFA}}_{x,x'}\ =\ \mathrm{diag}(\mathcal{D}^{\mathrm{MFA},u}_{x,x'}\mathcal{D}^{\mathrm{MFA},d}_{x,x'})\ , \quad (8)$$

where

$$\mathcal{D}^{\mathrm{MFA},f}_{x,x'}\ =\ \delta^{(4)}(x-x')\,(-i\not{\partial} - q_f B\,x_1\gamma_2 + m_c)\ + \\ \bar{\sigma}\,\mathcal{G}(x-x')\,\exp\left[i(q_f B/2)\,(x_1+x_1')\,(x_2-x_2')\right]\ . \quad (9)$$

To deal with this operator it is convenient to introduce its Ritus transform $\mathcal{D}^{\mathrm{MFA},f}_{\bar{p},\bar{p}\,'}$, defined by

$$\mathcal{D}^{\mathrm{MFA},f}_{\bar{p},\bar{p}\,'} = \int d^4x\ d^4x'\ \bar{\mathbb{E}}_{\bar{p}}(x)\ \mathcal{D}^{\mathrm{MFA},f}_{x,x'}\ \mathbb{E}_{\bar{p}\,'}(x')\ , \quad (10)$$

where $\mathbb{E}_{\bar{p}}(x)$ and $\bar{\mathbb{E}}_{\bar{p}}(x)$, with $\bar{p} = (k, p_2, p_3, p_4)$, are Ritus functions [11]. The index k is an integer that labels the Landau energy levels. Using the properties of Ritus functions, after some calculation we obtain [5, 6]

$$\mathcal{D}^{\mathrm{MFA},f}_{\bar{p},\bar{p}\,'} = (2\pi)^4\delta_{kk'}\,\delta(p_2-p_2')\,\delta(p_3-p_3')\,\delta(p_4-p_4')\ \mathcal{D}^f_{k,p_\parallel}\ , \quad (11)$$

where

$$\mathcal{D}^f_{k,p_\parallel}\ =\ P_{k,s_f}\left(-s_f\sqrt{2k|q_f B|}\ \gamma_2 + p_\parallel\cdot\gamma_\parallel\right) + \sum_{\lambda=\pm} M^{\lambda,f}_{k,p_\parallel}\ \Delta^\lambda\ . \quad (12)$$

Here we have introduced the definitions $s_f = \mathrm{sign}(q_f B)$, $p_\parallel = (p_3, p_4)$, $\gamma_\parallel = (\gamma_3, \gamma_4)$, $\Delta^+ = \mathrm{diag}(1,0,1,0)$, $\Delta^- = \mathrm{diag}(0,1,0,1)$ and $P_{k,\pm1} = (1-\delta_{k0})\,\mathcal{I} + \delta_{k0}\,\Delta^\pm$. In addition, we denote

$$M^{\lambda,f}_{k,p_\parallel} = \frac{4\pi}{|q_f B|}\,(-1)^{k_\lambda}\int\frac{d^2p_\perp}{(2\pi)^2}\ \left(m_c + \sigma\ g(p_\perp^2 + p_\parallel^2)\right)\ \times \\ \exp(-p_\perp^2/|q_f B|)\,L_{k_\lambda}(2p_\perp^2/|q_f B|)\ . \quad (13)$$

where we have used the definitions $k_\pm = k - 1/2 \pm s_f/2$ and $p_\perp = (p_1, p_2)$, while $g(p^2)$ is the Fourier transform of $\mathcal{G}(x)$ and $L_m(x)$ are Laguerre polynomials, with the usual convention $L_{-1}(x) = 0$.

Using the fact that $\mathcal{D}^{\mathrm{MFA},f}$ is diagonal in Ritus space the corresponding contribution to the MFA action can be readily calculated. We obtain

$$\frac{S^{\mathrm{MFA}}_{\mathrm{bos}}}{V^{(4)}} = \frac{\bar{\sigma}^2}{2G} - N_c\sum_{f=u,d}\frac{|q_f B|}{2\pi}\int\frac{d^2p_\parallel}{(2\pi)^2} \\ \left[\ln\left(p_\parallel^2 + {M^{\lambda_f,f}_{0,p_\parallel}}^2\right) + \sum_{k=1}^{\infty}\ \ln\Delta^f_{k,p_\parallel}\right]\ , \quad (14)$$

where $\lambda_f = +\,(-)$ for $s_f = +1\,(-1)$, and $\Delta^f_{k,p_\parallel}$ is defined by

$$\Delta^f_{k,p_\parallel} = \left(2k|q_f B| + p_\parallel^2 + M^{+,f}_{k,p_\parallel}\ M^{-,f}_{k,p_\parallel}\right)^2 + \\ p_\parallel^2\left(M^{+,f}_{k,p_\parallel} - M^{-,f}_{k,p_\parallel}\right)^2\ . \quad (15)$$

Here it is seen that the functions $M^{\pm,f}_{k,p_\parallel}$ play the role of constituent quark masses in the presence of the external magnetic field.

We extend now the analysis to a system at finite temperature. This is done by using the standard Matsubara formalism. To account for confinement effects, we also include the coupling of fermions to the Polyakov loop (PL), assuming that quarks move on a constant color background field $\phi = ig\,\delta_{\mu 0}\,G^\mu_a\lambda^a/2$, where G^μ_a are the SU(3) color gauge fields. We work in the so-called Polyakov gauge, in which the matrix ϕ is given a diagonal representation $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$, taking the traced Polyakov loop $\Phi = \frac{1}{3}\mathrm{Tr}\ \exp(i\phi/T)$ as an order parameter of the confinement/deconfinement transition. We also include in the

Lagrangian a Polyakov-loop potential $\mathcal{U}(\Phi, T)$, which accounts for effective gauge field self-interactions. The resulting scheme is usually denoted as nonlocal Polyakov-Nambu-Jona-Lasinio (nlPNJL) model [12, 13]. For definiteness we will consider here a polynomial PL potential of the form proposed e.g. in Ref. [14].

The grand canonical thermodynamic potential of the system under the external magnetic field is found to be given by

$$\Omega^{\rm MFA}_{B,T} = \frac{\bar\sigma^2}{2G} \; - \; T \sum_{n=-\infty}^{\infty} \sum_{c,f} \frac{|q_f B|}{2\pi} \int \frac{dp_3}{2\pi} \left[\ln \left({p_\parallel}_{nc}^{\,2} + M^{\lambda_f, f\;2}_{0,p_{\parallel_{nc}}} \right) + \sum_{k=1}^{\infty} \ln \left(\Delta^f_{k,p_{\parallel_{nc}}} \right) \right] + \; \mathcal{U}(\Phi,T) \;, \quad (16)$$

where we have defined ${p_\parallel}_{nc} = (p_3\,,\,(2n+1)\pi T + \phi_c)$. The sums over color and flavor indices run over $c = r, g, b$ and $f = u, d$, respectively, while the color background fields are $\phi_r = -\phi_g = \phi_3$, $\phi_b = 0$. As usual in nonlocal models, it is seen that $\Omega^{\rm MFA}$ turns out to be divergent, thus it has to be regularized. We take here a usual prescription in which we subtract a free contribution and add it in a regularized form. This "free" contribution is in fact the potential obtained in absence of the strong current-current coupling (i.e. setting $\bar\sigma = 0$), but keeping the interaction with the magnetic field and the PL. One has

$$\Omega^{\rm MFA,reg}_{B,T} \; = \; \Omega^{\rm MFA}_{B,T} \, - \, \Omega^{\rm free}_{B,T} \, + \, \Omega^{\rm free,reg}_{B,T} \; . \qquad (17)$$

The explicit form of $\Omega^{\rm free,reg}_{B,T}$ can be found in Ref. [6]. The values of $\bar\sigma$ and Φ can be obtained by minimization of $\Omega^{\rm MFA,reg}_{B,T}$, and the magnetic field dependent quark condensates $\langle \bar q_f q_f \rangle$ can be calculated by taking the derivatives of the thermodynamic potential with respect to the corresponding current quark masses. To make contact with LQCD results given in Ref. [4] we define the quantities

$$\Sigma^f_{B,T} \;\; = \;\; - \frac{2\, m_c}{S^4} \left[\langle \bar q_f q_f \rangle^{\rm reg}_{B,T} - \langle \bar q q \rangle^{\rm reg}_{0,0} \right] + 1 \;, \qquad (18)$$

where $S = (135 \times 86)^{1/2}$ MeV. We also introduce the definitions $\Delta\Sigma^f_{B,T} = \Sigma^f_{B,T} - \Sigma^f_{0,T}$, $\bar\Sigma_{B,T} = (\Sigma^u_{B,T} + \Sigma^d_{B,T})/2$ and $\Delta\bar\Sigma_{B,T} = (\Delta\Sigma^u_{B,T} + \Delta\Sigma^d_{B,T})/2$, which correspond to the subtracted normalized flavor condensate, the normalized flavor average condensate and the subtracted normalized flavor average condensate, respectively.

In addition, from the expansion of $\mathcal{D}_{x,x'}$ in powers of the fluctuations of meson fields around their mean field values it is possible to obtain the theoretical expressions for the π^0 meson mass (at $T = 0$) within our model. One has

$$-\log\det\mathcal{D} = -{\rm Tr}\,\log\mathcal{D}^{\rm MFA} \; - \; {\rm Tr}\,(\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D}) \; + \frac{1}{2}\,{\rm Tr}\,(\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D})^2 \; + \; \dots \qquad (19)$$

By writing the trace in momentum space one gets

$$S_{\rm bos}\big|_{(\delta\pi_3)^2} = \frac{1}{2}\,{\rm Tr}\,(\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D})^2\Big|_{(\delta\pi_3)^2} + \frac{1}{2G} \int \frac{d^4t}{(2\pi)^4}\; \delta\pi_3(t)\,\delta\pi_3(-t) = \frac{1}{2} \int \frac{d^4t}{(2\pi)^4} \Big[F(t_\perp^2, t_\parallel^2) + \frac{1}{G} \Big] \delta\pi_3(t)\,\delta\pi_3(-t) \;, \qquad (20)$$

where $F(t_\perp^2, t_\parallel^2)$ is a function that involves the external field B. Its explicit form can be found in Ref. [7]. Choosing the frame in which the π^0 meson is at rest, its mass can be obtained as the solution of the equation

$$F(0, -m^2_{\pi^0}) + \frac{1}{G} \;\; = \;\; 0 \;. \qquad (21)$$

On the other hand, in the absence of external fields, the π^0 decay constant is defined through the matrix element of the axial current $\mathcal{J}^\mu_{A3}$ between the vacuum and the physical pion state, taken at the pion pole. One has

$$\langle 0 |\, \mathcal{J}^\mu_{A3}(x)\, | \tilde\pi_3(t) \rangle \;\; = \;\; i\, e^{-i(t\cdot x)}\, f(t^2)\, t^\mu \;, \qquad (22)$$

where $\tilde\pi_3(t) = Z^{-1/2}_{\pi^0}\pi_3(t)$ is the renormalized field associated with the π^0 meson state, with $t^2 = -m^2_{\pi^0}$. In our framework it is possible to obtain an analytical expression for the form factor $f(t^2)$ under a static uniform magnetic field, defining the π^0 decay constant $f_{\pi^0}(B)$ as the value of this form factor at $t^2 = -m^2_{\pi^0}(B)$ (it should be noticed, however, that in the presence of the magnetic field further Lorentz structures are allowed for the matrix element in Eq. (22), and there could exist other nonzero form factors). The wave function renormalization factor $Z^{1/2}_{\pi^0}$ is given by the residue of the pion propagator at $t^2 = -m^2_{\pi^0}$, namely

$$Z^{-1}_{\pi^0} \;\; = \;\; \frac{dF(0, t^2_\parallel)}{dt^2_\parallel}\Bigg|_{t^2_\parallel = -m^2_{\pi^0}} \;. \qquad (23)$$

The matrix element in Eq. (22) can be obtained by introducing a coupling between the current $\mathcal{J}^\mu_{A3}$ and an auxiliary axial gauge field W^μ_3, and taking the corresponding functional derivative of the effective action. In the same way as discussed at the beginning of this section, gauge invariance requires the couplings to this auxiliary gauge field to be introduced through the covariant derivative and the parallel transport of the fermion fields, see Eqs. (3) and (4). Assuming that the mean field value of the π_3 field vanishes, the pion decay constant can be obtained by expanding the bosonized action up to first order in $W_{3\mu}$ and $\delta\pi_3$. Writing

$$S_{\rm bos}\big|_{W_3\,\delta\pi_3} \;\; = \;\; \int \frac{d^4t}{(2\pi)^4}\, F_\mu(t)\; W_{3\mu}(t)\; \delta\pi_3(-t) \;, \qquad (24)$$

one finds

$$f_{\pi^0} \;\; = \;\; f(-m^2_{\pi^0}) \;\; = \;\; i\, Z^{1/2}_{\pi^0}\, \frac{t_\mu F_\mu(t)}{t^2}\Bigg|_{t^2_\perp = 0, t^2_\parallel = -m^2_{\pi^0}} \;. \qquad (25)$$

To find the function $F_\mu(t)$ we consider once again the expansion in Eq. (19). In addition, we expand $\delta\mathcal{D}$ in powers of $\delta\pi_3$ and W_3,

$$\delta\mathcal{D} \;=\; \delta\mathcal{D}_W + \delta\mathcal{D}_\pi + \delta\mathcal{D}_{W\pi} + \;\ldots\;, \tag{26}$$

which leads to

$$S_{\rm bos}\big|_{W_3\,\delta\pi_3} = -\,{\rm Tr}\,(\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D}_{W\pi}) \;+ \\ {\rm Tr}\,(\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D}_W\mathcal{D}^{\rm MFA\,-1}\,\delta\mathcal{D}_\pi)\;. \tag{27}$$

The explicit expression for $t\cdot F(t)|_{t_\perp=0}$ is given in Ref. [7]. It is worth mentioning that the chiral Goldberger-Treiman and Gell-Mann-Oakes-Renner relations remain valid in our model in the presence of the external magnetic field [7].

III. NUMERICAL RESULTS

To obtain numerical predictions for the above defined quantities it is necessary to specify the particular shape of the nonlocal form factor $g(p^2)$. We will show here the results corresponding to the often-used Gaussian form

$$g(p^2) \;=\; \exp(-p^2/\Lambda^2)\;. \tag{28}$$

Notice that, owing to Lorentz invariance, we need to introduce in the form factor an energy scale Λ, which acts as an effective momentum cut-off and has to be taken as an additional parameter of the model. Thus, the free parameters to be determined are m_c, G and Λ. We have considered different parameter sets, obtained by requiring that the model leads to the empirical values of the pion mass and decay constant, as well as some phenomenologically acceptable value of the quark condensate at $B = 0$ and $T = 0$. Here we take in particular $(-\langle\bar{q}q\rangle^{\rm reg}_{0,0})^{1/3} = 220$, 230 and 240 MeV. The corresponding parameter sets can be found e.g. in Ref. [6].

Let us start by discussing our results for the condensates at zero temperature. In Fig. 1 we show the predictions of our model for $\Delta\bar{\Sigma}_{B,0}$ as functions of eB, together with LQCD data from Ref. [4]. Solid, dashed and dotted curves correspond to $(-\langle\bar{q}_f q_f\rangle^{\rm reg}_{0,0})^{1/3} = 220$, 230 and 240 MeV, respectively. It can be seen that the predictions for $\Delta\bar{\Sigma}_{B,0}$ are very similar for all parameter sets, and show a very good agreement with LQCD results.

Next, we consider the results for a system at finite temperature. In Fig. 2 we show the behavior of the averaged chiral condensate $\bar{\Sigma}_{B,T}$ and the traced Polyakov loop Φ as functions of the temperature, for three representative values of the external magnetic field B, namely $B = 0$, 0.6 and 1 GeV^2. The curves correspond to parameter sets leading to $(-\langle\bar{q}q\rangle^{\rm reg}_{0,0})^{1/3} = 230$ MeV. Given a value of B, it is seen from the figure that the chiral restoration and deconfinement transitions proceed as smooth crossovers, at approximately the same critical temperatures. For definiteness we take these temperatures from the maxima of the chiral and PL susceptibilities, which we define

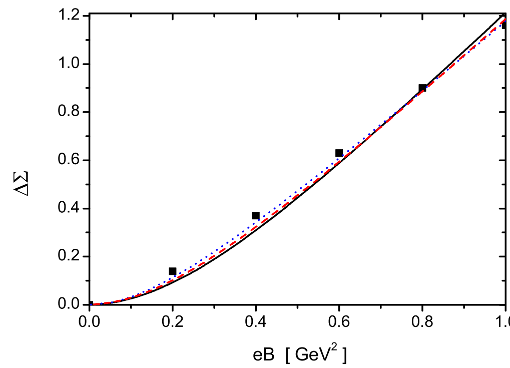

Figure 1. Normalized condensates as functions of the magnetic field at $T = 0$. Solid (black), dashed (red) and dotted (blue) curves correspond to parameterizations leading to $(-\langle\bar{q}q\rangle^{\rm reg}_{0,0})^{1/3} = 220$, 230 and 240 MeV, respectively. Full square symbols indicate LQCD results taken from Ref. [4].

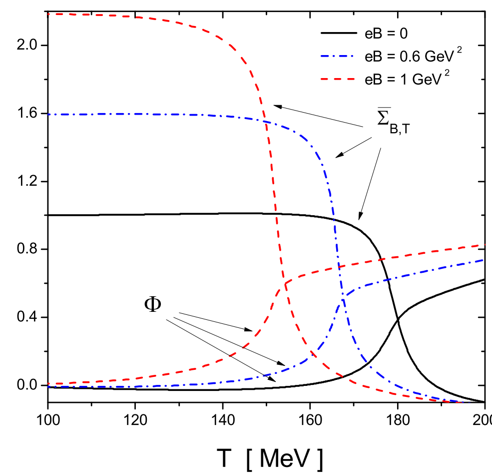

Figure 2. Normalized flavor average condensate and traced Polyakov loop as functions of the temperature, for three representative values of eB.

as the derivatives $\chi_{\rm ch} = -\partial[(\langle\bar{u}u\rangle^{\rm reg}_{B,T} + \langle\bar{d}d\rangle^{\rm reg}_{B,T})/2]/\partial T$ and $\chi_\Phi = \partial\Phi/\partial T$, respectively.

The chiral restoration and deconfinement critical temperatures obtained in absence of external magnetic field are quoted in Table I. It is seen that the splitting between both critical temperatures is below 5 MeV, which is consistent with the results obtained in LQCD, and the values of critical temperatures do not vary significantly with the parametrization. On the other hand, the critical temperatures in Table I are found to be somewhat higher than those obtained from LQCD, which lie around 160 MeV [15, 16]. It is worth mentioning that in absence

Table I. Critical temperatures for $B = 0$ and various parametrizations.

$(-\langle q\bar{q}\rangle_{0,0}^{\rm reg})^{1/3}$ (MeV)	220	230	240
Chiral T_c (MeV)	182.1	179.1	177.4
Deconfinement T_c (MeV)	182.1	178.0	175.8

of the interaction with the Polyakov loop the values of T_c drop down to about 130 MeV [5].

Let us discuss the effect of the magnetic field on the phase transition features. From Fig. 2 it is seen that the splitting between the chiral restoration and deconfinement critical temperatures remains very small in the presence of the external field. In addition, the effect of inverse magnetic catalysis is found. Indeed, contrary to what happens e.g. in the standard local NJL model [1, 2], in our model the chiral restoration critical temperature becomes lower as the external magnetic field is increased. This is related with the fact that the condensates do not show in general a monotonic increase with B for a fixed value of the temperature. In Fig. 3 we plot our results for the chiral restoration critical temperatures $T_c(B)$, normalized to the corresponding values at vanishing external magnetic field. The gray band in indicates the results obtained in LQCD, taken from Ref. [4]. It is seen that the critical temperatures decrease with B, i.e. IMC is observed. As a general conclusion, it can be stated that the behavior of the critical temperatures with the external magnetic field is compatible with LQCD results, for phenomenologically adequate values of the chiral condensate.

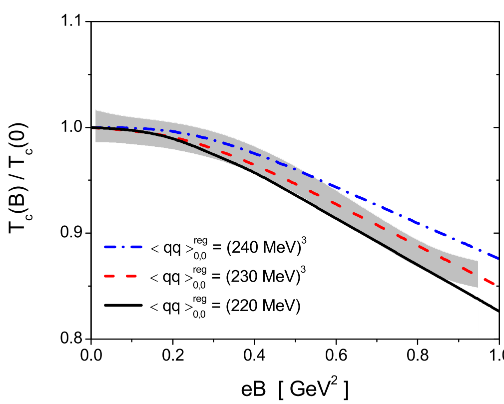

Figure 3. Normalized critical temperatures as functions of eB. For comparison, LQCD results of Ref. [4] are indicated by the gray band.

Finally, we study the behavior of the pion mass $m_{\pi^0}(B)$ and the squared pion decay constant $f_{\pi^0}^2(B)$ for the above mentioned parameter sets. Our results are shown in Fig. 4, where once again we consider the parameter set corresponding to $(-\langle\bar{q}q\rangle_{0,0}^{\rm reg})^{1/3} = 230$ MeV. The curves have been normalized to the $B = 0$ values $m_{\pi^0}(0) = 139$ MeV and $f_{\pi^0}^2 = (92.4\ {\rm MeV})^2$. As shown in the upper panel of Fig. 4, the π^0 mass is found to decrease when eB gets increased, reaching a value of about 65% of $m_{\pi^0}(0)$ at $eB \simeq 1.5$ GeV^2, which corresponds to a magnetic field of about 2.5×10^{20} G. We also include in the figure a gray band that corresponds to recently quoted results from lattice QCD [17]. The latter have been obtained from a continuum extrapolation of lattice spacing, considering a relatively large quark mass for which $m_\pi = 415$ MeV. For comparison, we also quote the results obtained within our model by shifting m_c to 56.3 MeV, which leads to this enhanced pion mass. In general it is seen from the figure that our predictions turn out to be in good agreement with LQCD calculations (notice that no extra adjustments have been required). Concerning the pion decay constant f_{π^0}, as shown in the lower panel of Fig. 4 we find that it behaves as an increasing function of B, which is fully consistent with the approximate validity of the Gell-Mann-Oakes-Renner relation. It is also worth mentioning that the curves in Fig. 4 are found to remain practically unchanged when the value of the $B = 0$ condensate used to fix the parameterization is varied within the range from $-(220\ {\rm MeV})^3$ to $-(250\ {\rm MeV})^3$.

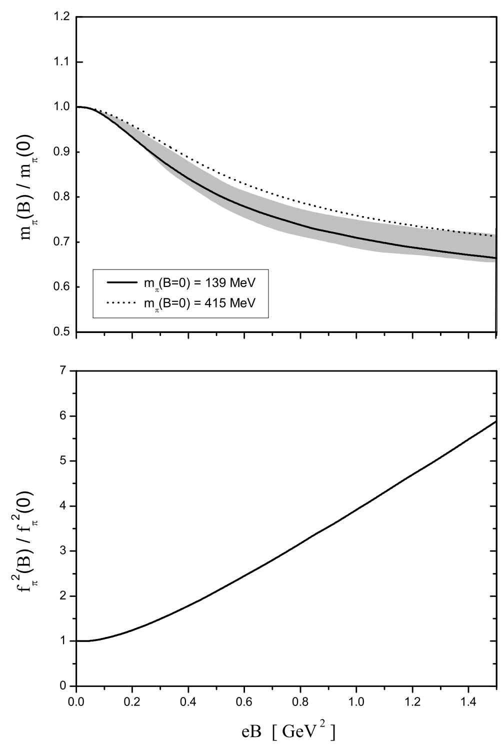

Figure 4. Mass (upper panel) and decay constant (lower panel) of the π^0 meson as functions of eB, normalized to their values for $B = 0$. In the upper panel, the dotted line is obtained for a parameterization in which $m_\pi = 415$ MeV, while the gray band corresponds to the results of lattice QCD calculations quoted in Ref. [17].

IV. SUMMARY & CONCLUSIONS

We have studied the behavior of strongly interacting matter under a uniform static external magnetic field in the context of a nonlocal chiral quark model. In this approach, which can be viewed as an extension of the Polyakov-Nambu-Jona-Lasinio model, the effective couplings between quark-antiquark currents include nonlocal form factors that regularize ultraviolet divergences in quark loop integrals and lead to a momentum-dependent effective mass in quark propagators. We have worked out the formalism introducing Ritus transforms of Dirac fields, which allows us to obtain closed analytical expressions for the gap equations, the chiral quark condensate and the quark propagator.

We have considered a Gaussian form factor, choosing some sets of model parameters that allow to reproduce the empirical values of the pion mass and decay constants. At zero temperature, for these parameterizations we have determined the behavior of the subtracted flavor average condensate $\Delta\bar{\Sigma}_{B,0}$ as a function of the external magnetic field B. Our results show the expected magnetic catalysis (condensates behave as growing functions of B), the curves being in quantitative agreement with lattice QCD calculations with slight dependence on the parametrization.

We have also extended the calculations to finite temperature systems, including the couplings of fermions to the Polyakov loop. We have defined chiral and PL susceptibilities in order to study the chiral restoration and deconfinement transitions, which turn out to proceed as smooth crossovers for the considered polynomial PL potential. From our numerical calculations, on one hand it is seen that, for all considered values of B, both transitions take place at approximately the same temperature, in agreement with LQCD predictions. On the other hand, it is found that for temperatures close to the transition region $\Delta\bar{\Sigma}_{B,T}$ becomes a nonmonotonic funtion of B, which eventually leads to the phenomenon of inverse magnetic catalysis, i.e., a decrease of the critical temperature when the magnetic field gets increased. This feature is also in qualitative agreement with LQCD expectations. Moreover, in general we find a good quantitative agreement with the results from LQCD calculations for the behavior of the normalized critical temperatures with B. It is interesting to compare the nonlocal models with approaches in which IMC is obtained by considering some dependence of the effective couplings on B and/or T [18, 19]. The naturalness of the IMC behavior in our framework can be understood by noticing that for a given Landau level the associated nonlocal form factor turns out to be a function of the external magnetic field, according to the convolution in Eq. (13).

Finally, we have studied the behavior of the π^0 mass and decay constant with the magnetic field. Both quantities are found to show a very mild dependence on the parametrization. It is worth noticing that our results for the pion mass turn out to be in good agreement with available lattice QCD calculations, with no need of extra ad-hoc assumptions.

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